Transport and thermalisation at NLO from kinetic theory

Jacopo Ghiglieri, SUBATECH, Nantes

INT, Seattle, September 16 2021

Outline

- Introduction
- The effective kinetic theory and
- its relation to transport coefficients
- its NLO corrections and
- their impact on isotropic thermalisation

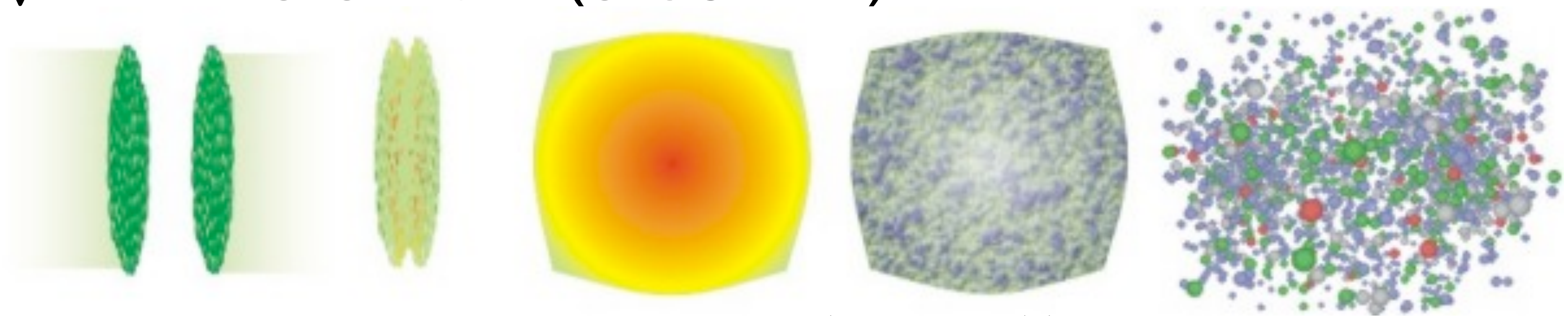
JG Moore Teaney (2015-2018)

Yu Fu, JG, Shahin Iqbal, Aleksi Kurkela **21xx.yyyyy**

Overview

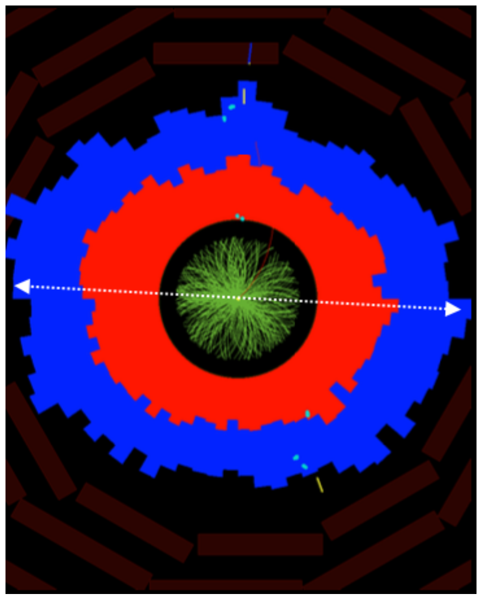
Heavy-ion collisions

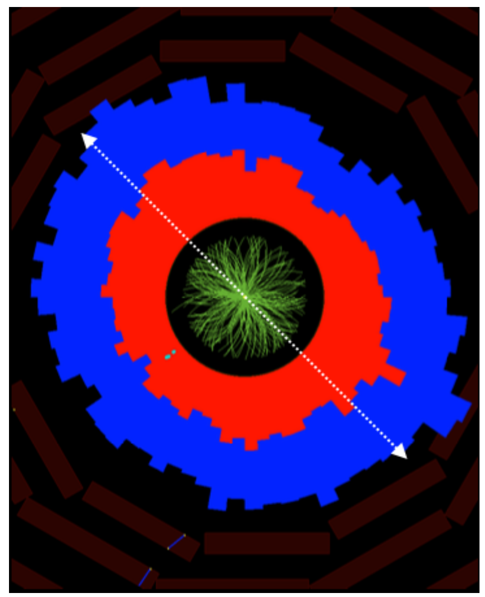
- A (transient) QGP can be formed in heavy ion collision experiments. RHIC (@BNL), up to $\sqrt{s_{NN}}$=200GeV. LHC up to $\sqrt{s_{NN}}$=5.5 TeV (5 so far).

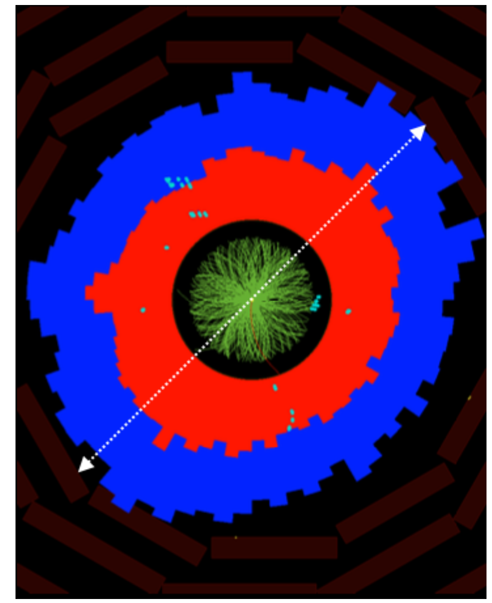


- Two Lorentz-contracted nuclei collide
- Rapid formation of a near-thermal QGP (~1 fm/c)
- Expansion and cooling for up to 5-10 fm/c, then
- Hadronizazion
- Lots of particles (dN_{ch}/dy $O(1000)$) stream to the detectors

Azimuthal anisotropies







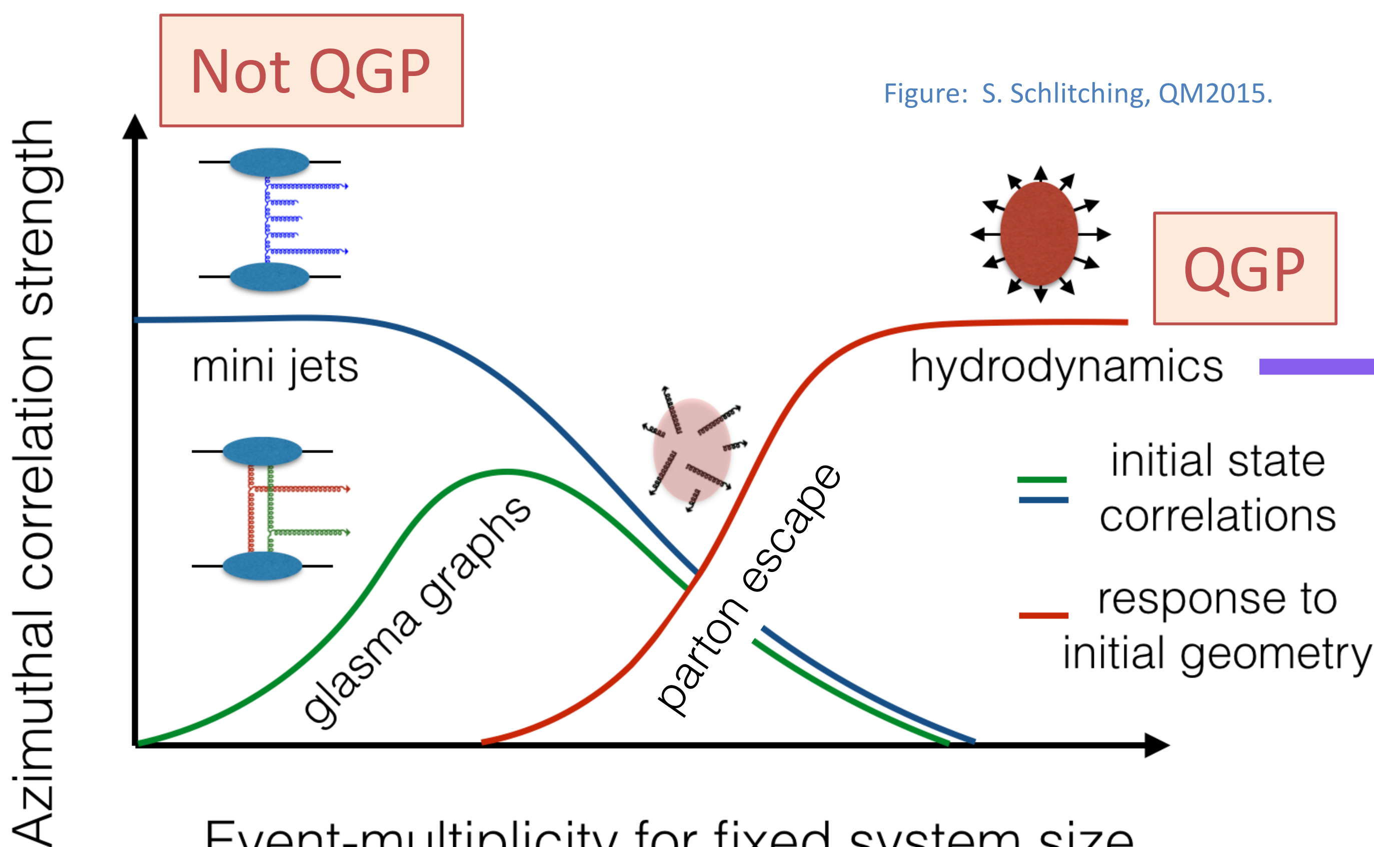


Figure: S. Schlitching, QM2015.

Expanding around infinite collisions in final state

Flow: a bulk property

- Initial asymmetries in position space are converted by collective, macroscopic (many body) processes into final state momentum space asymmetries

- Quantitatively: azimuthal Fourier decomposition of the final state particle spectra

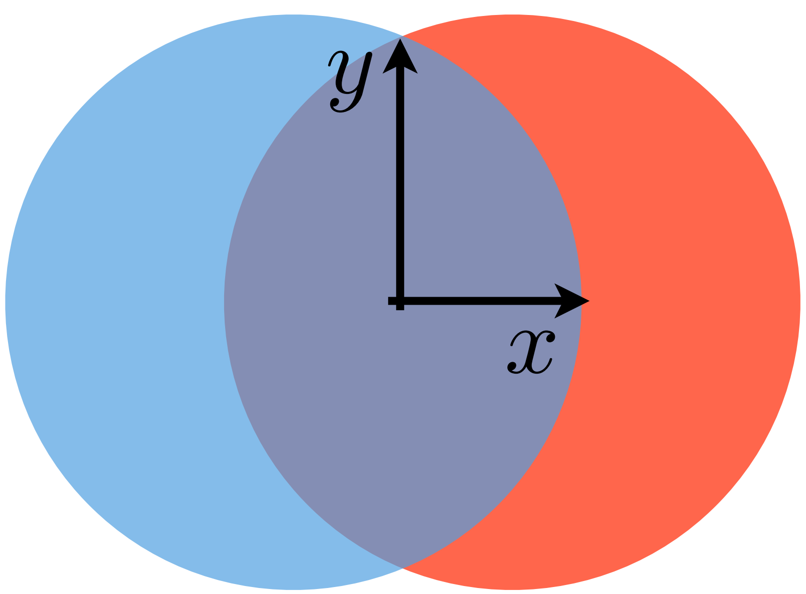
$$\frac{dN_i}{dy\, d^2p_T} = \frac{dN_i}{2\pi p_T dP_T dy}\left(1 + \sum_{n=1}^{\infty} 2 v_{i,n}(p_T, y) \cos(n\phi)\right)$$

vzero amplitude + v_n coefficients

- 2D analogue of the multipole expansion of the CMB

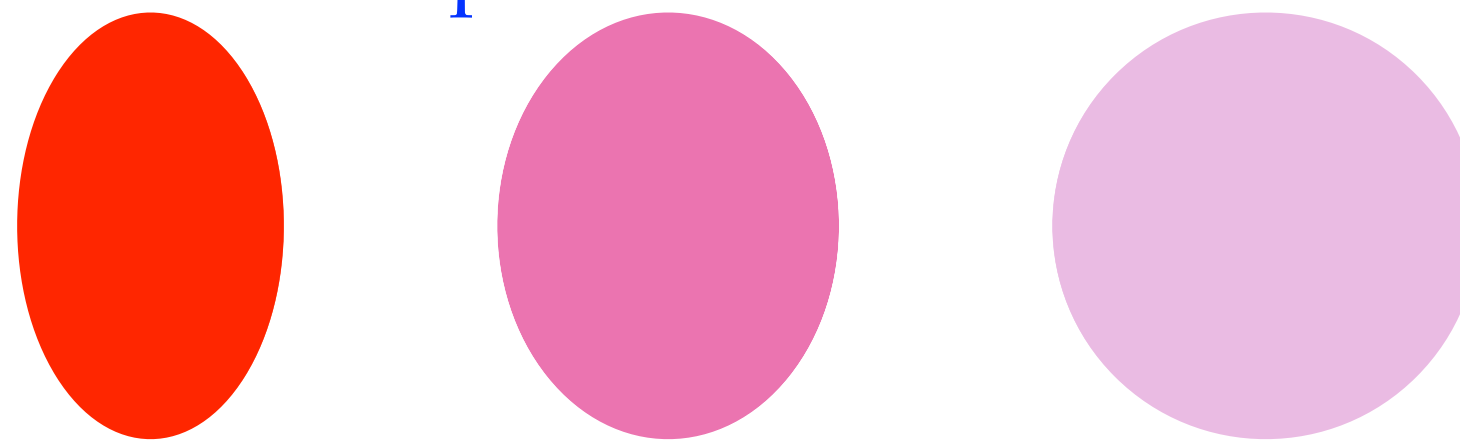
A famous example:elliptic flow

Initial asymmetry



Beam along z

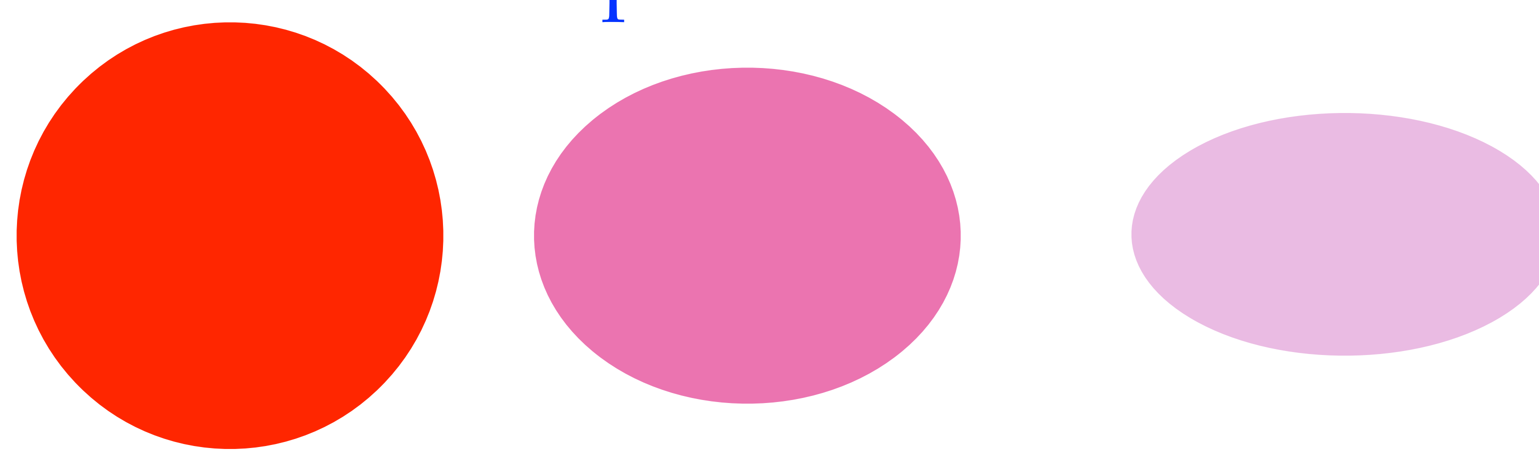
Position space



Large pressure gradients

No more flow

Momentum space



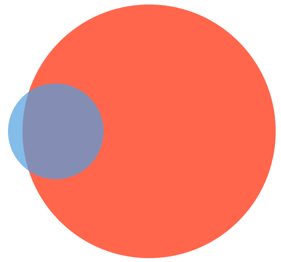
From initial symmetry

to final fixed anisotropy

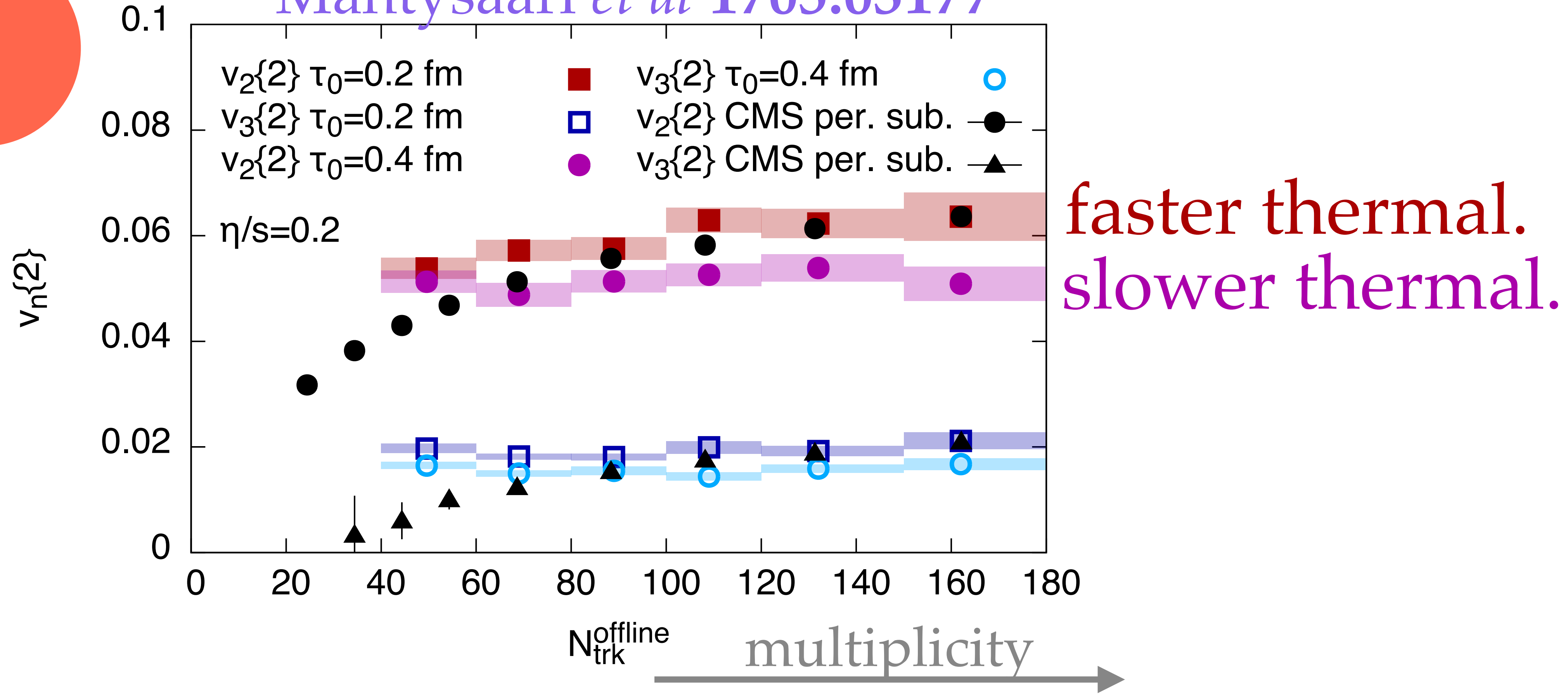
- Hydrodynamics describes the buildup of flow. The shear viscosity parametrizes the efficiency of the conversion

Understanding thermalisation

- Bulk observables are very successfully described by hydrodynamics
- Hydro needs a locally equilibrated medium at the initialisation time τ_0

