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ARIZONA STATE UNIVERSITY

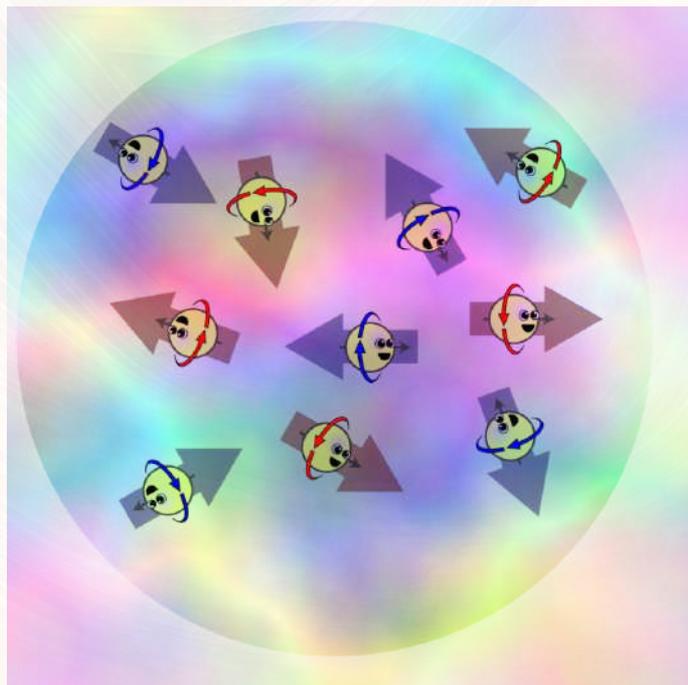


Chiral anomalous magnetospheres of magnetars

Igor Shovkovy

[Gorbar & Shovkovy, arXiv:2110.11380]

S@INT Seminar, INT, University of Washington



CHIRAL PLASMA

[Miransky & Shovkovy, Phys. Rep. 576, 1 (2015)]

[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]

Anomalous plasma

- Chiral relativistic plasma may allow $n_L \neq n_R$ to persist on *macroscopic* time/distance scales
- Slow evolution of $n = n_R + n_L$ and $n_5 = n_R - n_L$ is controlled by the continuity equations

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

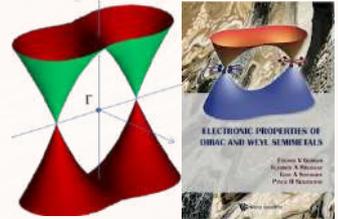
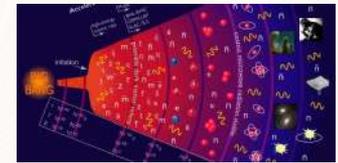
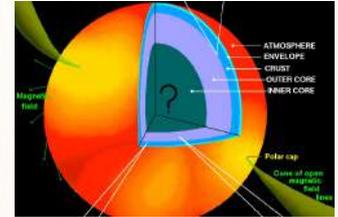
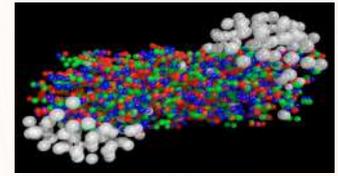
and

$$\frac{\partial n_5}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_m n_5$$

where the chirality flip rate: $\Gamma_m \propto \alpha^2 T (m/T)^2$

- Chiral anomaly can produce *macroscopic* effects in plasma

- **Heavy-ion collisions (high temperature)**
[Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)]
- **Super-dense matter in compact stars (high density)**
[Yamamoto, Phys. Rev. D 93, 065017 (2016)]
- **Early Universe (high temperature)**
[Boyarsky, Frohlich, Ruchayskiy, Phys. Rev. Lett. 108, 031301 (2012)]
- **Electron plasma in Dirac/Weyl (semi-)metals**
[Gorbar, Miransky, Shovkovy, Sukhachov, *Electronic Properties of Dirac and Weyl Semimetals* (World Scientific, Singapore, 2021)]
- **Other: cold atoms, superfluid $^3\text{He-A}$, etc.**
[Volovik, JETP Lett. 105, 34 (2017)]
- **Magnetospheres of magnetars** [Gorbar & Shovkovy, arXiv:2110.11380]
(electron-positron plasma at moderately high temperature)



