Stiffening of matter in **quark-hadron continuity**

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Ref) TK, 2106.06687 [nucl-th]

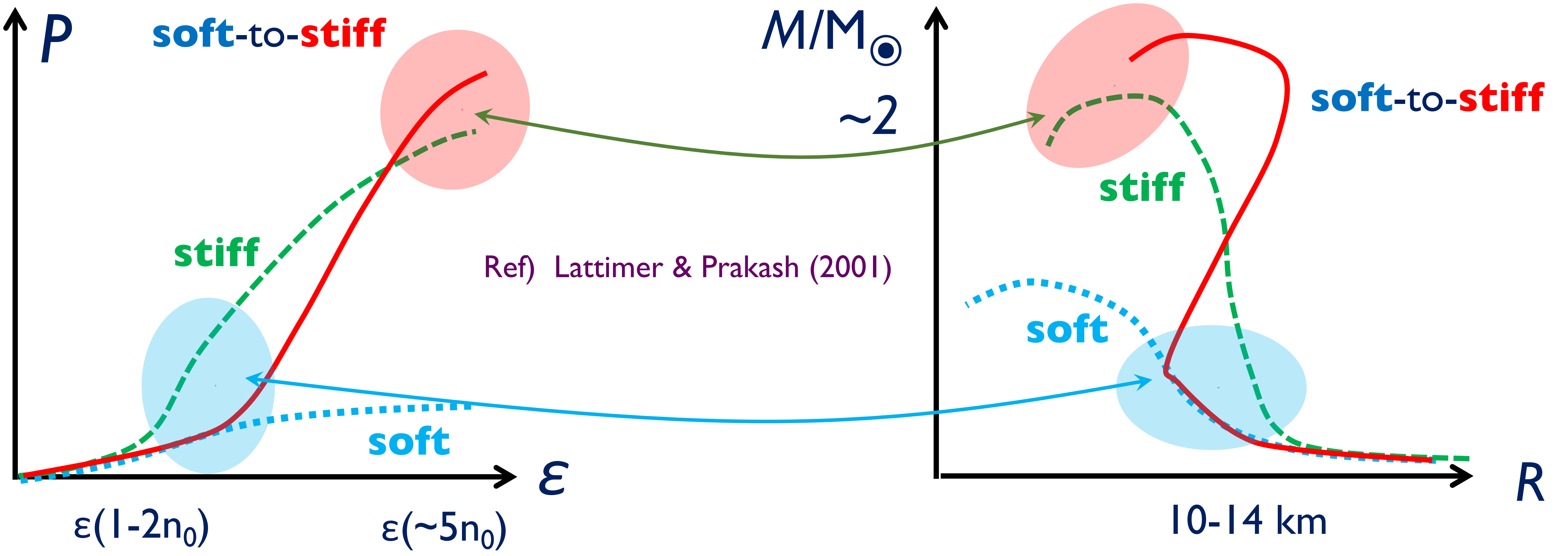
EoS & Neutron Star M-R relation

Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ QCD (+EW) EoS

gravity

(source: ε_{QCD})

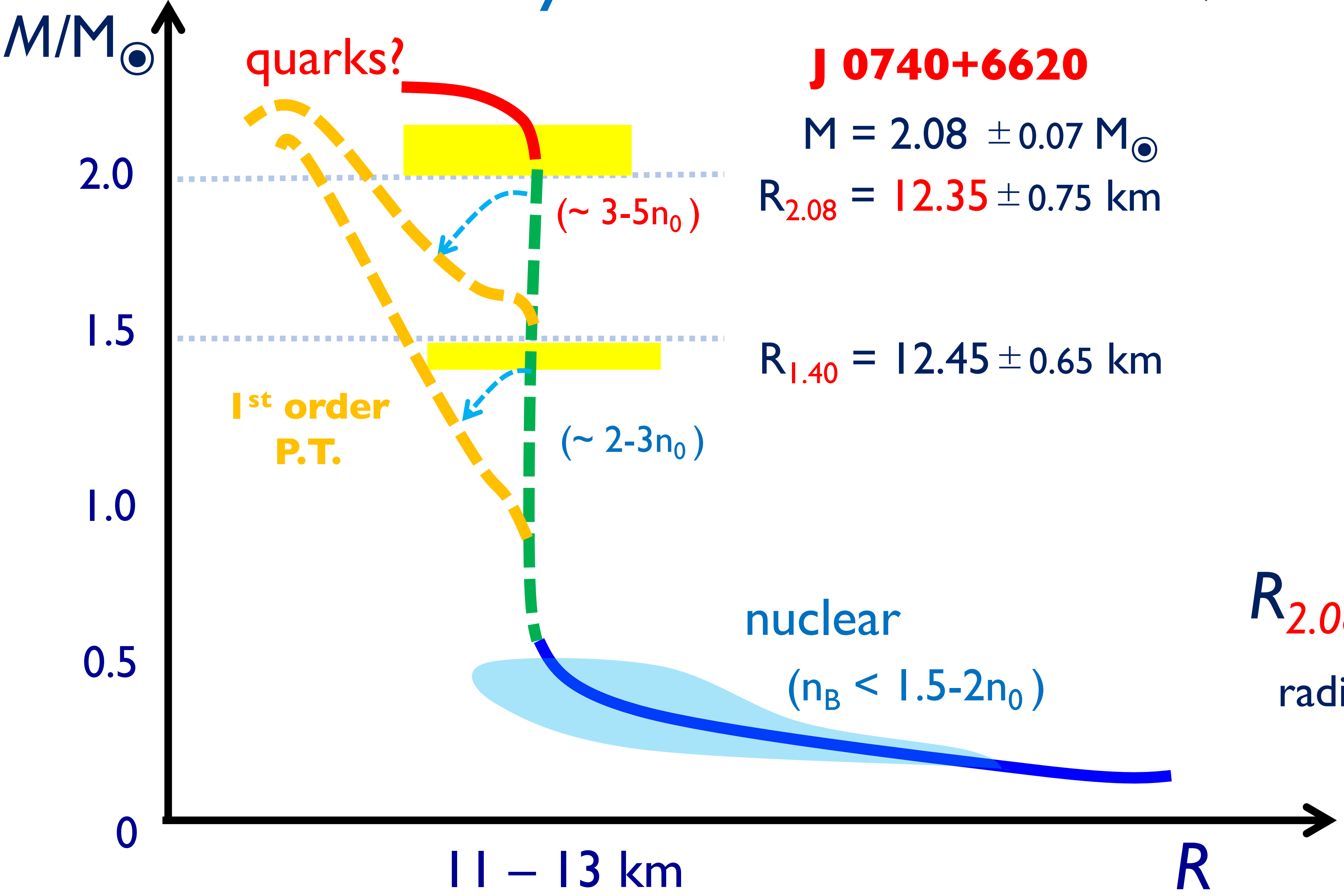
QCD pressure



Ref) Lattimer & Prakash (2001)

Density vs *M-R* curves

Ref) Lattimer & Prakash (2001)



J 0740+6620

$M = 2.08 \pm 0.07\, M_\odot$

$R_{2.08} = 12.35 \pm 0.75$ km

$R_{1.40} = 12.45 \pm 0.65$ km

based on

NICER

J0740+6620 data

+ J0030+0451 data

+ GW170817

+ *nuclear constraints*

[Miller+ '21]

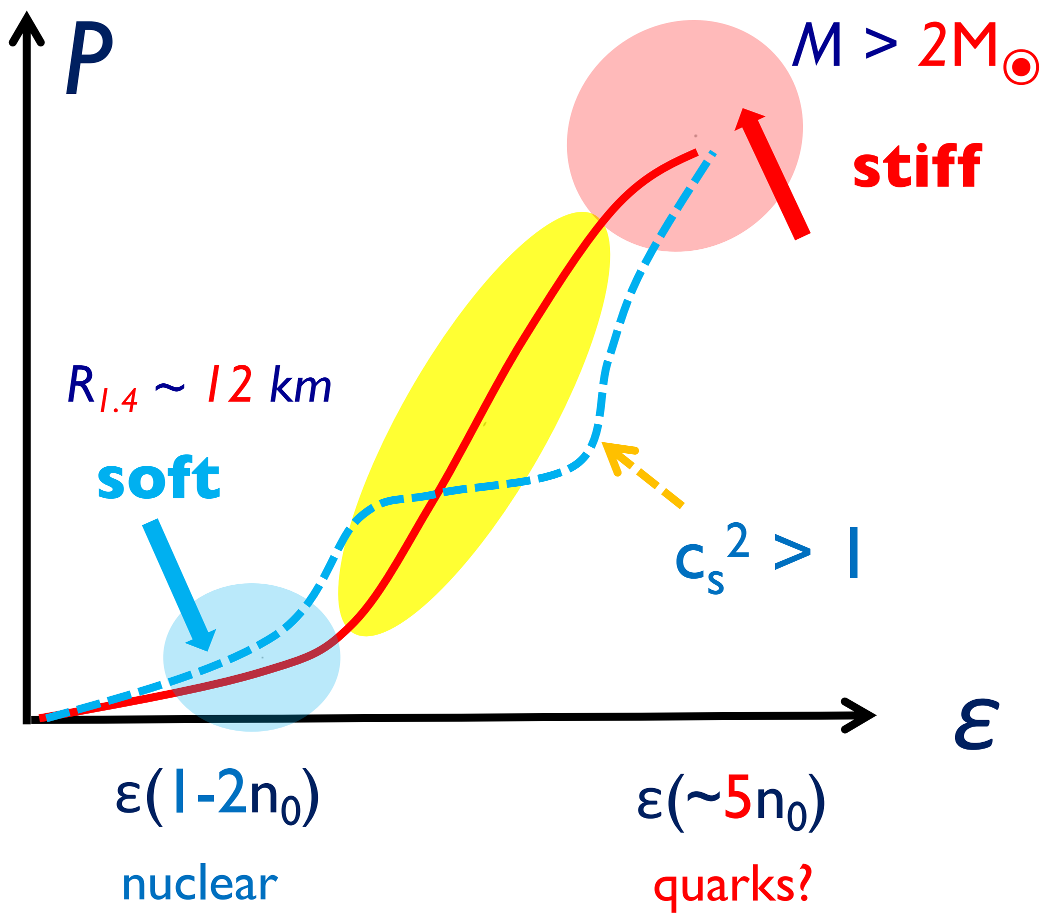
$R_{2.08} \sim R_{1.40}$ (!)

