Measuring the speed of sound in matter created in heavy-ion collisions

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In collaboration with
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Why the speed of sound in nuclear matter is interesting

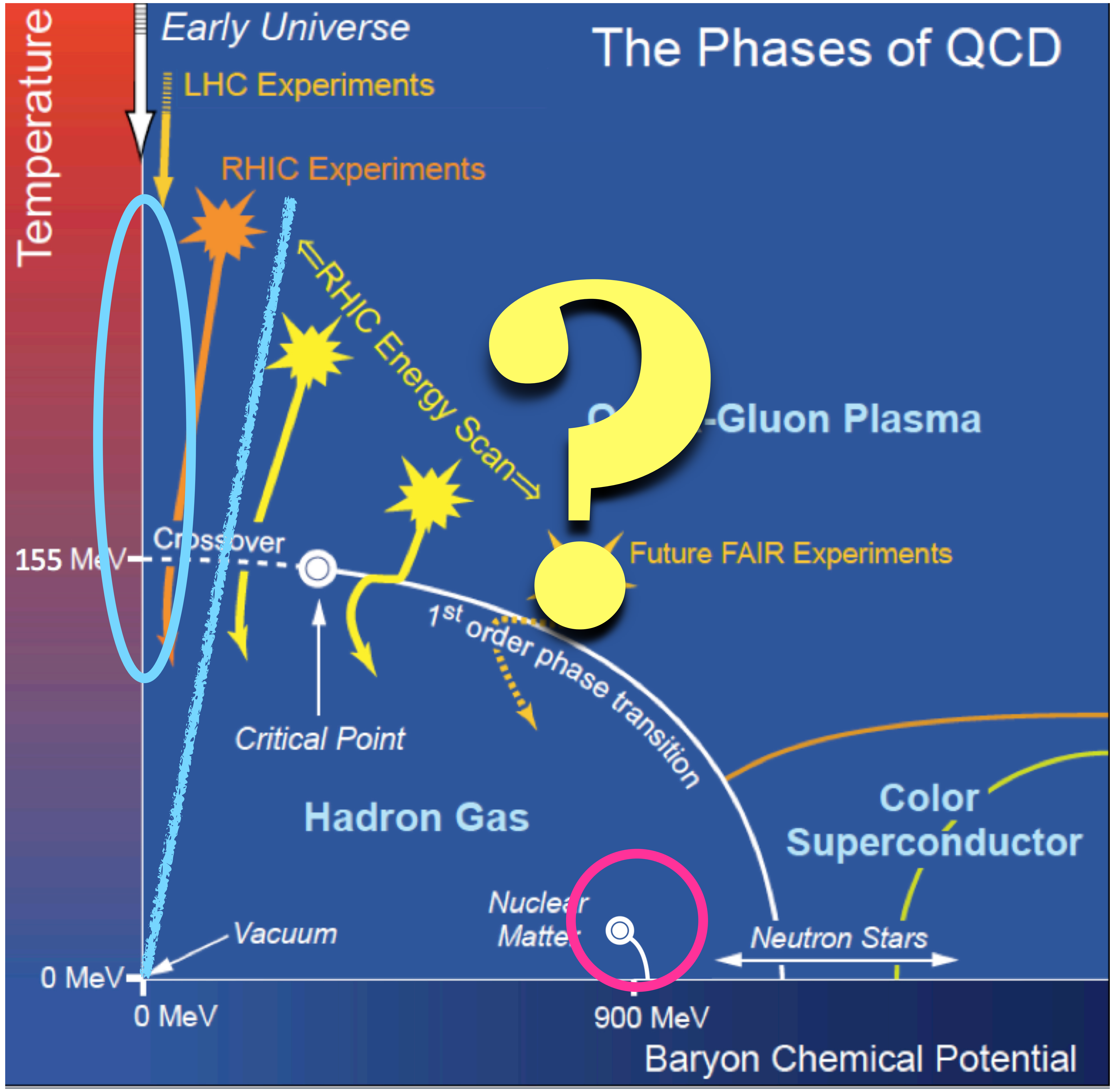
(besides being a fundamental property of nuclear matter)

The biggest take-away from high-energy heavy-ion collisions: quark-gluon plasma (QGP) can be studied in the laboratory

QGP can be produced by smashing hadrons ⇒ transition between hadrons and QGP can be studied

different collision energies:
⇒ varying the energy deposited in the collision region
⇒ varying net baryon number trapped in the collision region: probing different regions on the phase diagram

Models predict a **1st order phase transition** at large n_B ~ large μ_B



Y. Akiba *et al*, "The Hot QCD White Paper: Exploring the Phases of QCD at RHIC and the LHC", arXiv:1502.02730 (2015), Akiba:2015jwa

LQCD EOS ($\mu_B = 0$)
finite m_q = crossover
pseudocritical temperature
$T_{pc} \simeq 150$ MeV

valid only for $\frac{\mu_B}{T} \leq 2$

nuclear critical point

extrapolations of well-tested nuclear forces + experiments on nuclear fragmentation

Thermodynamics is encoded in the equation of state (EOS)

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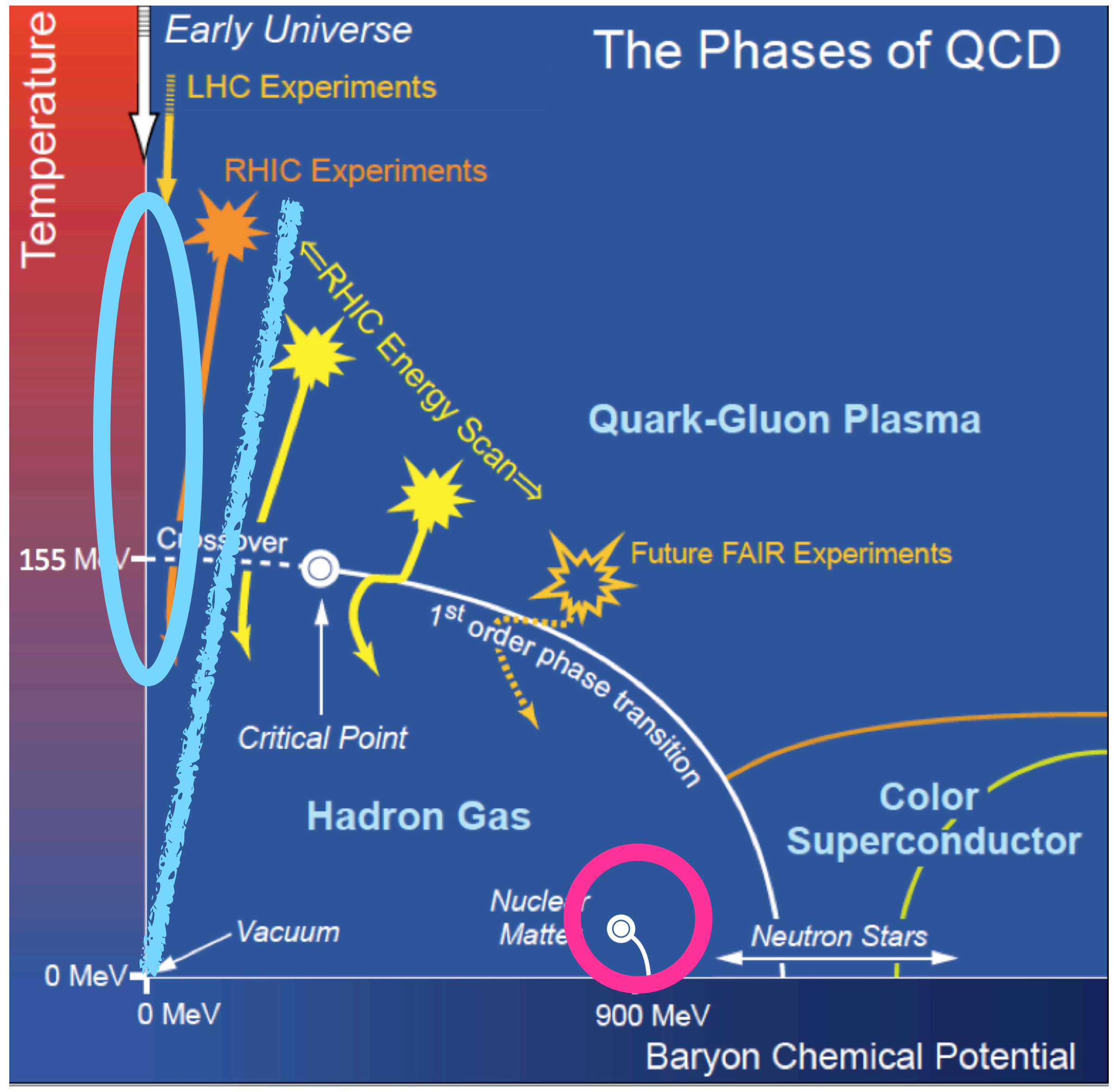


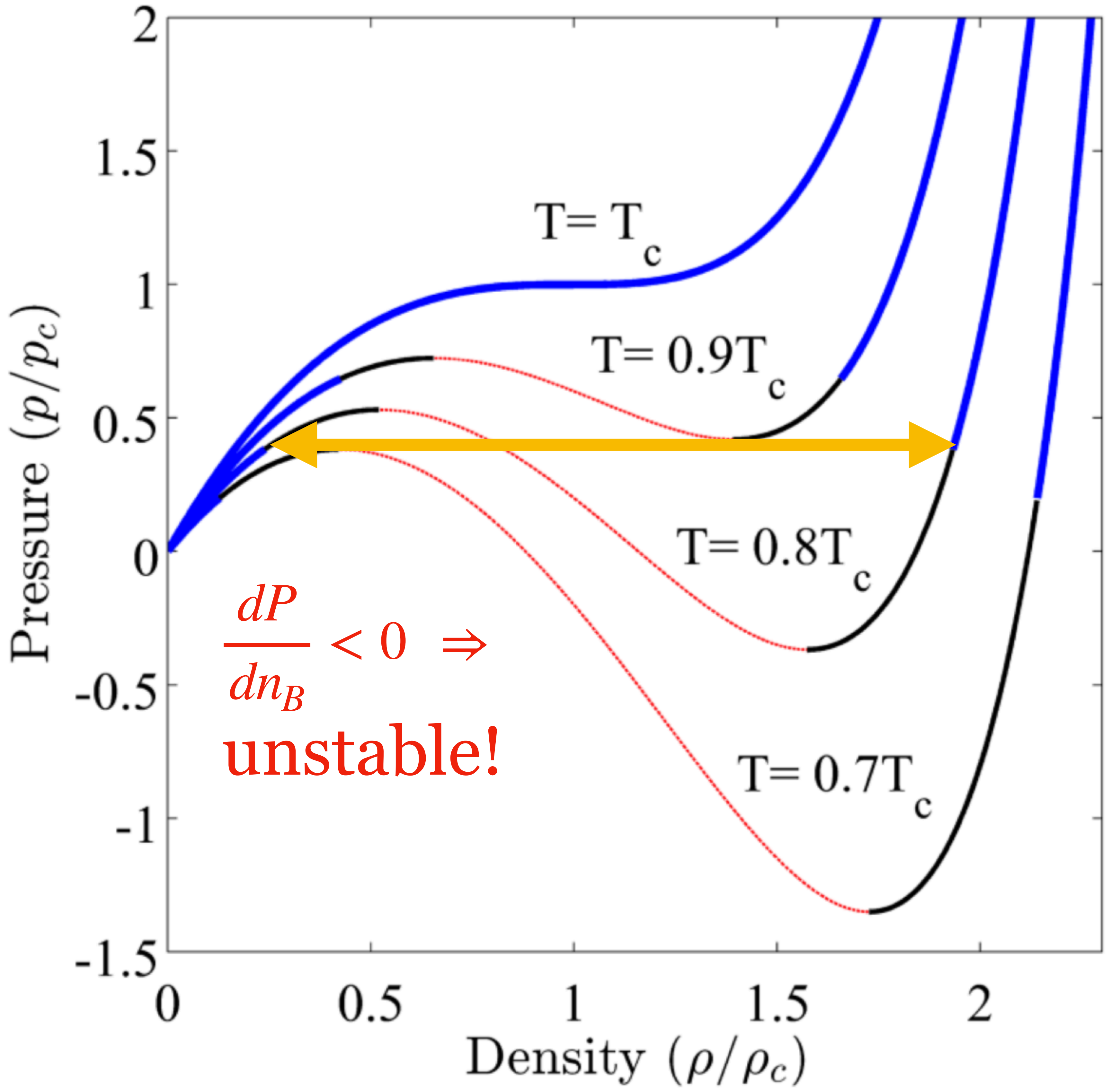
Example: Van der Waals EOS

$$P = R\frac{\rho T}{1 - b\rho} - a\rho^2$$

coexistence of phases:

$$T_1 = T_2, \quad P_1 = P_2, \quad \mu_1 = \mu_2$$

How does it all connect to the speed of sound?



L. Cueto-Felgueroso et al, Phys. Rev. Fluids **3**, 084302 (2018)

Like pressure, the behavior of the speed of sound signals a phase transition:

$$c_T^2 \equiv \left(\frac{dP}{d\mathcal{E}}\right)_T = \left(\frac{d\mathcal{E}}{dn_B}\right)_T^{-1} \left(\frac{dP}{dn_B}\right)_T$$

1st order phase transition = c_T^2 becomes negative/zero (Maxwell construction)

There are advantages in looking at a derivative of P instead of P

$c_s^2(n_B > 1.5n_0)$ may exceed the conformal limit of 1/3

P. Bedaque and A. W. Steiner, Phys. Rev. Lett. **114**, no.3, 031103 (2015), arXiv: 1408.5116, Bedaque:2014sqa
I. Tews, J. Carlson, S. Gandolfi and S. Reddy, Astrophys. J. **860**, no.2, 149 (2018), arXiv:1801.01923, Tews:2018kmu

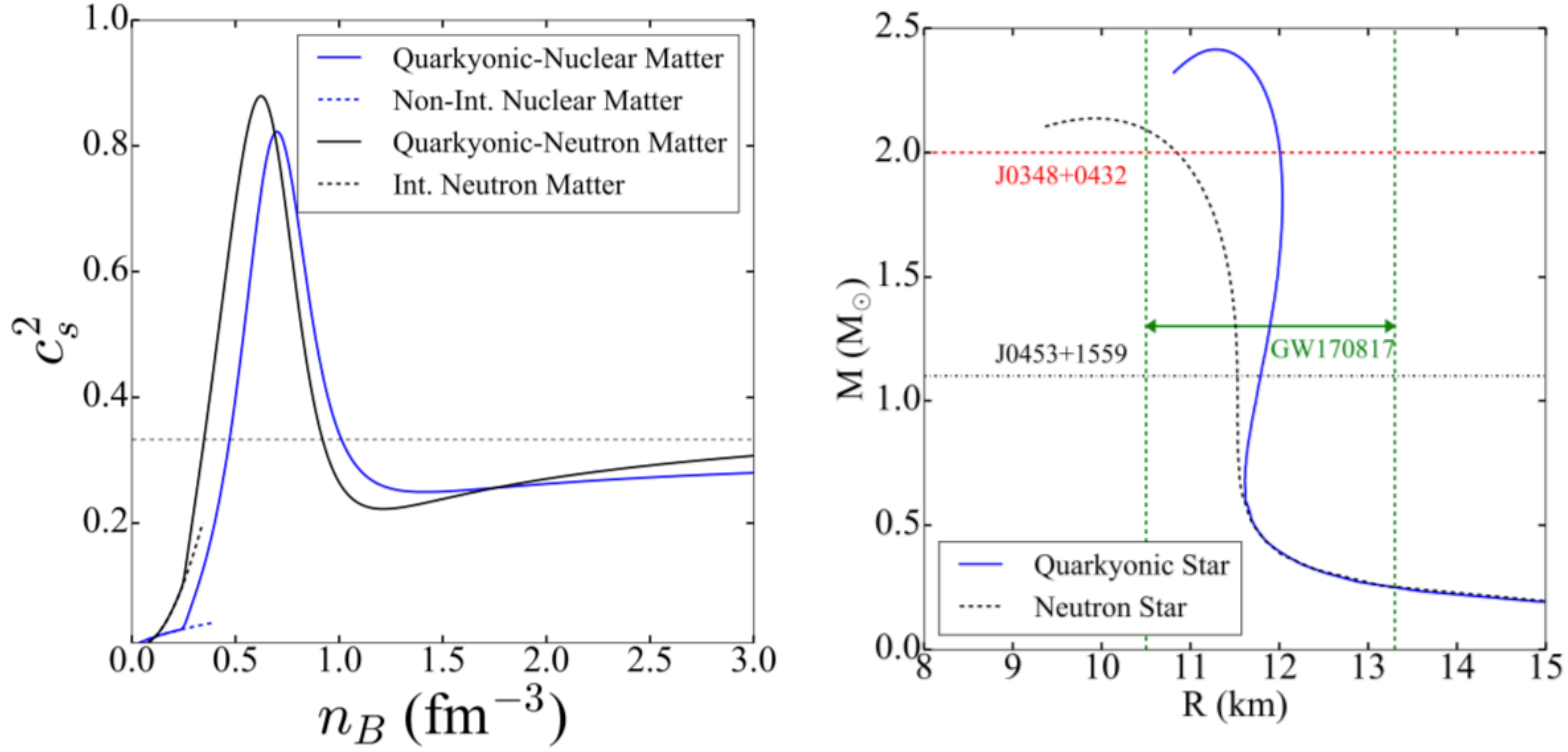
c_s^2 in any medium conjectured to be smaller than $c/3$:

- easily shown in non-relativistic and/or weakly coupled theories
- demonstrated in several classes of strongly coupled theories with gravity duals
- saturated only in conformal theories

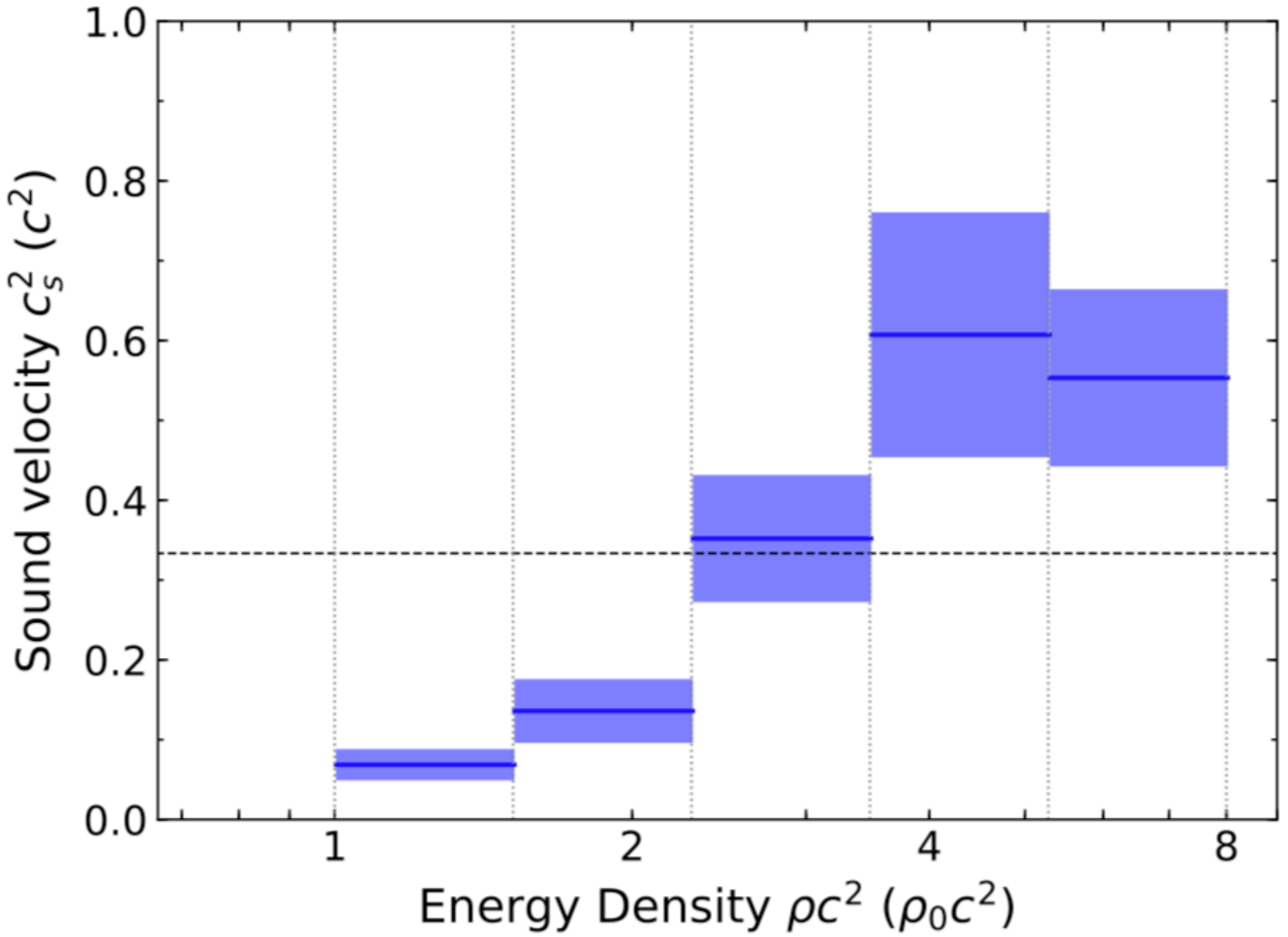
neutron stars with $M \gtrsim 2M_\odot$ + knowledge of the EOS of hadronic matter at "low" densities

$\Rightarrow$ **strong tension with this bound**

L. McLerran and S. Reddy, Phys. Rev. Lett. **122**, no.12, 122701 (2019), arXiv:1811.12503, McLerran:2018hbz



Y. Fujimoto, K. Fukushima and K. Murase, Phys. Rev. D **101**, no.5, 054016 (2020), arXiv:1903.03400, Fujimoto:2019hxv



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c_s^2 in any medium c
- easily shown in non-
- demonstrated in sev
- saturated only in co

neutron stars with

⇒ **strong tension**

This is a striking behavior!
Can it be studied in heavy-ion collisions?

L. McLerran and S. Reddy,
arXiv:1811.12503, McLerran:2018hbz

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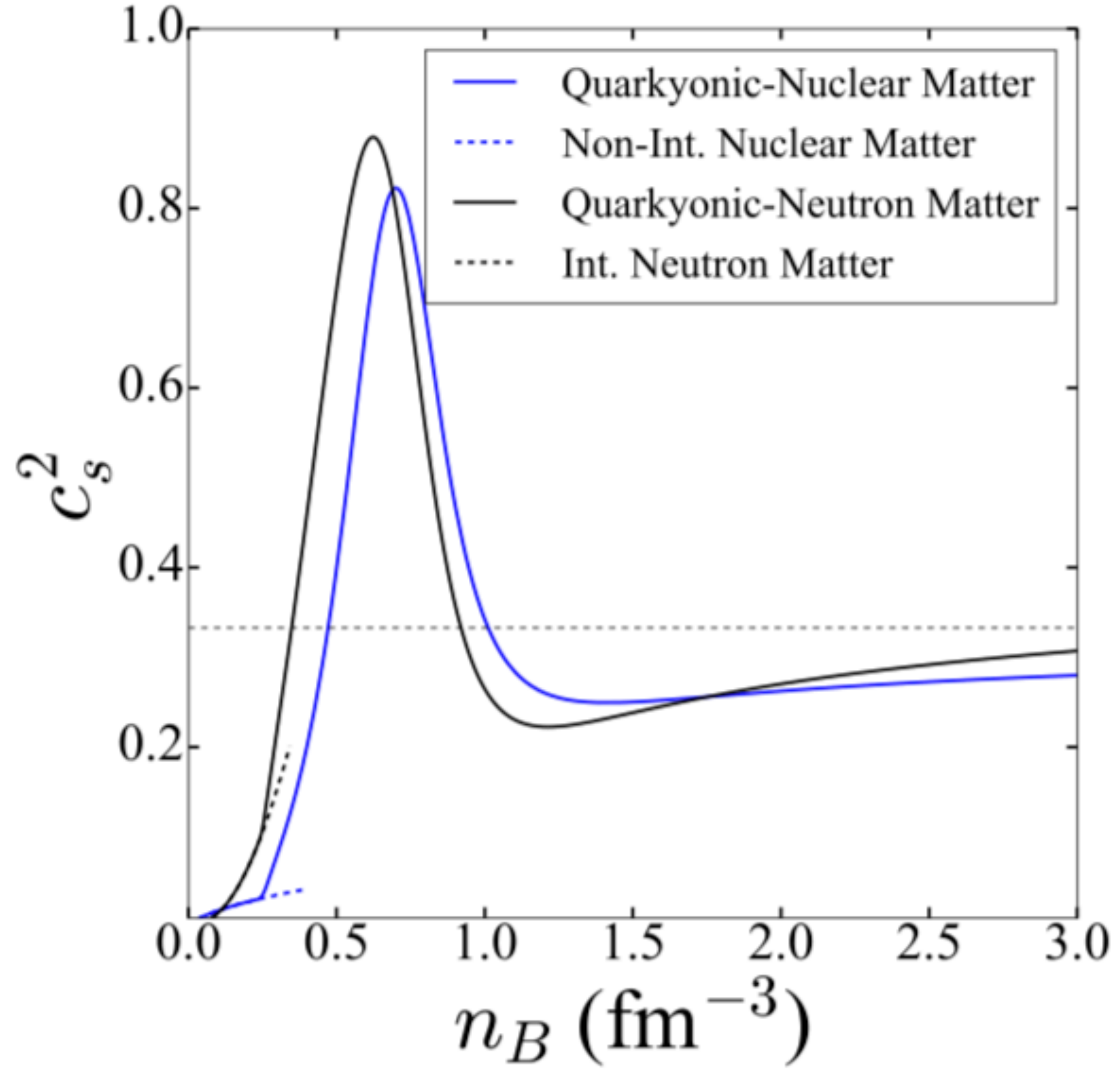
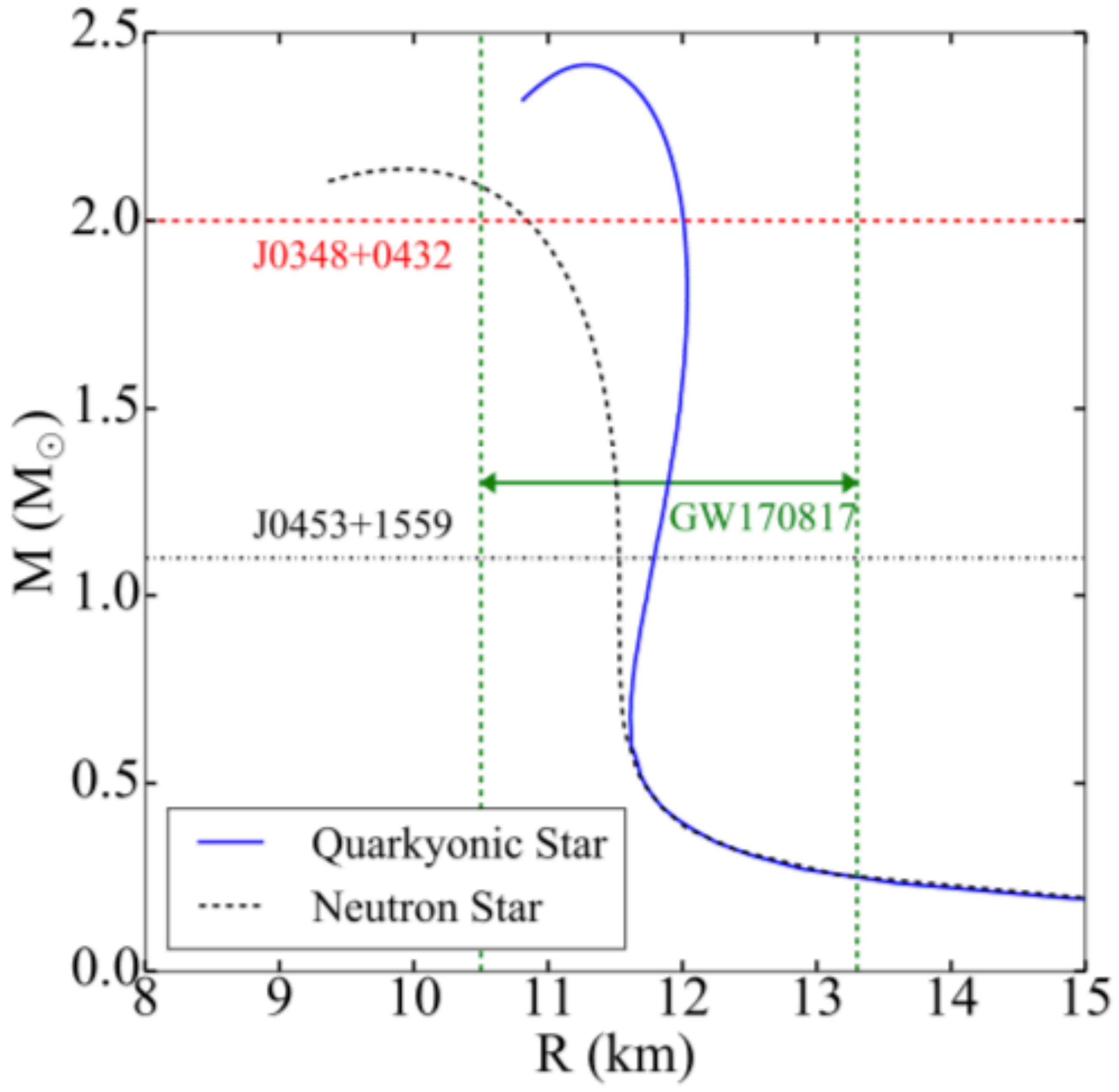
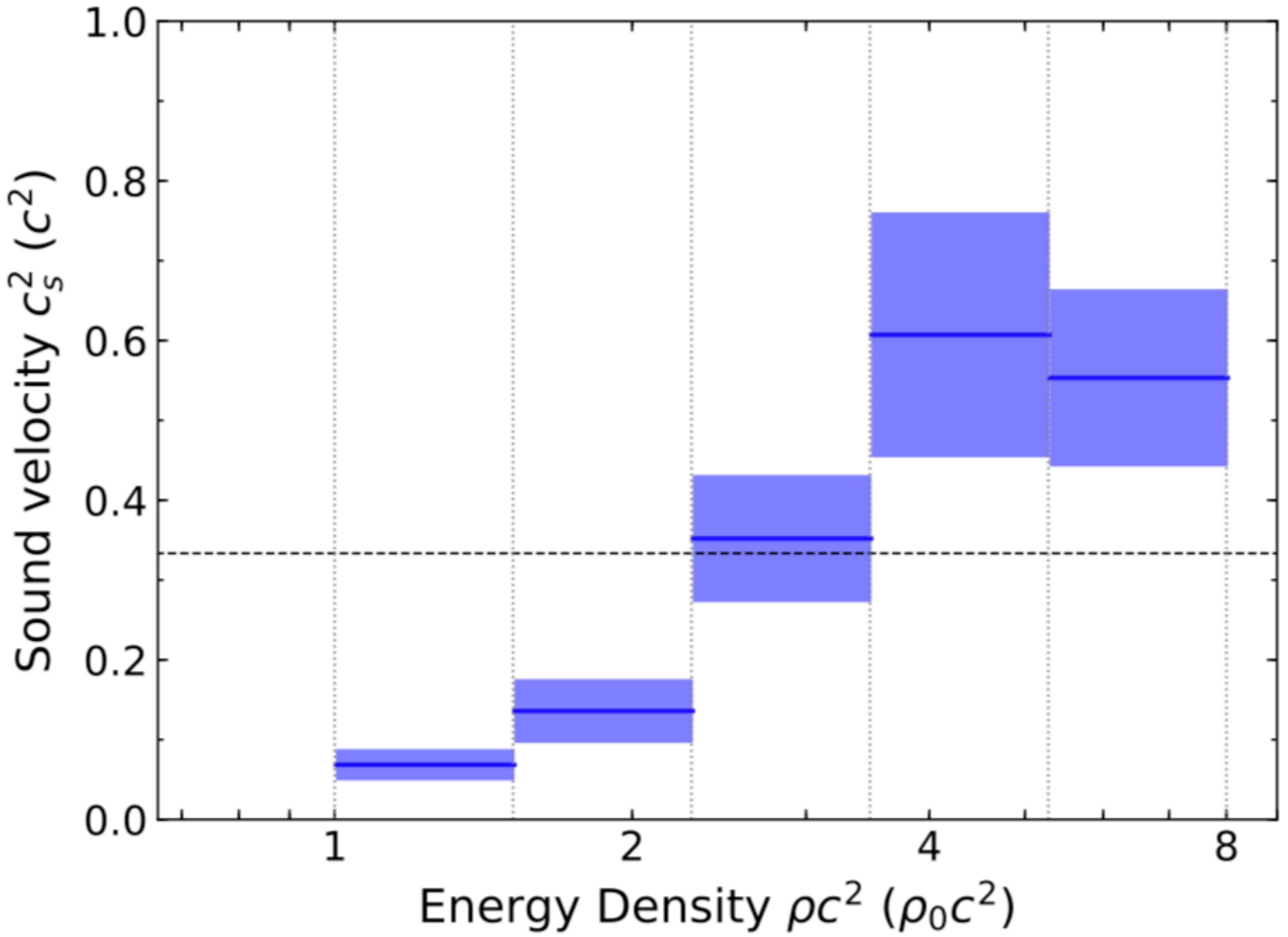