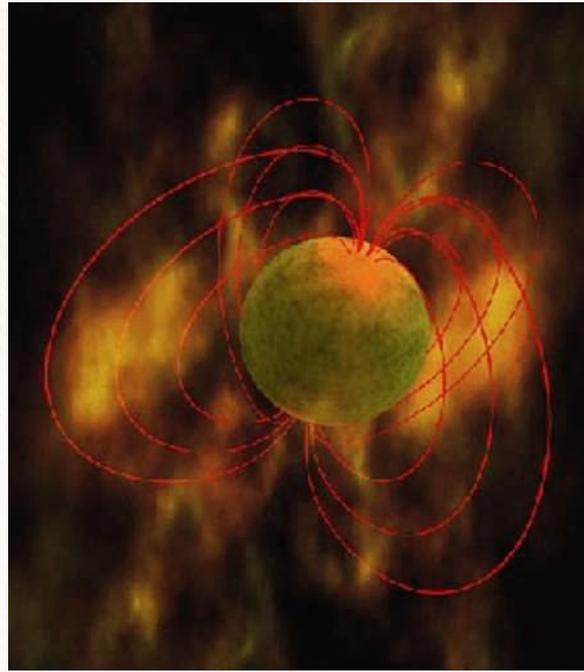


Image credit: NASA

PULSARS

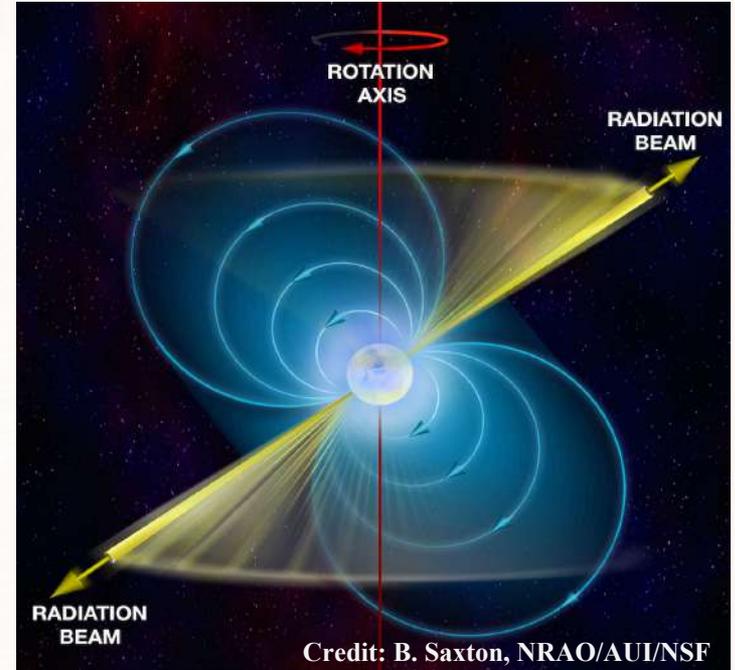
- **Neutron stars** are laboratories of matter under extreme conditions
- Prediction

[Baade & Zwicky, *Proc. Nat. Acad. Sci.* **20**, 259 (1934)]

- Observation

[Hewish, Bell, Pilkington, Scott & Collins, *Nature* **217**, 709 (1968)]

- **Pulsars** are neutron stars that are
 - rapidly rotating ($P \sim 1$ ms to 10 s)
 - strongly magnetized ($B \sim 10^8$ to 10^{15} G)
- Pulsar radiation is beamed along the magnetic field direction (the “lighthouse” effect)