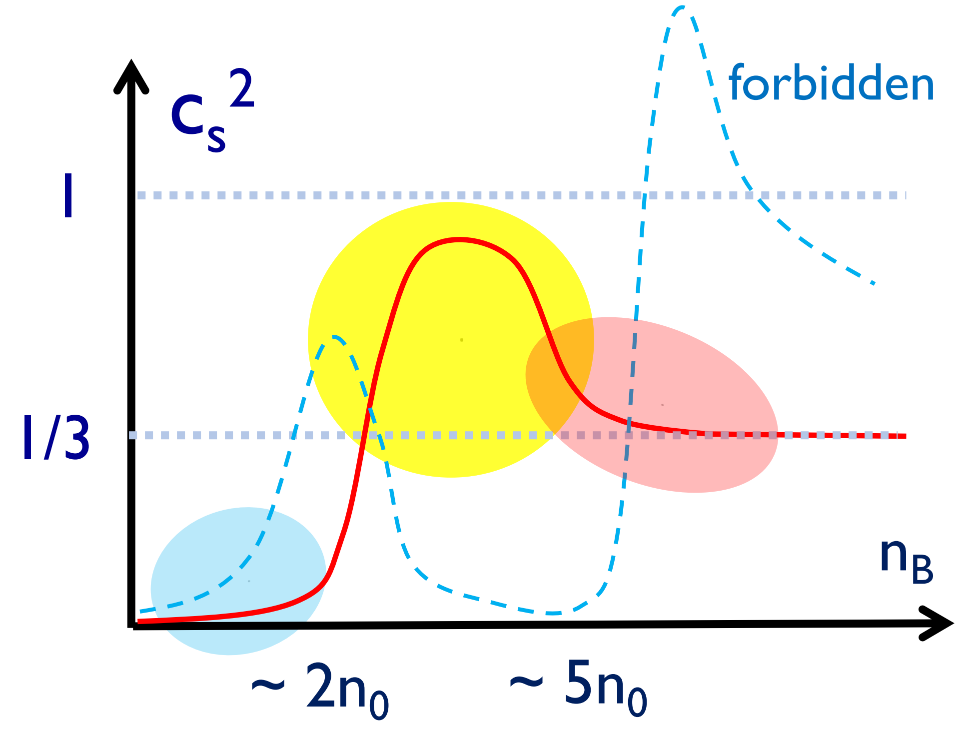
radical softening unlikely

for 2-5 n_0

Soft to stiff is challenging

speed of sound: $c_s^2 = dP/d\varepsilon < 1$ (*causality*)





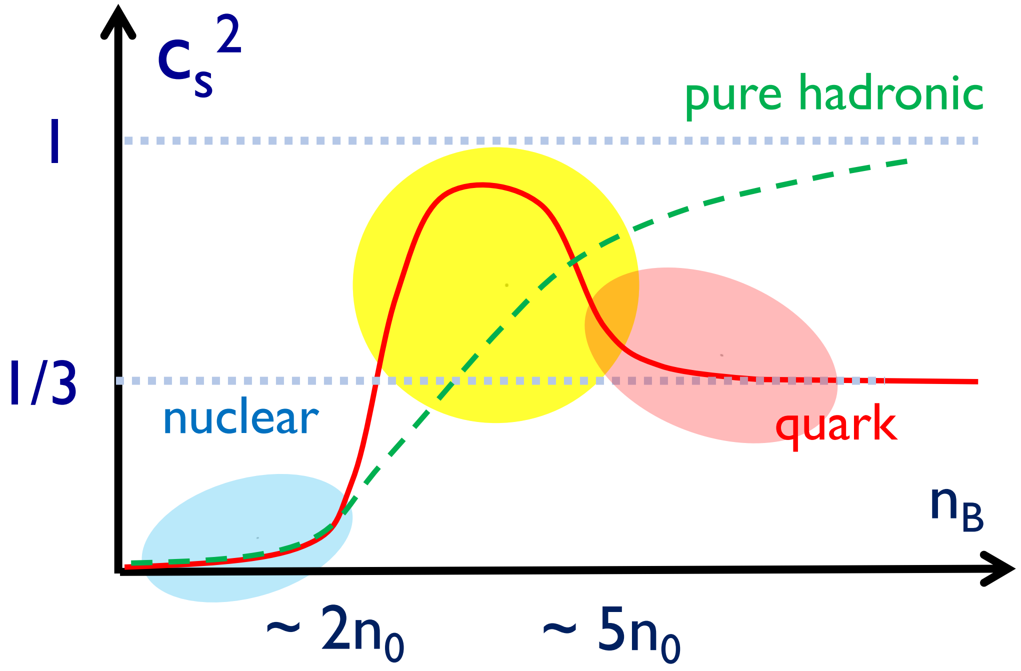
the simplest: *quark-hadron continuity*

→ we take it as our **baseline**

Goals in this talk

→ *Direct* descriptions of the peak in c_s from *microphysics*

follow up of [McLerran-Reddy (MR), PRL '19,...]



the statements include:

- *robust in a quark-hadron continuity model*
- *appears before baryon cores overlap*
- *nuclear repulsive forces are NOT major driving forces*

[nuclear int. will be ignored at LO]

Strategy : to clarify the physics

- first stick to *quark d.o.f only* ;   differing from [McLerran-Reddy, PRL '19,...]
  - remove worries about *double counting*
  - remove additional modeling for switching of d.o.f. (*baryon* → *quark*)
  - remove the issues of *the zero point energy*   [e.g. *baryonic Dirac sea* v.s. *quark Dirac sea*]

→ **EoS** in baryonic & quark matter have the **same normalization**

- after establishing the picture, we will replace *a part of quarks* with *baryons*

consistency check with [McLerran-Reddy, PRL '19,...]

contents

1, Quarks in a baryon

2, Quarks in baryonic matter

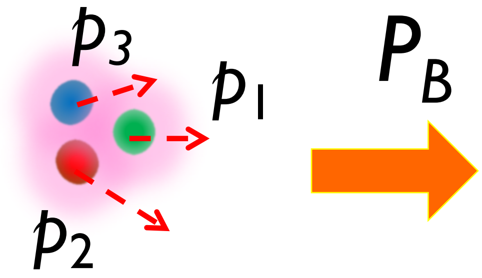
3, Quark matter formation

4, Baryons in quark matter

interactions (except confinement) neglected

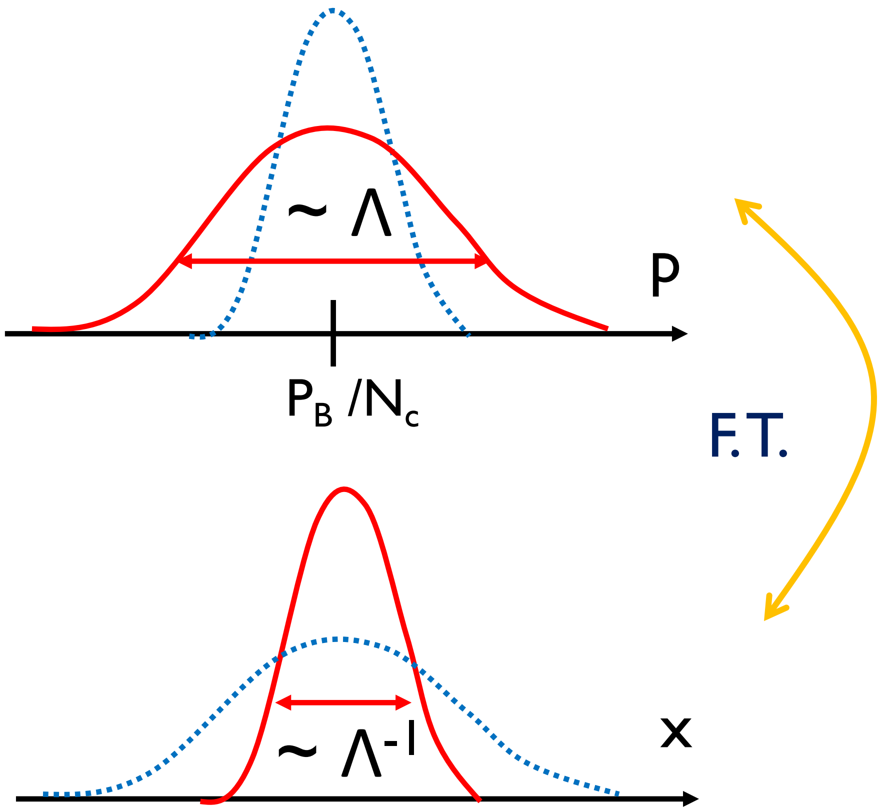
5, Quark interactions & stiff EoS

Quarks in a baryon



probability density:

$$Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B) = \mathcal{N} e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2} \qquad \mathcal{N} = \frac{8\pi^{3/2}}{\Lambda^3}$$



quarks are **localized** in both *p*- & *x*- space

mean

$$\langle \boldsymbol{P}_B \rangle = N_c \int_{\boldsymbol{p}} \boldsymbol{p} Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$

variance

$$\left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$$

energetic !

Single quark and baryon energies

$$\langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B} = \mathcal{N}\int_{\boldsymbol{p}} E_q(\boldsymbol{p}) \mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2} = \mathcal{N}\int_{\boldsymbol{p}'} E_q\left(\boldsymbol{p}' + \frac{\boldsymbol{P}_B}{N_c}\right)\mathrm{e}^{-\frac{1}{\Lambda^2}\boldsymbol{p}'^2}$$

(M_q ~300 MeV)

~ $M_q + \Lambda$ ~ $1/E_q$

$$\simeq \langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B=0} + \frac{1}{6}\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i}\right\rangle_{\boldsymbol{P}_B=0}\left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

x Nc x Nc

baryon mass baryon kin. energy

~ $N_c (M_q + \Lambda)$ ~ $P_B^2 / (N_c E_q)$

Quarks in a baryonic matter

Occupation probability of quark states

occupation **probability** of **quark** state with p

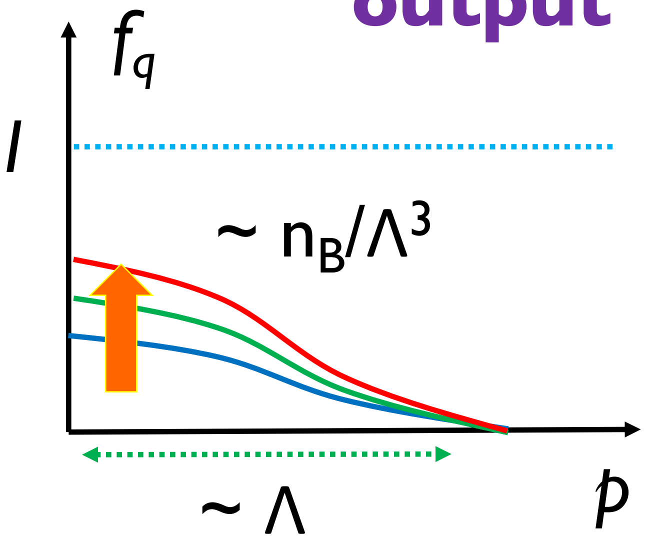
occupation **probability** of **baryon** state with P_B

quark mom. **pro. density** in a baryon

$$f_q(\underline{p}; n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(\underline{P_B}; n_B) Q_{\rm in}(\underline{\boldsymbol{p}}, \underline{\boldsymbol{P}_B})$$

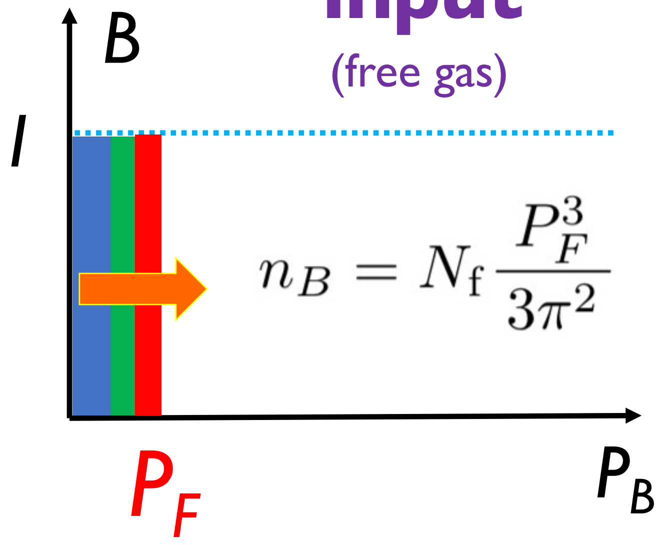
e.g.) in **dilute** baryonic matter

output



input

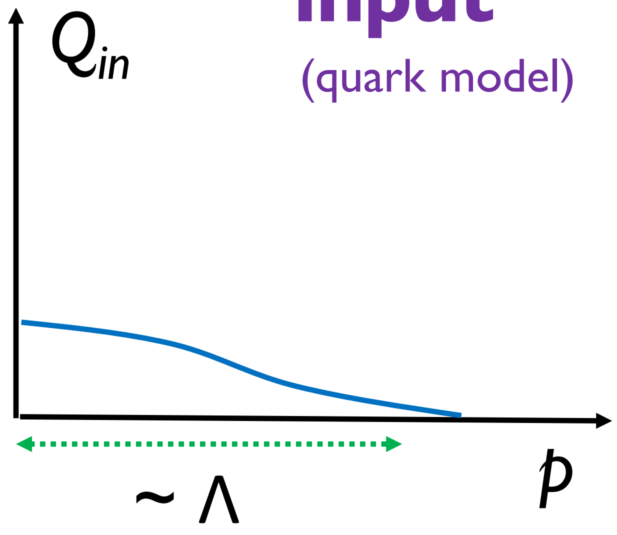
(free gas)