Studying QCD thermodynamics with heavy-ion collisions

A possible signature of a 1st order phase transition

Cumulants of the baryon number distribution

$$\kappa_j \equiv VT^{j-1}\left(\frac{d^jP}{d\mu_B^j}\right)_T$$

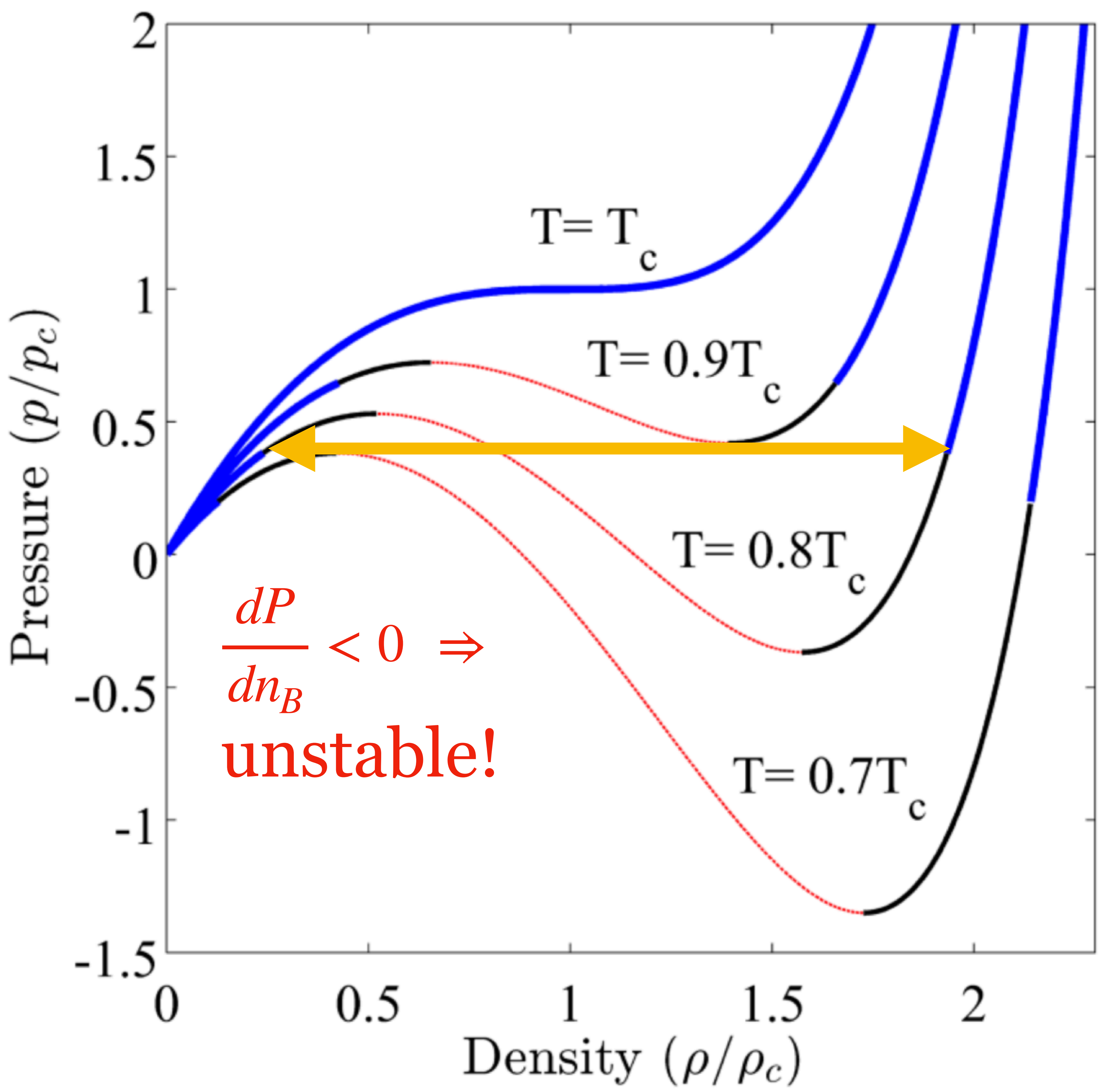
$$\kappa_1 = \langle N \rangle$$
$$\kappa_2 = \left\langle \left(N - \langle N \rangle\right)^2 \right\rangle = \sigma^2$$
$$\kappa_3 = \left\langle \left(N - \langle N \rangle\right)^3 \right\rangle$$
$$\kappa_4 = \left\langle \left(N - \langle N \rangle\right)^4 \right\rangle - 3\kappa_2^2$$

$$\kappa_1 = Vn_B$$

$$\kappa_2 = VT\left(\frac{dn_B}{d\mu_B}\right)_T = VT\left(\frac{d\mu_B}{dn_B}\right)_T^{-1} = VTn_B\left(\frac{dP}{dn_B}\right)_T^{-1}$$

$$\kappa_3 = VT^2\left(\frac{d^2n_B}{d\mu_B^2}\right)_T = \frac{VT^2n_B}{\left(\frac{dP}{dn_B}\right)_T^2}\left[1 - \frac{n_B}{\left(\frac{dP}{dn_B}\right)_T}\left(\frac{d^2P}{dn_B^2}\right)_T\right]$$

$$\kappa_4 = VT^3\left(\frac{d^3n_B}{d\mu_B^3}\right)_T = \frac{VT^3n_B}{\left(\frac{dP}{dn_B}\right)_T^3}\left[1 - \frac{4n_B}{\left(\frac{dP}{dn_B}\right)_T}\left(\frac{d^2P}{dn_B^2}\right)_T + \frac{3n_B^2}{\left(\frac{dP}{dn_B}\right)_T^2}\left(\frac{d^2P}{dn_B^2}\right)_T^2 - \frac{n_B^2}{\left(\frac{dP}{dn_B}\right)_T}\left(\frac{d^3P}{dn_B^3}\right)_T\right]$$



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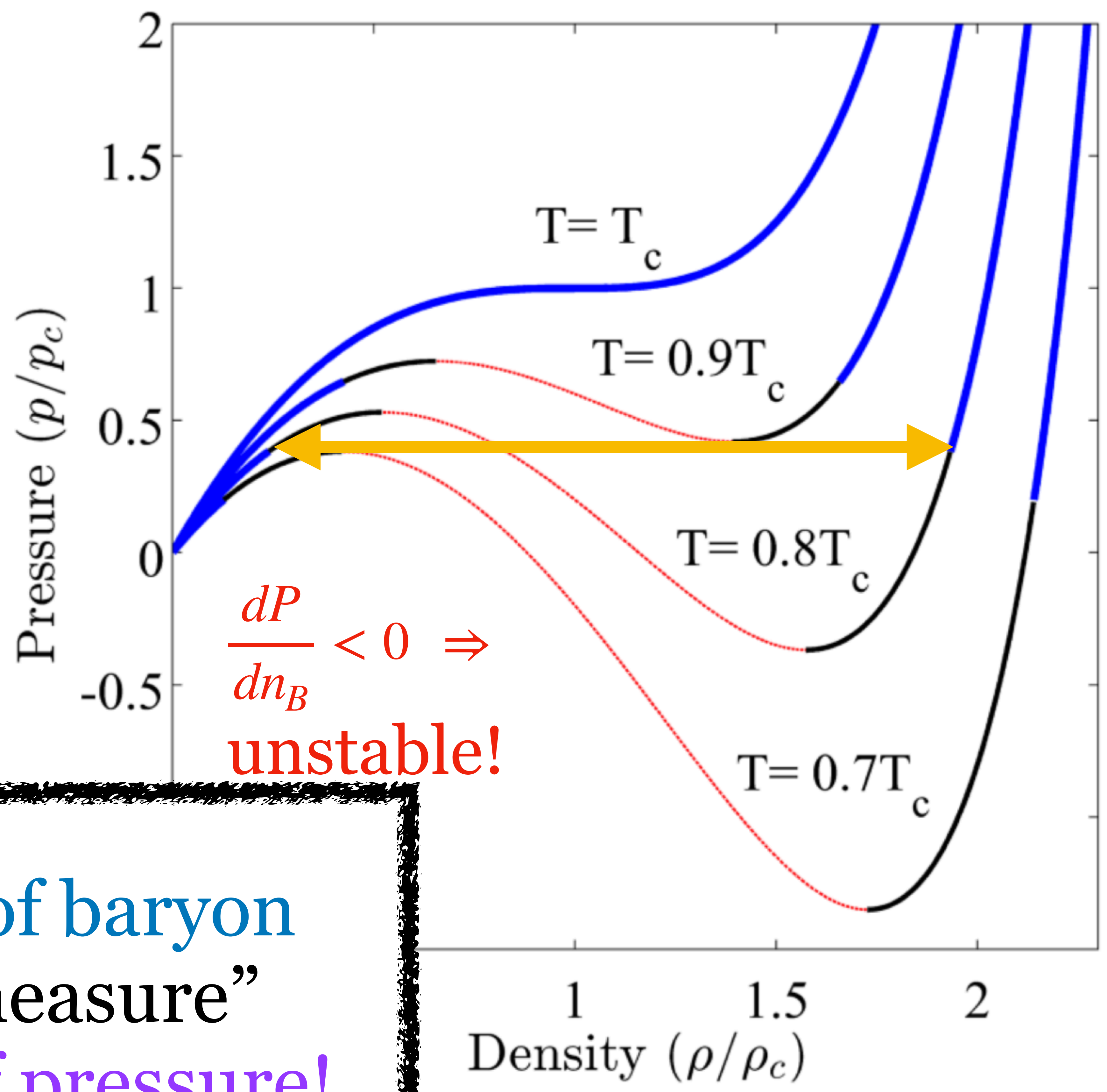

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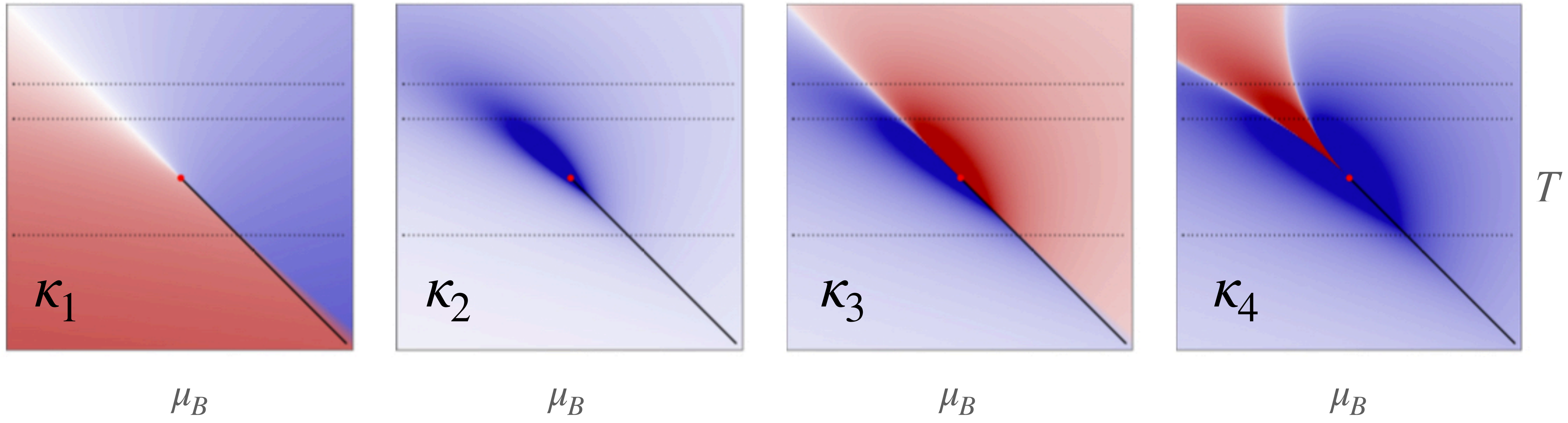
Cumulants of baryon number "measure" derivatives of pressure!

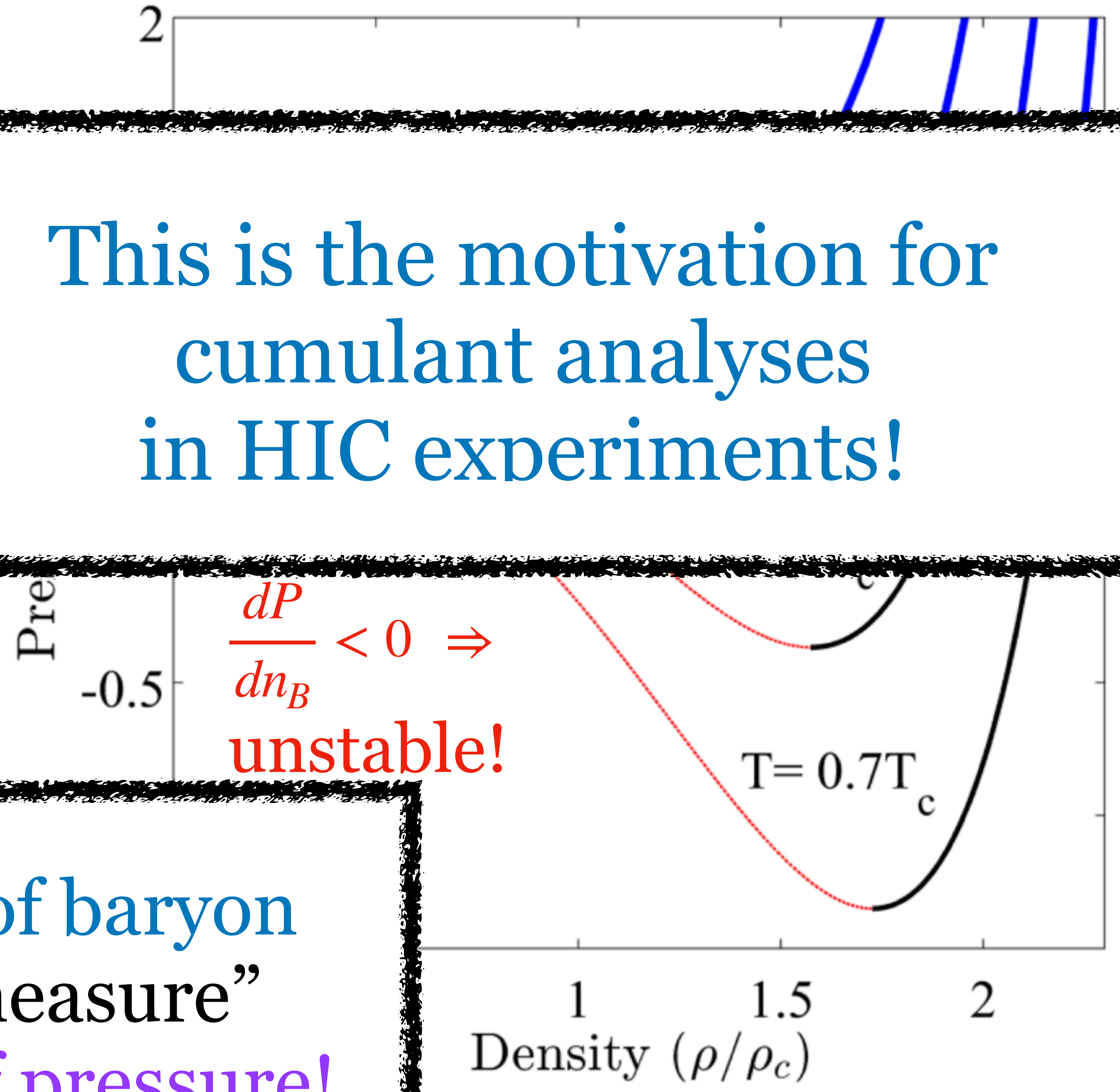
A possible signature of a 1st order phase transition

Cumulants expected to "signal" fluctuations near the CP:



A. Bzdak *et al.*, Physics Reports **853** (2020) 1-87, arXiv:1906.00936, Bzdak:2019pkr

This is the motivation for cumulant analyses in HIC experiments!



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see also
STAR, arXiv:2112.00240 (2021),
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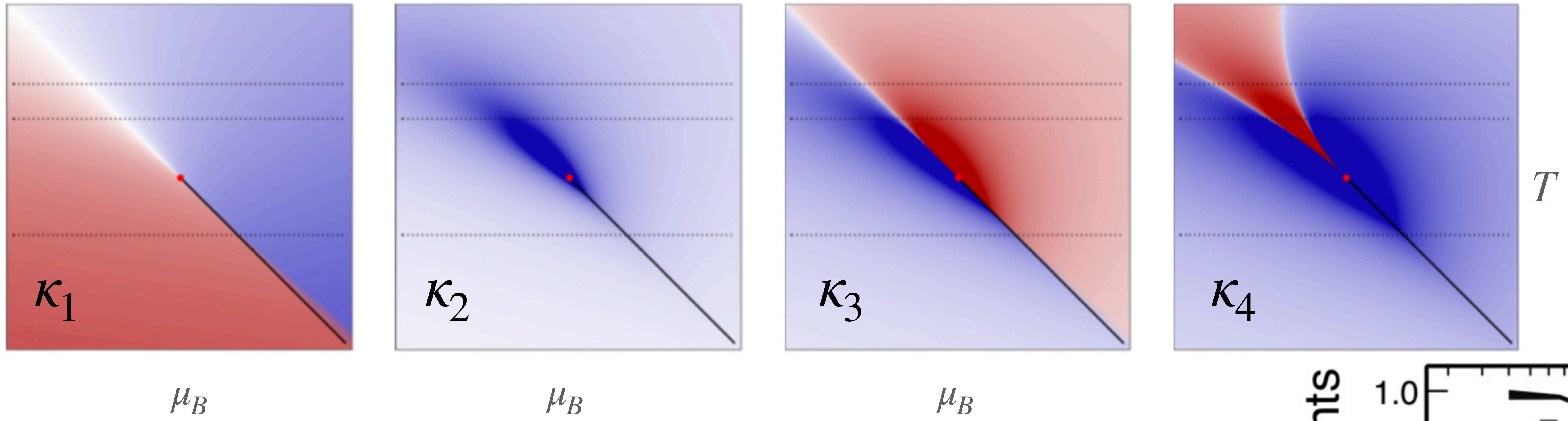

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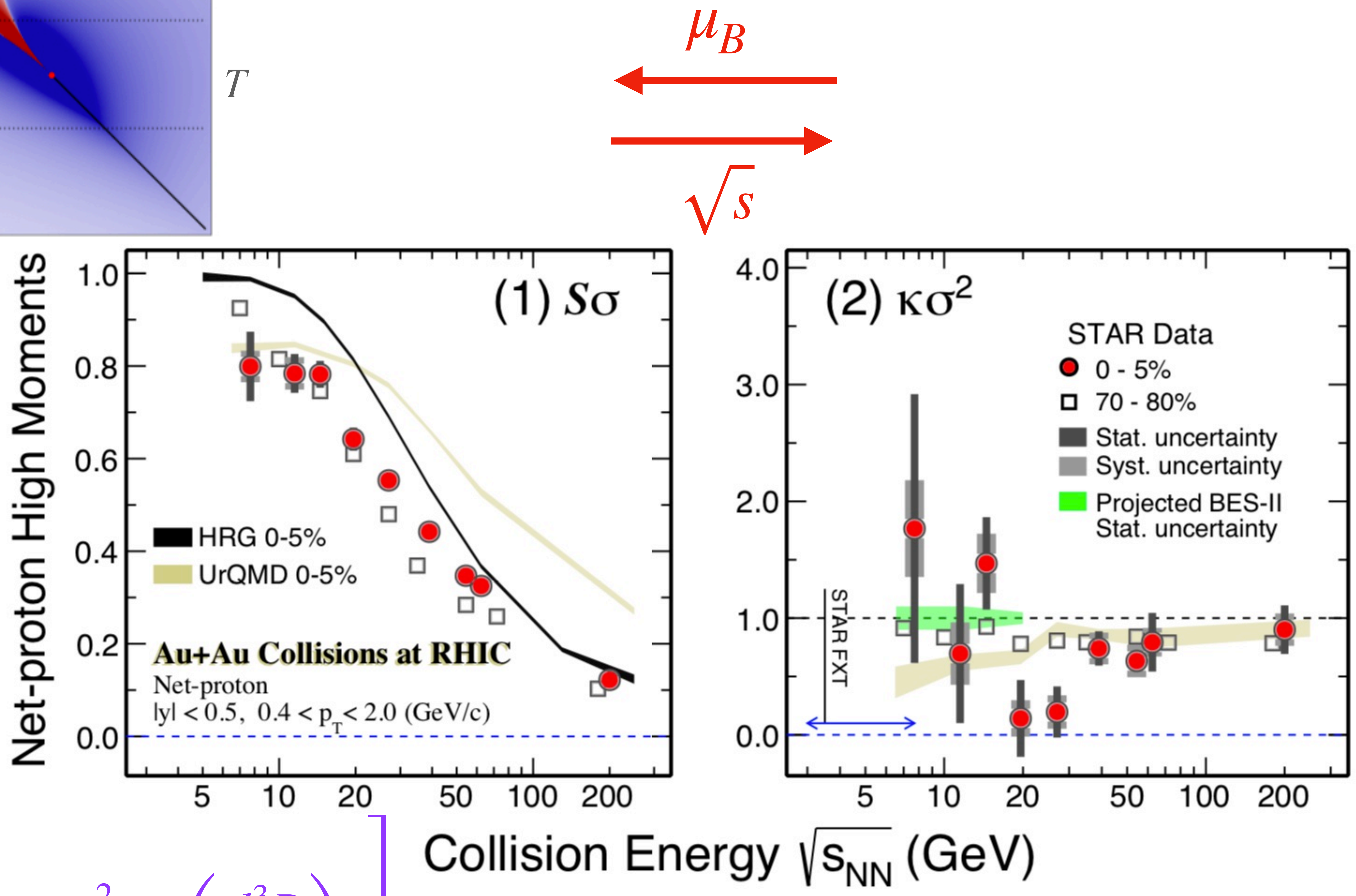
$$\frac{\kappa_2}{\kappa_1} = T\left(\frac{dP}{dn_B}\right)_T^{-1}$$

$$\frac{\kappa_3}{\kappa_2} = \frac{T}{\left(\frac{dP}{dn_B}\right)_T}\left[1 - \frac{n_B}{\left(\frac{dP}{dn_B}\right)_T}\left(\frac{d^2P}{dn_B^2}\right)_T\right]$$

$$\frac{\kappa_4}{\kappa_2} = \frac{T^2}{\left(\frac{dP}{dn_B}\right)_T^2}\left[1 - \frac{4n_B}{\left(\frac{dP}{dn_B}\right)_T}\left(\frac{d^2P}{dn_B^2}\right)_T + \frac{3n_B^2}{\left(\frac{dP}{dn_B}\right)_T^2}\left(\frac{d^2P}{dn_B^2}\right)_T^2 - \frac{n_B^2}{\left(\frac{dP}{dn_B}\right)_T}\left(\frac{d^3P}{dn_B^3}\right)_T\right]$$



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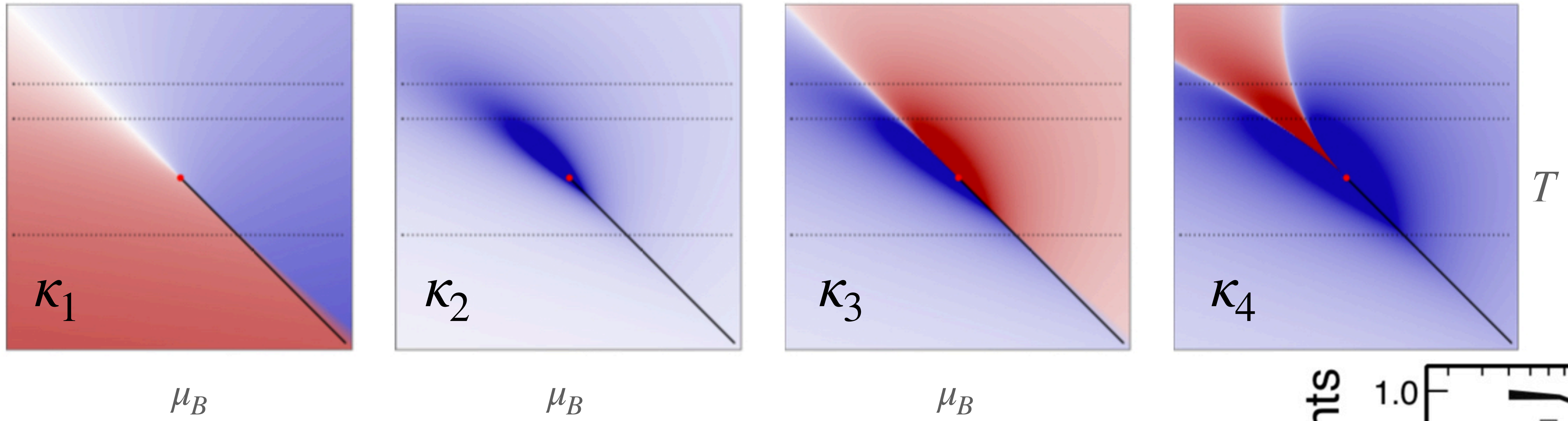

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More statistics is needed: RHIC Beam Energy Scan II!