Pulsars in $P-\dot{P}$ plane

- Characteristic age

$$\tau \simeq \frac{P}{2\dot{P}}$$

- Spin-down luminosity

$$-\dot{E} \simeq 4\pi^2 I \frac{\dot{P}}{P^3}$$

- Characteristic magnetic field

$$B \simeq 3 \times 10^{19} \left(\frac{P\dot{P}}{s} \right)^{1/2} \text{ G}$$

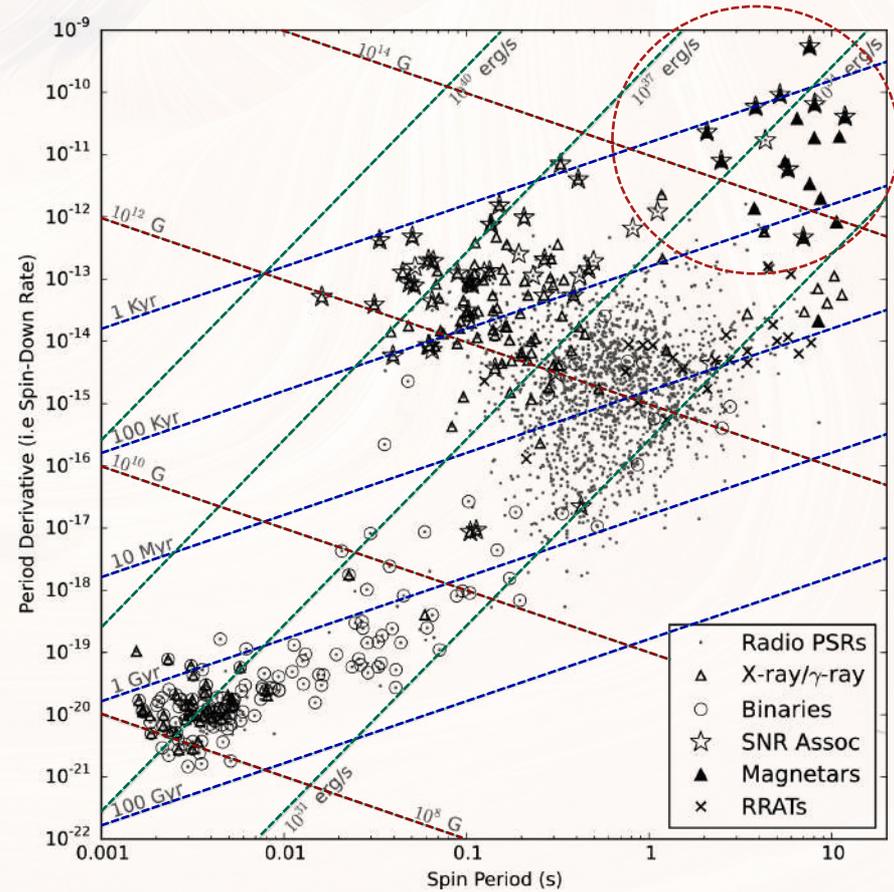


Image credit: Condon & Ransom, “Essential Radio Astronomy” (2016)



Image credit: Aurore Simonnet, Sonoma State University

MAGNETOSPHERES

Pulsar electrodynamics (VDM)

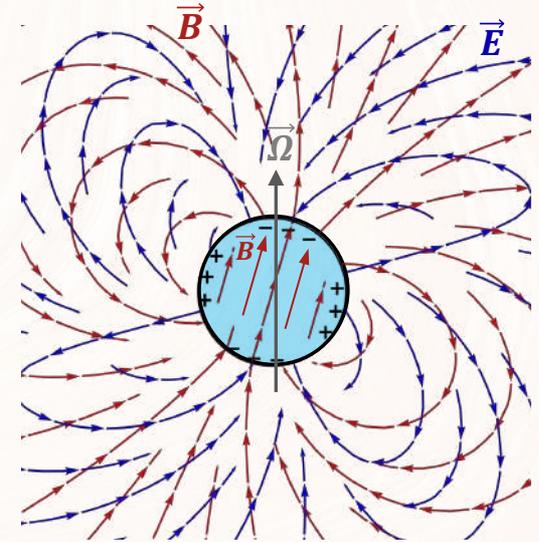
- **Vacuum dipole model (VDM)** ($\rho = 0$ & $J = 0$ outside the star)
- Stellar interior (good conductor):

$$\vec{E}'_{in} = \vec{E}_{in} + \frac{\vec{\Omega} \times \vec{r}}{c} \times \vec{B}_{in} = 0$$

