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Seattle, WA

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Virtual Compton Scattering and Polarizabilities of the Proton  
at High  $Q^2$

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Experimental Discussion and Theoretical Motivation for a new proposal to  
JLab

**Elastic Form Factors** (Nobel Prize 1961) are the **fourier transforms** of the **charge and current densities** of the nucleon as a function of the spatial coordinate transverse to the light cone axis of the infinite momentum frame (IMF).

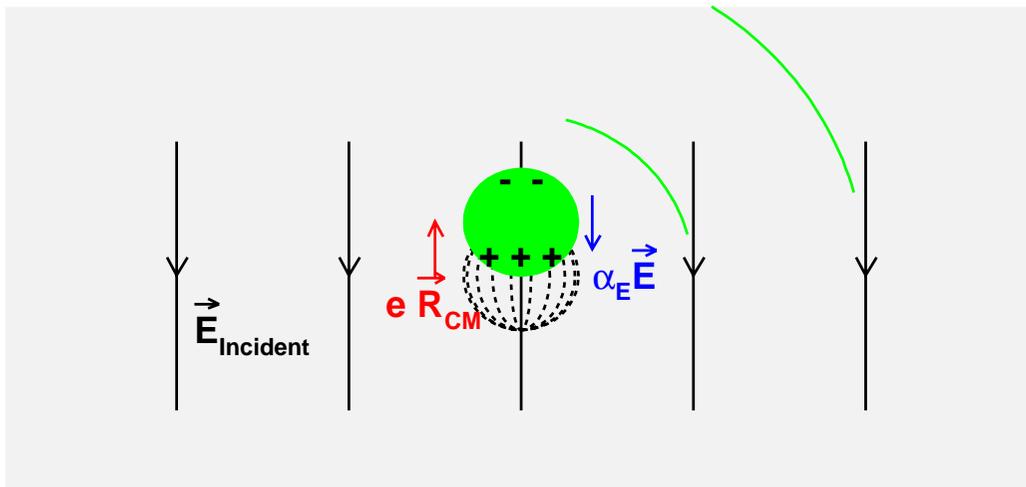
**Deep Inelastic Structure Functions** (Nobel Prize 1990) are the **parton densities** as a function of longitudinal momentum (in the IMF).

**Inelastic Form Factors** are the **fourier transforms** of the **transition densities** from the ground state to an excited state.

–But–

No model independent way to separate Resonant and Non-Resonant terms.

The  $Q^2$ -dependence of the **Generalized Polarizabilities (GP)** is the **fourier transform** of the **transition densities** from the ground state to the the virtual state created by a long wavelength electric-dipole or magnetic-dipole field.



In a single observable, **Generalized Polarizabilities** summarize information about the entire excitation spectrum of the proton.

With the GPs, we can form a picture of how the proton is distorted by an external field:

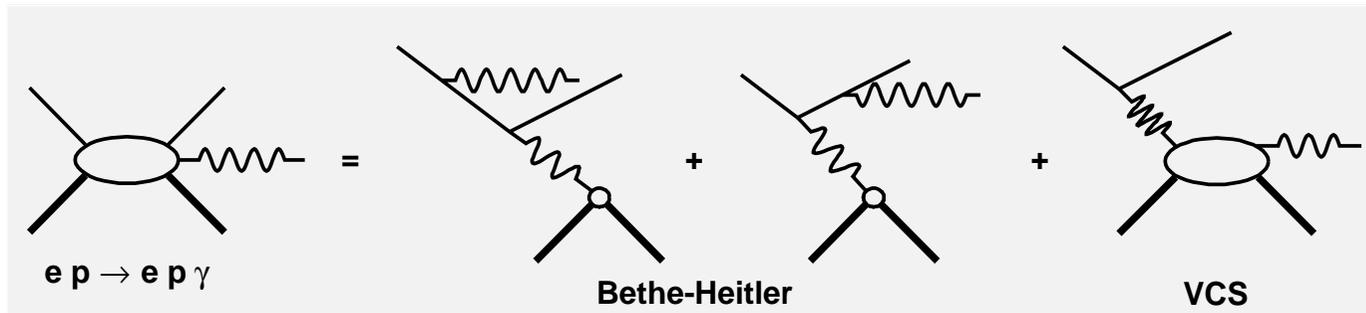
Does the periphery (pion cloud?) distort more readily than the core?

Do the electric and magnetic polarization responses differ from their respective elastic form factors?

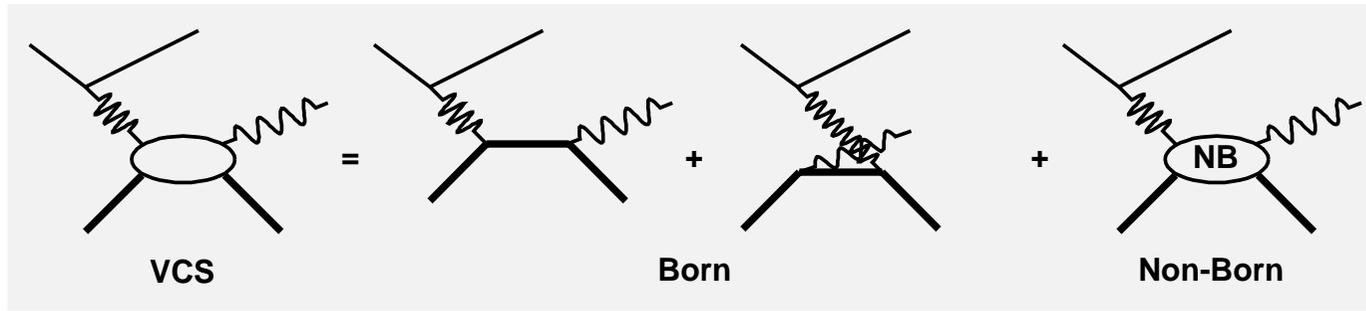
What is the relative magnitude of the paramagnetism (quark spin-flip?) and dia-magnetism (Lenz' law)?