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$$\frac{\kappa_3}{\kappa_2} = \frac{T}{\left(\frac{dP}{dn_B}\right)_T}\left[1 - \frac{n_B}{\left(\frac{dP}{dn_B}\right)_T}\left(\frac{d^2P}{dn_B^2}\right)_T\right]$$

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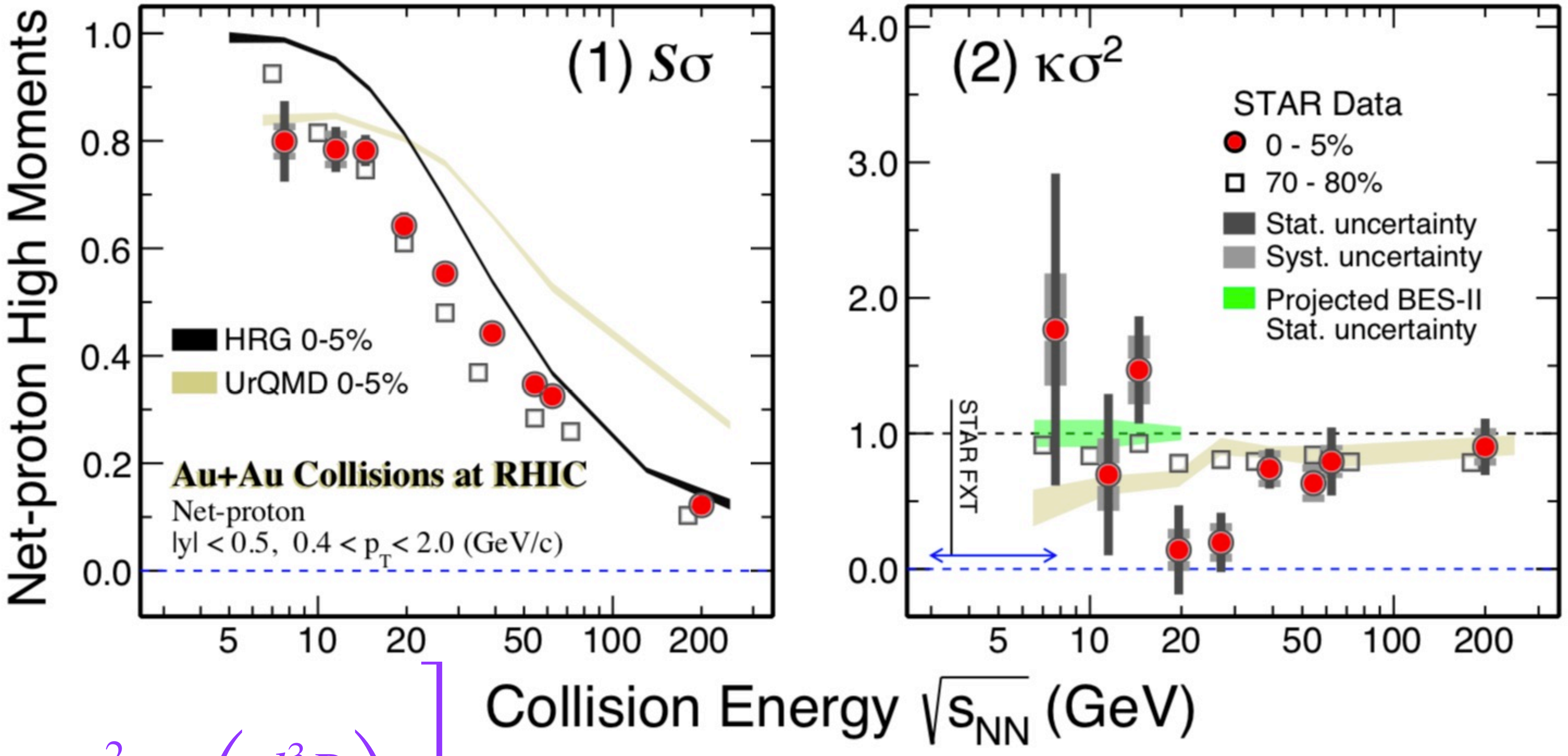

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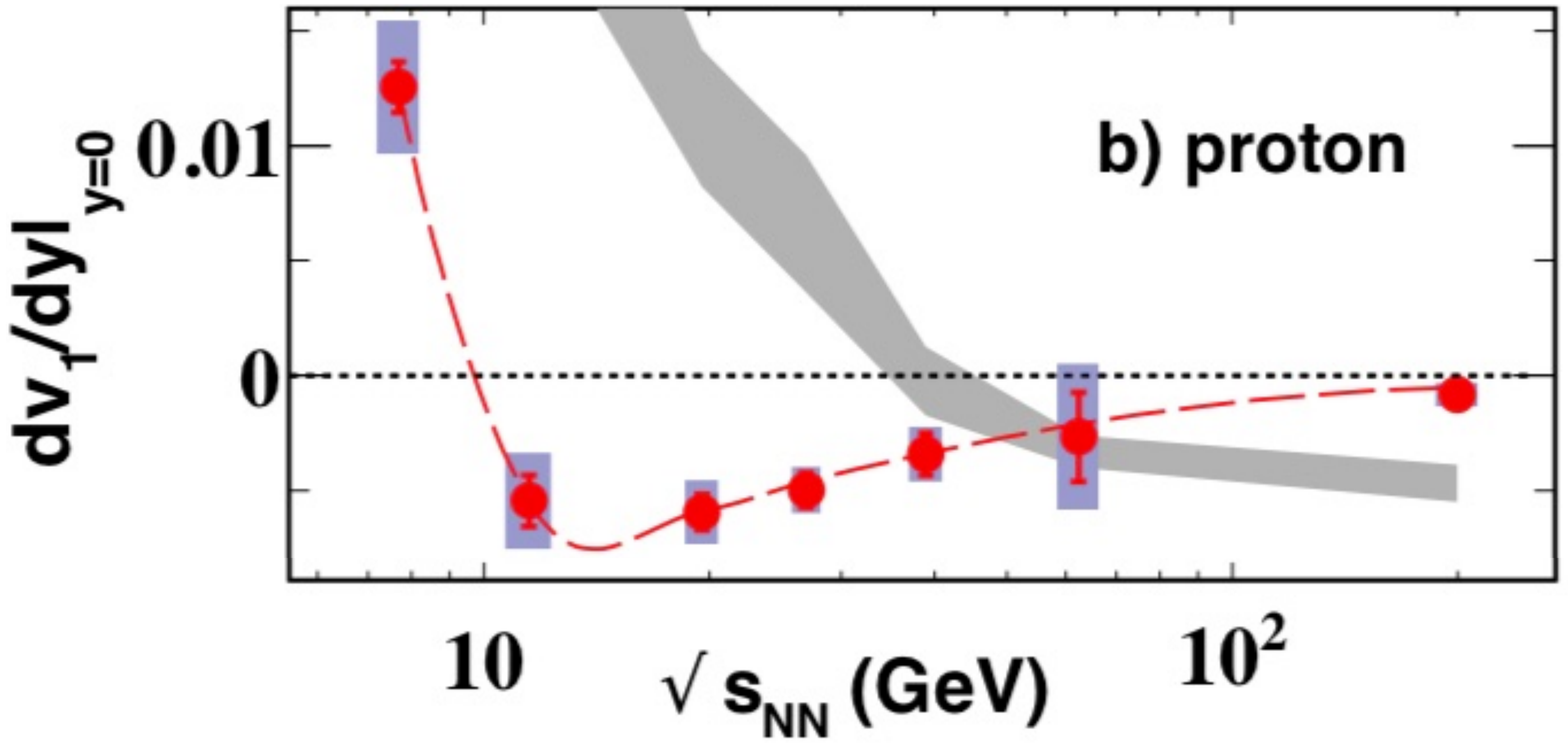
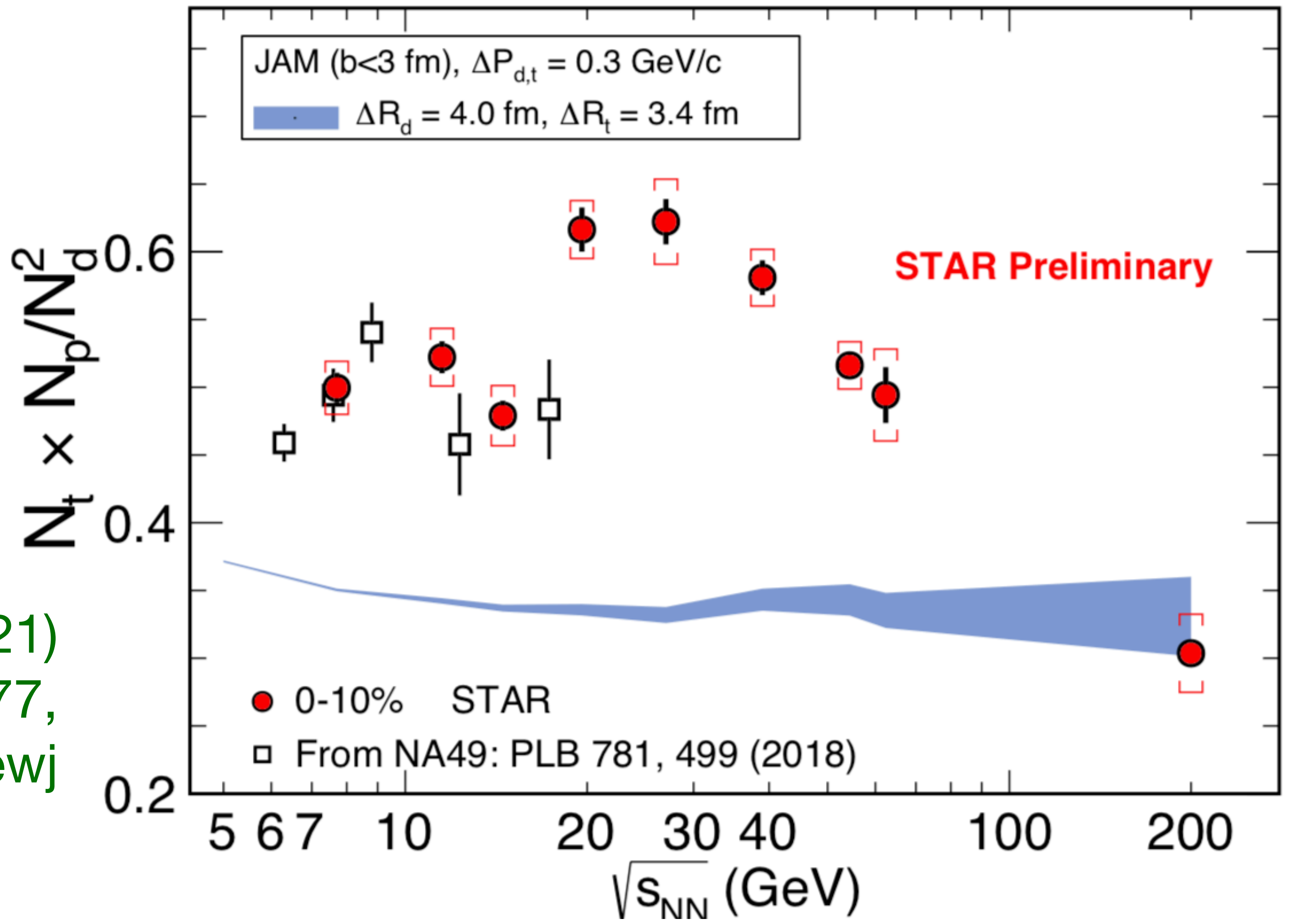
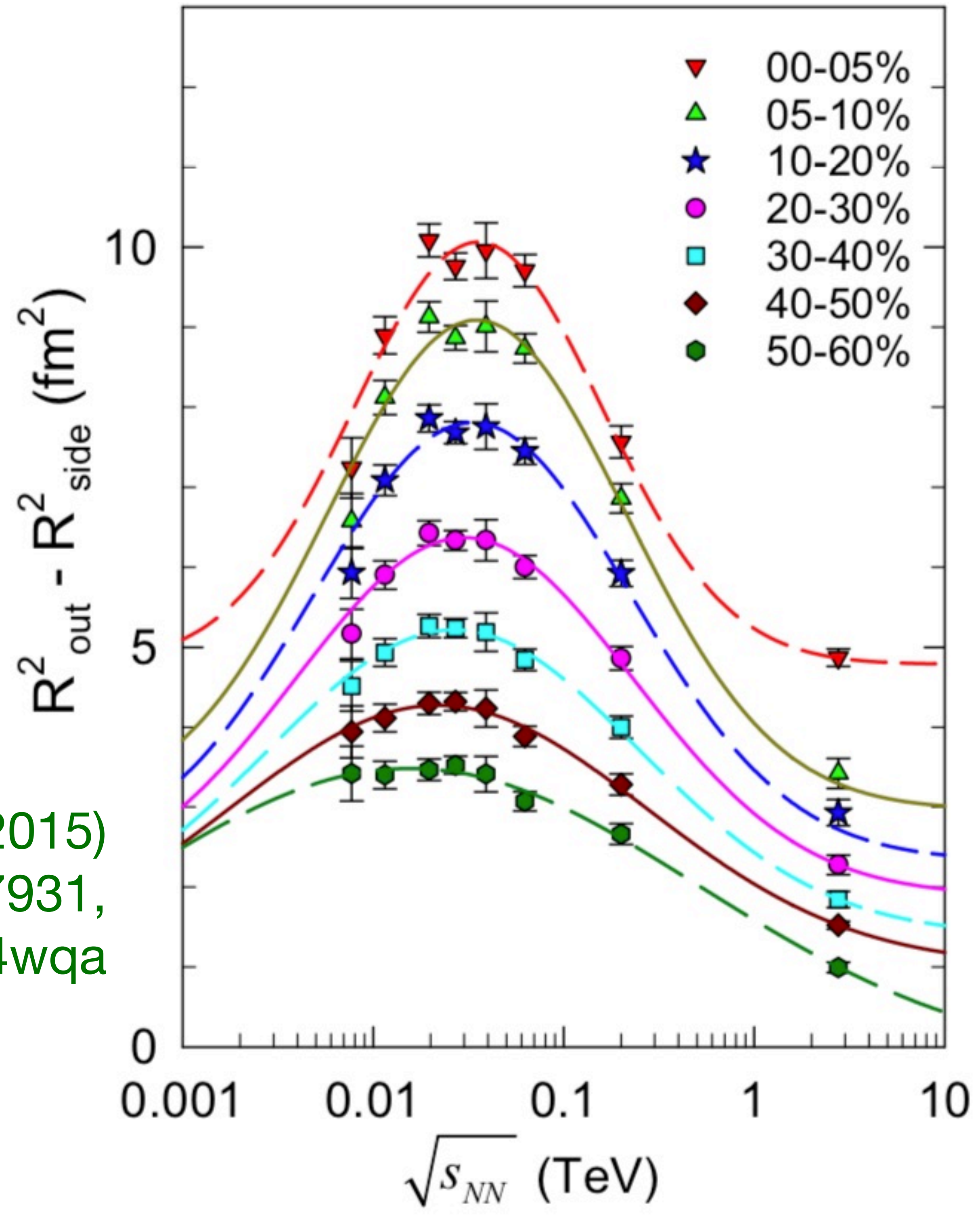
Many possible signatures of a critical point

STAR, Phys. Rev. Lett. **112** (2014) , no.16, 162301, arXiv:1401.3043, STAR:2014clz

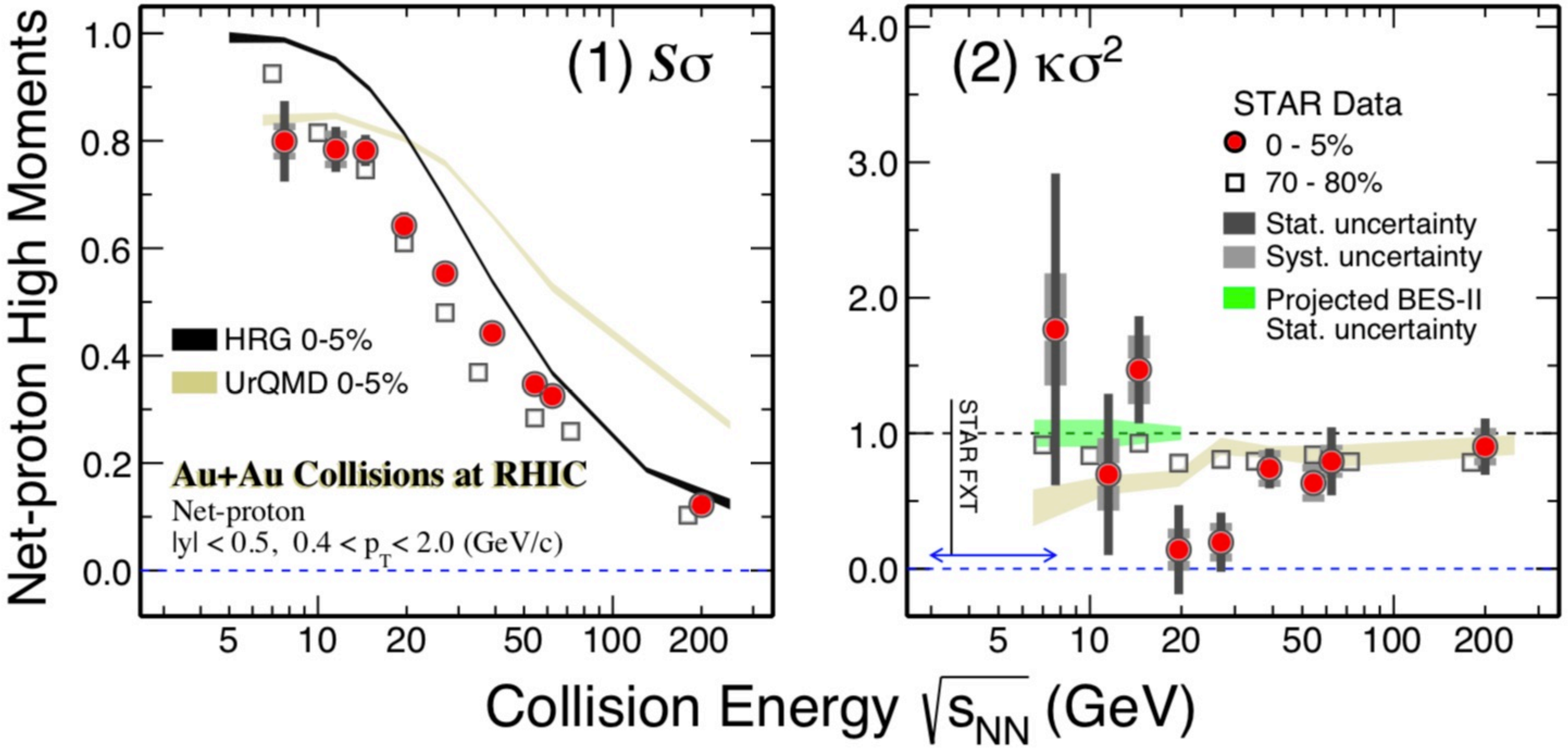




STAR, Nucl. Phys. A **1005** (2021) 121825, arXiv:2002.10677, Zhang:2020ewj



R. A. Lacey, Phys. Rev. Lett. **114** (2015) no.14, 142301, arXiv:1411.7931, Lacey:2014wqa



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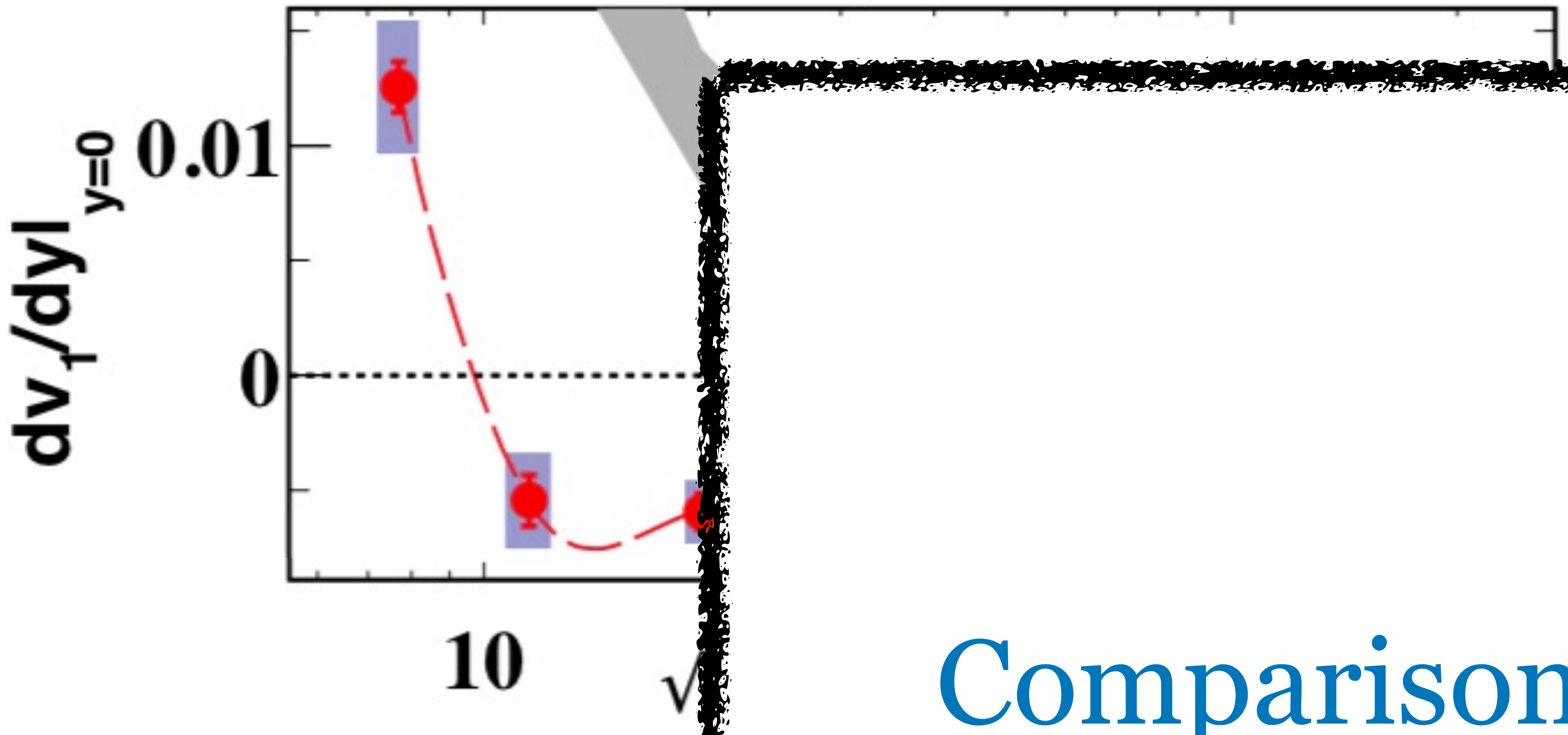
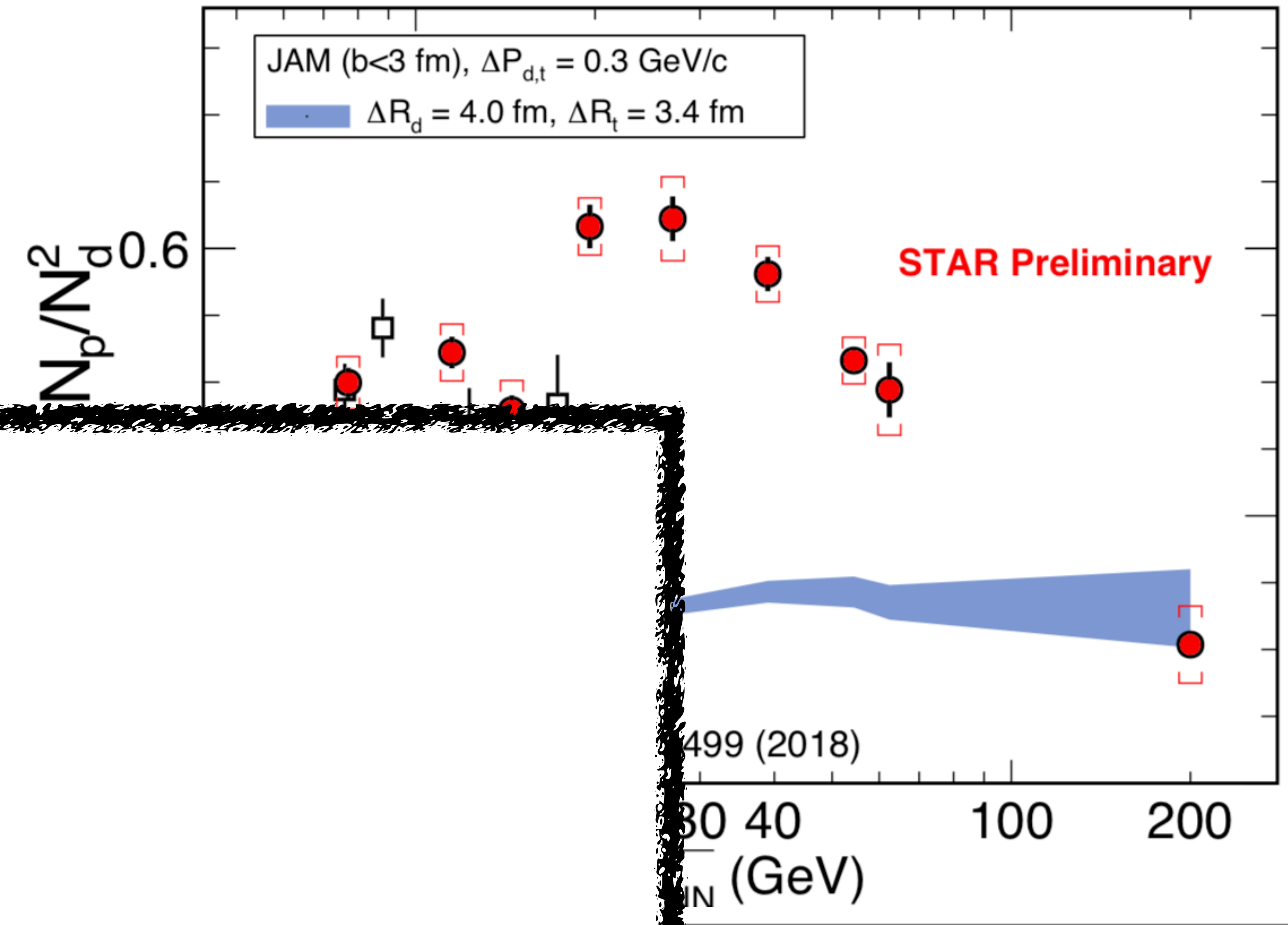
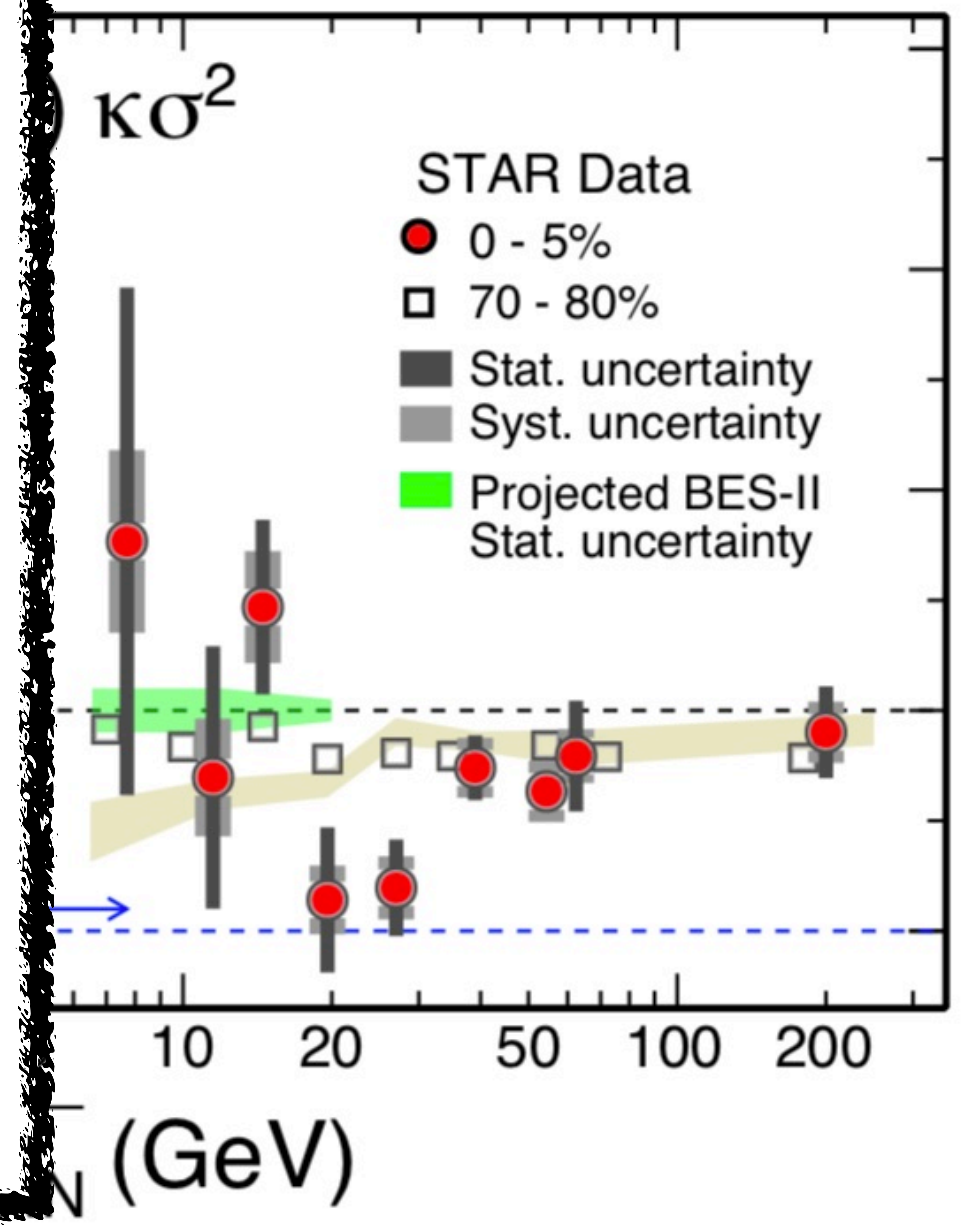




Comparisons with simulations will be crucial
to interpreting experimental data
and understanding the thermodynamics
of hot and dense nuclear matter



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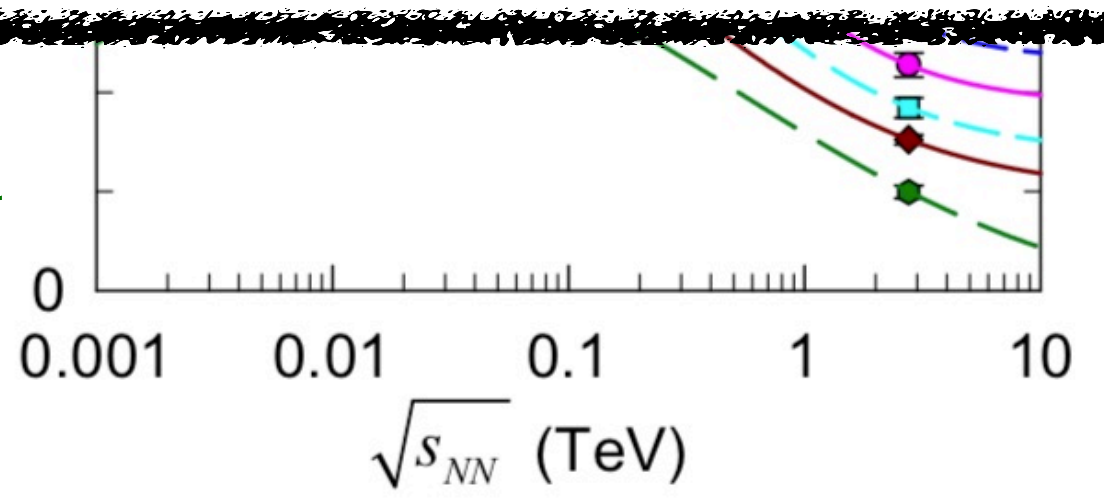

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Connecting the speed of sound to heavy-ion collision observables

How to measure c_s^2 in heavy-ion collisions?