- Fields outside the pulsar are

$$\vec{B} = \frac{B_0 R^3}{2r^3} (3(\hat{m} \cdot \hat{r})\hat{r} - \hat{m})$$

$$\vec{E} = \dots \quad [\text{see Deutsch, Ann. Astrophys. 18, 1 (1955)}]$$



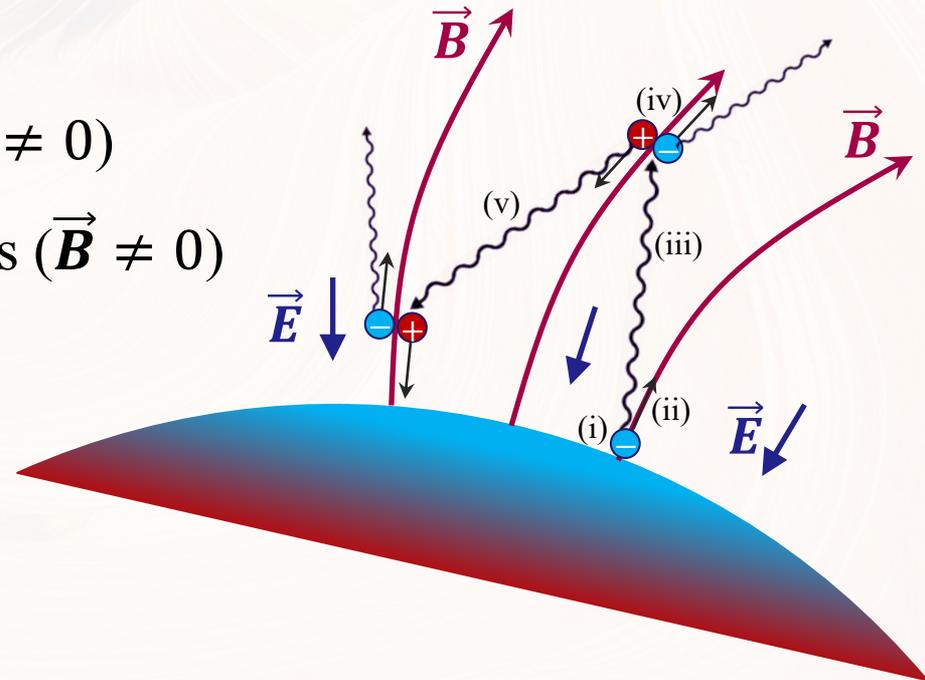
where \mathbf{m} is the magnetic moment and $\mathbf{\Omega}$ is the angular frequency

- There is a nonzero charge density and a strong electric field on the surface ($E_{\text{surf}} \sim \Omega R B_0 \sim 10^{12}$ to 10^{15} V/m)

- Charged particles
 - i. pulled up from the surface ($\vec{E} \neq 0$)
 - ii. move along curved trajectories ($\vec{B} \neq 0$)
 - iii. produce curvature radiation
 - iv. γ -quanta produce e^+e^- pairs

$$l_\gamma \simeq \frac{2R_c}{15} \frac{B_c}{B} \frac{m_e}{\epsilon_\gamma}$$

- v. Secondary particles produce synchrotron & curvature radiation
- **End result:** (I) magnetized vacuum is nontransparent for photons with $\epsilon_\gamma \gtrsim 2m_e$; (II) vacuum turns into plasma



- **Rotating magnetosphere model (RMM)** (assuming highly conducting plasma outside the star)

$$\vec{E}' = \vec{E} + \frac{\vec{\Omega} \times \vec{r}}{c} \times \vec{B} = 0$$

i.e., $E_{\parallel} = 0$

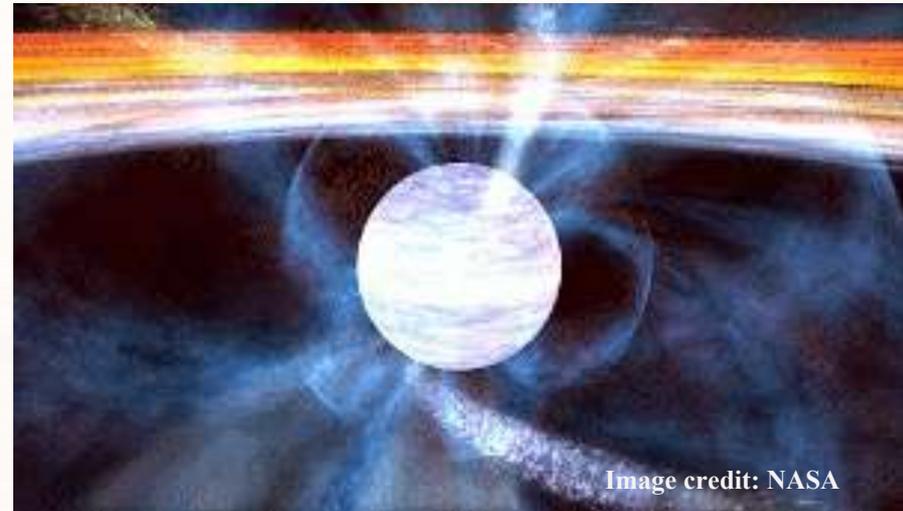
- Plasma motion is determined by

$$\vec{v}_{\text{drift}} = c \frac{\vec{E} \times \vec{B}}{B^2} = \vec{\Omega} \times \vec{r} + j_{\parallel} \vec{B}$$

- Corotating plasma is charged

$$\rho_{\text{GJ}} = \vec{\nabla} \cdot \vec{E} = -\frac{2}{c} \vec{\Omega} \cdot \vec{B}$$

[Goldreich & Julian, *Astrophys. J.* **157**, 869 (1969)]