Do the para- and dia-magnetic responses have different spatial distributions?

# Generalized Polarizabilities and Virtual Compton Scattering



VCS amplitude = [Bethe-Heitler + Born] amplitudes  
 + Low Energy Theorem (GP's)  
 + Higher order terms (DR model).



## Low Energy Expansion of Virtual Compton Scattering

P.A.M. Guichon, G.Q. Liu, A.W. Thomas, *Nucl. Phys. A*, **591** 606 (1995).

P.A.M. Guichon, M. Vanderhaeghen, *Prog. Part. Nucl. Phys.* **41** 125 (1998).

$$d\sigma(ep\gamma) = d\sigma(\text{BH} + \text{Born}) + (\text{Phase Space}) \times \\ \left\{ v_{LL} [P_{LL}(q_{cm}) - P_{TT}(q_{cm})/\epsilon] + v_{LT} P_{LT}(q_{cm}) + \mathcal{O}(q'_{cm}) \right\}$$

$P_{\Lambda}$  = Interference between leading **Non-Born** term with **BH + Born** term

$$P_{LL}(q_{cm}) = 4M G_E(\tilde{Q}^2) \frac{\alpha_E(q_{cm})}{\alpha_{QED}}$$

$$P_{TT}(q_{cm}) = 3q_{cm}^2 G_M(\tilde{Q}^2) \left[ \sqrt{2} P^{(C1,M2)1}(q_{cm}) - \frac{1}{\tilde{\nu}_{cm}} P^{(M1,M1)1}(q_{cm}) \right]$$

$$P_{LT}(q_{cm}) = -2 \frac{M q_{cm}}{\sqrt{\tilde{Q}^2}} G_E(\tilde{Q}^2) \frac{\beta_M(q_{cm})}{\alpha_{QED}} \\ + \frac{3q_{cm}}{2\tilde{\nu}_{cm}} \sqrt{\tilde{Q}^2} G_M(\tilde{Q}^2) P^{(C1,C1)1}(q_{cm})$$

Polarizabilities in Virtual and Real Compton Scattering  
 Spatial variation of response of proton to external  
 electric dipole or magnetic dipole field.

$$\begin{array}{l}
 P(\Lambda_f, \Lambda_i) \Delta S \xrightarrow{Q^2 \rightarrow 0} \text{Polarizabilities in RCS} \\
 P^{(C1, C1)0}(Q^2) \longrightarrow -\sqrt{2/3} \bar{\alpha}_E / \alpha_{\text{QED}} \\
 P^{(M1, M1)0}(Q^2) \longrightarrow -\sqrt{8/3} \bar{\beta}_M / \alpha_{\text{QED}} \\
 P^{(C1, C1)1}(Q^2) \longrightarrow 0 \\
 P^{(M1, M1)1}(Q^2) \longrightarrow 0 \\
 P^{(M1, C2)1}(Q^2) \longrightarrow -\sqrt{8/27} \gamma_{E2} / \alpha_{\text{QED}} \\
 P^{(C1, M2)1}(Q^2) \longrightarrow -\sqrt{2/3} \gamma_{M2} / \alpha_{\text{QED}}
 \end{array}$$

$(\Lambda_f, \Lambda_i) =$  (final, initial) Multipolarity

$\Delta S = 0, 1$ : Proton spin flip

## Dispersion Relations (VCS amplitude up to $\pi\pi N$ Threshold)

B. Pasquini *et al.*, Eur. Phys. J. A **11**, 185 (2001)

VCS Amplitude =  $\pi N$  Intermediate States +  $\pi^0$   $t$ -channel exchange  
(MAID2000)  
+ two Low Energy “constants” (functions of  $Q^2$ ).