⊗

input

(quark model)



Quarks in ideal baryon gas

$$\tilde{p} = p/\Lambda\,, \quad \tilde{P}_B = P_B/N_c\Lambda$$

$$f_q(p;n_B) = \mathcal{N}\frac{(N_c\Lambda)^3}{(2\pi)^2}\int_0^{\tilde{P}_F}\tilde{P}_B^2\,\mathrm{d}\tilde{P}_B\;\mathrm{e}^{-\left(\tilde{p}^2+\tilde{P}_B^2\right)}\int_{-1}^{1}\mathrm{d}\cos\theta\;\mathrm{e}^{2\tilde{p}\tilde{P}_B\cos\theta}$$

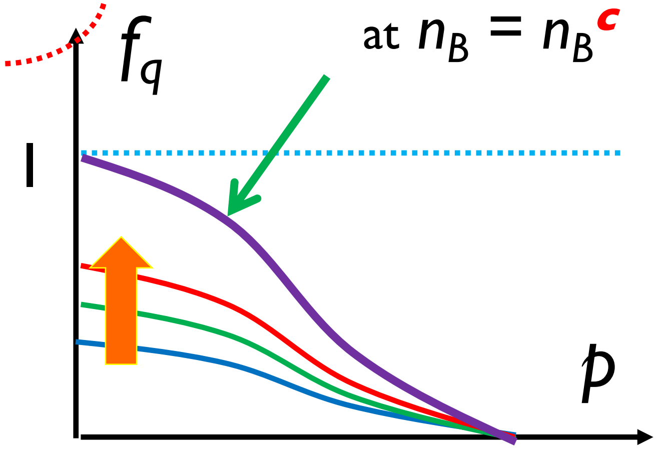
$1/N_c$ expansion

$$f_q \simeq \mathcal{N}\frac{P_F^3}{6\pi^2}\,\mathrm{e}^{-\tilde{p}^2}\left(1+\frac{-3+2\tilde{p}^2}{5}\tilde{P}_F^2+\cdots\right)$$

$1/N_c^2$ expansion

large N_c

$$f_q(p;n_B)\big|_{N_c\to\infty} = \mathcal{N}\frac{P_F^3}{6\pi^2}\,\mathrm{e}^{-\tilde{p}^2} = \frac{n_B}{n_B^c}\,\mathrm{e}^{-\tilde{p}^2}$$



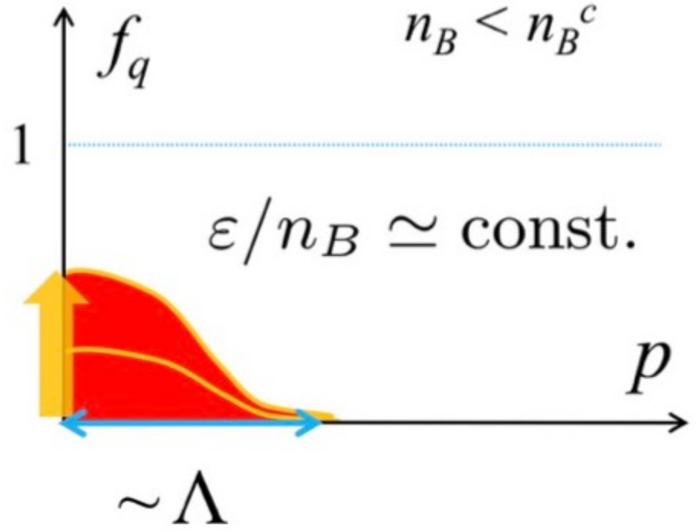
- the **shape** of f_q does not change
- the **height** of f_q grows linearly in n_B

Quarks in ideal baryon gas: EoS

$$f_q(p;n_B)\big|_{N_c\to\infty} = \mathcal{N}\frac{P_F^3}{6\pi^2}\,\mathrm{e}^{-\tilde{p}^2} = \frac{n_B}{n_B^c}\,\mathrm{e}^{-\tilde{p}^2}$$

energy density

$$\varepsilon(n_B) = 2N_c N_f \int_{\boldsymbol{p}} E_q(p) f_q(p;n_B)$$



large N_c $\quad$ n_B -indep.

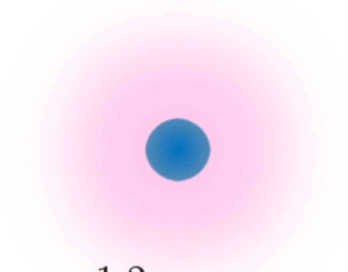
$$\varepsilon^{N_c\to\infty} = n_B \underline{N_c \mathcal{N}\int_{\boldsymbol{p}} E_q(p)\,\mathrm{e}^{-\tilde{p}^2}} = n_B \underline{M_B^{N_c\to\infty}}$$

pressure

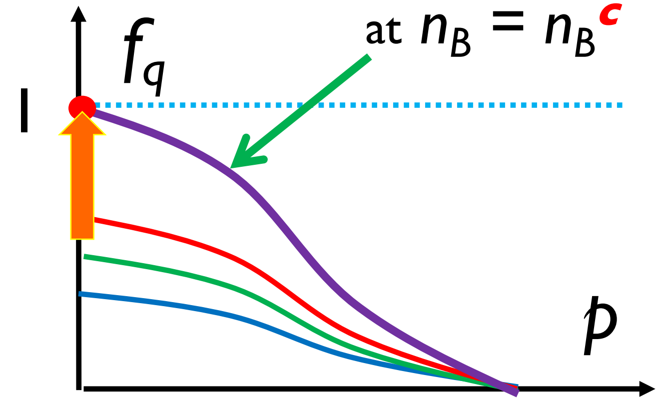
$$\mathcal{P} = n_B^2\,\partial(\underline{\varepsilon/n_B})/\partial n_B \sim 0 + O(1/N_c)$$

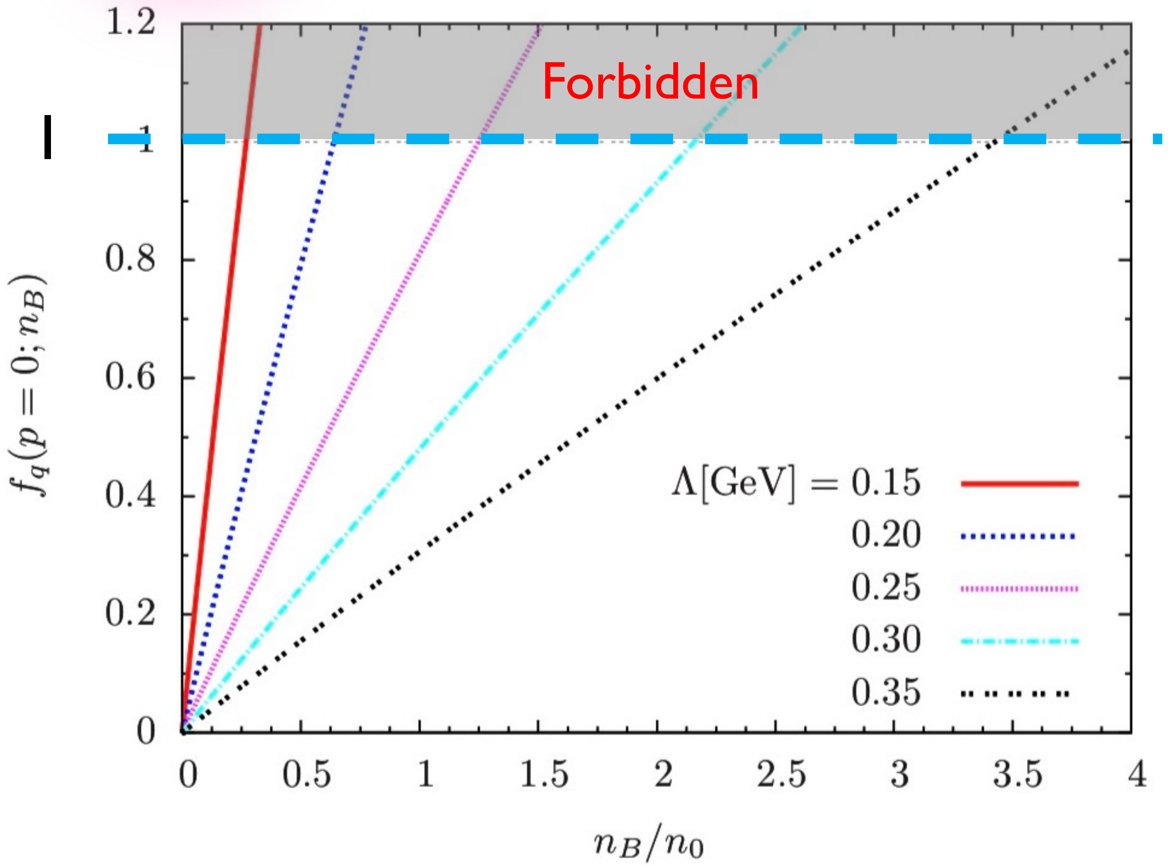
~ const. $\quad$ very soft

Saturation of quark states



baryon size dep. of n_B^c



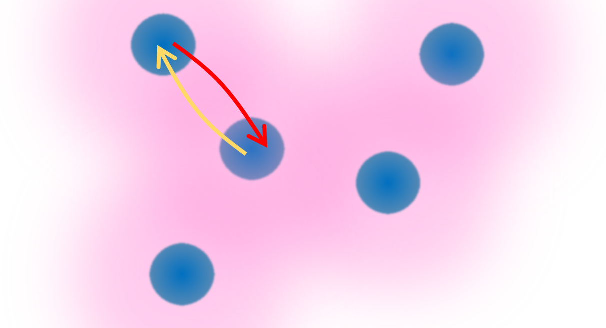


$\Lambda \sim$ 0.15-0.35 GeV $\rightarrow n_B^c \sim$ 0.2-3.5 n_0

(may happen before baryon cores overlap)

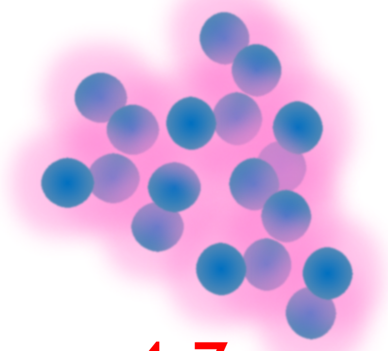
cf) [Fukushima-TK-Weise, '20]

Soft Deconf.



$\sim$ 0.2-3.5 n_0

Hard Deconf.



$\sim$ 4-7 n_0

Quantum numbers ?

quark quantum numbers; N_c, N_f, 2-spins (for a given spatial w.f.)

how many baryon species are needed to saturate quark states?