Gaps in magnetosphere

- If one assumes that $E_{\parallel}=0$ everywhere, the magnetic field lines are equipotential ($V = \text{const}$)



- Then,

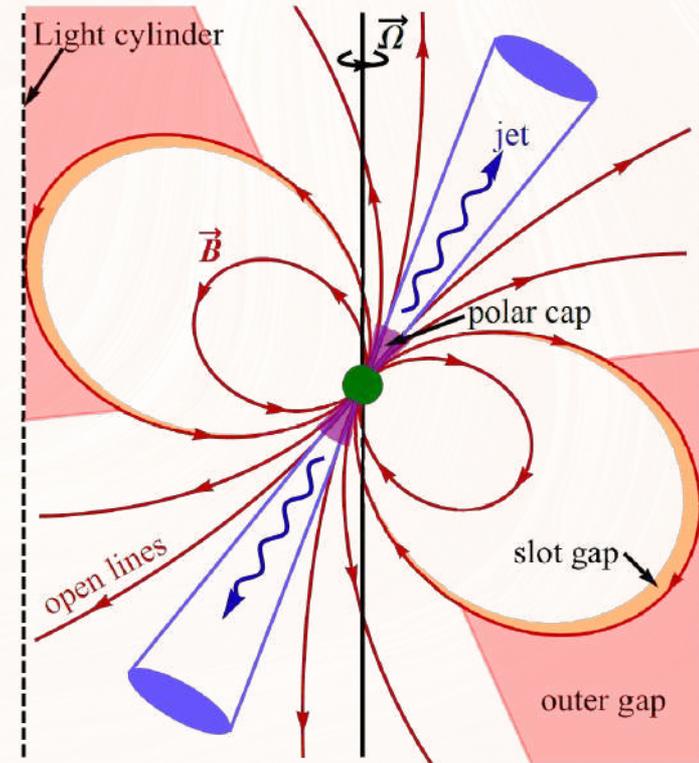
$$0 = \oint \vec{E} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{s}$$

- Thus, $E_{\parallel}=0$ cannot be enforced everywhere if \vec{B} changes in time
- Regions (“gaps”) with unscreened E_{\parallel} will necessarily develop (they result from dynamical charge/current starvation)

[Ruderman & Sutherland, *Astrophys. J.* **196**, 51 (1975)]

Gaps in magnetosphere

- Gaps can develop at various locations
- Intermittent gaps are caused by rapid outflow of charge
- The **gap size** h grows at a speed close to the speed of light
- Electric **potential** difference grows like $\Delta V = E_{\parallel} h \propto h^2$
- ΔV & photon flux cause an avalanche production of **electron-positron pairs**
- Since $B \propto 1/r^3$, anomalous effects are strongest near **polar caps**



[Ruderman & Sutherland, *Astrophys. J.* **196**, 51 (1975)]

Pulsar gaps

- Estimate of the gap size and the electric field

$$E_{\parallel} \simeq Bh/R_{LC}$$

where $R_{LC} = c/\Omega$ is the radius of light cylinder and

$$h \simeq 3.6 \text{ m} \left(\frac{R}{10 \text{ km}} \right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}} \right)^{-3/7} \left(\frac{B}{10^{14} \text{ G}} \right)^{-4/7}$$

The field scales with pulsar parameters as follows

$$E_{\parallel} \approx 2.7 \times 10^{-8} E_c \left(\frac{R}{10 \text{ km}} \right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left(\frac{B}{10^{14} \text{ G}} \right)^{3/7}$$

where $E_c = m_e^2/e = 1.3 \times 10^{18} \text{ V/m}$.

[Ruderman & Sutherland, *Astrophys. J.* **196**, 51 (1975)]

Gap parameters

- Quantitative estimate of the gap size and fields

B	10^{12} G	10^{13} G	10^{14} G	10^{15} G
h	50 m	13.4 m	3.6 m	0.97 m
$\frac{E_{\parallel}}{E_c}$	3.8×10^{-9}	1.0×10^{-8}	2.7×10^{-8}	7.3×10^{-8}
$\frac{\mathbf{E} \cdot \mathbf{B}}{E_c B_c}$	8.6×10^{-11}	2.3×10^{-9}	6.2×10^{-8}	1.7×10^{-6}

where

$$E_c = m_e^2/e = 1.3 \times 10^{18} \text{ V/m}$$

$$B_c = m_e^2/e = 4.4 \times 10^{13} \text{ G}$$

Chiral charge production

- The evolution of the chiral charge is determined by

$$\frac{\partial n_5}{\partial t} + \vec{\nabla} \cdot \vec{J}_5 = \frac{e^2 \vec{E} \cdot \vec{B}}{2\pi^2} - \Gamma_m n_5$$

