Soft-Collinear Effective Theory: From B decays to the pion form factor

Thorsten Feldmann (CERN)

"Generalized Parton Distribution & Hard Exclusive Reactions", INT, Seattle, 25/June/2003

- effective field theory formulation of factorization, identifying relevant degrees of freedom
- toy integral for $B \to \gamma$: "QCD" vs. effective theory
- drawing conclusions

[based on work in progress by Beneke/TF and Beneke/Diehl/TF]

Motivation for SCET:

- Field-theoretical basis for BBNS factorization approach to exclusive B decays
  ultimate goal: constrain SM and beyond-SM parameters

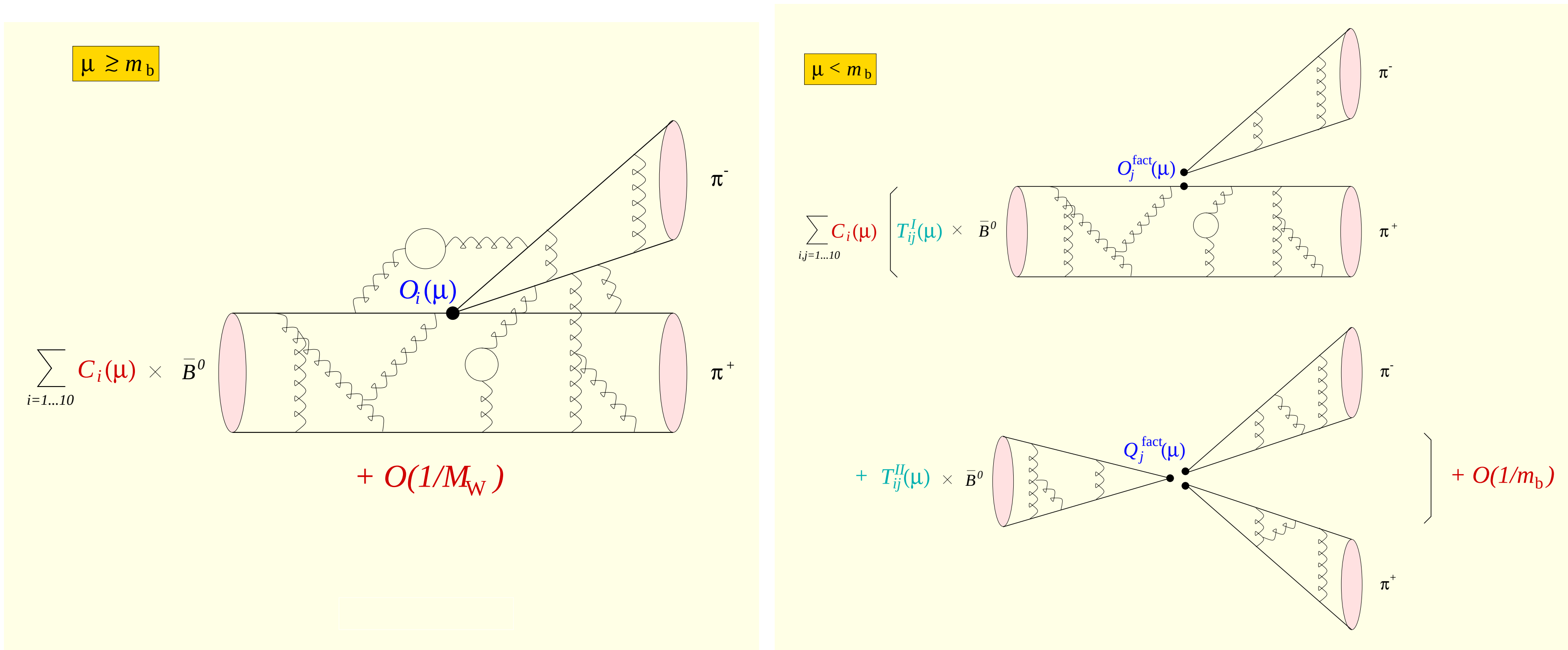
- Alternative proof/formulation of factorisation theorems
  in various hard-scattering reactions

- Better understanding of Sudakov effects, power-corrections, "soft overlap" etc.
  (simplest objects to study: transition form factors (e.g. $B \to \pi$, or $\pi \to \pi \ldots$)
  – important non-perturbative input for many processes –)

[Beneke/Buchalla/Neubert/Sachrajda]
[Bauer/Stewart et al.]
[Chay/Kim]
[Beneke/Diehl/Chapovsky/TF]
[Descotes-Genon/Sachrajda]
[Lunghi/Wyler et al.]
[Neubert et al.]