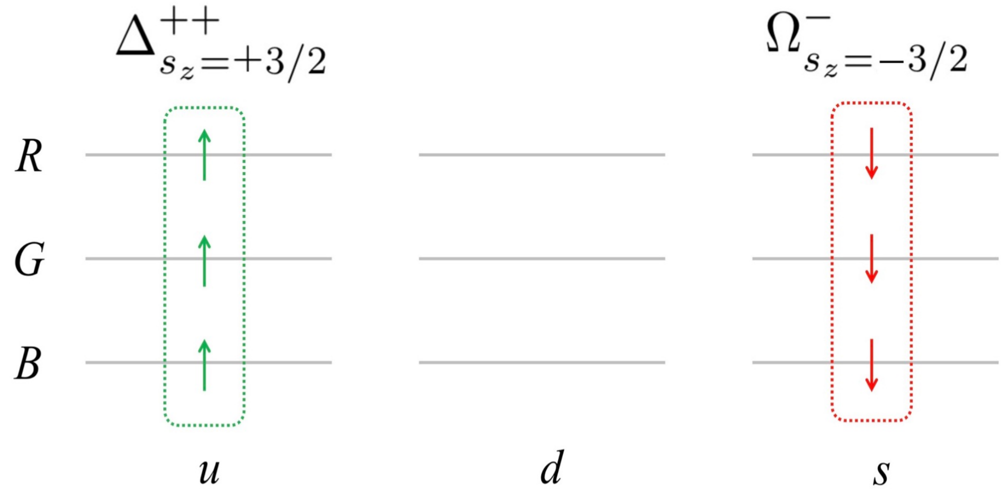
→ we need only $\mathbf{2N_f}$ = **6** species for N_f = 3

(full members of singlet, octet, decuplet are NOT necessary)

convenient **color-flavor-spin** bases

[neglect N-Δ splitting etc. for simplicity]

$$\Delta^{++}_{s_z=\pm 3/2} = [u_R{\uparrow}\, u_G{\uparrow}\, u_B{\uparrow}]\,, \quad [u_R{\downarrow}\, u_G{\downarrow}\, u_B{\downarrow}]\,,$$
$$\Delta^{-}_{s_z=\pm 3/2} = [d_R{\uparrow}\, d_G{\uparrow}\, d_B{\uparrow}]\,, \quad [d_R{\downarrow}\, d_G{\downarrow}\, d_B{\downarrow}]\,,$$
$$\Omega^{-}_{s_z=\pm 3/2} = [s_R{\uparrow}\, s_G{\uparrow}\, s_B{\uparrow}]\,, \quad [s_R{\downarrow}\, s_G{\downarrow}\, s_B{\downarrow}]\,,$$



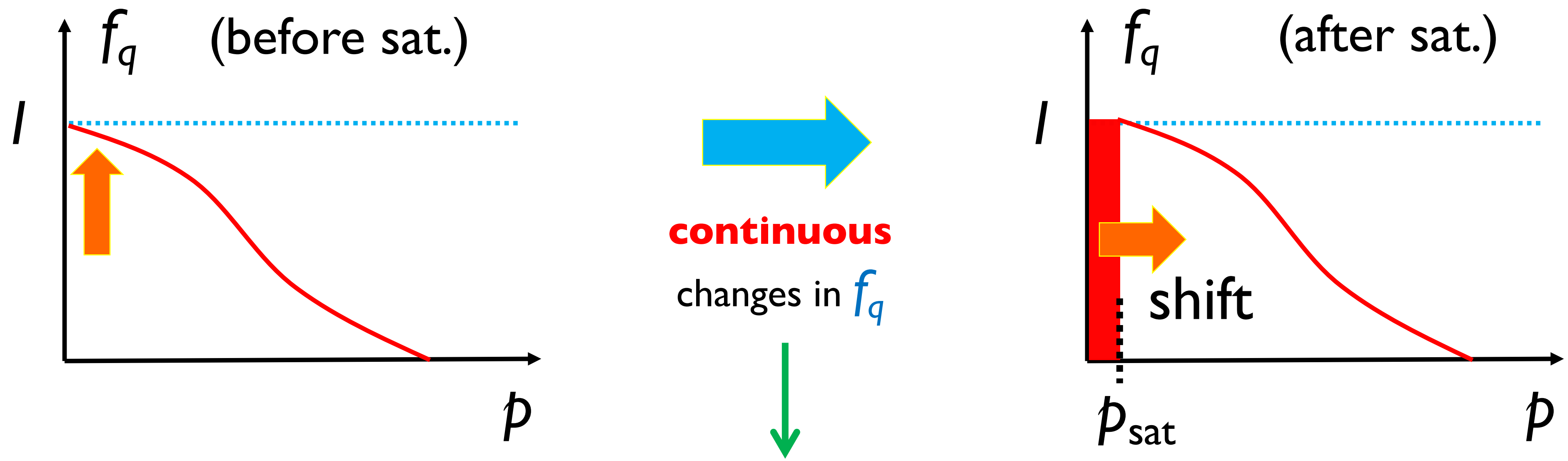
Quark matter formation

Quark matter formation: $f_q\,(p\,;n_B)$

a **model** after the saturation:

same shape

$$f_q^{\text{after}} = \theta(p_{\text{sat}} - p) + \theta(p - p_{\text{sat}})\,\underline{f_q(p - p_{\text{sat}}; n_B^c)}$$

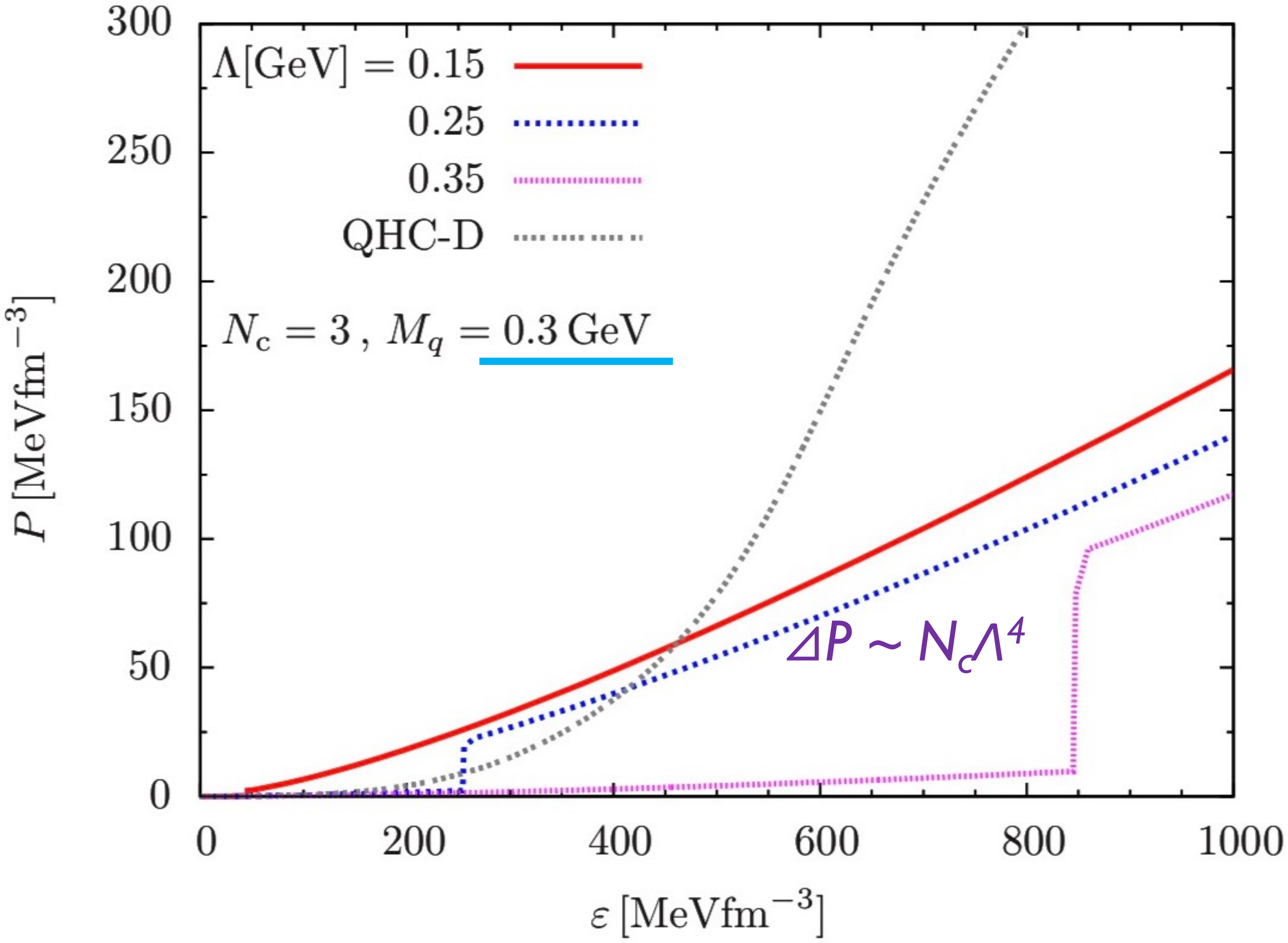


ε, n_B are **continuous** before and after the saturation.

any nontrivial consequences ?

jumps in μ_B, P

(Nc = 3, M_q = 0.3 GeV)



a model of f_q looks innocent,

but its consequence is unphysical:

ε, n_B are **continuous,**

jumps in μ_B & P

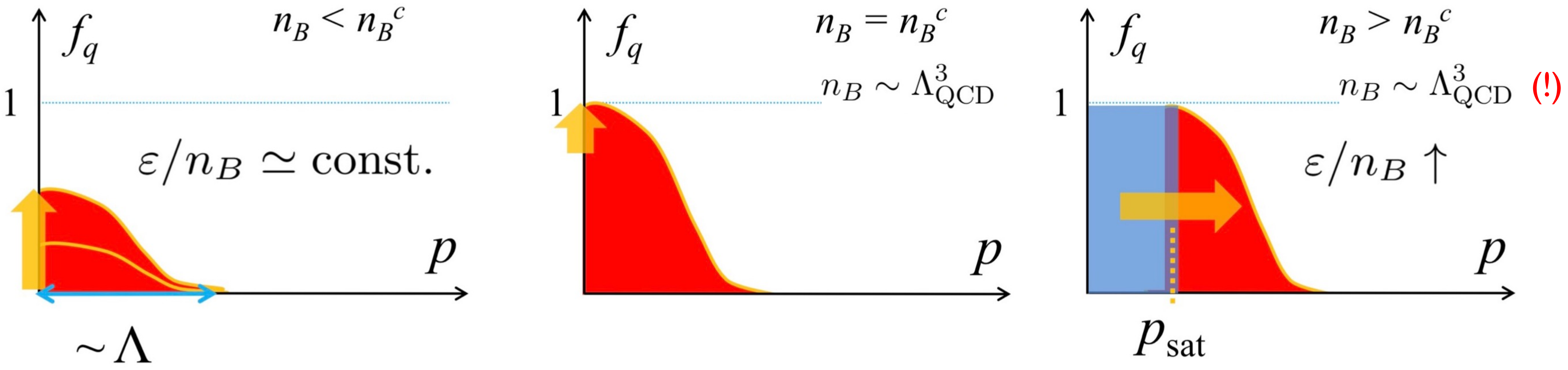
($\Delta\mu_B \sim N_c\Lambda$)

opposite to usual 1st order P.T.

