Constraining the EoS for neutron star from theory and data

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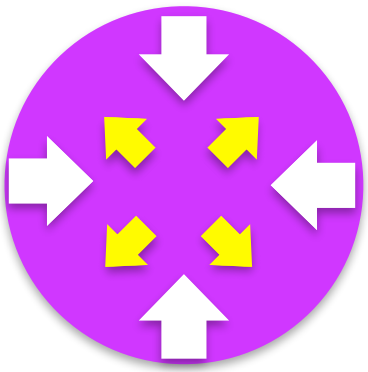
The University of Tokyo

— Online Talk at S@INT Seminar —

Equation of State for Neutron Stars

Compact Stars



Gravitational force is sustained by the pressure from inside.

Hydrostatic condition for $r \sim r + dr$

$$\frac{dp(r)}{dr} = -G\frac{M(r)}{r^2}\varepsilon(r)$$

***M*(*r*) represents the integrated mass in *r*-sphere.**

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

One condition is missing!

Equation of State for Neutron Stars

$$\frac{dp(r)}{dr} = -G\frac{M(r)}{r^2}\varepsilon(r)$$

TOV Equations

General Relativistic

$$\frac{dp(r)}{dr} = -G\frac{M}{r^2}(\varepsilon + p)\left(1 + \frac{4\pi r^3 p}{M}\right)\left(1 - \frac{2GM}{r}\right)^{-1}$$

Missing condition

A relation between p and ε → **Equation of State (EoS)**

$\boxed{\text{Initial}}$

$r = 0$

$\varepsilon(r=0) = \varepsilon_{\text{max}}$ **free parameter**

$p(r=0) = p_{\text{max}} = p(\varepsilon_{\text{max}})$

$\boxed{\text{Final}}$

$r = R$

$p(r=R) = 0$

$M = \int dr 4\pi r^2 \varepsilon(r)$

EoS from Deep Neural Network

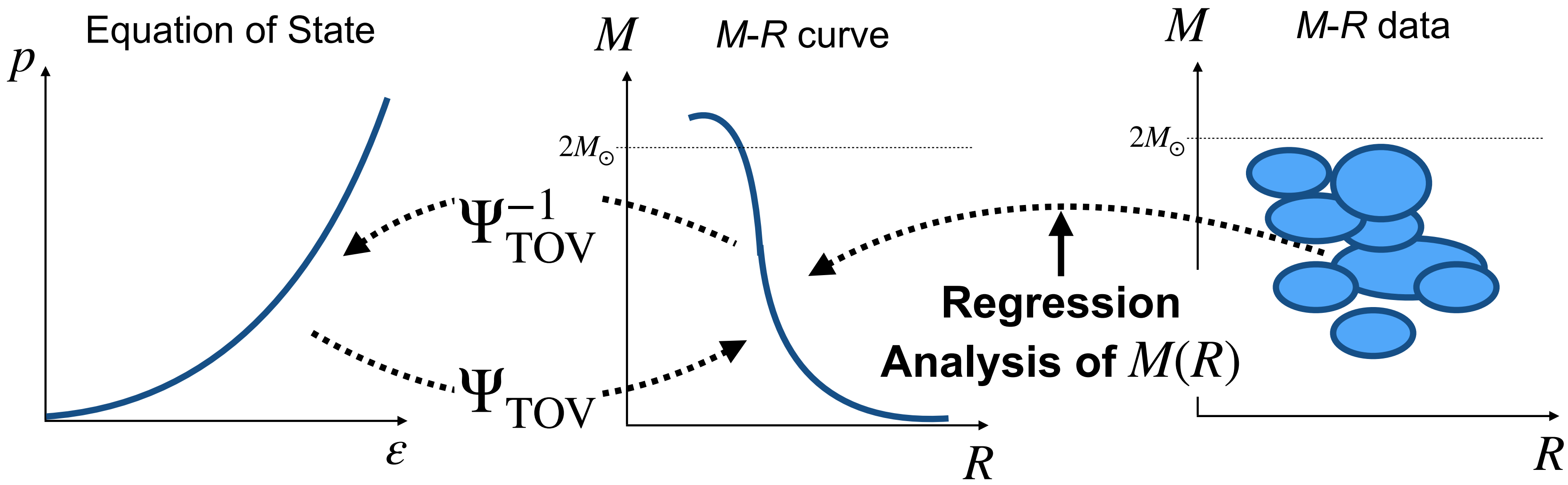
EoS from resummed pQCD

EoS and gravitational waves

EoS Inference

Conventional Model Approach

Model → Solving TOV → *M-R* Curve → Observation



Initial condition: $p(r \simeq 0) = p(\rho_{\rm max}), \quad \varepsilon(r \simeq 0) = \varepsilon(\rho_{\rm max})$