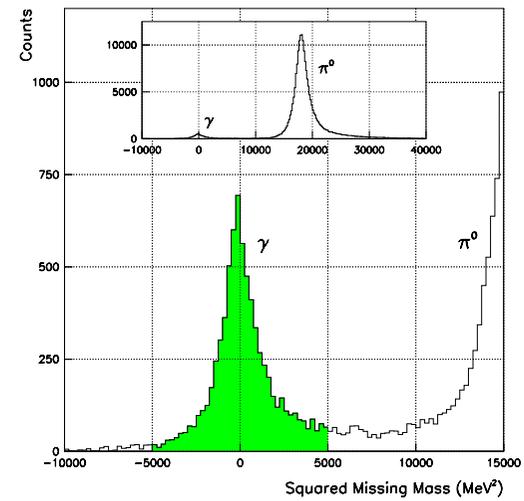
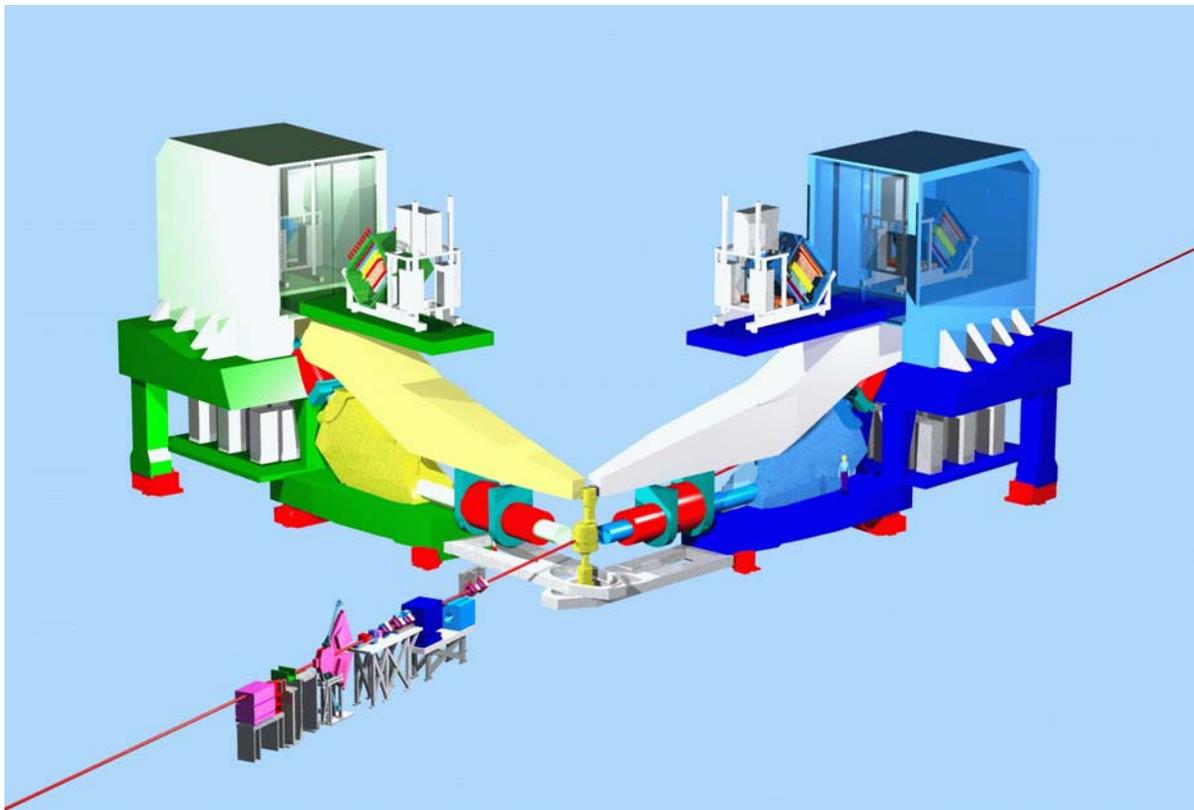
$$\Delta\beta(Q^2) = \sigma\text{-meson } t\text{-channel exchange} \\ \sim \Delta\beta(0) / [1 + Q^2/\Lambda_\beta^2]^2$$

$$\Delta[\alpha + \beta](Q^2) = \text{contact term}$$

$$\Delta\alpha(Q^2) \sim \Delta\alpha(0) / [1 + Q^2/\Lambda_\alpha^2]^2$$

Dipole parameterizations not essential to analysis.

Results from JLab VCS experiment E93-050:  $H(e, e'p)\gamma$



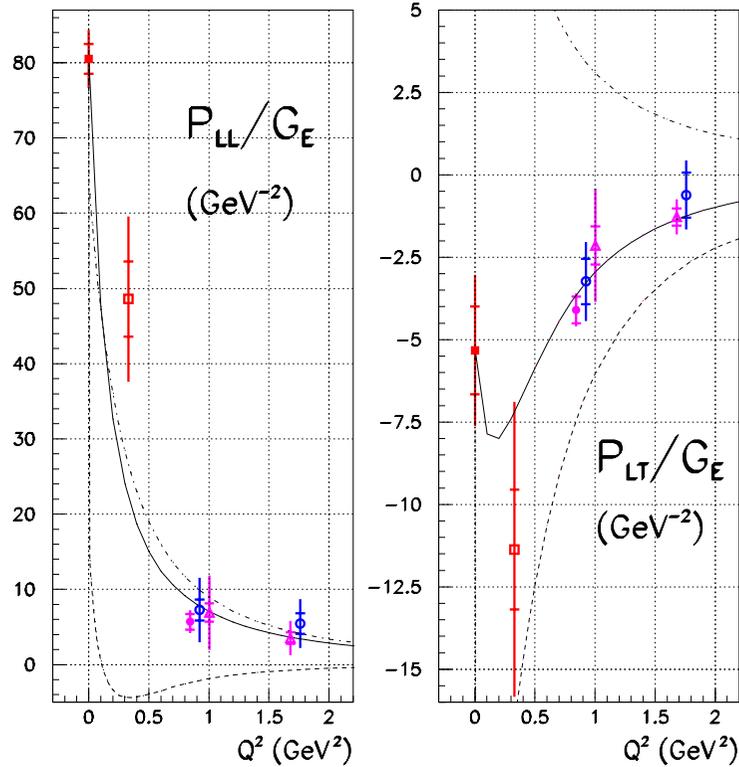


FIG. 3: World data on the structure functions  $P_{LL}/G_E^p$  and  $P_{LT}/G_E^p$ . The points at  $Q^2=0$  ( $0.33$ )  $\text{GeV}^2$  are taken from ref. [12] ([13]), the other ones are from this experiment. Together with the results presented in this letter ( $\bullet, \Delta$ ) we have also included our results based on the Low Energy Theorem ( $\circ$ ) [4]. Some points are shifted in abscissa for better visibility.

