Generalized Parton Distributions, Orbital Angular Momentum and recent results from Lattice QCD

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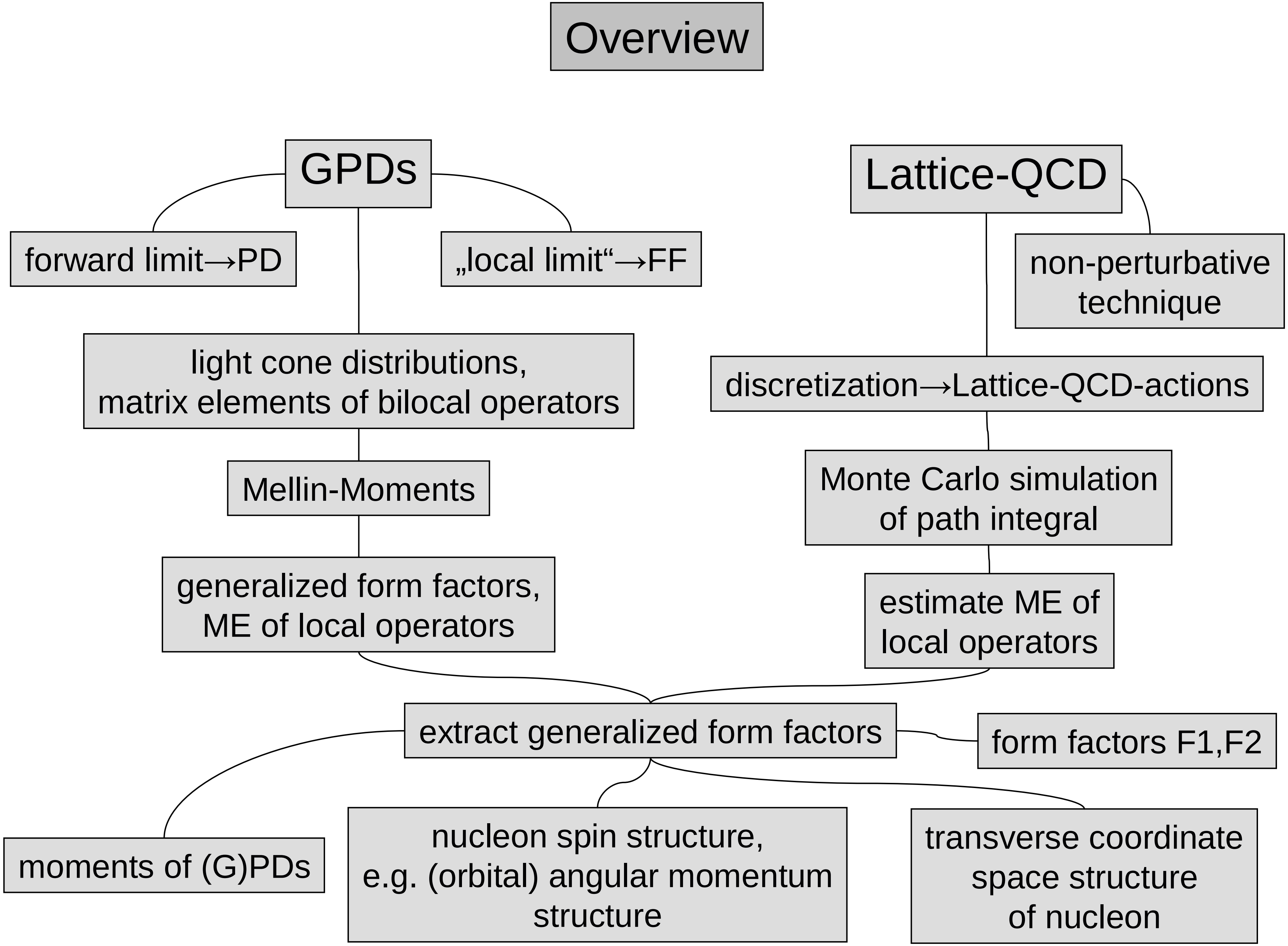
Hägler/Negele/Renner/Schroers/…., hep-lat/0304018, accepted for publ. in PRD


Overview
GPDs
Lattice-QCD
forward limit→PD
„local limit“→FF
non-perturbative technique
light cone distributions, matrix elements of bilocal operators
discretization→Lattice-QCD-actions
Mellin-Moments
Monte Carlo simulation of path integral
generalized form factors, ME of local operators
estimate ME of local operators
extract generalized form factors
form factors F1,F2
moments of (G)PDs
nucleon spin structure, e.g. (orbital) angular momentum structure
transverse coordinate space structure of nucleon

GPDs: Short overview

Ji, 1997
Radyushkin, 1997

off-forward matrix elements of bilocal operators

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \overline{\psi}(-\frac{\lambda n}{2}) \Gamma \mathcal{U} \psi(\frac{\lambda n}{2}) | P \rangle = h_{\Gamma}\{H, \widetilde{H}\}(x, \xi, \Delta^2) + e_{\Gamma}\{E, \widetilde{E}\}(x, \xi, \Delta^2)$$

forward limit

longitudinal momentum transfer $\xi \to 0$
momentum transfer squared
(transverse momentum transfer) $\Delta^2 \to 0$