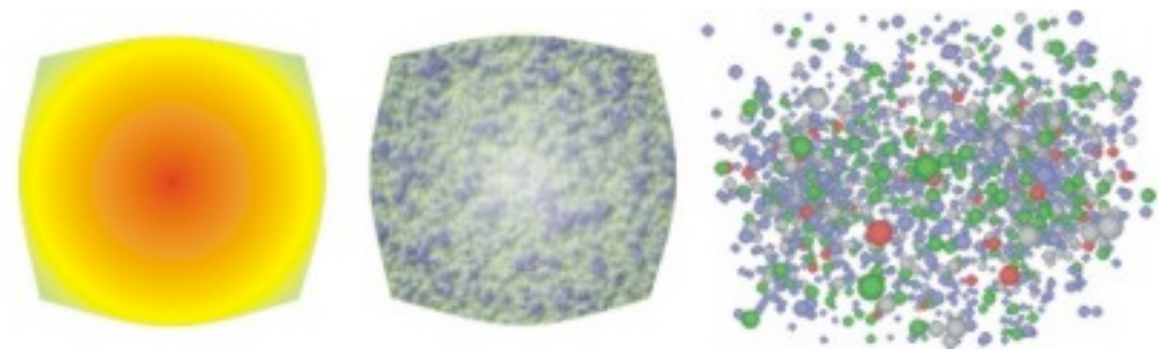


Mäntysaari *et al* **1705.03177**



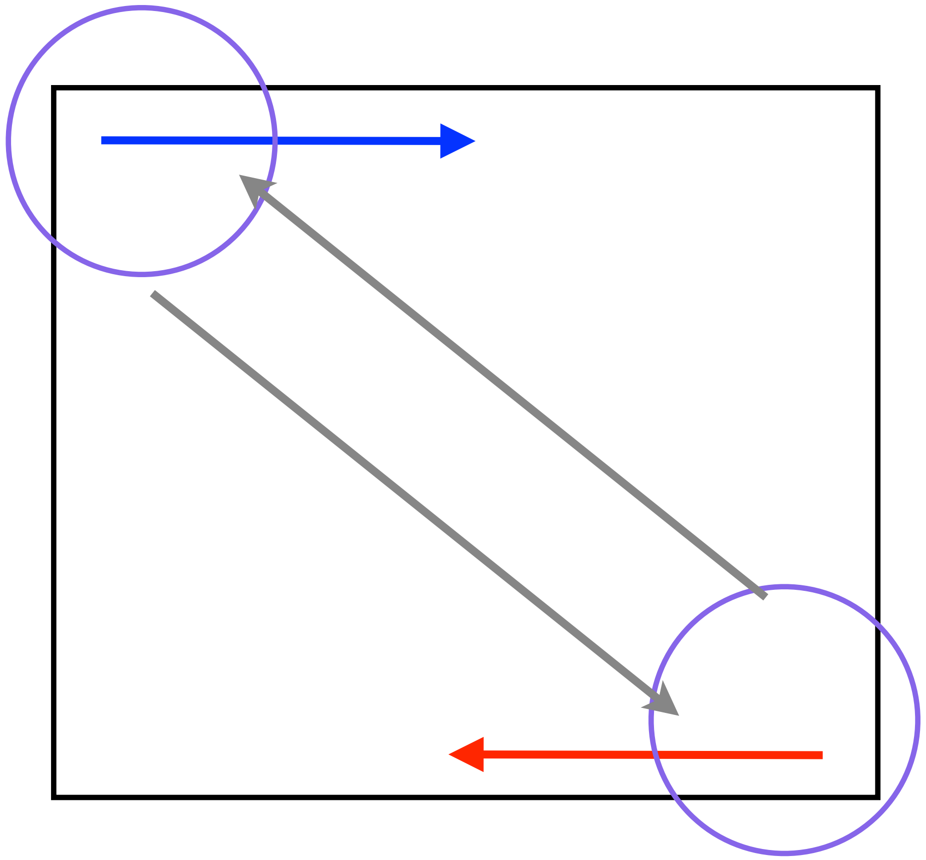
Understanding thermalisation

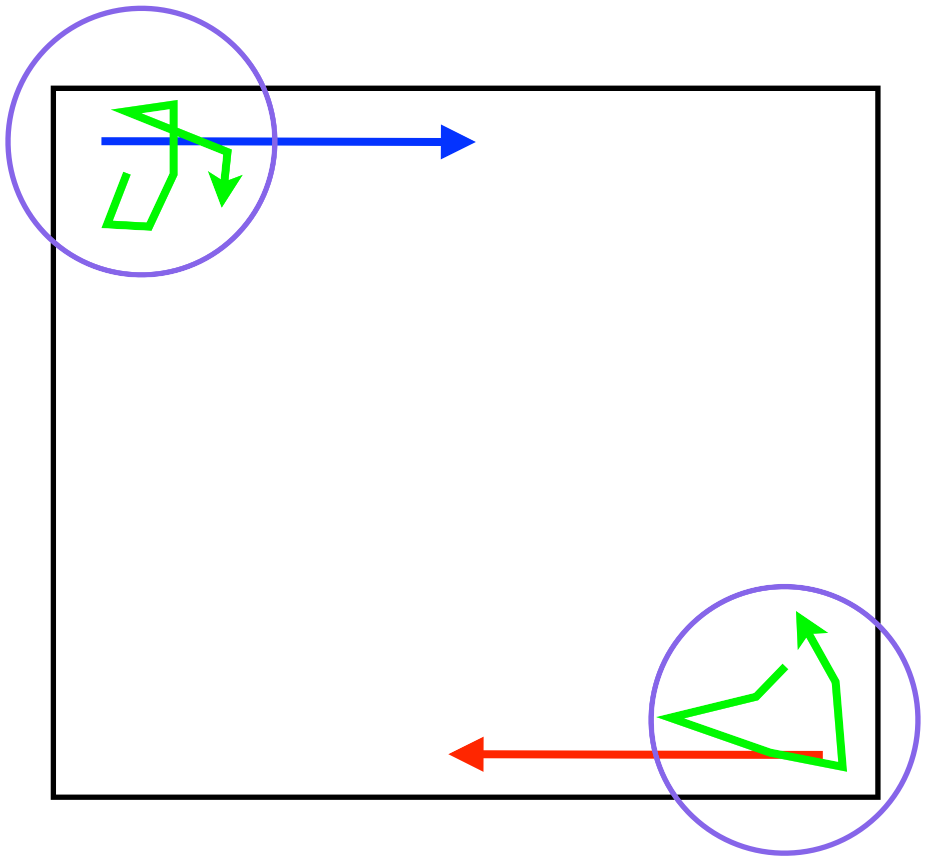
?



- How can we describe this rapid transition from a confined hadronic initial state to a near-thermal deconfined one?

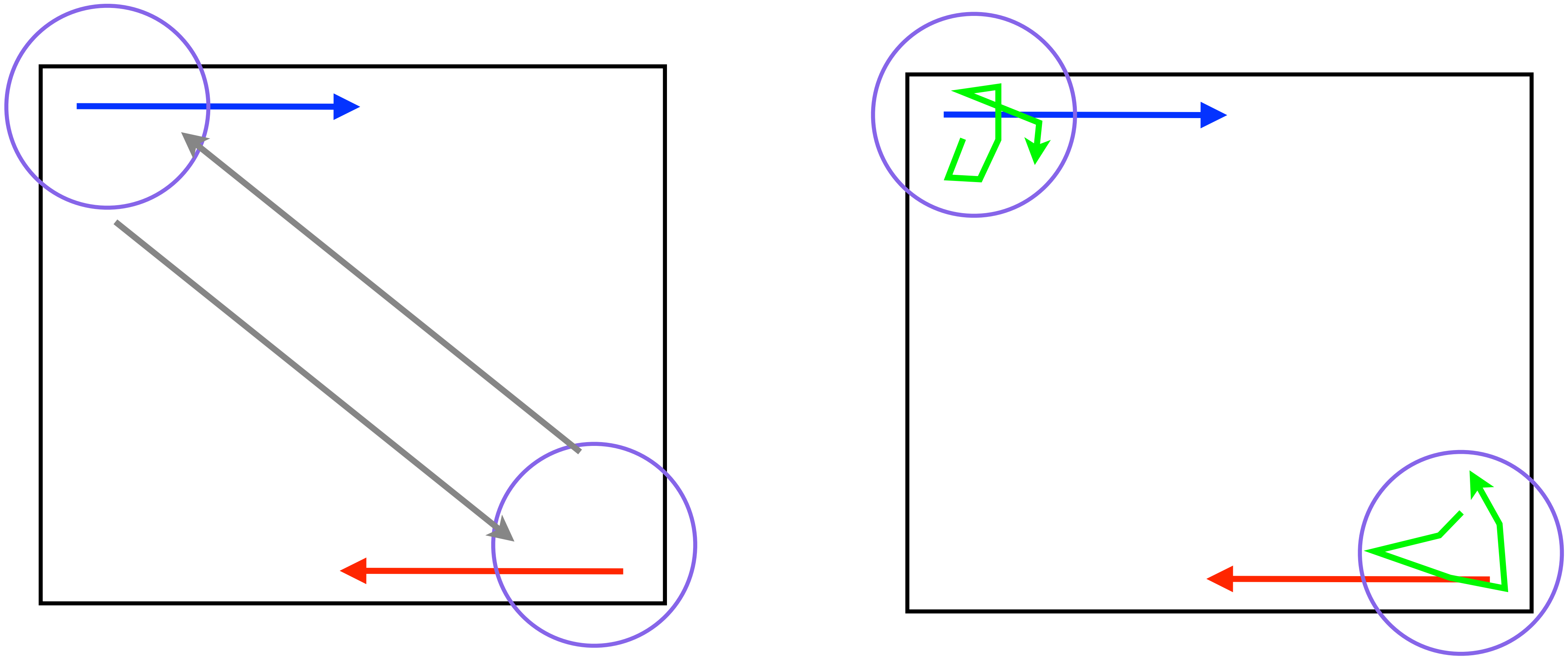
Understanding transport





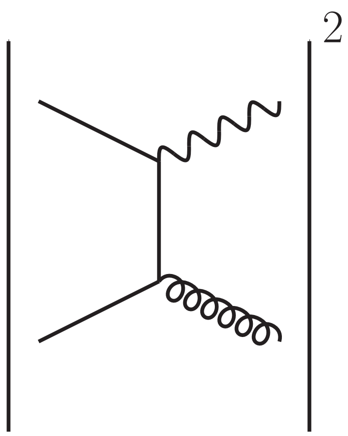
- Weak coupling: long distances between collisions, easy diffusion. **Large** η/s
- Strong coupling: short distances between collisions, little diffusion. **Small** η/s

Understanding transport

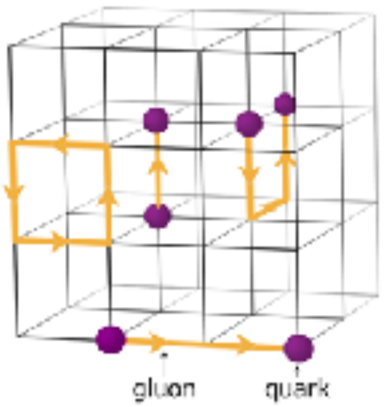


- Can we understand the microscopic physics giving rise to thermalisation and transport within the same approach?

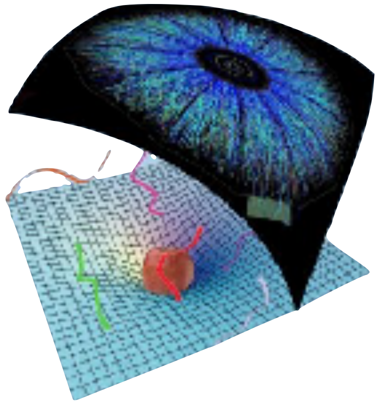
Theory approaches to transport and thermalisation



pQCD, EFTs and kinetic theories thereof: QCD action. Real world: extrapolate from $g \ll 1$ to $\alpha_s \sim 0.3$

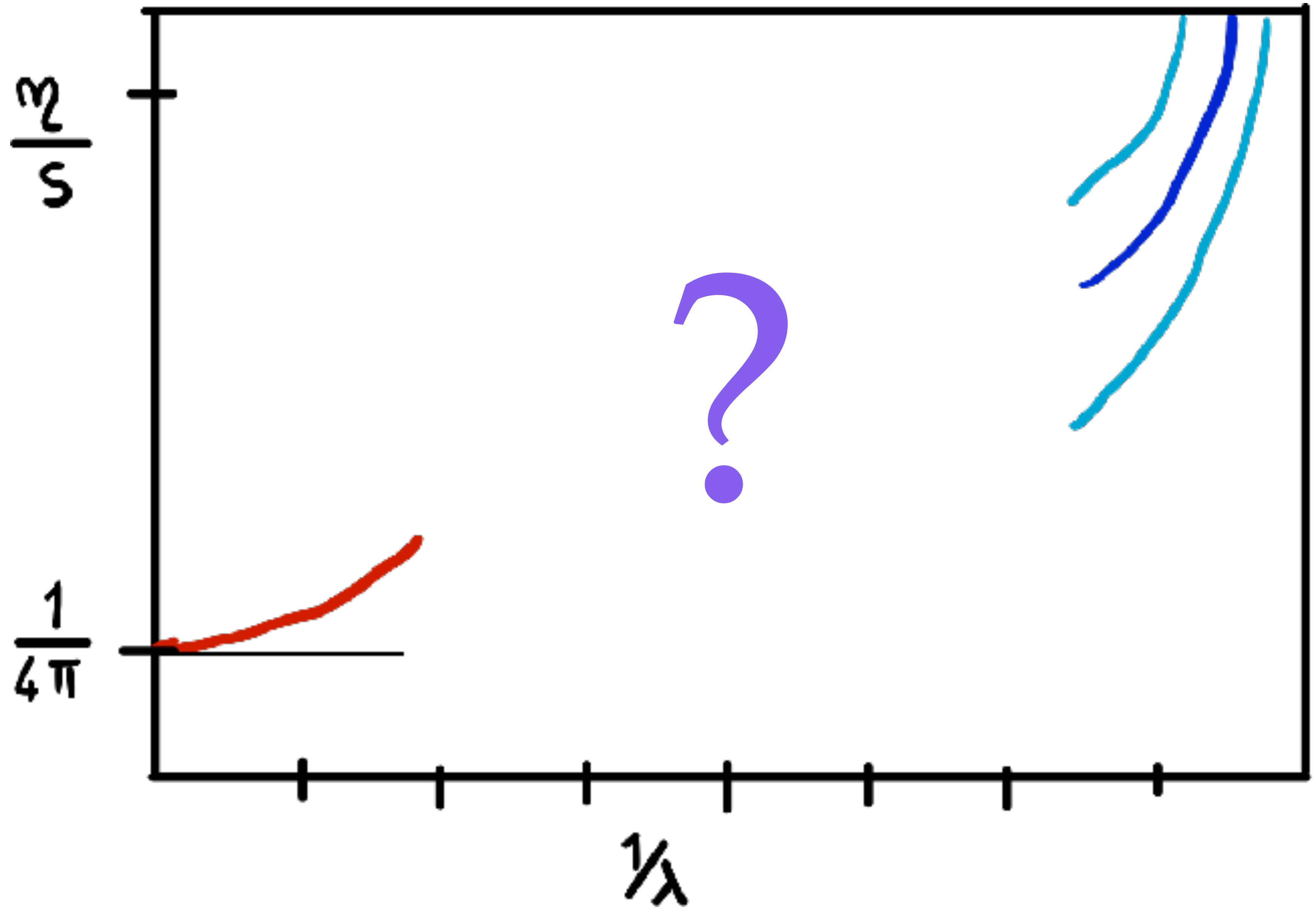


lattice QCD: Euclidean QCD action. Real world: analytically continue to Minkowskian domain, or, if classical, to the quantum world



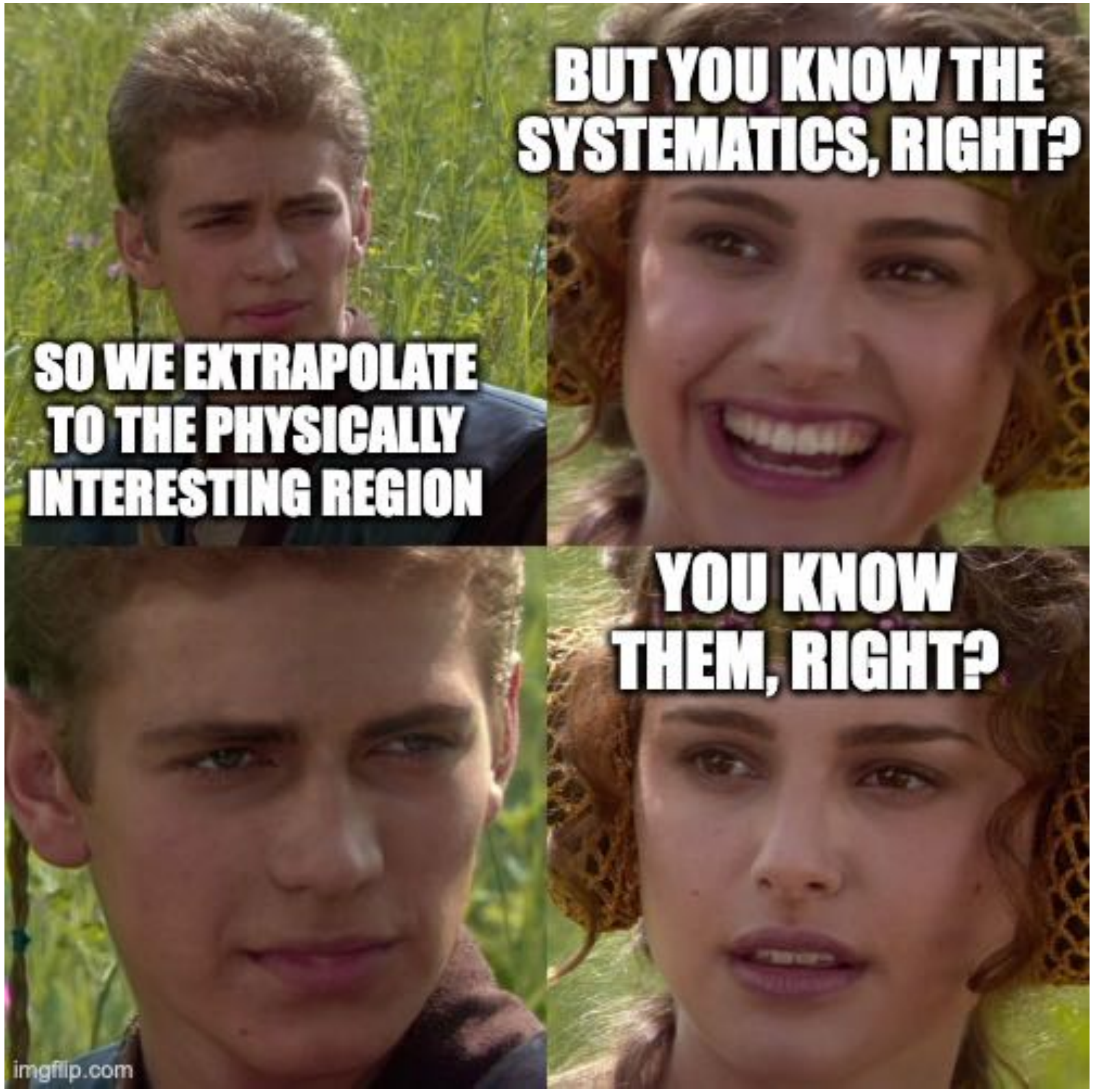
AdS/CFT: $\mathcal{N}=4$ action, weak and strong coupling. Real world: extrapolate to QCD

Motivation



- Holography: Kovtun Son Starinets Policastro **PRL87** (2001) **PLR94** (2004), $\lambda^{-3/2}$ corrections: Buchel et al (2005-2008)
- pQCD: Arnold Moore Yaffe (AMY) (2000-2003)

Motivation


SO WE EXTRAPOLATE TO THE PHYSICALLY INTERESTING REGION
BUT YOU KNOW THE SYSTEMATICS, RIGHT?
YOU KNOW THEM, RIGHT?
imgflip.com