Quark matter formation: *EoS*

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

energy per particle



$\varepsilon \sim n_B \times N_{\rm c}\Lambda_{\rm QCD}$

$\varepsilon \sim N_{\rm c}\Lambda_{\rm QCD}^4$

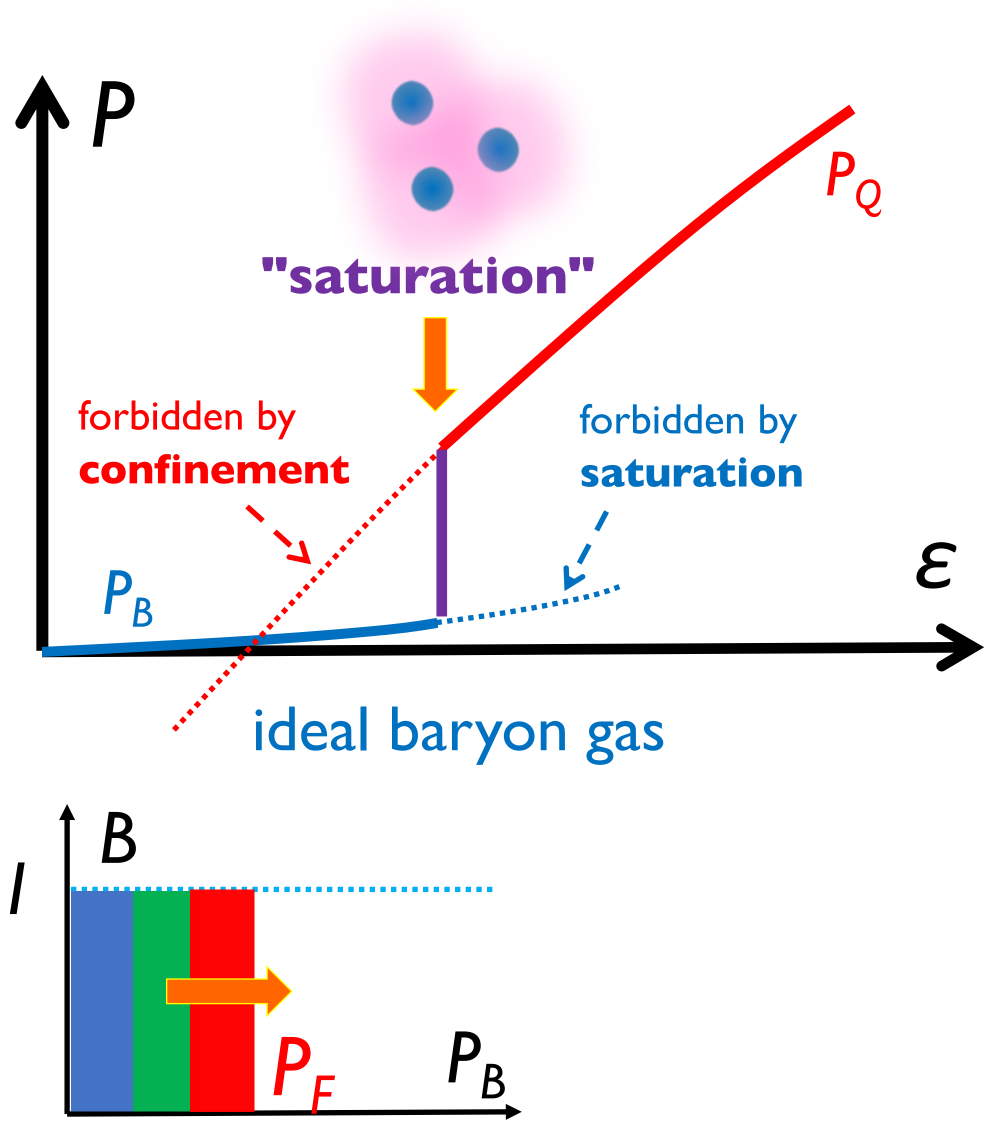
$\mathcal{P} \sim n_B^{5/3}/N_{\rm c}\Lambda_{\rm QCD}$

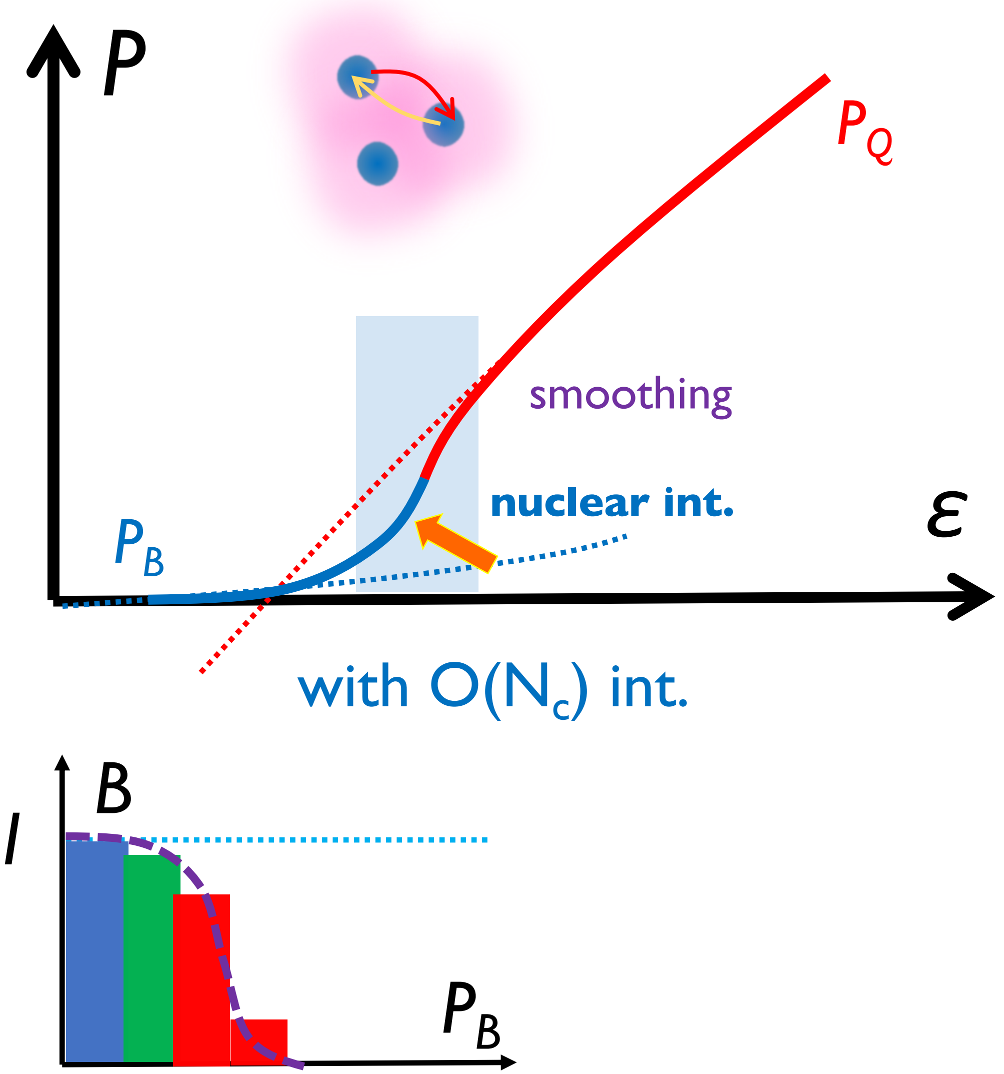
$\mathcal{P} \sim \Lambda_{\rm QCD}^4/N_{\rm c}$

$\mathcal{P} \sim N_{\rm c}\Lambda_{\rm QCD}^4$ (!)

More realistic picture

[P_B & P_Q from a **single** model]

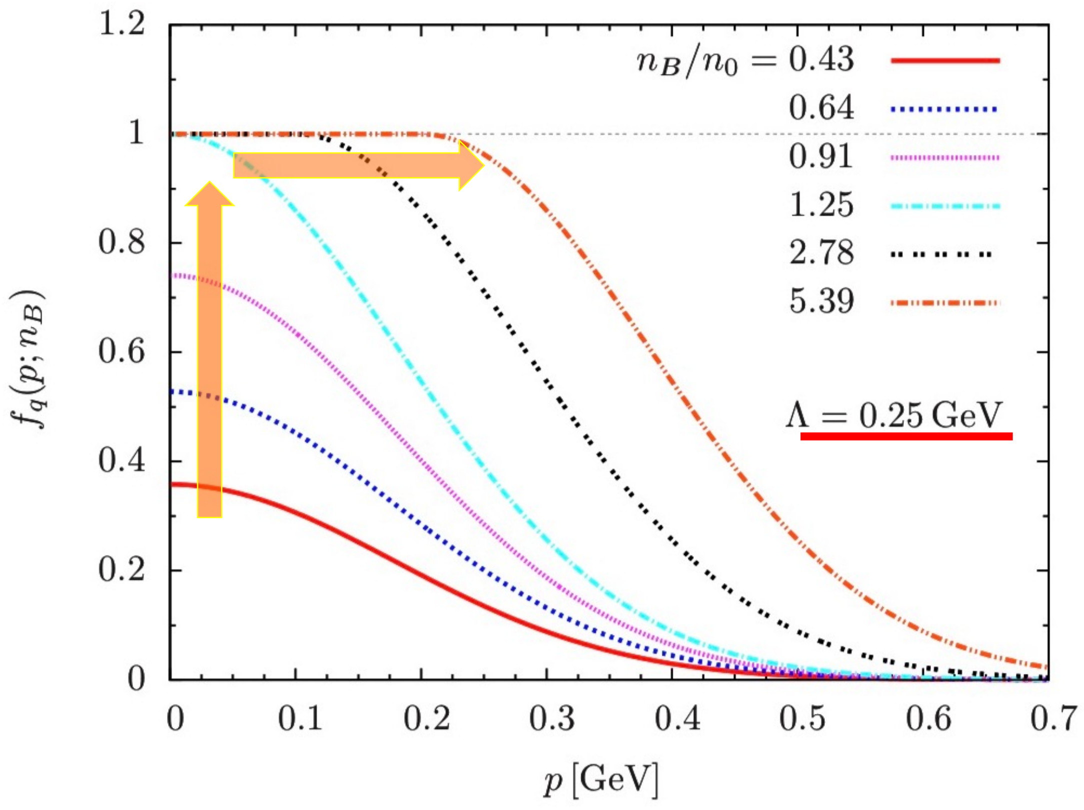




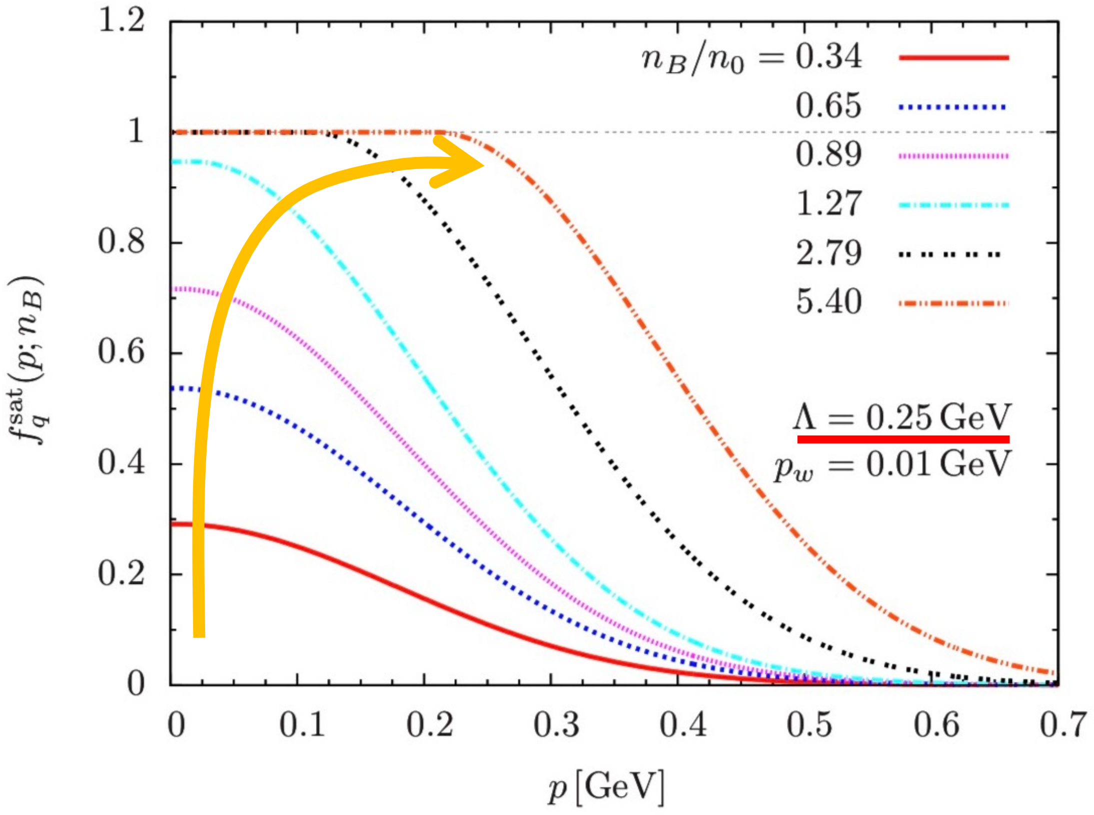
Smooth version of f_q (ad hoc smearing)