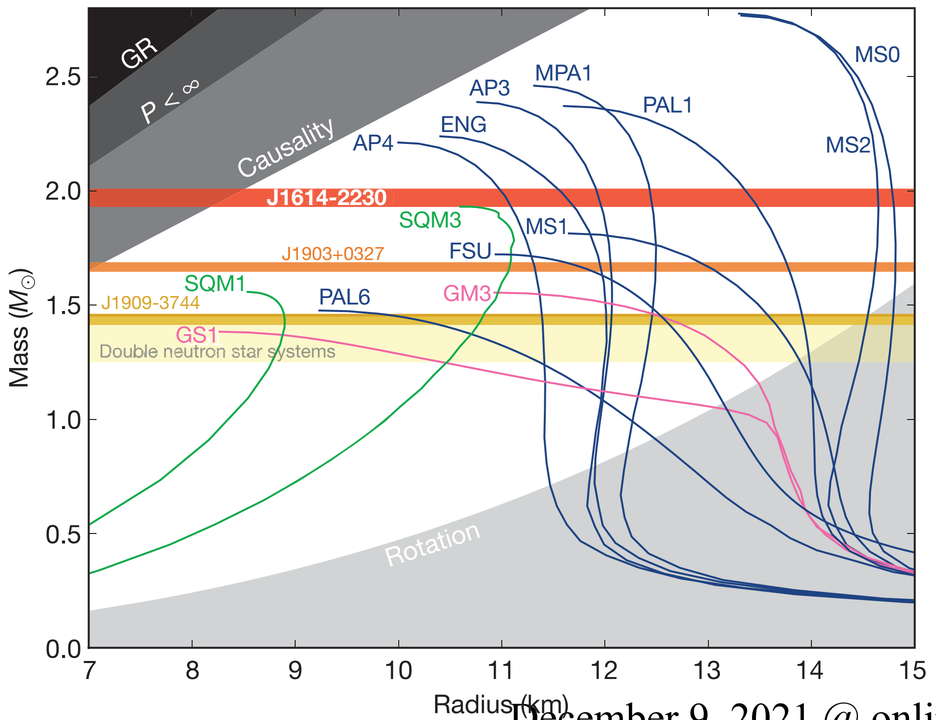
***M-R*:** $p(r = R) = 0, \quad M = \int^R d^3x\, \varepsilon(r)$

EoS Inference

Conventional Model Approach

Model → Solving TOV → *M-R* Curve → Observation

[Very Famous Example]



Demorest et al. (2010-2016)
1.928(17) M_{sun} (J1614-2230)

Some models excluded from observations

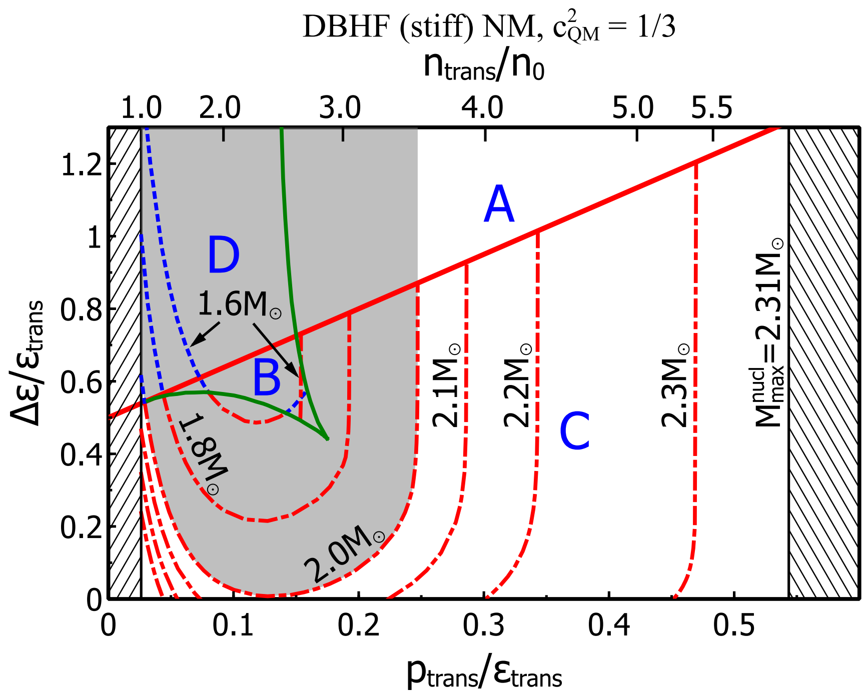
Even more massive NSs have been discovered later.