- While the chiral anomaly produces n_5 , the chirality flipping tries to wash it away
- The chiral charge n_5 approaches the following steady-state value:

$$n_5 = \frac{e^2}{2\pi^2 \Gamma_m} \vec{E} \cdot \vec{B}$$

- The estimates for the chirality flip rate in a hot plasma

$$\Gamma_m \simeq \frac{\alpha^2 m_e^2}{T} \quad (T \lesssim m_e / \sqrt{\alpha}) \quad \text{and} \quad \Gamma_m \simeq \frac{\alpha m_e^2}{T} \quad (T \gg m_e / \sqrt{\alpha})$$

[Boyersky, Cheianov, Ruchayskiy, Sobol, Phys. Rev. Lett. **126**, 021801 (2021)]

- The gap formation time

$$t_h \sim h/c \sim 10^{-8} \text{ s}$$

- Timescale for chiral charge production

$$t^* \sim 1/\Gamma_m \sim 10^{-17} \text{ s}$$

- Note that

$$t_h \gg t^*$$

- Thus, the chirality production is nearly instantaneous

Estimate for n_5 in magnetars

- The estimate for the chiral charge is given by

$$n_5 \simeq \frac{e^2 E_{\parallel} B}{2\pi^2 \Gamma_m} \simeq 1.5 \times 10^{-5} \text{ MeV}^3 \left(\frac{T}{1 \text{ MeV}} \right) \\ \times \left(\frac{R}{10 \text{ km}} \right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left(\frac{B}{10^{14} \text{ G}} \right)^{10/7}$$

- The corresponding chiral chemical potential is

$$\mu_5 \simeq \frac{3n_5}{T^2} \simeq 4.6 \times 10^{-5} \text{ MeV} \left(\frac{T}{1 \text{ MeV}} \right)^{-1} \\ \times \left(\frac{R}{10 \text{ km}} \right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left(\frac{B}{10^{14} \text{ G}} \right)^{10/7}$$

- The corresponding numerical values for chiral charge and chiral chemical potential are

B	10^{12} G	10^{13} G	10^{14} G	10^{15} G
h	50 m	13.4 m	3.6 m	0.97 m
$\frac{E_{\parallel}}{E_c}$	3.8×10^{-9}	1.0×10^{-8}	2.7×10^{-8}	7.3×10^{-8}
$\frac{\mathbf{E} \cdot \mathbf{B}}{E_c B_c}$	8.6×10^{-11}	2.3×10^{-9}	6.2×10^{-8}	1.7×10^{-6}
$\frac{n_5}{m_e^3}$	1.6×10^{-7}	4.3×10^{-6}	1.1×10^{-4}	3.1×10^{-3}
$\frac{\mu_5}{m_e}$	1.2×10^{-7}	3.4×10^{-6}	9.0×10^{-5}	2.4×10^{-3}