"still in progress"

QCD factorization à la BBNS

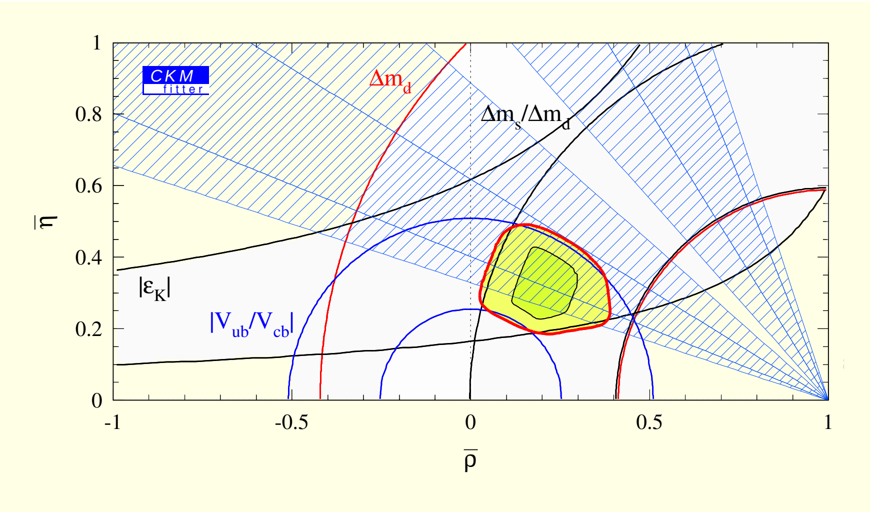


[Fig. taken from M.Neubert]

- hard gluon effects can be factorized into hard-scattering kernels (Wilson coefficient functions)
- soft gluons can be absorbed into $B \longrightarrow \pi$ form factors, or light-cone distribution amplitudes
- non-factorizable contributions are power-suppressed

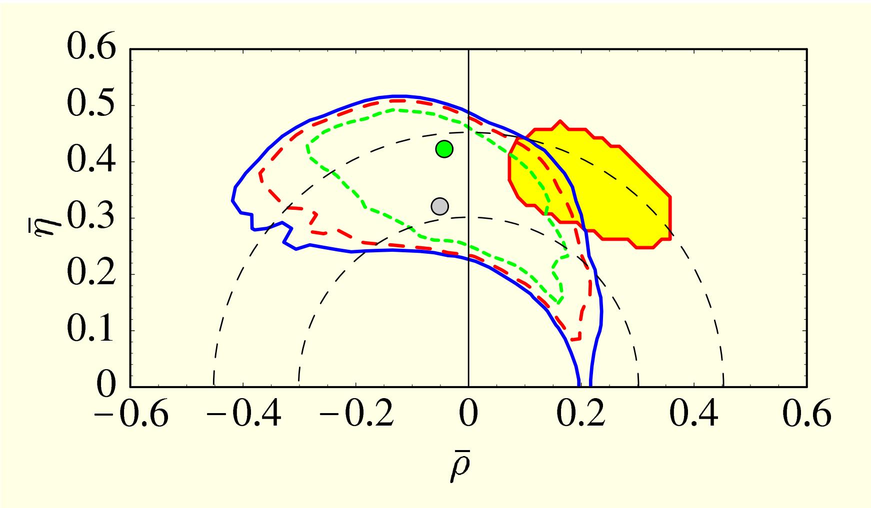
Constraining the CKM unitarity triangle

"standard" analysis:



[taken from Hoecker et al.]

using $B \to \pi\pi$ and $B \to \pi K$ data
theory based on QCD factorization approach:



[taken from M.Beneke]

[includes error estimate on all kind of hadronic uncertainties]

Effective theory formulation of factorization

1. integrate out short-distance modes related to highest scale μ_1 (“matching“)

$\longrightarrow$ short-distance physics is encoded in coefficient functions of effective operators; in general infinite series of operators suppressed by increasing powers of $1/\mu_1$

2. coefficient functions obey RG equation in effective theory; RG evolution from μ_1 to $\mu < \mu_1$ resums large logarithms $\ln \mu_1/\mu$ (“running”)

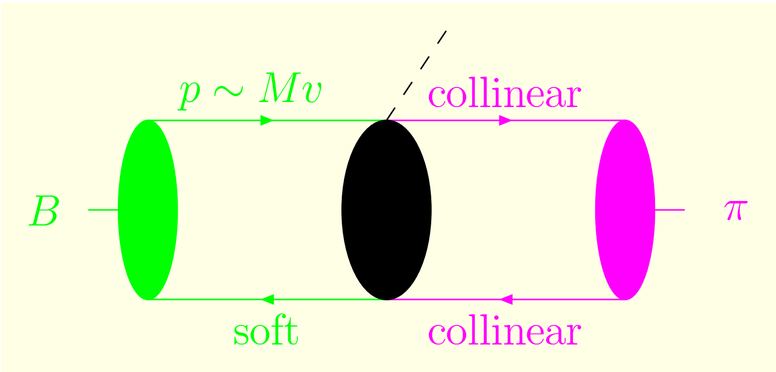
3. if necessary/possible, repeat 1.+2. to integrate out further modes (μ_2 etc.)

Matrix elements of operators in effective theory contain dynamics below factorization scale μ

Field content of effective Lagrangian reproduces IR behavior of “full” theory

Identifying the relevant degrees of freedom (A)

Example: heavy-to-light form factors at large recoil $(E_\pi \sim m_b/2 \gg \Lambda)$



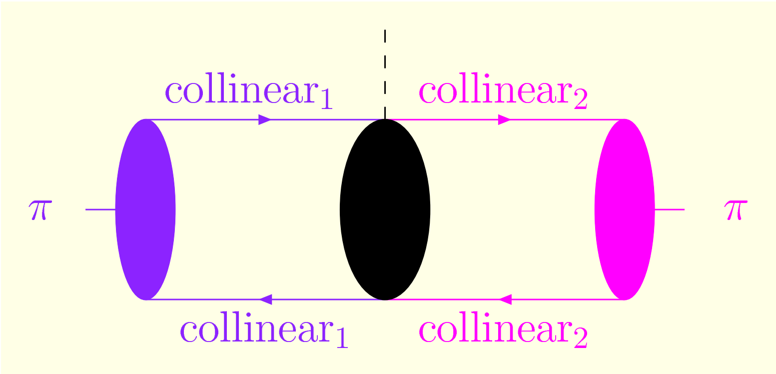
(similar $B \to \rho$, $B \to \gamma$)

- external modes:
  - ⋆ almost on-shell heavy quark
    $p = m_b v + k, \quad k^\mu \sim \Lambda \ll m_b$
    (soft residual momentum)
  - ⋆ soft B meson spectators
    $l^\mu \sim \Lambda \ll m_b, \quad l^2 \sim \Lambda^2$
  - ⋆ collinear partons in fast pion
    $E \sim m_b, \quad p^2 \sim \Lambda^2$
- internal modes:
  - ⋆ hard modes
    (heavy quark + collinear)$^2 \sim \mathcal{O}(m_b^2)$
  - ⋆ hard-collinear modes
    (soft + collinear)$^2 \sim \mathcal{O}(m_b \Lambda)$

[B meson rest frame]

Identifying the relevant degrees of freedom (B)

Example: pion form factor at large momentum transfer $(Q^2 = -q^2 \gg \Lambda^2)$



(similar $\pi \longrightarrow \gamma$)

- external modes:
  - ⋆ collinear partons in incoming pion $E \sim Q, \quad p^2 \sim \Lambda^2$
  - ⋆ collinear partons in outgoing pion $E' \sim Q, \quad (p')^2 \sim \Lambda^2$
- internal modes:
  - ⋆ hard modes (collinear$_1$ - collinear$_2$)^2 $\sim$ $\mathcal{O}(Q^2)$
  - ⋆ soft modes?