$$H(x,0,0) = q(x)$$

$$\widetilde{H}(x,0,0) = \Delta q(x) = \Delta\Sigma(x)$$

„local“ limit, Mellin transformation

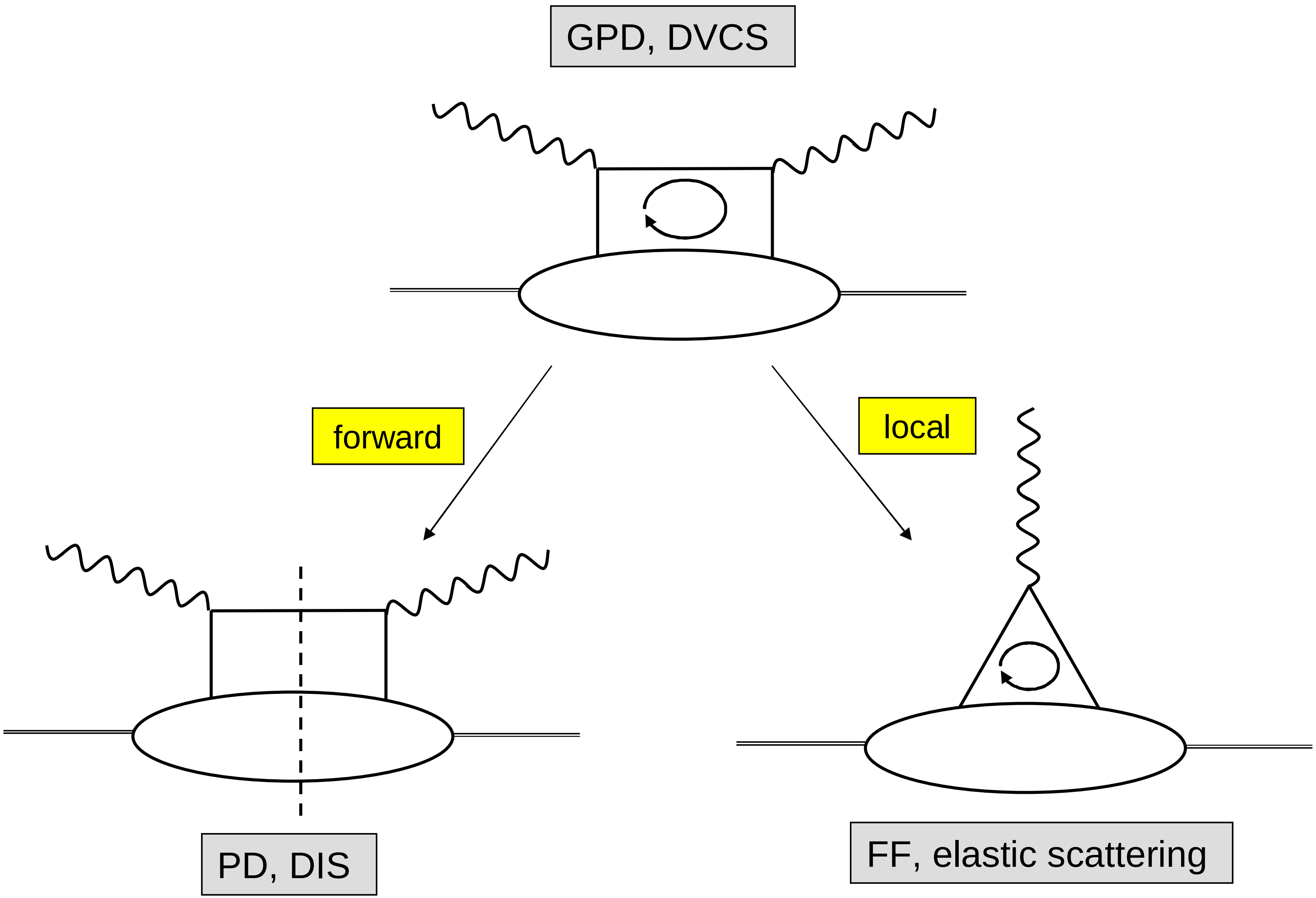
$$\int_{-1}^{1} dx x^{n-1}$$

$$\langle P' | \overline{\psi}(0) \Gamma D^{\mu_1} D^{\mu_2} \ldots \psi(0) | P \rangle$$

$$\propto \sum_i \left(a_i^{\mu_1 \mu_2 \cdots} A_{i0}(\Delta^2) + b_i^{\mu_1 \mu_2 \cdots} B_{i0}(\Delta^2) \right) + c^{\mu_1 \mu_2 \cdots} C(\Delta^2)$$

$n = 1$ — $F_1(\Delta^2), F_2(\Delta^2), \ldots$

GPDs and basic experimental probes



GPDs and (O)AM

(orbital) angular momentum and off-forward distributions

$$\langle P' | T_{q+g}{}^{\mu_1\mu_2} | P \rangle = a^{\mu_1\mu_2} A(\Delta^2) + b^{\mu_1\mu_2} B(\Delta^2) + c^{\mu_1\mu_2} C(\Delta^2)$$

Noether

nucleon spin sumrule

related sumrules

$A(0) = 1$

$B(0) = 0$

$$\frac{1}{2} = J_N = \frac{1}{2}\left(A(0) + B(0)\right) \stackrel{!}{=} \frac{1}{2}\left(H^{n=2}(0) + E^{n=2}(0)\right) = \frac{1}{2}\left(A_{20}(0) + B_{20}(0)\right)$$

Is the relation GPDs↔nucleon spin trivial?

spin and orbital decomposition

$$\frac{1}{2} = S + L = S + (J - S)$$

flavour and gluon decomposition

$$\frac{1}{2} = \sum_q J_q + J_g$$

Quark OAM and twist-3 GPDs

parametrization including transverse parts in terms of twist-3-distributions

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}(-\frac{\lambda n}{2}) \gamma_{\mu\perp} \psi(\frac{\lambda n}{2}) | P \rangle = \langle\langle \gamma_{\mu\perp} \rangle\rangle \{H + E\} + \langle\langle \gamma_{\mu\perp} \rangle\rangle G_2 + \langle\langle \not{n} \gamma_5 \rangle\rangle i\varepsilon_{\mu\nu\perp} \Delta^{\nu}_{\perp} G_4 + (\propto \Delta_{\mu\perp})$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}(-\frac{\lambda n}{2}) \gamma_{\mu\perp} \gamma_5 \psi(\frac{\lambda n}{2}) | P \rangle = \langle\langle \gamma_{\mu\perp} \gamma_5 \rangle\rangle \tilde{H} + \ldots$$

Kiptily/Polyakov, hep-ph/0212372

use EOF

derive an integrated sumrule for the twist three distributions and OAM

Pentinnen et al, PLB 491, 2000

$$\int dx x [G_2(x,0,0) + 2G_4(x,0,0)] =$$

$$-\frac{1}{2} \int dx x [H(x,0,0) + E(x,0,0)] + \frac{1}{2} \int dx \tilde{H}(x,0,0) = -J_q + \frac{1}{2} \Delta\Sigma \equiv -L_q$$

sheds some new light on quark OAM compared to standard definition $\{DVCS \to J_q, DIS \to \Delta\Sigma\} \to L_q = J_q - \frac{1}{2}\Delta\Sigma$

Is a generalization to distributions $L_q(x), \ldots$ possible?

start with definition of OAM-distribution

$$L_q(x) \equiv \frac{1}{NV}\int dx^- e^{ixx^-P^+/2}\langle P|\int d^2x_\perp \psi_+^\dagger(x_\perp)[ix_1\mathcal{D}_2 - ix_2\mathcal{D}_1]\psi_+(x_\perp + x^-)|P\rangle$$