$$\alpha_E(Q^2) = \frac{P_{LL}(Q^2)}{G_E(Q^2)} \frac{\alpha_{QED}}{4M}$$

$$\beta_M(Q^2) = -\frac{P_{LT}(Q^2)}{G_E(Q^2)} \frac{\alpha_{QED}}{2M} \frac{\sqrt{\tilde{Q}^2}}{|q^{CM}|} + (\dots) P_{(C1,C1)1}(Q^2)$$

Dispersion Relations:

$$\alpha_E^{\pi N}(Q^2) \ll \alpha_E(Q^2).$$

$\beta_M^{\pi N}(Q^2)$  strongly paramagnetic.

Diamagnetism dominated by polarizability of pion in pion cloud?

What do we learn from these results?

$\alpha_E \ll$  proton volume: Proton is intrinsically relativistic

$\beta \ll \alpha$ : Strong diamagnetism

Evidence for sharp decrease in diamagnetism at low  $Q^2$ .

From Dispersion Relations, we see that both  $\alpha$  and  $\beta$  have strong contributions from degrees of freedom beyond  $\pi N$  intermediate states—high energy degrees of freedom are responsible for the low-energy phenomenon of polarizabilities

For  $\beta_M$ , DR suggest diamagnetism is dominated by  $\sigma$ -meson exchange in  $t$ -channel, which models pion-polarizability in pion-cloud.

## Polarizabilities at High $Q^2$

Spatial Distribution of Polarization Response

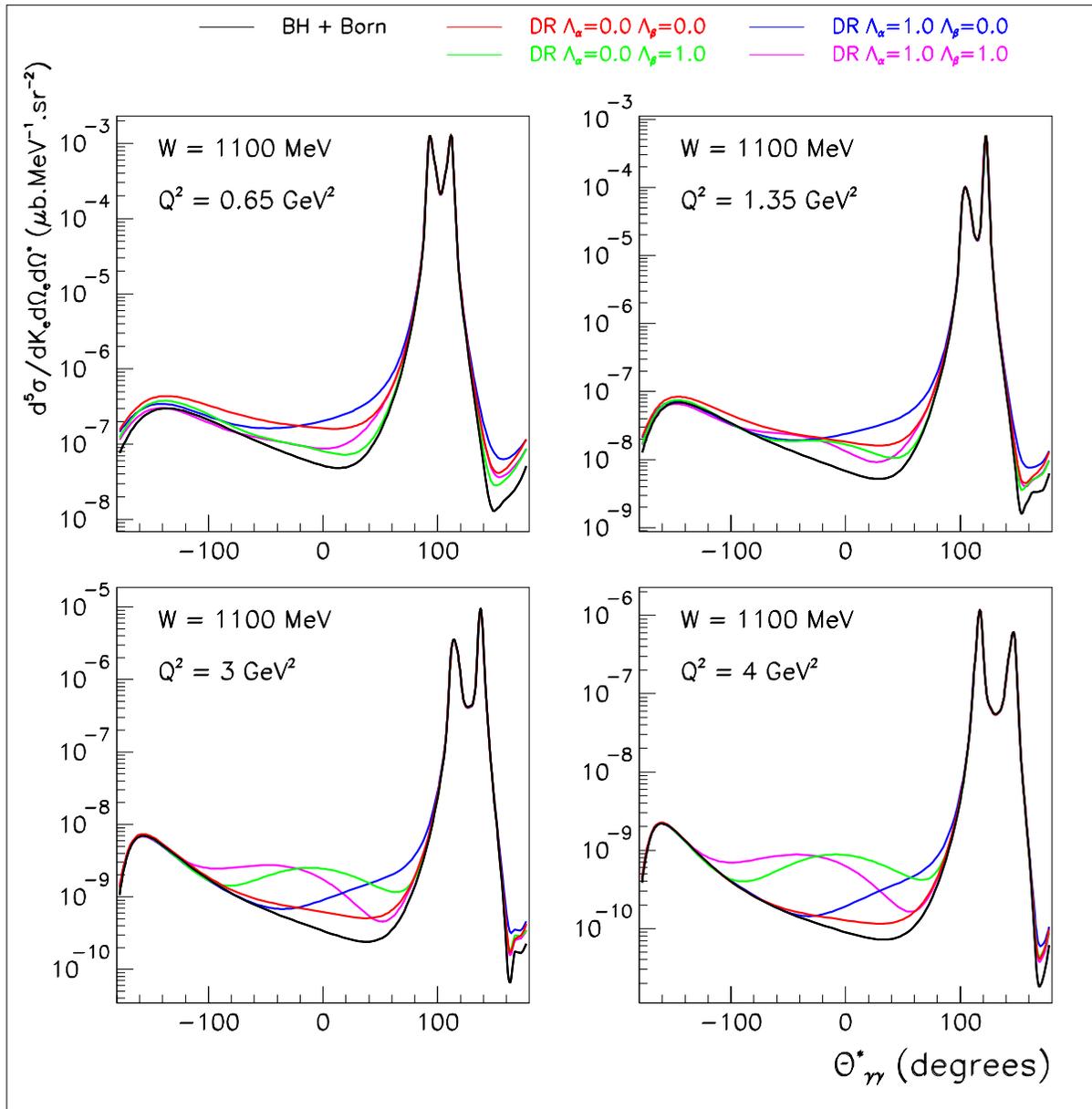
Duality?  $\longrightarrow$  GPD as  $\xi \rightarrow 1, t \rightarrow -Q^2$

$$\mathcal{T}^{\text{VCS}}(q' \rightarrow 0) \propto \int \frac{H(x, \xi \rightarrow 1, t \rightarrow -Q^2)}{1-x} dx$$