The weak-coupling picture

The weak-coupling picture

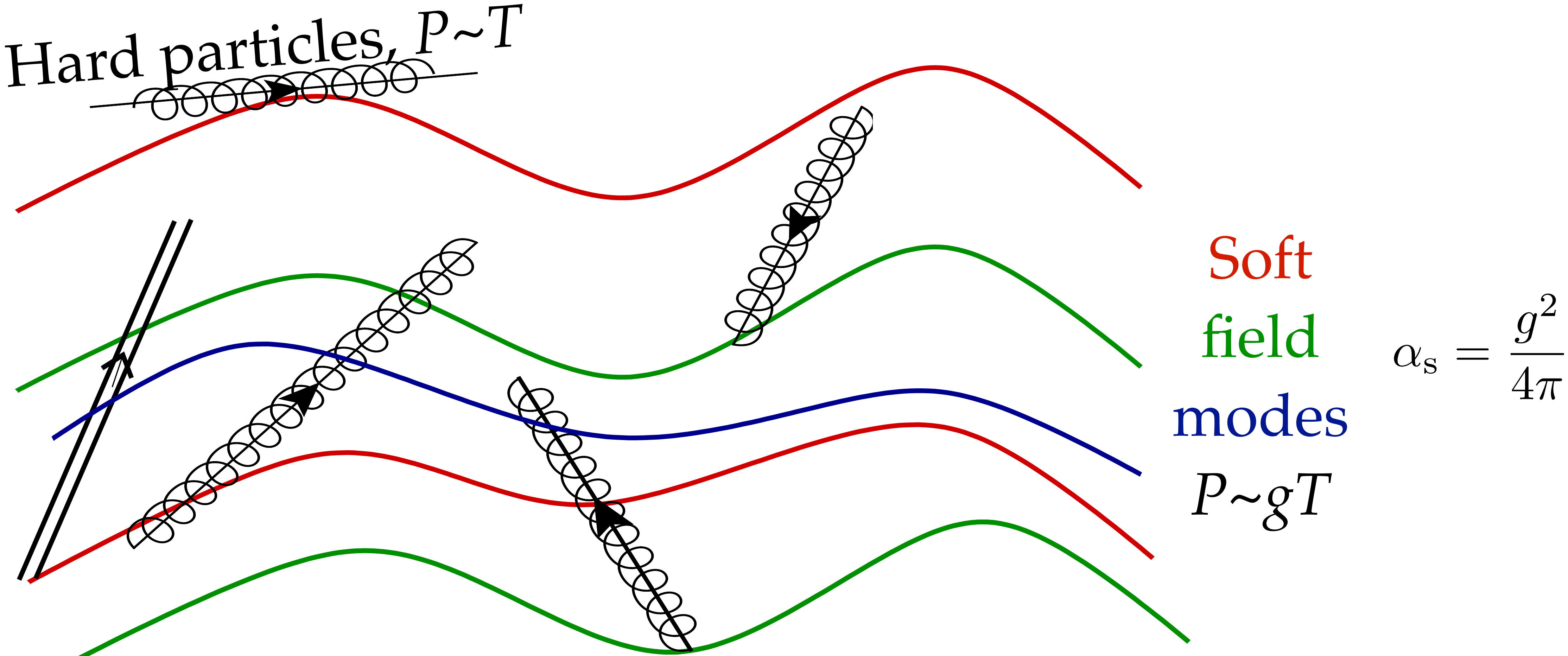
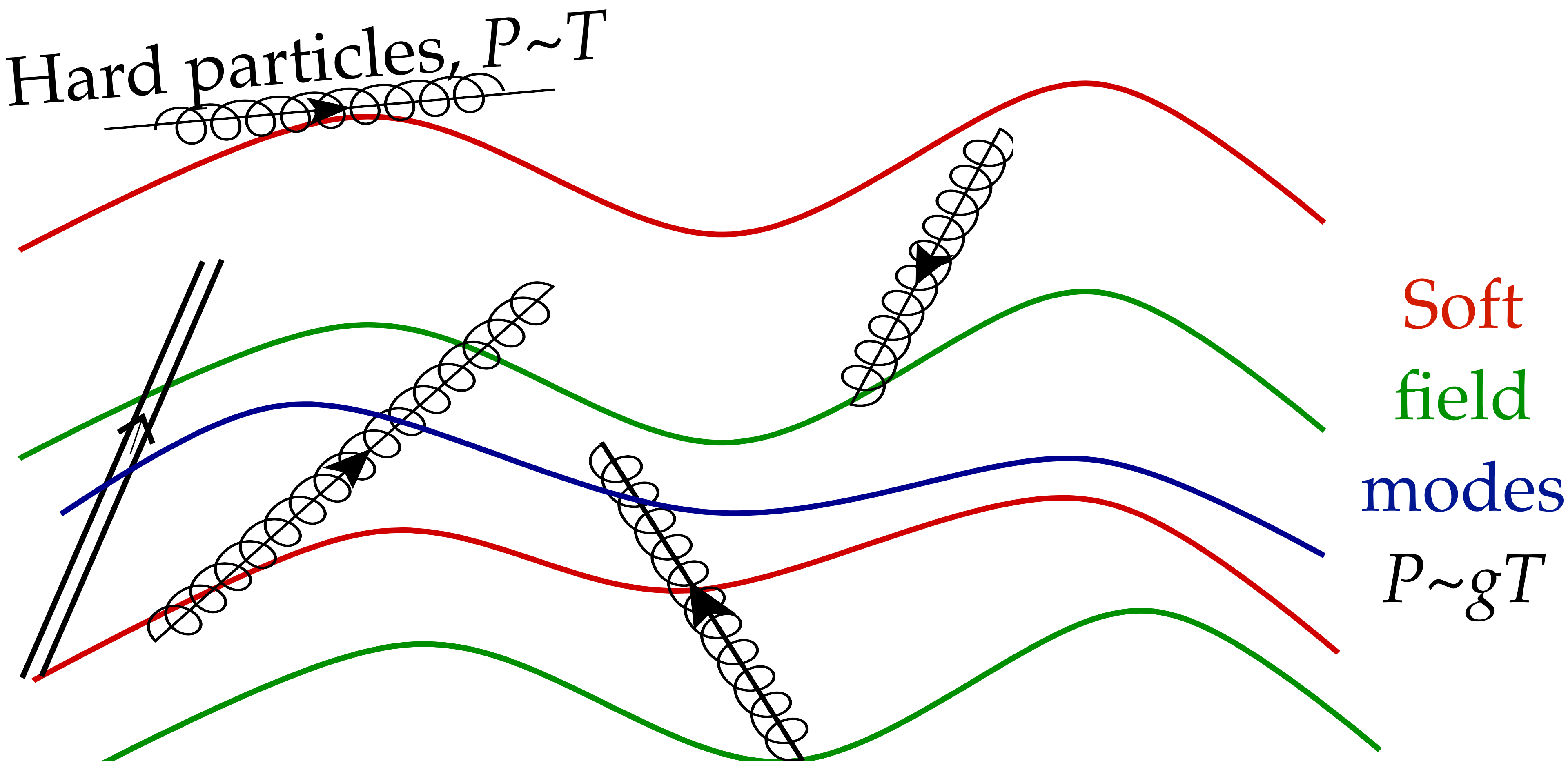


Figure by D. Teaney

- Hard (quasi)-particles carry most of the stress-energy tensor. (Parametrically) largest contribution to thermodynamics

The weak-coupling picture



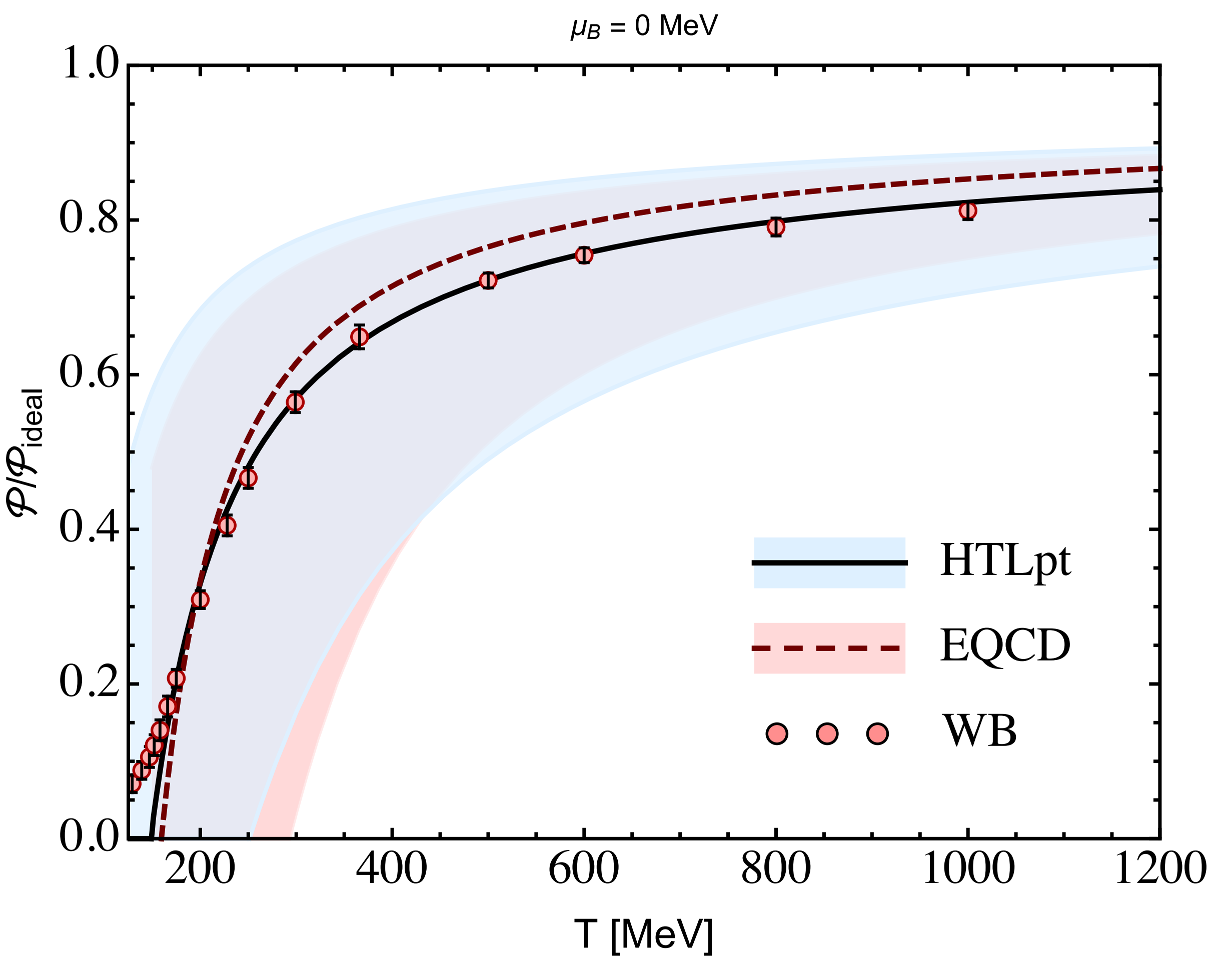
$\alpha_s = \frac{g^2}{4\pi}$

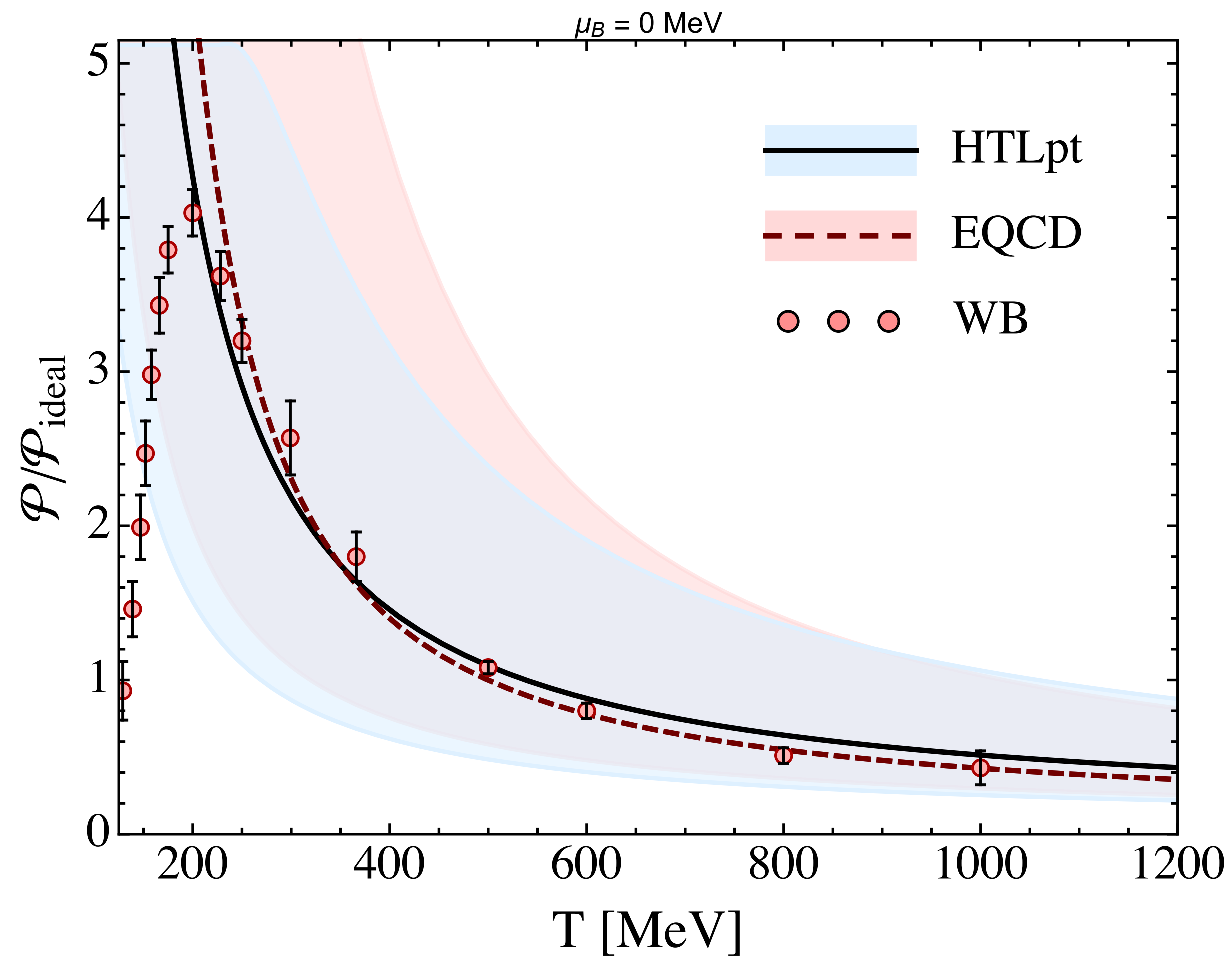
Figure by D. Teaney

- The gluonic soft fields have large occupation numbers ⇒ they can be treated classically

$$n_B(\omega) = \frac{1}{e^{\omega/T}-1} \overset{\omega\sim gT}{\simeq} \frac{T}{\omega} \sim \frac{1}{g}$$

Weak-coupling thermodynamics



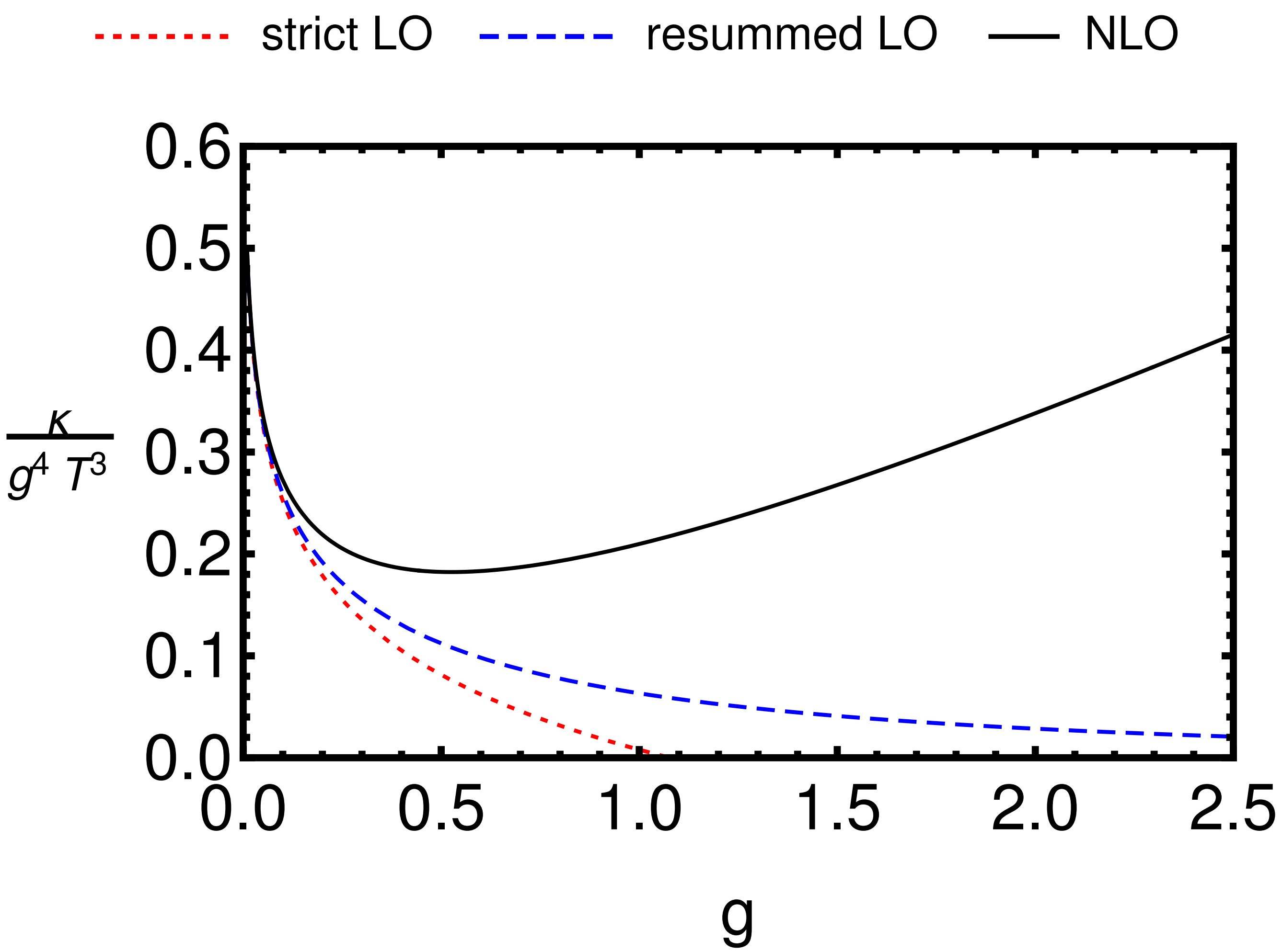


Review: JG Kurkela Strickland Vuorinen **Phys. Rep. 880** (2020)

- Successful for static (thermodynamical) quantities. Possibility of solving the soft sector non-perturbatively (3D theory EQCD on the lattice).

Weak-coupling dynamics

- Starting to scratch the surface of beyond leading-order calculations

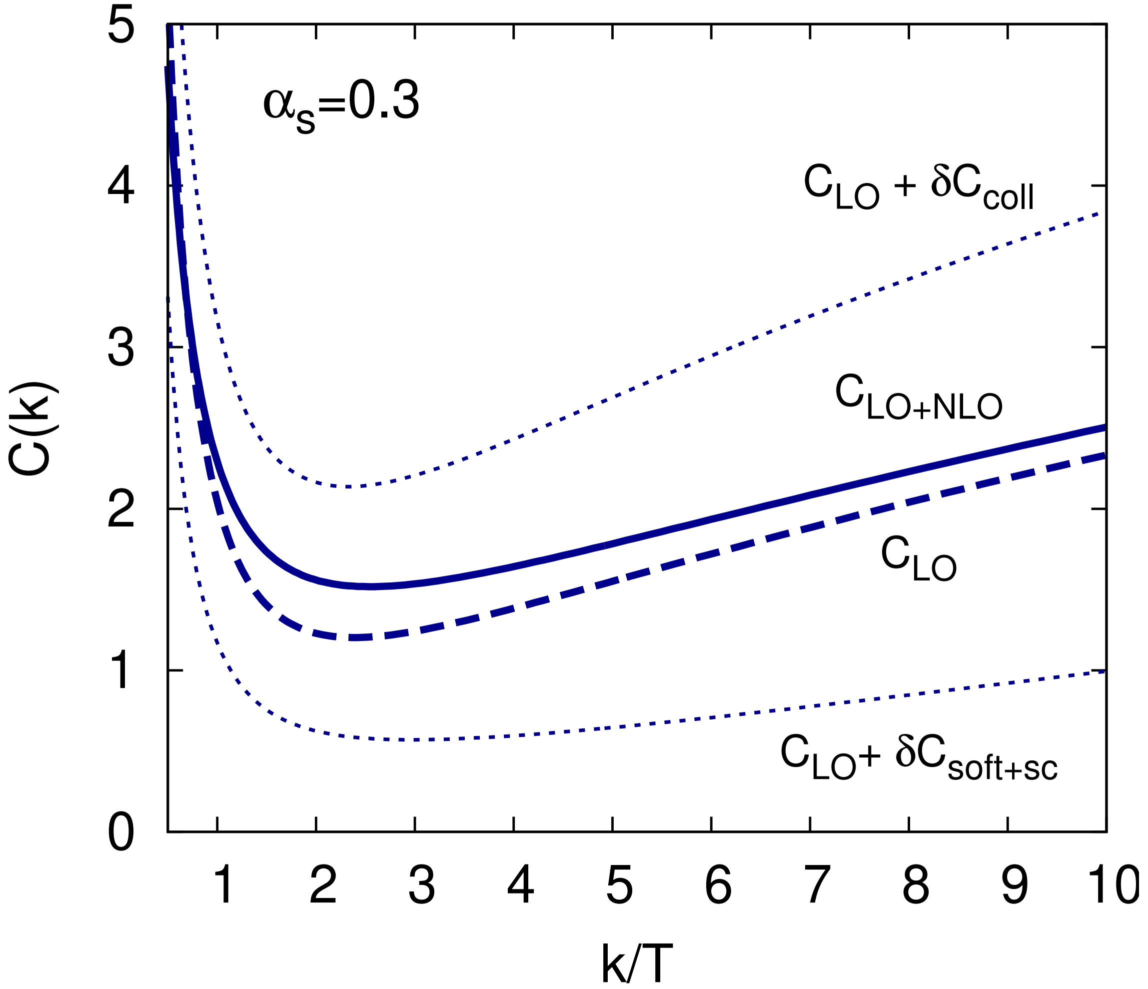


- Heavy-quark momentum diffusion
  Caron-Huot Moore **PRL100** (2007)

Weak-coupling dynamics

- Starting to scratch the surface of beyond leading-order calculations

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{2\alpha\alpha_s T^2 n_{\rm F}(k)}{9k\pi^2} C(k)$$



- Thermal photon rate

JG Hong Lu Kurkela Moore Teaney **JHEP1305** (2013)

The effective kinetic theory

Baym Braaten Pisarski Arnold Moore Yaffe Baier Dokshitzer Mueller Schiff Son Peigné Wiedemann Gyulassy Wang Aurenche Gelis Zaraket Blaizot Iancu . . .

The effective kinetic theory

- Justified at weak coupling, but could be extended to factor in non-perturbative contributions

- The effective theory is obtained by integrating out (off-shell) quantum fluctuations (for instance from Kadanoff-Baym equations). Appropriate for describing the dynamics of excitations on scales large compared to *1/T*, which is the size of the typical de Broglie wavelength of an excitation.

- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator

$$(\partial_t + \mathbf{v}_\mathbf{p} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$$

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- Boltzmann equation for the single-particle phase space-distribution: its convective derivative equals a collision operator $$(\partial_t + \mathbf{v}_{\mathbf{p}} \cdot \boldsymbol{\nabla}) f(\mathbf{p}, \mathbf{x}, t) = C[f]$$

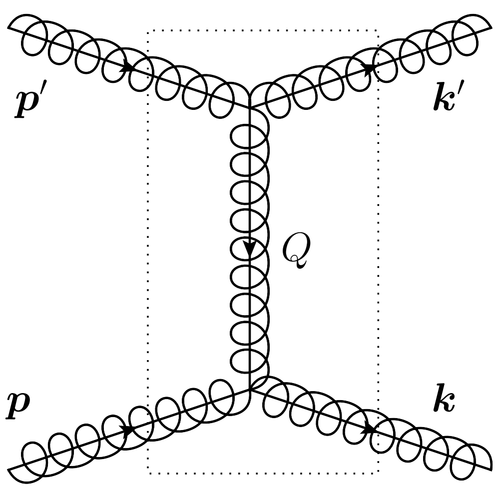
- In other words at weak coupling the underlying QFT has well-defined quasi-particles. These are weakly interacting with a *mean free time* ($1/g^4T$) *large compared to the actual duration of an individual collision* ($1/T$)

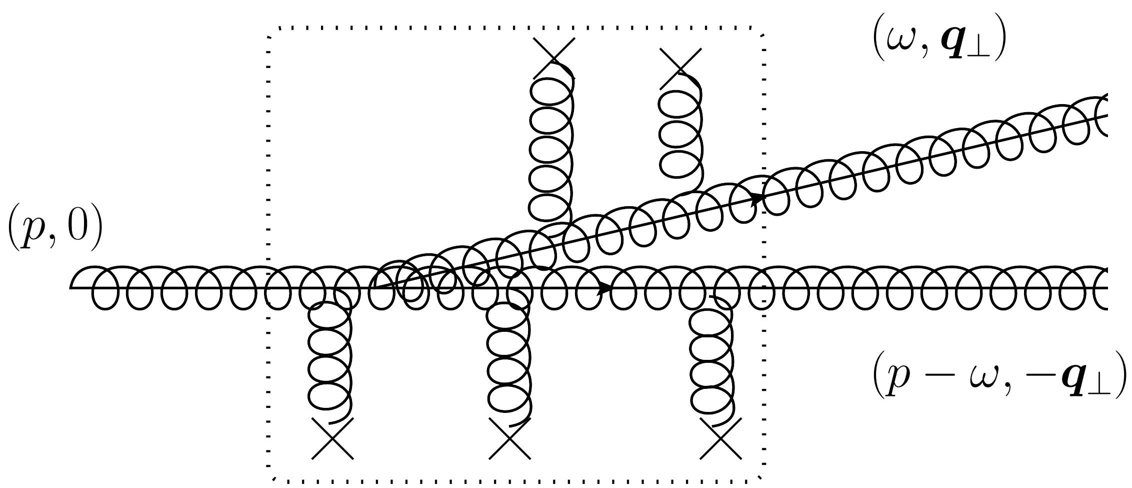
The AMY kinetic theory

- Effective Kinetic Theory (EKT) for the phase space density of quarks and gluons

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

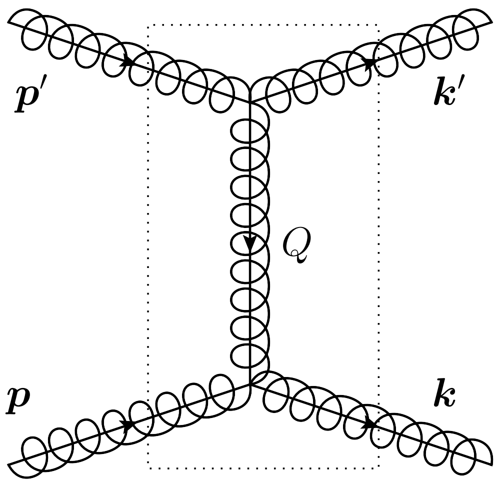
- At **leading order**: elastic, number-preserving 2↔2 processes and collinear, number-changing 1↔2 processes (LPM, AMY, all that) AMY (2003)





The AMY kinetic theory

- How to deal with the soft Q region?
- Standard approach: dressing the intermediate propagator with Hard Thermal Loops (HTL) for IR finiteness
  Braaten Pisarski 1990-92, Arnold Moore Yaffe



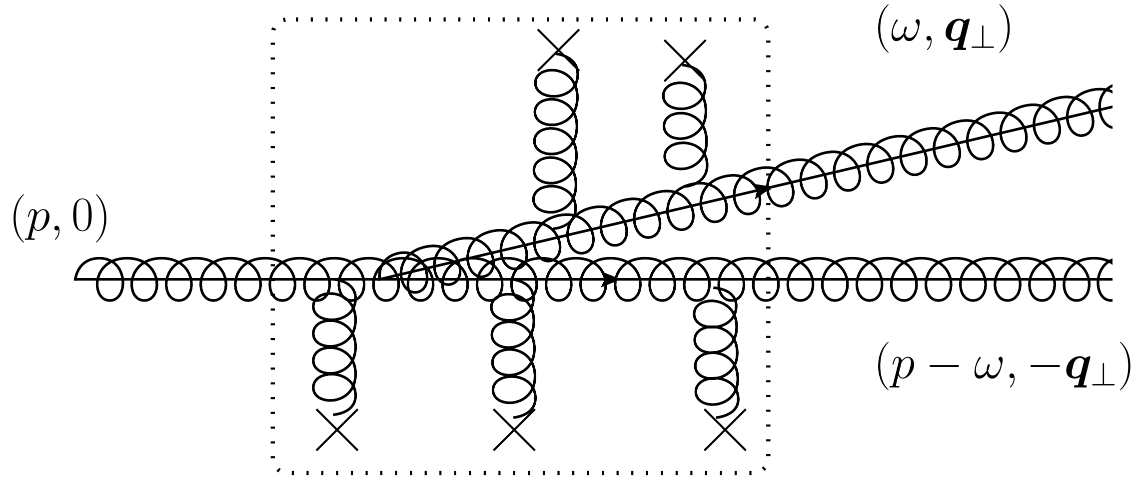
- Hard Thermal Loops: resummation of 1-loop hard off-shell loops into soft propagators (and vertices). Rich structure, collective effects (plasmons, Landau damping)

$$m_D^2 = g^2T^2(N_c/3 + n_f/6)$$

$$G_R^{00}(\omega,\mathbf{q}) = \frac{-i\eta^{00}}{q^2+\Pi_L(\omega/q)} \qquad \Pi_L = m_D^2\left(1-\frac{\omega}{2q}\log\left(\frac{\omega+q}{\omega-q}\right)\right)$$

$$G_R^{ij}(\omega,\mathbf{q}) = \frac{-i(\delta^{ij}-\hat{q}^i\hat{q}^j)}{-(q^0)^2+q^2+\Pi_T(\omega/q)} \qquad \Pi_T = \frac{m_D^2}{2}\left(\left(\frac{\omega}{q}\right)^2 - \frac{(\omega^2-q^2)\omega}{2q^3}\log\left(\frac{\omega+q}{\omega-q}\right)\right)$$

The AMY kinetic theory



- Apparently suppressed by powers of g but
  - Soft and collinear enhancements cancel the suppression
  - Mean free time between soft collisions ($1/g^2T$) of the same order of formation time ⇒ interference of many such scatterings (Landau-Pomeranchuk-Migdal effect)
    Baier Dokshitzer Mueller Peigné Schiff Son Zakharov Arnold Moore Yaffe

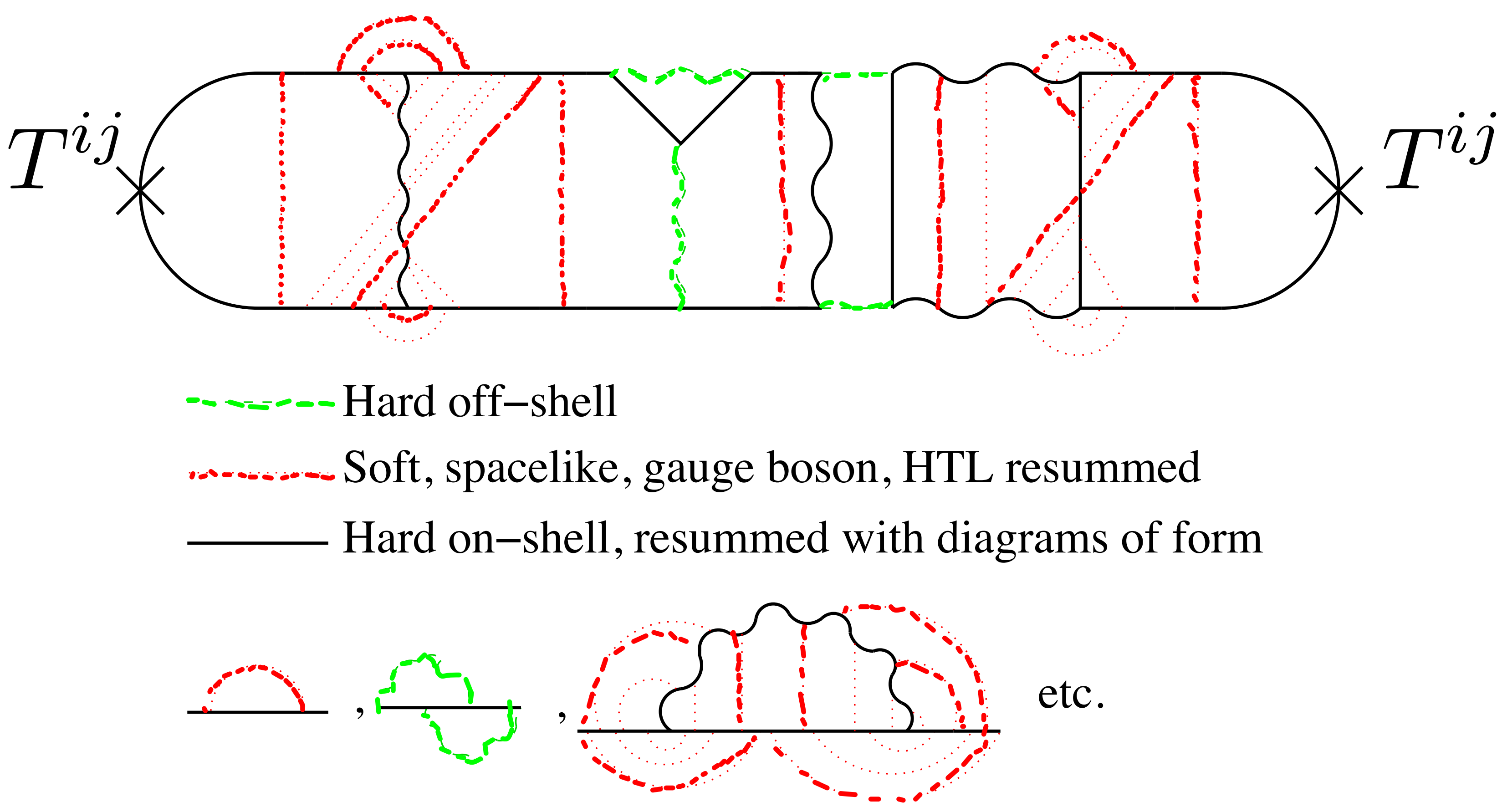
Transport coeffs from the EKT

Transport coeffs from the EKT

- Linearized EKT equivalent to Kubo formula

$$\eta = \frac{1}{20}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\left\langle\left[T^{ij}(t,\mathbf{x}),T^{ij}(0,\mathbf{0})\right]\right\rangle\theta(t)$$

- Not practical at weak coupling: loop expansion breaks down AMY (2000-2003)



Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\mathrm{EQ}}(\mathbf{p}) + \sum_{\ell} \delta f_{\ell}(\mathbf{p})$$

- Source term equates linearized collision operator

$$\mathcal{S}_{\ell} = \mathcal{C}\delta f_{\ell}$$

$$\mathcal{S}_{\ell} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}\right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu)$$

- Since $\langle T^{i\neq j}\rangle \propto \eta$ η requires ℓ=2

- Transport coefficients obtained by the kinetic theory definitions of T once δf_{ℓ} has been obtained

Transport coeffs from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\mathrm{EQ}}(\mathbf{p}) + \sum_{\ell} \delta f_{\ell}(\mathbf{p})$$

- Source term equates linearized collision operator

$$\mathcal{S}_{\ell} = \mathcal{C} \delta f_{\ell}$$

- To solve the linear equation, introduce the inner product

$$(f, g) \equiv \int_{\mathbf{p}} f(\mathbf{p})\, g(\mathbf{p})$$

and minimize

$$(\delta f_{\ell}, \mathcal{S}_{\ell}) - \tfrac{1}{2}(\delta f_{\ell}, \mathcal{C} f_{\ell})$$

AMY (2000-03)

Thermalisation from the EKT: bottom-up

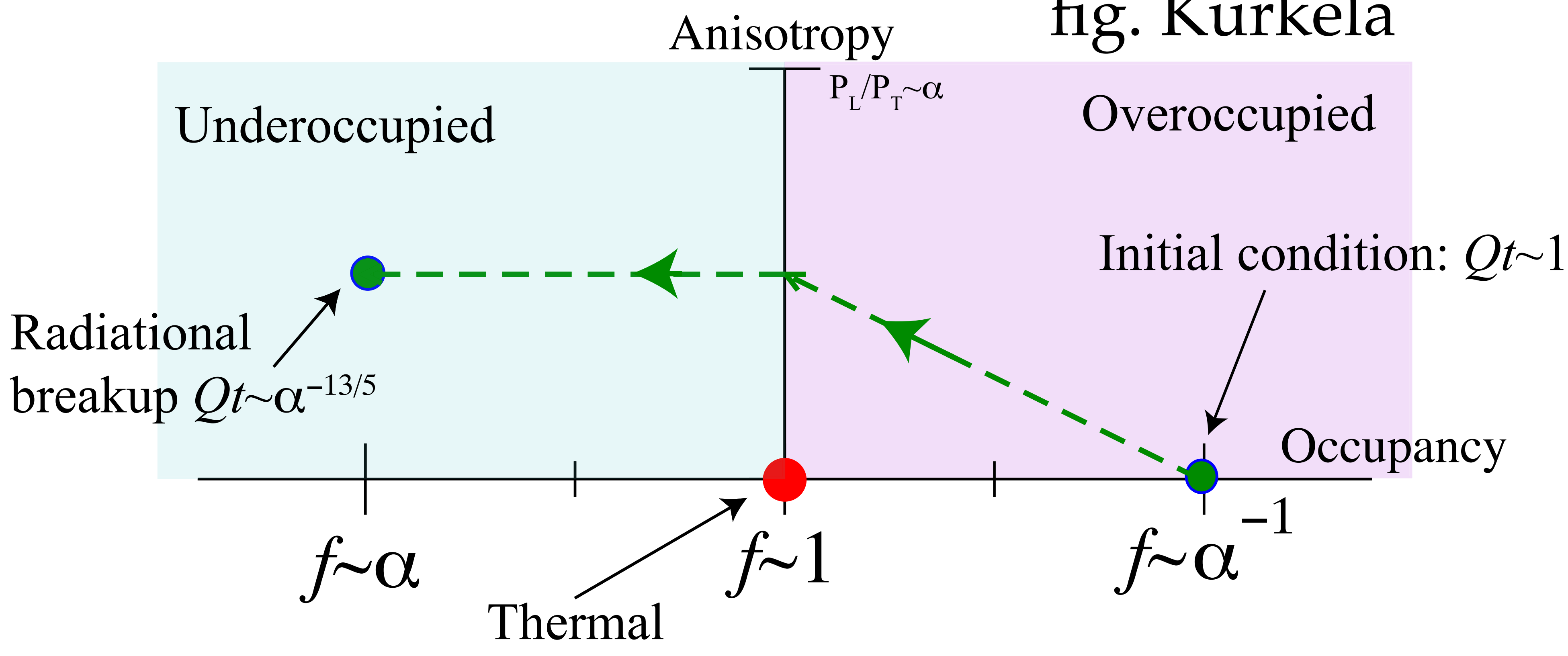
reviews in Schlichting Teaney **Ann.Rev.Nucl.Part.Sci. 69** (2019)
Berges Heller Mazeliauskas Venugopalan **2005.12299**

Thermalisation from the EKT: bottom-up

- Competition between expansion and collision, attractor solution when they balance out

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

fig. Kurkela



Baier Mueller Schiff Son (2001) Kurkela Moore (2011) fig. Mazeliauskas

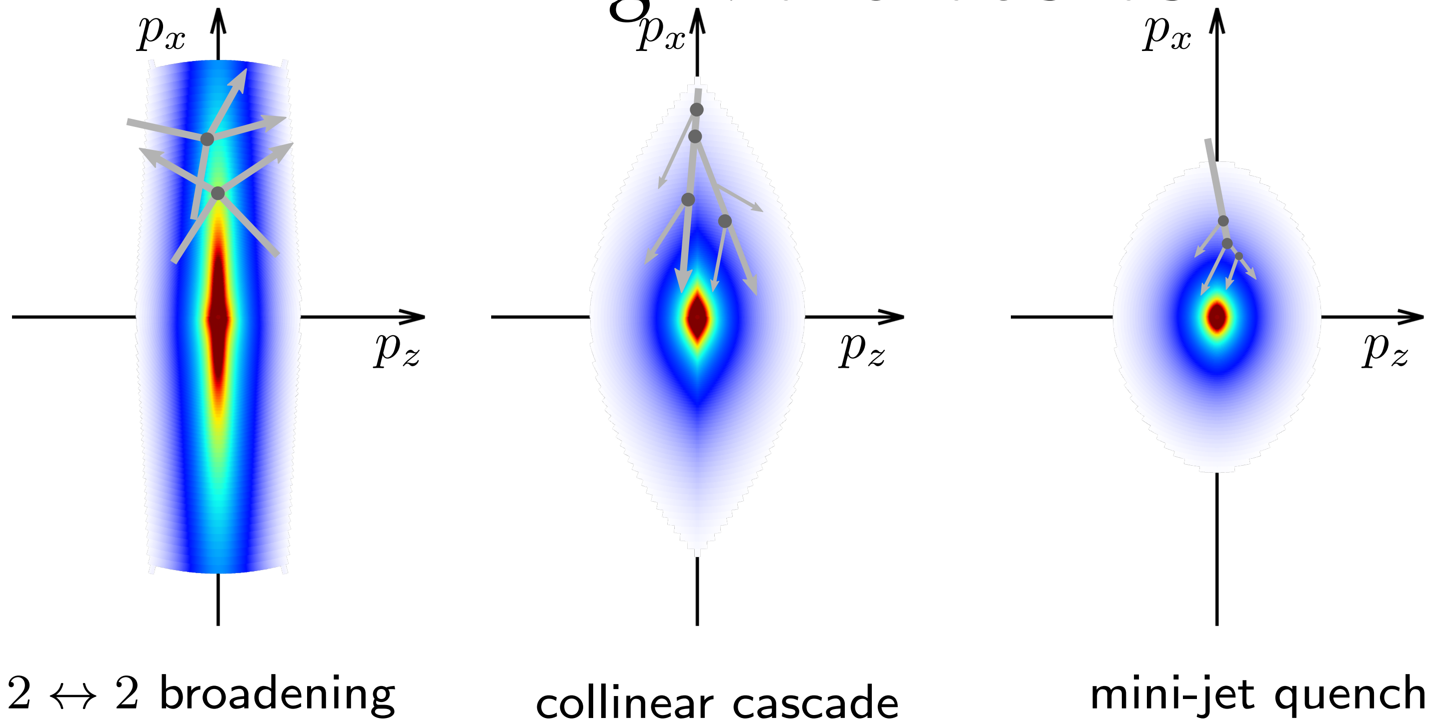
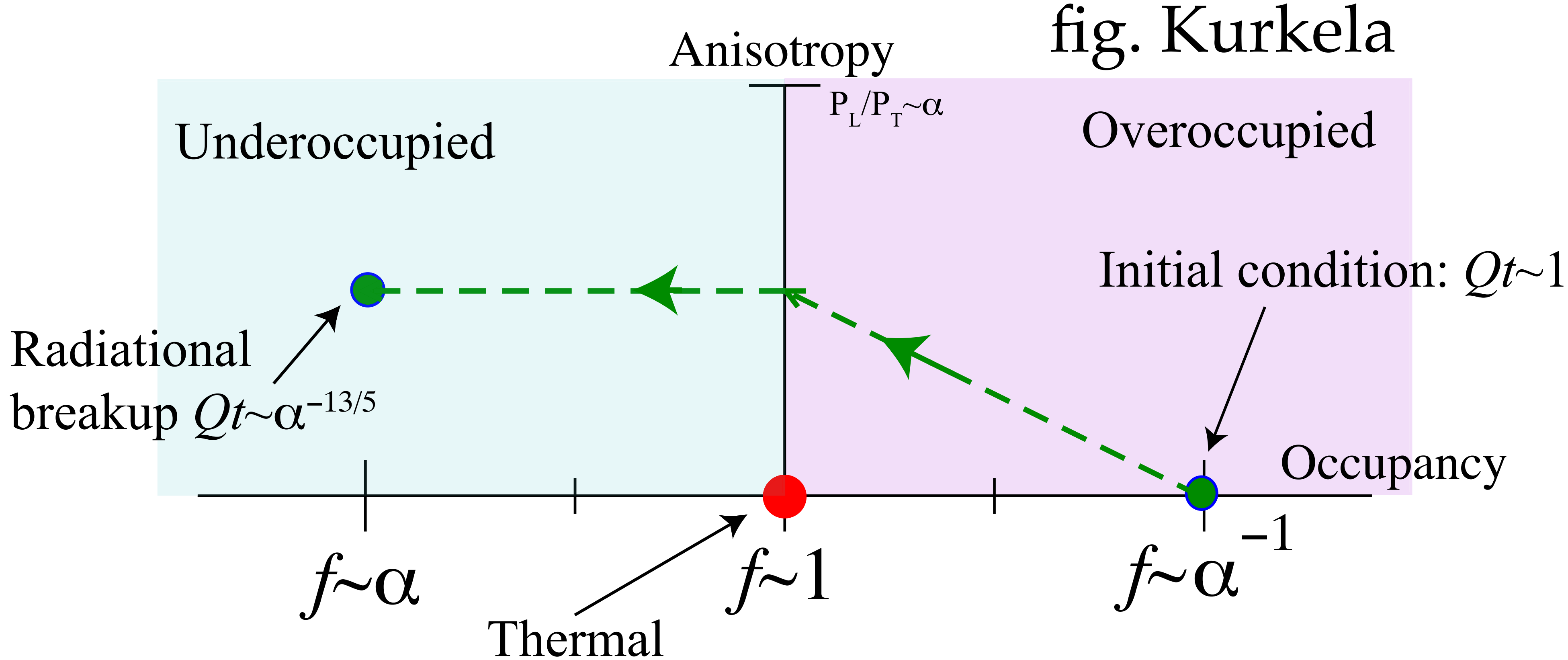