$$f_q^{\rm sat}(p;p_{\rm sat}) = \tanh(p_{\rm sat}/p_w) \times \left[\theta(p_{\rm sat}-p) + \theta(p-p_{\rm sat})\,{\rm e}^{-(\tilde{p}-\tilde{p}_{\rm sat})^2}\right]$$

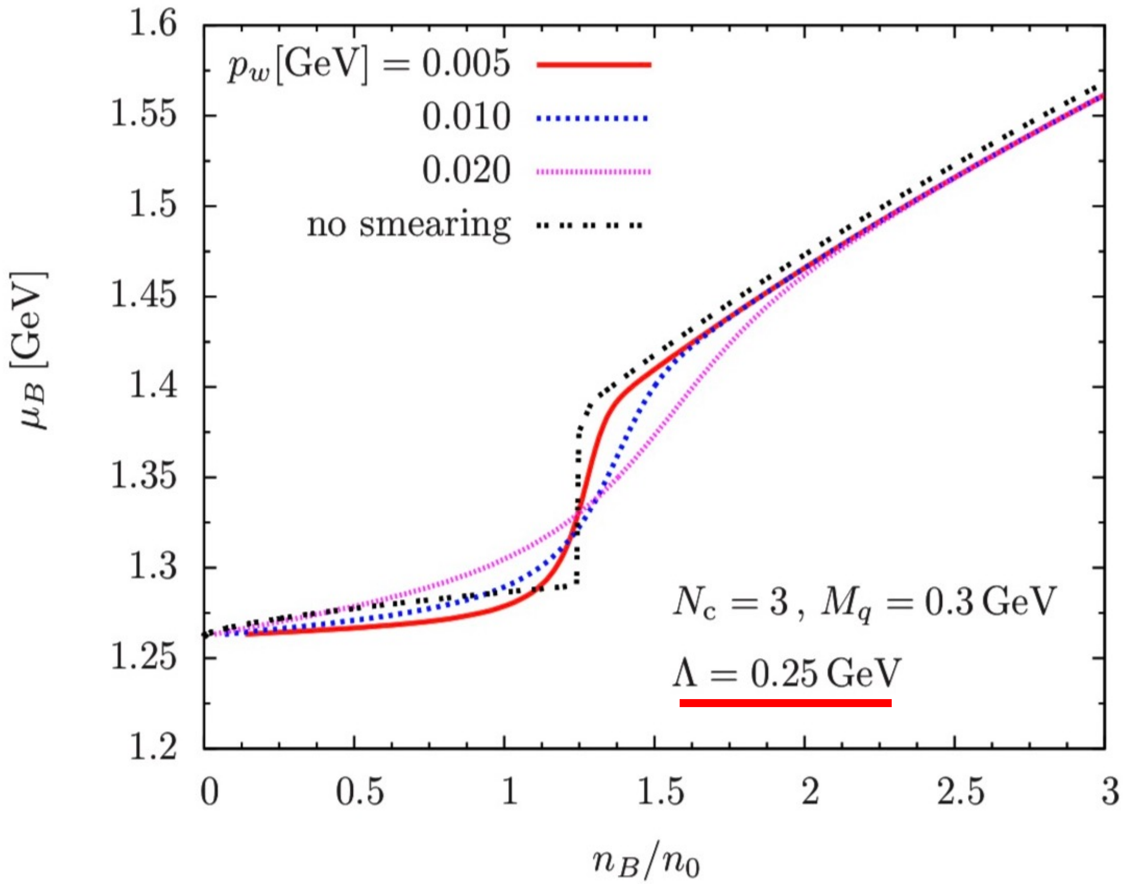
no smearing

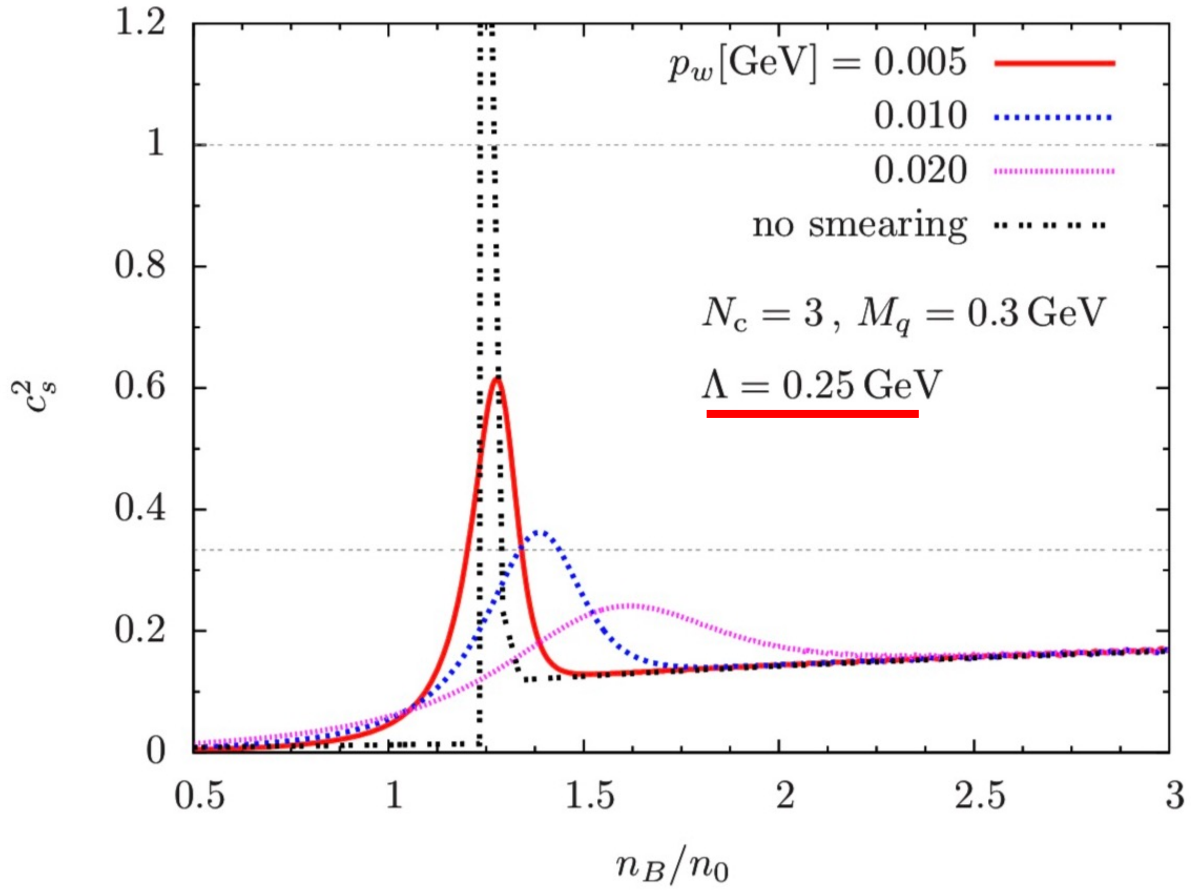


smooth version (p_w ~ 10 MeV)



Smooth version: EoS





μ_B physical now

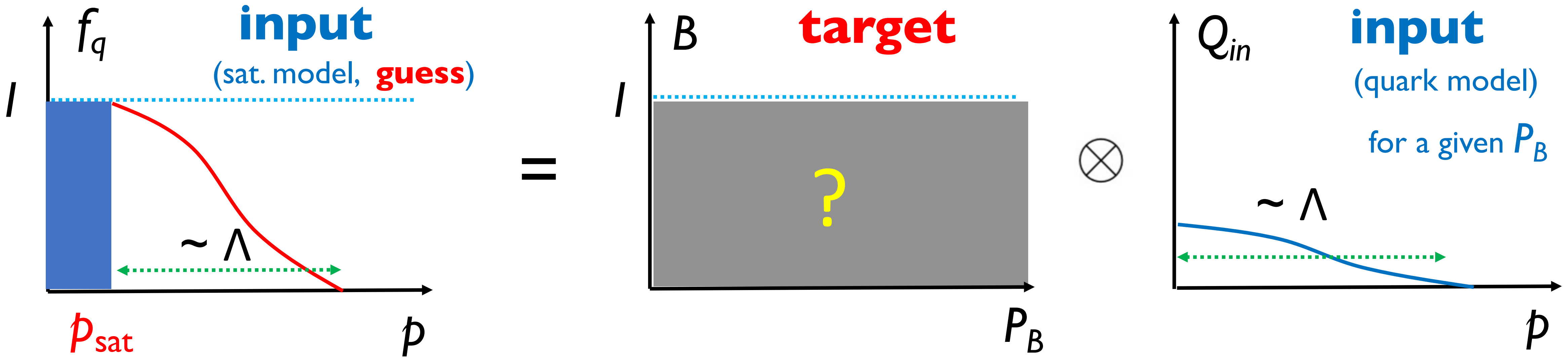
peak in *c_s* :

- *location* primarily determined by *Λ*
- *width* primarily determined by *p_w* ...

Baryons in quark matter

Inversion problem: from f_q to B

$$f_q(p;n_B) = \int_{\boldsymbol{P}_B} \underline{\mathcal{B}(P_B;n_B)} Q_{\rm in}(\boldsymbol{p},\boldsymbol{P}_B)$$



How does *baryon occ. probability* look *after* the saturation ?