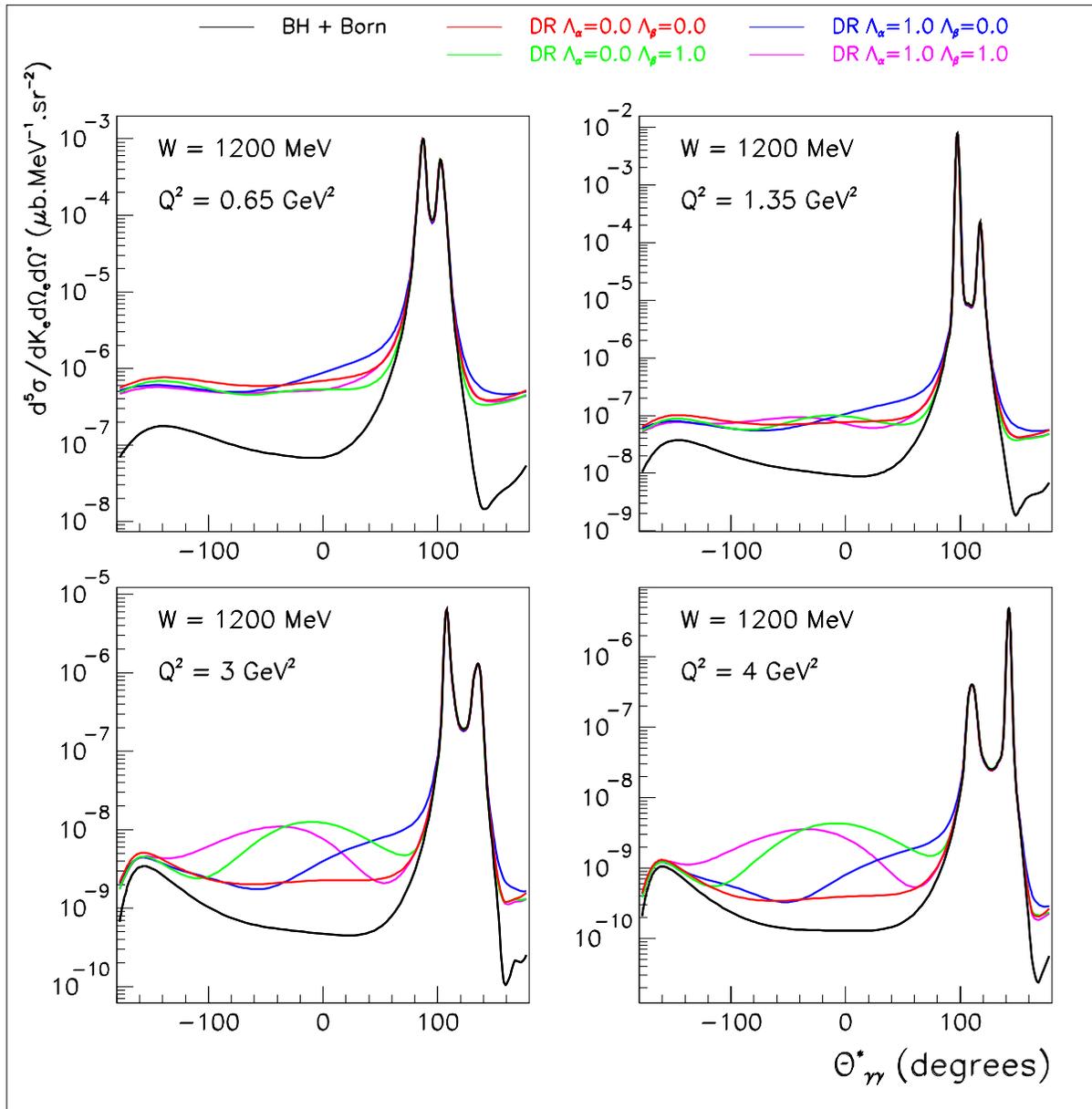
Compare with RCS

$$R_V = \int \frac{H(x, \xi = 0, t)}{x} dx$$

Dispersive Effects in Elastic e-p Scattering

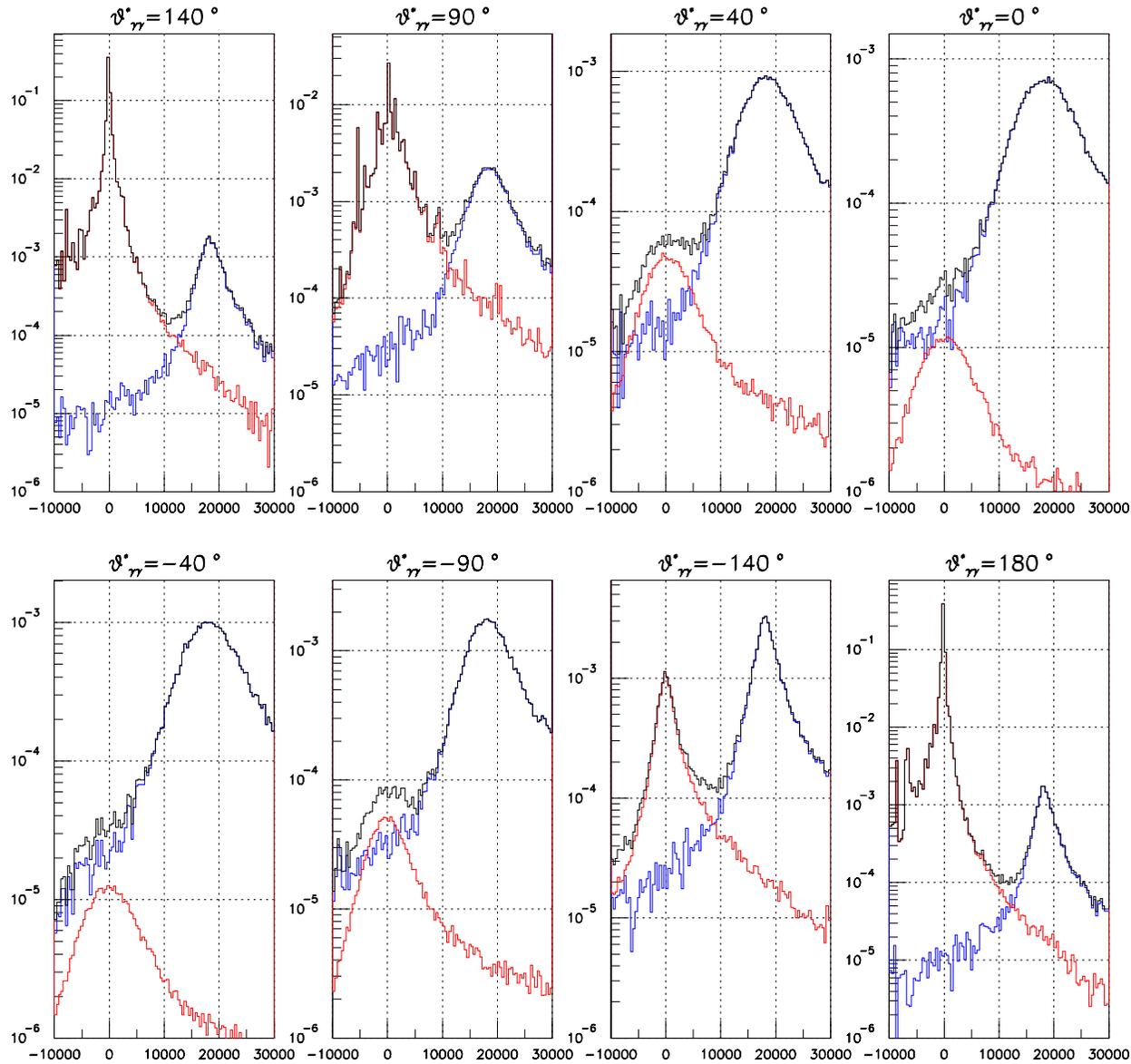


Differential VCS Cross sections at  $W = 1100 \text{ MeV}$  and the four  $Q^2$  values, vs.  $\theta_{\gamma\gamma}^{CM}$  in the electron scattering plane.  $\theta_{\gamma\gamma}^{CM} < 0$  corresponds to the azimuth  $\phi_{\gamma\gamma} = 180^\circ$ . The curves include the BH+B curves, and the DR curves with four values for the dipole parameters in the DR amplitudes.