[Breit frame]

Characterization of relevant modes

- define two light-like vectors: $n_+^2 = n_-^2 = 0, \quad n_+ n_- = 2$
- decompose any vector as $$p^\mu = (n_+ p)\,\frac{n_-^\mu}{2} + p_\perp^\mu + (n_- p)\,\frac{n_+^\mu}{2}$$
- consider expansion parameter $\lambda = \sqrt{\Lambda/M} \ll 1$ $\quad (M = m_b, Q)$

$(\frac{n_+ p}{M}, \frac{p_\perp}{M}, \frac{n_- p}{M})$	Terminology	$B \to \pi$	$\pi \to \pi$
$(1, 1, 1)$	hard	$\surd$	$\surd$
$(1, \lambda, \lambda^2)$	hard-collinear	$\surd$	–
$(\lambda, \lambda, \lambda)$	semi-hard*	–	–
$(\lambda^2, \lambda^2, \lambda^2)$	soft	$\surd$	?
$(\lambda^4, \lambda^2, 1)$	collinear$_1$	–	$\surd$
$(1, \lambda^2, \lambda^4)$	collinear$_{(2)}$	$\surd$	$\surd$

* irrelevant for matching

Effective theory description

$B \to \pi, \gamma, \ldots$	$\pi \to \pi, \gamma, \ldots$
introduce separate fields for different quark and gluon modes in QCD integrate out hard modes at $\mu^2 \sim M^2$	
effective theory of hard-collinear, collinear, and soft fields	$\ldots$
integrate out hard-collinear modes at $\mu^2 \sim M\Lambda$	$\ldots$
effective theory of collinear, and soft fields	effective theory of collinear_1, collinear_2 (and soft?) fields
matrix elements in effective theory only contain modes with virtuality $k^2 \sim \Lambda^2$ separate fields are decoupled in (leading power) effective Lagrangian	

Power-counting in the effective theory

- soft fields: $[h_v = \frac{1+\not v}{2}\, e^{i m_b\, v\cdot x}\, Q]$

$$\mathcal{L}_s \quad = \quad \bar q_s\,(i\not D_s - m)\,q_s - \frac{1}{2}\,\mathrm{tr}\,[F_s^{\mu\nu}\,F_{\mu\nu\,s}] \;\;, \quad \mathcal{L}_{\rm HQET} \;=\; \bar h_v\, iv\cdot D_s\, h_v + \ldots$$

$$\boxed{\Rightarrow A_s^\mu \sim \lambda^2, \quad q_s \sim \lambda^3, \quad h_v \sim \lambda^3}$$

- collinear fields: $[\,\xi_c = \frac{\not n_- \not n_+}{4}\, q_c]$

$$\mathcal{L}_c \quad = \quad \bar\xi_c \left\{ in_- D_c + (i\not D_{\perp c} - m)\,\frac{1}{in_+ D_c}(i\not D_{\perp c} + m)\right\} \frac{\not n_+}{2}\,\xi_c - \frac{1}{2}\,\mathrm{tr}\,[F_c^{\mu\nu}\,F_{\mu\nu\,c}]$$