EoS Inference

Conventional Model Approach

Model → Solving TOV → *M-R* Curve → Observation

[Model Independent Analysis]



Alford-Burgio-Han-Taranto-Zappala (2015)

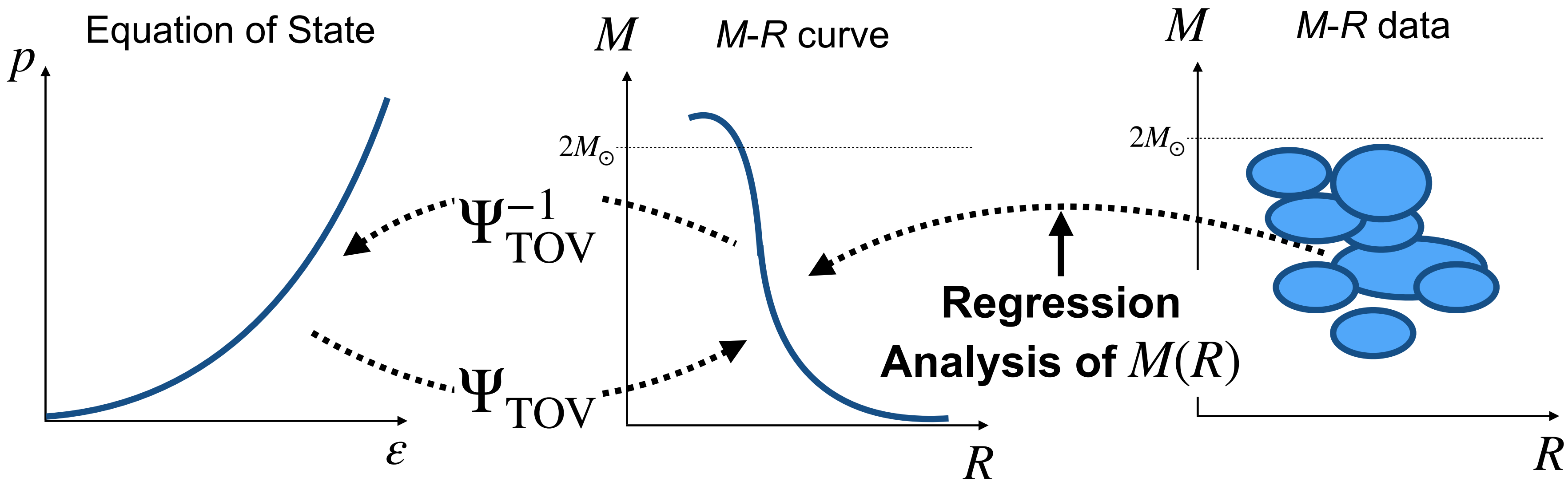
$$\varepsilon(p) = \begin{cases} \varepsilon_{NM}(p) & p < p_{trans} \\ \varepsilon_{NM}(p_{trans}) + \Delta\varepsilon + c_{QM}^{-2}(p - p_{trans}) & p > p_{trans} \end{cases}$$

Still relies on several scenarios…

EoS Inference

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation



Once one *M-R* curve is identified, one EoS is concluded.

The best we can do is to find the "likely" *M-R* curve.

EoS Inference

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation

Bayesian Analysis

Ozel et al., Steiner et al. (2015~)

A : EoS Parameters B : M-R Observation

(Bayes' theorem)

Normalization

$$\underbrace{P(A|B)}_{\text{Want to know}}\cancel{P(B)} = \underbrace{P(B|A)}_{\text{Likelihood}}\underbrace{P(A)}_{\text{prior}}$$