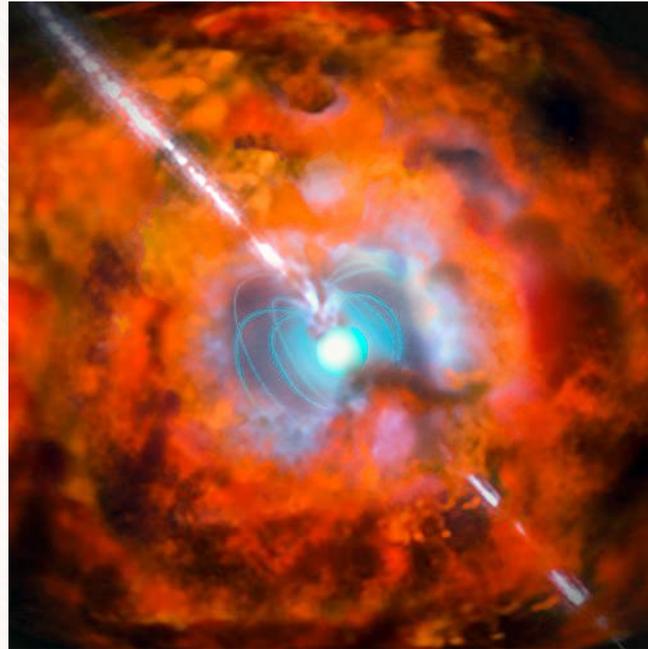


Image credit: European Southern Observatory

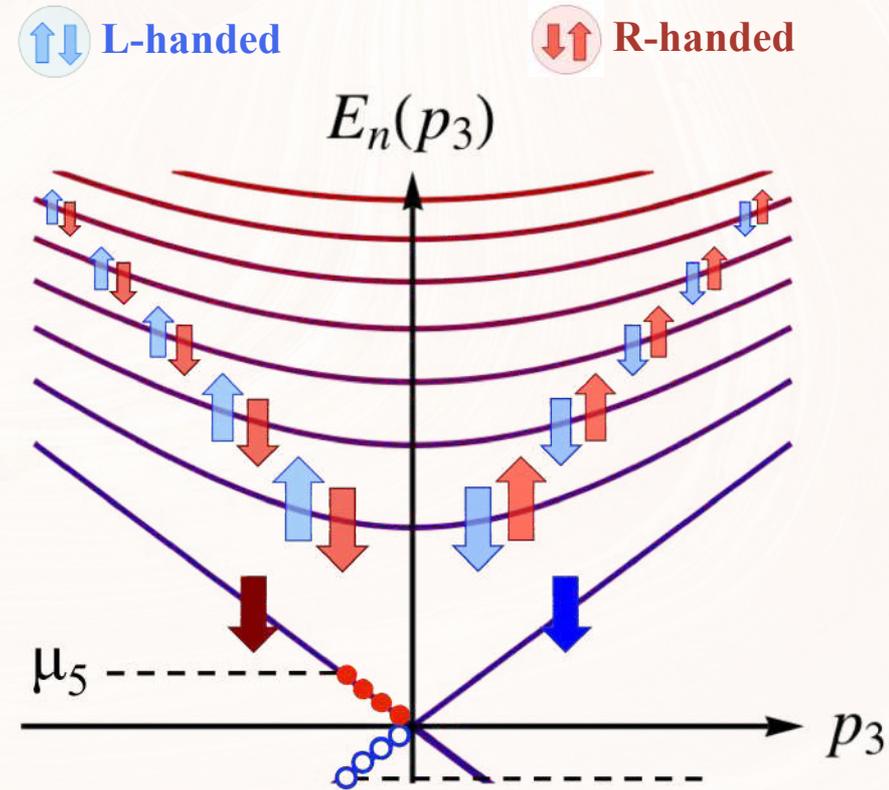
CHIRAL PLASMA INSTABILITY

□

- Nonzero μ_5 and \vec{B} drive the chiral magnetic effect (CME)

$$\vec{j} = \frac{e^2 \vec{B}}{2\pi^2} \mu_5$$

- The effect comes from the spin-polarized LLL ($s=\downarrow$)
 - **L-handed** states ($p_3 < 0$ & $|E| < \mu_5$) are empty (holes with $p_3 > 0$)
 - **R-handed** states ($p_3 < 0$ & $E < \mu_5$) are occupied



- However, plasma at $\mu_5 \neq 0$ is unstable

- The total current (CME + Ohm)

$$\mathbf{j} = \frac{2\alpha}{\pi} \mu_5 \mathbf{B} + \sigma \mathbf{E}$$

- By substituting \mathbf{j} into Ampere's law

$$\nabla \times \mathbf{B} = \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}$$

and solving for the electric field, one derives

$$\mathbf{E} = \frac{1}{\sigma} \left(\nabla \times \mathbf{B} - k_\star \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} \right) \quad \text{where } k_\star = \frac{2\alpha \mu_5}{\pi}$$

- Finally, by calculating the curl and using Faraday's law,

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{\sigma} \left(\nabla \times (\nabla \times \mathbf{B}) - k_\star \nabla \times \mathbf{B} + \frac{\partial^2 \mathbf{B}}{\partial t^2} \right)$$

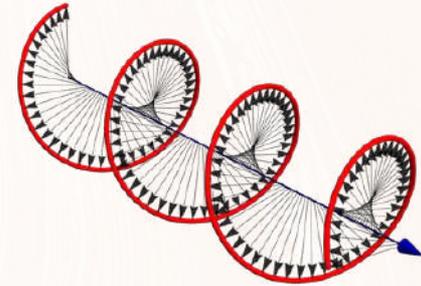
Helical modes at $\mu_5 \neq 0$

- Search for a solution as a superposition of helical eigenstates

$$\nabla \times \mathbf{B}_{\lambda,k} = \lambda k \mathbf{B}_{\lambda,k}$$

e.g.,

$$\mathbf{B}_{\lambda,k} = B_0 (\hat{\mathbf{x}} + i\lambda\hat{\mathbf{y}}) e^{-i\omega t + ikz}$$