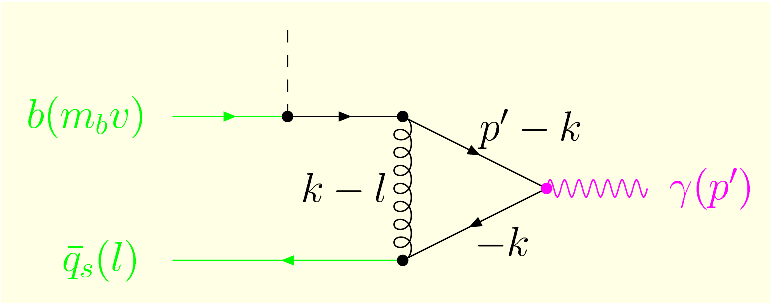
$$\boxed{\Rightarrow A_c^\mu \sim (1,\lambda^2,\lambda^4), \quad \xi_c \sim \lambda^2}$$

Questions

- How can one formally understand the relevance of modes?
- How does ERBL-type factorization show up?
- Where is the "soft overlap"?

Let's study a toy integral . . .

A simple toy integral in "QCD" $(B \to \gamma)$



$$I = \int [dk] \, \frac{1}{[(k-l)^2]\,[k^2-m^2]\,[(p'-k)^2-m^2]}$$

$$= \frac{1}{2\,p'\cdot l} \left\{ \mathrm{Li}_2[-\frac{2\,p'\cdot l}{m^2}] - \frac{\pi^2}{6} \right\}$$

$$[dk] = \mu^{2\epsilon}\, e^{\epsilon\,\gamma_E}\, \frac{d^d k}{i\pi^{d/2}}, \qquad (d = 4-2\epsilon)$$

- for simplicity only scalar propagators

$\longrightarrow$ toy integral is UV finite

- IR regularization via finite quark mass

- $p'^2 = 0$ (external energetic 'photon')
- $l^2 = m^2 \sim \lambda^4$ (soft spectator)
- $p' \cdot l \sim \lambda^2$ (hard-collinear scale)

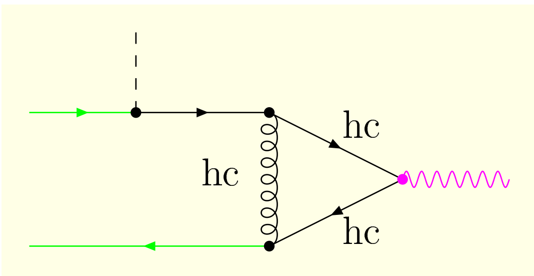
$\longrightarrow$ power-expansion in $\frac{m^2}{p'\cdot l} \sim \lambda^2$:

$$\boxed{I = -\frac{1}{2\,p'\cdot l}\left\{\frac{1}{2}\,\ln^2\frac{2\,p'\cdot l}{m^2} + \frac{\pi^2}{3} + \mathcal{O}(\lambda^2)\right\}}$$

Reproducing the full integral by regions

[Beneke/Smirnov]

- hard-collinear region: expand integrand, assuming all propagators are hard-collinear



$$I_{\rm hc} = \int [dk]\, \frac{1}{[k^2 - n_+k\, n_-l]\,[k^2]\,[k^2 - n_+p'\, n_-k]} + \ldots$$

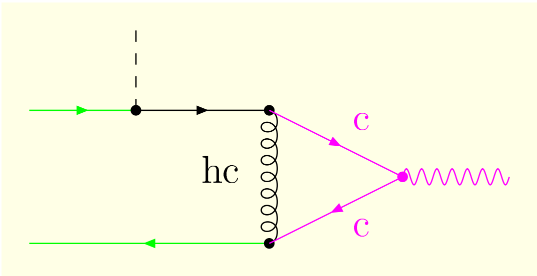
$$= -\frac{1}{2\,p'\cdot l}\left\{ \frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\frac{2\,p'\cdot l}{\mu^2} + \frac{1}{2}\ln^2\frac{2p'\cdot l}{\mu^2} - \frac{\pi^2}{12}\right\}$$

⋆ 1-loop correction to the tree-level hard-scattering kernel for $b\bar{q}_s \longrightarrow \gamma$ $\surd$

⋆ insensitive to IR regulator m^2 $\longrightarrow$ regulated by dim-reg $\longrightarrow$ factorization scale μ $\surd$

⋆ scale-dependence should match with low-virtuality regions (see below)