At finite T, one often considers the following two expressions for the speed of sound:

the isothermal speed of sound ($T = const$):

$$c_T^2 \equiv \left(\frac{dP}{d\mathcal{E}}\right)_T = \frac{\left(\frac{dP}{dn_B}\right)_T}{T\left(\frac{ds}{dn_B}\right)_T + \mu_B}$$

the isentropic speed of sound ($\sigma = s/n_B = const$):

$$c_\sigma^2 \equiv \left(\frac{dP}{d\mathcal{E}}\right)_\sigma = \frac{\frac{s}{n_B}\left(\frac{dP}{dT}\right)_{n_B} + \left(\frac{dP}{dn_B}\right)_T\left(\frac{ds}{dT}\right)_{n_B} - \left(\frac{dP}{dT}\right)_{n_B}\left(\frac{ds}{dn_B}\right)_T}{\left(\frac{\mathcal{E}+P}{n_B}\right)\left(\frac{ds}{dT}\right)_{n_B}}$$

common $T = 0$ limit:

$$c_T^2\Big|_{T=0} = c_\sigma^2\Big|_{T=0} = \frac{n_B}{\mu_B}\left(\frac{d\mu_B}{dn_B}\right)_T = \frac{1}{\mu_B}\left(\frac{dP}{dn_B}\right)_T$$

Easy to notice: the speed of sound ~ derivatives of pressure

What "measures" derivatives of pressure? **Cumulants of baryon number!**

Can one connect the speed of sound with the cumulants?

$$c_T^2 \equiv \left(\frac{dP}{d\mathcal{E}}\right)_T = \left(\frac{dP}{d\mu_B}\right)_T \left(\frac{d\mathcal{E}}{d\mu_B}\right)_T^{-1} = \frac{n_B}{T\left(\frac{ds}{d\mu_B}\right)_T + \mu_B\left(\frac{dn_B}{d\mu_B}\right)_T}$$

Maxwell relations

$$= \frac{n_B}{T\left(\frac{dn_B}{dT}\right)_{\mu_B} + \mu_B\left(\frac{dn_B}{d\mu_B}\right)_T}$$

$n_B = \kappa_1/V$

$$= \left[\frac{T}{\kappa_1}\left(\frac{d\kappa_1}{dT}\right)_{\mu_B} + \frac{\mu_B}{\kappa_1}\left(\frac{d\kappa_1}{d\mu_B}\right)_T\right]^{-1}$$

Problematic: difficult to estimate T derivatives of cumulants from experiment

$T/\mu_B \ll 1$

$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2}$$

an upper limit to c_T^2

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303, arXiv:2103.07365, Sorensen:2021zme

Can one connect the speed of sound with the cumulants?

Does it make sense intuitively that $c_T^2 \propto \kappa_2^{-1}$?

In the nonrelativistic case: $c_T^2\Big|_{nonrel} = \left(\dfrac{dP}{dn_B}\right)_T$

Given local change in density dn_B: dP is large $\Rightarrow$ c_T^2 is large.

But, if dP is large for a given dn_B, that produces large pressure gradients $\Rightarrow$ large restoring forces F_r .

Large restoring forces will work against large local changes in density $\Rightarrow$ suppression in local density fluctuations $\Rightarrow$ small κ_2.

So yes, it makes sense!

$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2}$$

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Maxwell relations

$$= \frac{n_B}{T\left(\frac{dn_B}{dT}\right)_{\mu_B} + \mu_B\left(\frac{dn_B}{d\mu_B}\right)_T}$$

$n_B = \kappa_1/V$

$$= \left[\frac{T}{\kappa_1}\left(\frac{d\kappa_1}{dT}\right)_{\mu_B} + \frac{\mu_B}{\kappa_1}\left(\frac{d\kappa_1}{d\mu_B}\right)_T\right]^{-1}$$

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$T/\mu_B \ll 1$

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an upper limit to c_T^2

$$\left(\frac{d\ln c_T^2}{d\ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3\kappa_1}{\kappa_2^2}$$

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303, arXiv:2103.07365, Sorensen:2021zme

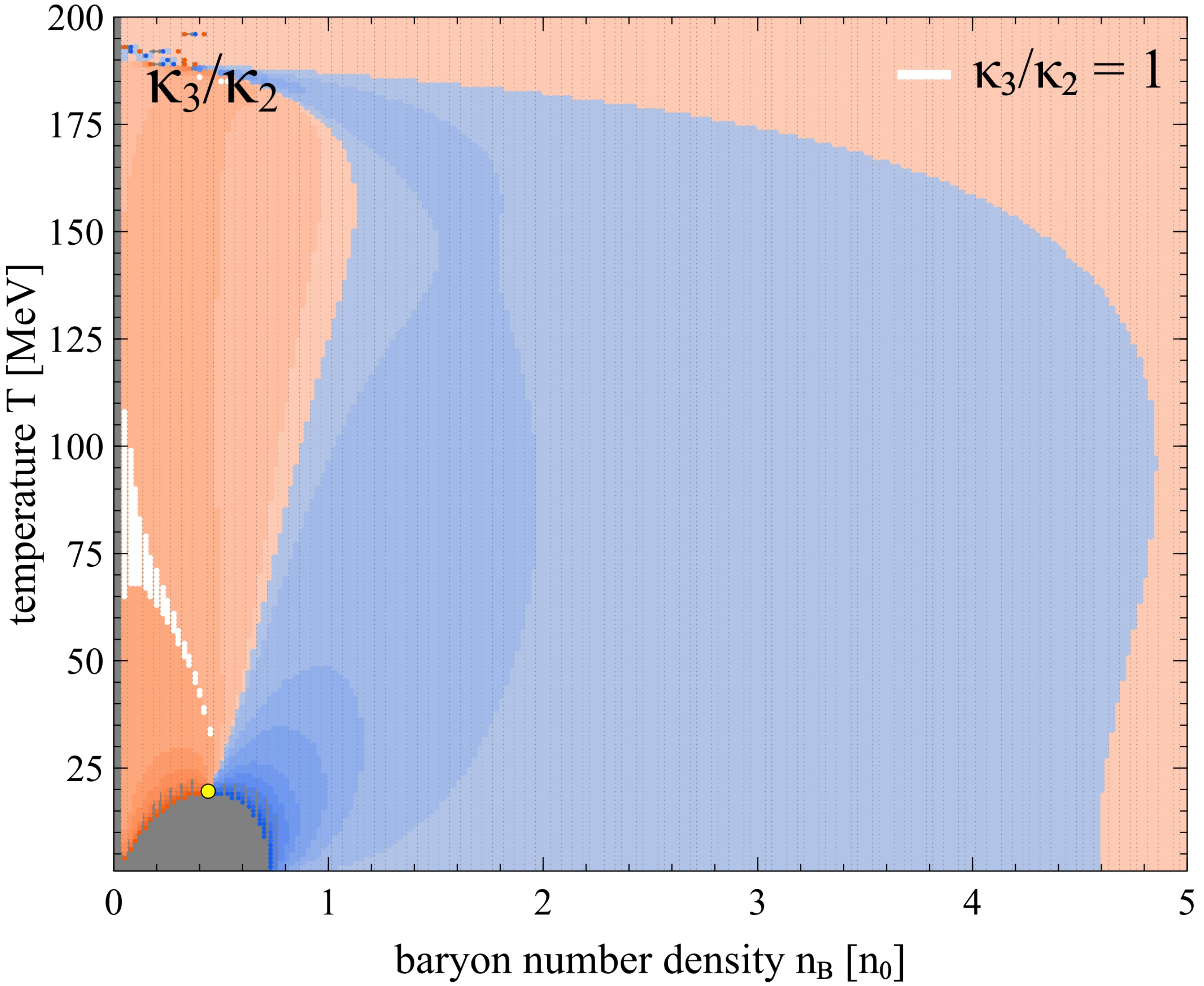
Tests in model calculations

Tests in models

AS, V. Koch, Phys. Rev. C **104** no. 3 (2021) 034904, arXiv:2011.06635, Sorensen:2020ygf

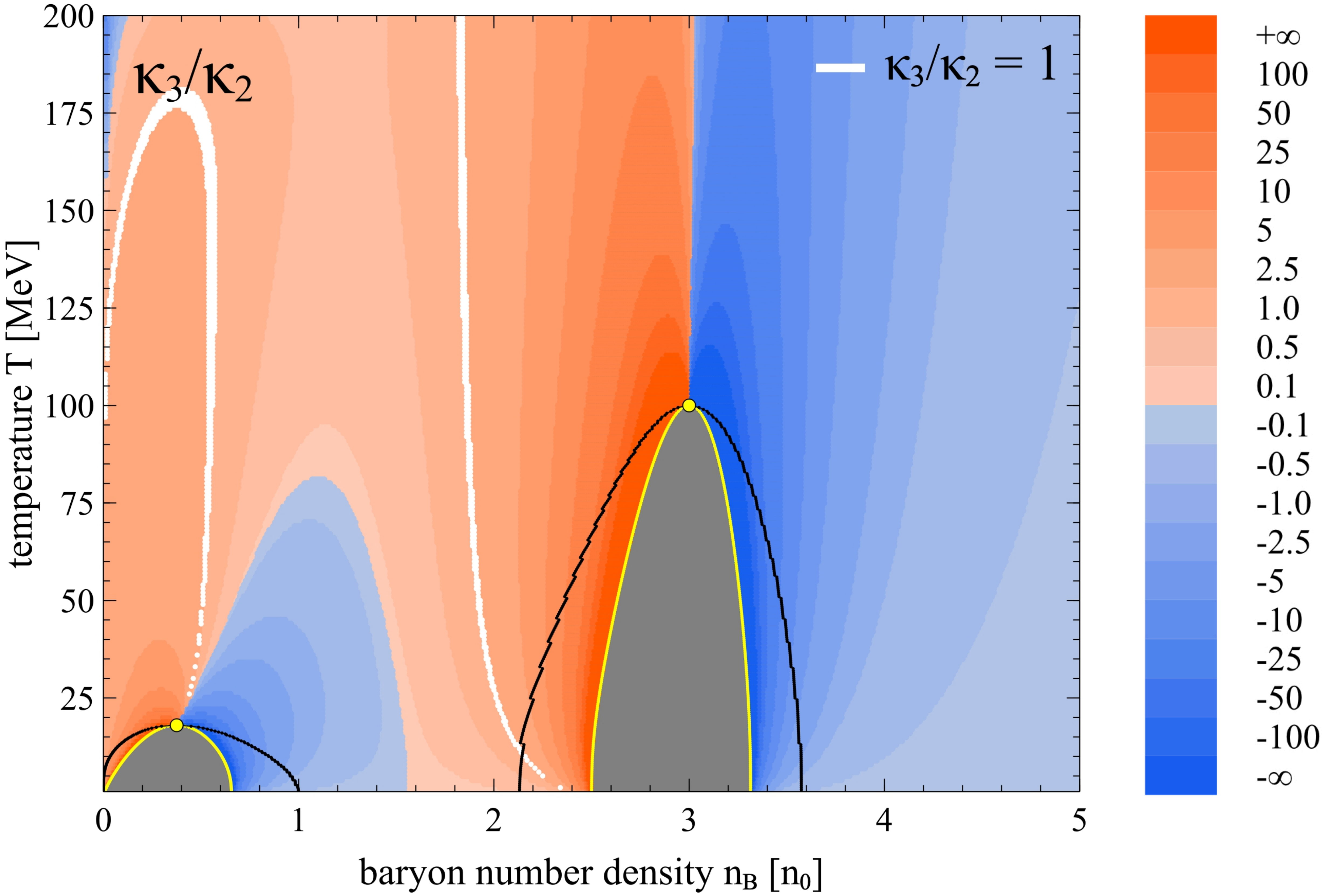
the Walecka model

$$P = g\int \frac{d^3p}{(2\pi)^3}\,\frac{p^2}{3\sqrt{p^2+m^{*2}}}\,f_{\mathbf{p}} - \frac{1}{2}G_s^2 n_s^2 + \frac{1}{2}G_v^2 n_B^2$$



the VDF (vector density functional) model

$$P = g\int \frac{d^3p}{(2\pi)^3}\,\frac{p^2}{3\sqrt{p^2+m^2}}\,f_{\mathbf{p}} + \sum_{i=1}^{4} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$



orange = positive values, blue = negative values, white lines for values = 1 (Poisson limit), yellow lines = spinodal regions, black lines for coexistence regions

Tests in models

AS, V. Koch, Phys. Rev. C **104** no. 3 (2021) 034904, arXiv:2011.06635, Sorensen:2020ygf

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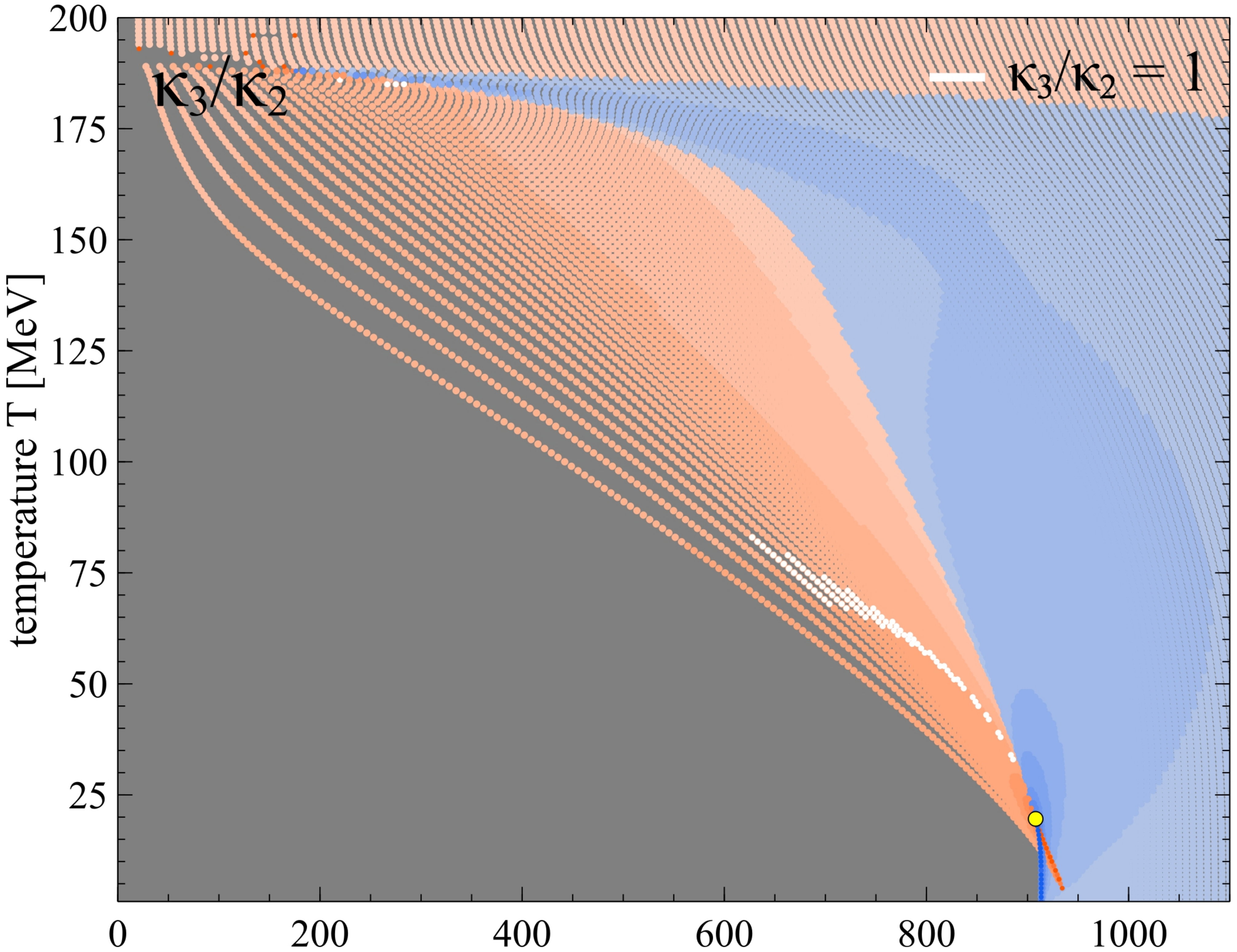

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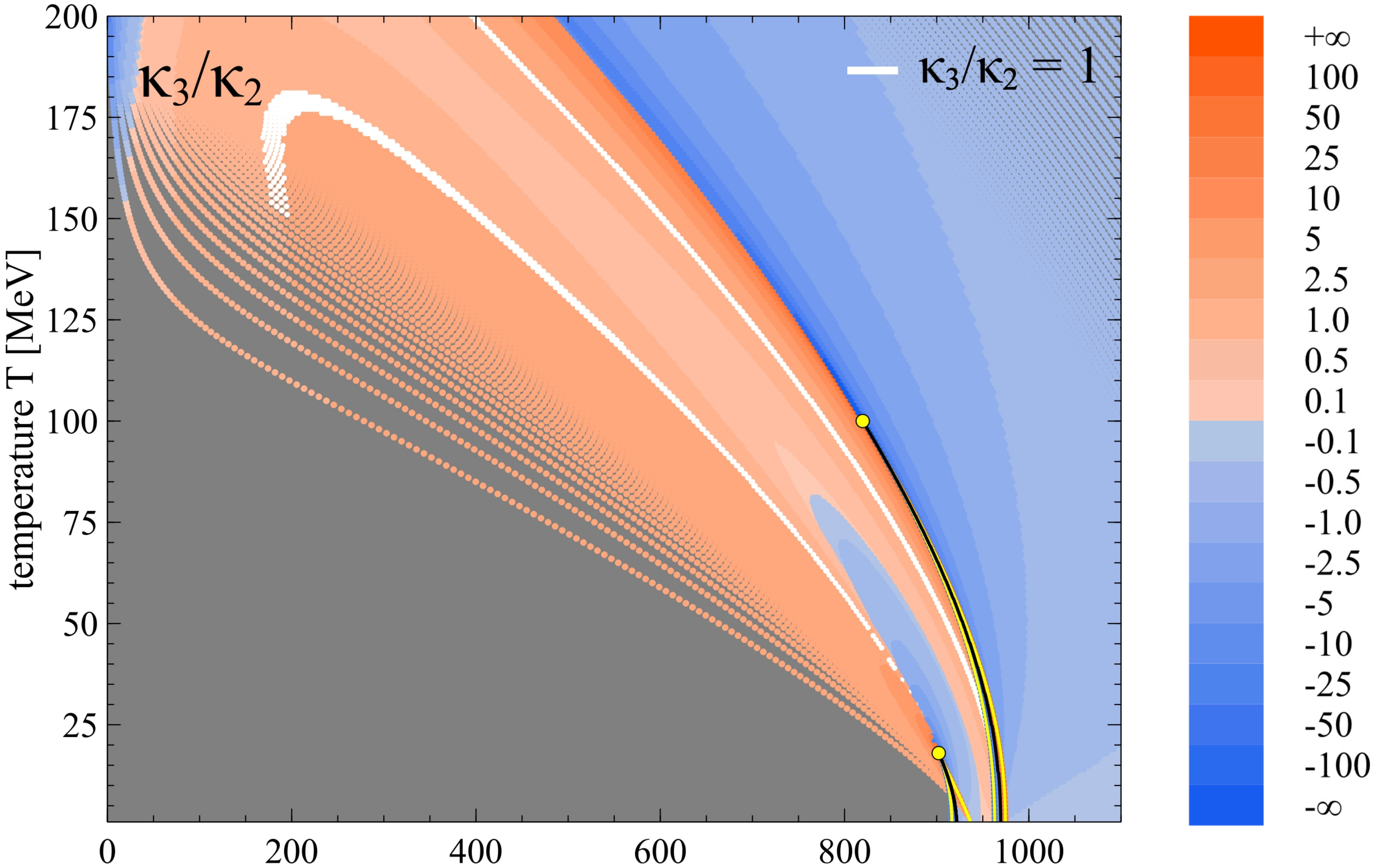

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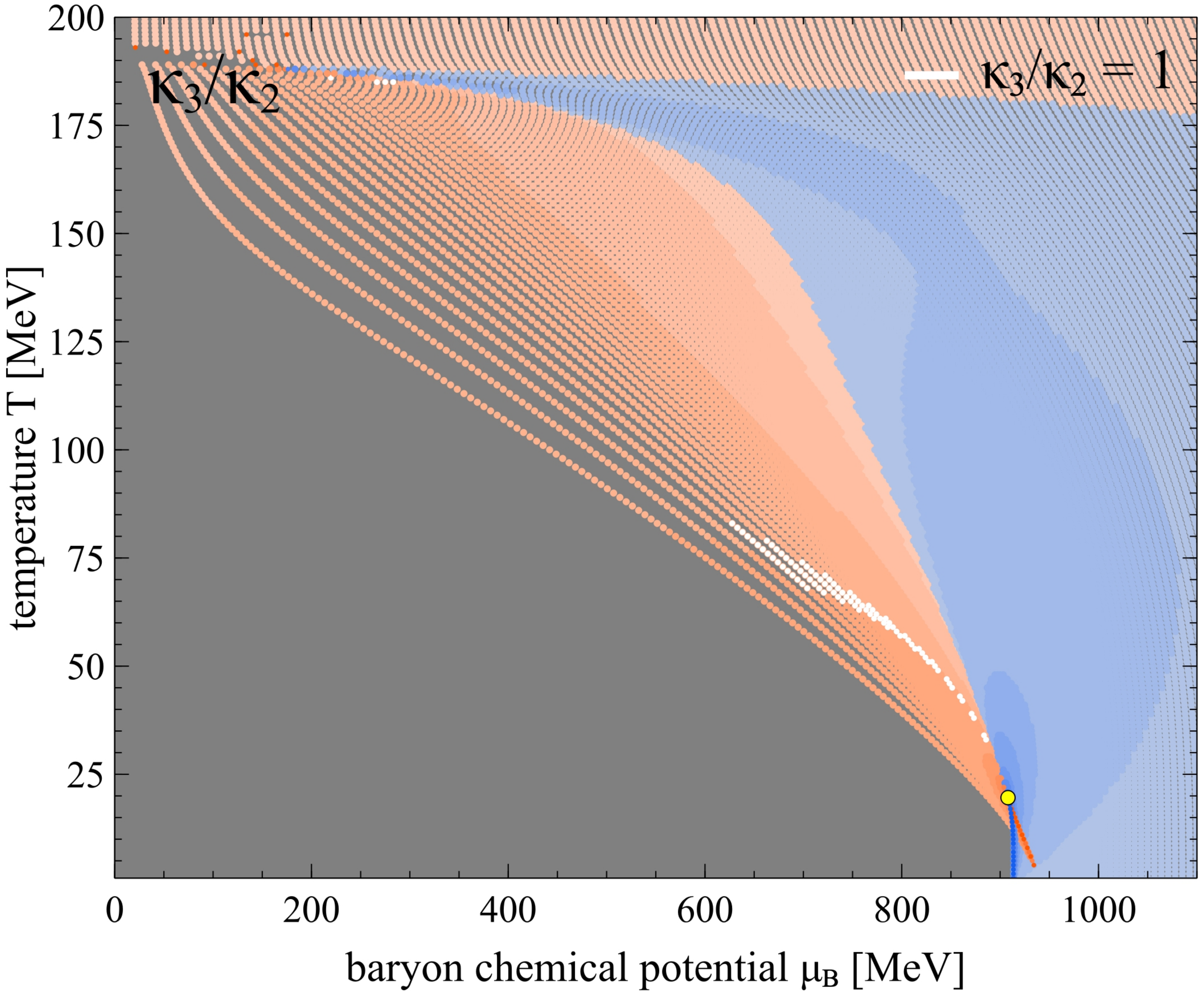

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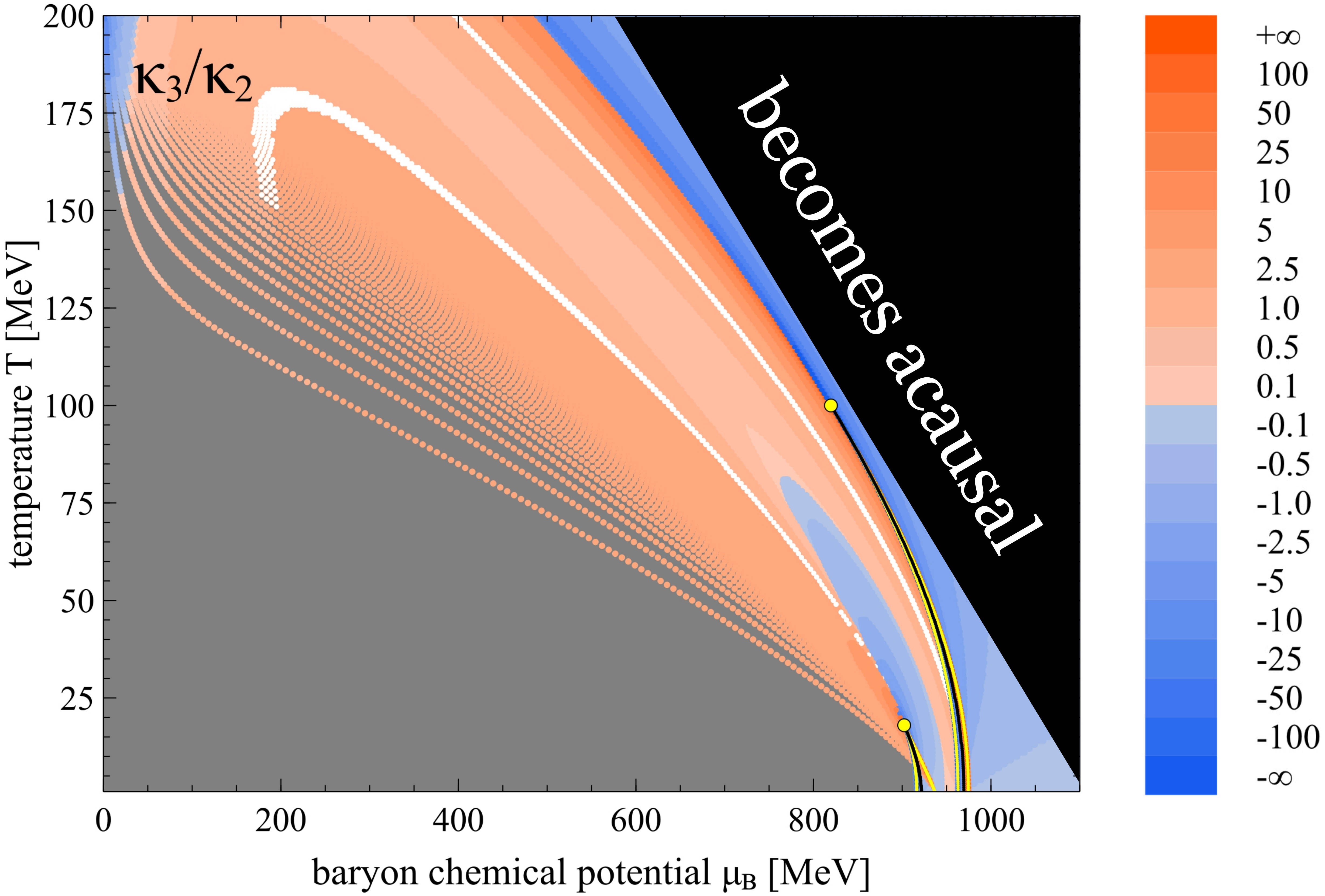