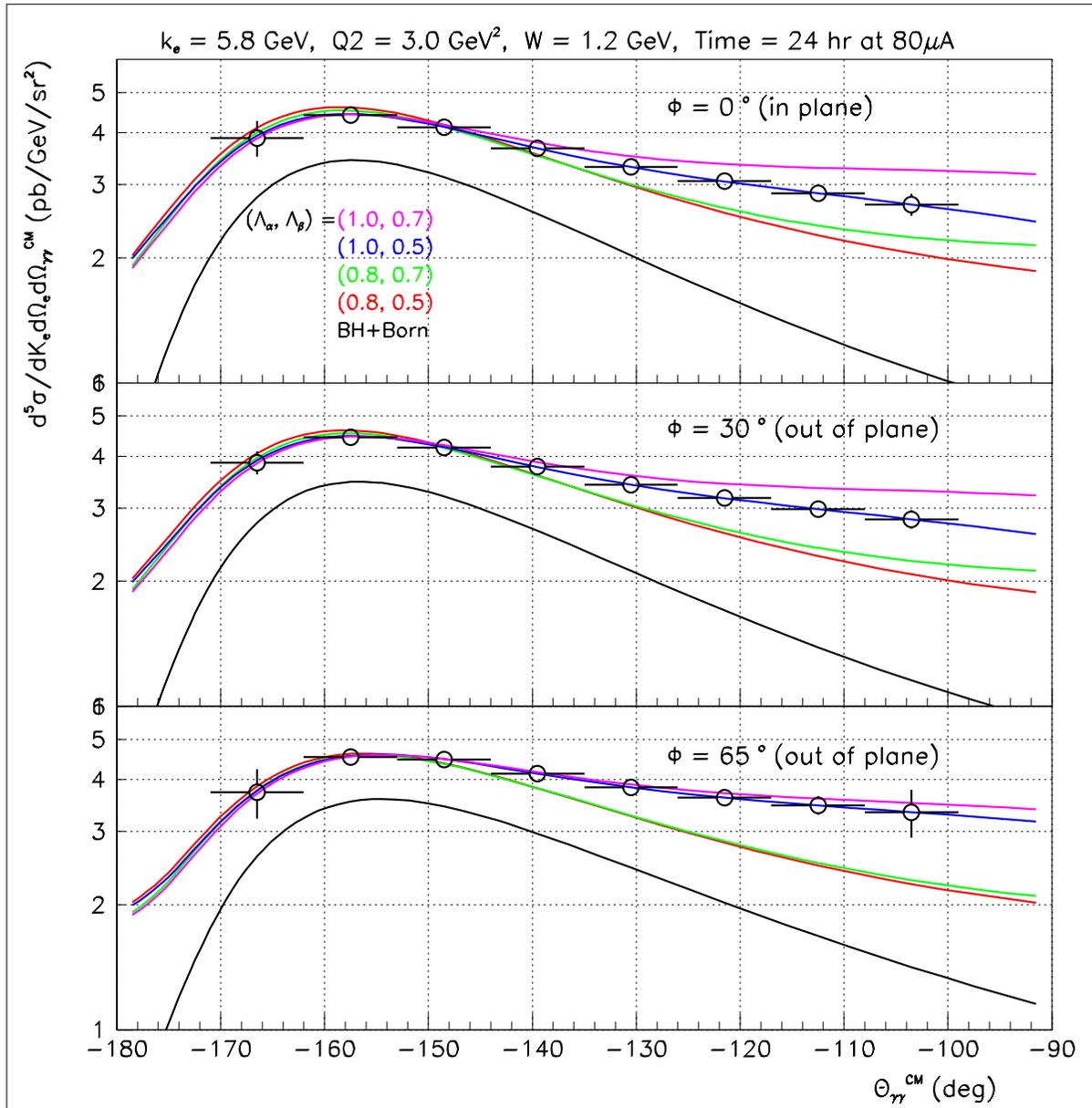


Differential VCS  
 Cross sections at  
 $W = 1200 \text{ MeV}$   
 and four  $Q^2$  values

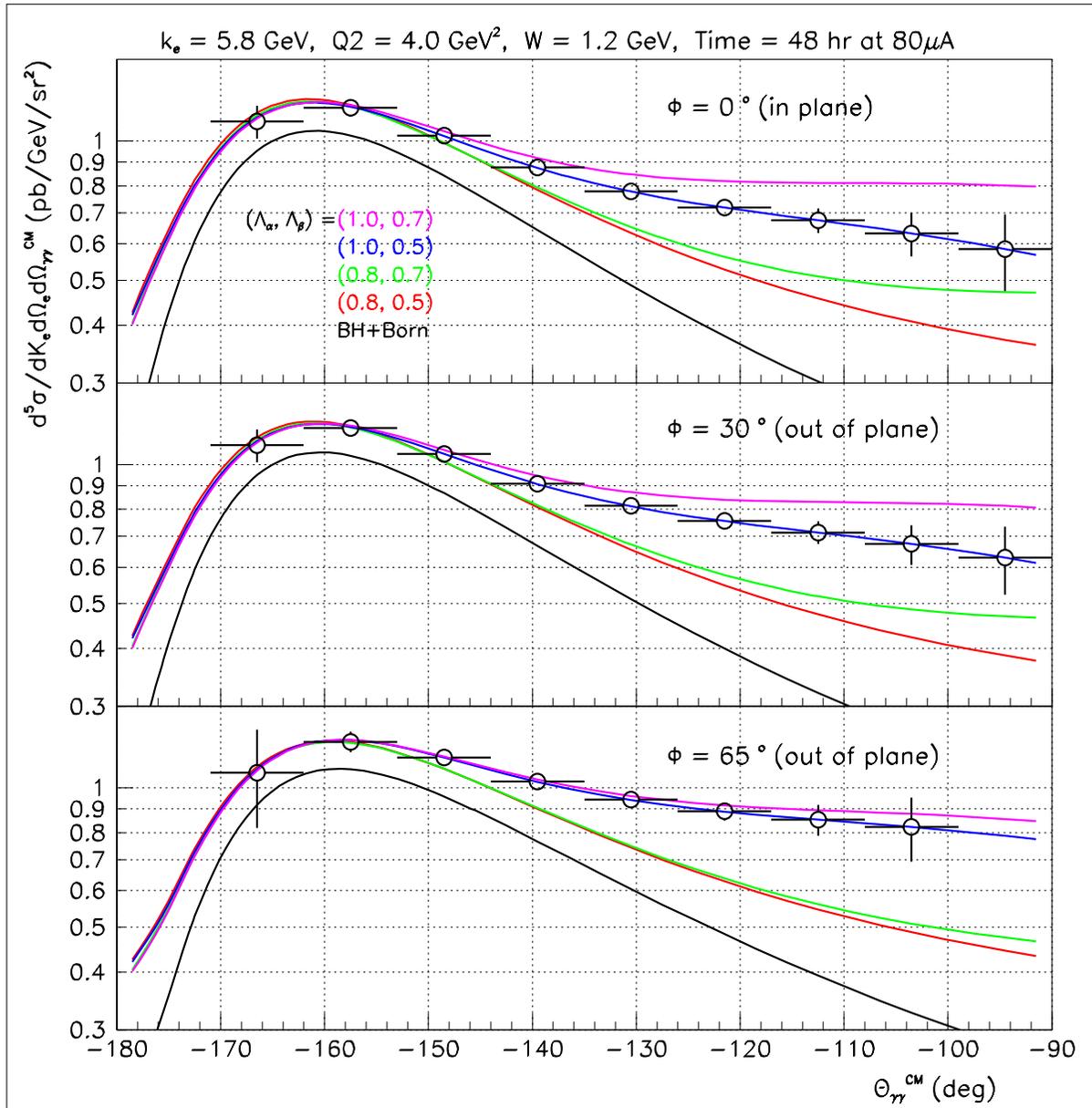
Missing Mass  $W=1.1$  GeV and  $Q^2=4$  GeV<sup>2</sup>



Projected missing mass resolution for the  $H(e, e'p)\gamma$  and  $H(e, e'p)\pi^0$  reactions with JLab Hall A HRS<sup>2</sup> at  $Q^2 = 4$  GeV<sup>2</sup> and  $W = 1.1$  GeV. The important range for VCS is from  $-180$  to  $+60^\circ$ .