Want to know **Likelihood** **prior**

Model

EoS Inference

Bayesian Analysis

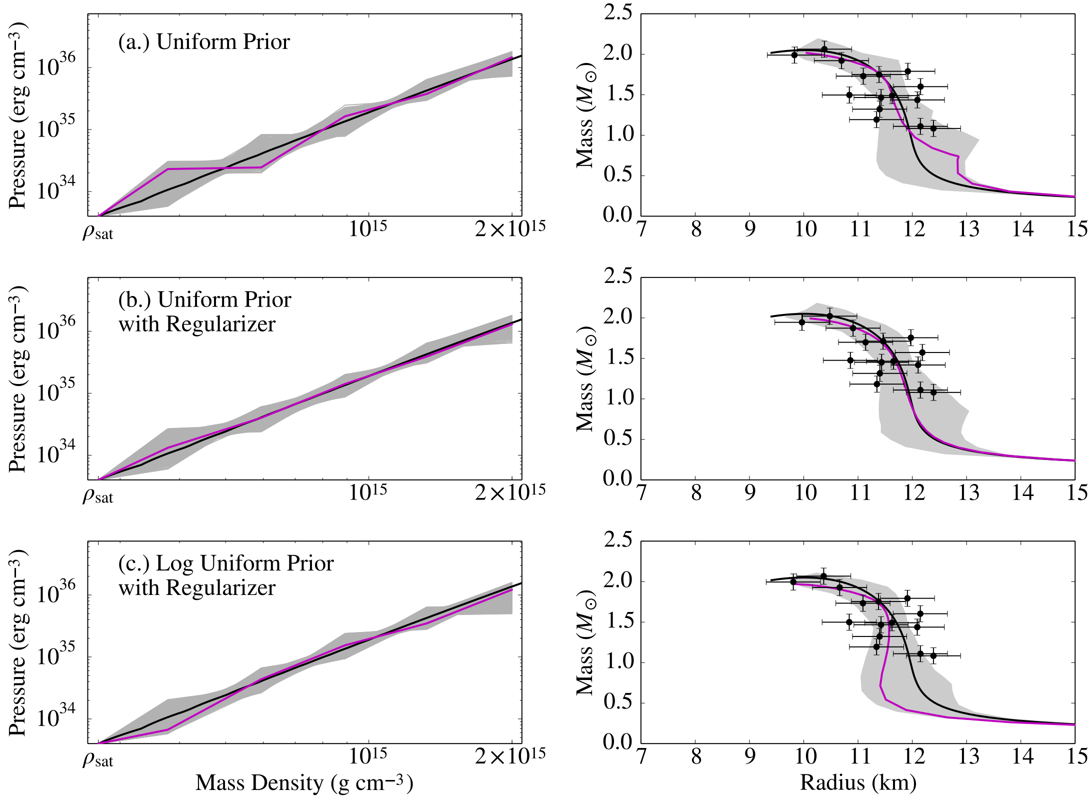
Ozel et al., Steiner et al. (2015~)

If B (observation data) is abundant, the likelihood would become sharper → the prior dependence can be reduced… but…

Raithel-Ozel-Psaltis (2017)

Mock data (SLy + Noises)

Very powerful approach but a complementary is desirable…



EoS Inference

Model Independent Approach

EoS ⟵ Solving TOV ⟵ *M-R* Curve ⟵ Observation

Machine Learning Inference **Fujimoto-Fukushima-Murase (2018,19,20)**

Several *M-R* observation points with errors ⟵ **Nonlinear Mapping** ⟶ Several parameters to characterize EoS

$$\{M_i, R_i\} \qquad \{P_i\} = F(\{M_i, R_i\}) \qquad \{P_i\}$$

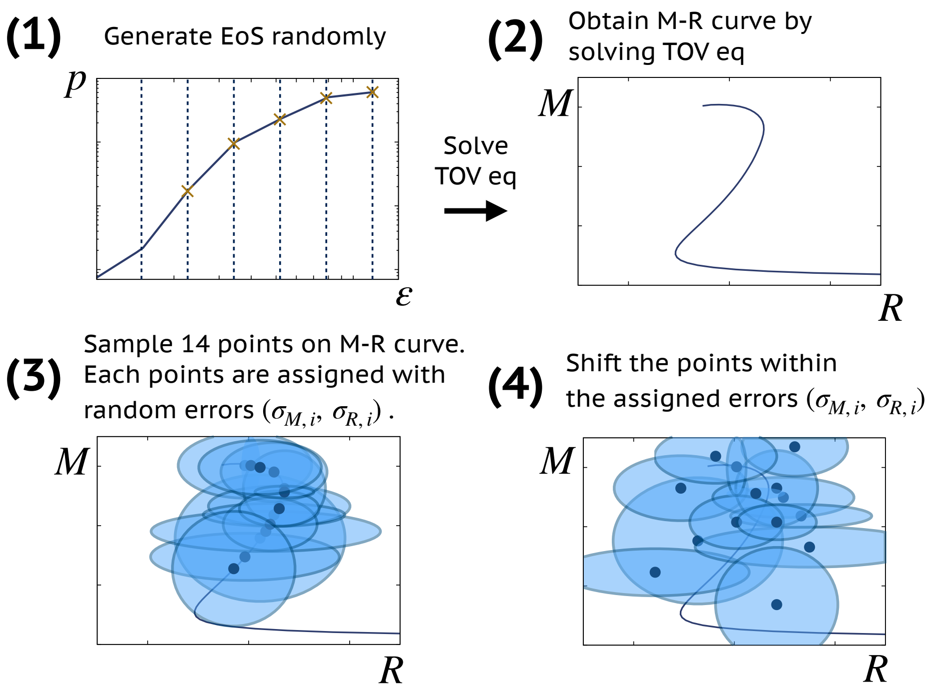
EoS Inference

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation

Machine Learning Inference

Fujimoto-Fukushima-Murase (2018,19,21)