Inversion problem: motivations to study *B*

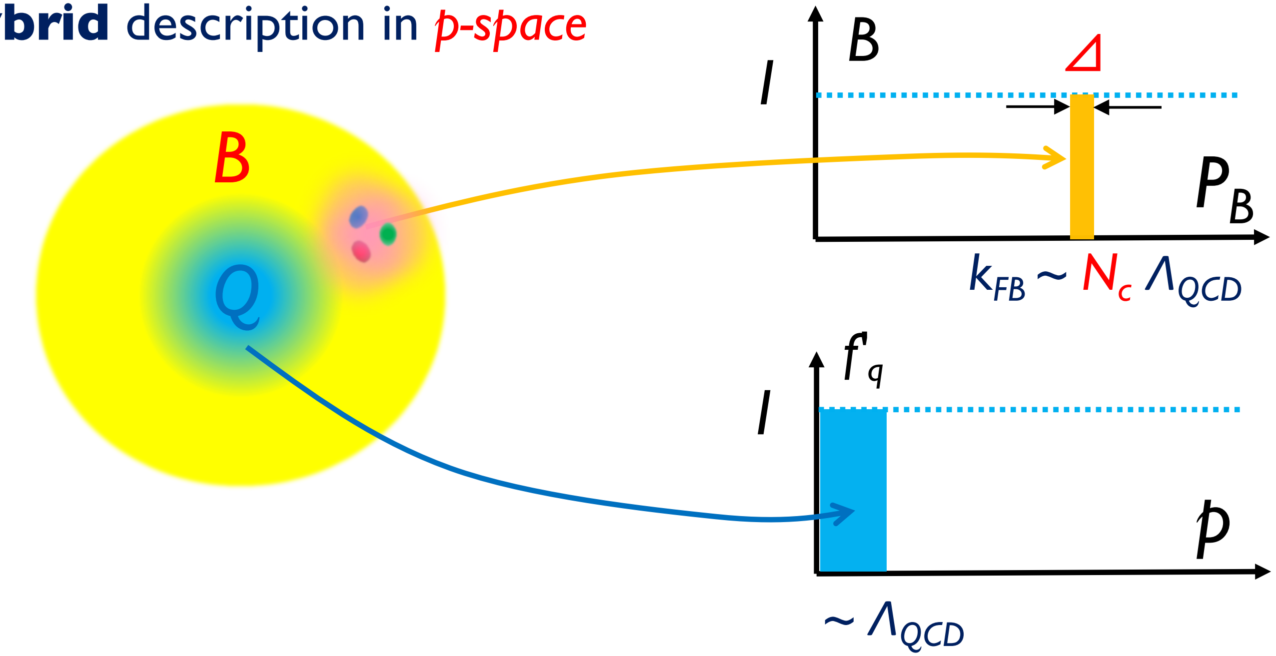
- perhaps convenient to *use the baryonic bases* for *low E* physics

$$P(\mu_B)|_{\beta\text{-eq}} \Rightarrow P(\mu_B, \mu_Q, T, \dots)$$

extensions of the *quark-hadron continuity*

- relations to the *McLerran-Reddy* (*MR*) model

hybrid description in *p-space*



important parameter

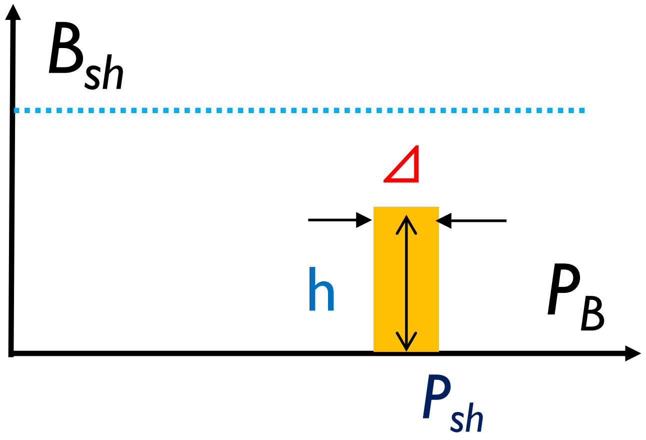
$$\Delta = \frac{\Lambda^3}{k_{\rm FB}^2} + \kappa \frac{\Lambda}{N_c^2}$$

why this form?

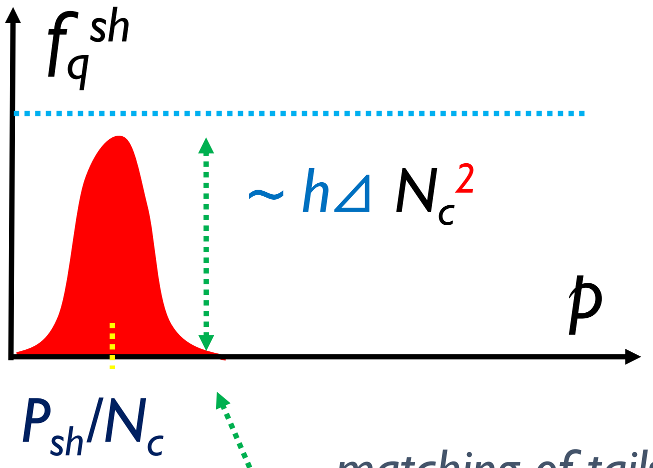
- phenomenological [McLerran-Reddy, PRL '19]
- derivation in excluded vol. model [Jeong-McLerran-Sen, '19]

A trial: *shell form*

$$\mathcal{B}^{\rm sh}(P_B;P_{\rm sh}) = h\,\theta(P_{\rm sh}-P_B)\,\theta(P_B-P_{\rm sh}-\Delta)$$



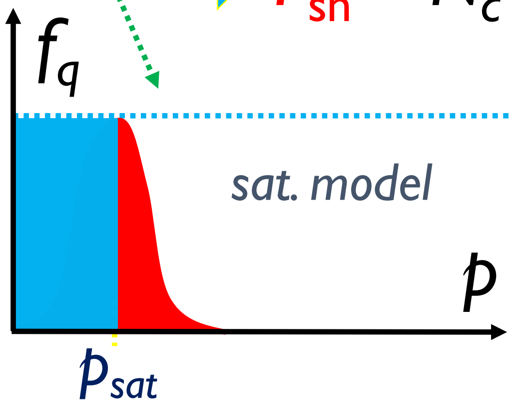
Q_{in}



$$f_q^{\rm sh}(p) \simeq h\Delta \frac{N_{\rm c}^3}{\sqrt{\pi}}\frac{\tilde P_{\rm sh}}{\tilde p}\, {\rm e}^{-\tilde p^2-\tilde P_{\rm sh}^2}\left({\rm e}^{2\tilde p\tilde P_{\rm sh}} - {\rm e}^{-2\tilde p \tilde P_{\rm sh}}\right)$$

matching of tails

$\Rightarrow P_{sh} \sim N_c\, p_{sat}$



$$\Rightarrow \quad f_q^{\rm sh}(p) \sim h\Delta N_{\rm c}^2\, {\rm e}^{-(\tilde p-\tilde P_{\rm sh})^2}$$

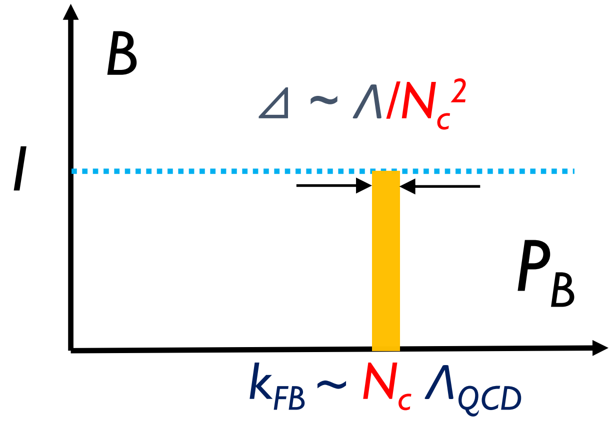
$P_{sh} \sim N_c \Lambda$

Constraints from f_q (*for $P_{sh} \sim N_c \Lambda$*)

$$f_q^{\rm sh}(p) \sim h\Delta N_{\rm c}^2\, {\rm e}^{-(\tilde{p}-\tilde{P}_{\rm sh})^2}$$

constraint : $f_q^{sh} < 1$ ➡ $h\Delta < \Lambda/N_c^2$

a possible scaling form: $$[h\Delta](P_{\rm sh}) \sim c_0 \Lambda \left(\frac{\Lambda^2}{P_{\rm sh}^2} + \frac{c_1}{N_{\rm c}} \frac{\Lambda}{P_{\rm sh}} + \frac{c_2}{N_{\rm c}^2} \right)$$



MR-model (thin shell model)

$h = 1$ & $\Delta = \frac{\Lambda^3}{k_{\rm FB}^2} + \kappa \frac{\Lambda}{N_c^2}$ $(c_1 = 0)$

MR-model: *EoS*

$P_{sh} \sim N_c \Lambda$ baryon relativistic

but $n_B^{(shell)} \simeq \frac{h}{\pi^2}\left(P_{\rm sh}^3 - (P_{\rm sh}-\Delta)^3\right) \sim h\Delta P_{\rm sh}^2$ $n_B^{(bulk)} \sim \Lambda^3$

$$\simeq c_0\Lambda^3 + c_1\Lambda^2 \frac{P_{\rm sh}}{N_c} + c_2\Lambda\left(\frac{P_{\rm sh}}{N_c}\right)^2$$

$n_B \sim \Lambda^3$ (!) << $(N_c\Lambda)^3$

(kin.) energy density:

$\varepsilon - m_B n_B \sim h\Delta \times [\, E(P_{sh}) - m_B \,] \times 4\pi P_{sh}^2$

consistent with quark's

$\sim \Lambda/N_c^2 \times (N_c\Lambda)^2/m_B \times (N_c\Lambda)^2 \sim N_c\Lambda^4$

relativistic pressure ~ $N_c\Lambda^4$ within $n_B \sim \Lambda^3$ $\rightarrow$ *stiff* EoS

Quark interactions

Interactions for stiff EoS : *a guide*

cf) [TK-Powell-Song-Baym, '14]

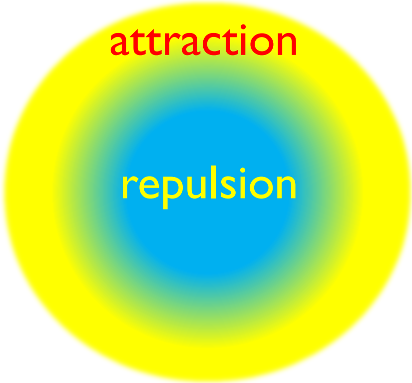
kin. energy — interactions

$$\varepsilon(n) = \underline{a} n^{4/3} + \underline{b} n^{\alpha} \xrightarrow{\mu = \frac{\partial \varepsilon}{\partial n}} \mu = \frac{4}{3} \underline{a} n^{1/3} + \alpha \underline{b} n^{\alpha-1}$$

conformal — interactions

$$\xrightarrow{P = \mu n - \varepsilon} \quad P = \frac{\varepsilon}{3} + \underline{b}\left(\alpha - \frac{4}{3}\right) n^{\alpha}$$

ideal combination



both the *sign* & *density dep.* are important

For $\boldsymbol{\alpha} > \mathbf{4/3}$: b > 0 (repulsion) → stiff EoS (e.g. bulk repulsion, ~ + $n_B^{\mathbf{2}}/\Lambda^2$)

For $\boldsymbol{\alpha} < \mathbf{4/3}$: b < 0 (attraction) → stiff EoS (e.g. surface pairings, ~ $-\Lambda^2 n_B^{\mathbf{2/3}}$)