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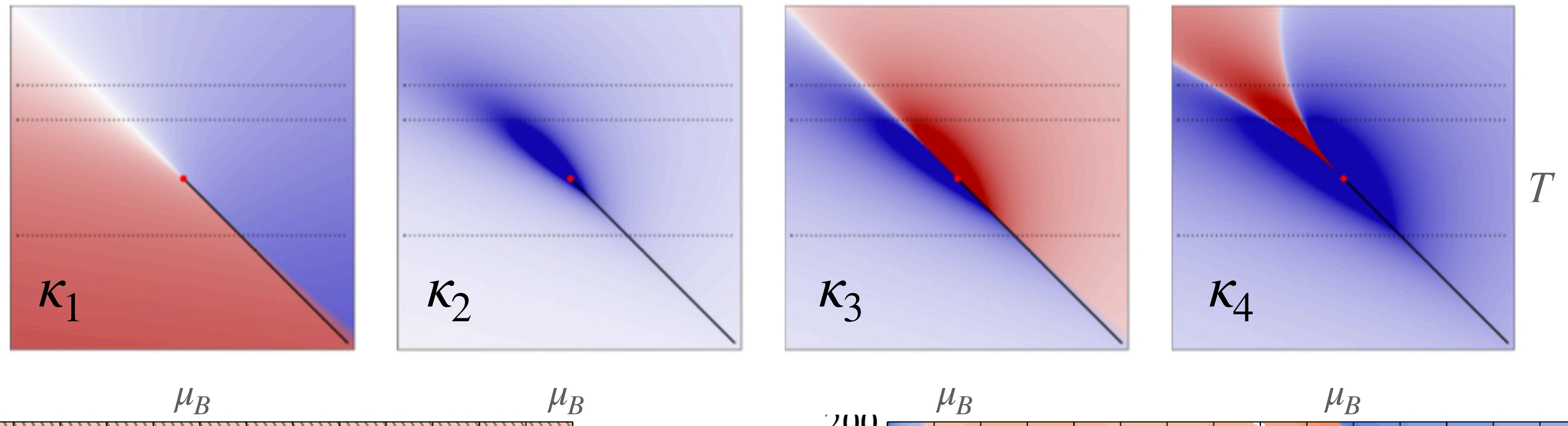
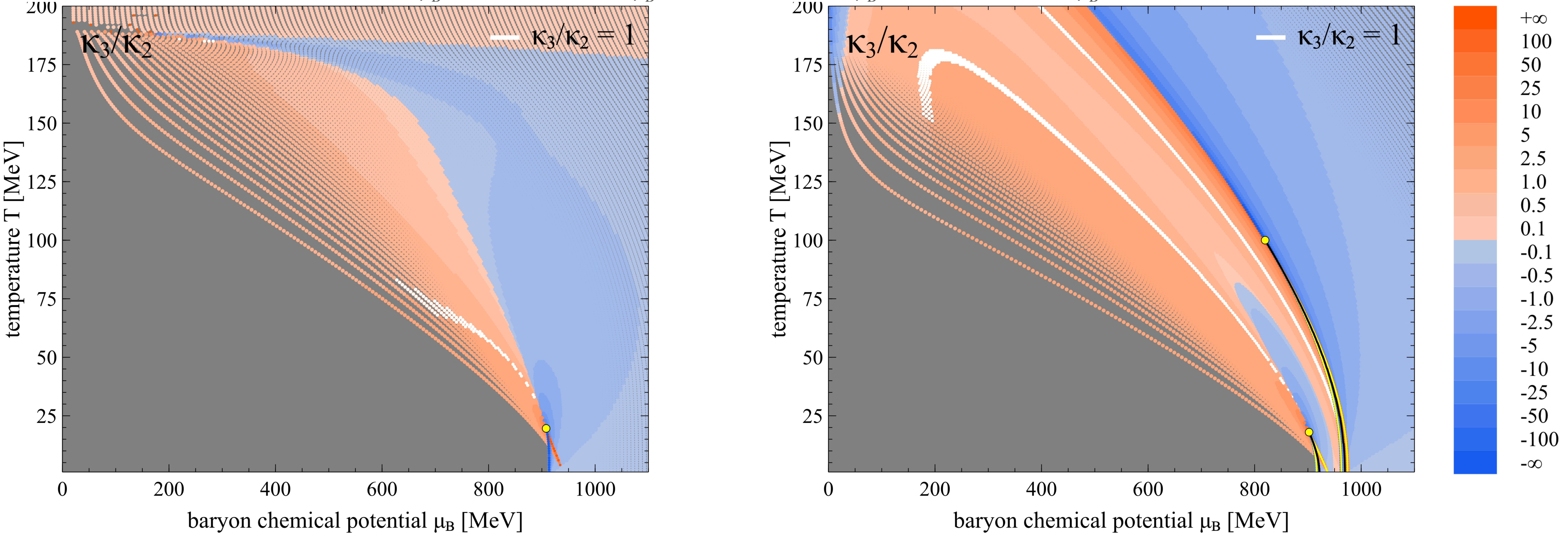


A. Bzdak *et al.*, Physics Reports **853** (2020) 1-87, arXiv:1906.00936, Bzdak:2019pkr



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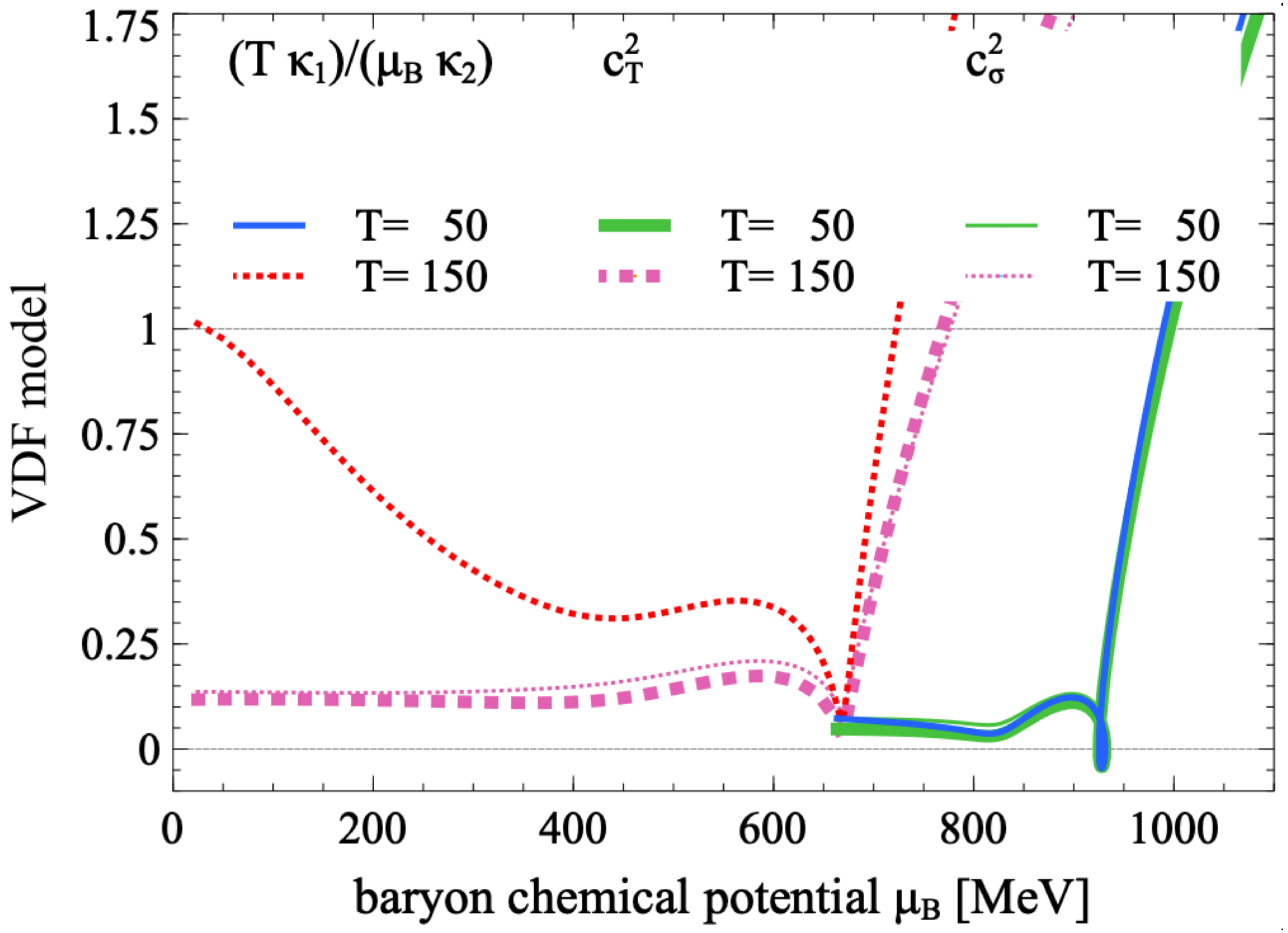
Tests in the VDF model

$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2}$$



AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
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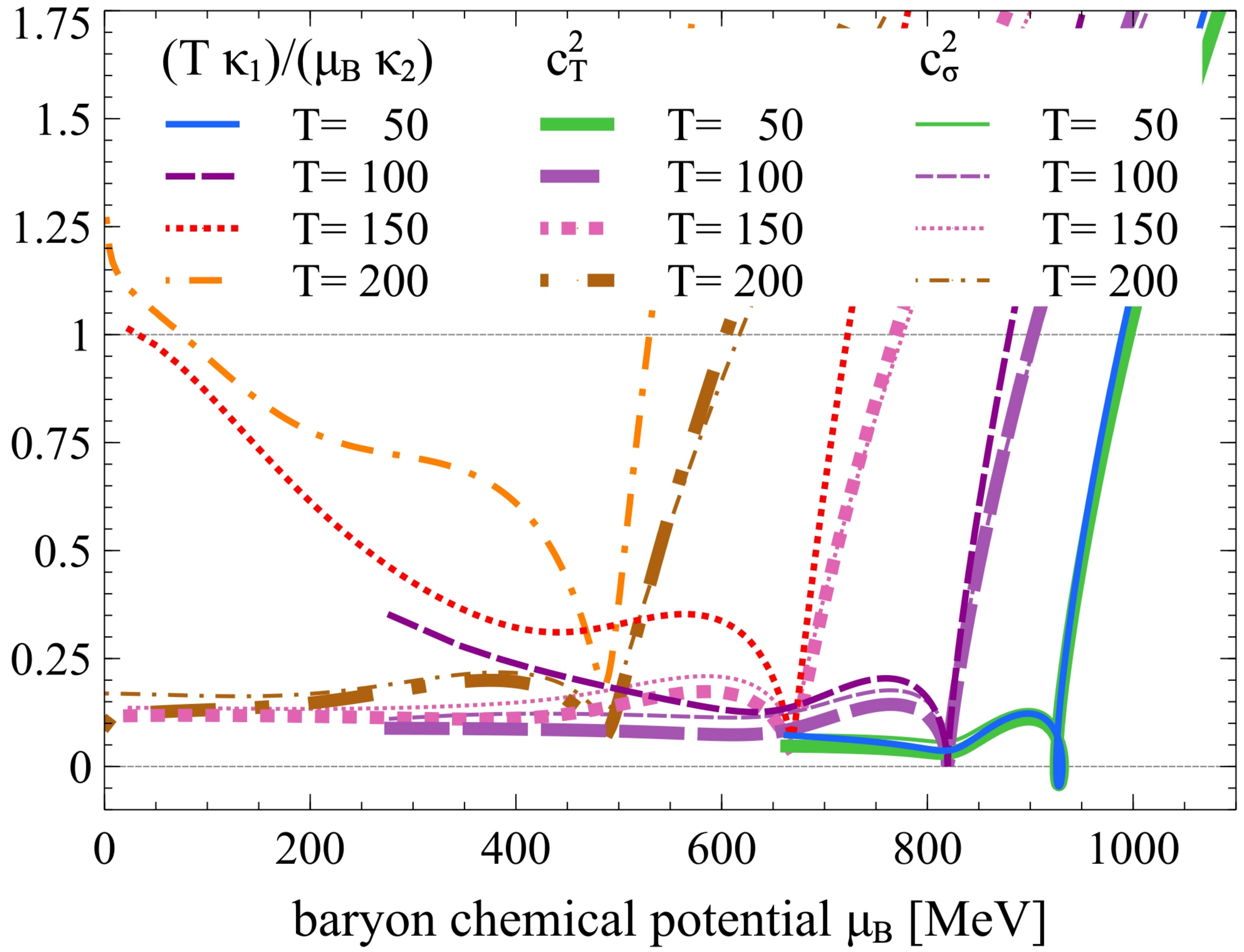

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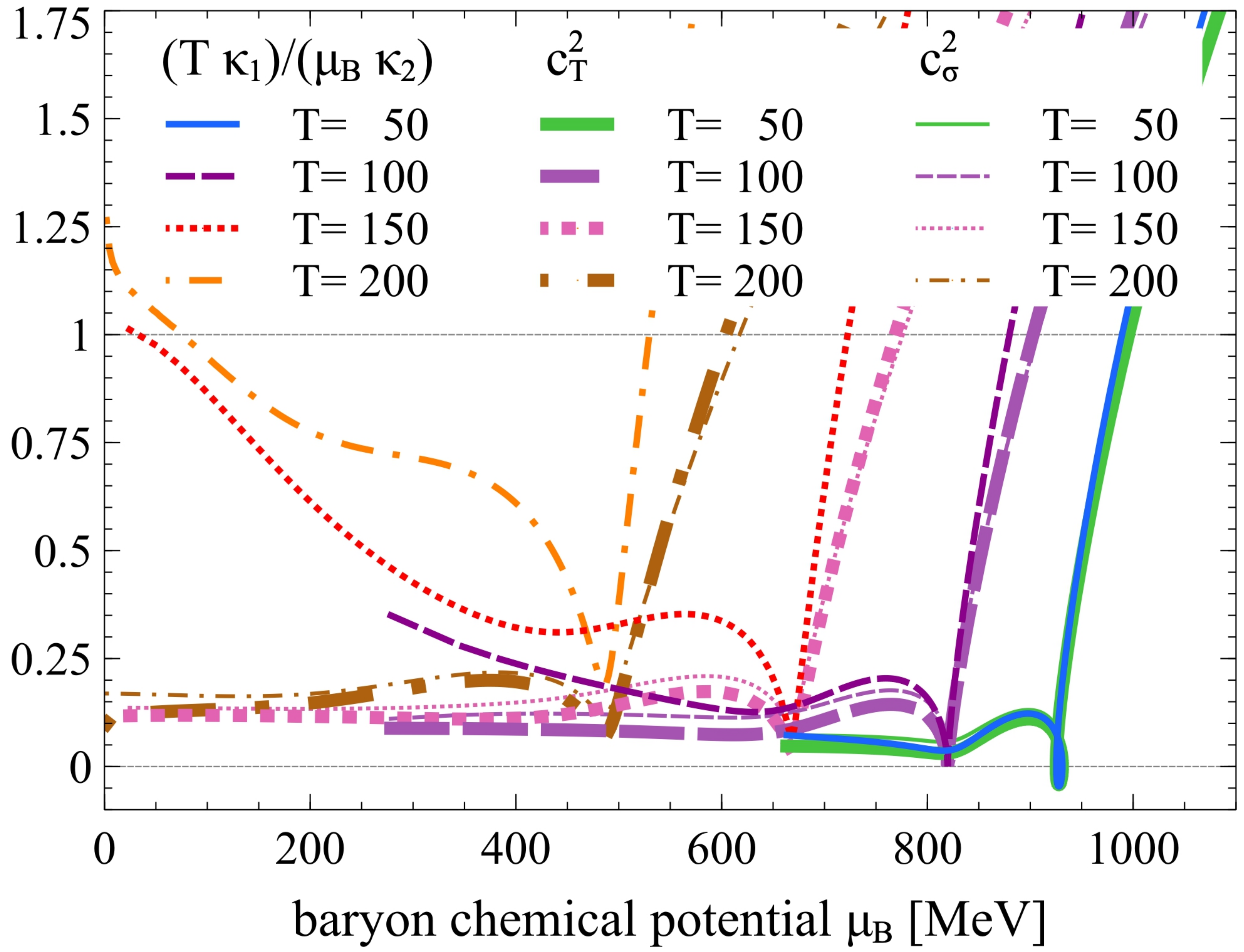

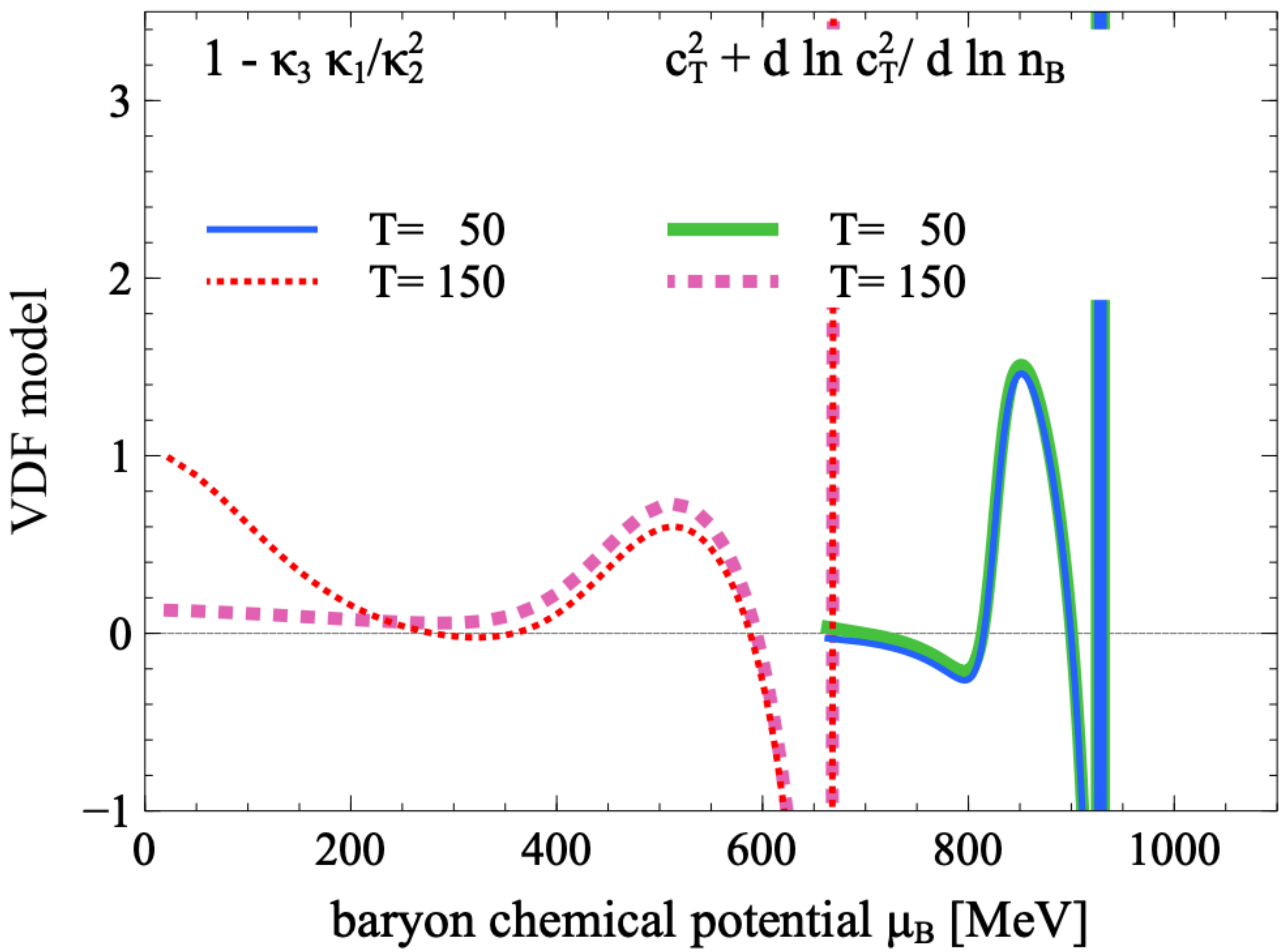

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
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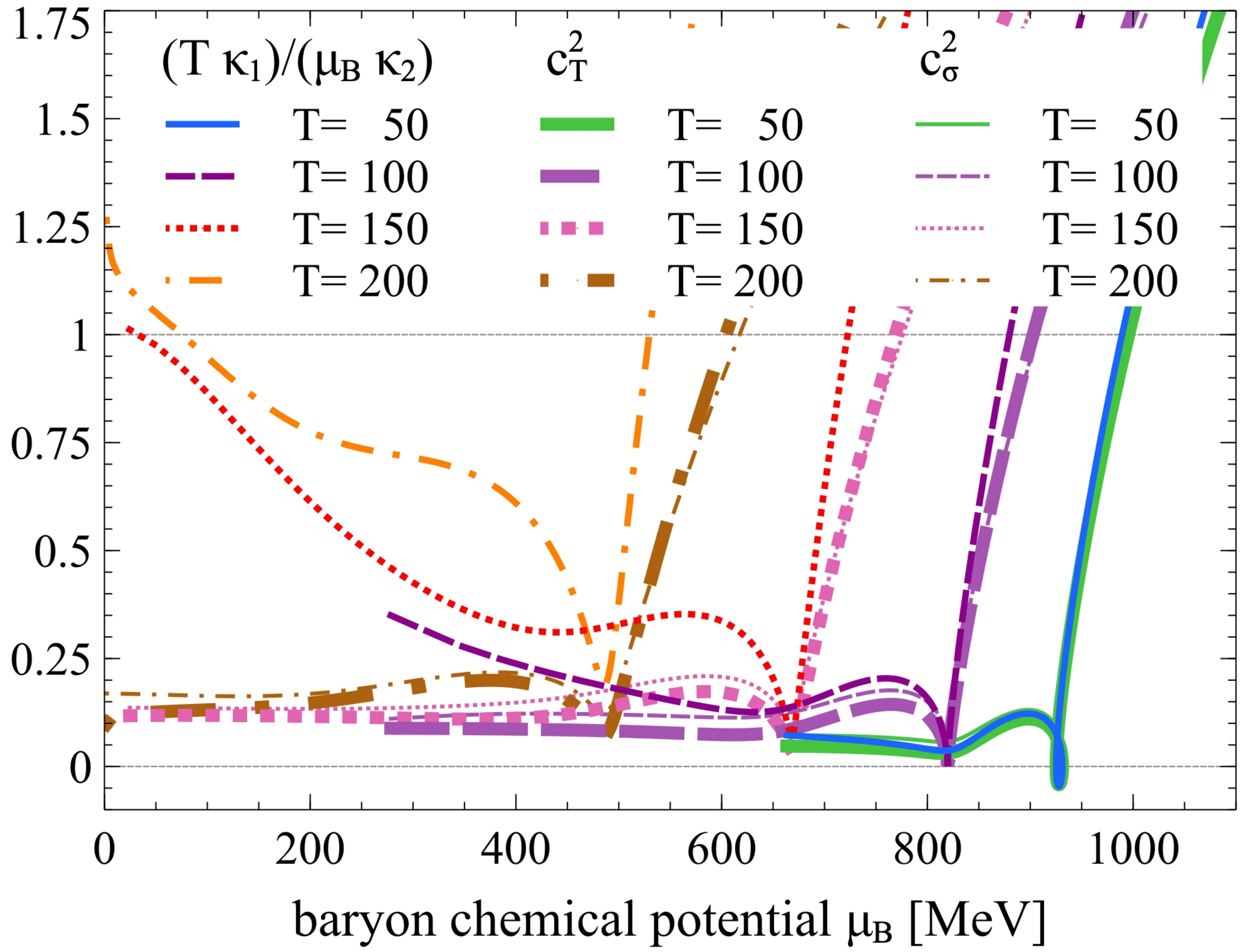
Tests in the VDF model