fig. Kurkela

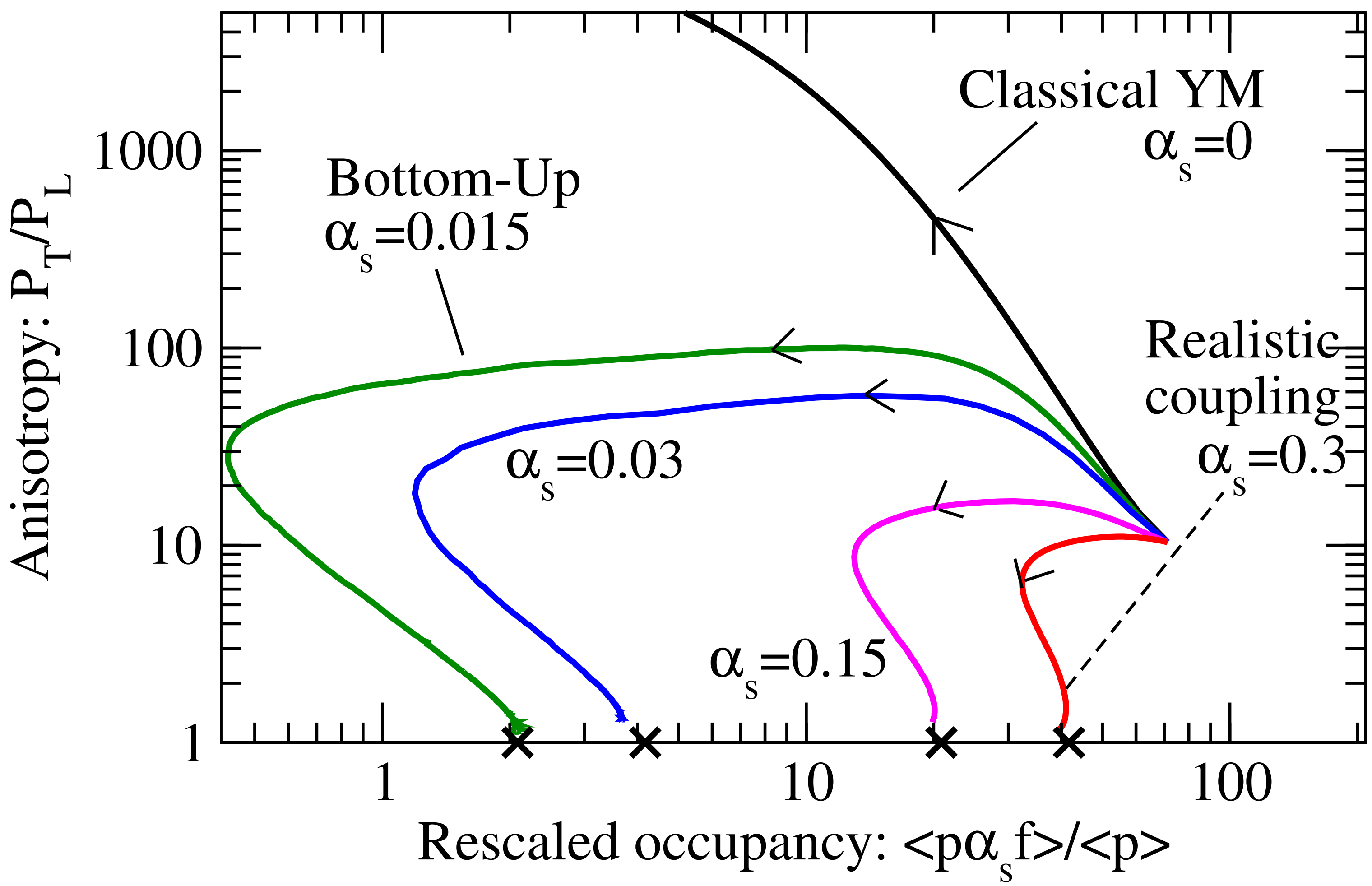


Baier Mueller Schiff Son (2001) Kurkela Moore (2011)

Thermalisation from the EKT: bottom-up

- From numerical solution of LO* kinetic theory

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$



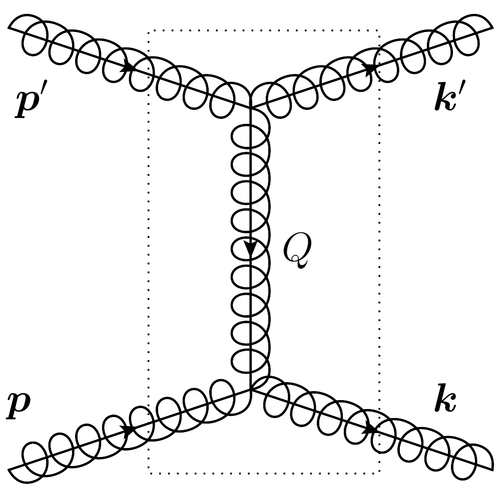
Kurkela Zhu **PRL115** (2015)

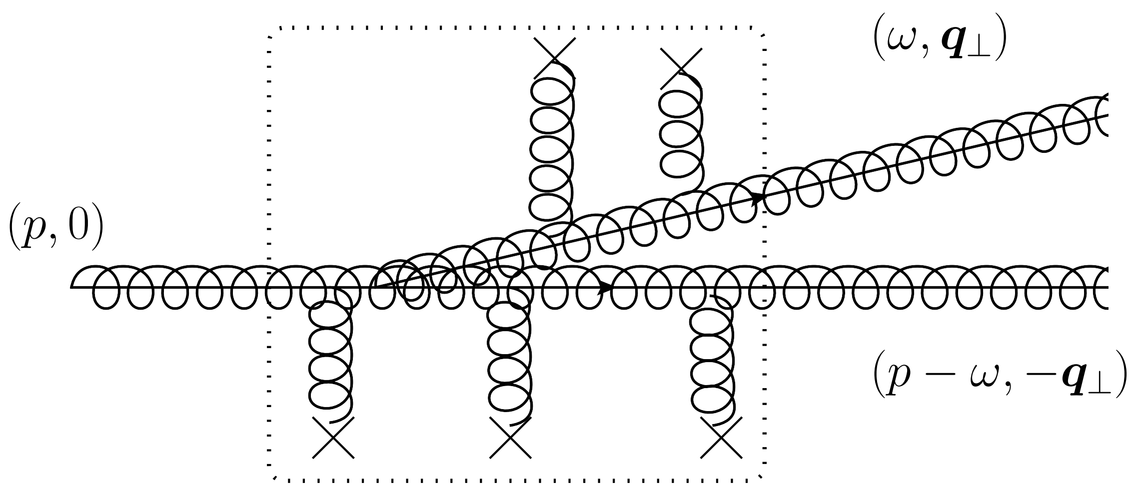
Bottom-up thermalisation

- From numerical solution of LO* kinetic theory

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

- If distributions are anisotropic (because of initial conditions and/or expansion) no consistent LO determination exists, because of plasma instabilities

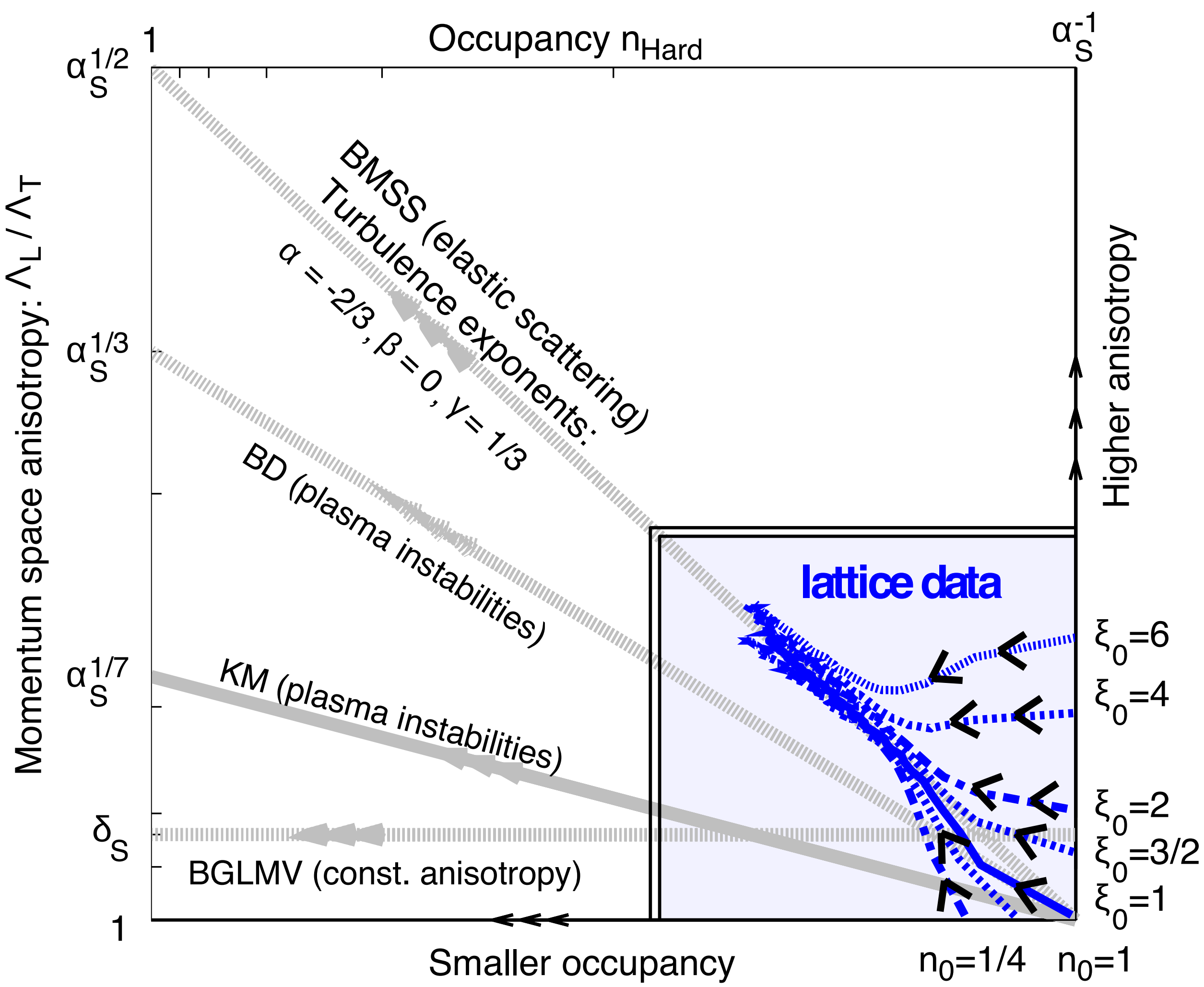




Thermalisation from the EKT: bottom-up

- From numerical solution of classical lattice theory

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\mathbf{p}) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$



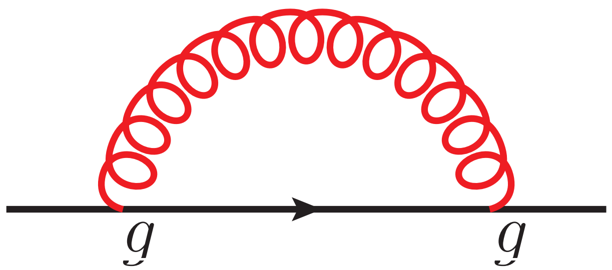
Berges Boguslavski Schlichting Venugopalan **PRD89** (2013)

Soft momenta

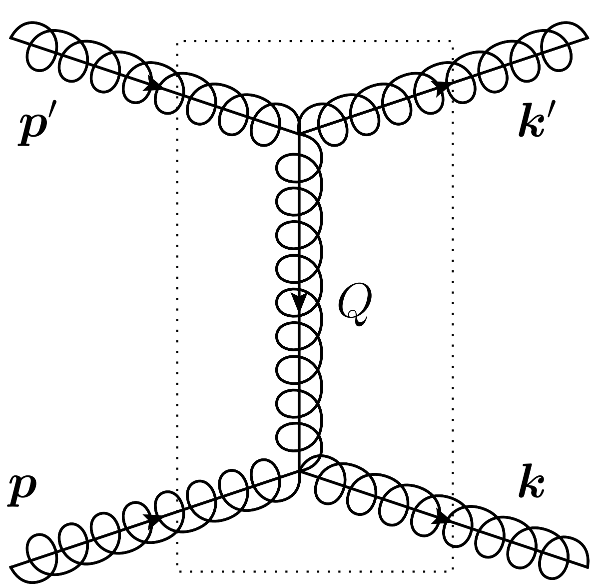
Soft momenta

- The NLO corrections come from regions sensitive to soft gluons (no quarks in this illustration)

$n_B(p) \sim T/p \sim 1/g$



- Before we get there, let's see these regions at LO
- Look at 2↔2 scattering

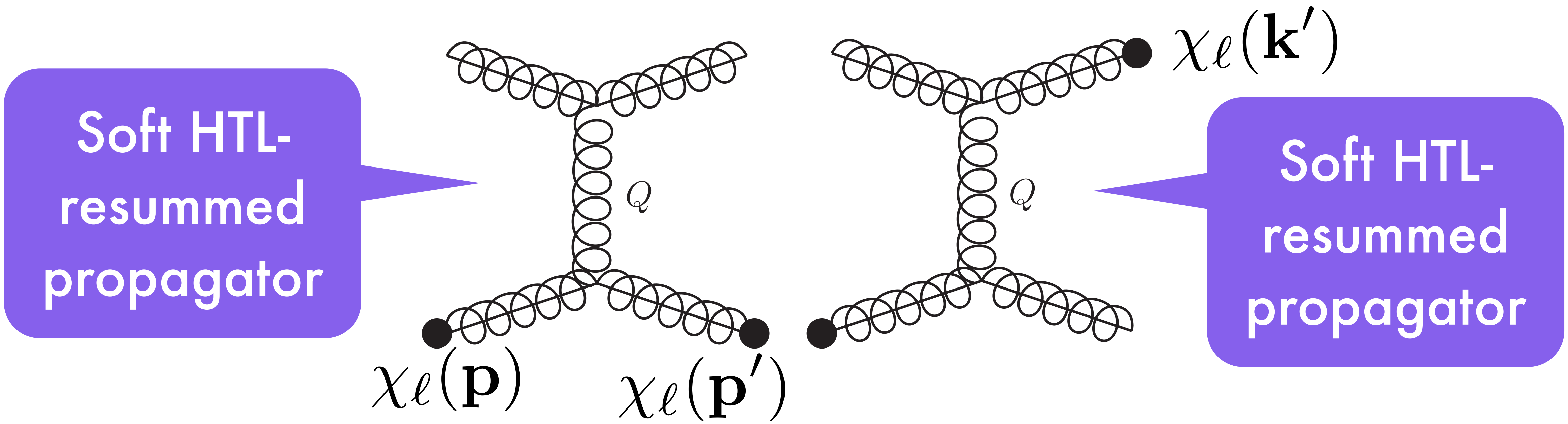


$$\int_{\mathbf{pkp'k'}} \left|\mathcal{M}(\mathbf{p},\mathbf{k};\mathbf{p}',\mathbf{k}')\right|^2 (2\pi)^4\, \delta^{(4)}(P+K-P'-K')$$
$$\times\, f_{\rm EQ}(p)\, f_{\rm EQ}(k)\, [1+f_{\rm EQ}(p')]\, [1+f_{\rm EQ}(k')]$$
$$\times \Big[\chi_\ell(\mathbf{p}) + \chi_\ell(\mathbf{k}) - \chi_\ell(\mathbf{p}') - \chi_\ell(\mathbf{k}')\Big]^2$$

$$\delta f_\ell(\mathbf{p}) \equiv f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\chi_\ell(\mathbf{p})$$

LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$



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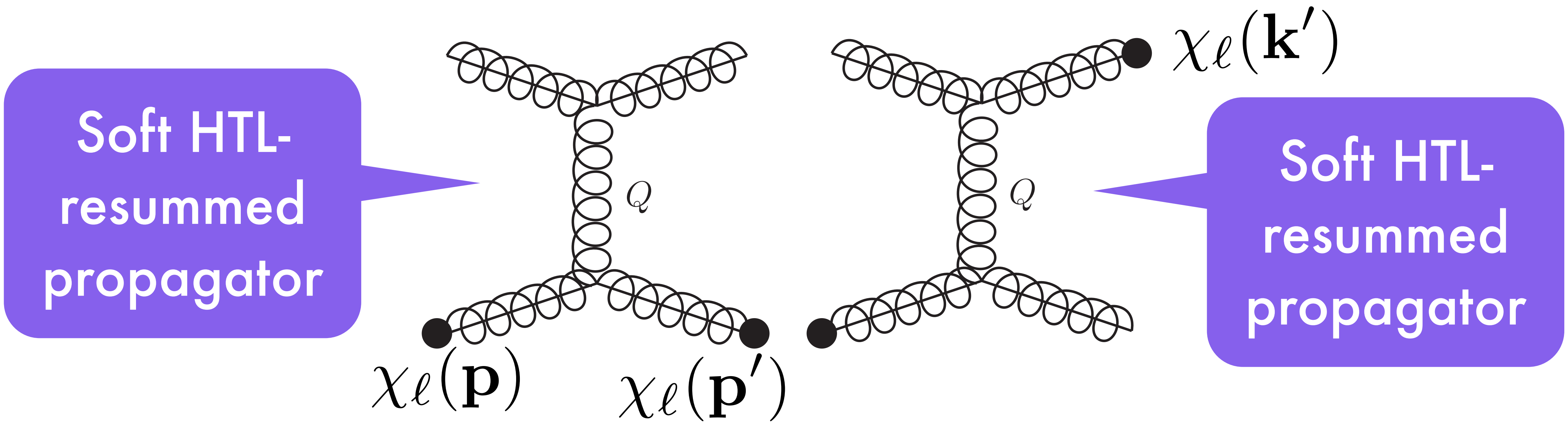


- Left: **diffusion terms**, **p** and **p′** strongly correlated

$$\left(\chi_\ell(\mathbf{p})-\chi_\ell(\mathbf{p}')\right)^2=(\hat{\mathbf{p}}\cdot\mathbf{q})^2[\chi'(p)]^2+\frac{\ell(\ell+1)}{2}\frac{q^2-(\hat{\mathbf{p}}\cdot\mathbf{q})^2}{p^2}[\chi(p)]^2$$

identify a longitudinal and a transverse momentum broadening contribution, $\hat{q}_L$ and $\hat{q}$

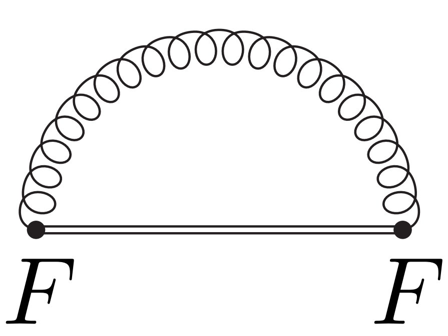
Light-cone techniques

- Key advancement over the past decade: analytical properties of soft thermal amplitudes at light-like separations. Heuristically, the hard, light-like parton sees undisturbed soft modes, which *"can't keep up"* with it
  Caron-Huot **PRD82** (2008) *Review*: JG Teaney **QGP5** (2015)

- In a nutshell, retarded functions are analytical in the upper half plane in **any** time-like variable. In the soft sector to NLO also for **light-like variables** ($q^+=(q^0+q^z)/2$).