Bashinsky/Jaffe, NPB 536, 1998

define new off-forward function

$$f_{L_q}(x,\Delta_\perp) \equiv \frac{1}{4\pi}\int dx^- e^{ix(x^-P^+)/2}\langle P'|\int d^2x_\perp \psi_+^\dagger(x_\perp)[ix_1\partial_2 - ix_2\partial_1]\psi_+(x_\perp + x^-)|P\rangle$$

with a testfunction T we have

$$f_{L_q}(x) \equiv \frac{\int d^2\Delta_\perp T(\Delta_\perp) f_{L_q}(x,\Delta_\perp)}{(2\pi)^2 T(0)}$$

if $T(\Delta_\perp) \to \delta^2(\Delta_\perp) \Rightarrow f_{L_q}(x) = L_q(x)$

now rewrite $x_\perp$ as $\partial_{\Delta\perp}$

$$f_{L_q}(x) \equiv \frac{1}{4\pi}\int dx^- e^{ix(x^-P^+)/2}\varepsilon_{jk}\partial_{\Delta\perp j}\left\{\langle P'|\psi_+^\dagger(0)\partial_{x\perp k}\psi_+(x^-)|P\rangle\right\}_{\Delta=0}$$

consider a general off-forward correlator

$$\int \frac{d\lambda d^2 x_\perp}{(2\pi)^3} e^{i\lambda x - i x_\perp \bullet k_\perp} \langle P' | \overline{\psi}(-\frac{\lambda n}{2} - \frac{x_\perp}{2}) \not{n} \psi(\frac{\lambda n}{2} + \frac{x_\perp}{2}) | P \rangle$$

take one unit of intrinsic quark $k^{\nu\perp}$ into account

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \overline{\psi}(-\frac{\lambda n}{2}) \not{n} \overset{\leftrightarrow}{\partial}{}^{\nu\perp} \psi(\frac{\lambda n}{2}) | P \rangle = a^{\nu\perp} A(x, \Delta_\perp^2) + \ldots$$

comparing with our expression for $f_{L_q}(x)$, it's straightforward to see that

$$\varepsilon_{\mu\nu\perp} \partial^\mu_{\Delta\perp} \left(a^{\nu\perp} A(x, \Delta_\perp^2) + \ldots \right)_{\Delta=0} = 2 f_{L_q}(x)$$

use EOF and (dropping gluons

$$\overline{\psi}(x_2) \gamma^{\{\mu} \overset{\leftrightarrow}{\partial}{}^{\nu\}} \psi(x_1) = i\varepsilon^{\nu\mu\alpha\beta} \overline{\psi}(x_2) \gamma_{\{\alpha} \underset{\leftrightarrow}{\partial}{}_{\beta\}} \gamma_5 \psi(x_1)$$

finally, replace

$$n \cdot \overset{\leftrightarrow}{\partial} \to -ix$$
$$\underset{\leftrightarrow}{\partial}{}_{\beta\perp} \to i\Delta_{\beta\perp}/2$$
$$n \cdot \underset{\leftrightarrow}{\partial} \to i\, n \cdot \Delta / 2$$

and use the standard twist-2 and 3 paramterization

in the WW-approximation

$$L_q(x) = x\left(H(x) + E(x) + G_2(x) + 2G_4(x)\right) - \Delta q(x)$$
$$= 2J_q(x) - \Delta q(x) + x\left(G_2(x) + 2G_4(x)\right) = 2L_q(x) + x\left(G_2(x) + 2G_4(x)\right)$$
$$\Rightarrow \quad L_q(x) = -x\left(G_2(x) + 2G_4(x)\right)$$

GPDs as transverse-coordinate space distributions

Burkardt, 2000

$\xi \to 0$

infinite momentum frame→Galileian structure in transverse plane

Fourier-transform in transverse plane

$$\int d^2\Delta_\perp e^{i\Delta_\perp \cdot b_\perp} H(x,\xi=0,\Delta_\perp^2) = H(x,b_\perp^2)$$

distributions in x and $b_\perp$ of quarks in the nucleon

x-moments

- charge $F_1(b_\perp), F_2(b_\perp)$
- momentum $q(b_\perp)$
- spin $\Delta\Sigma(b_\perp)$
- total angular momentum $J_q(b_\perp)$ (?)

distributions in $b_\perp$ of quarks in the nucleon

$\xi \neq 0$

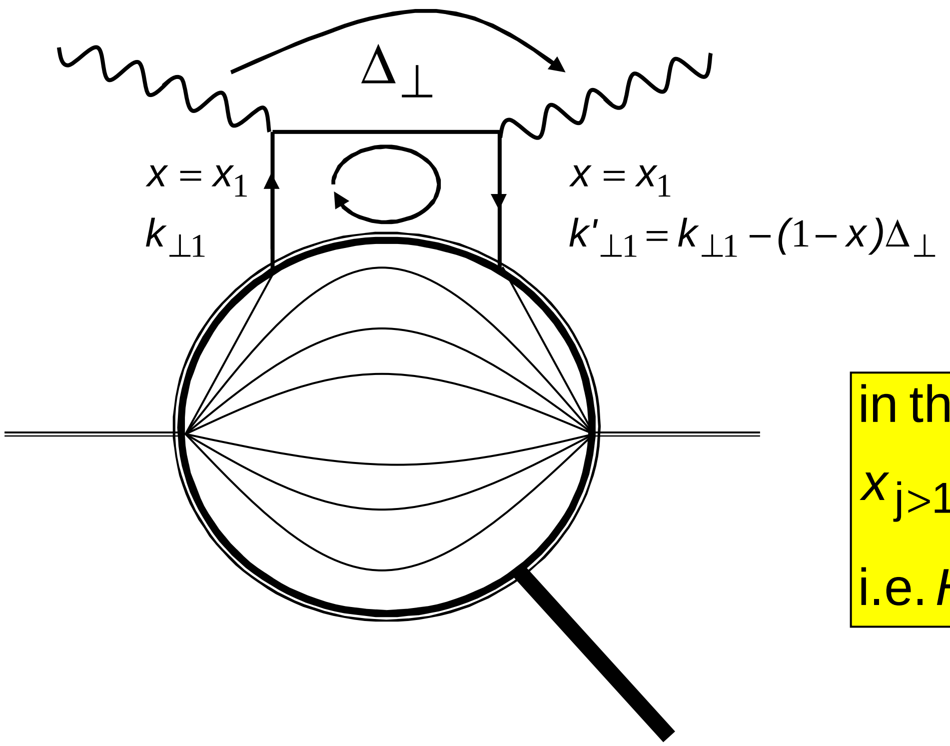
Diehl, 2002

GPDs in terms of light-cone wave functions 1

Brodsky et al, NPB596, 2001

$$H(x,\xi\to 0,\Delta^2)\propto \sum_n \int \prod_{i=1..n} dx_i d^2k_{\perp i}\,\delta(1-\sum_j x_j)\,\delta^2(\sum_j k_{\perp j})\,\Psi_n^*(x'_m,k'_{\perp m})\,\Psi_n(x_p,k_{\perp p})$$

momentum space



$x = x_1$
$k'_{\perp 1} = k_{\perp 1} - (1-x)\Delta_\perp$
$x'_j = x_j$
$k'_{\perp j>1} = k_{\perp j} + x_j \Delta_\perp$

in the limit $x\to 1$, apparently
$x_{j>1}\to 0$, and therefore $k'_{\perp j} = k_{\perp j}$,
i.e. $H(x\to 1,\xi\to 0,t=\Delta^2) = const.$

how does this translate to coordinate (impact parameter) space?