Projected statistical precision from JLab P03-010 (deferred) in  $9^\circ$  bins in  $\theta_{\gamma\gamma}^{CM}$  and three bins in  $\phi_{\gamma\gamma}$  spanning  $180^\circ - [0, 90^\circ]$  out-of-plane.



Projected statistical precision from JLab P03-010 (deferred) in  $9^\circ$  bins in  $\theta_{\gamma\gamma}^{CM}$  and three bins in  $\phi_{\gamma\gamma}$  spanning  $180^\circ - [0, 90^\circ]$  out-of-plane. With a triple  $H(e, e'p\gamma)$  coincidence using the Hall A DVCS  $\text{PbF}_2$  array, the angular range can be extended to  $\theta_{\gamma\gamma}^{CM} < -60^\circ$

# Radiative Corrections to Elastic e-p Scattering

Discrepancy between Rosenbluth and Polarization measurements of  $G_E/G_M$  is sensitive to Real part of dispersive amplitude  $\mathcal{O}(\alpha_{\text{QED}})$

Guichon & Vanderhaeghen hep-ph/0306007

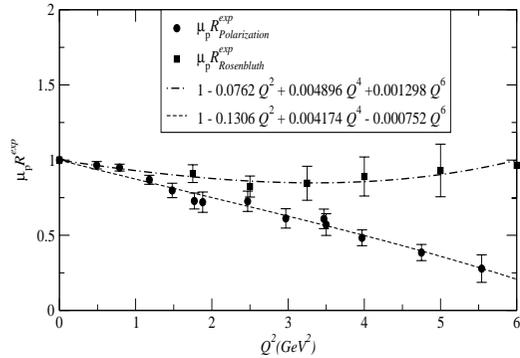


FIG. 1: Experimental values of  $R_{\text{Rosenbluth}}^{\text{exp}}$  and  $R_{\text{Polarization}}^{\text{exp}}$  and their polynomial fits.

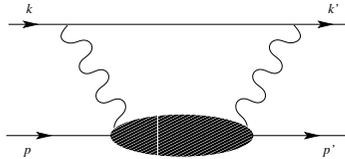


FIG. 2: The box diagram. The filled blob represents the response of the nucleon to the scattering of the virtual photon.

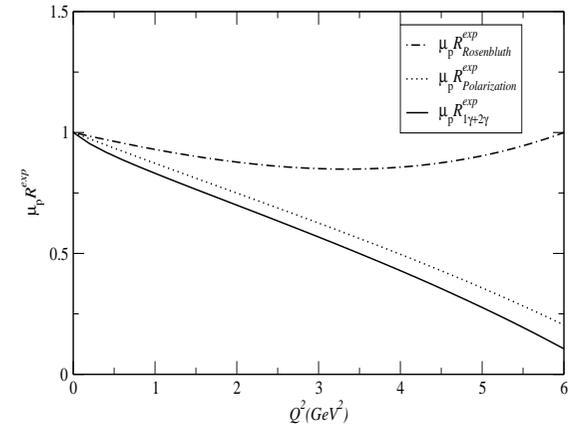


FIG. 4: Comparison of the experimental ratios  $\mu_p R_{\text{Rosenbluth}}^{\text{exp}}$  and  $\mu_p R_{\text{Polarization}}^{\text{exp}}$  with the value of  $\mu_p R_{1\gamma+2\gamma}^{\text{exp}}$  deduced from our analysis.

## Two photon amplitude in elastic e-p Scattering

Imaginary part of  $2\gamma$  amplitude in  $ep \rightarrow ep$

Afanasev, Akushevich, Merenkov, hep-ph/0208260

Measureable in elastic single spin asymmetries

Gorchtein & Vanderhaeghen 2003

(Transverse Beam Asymmetry  $5 \cdot 10^{-5}$  at  $Q^2 = 1 \text{ GeV}^2$ )

Calculable (?) via Optical Theorem from  $ep \rightarrow N\pi \dots$

Real part of  $2\gamma$  amplitude in  $ep \rightarrow ep$

Calculable (?) from Imaginary part via Dispersion Relations plus low energy terms (**Polarizabilities**).

## Conclusions

Generalized Polarizabilities give important insight to proton structure

High precision study of  $ep \rightarrow ep\gamma$  possible with modest beam time and existing equipment at JLab Hall A at  $Q^2 = 3, 4 \text{ GeV}^2$ .

Is there a connection between Dispersion Relations for Generalized Polarizabilities at high  $Q^2$  and the Generalized Parton Distributions in the  $t = -Q^2$  limit?

$ep \rightarrow ep\gamma$  and Generalized Polarizabilities can provide important input to dispersive effects in elastic e-p scattering.