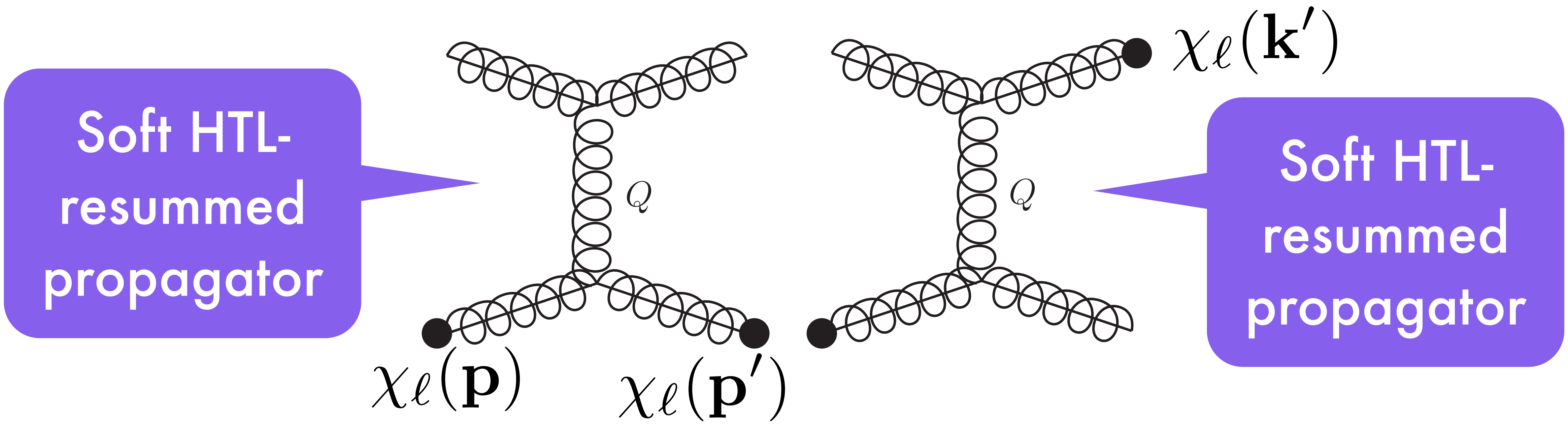
$$\hat{q}(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-\perp}(Q) F^{-}_{\perp} \rangle_{q^-=0}$$

$$\hat{q}_L(\mu_\perp) = g^2 C_A \int^{\mu_\perp} \frac{d^2 q_\perp}{(2\pi)^2} \int \frac{dq^+}{2\pi} \langle F^{-+}(Q) F^{-+} \rangle_{q^-=0}$$



LO soft gluon scattering

- When Q=P'-P becomes soft there are two possibilities for $\color{red}{\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p}')-\chi_\ell(\mathbf{k}')\right]^2}$ $\quad (\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$



- **Diffusion terms:** summing up, light-cone techniques

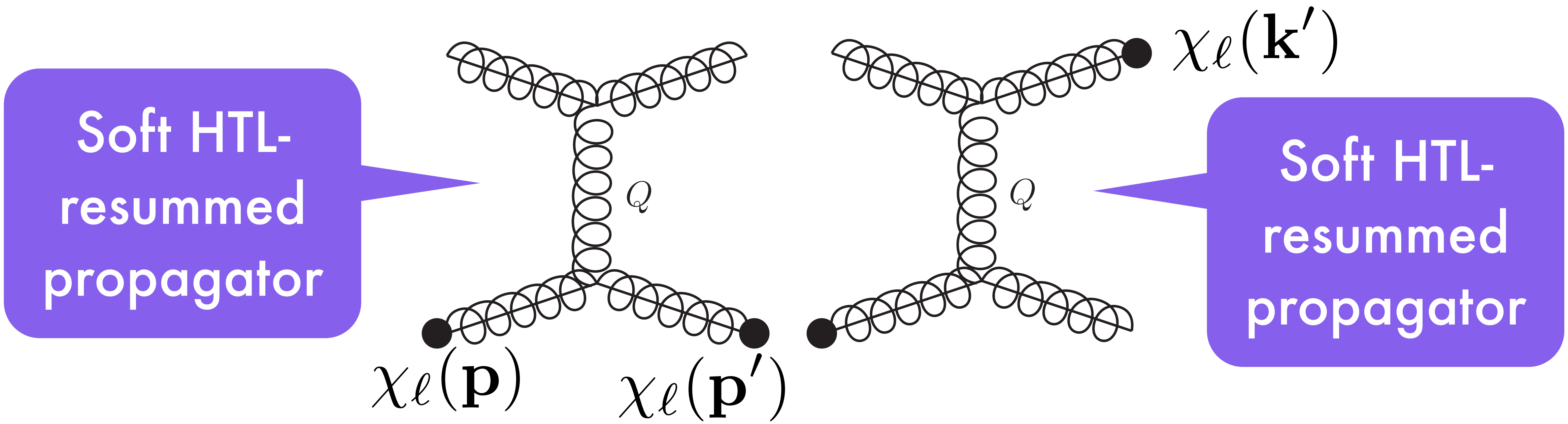
$$\hat{q}_L^a\Big|_{\text{soft}} = \frac{g^2 C_{R_a} T m_D^2}{4\pi}\ln\frac{\sqrt{2}\mu_\perp}{m_D} \qquad \hat{q}^a\Big|_{\text{soft}} = \frac{g^2 C_{R_a} T m_D^2}{2\pi}\ln\frac{\mu_\perp}{m_D}$$

give rise to the leading log contribution

Caron-Huot **PRD82** (2008) JG Moore Teaney **JHEP1603** (2015)

LO soft gluon scattering

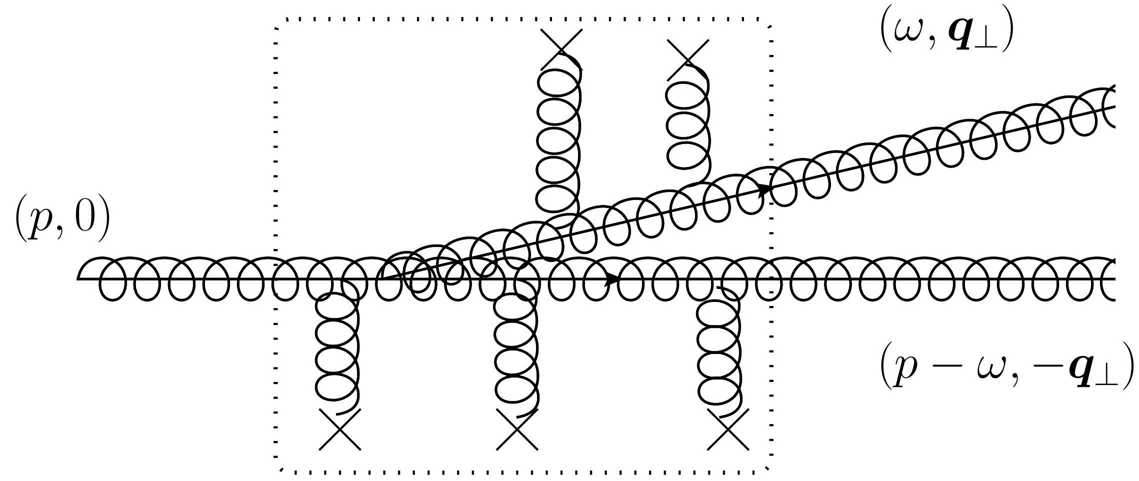
- When Q=P'-P becomes soft there are two possibilities for $\left[\chi_\ell(\mathbf{p})+\chi_\ell(\mathbf{k})-\chi_\ell(\mathbf{p'})-\chi_\ell(\mathbf{k'})\right]^2$ $\quad(\chi_\ell(\mathbf{p})=f_\ell(\hat{\mathbf{p}})\chi(p))$



- Right: gain terms, **p**,**p'** and **k**,**k'** not correlated. Two-point function of two uncorrelated deviations from equilibrium
Light-cone techniques not applicable, have to use numerical integration.

Reorganization

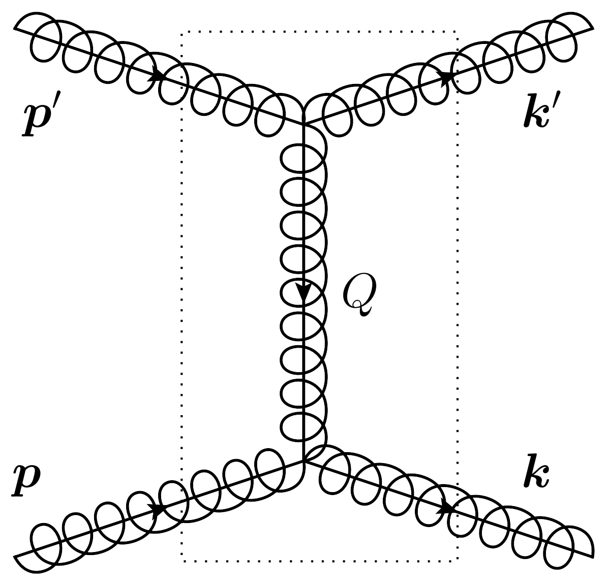
- 1↔2 processes: well separated at LO from 2↔2



- Reorganization of the LO collision operator

$$\int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu) = \int_{\mathbf{p}} \delta f_\ell(\mathbf{p}) \left[C^{\mathrm{large}}[\mu_\perp] + C^{\mathrm{diff}}[\mu_\perp] + C^{\mathrm{cross}} + C^{\mathrm{coll}} \right]$$

- Final ingredient: 2↔2 large angle scatterings, IR-regulated to avoid the soft region



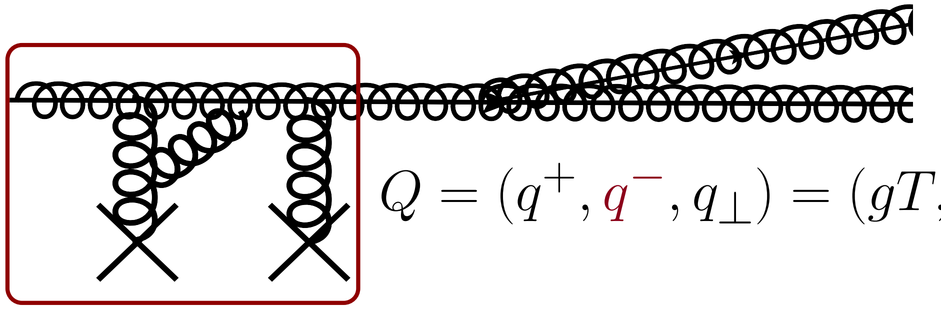
Going to NLO

- The diffusion, gain and collinear terms receive $O(g)$ corrections
- There is a new semi-collinear region

Collinear corrections

- The differential eq. for LPM resummation gets correction from NLO $C(q_\perp)$ and from the thermal asymptotic mass at NLO (Caron-Huot 2009)

$$\frac{d\Gamma}{d^2q_\perp} = \mathcal{C}_{\rm LO}(q_\perp) = \frac{g^2 C_A T m_D^2}{q_\perp^2(q_\perp^2+m_D^2)}$$



$$Q = (q^+, q^-, q_\perp) = (gT,\, g^2T,\, gT)$$

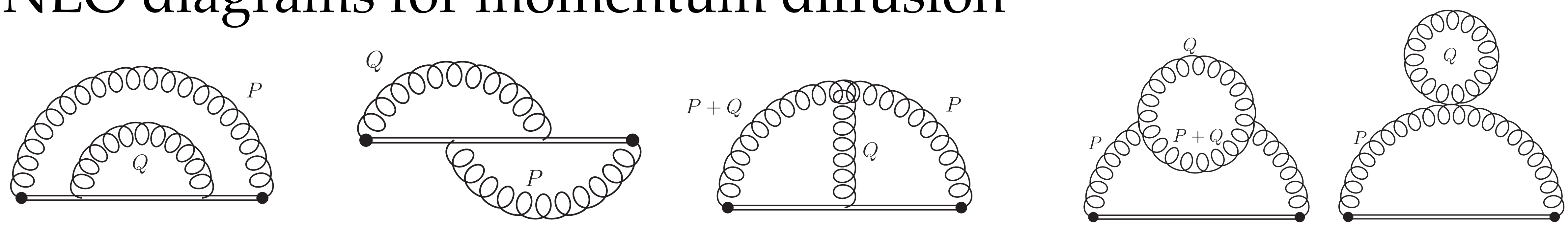
$\mathcal{C}_{\rm NLO}(q_\perp)$ complicated but analytical (Euclidean tech)

Caron-Huot **PRD79** (2009), Lattice: Panero *et al.* (2013), Moore Schlusser (2019-2020) Moore Schlichting Schlusser Soudi (2021)

mass JG Moore Schicho Schlusser, in progress

Diffusion and semi-collinear corrections

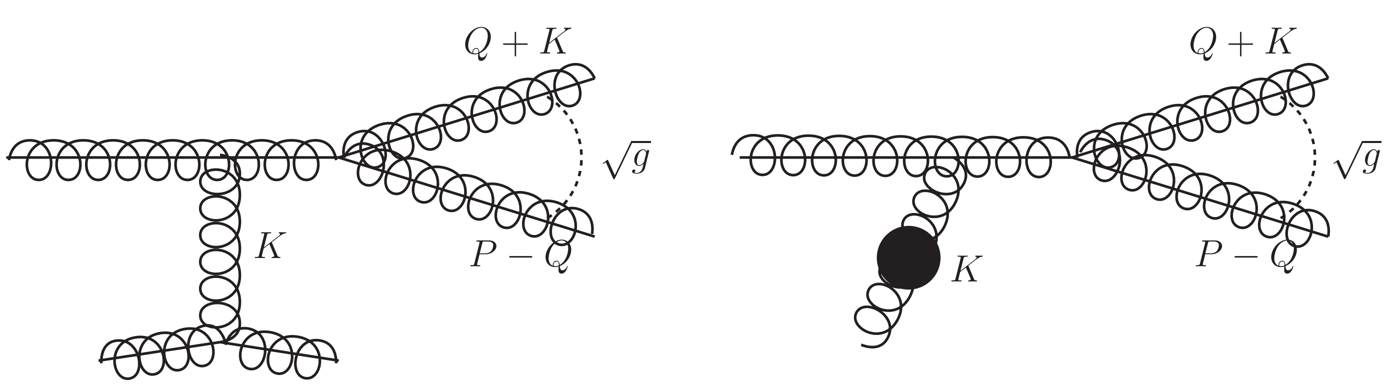
- NLO diagrams for momentum diffusion



- Transverse and longitudinal analytical with light-cone techniques

  Caron-Huot **PRD79** (2009) JG Moore Teaney **JHEP1603** (2015)

- Semi-collinear processes (wider angle radiation) need to be included JG Moore Teaney **JHEP1603** (2015)



K soft cut, spacelike

K soft plasmon, timelike

Results for transport (and their fine print)

- Inversion of the collision operator using variational Ansatz
- At NLO just add $O(g)$ corrections to the LO collision operator, do not treat them as perturbations in the inversion

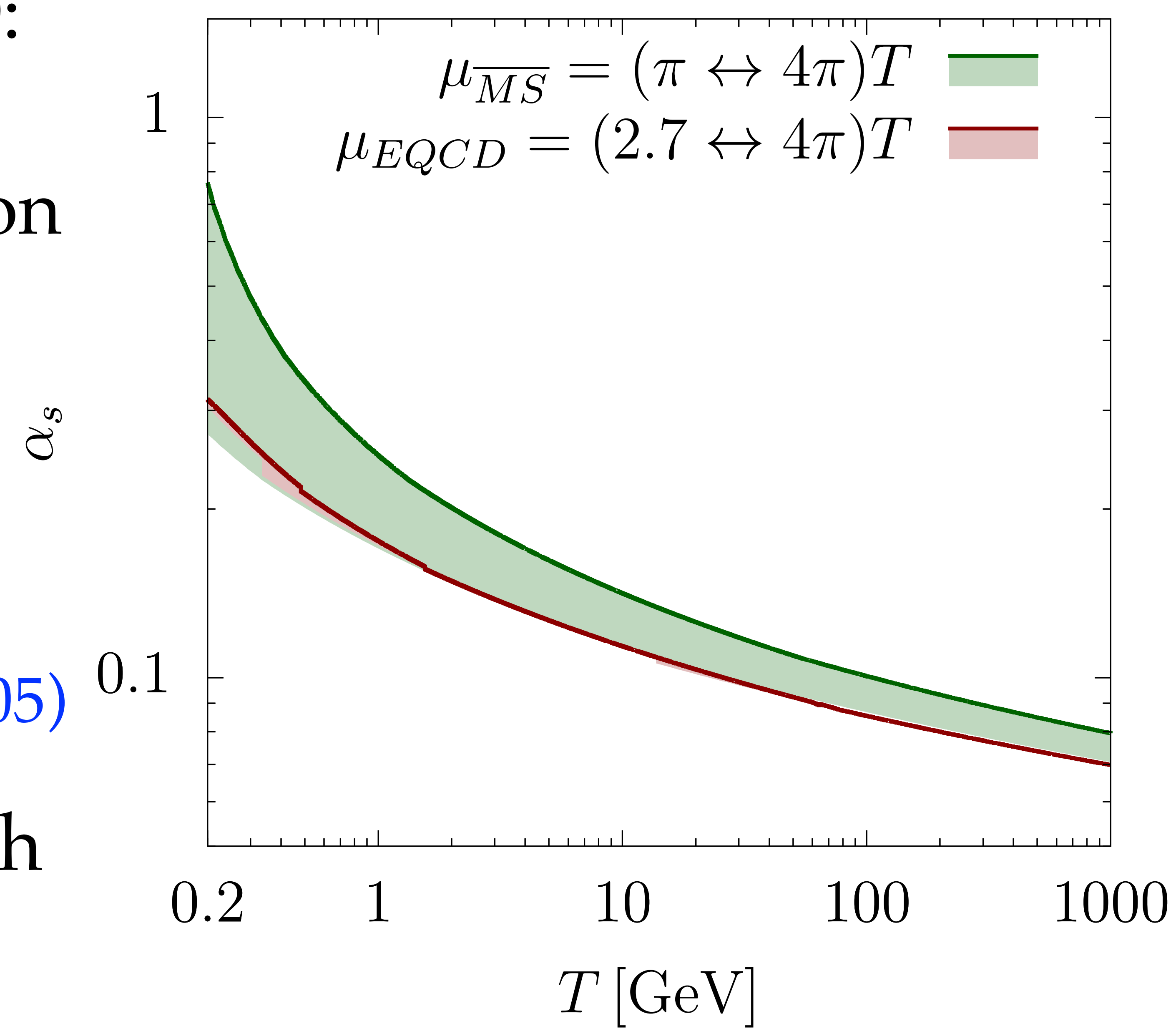
$$\mathcal{S}_\ell = [\mathcal{C} + \delta\mathcal{C}]\delta f_\ell \implies \delta f_\ell = \frac{1}{\mathcal{C} + \delta\mathcal{C}}\mathcal{S}_\ell$$

- First perturbative corrections to s are of order g^2. Including them would be inconsistent with the treatment of the collision operator
- New implementation for semi-collinear processes. Better behaved as g grows due to resummations of $\mathrm{N}^n\mathrm{LO}$ terms ($n \geq 2$)

JG Moore Teaney **JHEP1803** (2018)

Results for transport (and their fine print)

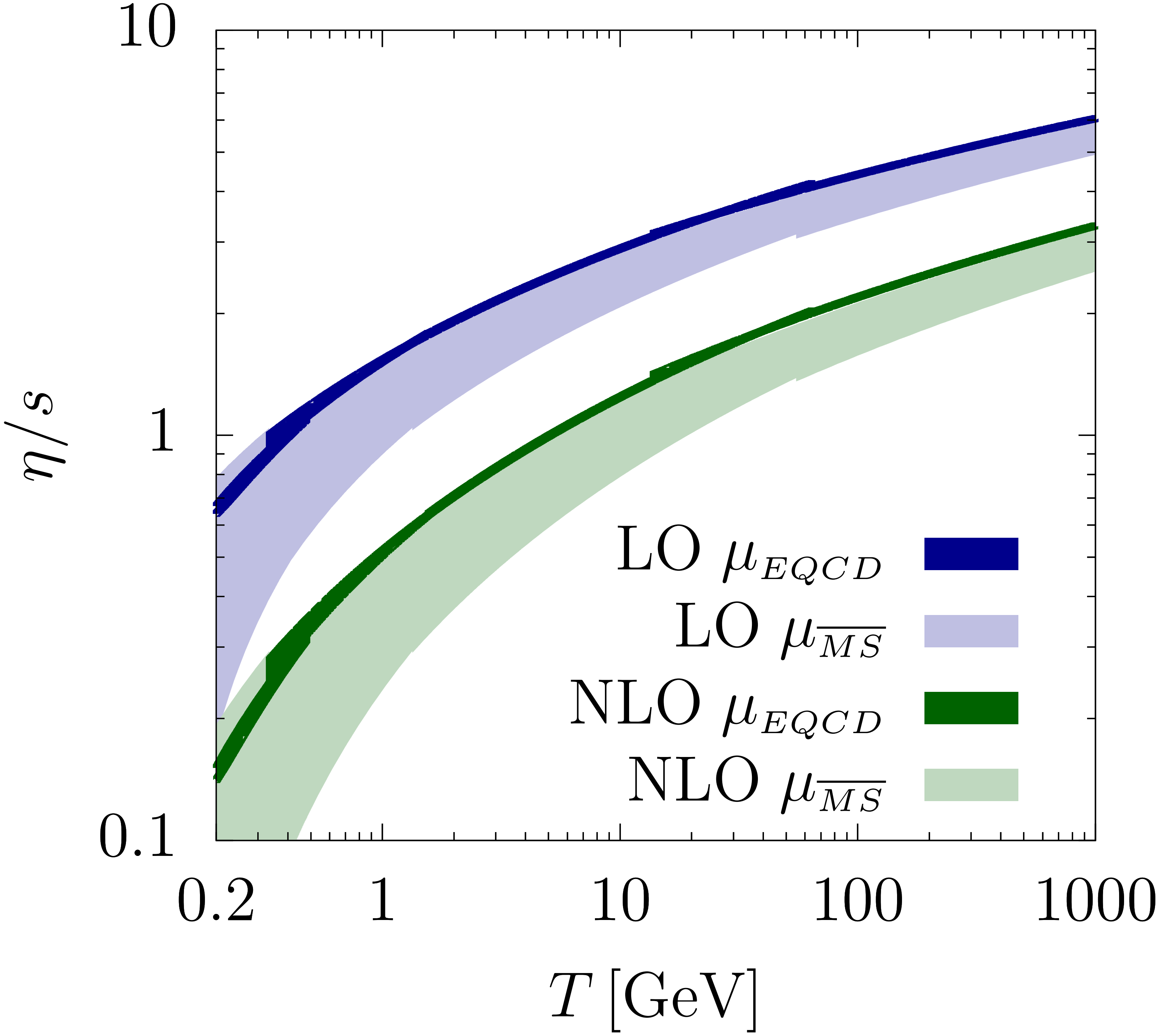
- A plot of $\eta(T)$ needs $g(T)$. But kinetic theory with massless quarks is conformal to NLO: no UV divergences and no guidance from the calculation on how to set the scale