GPDs in terms of light-cone wave functions 2

$$k'_{\perp 1} = k_{\perp 1} - (1-x)\Delta_\perp$$
$$k'_{\perp j>1} = k_{\perp j} + x_j \Delta_\perp$$

$$\int d^2 k'_{\perp 1}\, \delta^2(k'_{\perp 1} - (k_{\perp 1} - (1-x)\Delta_\perp))$$
$$\int d^2 k'_{\perp j>1}\, \delta^2(k'_{\perp j>1} - k_{\perp j>1} - x_j \Delta_\perp)$$

Fourier - transform

$$H(x,0,b_\perp) = \int d^2\Delta_\perp e^{-ib_\perp \bullet \Delta_\perp} H(x,0,\Delta_\perp)$$

$$H(x,0,b_\perp) \propto \sum_n \prod_{m=1..n} \int dx_m d^2 r_{\perp m} d^2 r'_{\perp m}\, \delta^2(b_\perp + (1-x) r'_{\perp 1} - \sum_i x_i r'_{\perp i})$$
$$\times \delta(1 - \sum x) \prod_j \delta^2(r'_{\perp 1} - r_{\perp 1} + r_{\perp j} - r'_{\perp j}) \Psi^*_n(x, r'_\perp) \Psi_n(x, r_\perp)$$

coordinate space

i.e. $H(x \to 1, 0, b_\perp) \propto \delta^2(b_\perp)$

distribution in impact parameter space
is in fact a delta - function in the limit $x \to 1$

GPDs in Lattice QCD

discretization, change of language

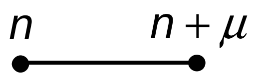
infinite continuous space-time → finite space-time lattice

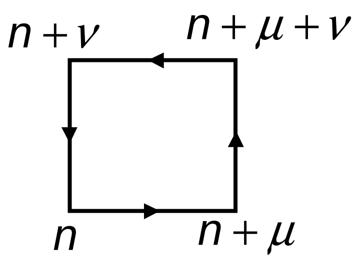
$\psi(x) \rightarrow \psi(n), \; n = (n_1, n_2, n_3, n_4)$, "sites"

$A^{\mu}(x) \rightarrow U_{n,n+\mu} = U_{\mu}(n)$, "links"

$F^{\mu\nu}(x) \rightarrow U_{\mu\nu}(n)$, "plaquettes"

$$\int dx \rightarrow \sum_n$$





lattice QCD-action is not unique

$$S_{QCD} = S_G[U] + S_F[U, \psi, \overline{\psi}],$$

$$S_G[U] \propto \sum_P \left[1 - \frac{1}{6} Tr\left(U_P + U_P^{\dagger}\right)\right]$$

general correlation functions

$$\left\langle \psi_{\alpha}(n) \ldots U_{\beta}(m) \ldots \overline{\psi}_{\gamma}(j) \ldots \right\rangle$$

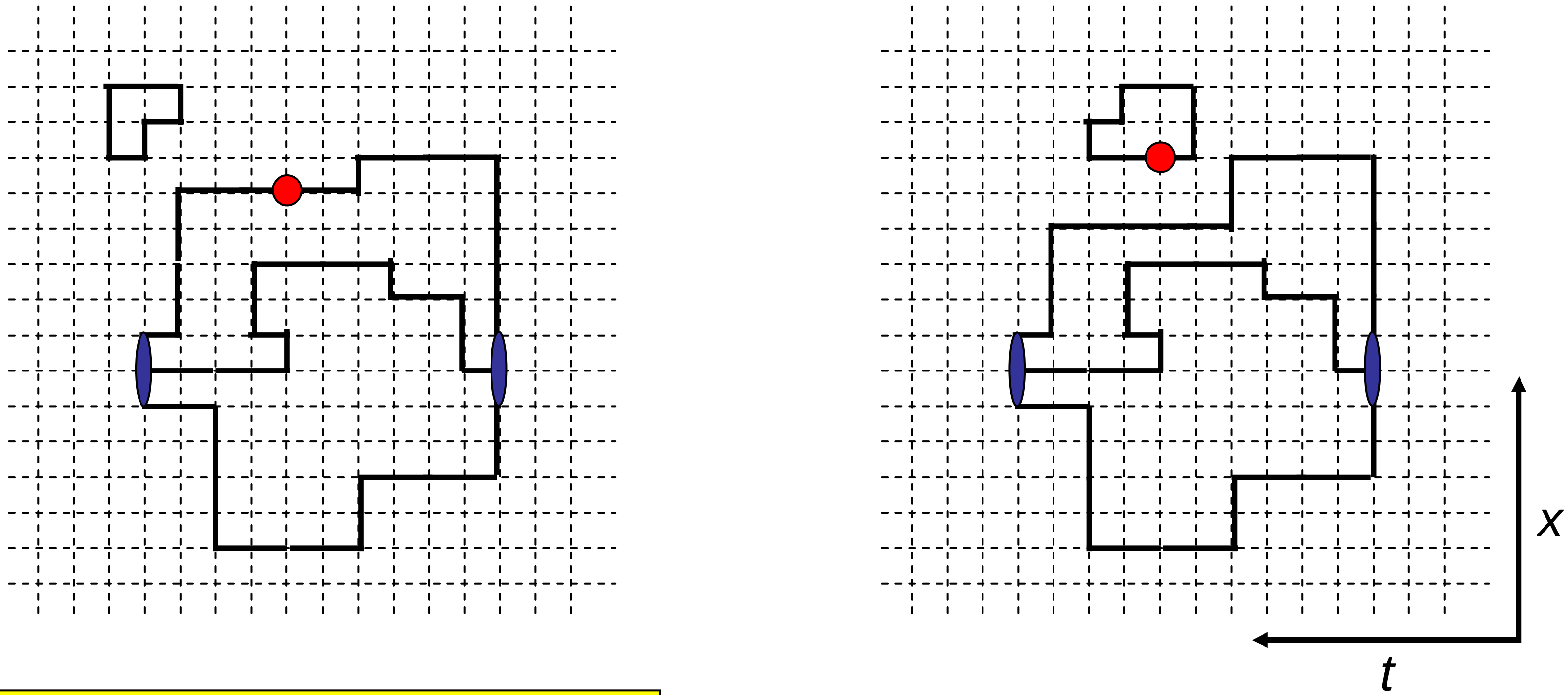
$$\propto \int [DU][D\psi][D\overline{\psi}] \left(\psi_{\alpha}(n) \ldots U_{\beta}(m) \ldots \overline{\psi}_{\gamma}(j) \ldots\right) e^{-S_{QCD}}$$

path integrals are numerically evaluated using Monte Carlo techniques

GPDs in Lattice QCD…continued

$$\langle N(P')|\overline{\psi}(0)\Gamma D^{\mu_1} D^{\mu_2} \ldots \psi(0)|N(P)\rangle \rightarrow C_{3pt}(\tau,P',P) = Tr\{\Gamma_{pol}\langle 0|N(\tau_{snk},P')O_{\Gamma}(\tau)\overline{N}(\tau_{src},P)|0\rangle\}$$