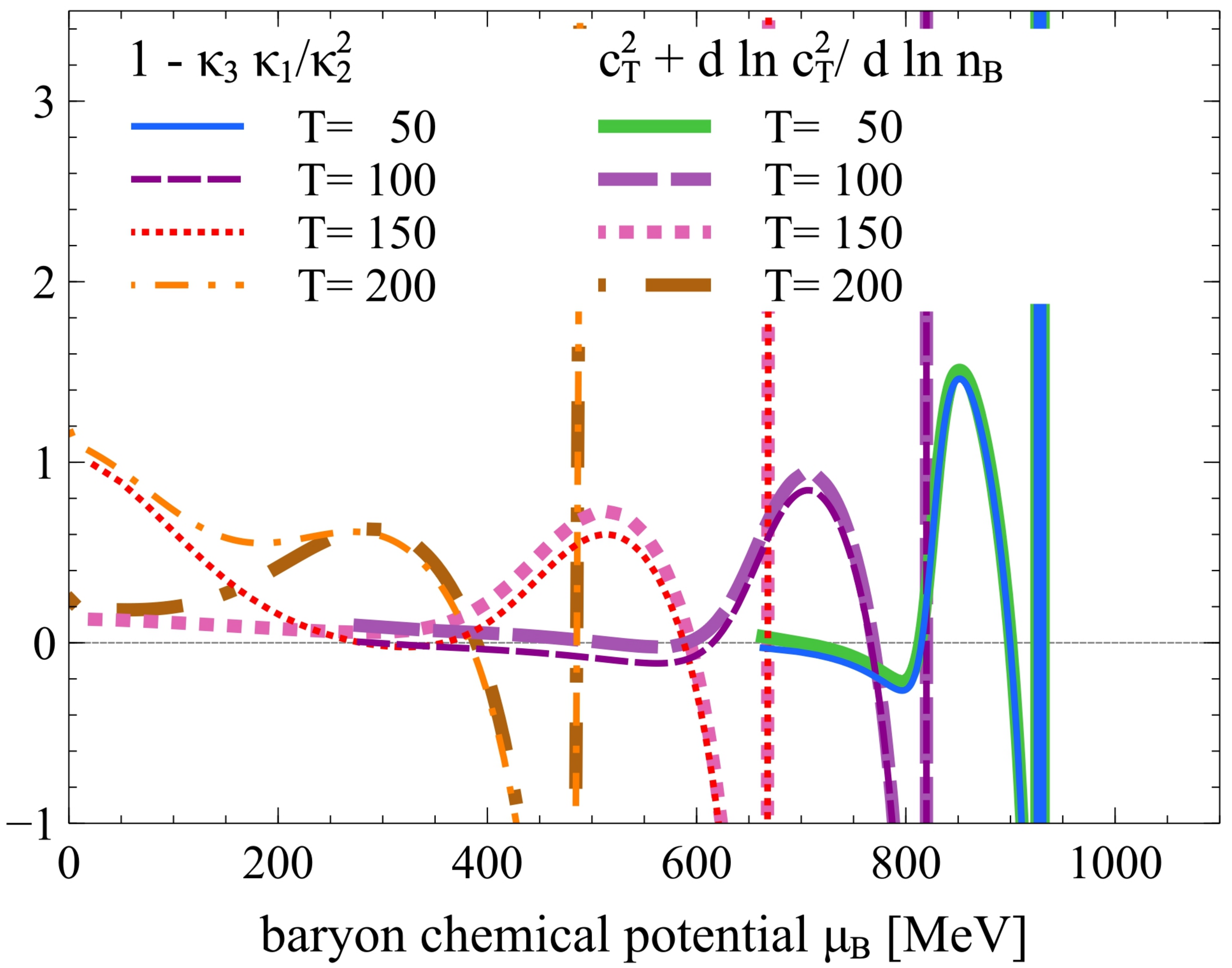
$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2}$$

works for $\mu_B \gtrsim 600$ MeV and $T < 150$

$$\left(\frac{d\ln c_T^2}{d\ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3\kappa_1}{\kappa_2^2}$$

works for $\mu_B \gtrsim 200$ MeV





AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

Tests in the Walecka model

$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2}$$

works for

$\mu_B \gtrsim 600$ MeV and $T < 150$

$$\left(\frac{d\ln c_T^2}{d\ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3\kappa_1}{\kappa_2^2}$$

works for

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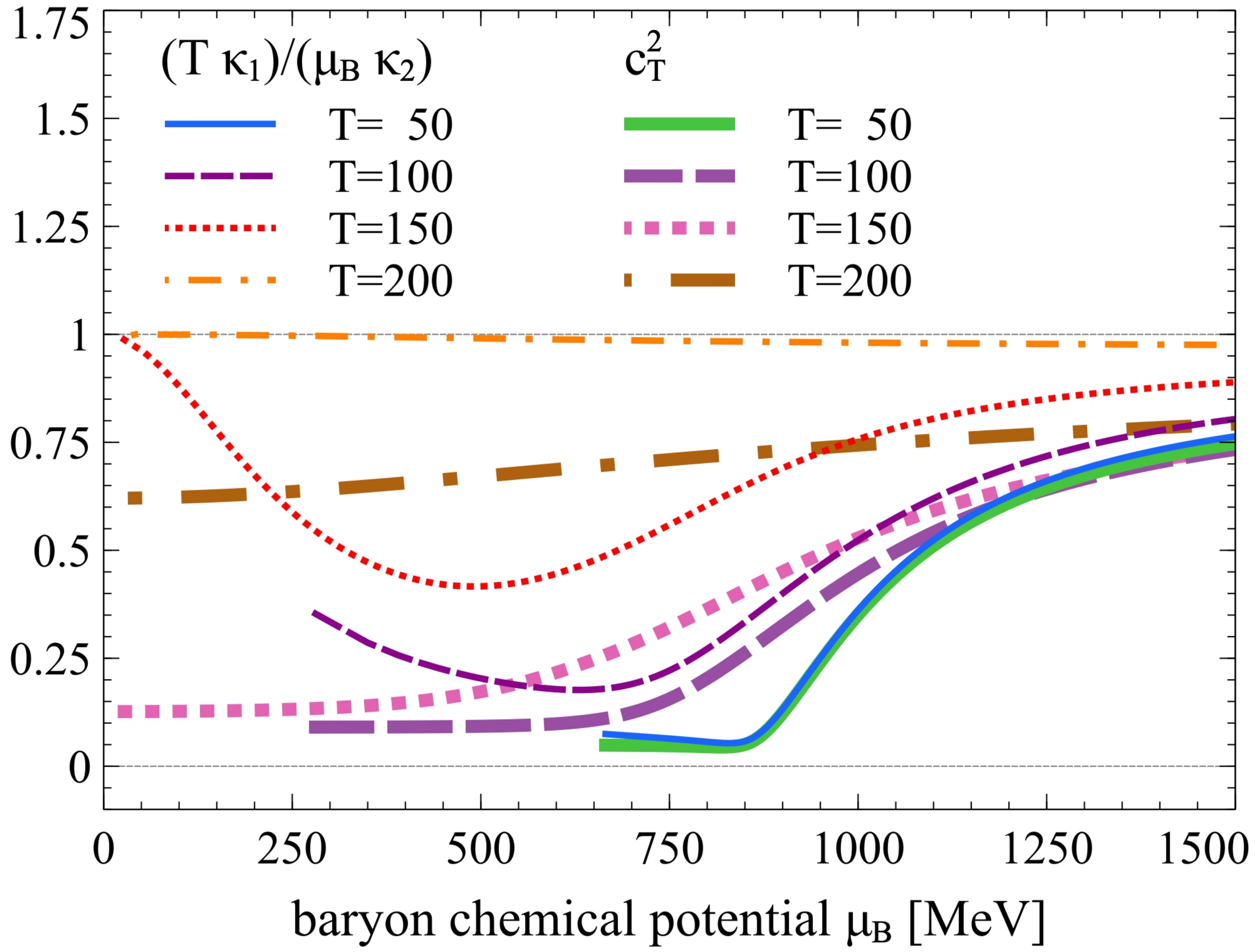

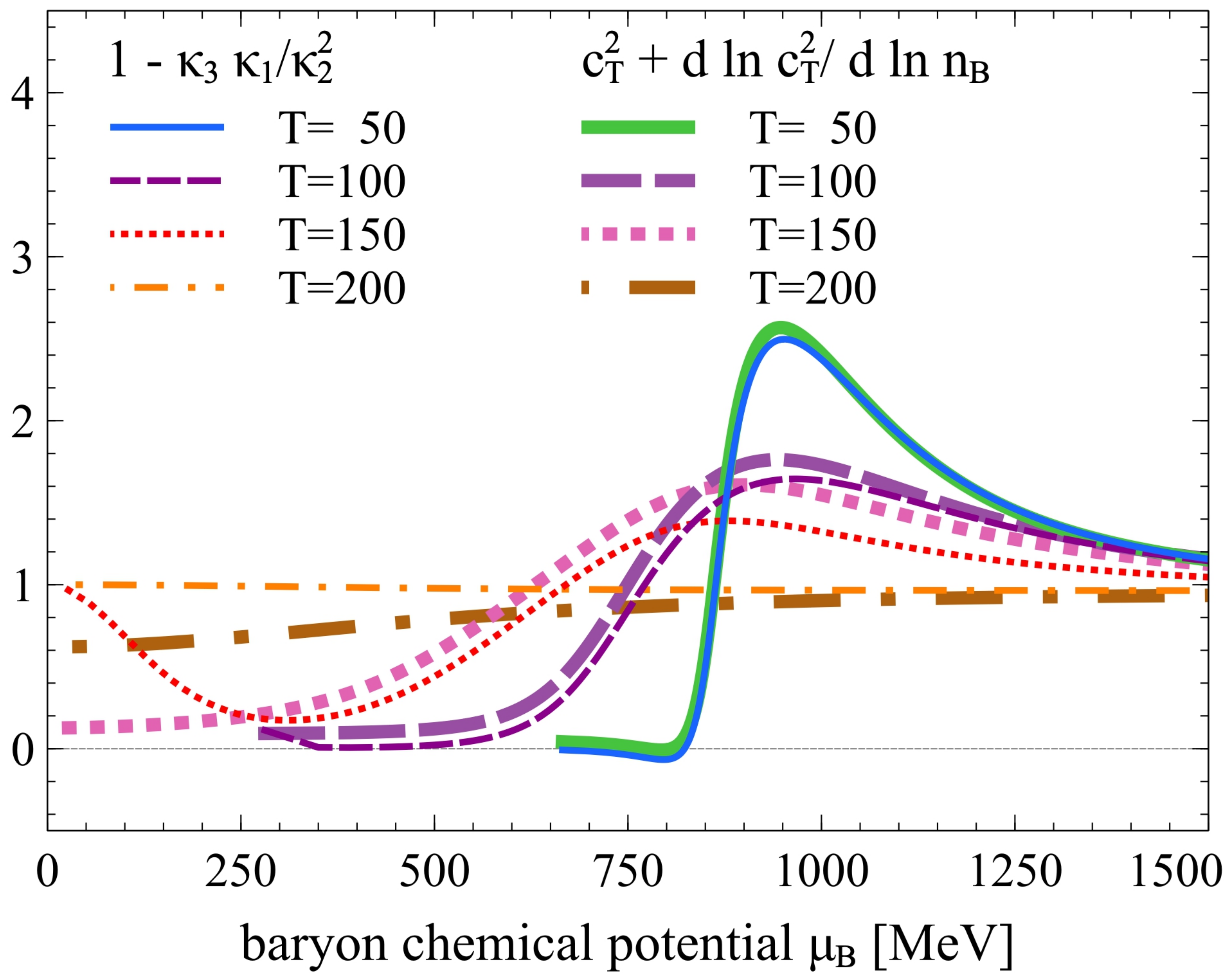

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
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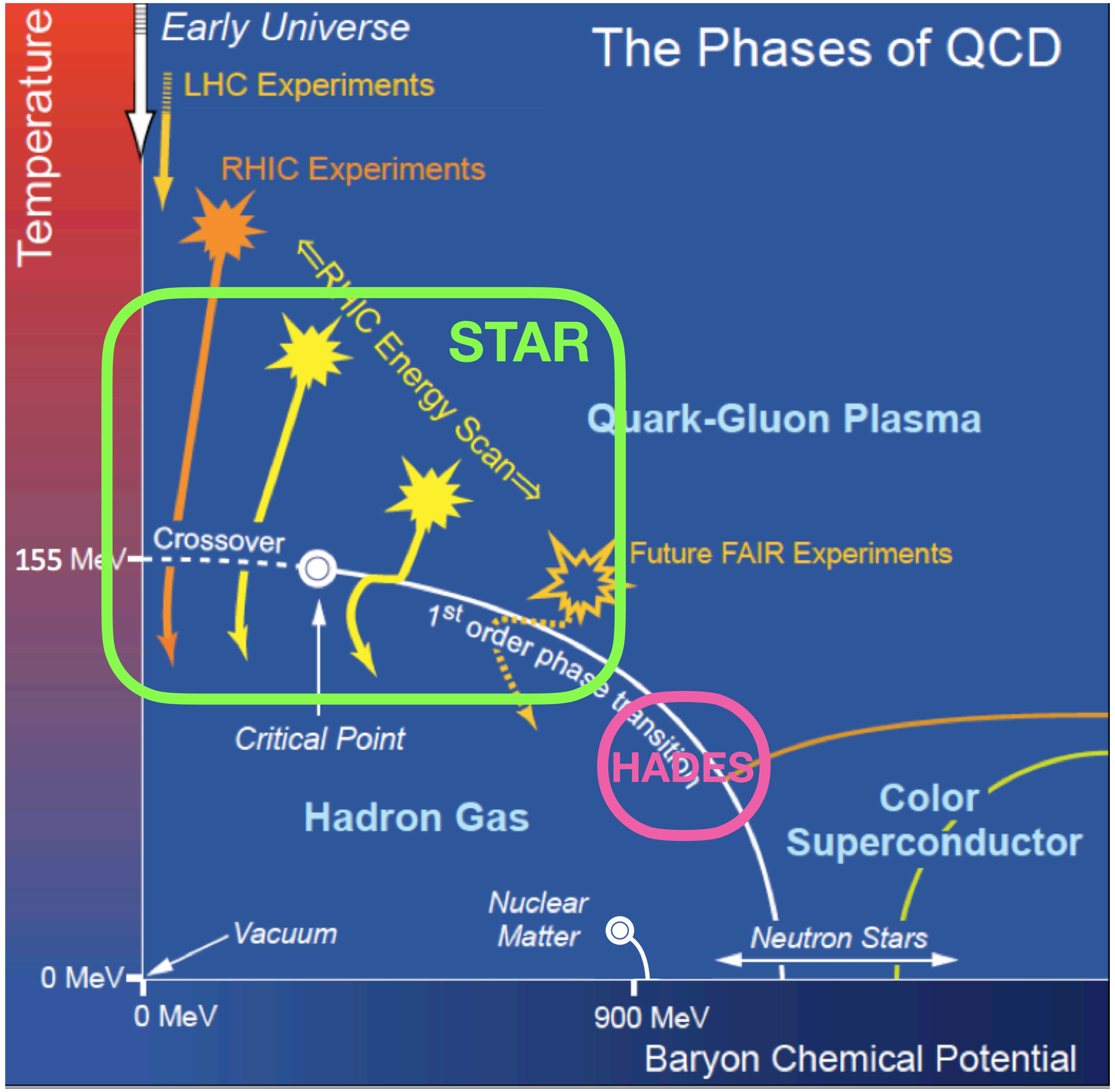
What we see in experimental data

Experimental data



The freeze-out parameters (T_{fo}, μ_{fo}) are obtained from particle yields:

$\sqrt{s}$ [GeV]	$T_{\rm fo}$ [MeV]	$\mu_{\rm fo}$ [MeV]
200	164.3	28
62.4	160.3	70
54.4	160.0	83
39	156.4	103
27	155.0	144
19.6	153.9	188
14.5	151.6	264
11.5	149.4	287
7.7	144.3	398
2.4	65	784

Cumulants κ_2, κ_3, ... are fluctuations around the means

M. Abdallah *et al.* (STAR), Phys. Rev. C **104** (2021) no. 2 024902, arXiv:2101.12413, STAR:2021iop
M. L. for the HADES collaboration (2019), 3rd EMMI Workshop

Experimental data

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

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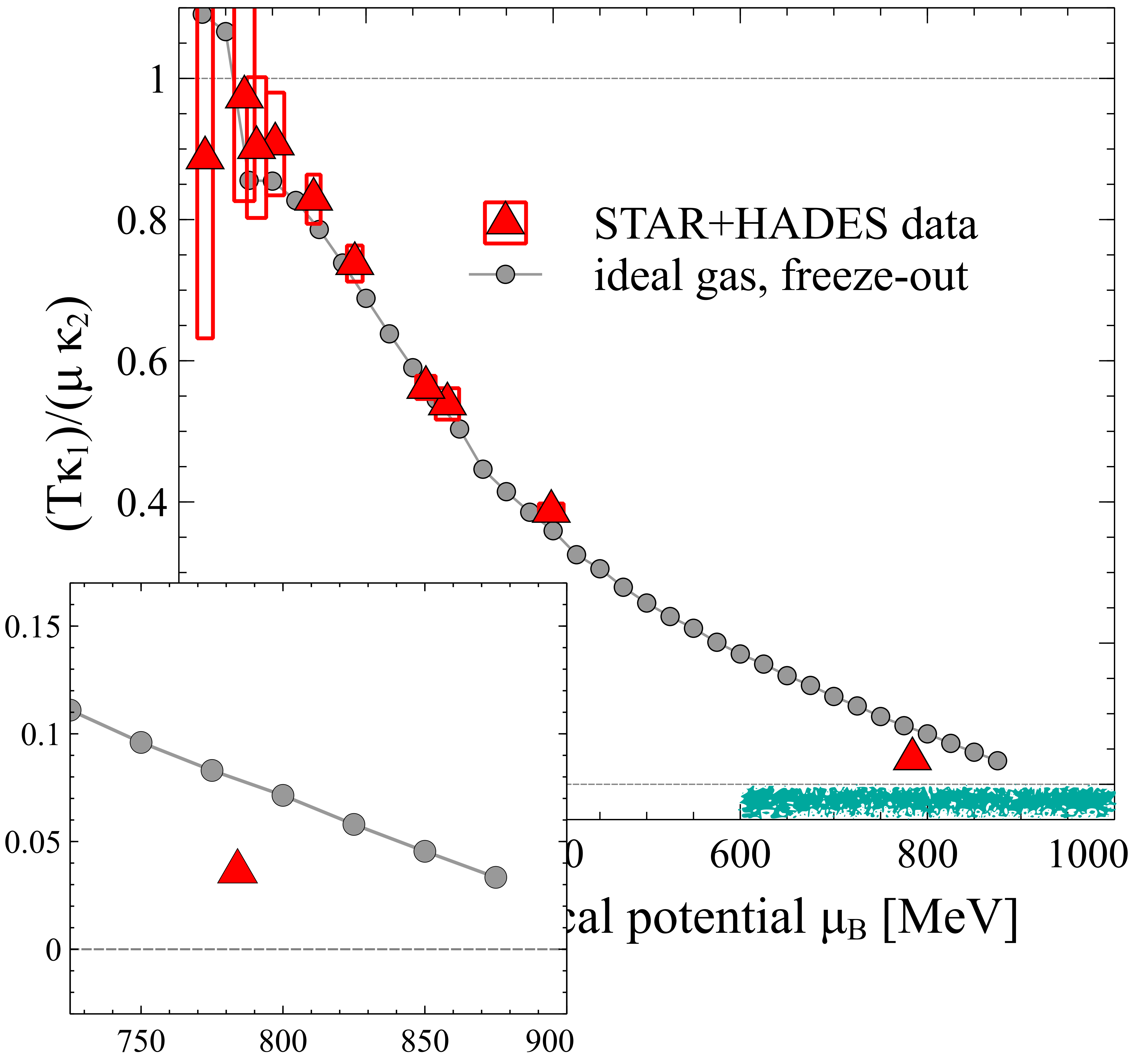
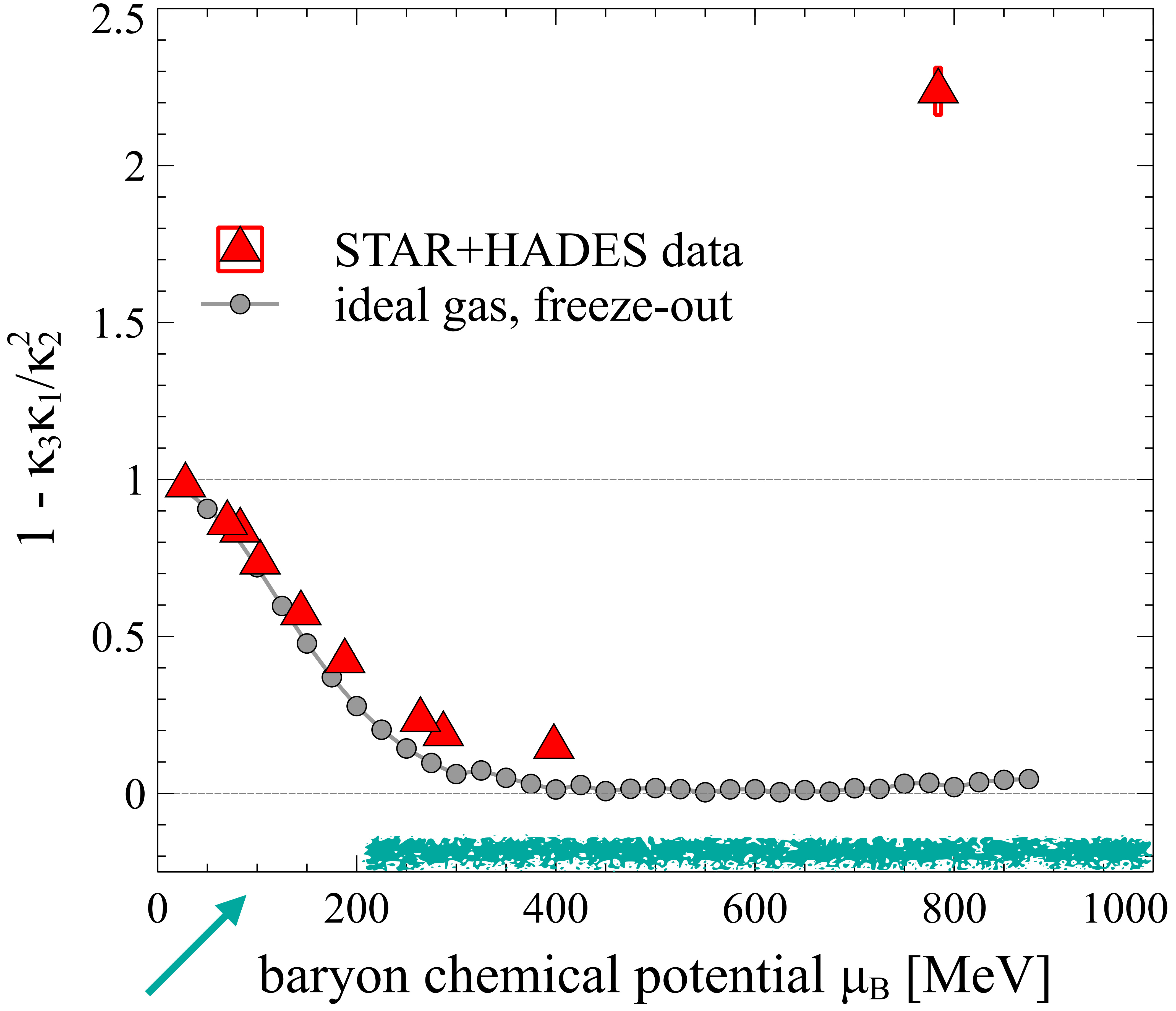
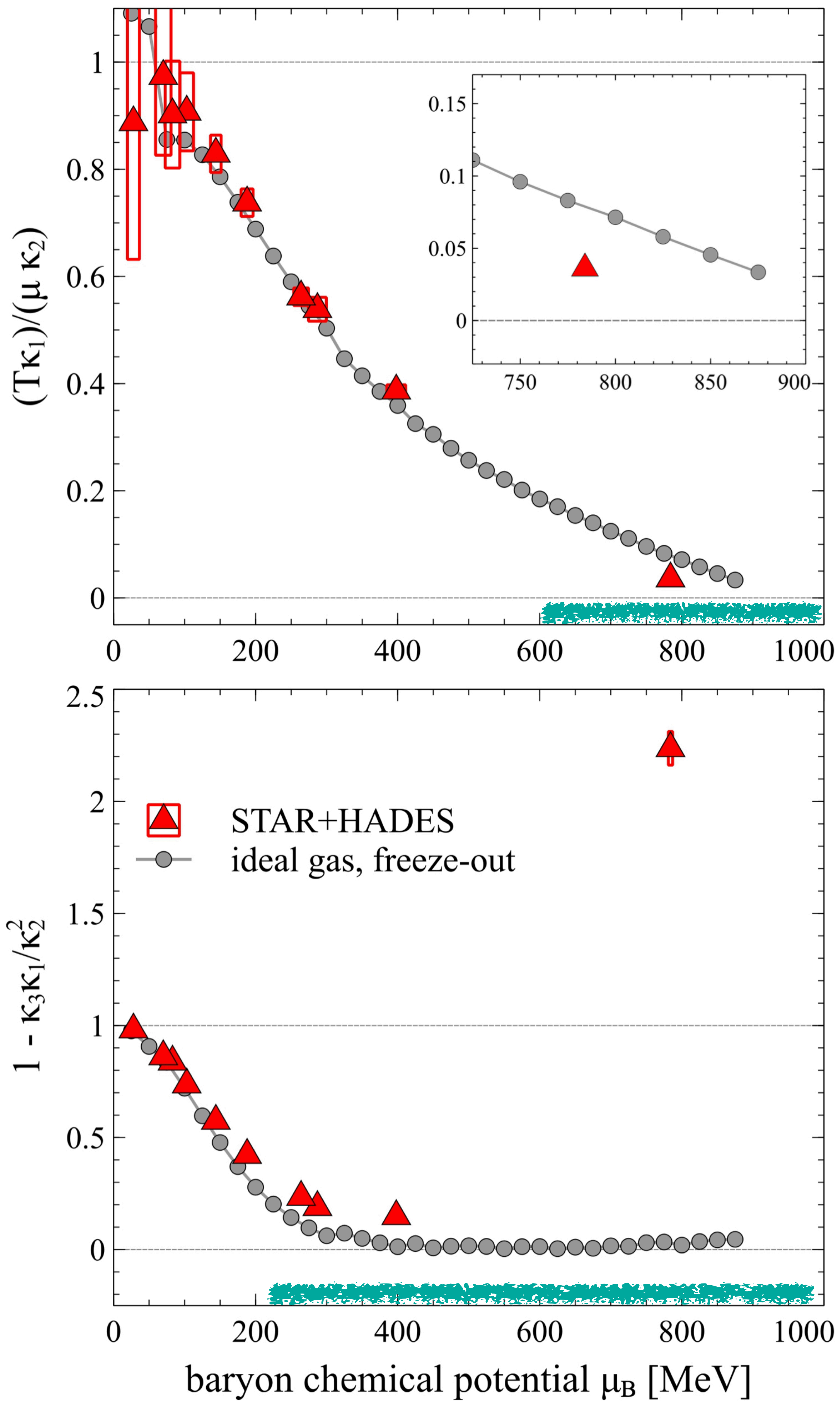




region of validity

M. Abdallah *et al.* (STAR), Phys. Rev. C **104** (2021) 2, 024902,
arXiv:2101.12413 ,STAR:2021iop
J. Adamczewski-Musch *et al.* (HADES), Phys. Rev. C **102** (2020),
arXiv:2002.08701, HADES:2020wpc

Experimental data: can we understand what is happening?



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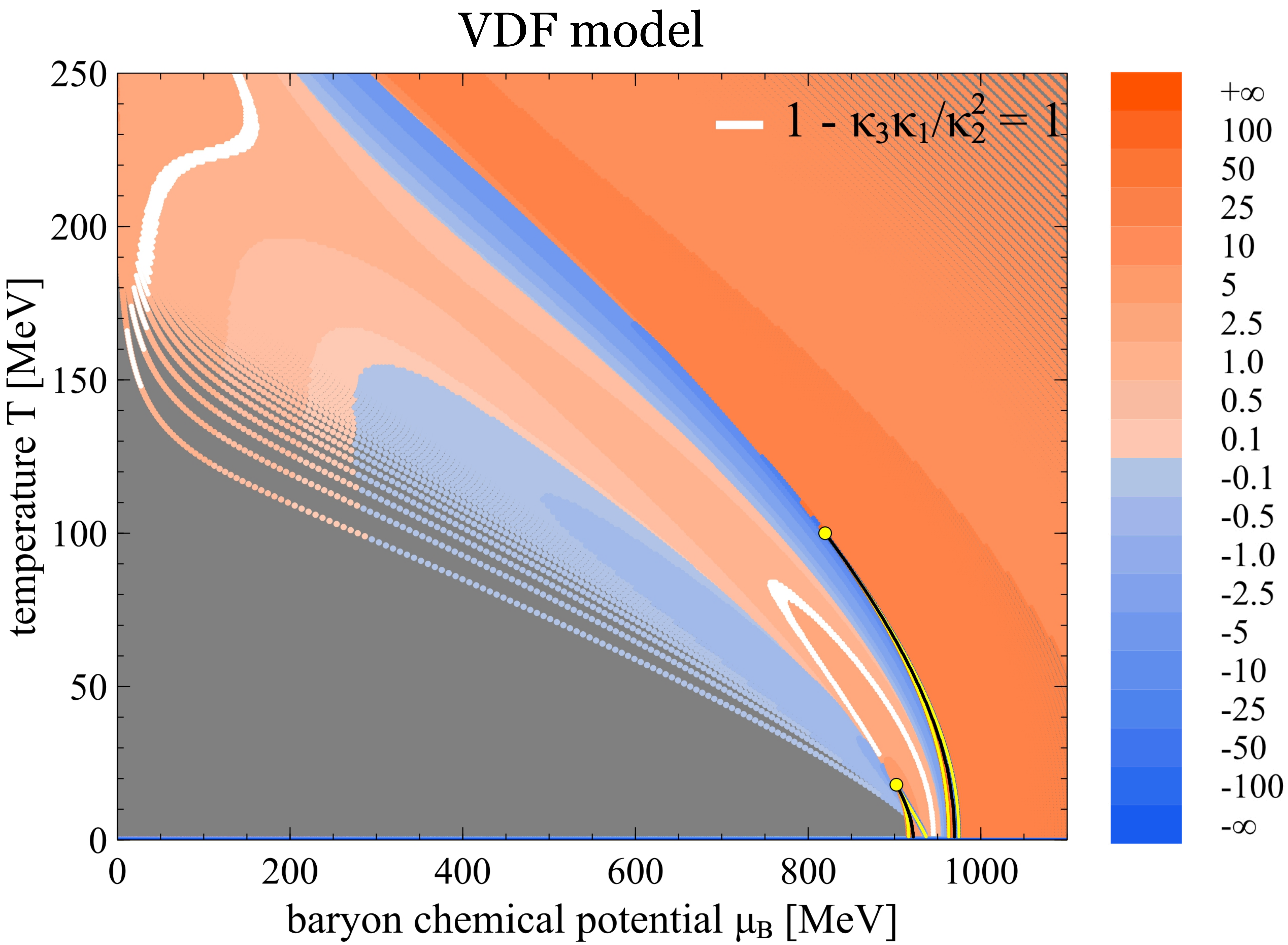