- collinear region:



$$I_c = \int [dk] \frac{[-\nu^2]^\delta}{[-n_+k\, n_-l]^{1+\delta}\, [k^2 - m^2]\, [k^2 - m^2 - 2\, p' \cdot k]}$$

$$= -\frac{1}{2\, p' \cdot l} \left\{ -\frac{1}{\delta} + \ln \frac{2\, p' \cdot l}{\nu^2} \right\} \left\{ \frac{1}{\epsilon} - \ln \frac{m^2}{\mu^2} \right\}$$

⋆ Surprise:

collinear (toy) integral has an additional endpoint-singularity from $n_+k \longrightarrow 0$

that is not regulated in dimensional regularization! ?

(here we introduced an analytical regularization of the 'gluon' propagator)

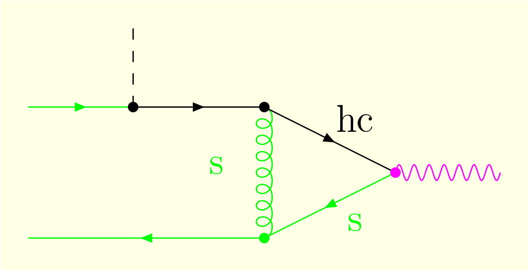
⋆ On the other hand:

$1/\epsilon$ (collinear) divergence and associated μ–dependence are expected.

Usually, one would absorb the divergence into light-cone distribution amplitude

$\longrightarrow$ ERBL formalism

- soft region:



$$I_{\rm s} = \int [dk] \, \frac{[-\nu^2]^\delta}{[(k-l)^2]^{1+\delta} \, [k^2 - m^2] \, [-n_+p' \, n_-k]}$$
$$= -\frac{1}{2\,p'\cdot l} \left\{ \left\{ \frac{1}{\delta} - \ln \frac{m^2}{\nu^2} \right\} \left\{ \frac{1}{\epsilon} - \ln \frac{m^2}{\mu^2} \right\} - \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln^2 \frac{m^2}{\mu^2} + \frac{5\pi^2}{12} \right\}$$

⋆ this time the endpoint divergence occurs for $n_+k \longrightarrow \infty$

⋆ endpoint-divergence cancels between collinear and soft integral

$$I_c + I_s = -\frac{1}{2\,p'\cdot l} \left\{ -\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{2\,p'\cdot l}{\mu^2} - \ln \frac{2\,p'\cdot l}{\mu^2} \ln \frac{m^2}{\mu^2} + \frac{1}{2} \ln^2 \frac{m^2}{\mu^2} + \frac{5\pi^2}{12} \right\}$$

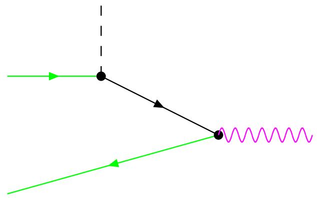
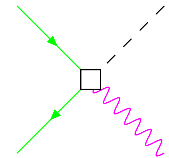
$\surd$

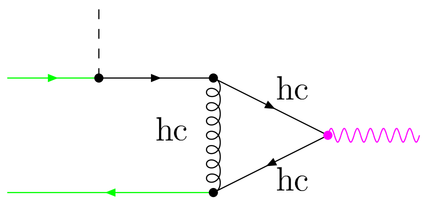
- Sum of regions $I_{\rm hc} + I_c + I_s$ reproduces the leading term in the full integral I

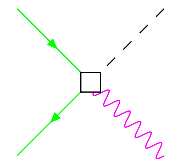
⋆ Contributions from other regions vanish, either because scale-less integrals in dim-reg. vanish, or because all poles in $(n_\pm \cdot k)$ appear on the same side of the real axis.