$\hat{=}$ nucleon source/sink $\hat{=}$ operator insertion $\hat{=}$ quark-path through lattice



connected, unquenched/full QCD

disconnected, not included so far

Horsley, PhD thesis, 2000

GPDs in Lattice QCD…continued

in the limit $\tau - \tau_{src} >> \frac{1}{E}$ and $\tau_{snk} - \tau >> \frac{1}{E}$, excited states drop out, and we have

$$\frac{\langle N(P')|\mathcal{O}_\Gamma|N(P)\rangle}{\sqrt{E(P')E(P)}} = \frac{C_{3pt}(\tau,P,P')}{C_{2pt}(\tau_{snk},P')}\left(\frac{C_{2pt}(\tau_{snk}-\tau+\tau_{src},P)C_{2pt}(\tau,P')C_{2pt}(\tau_{snk},P')}{C_{2pt}(\tau_{snk}-\tau+\tau_{src},P')C_{2pt}(\tau,P)C_{2pt}(\tau_{snk},P)}\right)^{\frac{1}{2}} \equiv \frac{R(\tau,P',P)}{\sqrt{E(P')E(P)}}$$

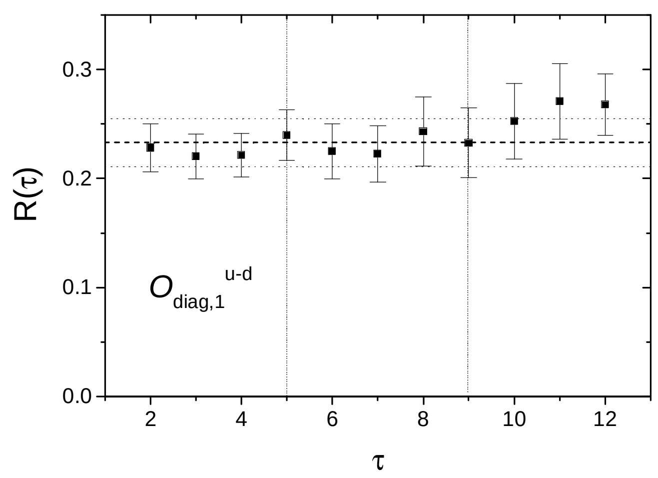
finally, we equate lattice result and continuum parametrization for different momenta and indices

$$\overline{R}^{\mu_1\mu_2\cdots}(P',P) = \langle P'|\overline{\psi}(0)\Gamma D^{\mu_1}D^{\mu_2}\ldots\psi(0)|P\rangle$$
$$= \sum_i \left(a_i^{\mu_1\mu_2\cdots}A_{i0}(\Delta^2) + b_i^{\mu_1\mu_2\cdots}B_{i0}(\Delta^2)\right) + c^{\mu_1\mu_2\cdots}C(\Delta^2)$$

this gives an (overdetermined) set of linear equations which is solved to get the GFFs

$A_{i0}(\Delta^2), B_{i0}(\Delta^2), C(\Delta^2)$

plateau



Numerical results for heavy and „light“ pions

SESAM - lattice - parameters :

- unimproved Wilson action
- unquenched calculation, only connected contributions
- lattice - size is $16^3 * 32$
- lattice - spacing is roughly $a^{-1} = 2$ GeV (i.e. the scale is $\mu^2 \approx 4$ GeV^2)
- quarks, pions, nucleons are quite heavy, $m_\pi \approx 700 \ldots 900$ MeV
- roughly 200 configurations / κ
- 3 κ available, corresponding to $m_\pi \approx 700, 800$ and 900 MeV

off-forward matrix element

see also QCDSF (Göckeler et al.), hep-ph/030424

MILC - lattice - parameters :

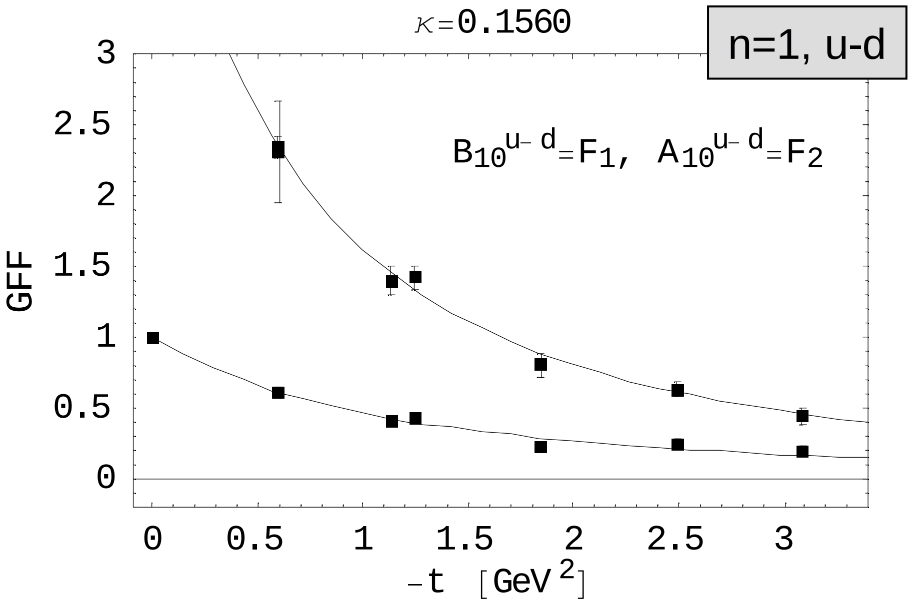
- domain - wall (chiral) fermions with staggered kernel
- improved gauge action with HYP - smearing
- unquenched calculation, only connected contributions
- lattice - size is $20^3 * 32$
- lattice - spacing is roughly $a^{-1} = 1.5$ GeV (i.e. the scale is $\mu^2 \approx 2.3$ GeV^2)
- quarks, pions, nucleons are closer to chiral limit, $m_\pi \approx 313$ (and 580) MeV
- roughly 100 (eventually 200) configurations / m_π