Then, for a fixed eigenmode, the evolution equation reads

$$\frac{d\mathbf{B}_{\lambda,k}}{dt} = \frac{1}{\sigma} \left(\lambda k_{\star} k - k^2 - \frac{\partial^2}{\partial t^2} \right) \mathbf{B}_{\lambda,k}$$

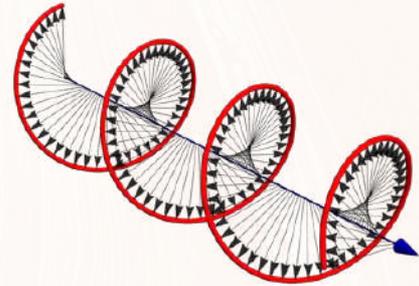
- The two solutions for the frequency are

$$\omega_{1,2} = -\frac{i}{2} \left(\sigma \pm \sqrt{\sigma^2 + 4k(\lambda k_{\star} - k)} \right)$$

Long-wavelength modes

- For a plasma with high conductivity

$$\omega_{1,2} \simeq \begin{cases} -i \left(\sigma + \frac{k(\lambda k_* - k)}{\sigma} \right) \\ i \frac{k(\lambda k_* - k)}{\sigma} \end{cases}$$



- The 1st mode is damped by charge screening:

$$B_{k,1} \propto B_0 e^{-\sigma t}$$

- The 2nd mode is unstable when $k < \lambda k_*$:

$$B_{k,2} \propto B_0 e^{+tk(\lambda k_* - k)/\sigma}$$

- The momentum of the fastest growing mode $B_{k,2}$ is

$$\frac{1}{2} k_*$$

Instability in pulsars

- The estimate for k_*

$$k_* \simeq 2.2 \times 10^{-7} \text{ MeV} \left(\frac{T}{1 \text{ MeV}} \right)^{-1} \left(\frac{R}{10 \text{ km}} \right)^{2/7} \left(\frac{\Omega}{1 \text{ s}^{-1}} \right)^{4/7} \left(\frac{B}{10^{14} \text{ G}} \right)^{10/7}$$

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$\frac{n_5}{m_e^3}$	1.6×10^{-7}	4.3×10^{-6}	1.1×10^{-4}	3.1×10^{-3}
$\frac{\mu_5}{m_e}$	1.2×10^{-7}	3.4×10^{-6}	9.0×10^{-5}	2.4×10^{-3}
$\frac{k_*}{m_e}$	5.8×10^{-10}	1.6×10^{-8}	4.2×10^{-7}	1.1×10^{-5}

Observational consequences

- Unstable plasma in the gaps produces **helical** (circularly polarized) **modes** in the frequency range

$$0 \lesssim \omega \lesssim k_*$$

- For magnetars, these span **radio frequencies** and may reach into the **near-infrared** range
- Available energy is of the order of $\Delta\mathcal{E} \sim \mu_5^2 T^2 h^3$, i.e.,

$$\Delta\mathcal{E} \simeq 2.1 \times 10^{25} \text{ erg} \left(\frac{T}{1 \text{ MeV}} \right) \left(\frac{R}{10 \text{ km}} \right)^{6/7} \\ \times \left(\frac{\Omega}{1 \text{ s}^{-1}} \right)^{-9/7} \left(\frac{B}{10^{14} \text{ G}} \right)^{2/7}$$

- The energy is sufficient to feed the **fast radio bursts (FRB)**

Outstanding problems

- Interplay of chiral charge and electron-positron pair **production** induced by energetic photons should be studied in detail
- The modification of the **chiral flip rate** $\Gamma_m \simeq \frac{\alpha^2 m_e^2}{T}$ by the strong magnetic field (extra suppression?)
- The role of the **inverse magnetic cascade** and the **chiral-magnetic turbulence** should be quantified
- Self-consistent **dynamics** of chiral plasma in the gap regions should be simulated in detail
- Detailed mechanism of the **energy transfer** from unstable helical modes to radio emission in FRBs

- Chiral anomaly can have *macroscopic* implications in pulsars
- It leads to a *significant* chiral charge production (up to 10^{34} m^{-3}) in strongly magnetized magnetospheres
- The chiral chemical potential μ_5 can be up to 10^{-3} MeV
- This is sufficient to trigger emission of helical waves with frequencies up to about $k_\star \simeq \frac{2}{\pi} \alpha \mu_5$ (radio to infrared range)
- Helical waves can affect the pulsar jets and observable features of the fast radio bursts
- For quantitative effects, further detailed studies are needed