$\surd$

Reproducing the toy integral in effective theory

tree-level:  $\longrightarrow$ $\frac{1}{p'\cdot l}\, C_1^{(0)}$ 

hard-collinear: 

$\longrightarrow$ $\frac{1}{p'\cdot l}\, C_1^{(1)} (\ln \frac{p'\cdot l}{\mu^2})$ 

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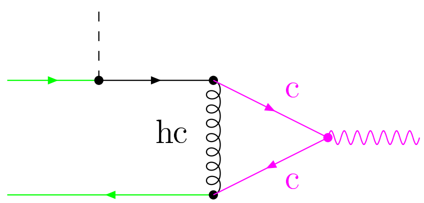
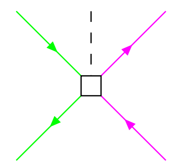
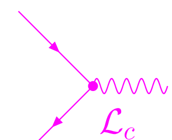
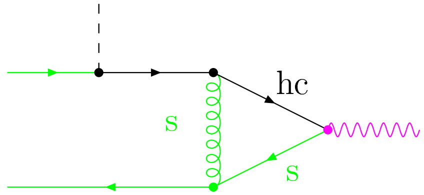
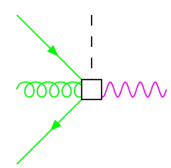
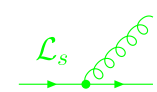
collinear: 

$\longrightarrow$ $\frac{1}{p'\cdot l}\, C_2^{(0)}$  and 

soft: 

$\longrightarrow$ $\frac{1}{p'\cdot l}\, C_3^{(0)}$  and 

Drawing Conclusions:

- relevant momentum modes are determined from regions of Feynman integrals $\surd$

- large-virtuality modes are absorbed into effective current operators $\surd$

- soft and collinear loops are reproduced by low-virtuality effective action $\surd$

- traditional "soft overlap" picture ("Feynman mechanism") is related to non-factorisable part of hadronic matrix elements in effective theory, e.g.

$$\langle \pi(p') | J_{\rm eff} | B(p) \rangle \neq \Phi_\pi(u,\mu) \star T(u,\omega,\mu) \star \Phi_B(\omega,\mu)$$

(cancellation of endpoint-divergences goes beyond ERBL formalism)

$(\ldots)$

ERBL factorization as exception to the rule:

In some cases endpoint-divergences are suppressed
by spin structures in *numerators* of integral $\to$ ERBL is applicable:

- $B \longrightarrow \gamma$ form factor at leading power [Bosch/Hill/Lange/Neubert 03]

$$\mathcal{A}(B \to \gamma \ell \nu) \sim \int_0^\infty d\omega \, \frac{\phi_B(\omega,\mu)}{\omega} \, T\left(\ln \frac{m_b}{\mu}, \, \ln \frac{E_\gamma \omega}{\mu^2}, \, \frac{q^2}{m_b^2} \right) \qquad \surd$$

important aspect of proof: hard-scattering kernel only depends on logarithms of ω $\longrightarrow$ no endpoint-divergences

- corrections to form factor relations for $B \longrightarrow \pi$, $B \longrightarrow K^*$ etc. [Beneke/TF]

$$\langle \pi(p') | J_{\rm eff} | B(p) \rangle \sim C(q^2,\mu) \, \xi_\pi(q^2,\mu) + \int_0^\infty d\omega \int_0^1 du \, \frac{\phi_\pi(u,\mu)}{\bar u} \, \frac{\phi_B(\omega,\mu)}{\omega} \, T(u, \, \omega, \, q^2, \, \mu)$$

non-trivial result: "non-factorisable" part $\xi_\pi(q^2)$ is independent of weak decay Dirac structure,

still to be shown: hard-scattering kernel induces no new endpoint-divergences at higher orders in α_s

More Examples:

- pion form factor, and $\pi \to \gamma$ form factor, . . . at leading-power

  ["standard" ERBL]

- corrections to "naive" factorization in $B \to \pi\pi$, $B \to K^*\gamma$, $B \to K^*\ell^+\ell^-$ . . .

  [BBNS, Bosch/Buchalla, Ali et al., Beneke/TF/Seidel, . . .]

- . . .

Translation to SCET formalism discussed by [Bauer/Stewart et al.] and [Chay/Kim].

Careful analysis of endpoint behavior for arbitrary orders of α_s still incomplete.

[$\longrightarrow$ work in progress]