- Fix $g(T)$ as
  - Two-loop EQCD as in Laine Schröder **JHEP0503** (2005)
  - Simple two-loop $\overline{MS}$ with various μ/T

JG Moore Teaney **JHEP1803** (2018)

η/s(T) of QCD



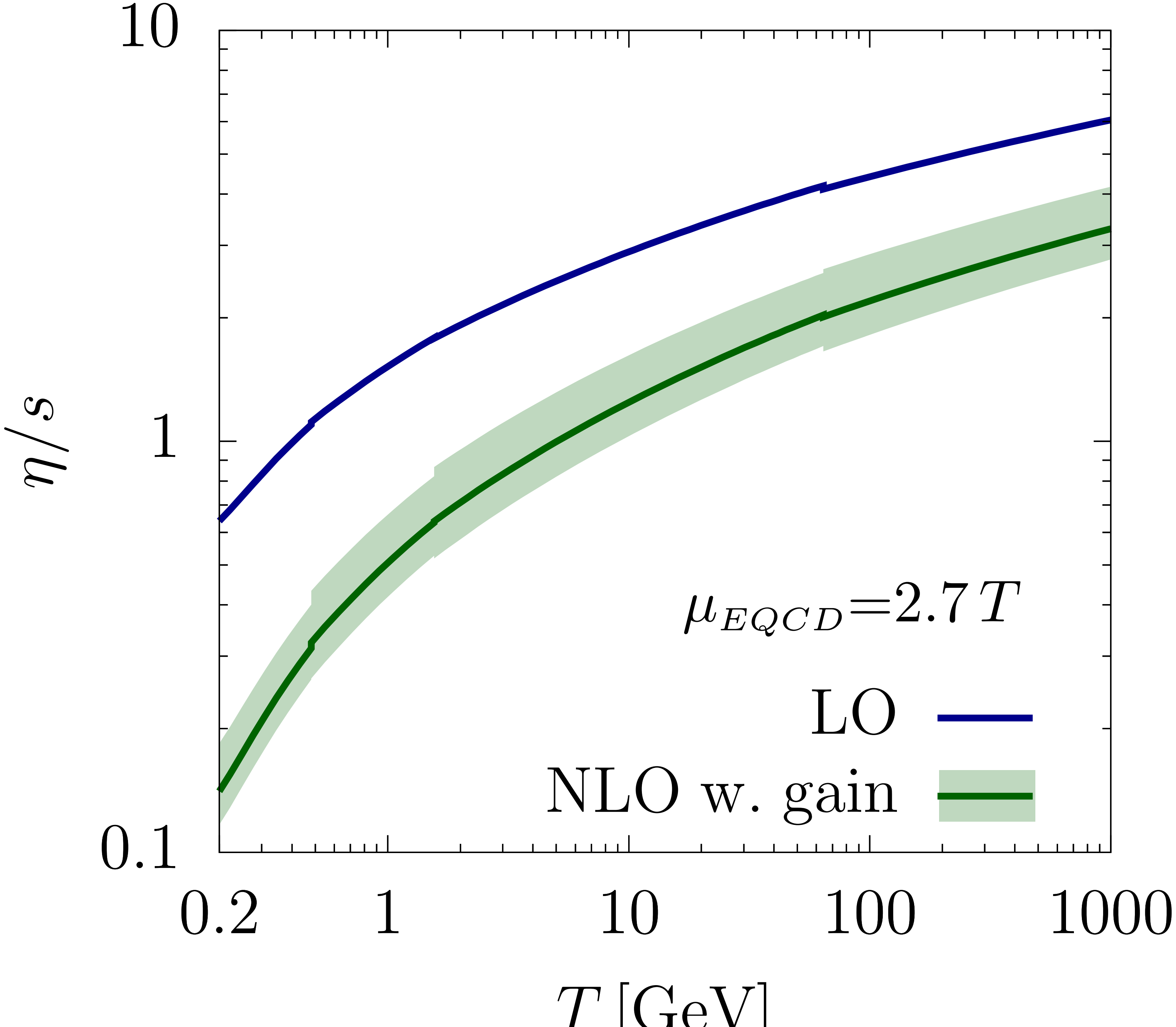
- (almost) NLO has larger effect than running (NNLO)

NLO gain terms

- For gain: no diffusion picture = no "easy" light-cone sum rules, only way would be bruteforce HTL (and understanding how to deal with soft legs in kinetic theory).
- **Missing**, but silver lining: they're finite, so just estimate the number and vary it
- NLO test ansatz: LO gain x m_D/T(~g) x arbitrary constant that we vary

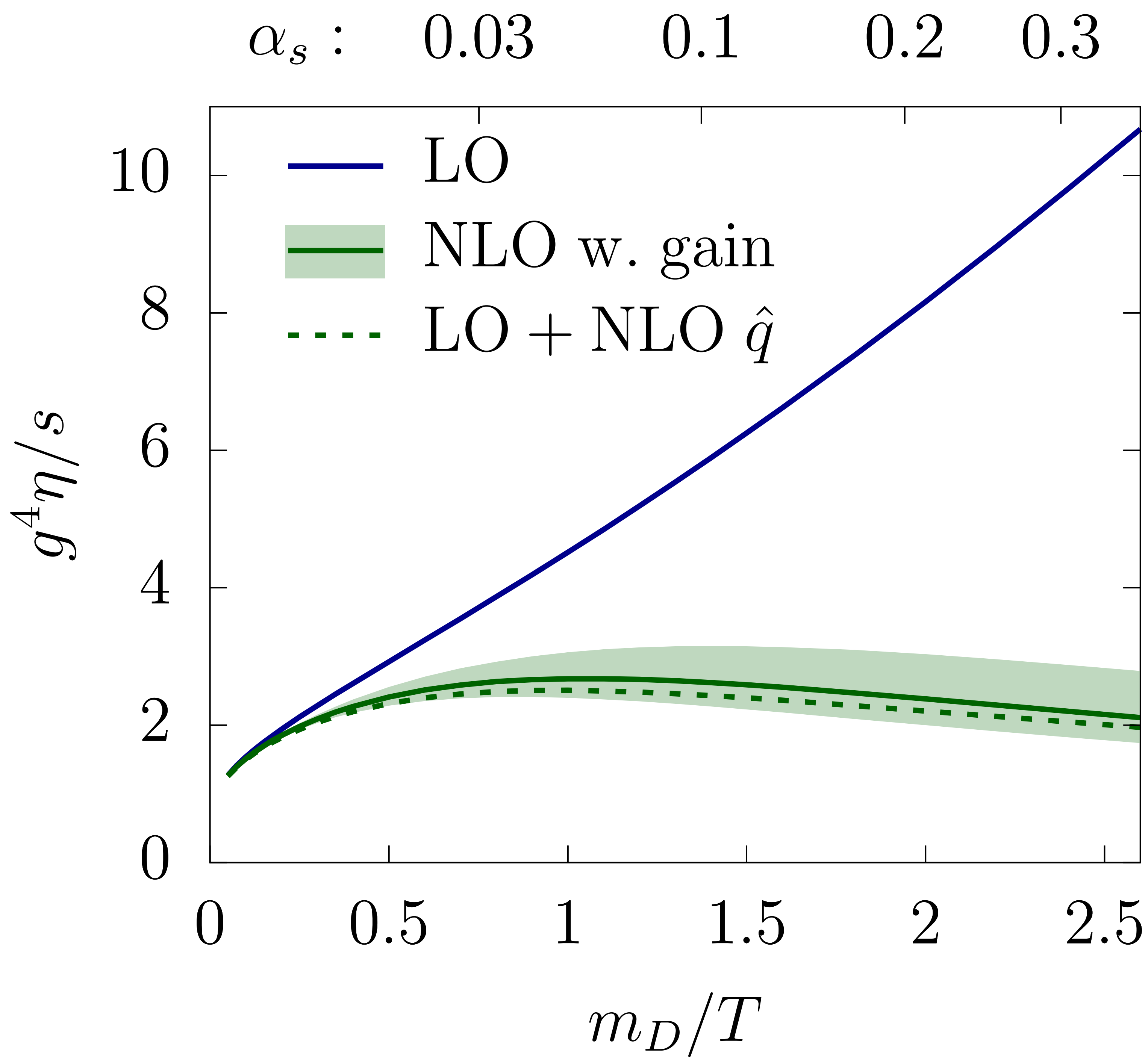
$$C_{\mathrm{NLO}}^{\mathrm{gain}} = C_{\mathrm{LO}}^{\mathrm{gain}} \times \frac{m_D}{T} \times c_{\mathrm{gain}}$$

η/s(T) of QCD



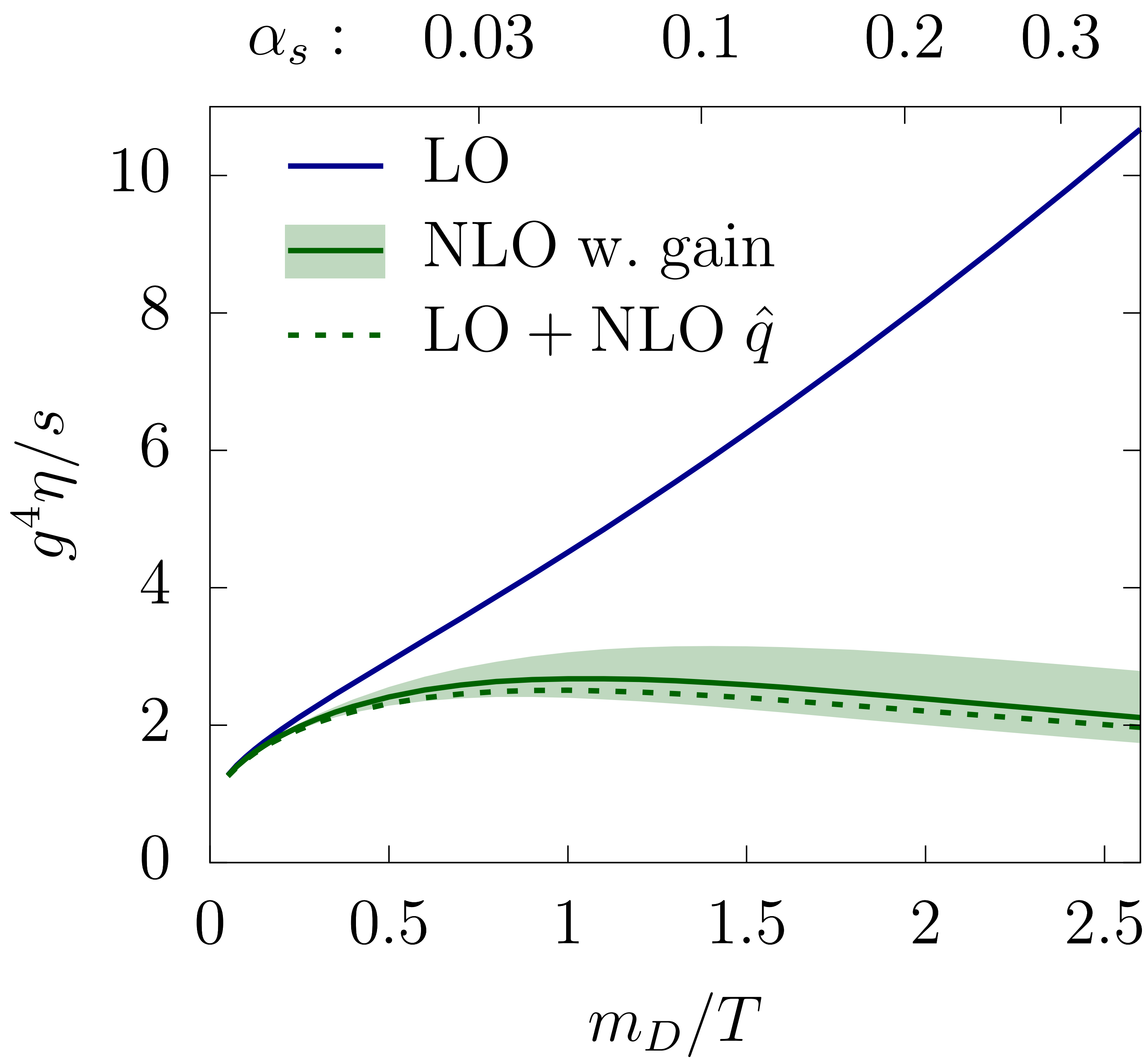
- Gain ansatz ($-2<C_{\text{gain}}<2$) introduces $O(\pm 30\%)$ uncertainty

η/s convergence



- Convergence realized at m_D~$0.5T$

η/s convergence



- The ~entirety of the downward shift comes from NLO $\hat{q}$

Isotropic thermalisation

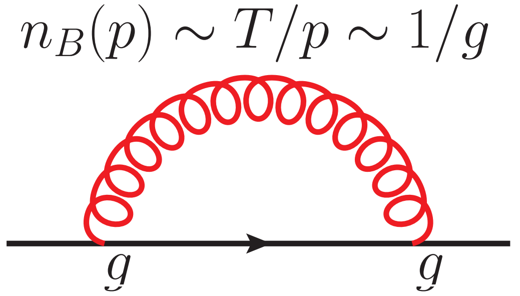
- In this talk we then concentrate on the idealised case where the distribution is isotropic, $f(\mathbf{p})=f(p)$, and there is no expansion

- This is a good description of the latest thermalisation stage, and can also be a toy model for the early stage

- Full leading-order results presented in
  Aabrao York Kurkela Lu Moore **PRD89** (2014)
  Kurkela Lu **PRL113** (2014)

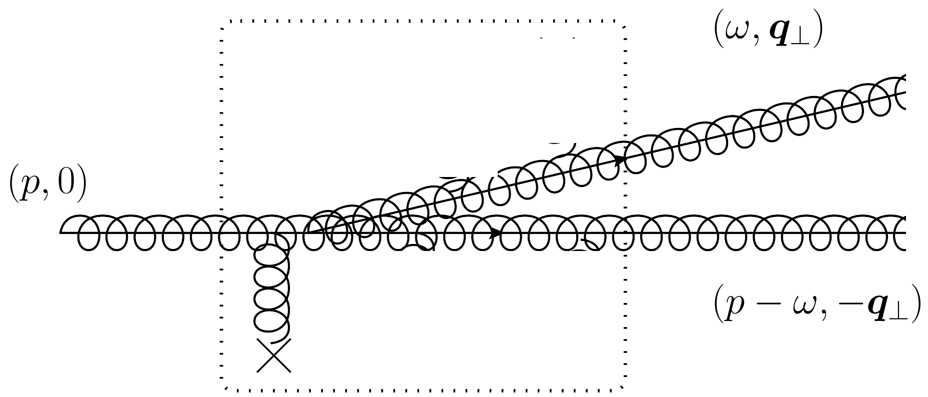
NLO kinetics and transport

- The NLO $O(g)$ corrections come from soft gluons. Out-of-equilibrium isotropic KMS enforced by Bethe-Heitler



$$\frac{T}{p} \to \frac{T_*}{p} \qquad T_* \equiv \frac{\int_{\mathbf{p}} f_p(1+f_p)}{2\int_{\mathbf{p}} f_p/p}$$

$$\begin{aligned}\mathcal{C}_{1\leftrightarrow 2}[f](p) = &\frac{(2\pi)^3}{2p^2}\int_0^\infty dp'dk'\,\gamma^{p}_{p',k'}(m,T_*) \\ &\times\{f_p[1+f_{p'}][1+f_{k'}] - f_{p'}f_{k'}[1+f_p]\}\delta(p-p'-k') \\ &+\frac{(2\pi)^3}{p^2}\int_0^\infty dp'dk\,\gamma^{p'}_{p,k}(m,T_*)\,\delta(p+k-p') \\ &\times\{f_pf_k[1+f_{p'}] - f_{p'}[1+f_p][1+f_k]\}. \qquad \text{(A2)}\end{aligned}$$



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NLO kinetics and transport

- The NLO corrections are then those we just saw, with two simplifications
  - no qhat in the 2↔2 processes, because of **isotropy**

$$\mathcal{C}_{2\leftrightarrow 2}[f](p) = \int_{\mathbf{k},\mathbf{p}',\mathbf{k}'} \frac{|\mathcal{M}(m)|^2 (2\pi)^4 \delta^{(4)}(p+k-p'-k')}{2\,2k\,2k'\,2p\,2p'}$$
$$\times \{f_p f_k[1+f_{p'}][1+f_{k'}] - f_{p'} f_{k'}[1+f_p][1+f_k]\},$$

$$p' \approx p + \hat{p}\cdot\mathbf{q}$$

  - no gain terms, because of **isotropy**
- We thus have all corrections of order $g^2 T_* / m$

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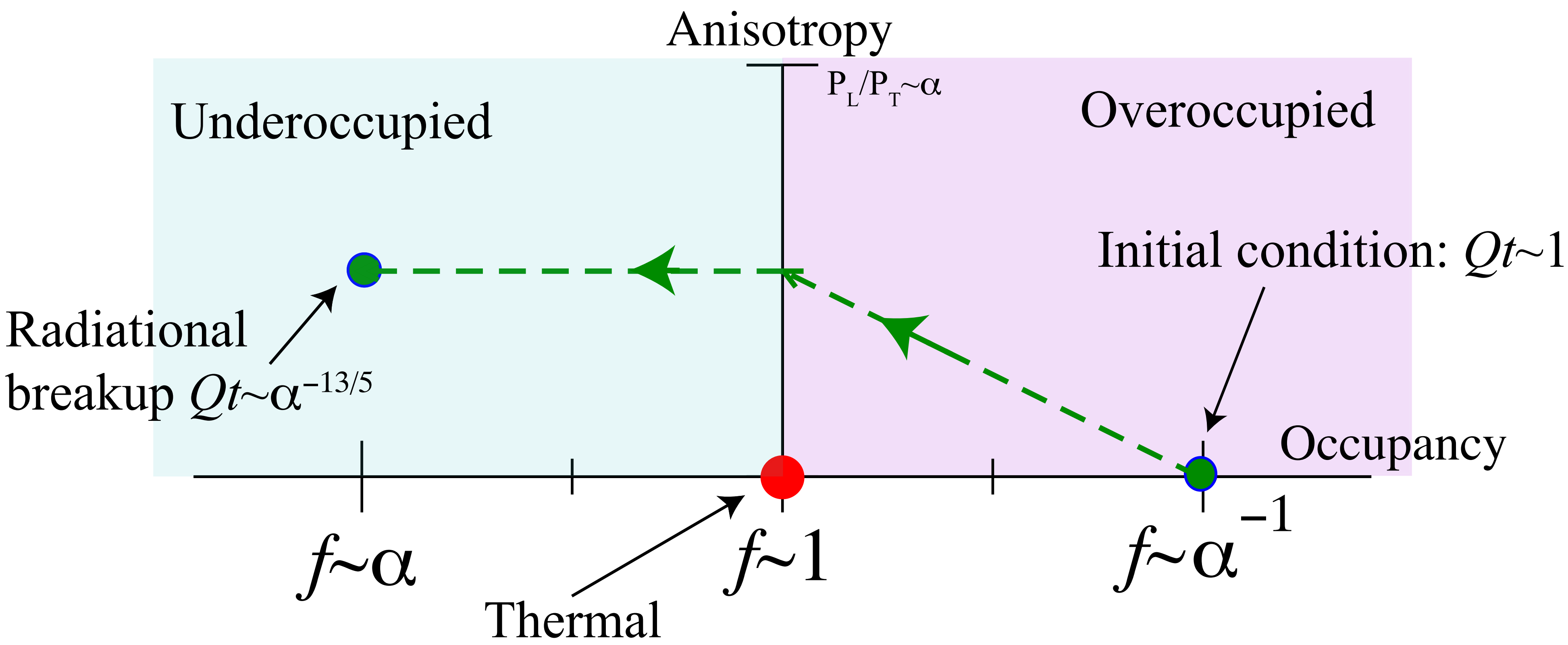
Results for thermalisation

- We consider underoccupied and overoccupied initial conditions



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Results for thermalisation

- We consider underoccupied and overoccupied initial conditions for a system of gluons only and we solve the EKT numerically
- In the underoccupied case a large-momentum gaussian ($Q \gg T_{\text{final}} \equiv T$) with a thermal bath carrying 10% of the initial energy

$$f(p) = Ae^{-\frac{(p-Q)^2}{(Q/10)^2}} + n_B(p, T_{\text{init}})$$

- In the overoccupied case the scaling solution arising from the classical lattice theory