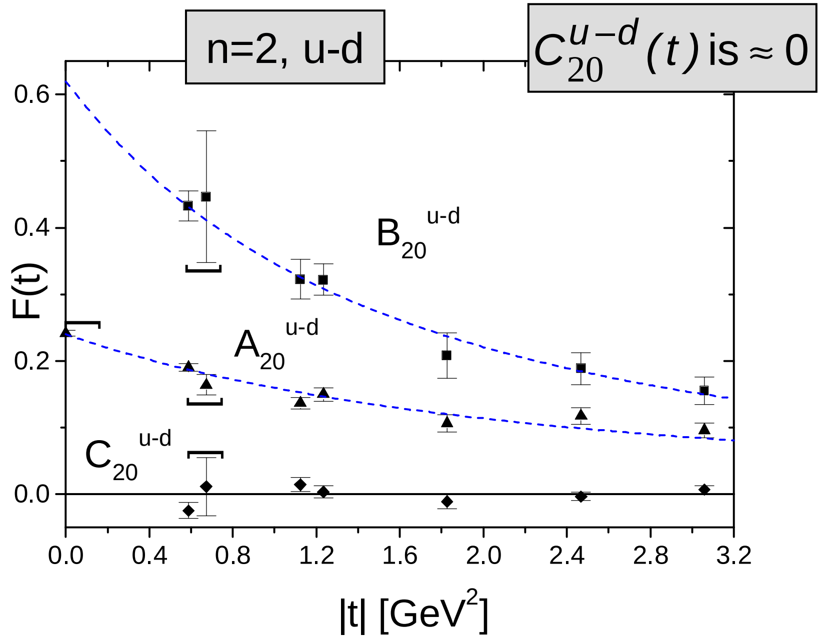
only forward matrix element so far

Results for the GFF A(t), B(t), C(t)

concentrate on u-d in order to cancel disconnected pieces

dipole - fits work well $GFF(t) \approx \frac{a}{\left(1 - t/m_D^2\right)^2}$





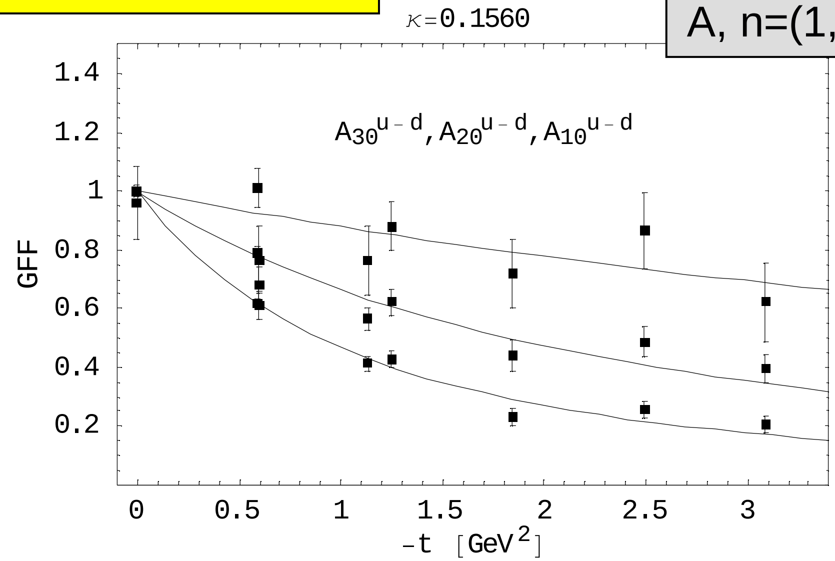
$$\int_{-1}^{1} dx x^0 H(x,0,t=0) = \int_{-1}^{1} dx q(x) = A_{10}(t=0)$$

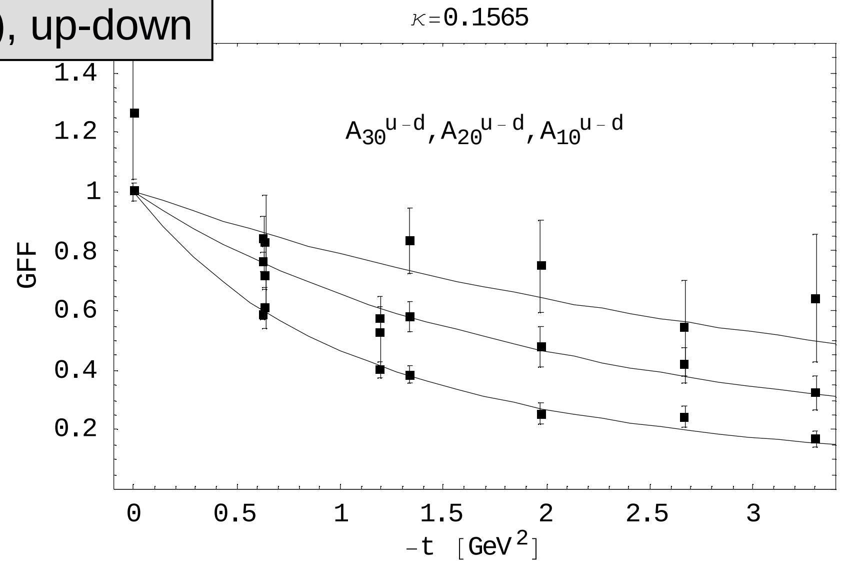
$$\int_{-1}^{1} dx x^1 H(x,0,t=0) = \int_{-1}^{1} dx x q(x) = A_{20}(t=0)$$

Flattening of t-slope for higher moments of H

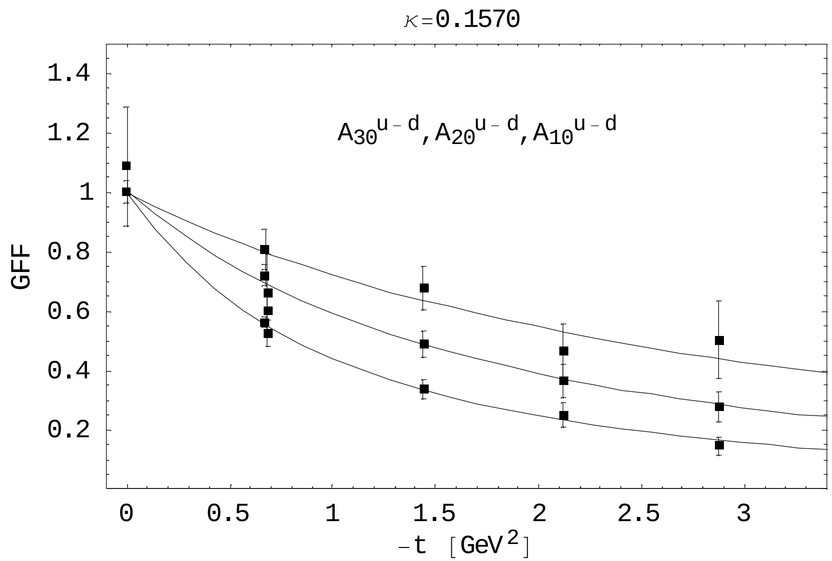
$\int dx x^{n-1} H(x, \xi = 0, t) = A_{n0}(t)$