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
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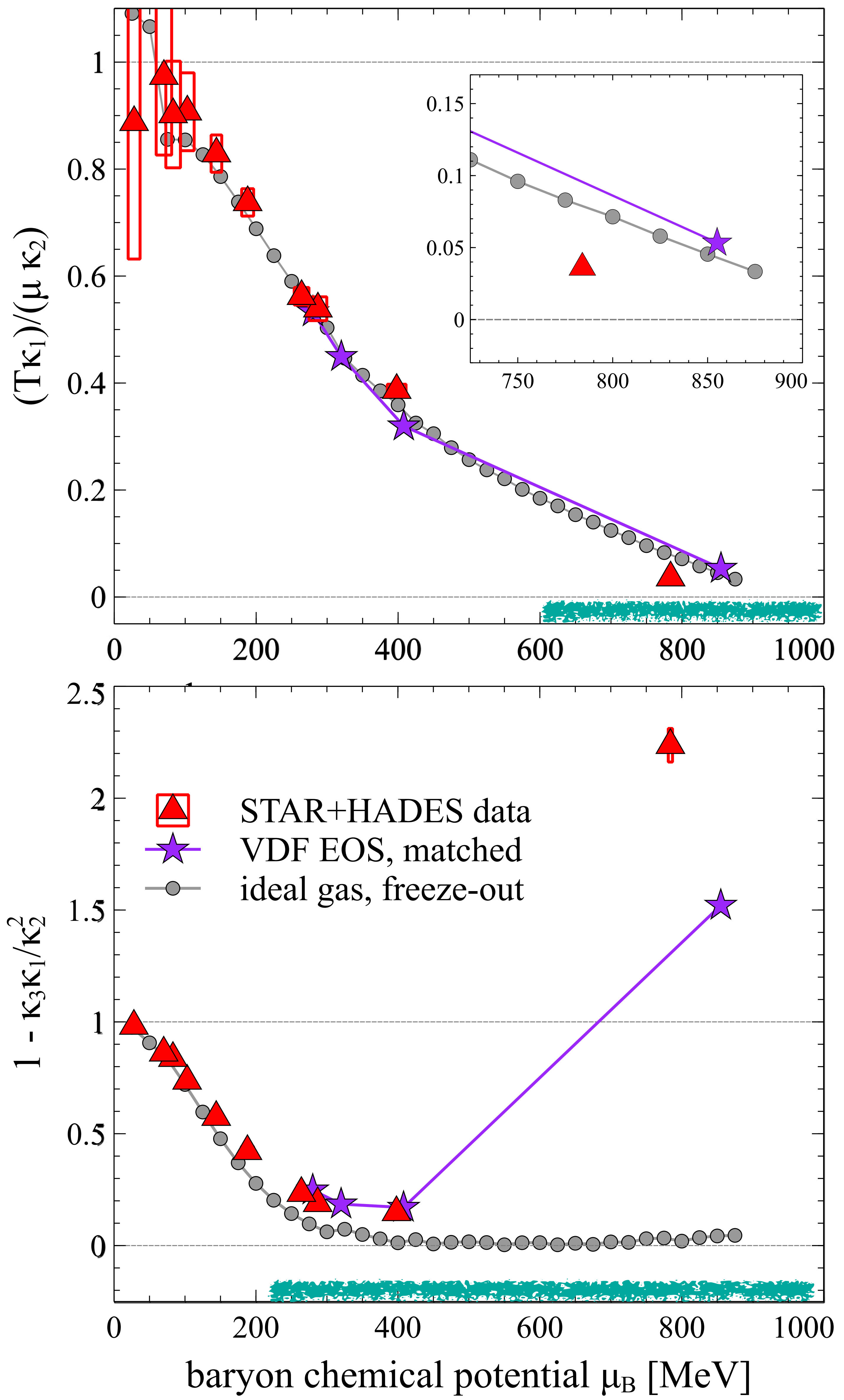
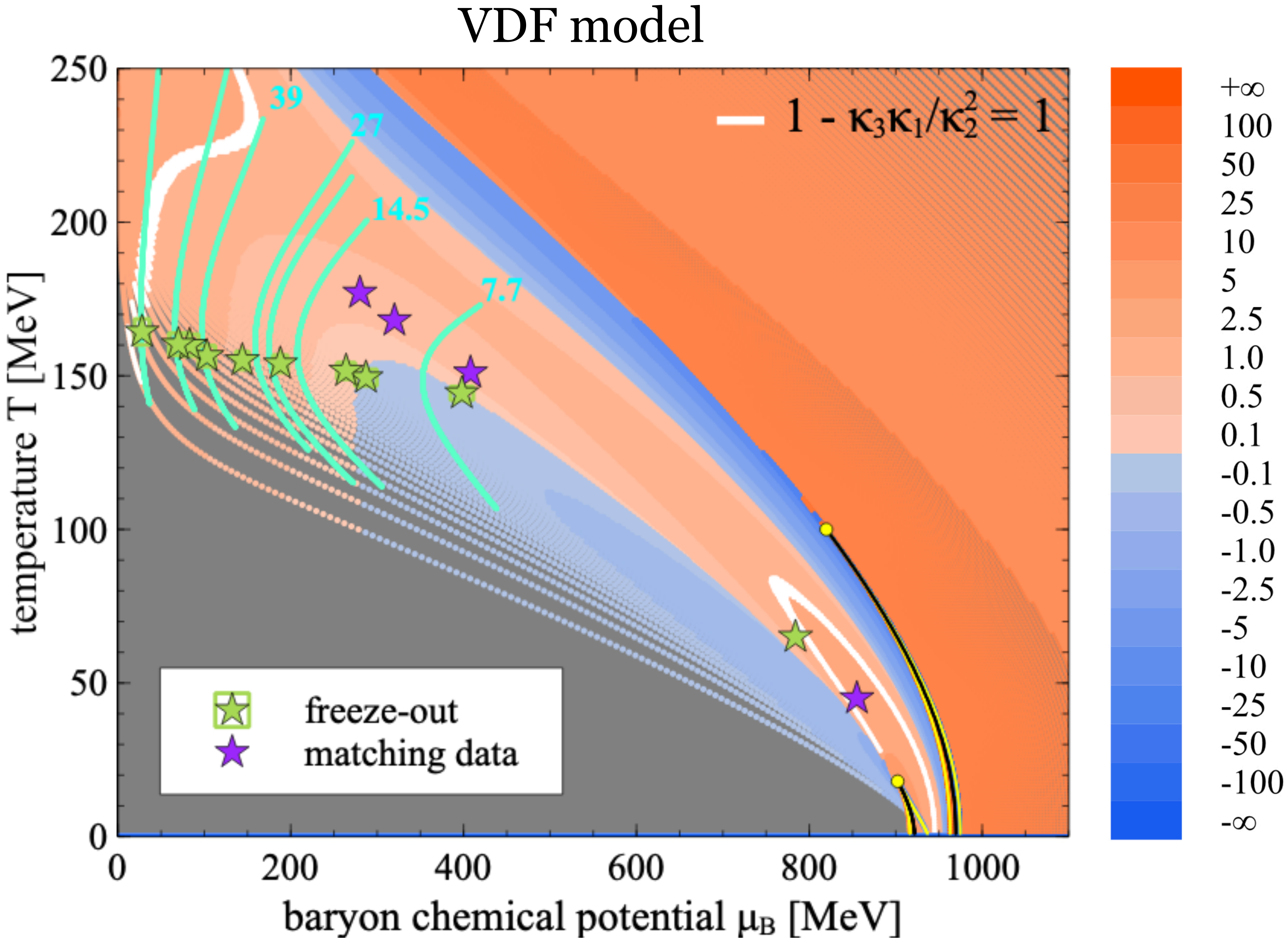


$$c_T^2 \approx \frac{T\kappa_1}{\mu_B \kappa_2} \qquad \left(\frac{d \ln c_T^2}{d \ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3 \kappa_1}{\kappa_2^2}$$



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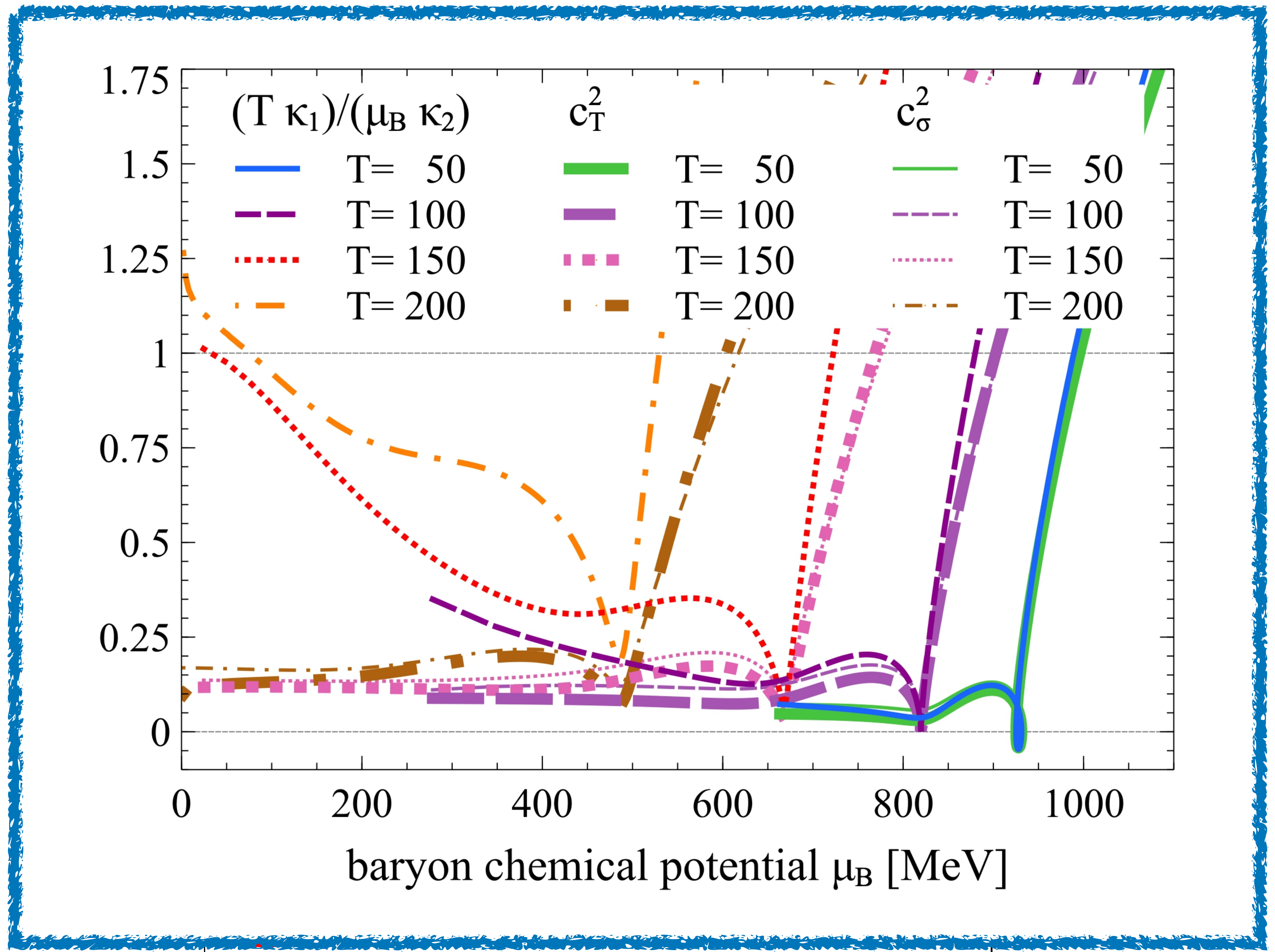

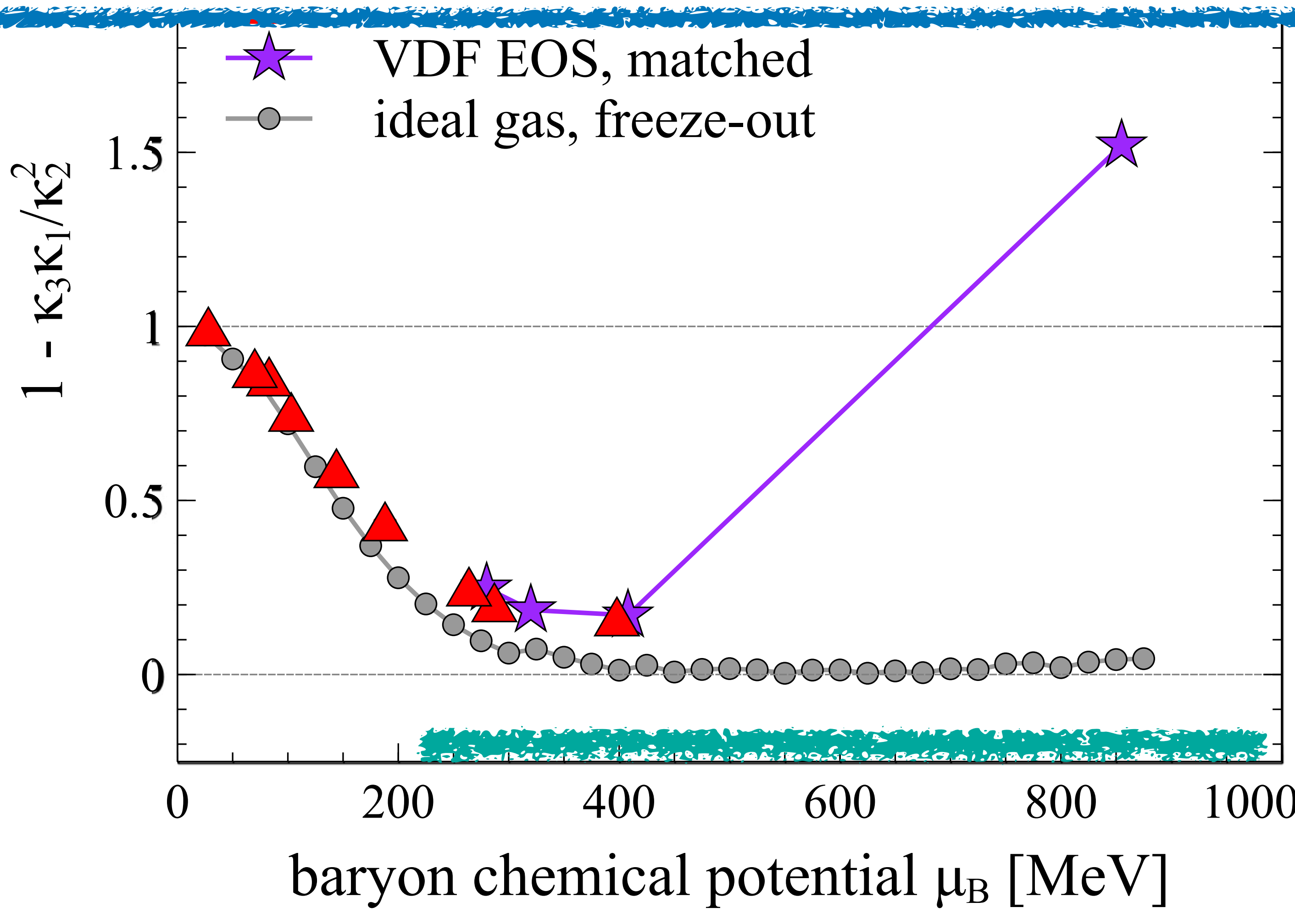

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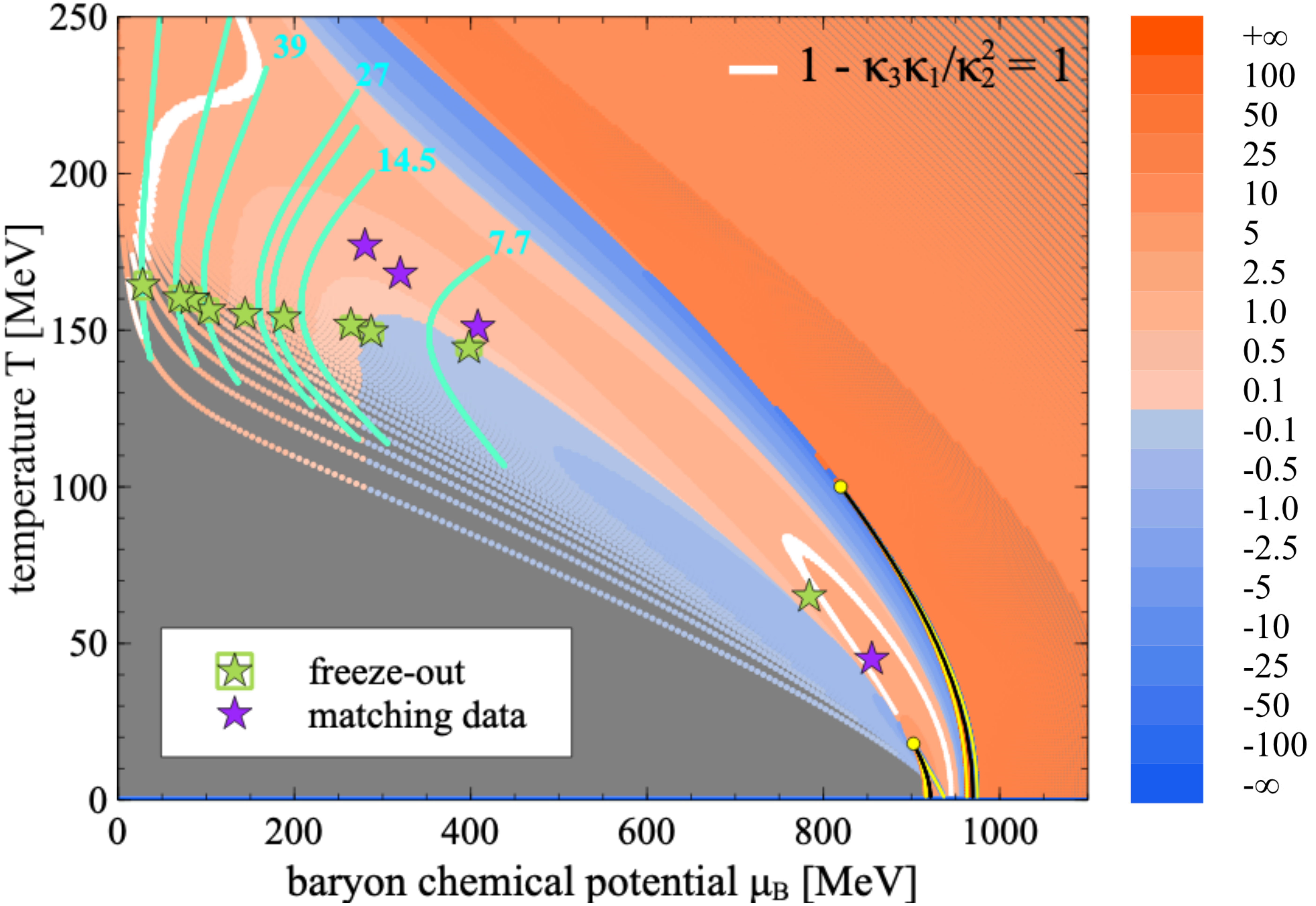
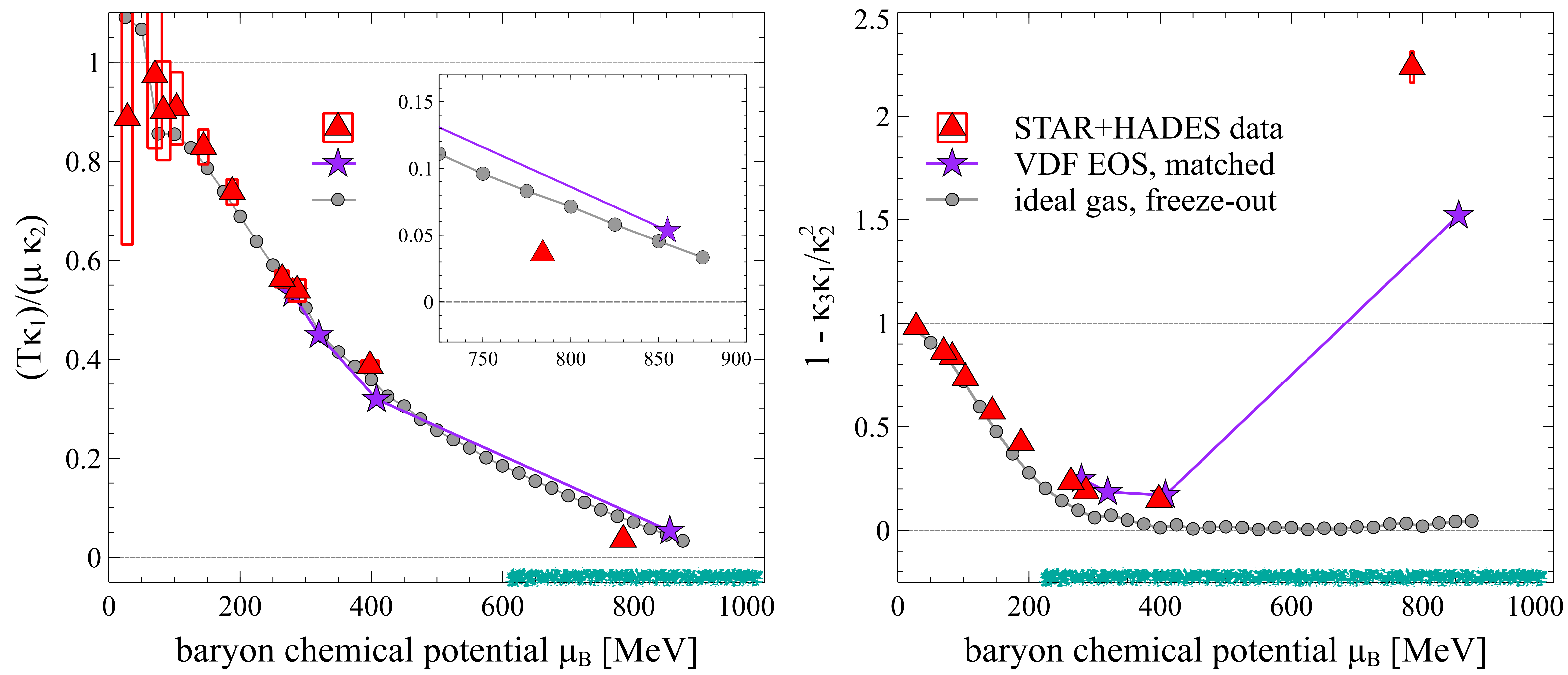
VDF model



AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

The results prompt more questions

STAR + HADES data



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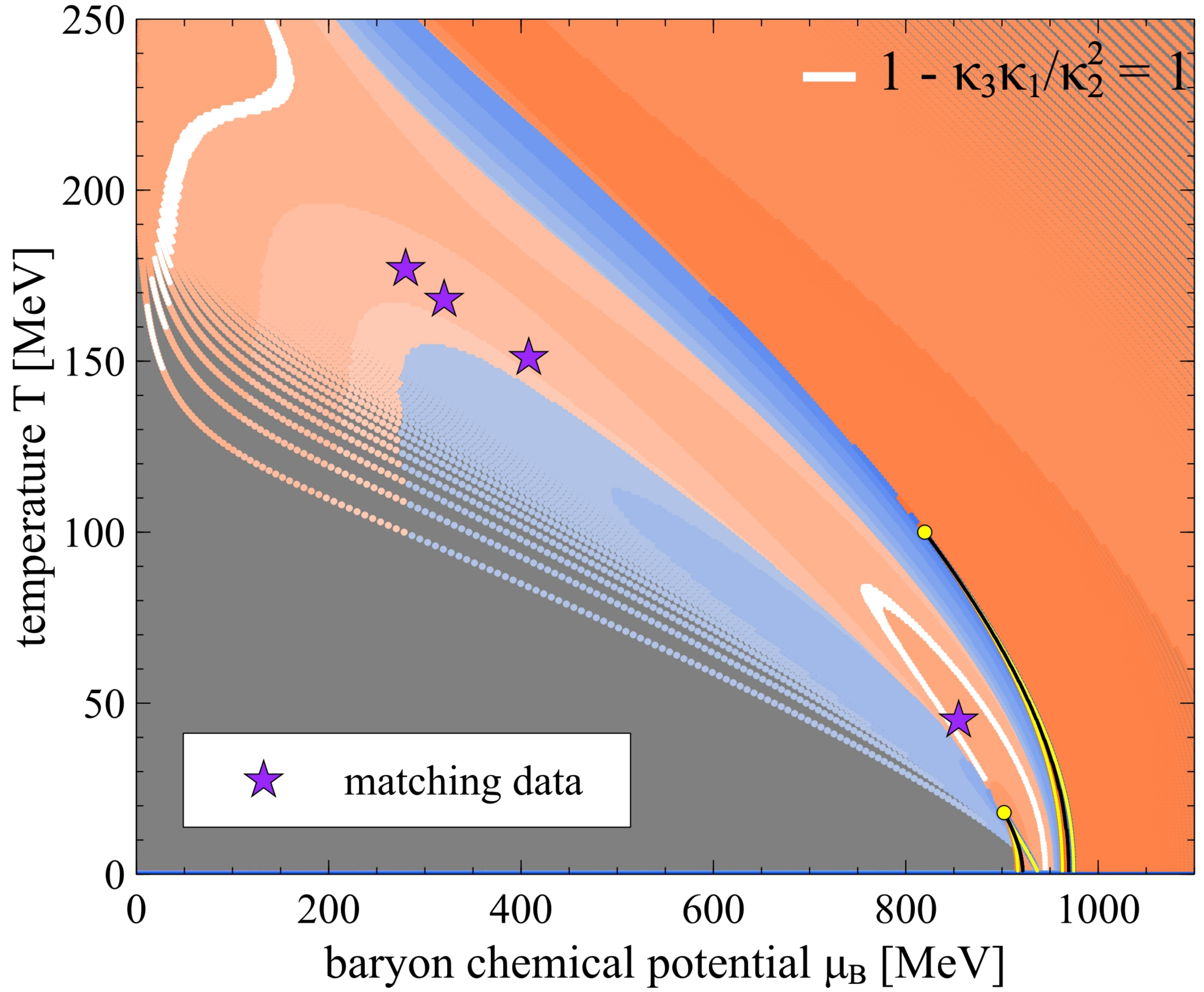


- is behavior of the cumulants at low energies dominated by hadronic effects and the nuclear liquid-gas phase transition?
- can we study c_T^2 in ordinary nuclear matter using very low-energy collisions?
- *something* significant is happening: κ_3 changes sign!

Need more experimental data and comparisons with simulations!

AS, D. Oliinychenko, V. Koch, L. McLerran, Phys. Rev. Lett. **127** (2021) 042303
arXiv:2103.07365, Sorensen:2021zme

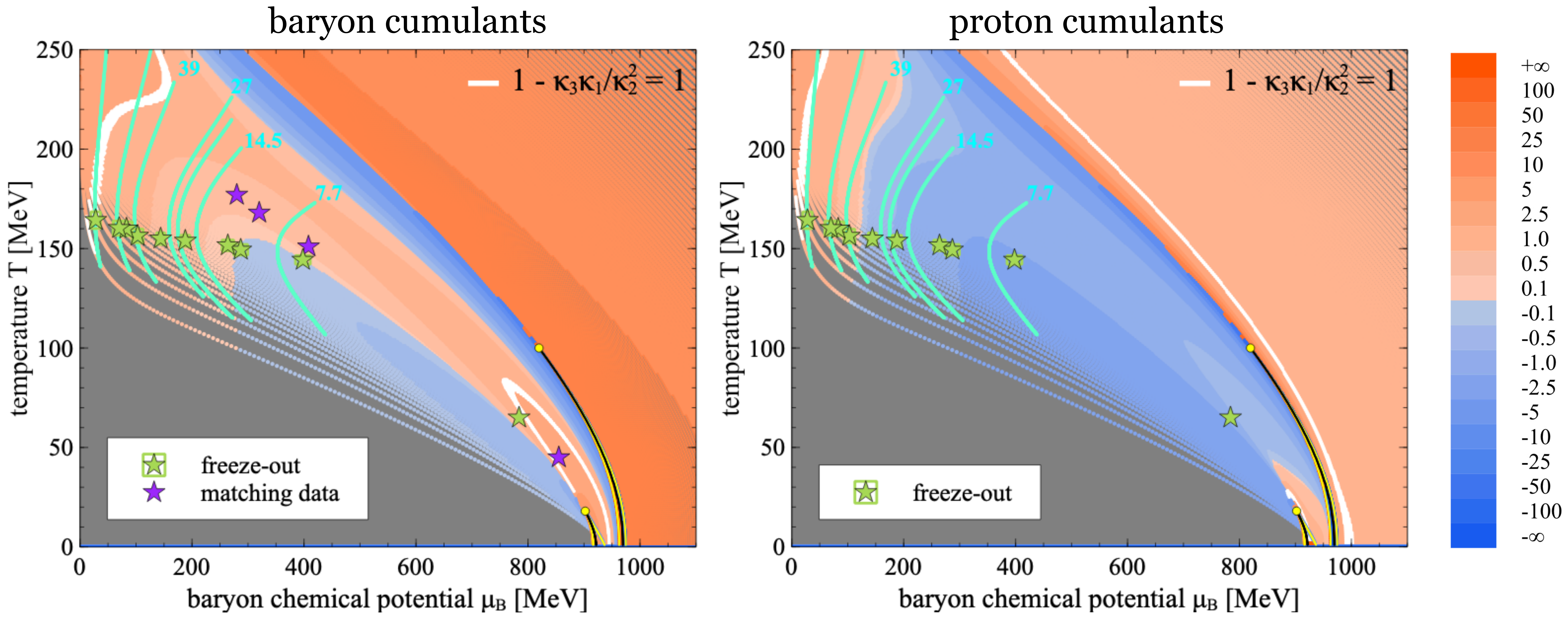
A couple of elephants in the room, and what to do with them

Proton vs. baryon cumulants

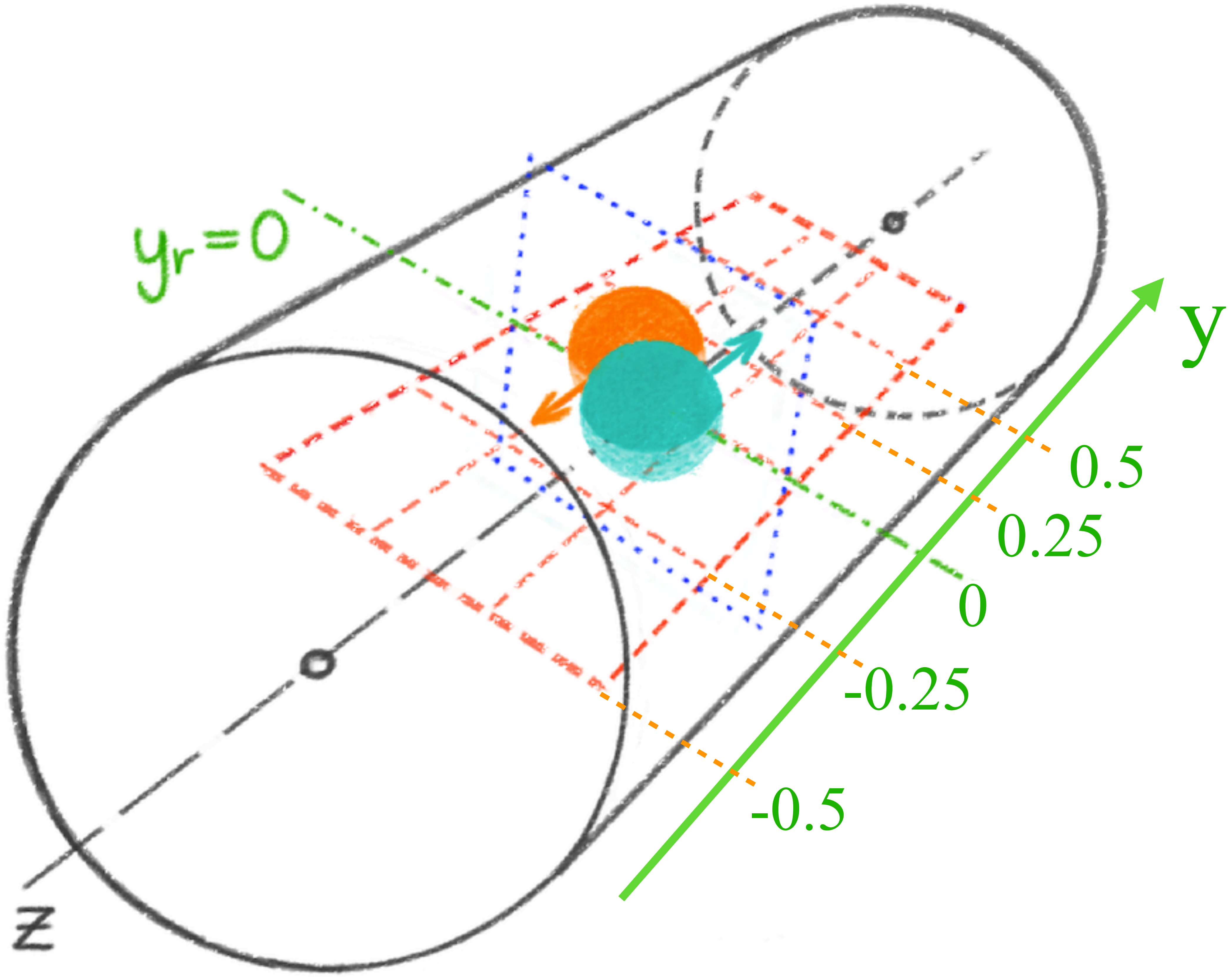
Y. Hatta, Y. and M. A. Stephanov, Phys. Rev. Lett. **91** (2003) 102003, arXiv:0302002, Hatta:2003wn

Proton cumulants said to be a good proxy for baryon cumulants.
Are they?