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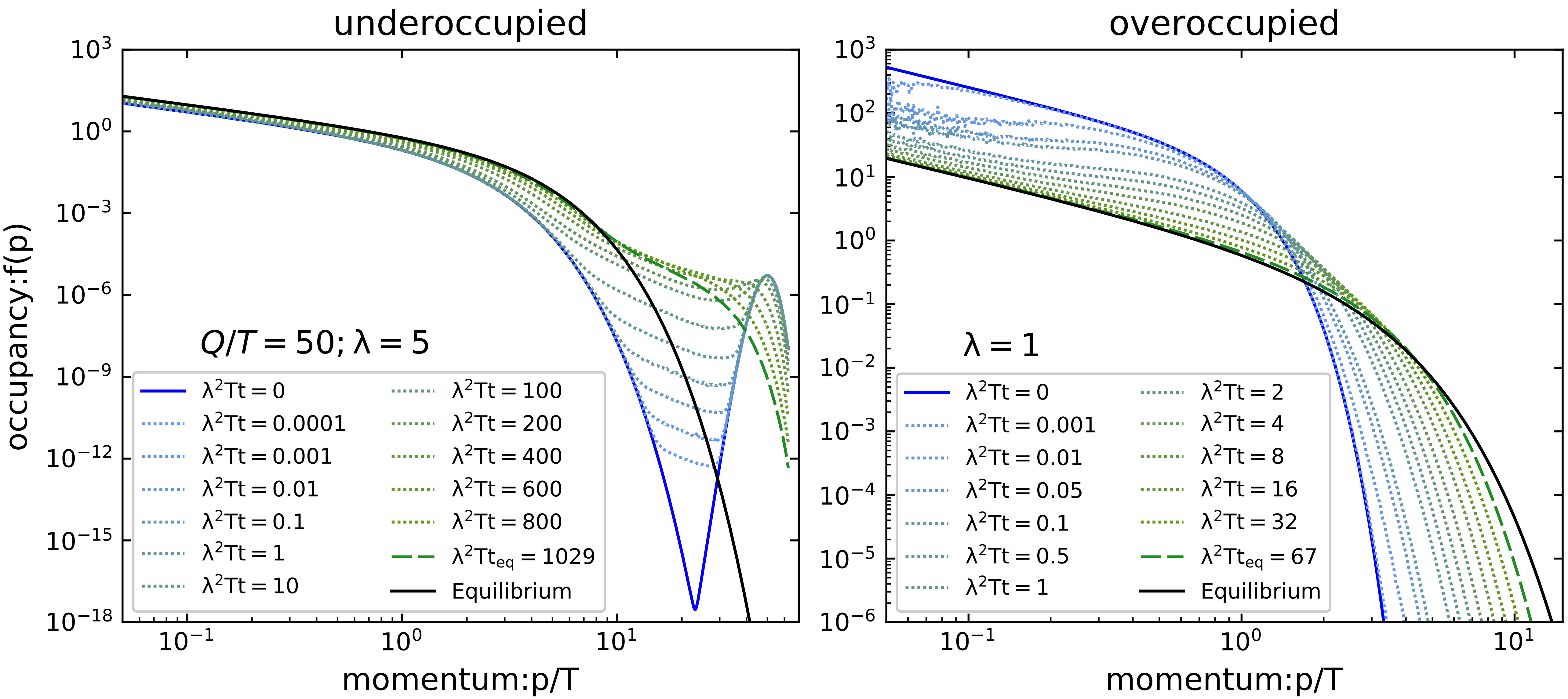
Results for thermalisation



$$\lambda = g^2 N_c$$

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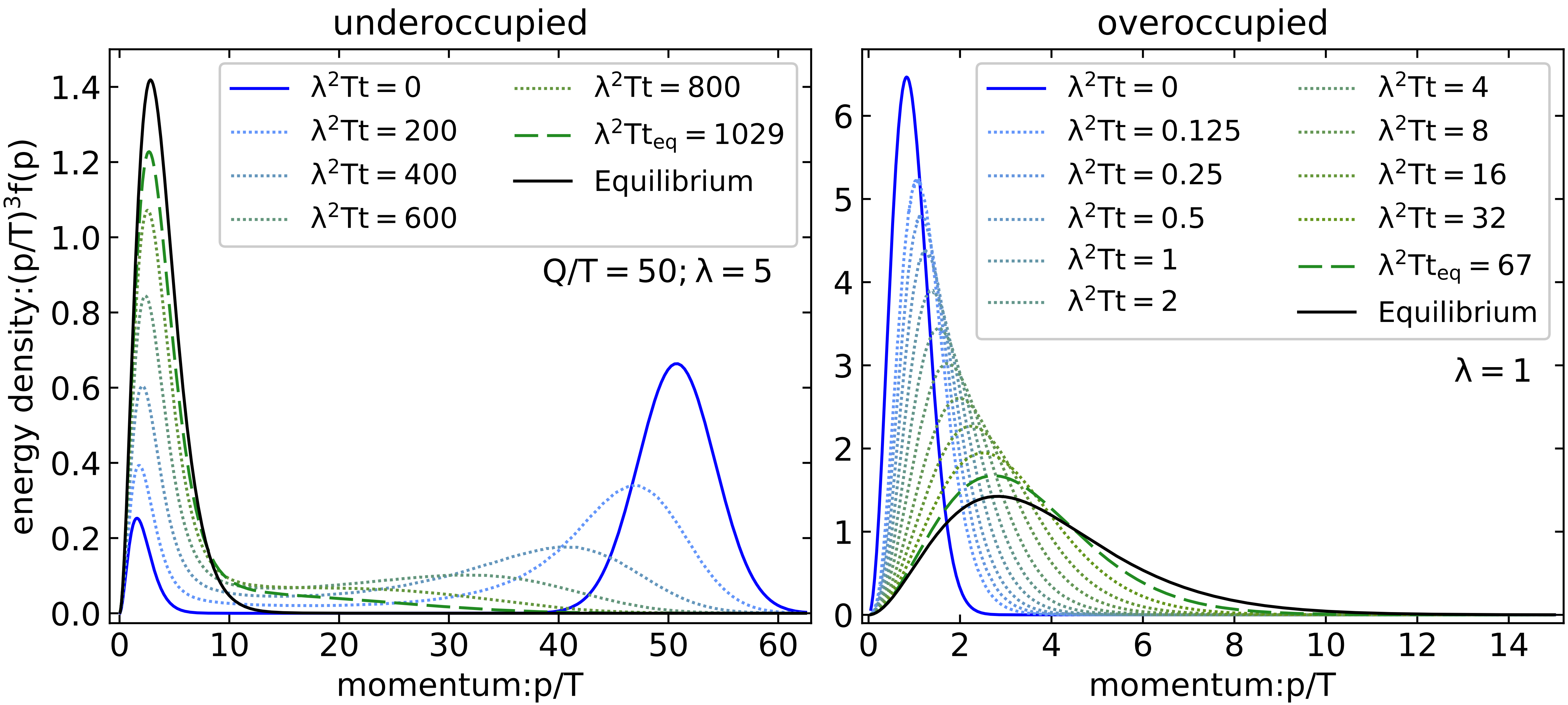
Results for thermalisation



- Direct (overoccupied) and inverse (under) cascade

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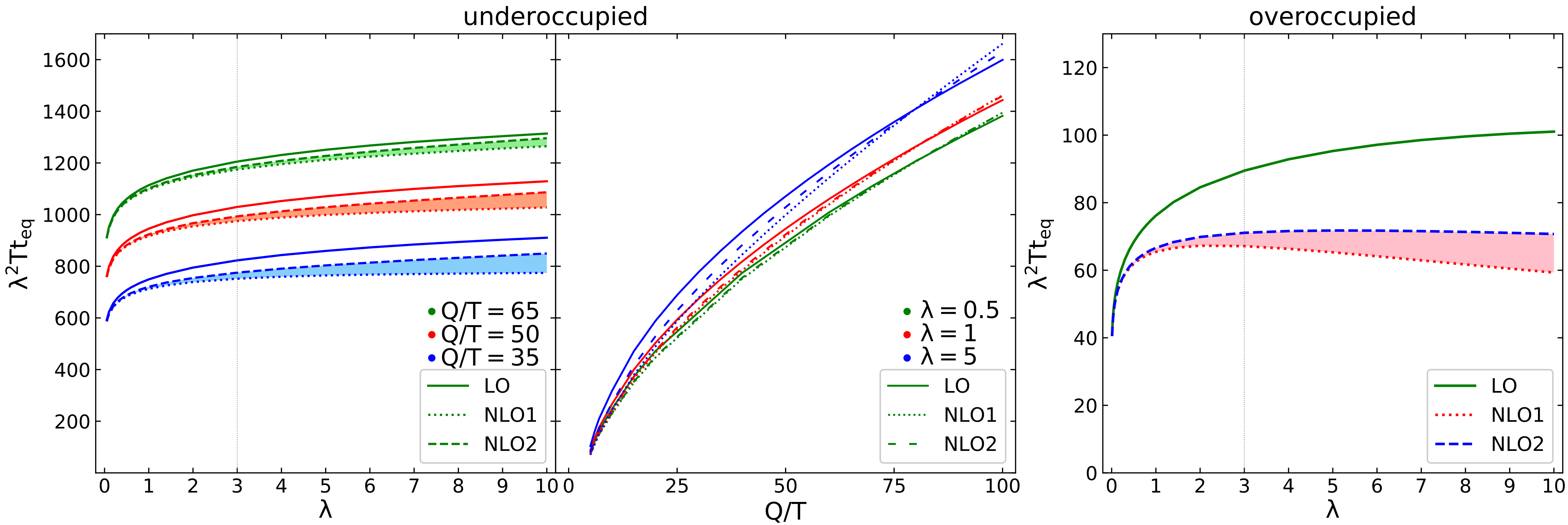
Results for thermalisation



- At the thermalisation time the ratio between two moments of f (both equal to T in equilibrium) is 0.9

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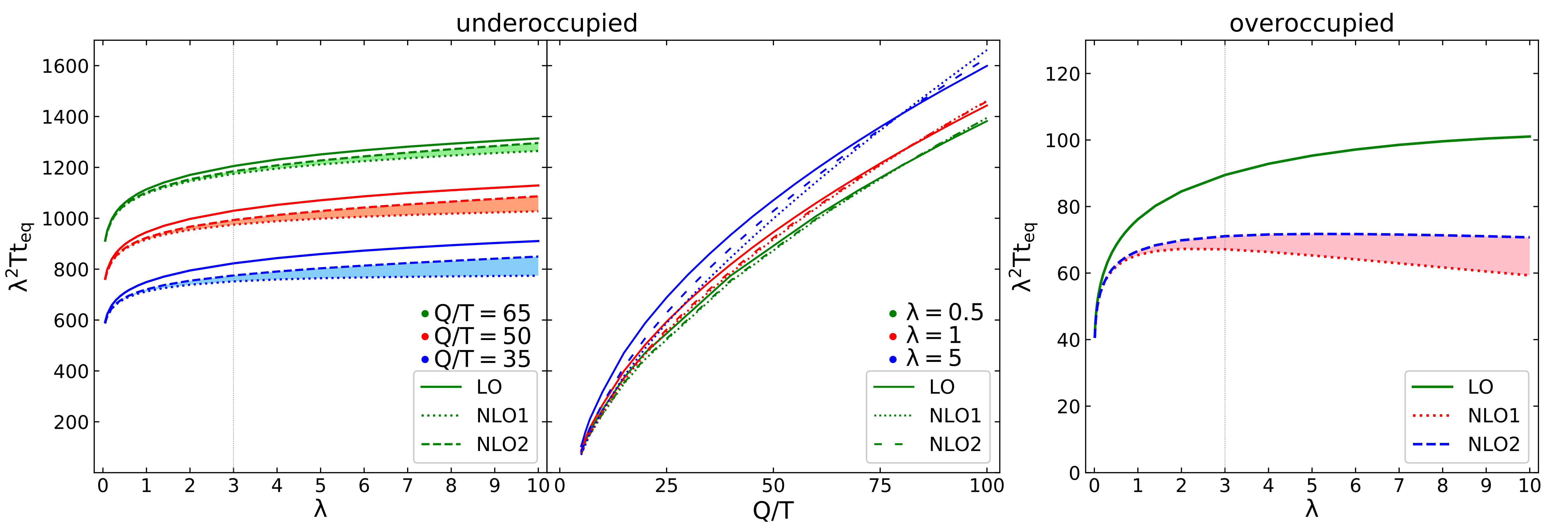
Results for thermalisation



- Two different NLO schemes which resum in different ways higher-order effects

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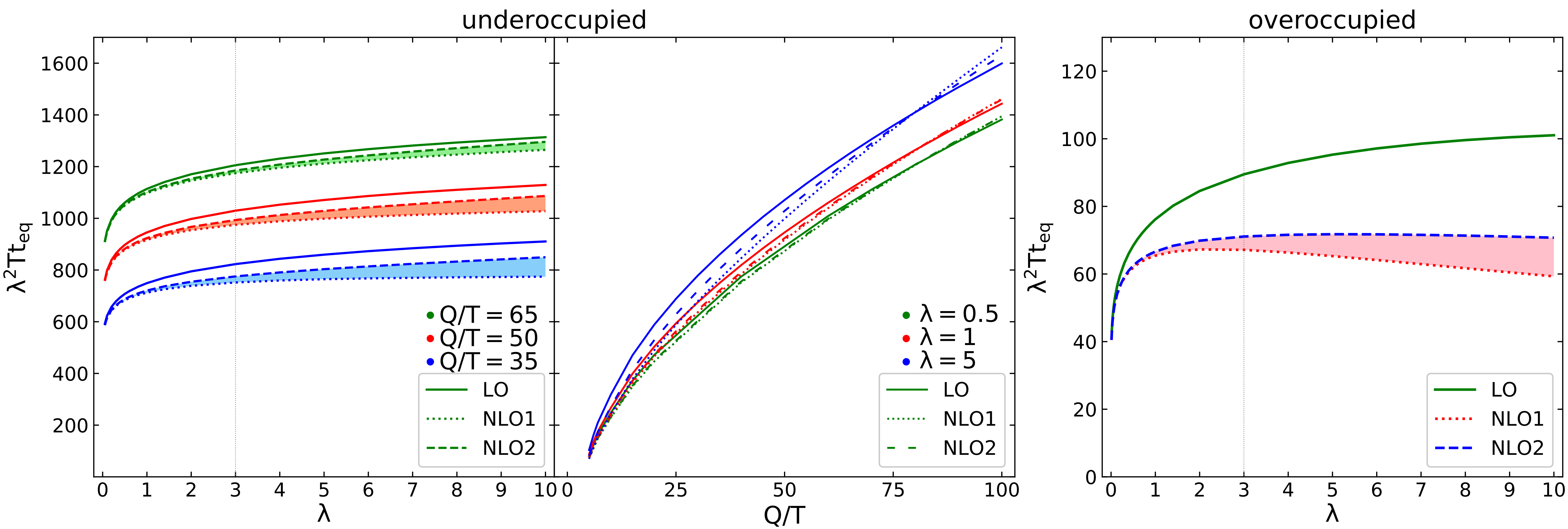
Results for thermalisation



- NLO corrections under reasonable control, at most 40% at λ=10

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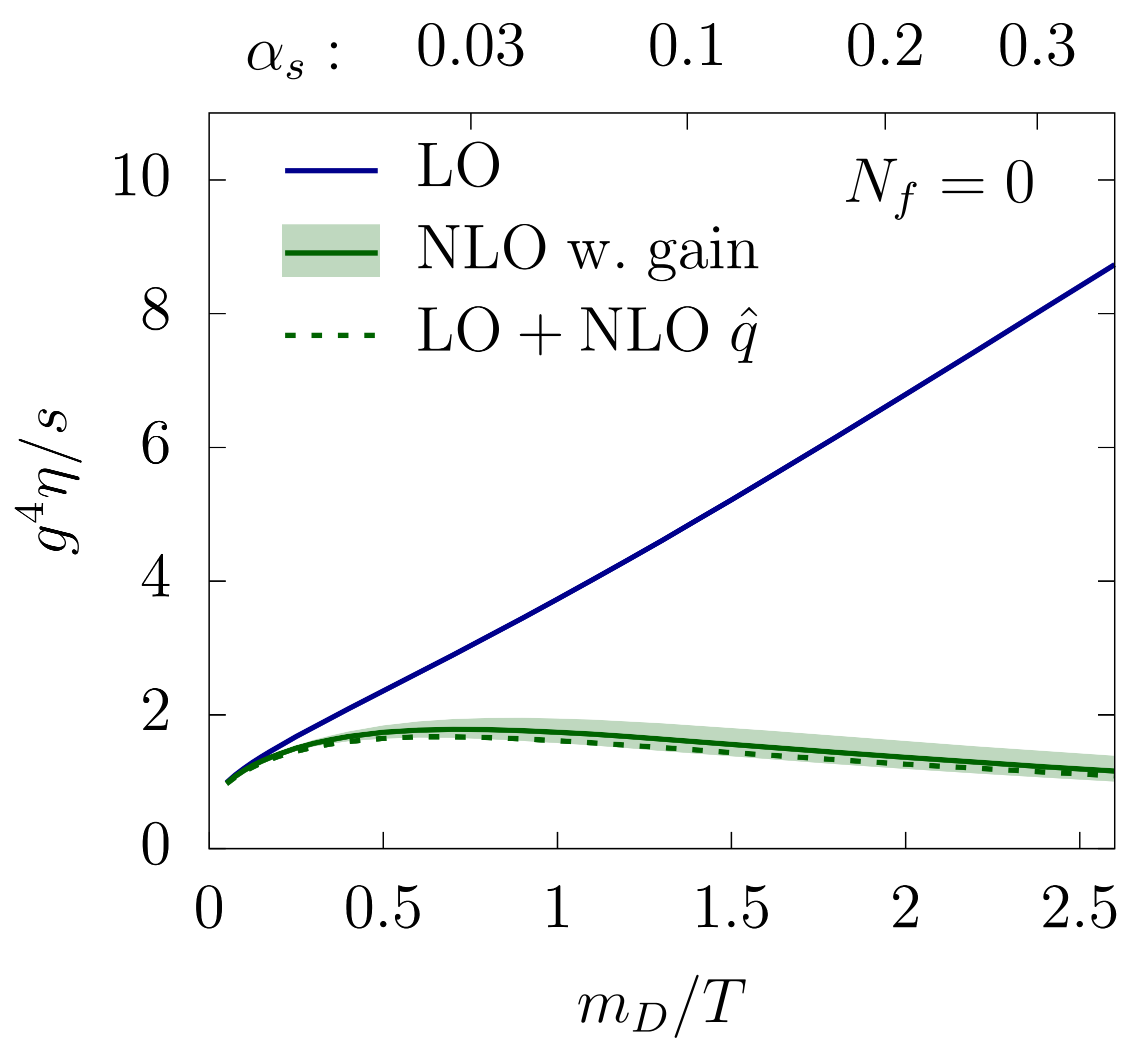
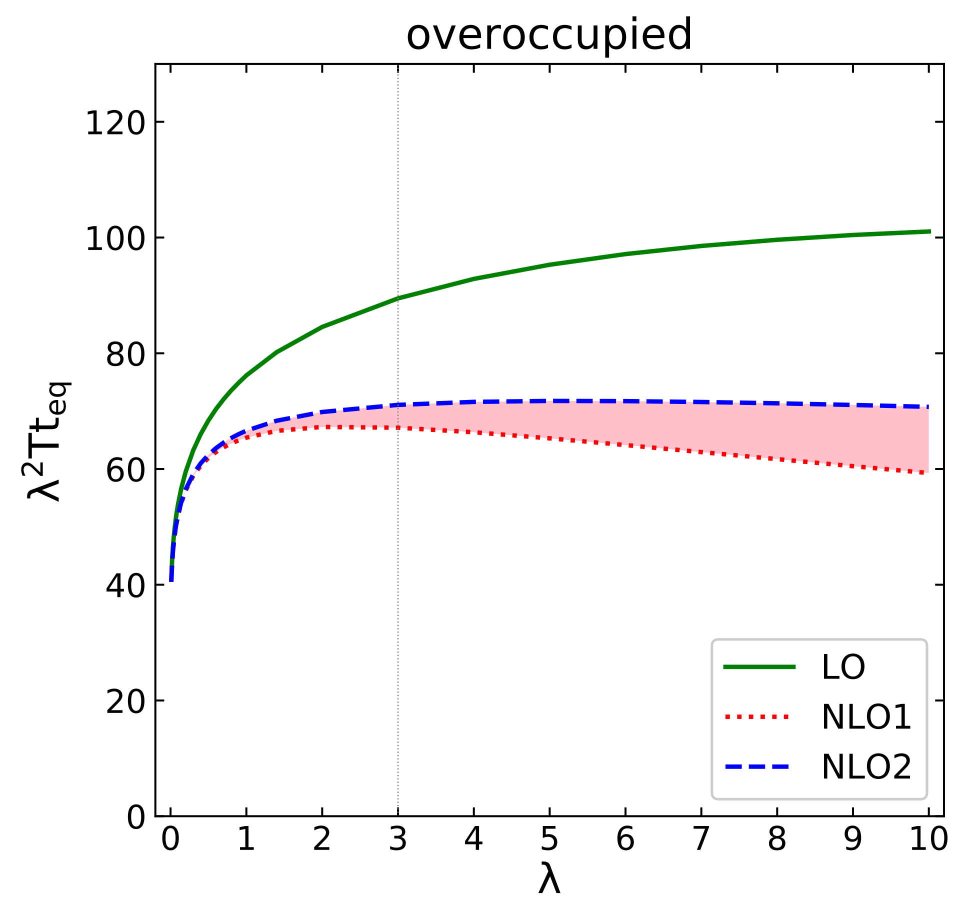
Results for thermalisation





- NLO qhat does not contribute directly in this case, as the kinetic theory is not directly sensitive to transverse momentum broadening

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Summary

- The effective kinetic theory as a tool for transport, thermalisation and jets
- In the transport case, NLO corrections are large
- In isotropic thermalisation case, they are reasonably well-behaved. “no qhat” pattern?
- Care needed when using LO kinetic theories with $g^2 T_* / m \gtrsim 1$