A, n=(1,2,3), up-down





t-slopes effectively flatten for larger n



$\int_0^1 dx x^{n-1} \xrightarrow{n \to \infty} x \approx 1$

$H(x \approx 1, b_\perp) \propto \text{"} \delta(b_\perp) \text{"}$

$H(x \approx 1, t) \propto const.$

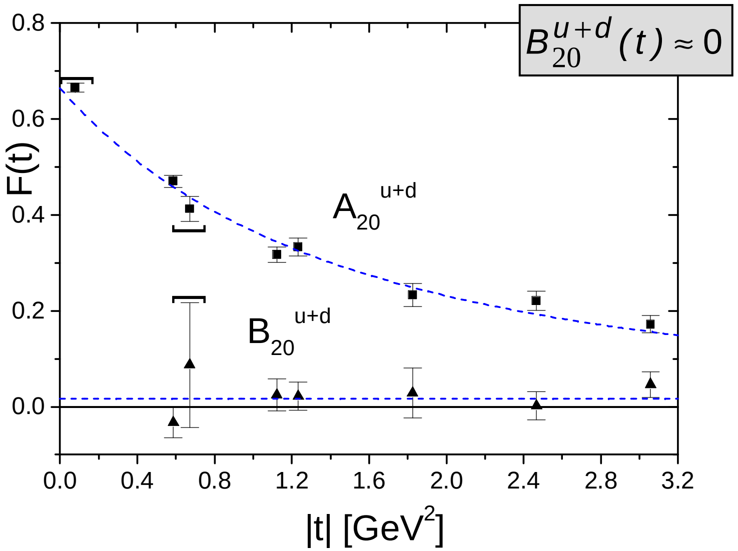
factorized ansätze like

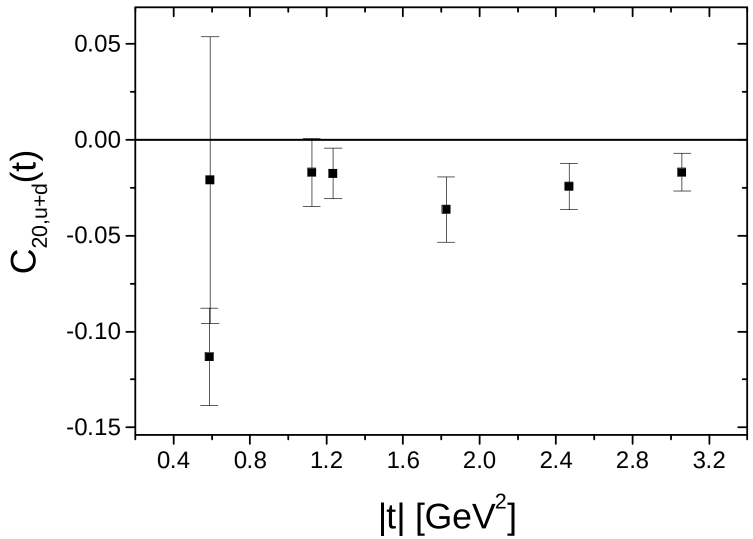
$H(x,t) = q(x) f(t)$

can be ruled out for these pion masses

(Orbital) Angular Momentum

n=2, up+down, disconnected pieces missing





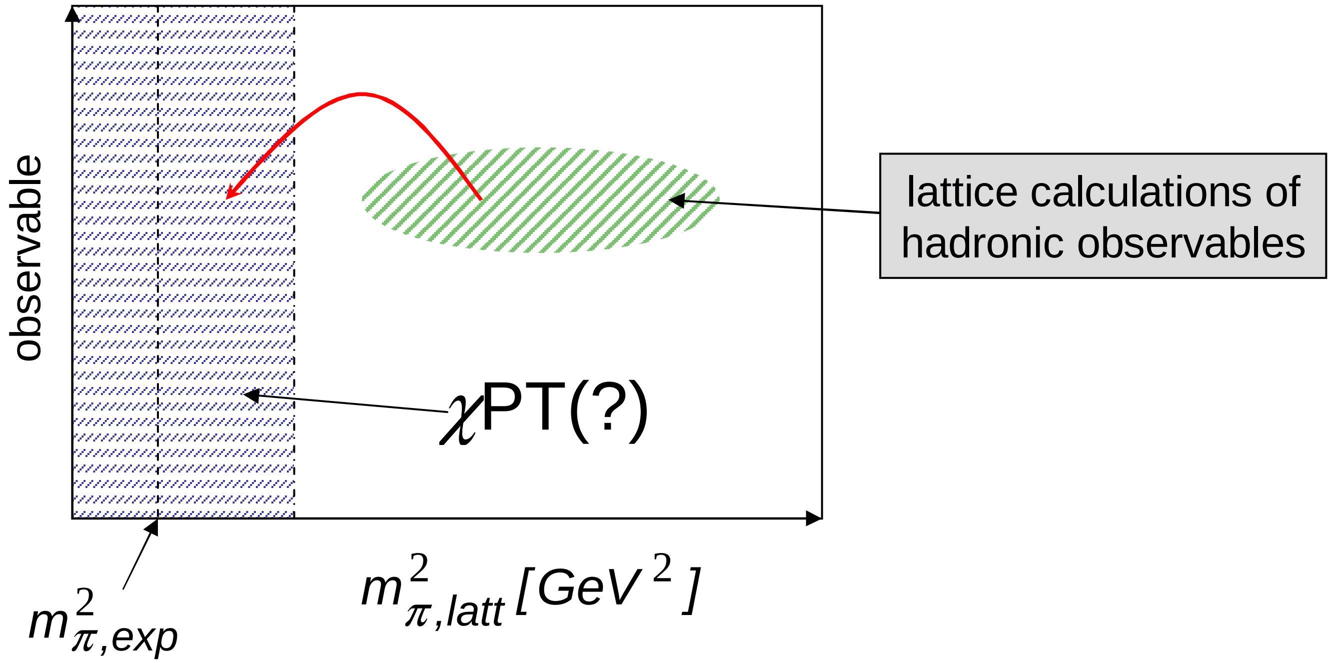
$$J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\big(H(x,0,t=0) + E(x,0,t=0)\big)$$

$$= \frac{1}{2}\int_{-1}^{1} dx\, x\big(q(x) + E(x,0,t=0)\big) = \frac{1}{2}\big(A_{20}(0) + B_{20}(0)\big)$$