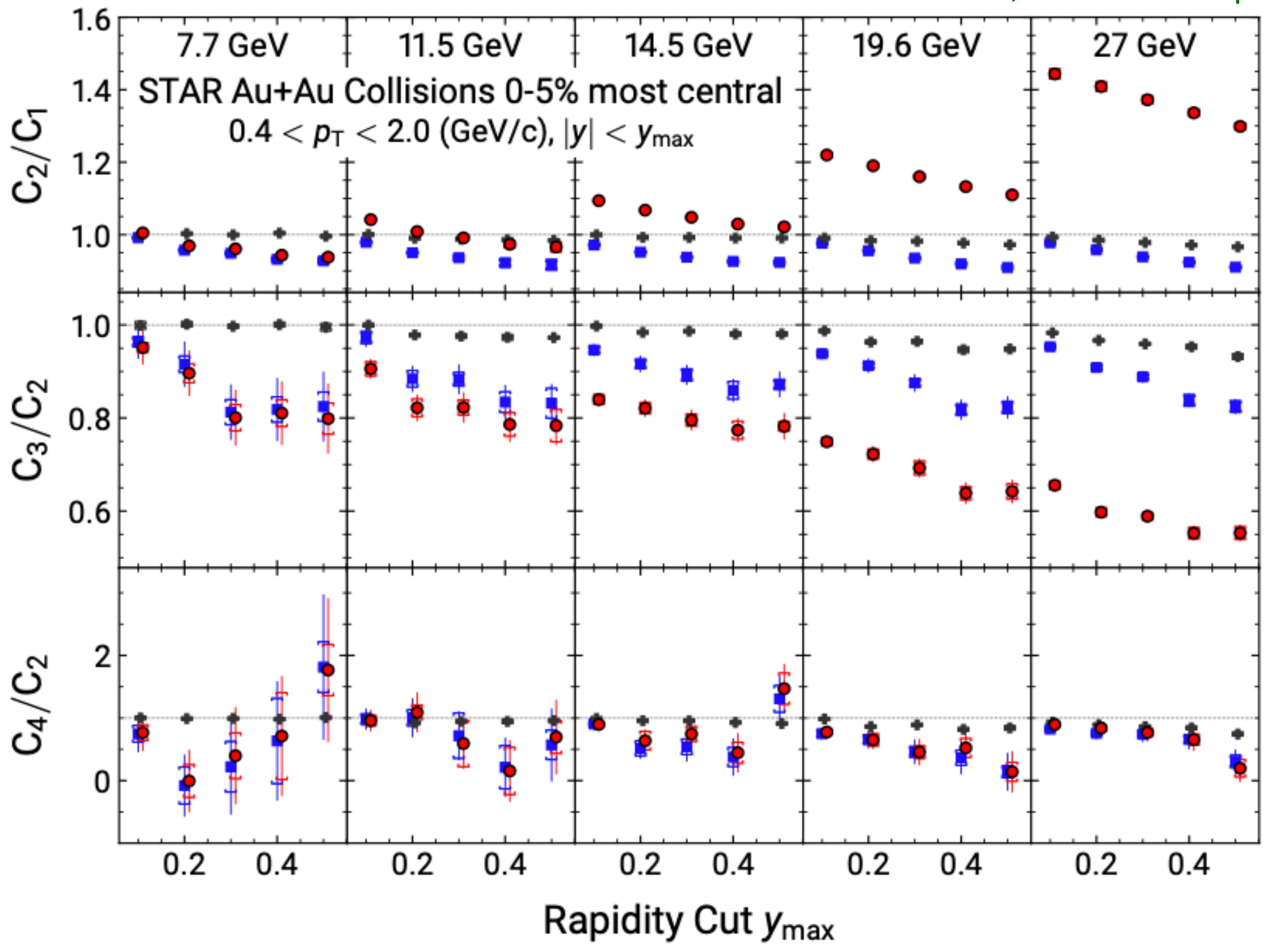
Definitely NOT in the VDF model:



Rapidity window dependence vs. theory calculations



STAR, Phys. Rev. C **104** (2021) no. 2, 024902, arXiv:2101.12413, STAR:2021iop



Changing the rapidity width:

⇒ changes the probed scale (what bin width will “capture” the correlations?)

⇒ can increase/reduce baryon number conservation effects

V. Vovchenko, O. Savchuk, R. Poberezhnyuk, M. Gorenstein, V. Koch, Phys. Lett. B **811** (2020) 135868, arXiv:2003.13905, Vovchenko:2020tsr

Results at which bin width should be compared with the theory?

Dynamically evolving systems vs. calculations in equilibrium

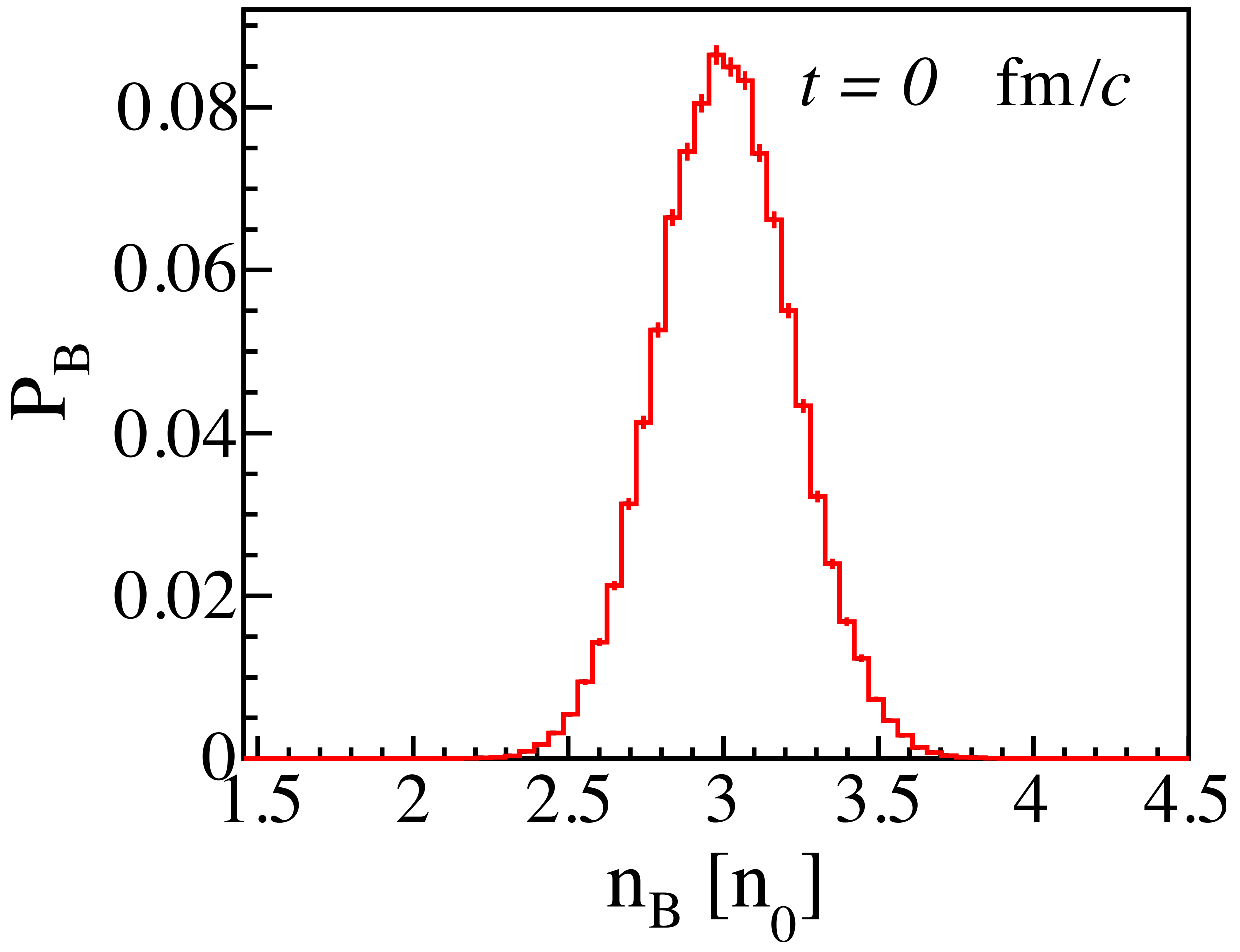
Heavy-ion collisions are dynamical, messy phenomena occurring in a finite volume and within a finite time span.
How to distinguish measuring thermodynamic properties from fooling ourselves?

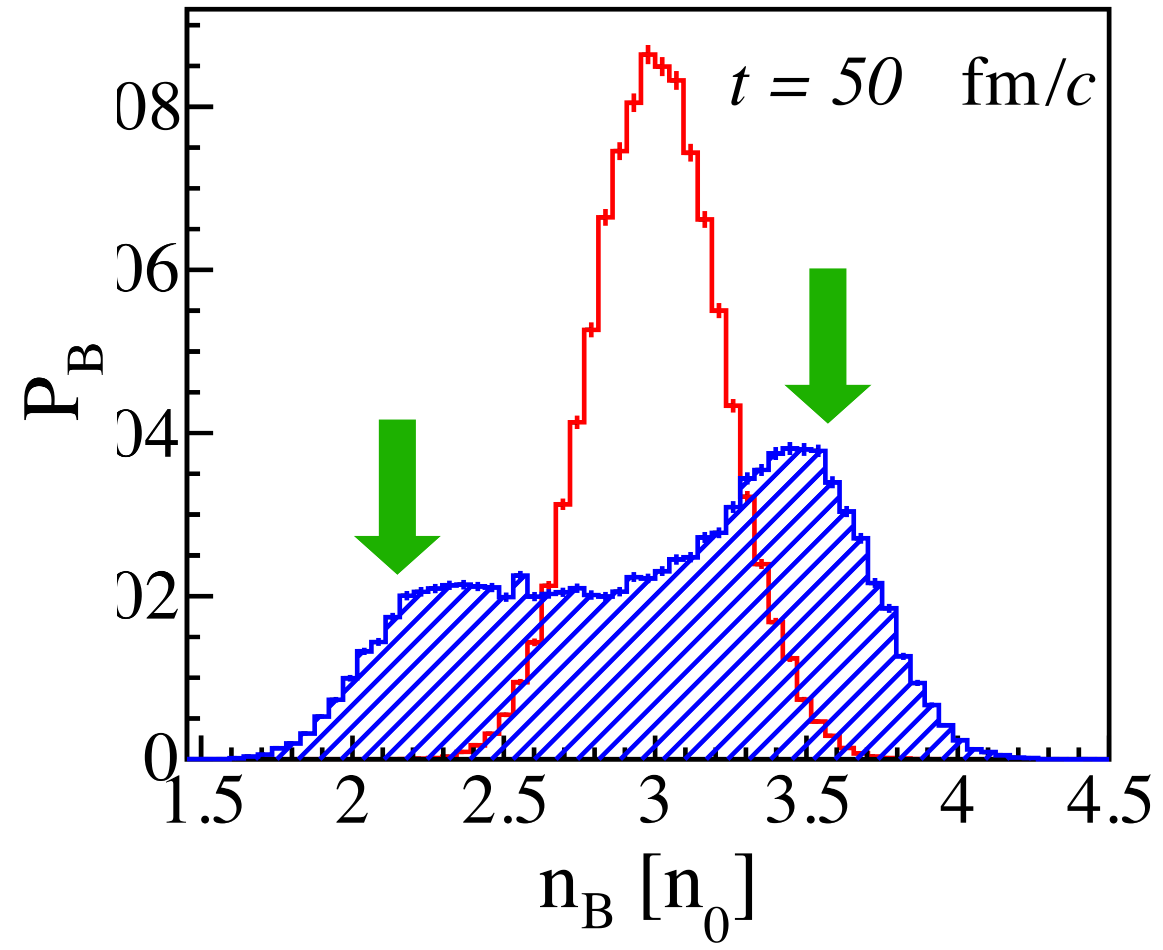
The way forward: simulations!

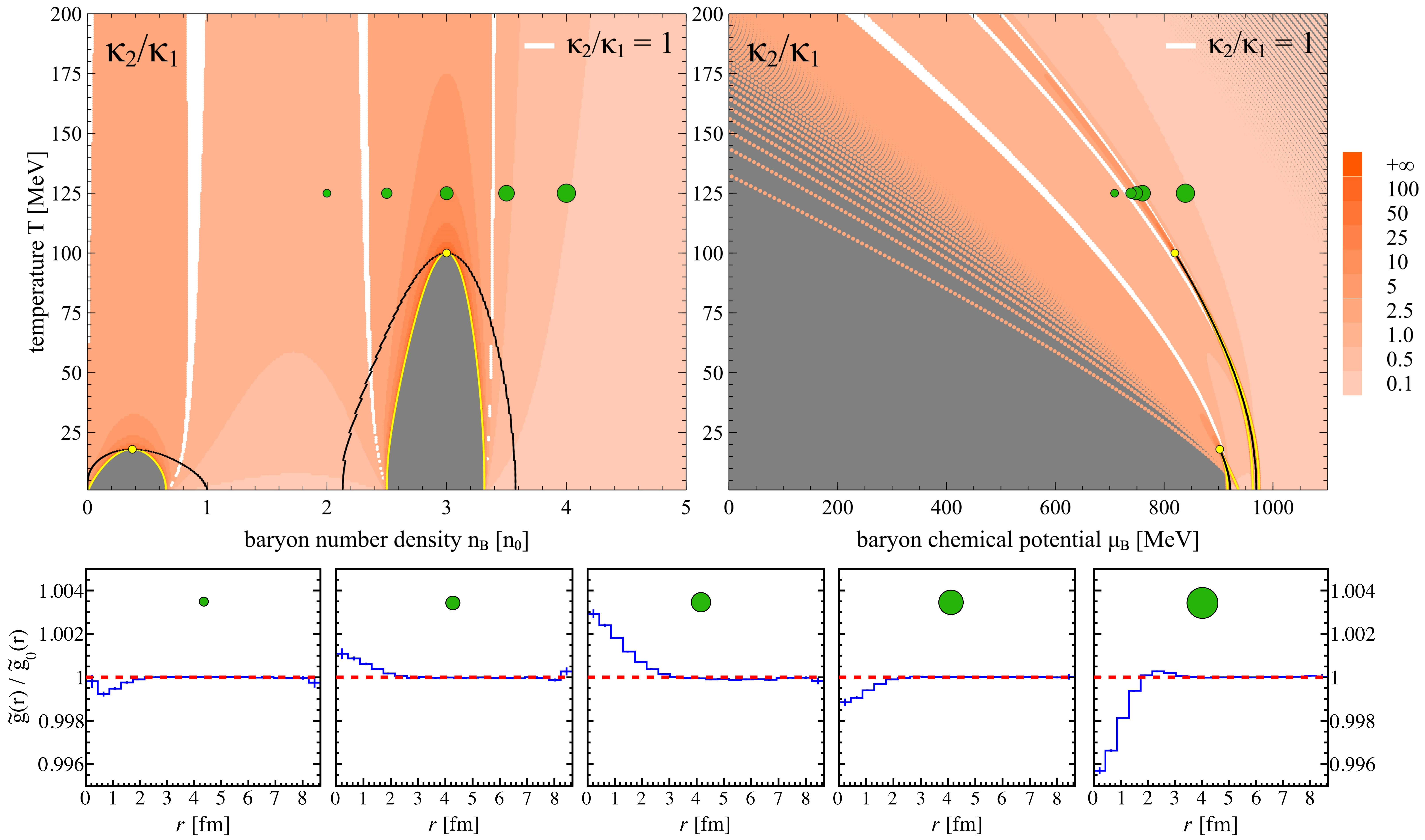
- At low energies hadronic transport is the appropriate tool to study this
- Simulations with flexible interactions to study a variety of possible EOSs

VDF model in SMASH

- Hadronic transport code SMASH with implemented VDF potentials is sensitive to thermodynamics of phase transitions described by an infinite family of possible EOSs



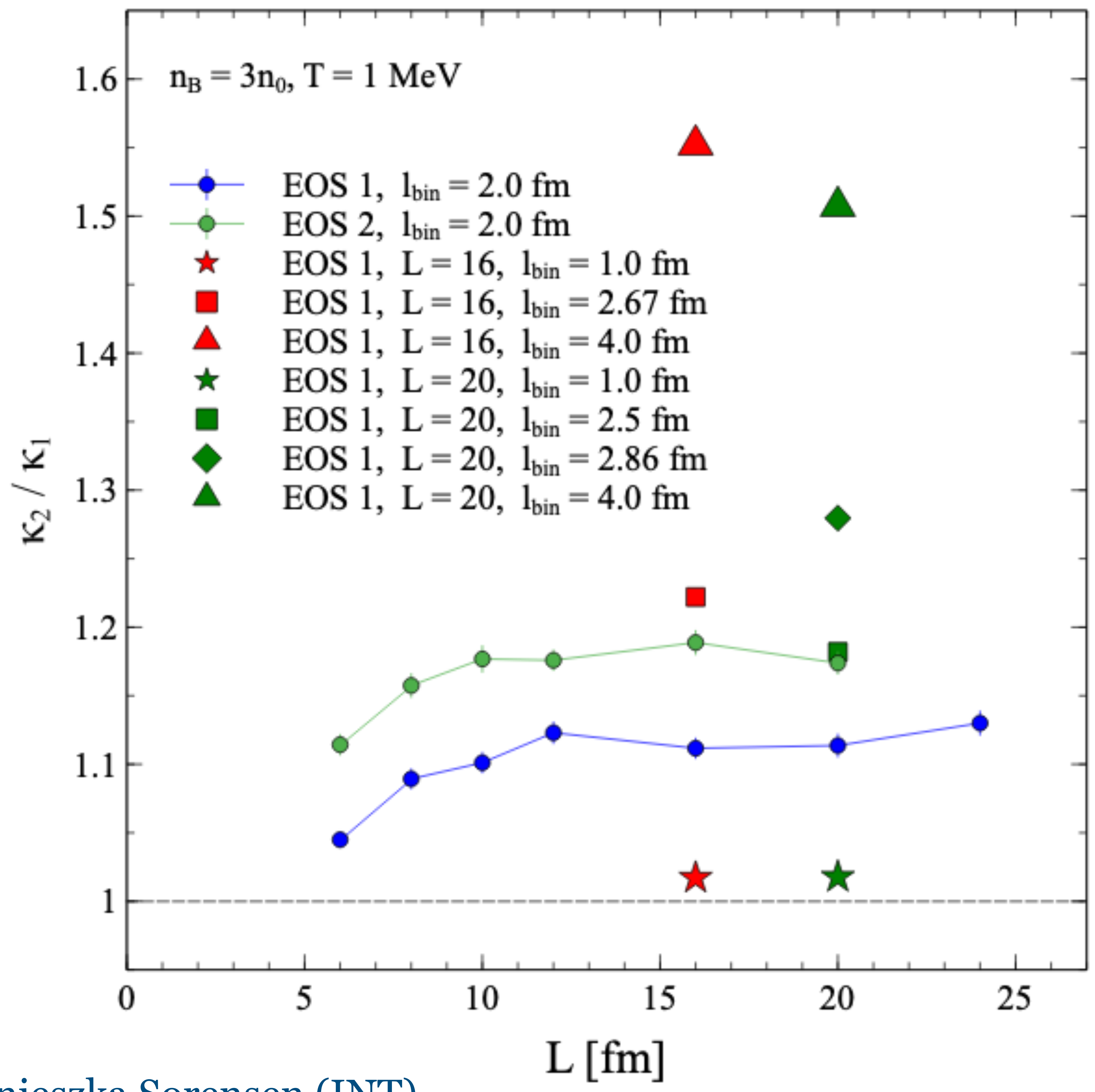


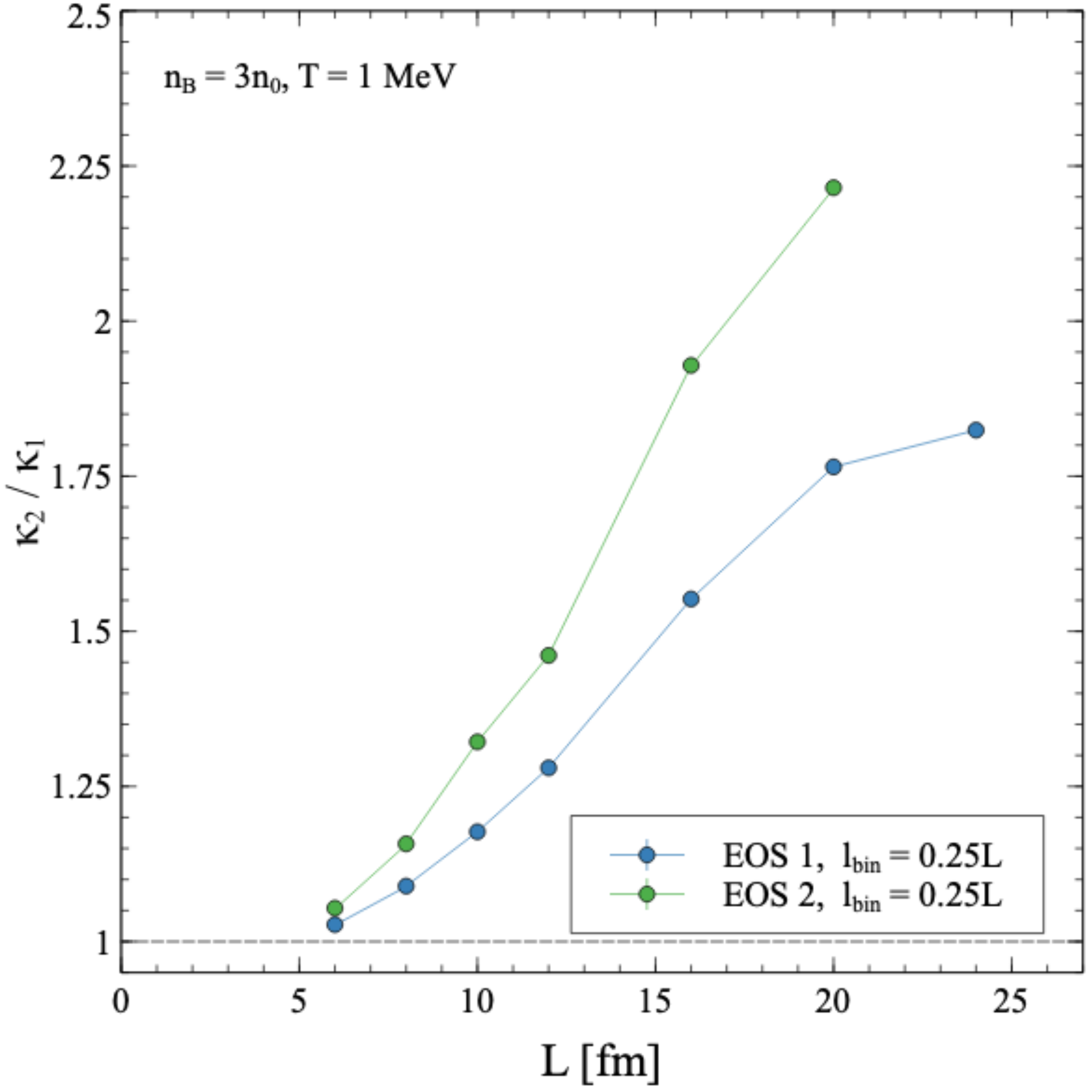


AS, V. Koch, Phys. Rev. C **104** no. 3 (2021) 034904, arXiv:2011.06635, Sorensen:2020ygf

Work in progress

- The connection between cumulants calculated in theory and measured in experiment can be directly studied

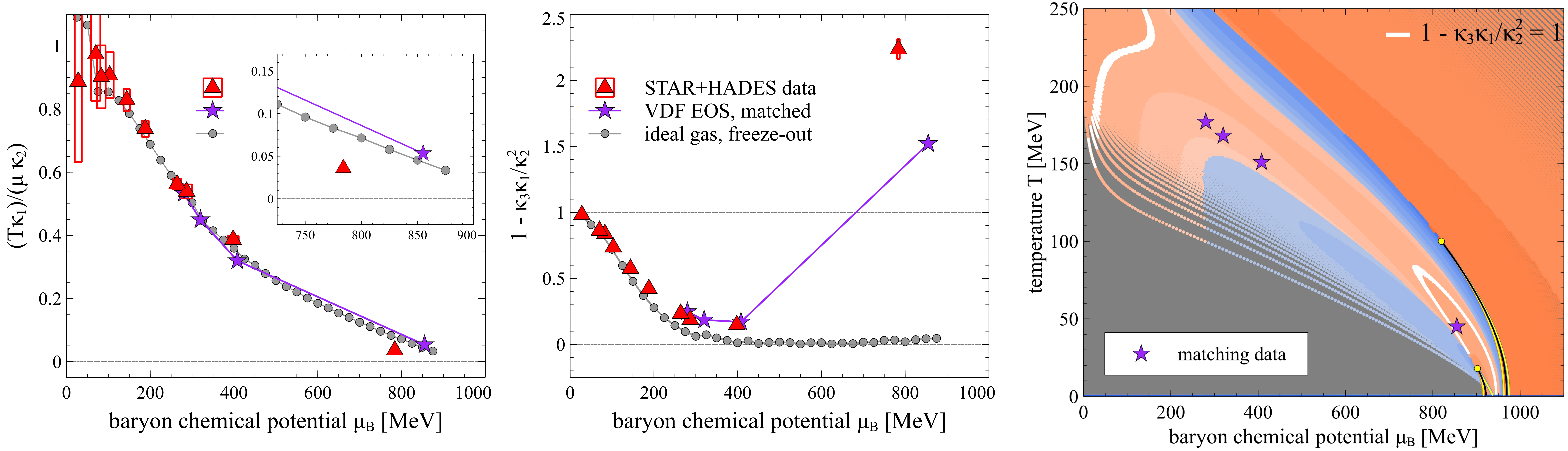




Summary

- Cumulants reimagined: they can be used to study the speed of sound
- Independently of the model interpretation, the data points towards interesting behavior
- The path forward is to understand finite number, binning, conservation, … effects in fluctuation observables

Thank you for your attention



This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under contract number DE-AC02-05CH11231.