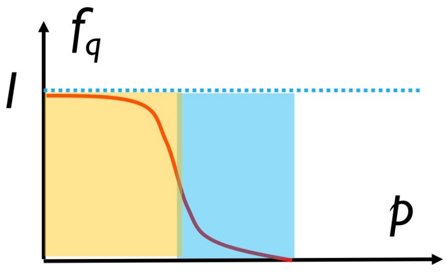
quark energy; *parameterization*

$$\mathcal{V}_{\rm CE}[f_q] = -C_E^{\mathcal{A}} \times \left(1 - \underline{f_q^\beta}\right) + C_E^{\mathcal{S}} \underline{f_q^\beta}$$

for f_q (p) << 1

$$\mathcal{V}_{\rm CE}[f_q] \simeq -C_E^{\mathcal{A}}$$

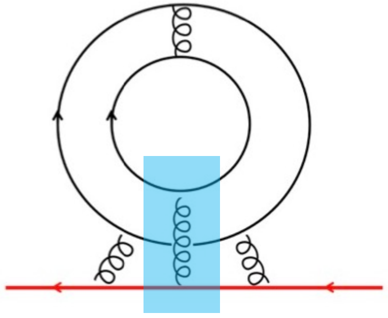
dilute in momentum space



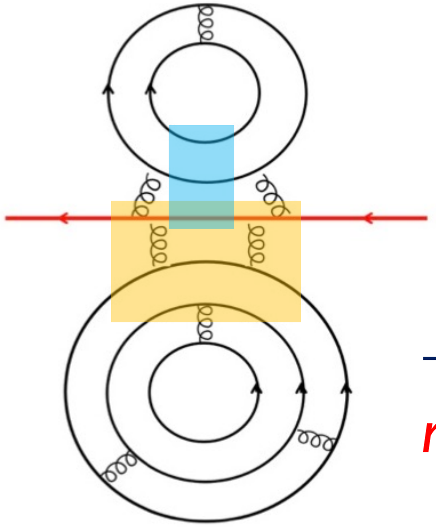
for f_q (p) ~ 1

$$\mathcal{V}_{\rm CE}[f_q] \simeq C_E^{\mathcal{S}}$$

for *saturated levels*



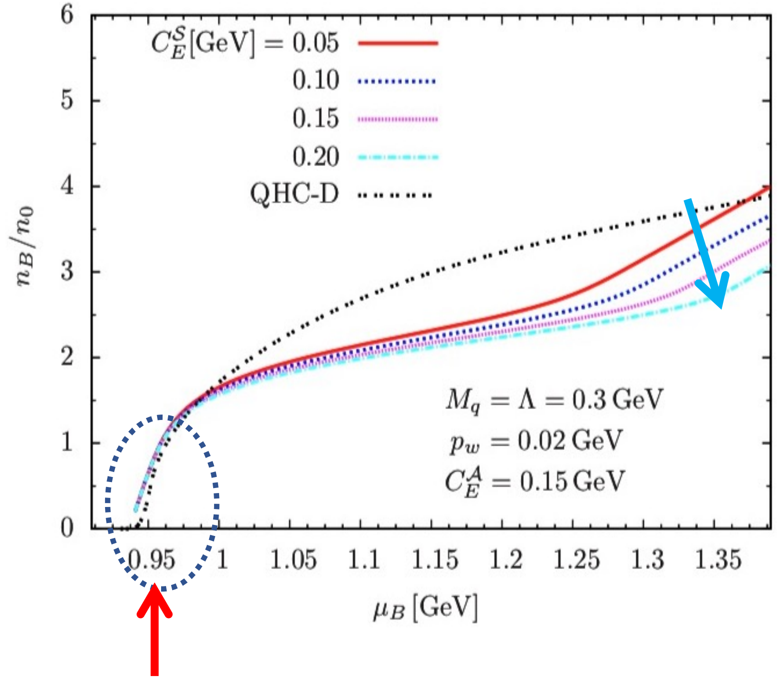
color-*antisym.* channels dominate
→ the quark feels *attractive* correlations

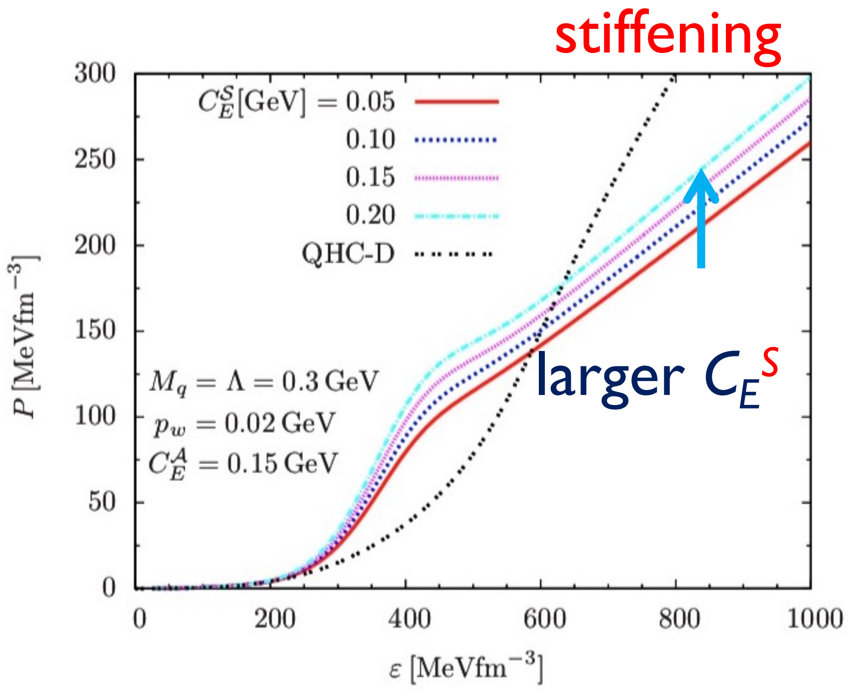


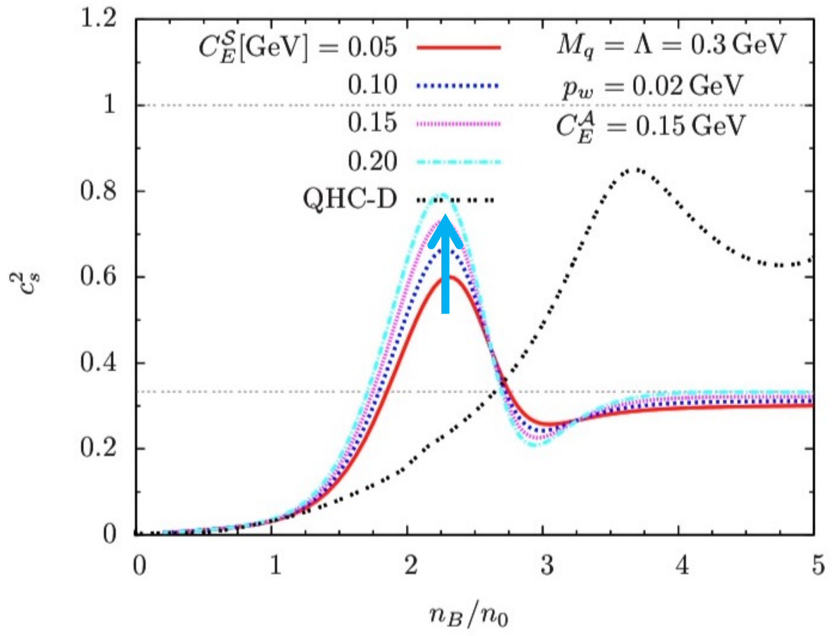
color-*sym.* channels also enter

→ the quark feels *repulsive* correlations also

EoS with interactions







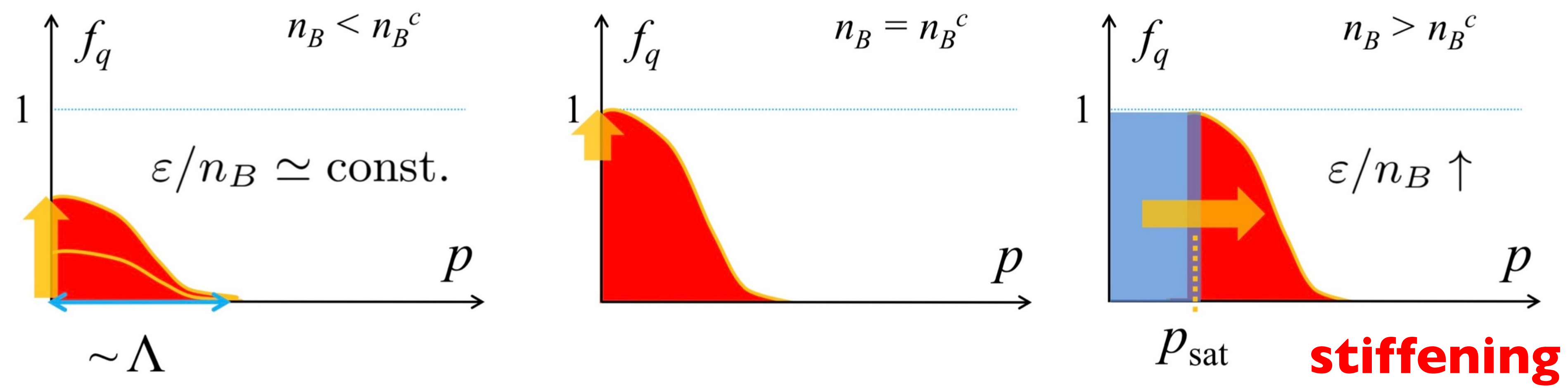
adjust C_E^A
to fit M_B = 939 MeV

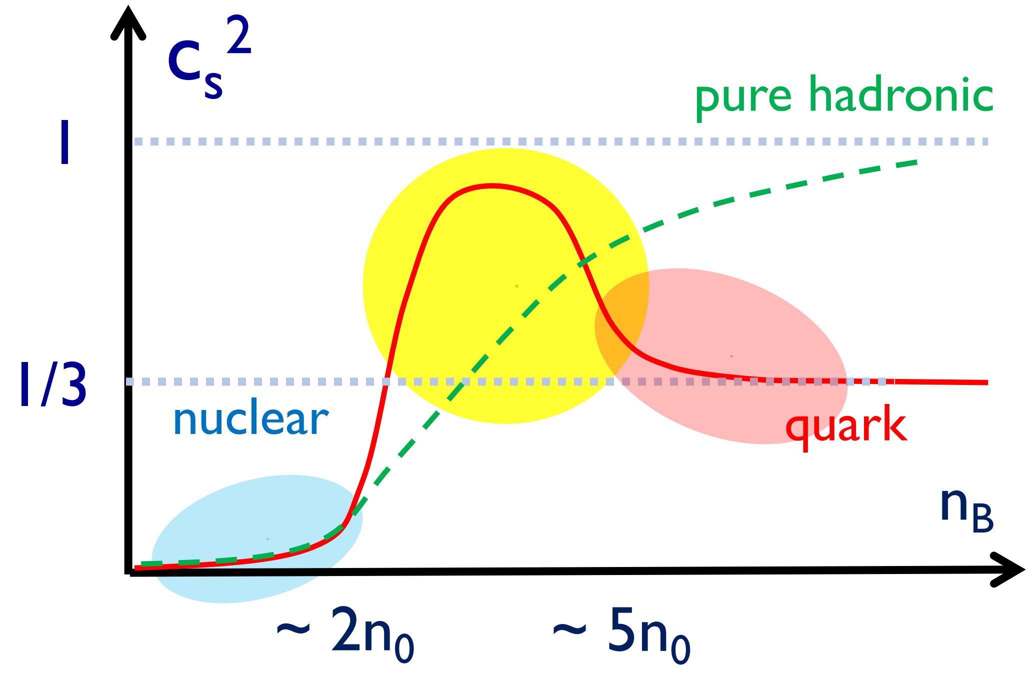
high density stiffening

peak in c_s

- existence robust
- height depends on C_E^S

Summary





- *robust in a quark-hadron continuity model*
- *appears before baryon cores overlap*
- *nuclear forces are NOT driving forces for the peak*