κ	0.1570	0.1565	0.1560
$\Delta\Sigma$	0.666 ± 0.033	0.727 ± 0.028	0.684 ± 0.018
$2\ J_q$	0.730 ± 0.035	0.688 ± 0.024	0.682 ± 0.029
$2\ L_q$	0.064 ± 0.048	-0.039 ± 0.037	-0.002 ± 0.034

$C_{20}^{u+d}(t)$ is small and negative $\rightarrow$
extrapolated $C_{20}^{u+d}(t=0) = ?$
seems to be at least compatible
with the chiral quark soliton model
(Kivel et al., 2000)

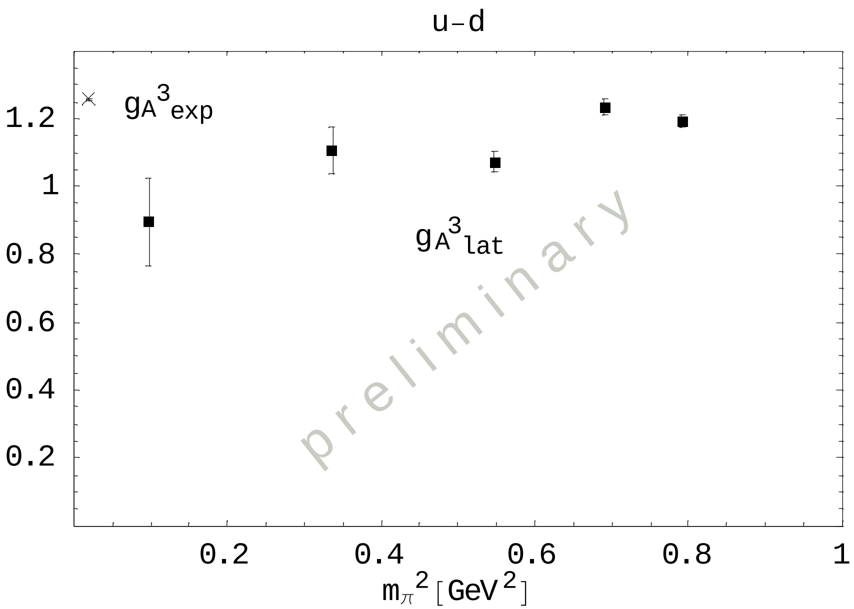
Towards the chiral region

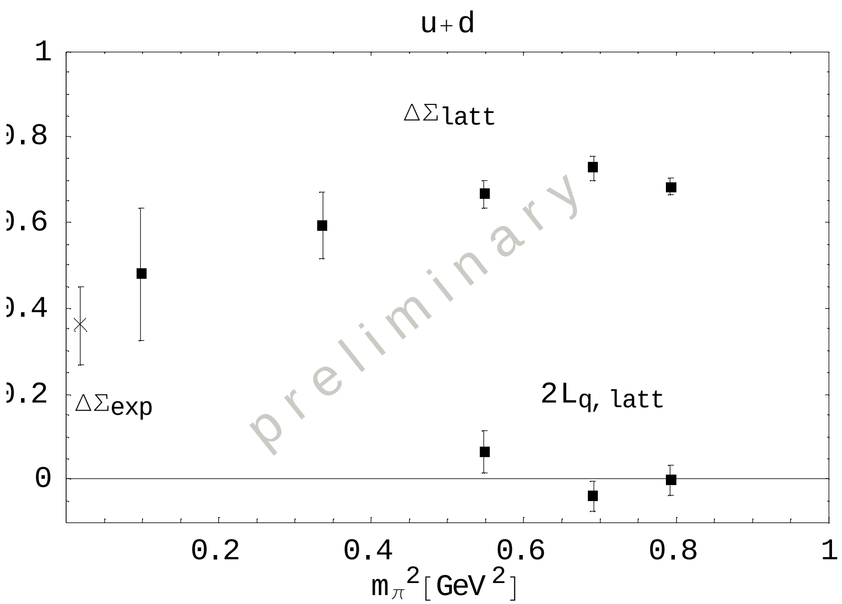


see e.g.
Göckeler et al., hep-lat/0303019
Detmold et al, Phys.Rev.Lett.87 2001

Towards the chiral region…continued

axial coupling/charge//form factor and the quark spin contribution





should be a clean observable…
„delocalization of axial charge“?

looks not bad, probably accidentally:
disconnected pieces are missing!

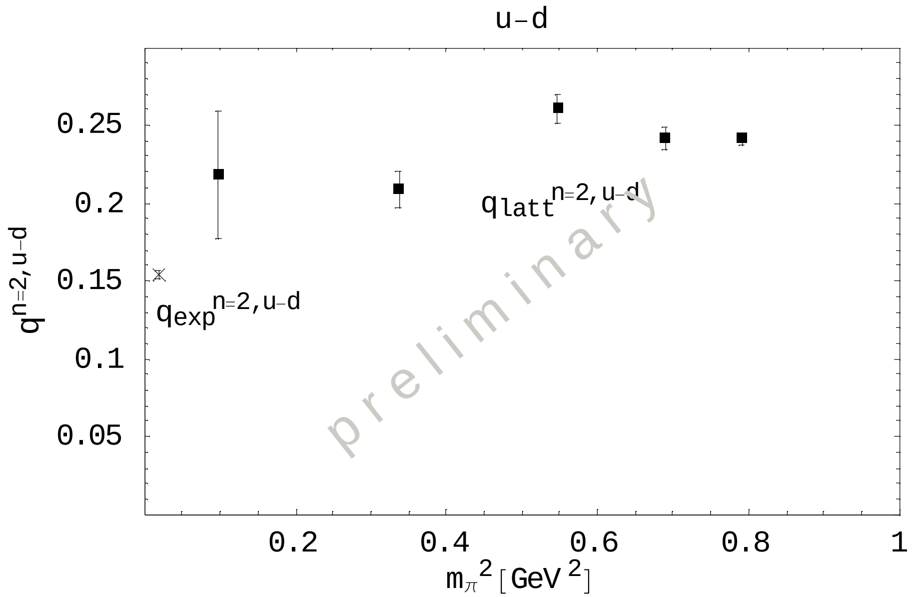
Jaffe PLB 529, 2002

chiral logs for (O)AM Chen/Ji, PRL88,2002

results for the lowest two pion masses are not renormalized,
I expect Z≈1±0.05 for the FF

Towards the chiral region…continued

quark longitudinal momentum fraction, u-d



error pretty large, inconclusive

n=3, u-d

work in progress

Outlook

-better stastistics for MILC is on the way
-extract GPDs from the recent MILC-configurations
-extract the spin-dependent GFFs for n=3
-(perturbative) renormalization with
domain wall fermions + HYP-smeared links
-chiral extrapolation, if possible
-what is going on with the axial coupling?
-non-factorized models for H,E
-disconnected diagrams
-lower pion masses/larger lattices