EoS parametrized by
speed of sound c_s^2

Convoluted with error bands
(**Data Augmentation**)

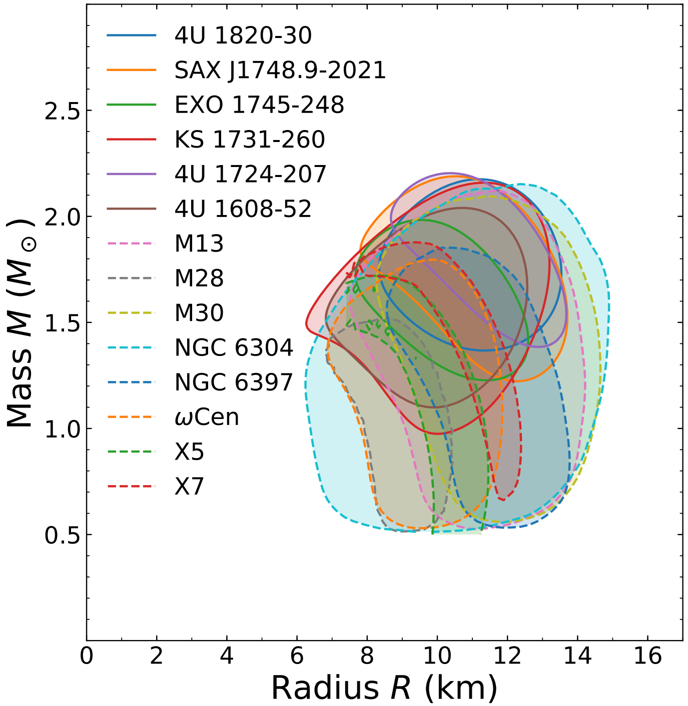
EoS Inference

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation

Machine Learning Inference

Fujimoto-Fukushima-Murase (2018,19,21)



We took 14 NS data and approximated the data with 4 parameters (M, R, σ_M, σ_R, neglecting distribution shapes).

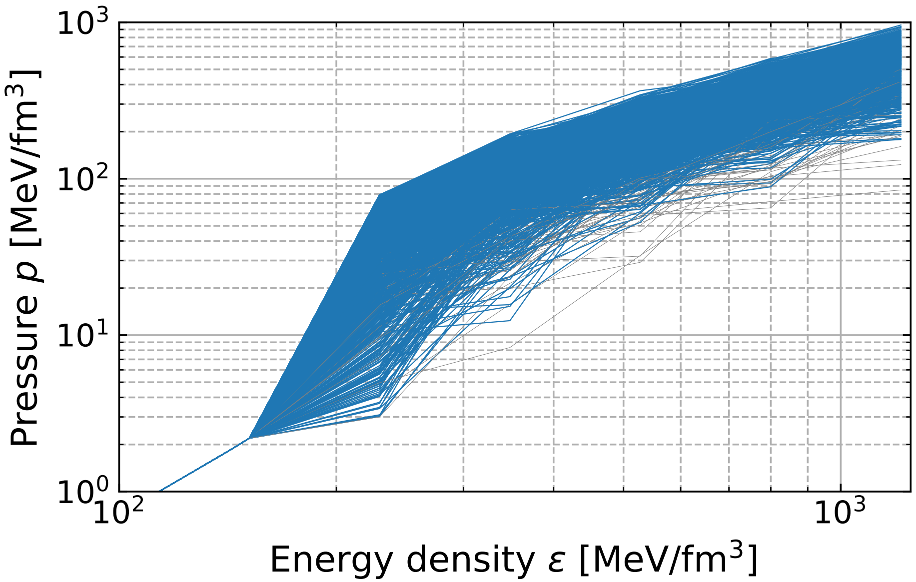
How to infer the most likely EoS?

EoS Inference

Machine Learning Inference

Fujimoto-Fukushima-Murase (2018,19,21)

Training Data and NN Architecture



2000EoSs times 10000 randomly sampled data

(Prior dependence hidden here)

Layer index	Neurons	Activation Function
0	56	N/A
1	60	ReLU
2	40	ReLU
3	40	ReLU
4	5	tanh

[Input Layer]

14 NSs times 4 parameter $(M, R, \sigma_M, \sigma_R) = 56$ neurons

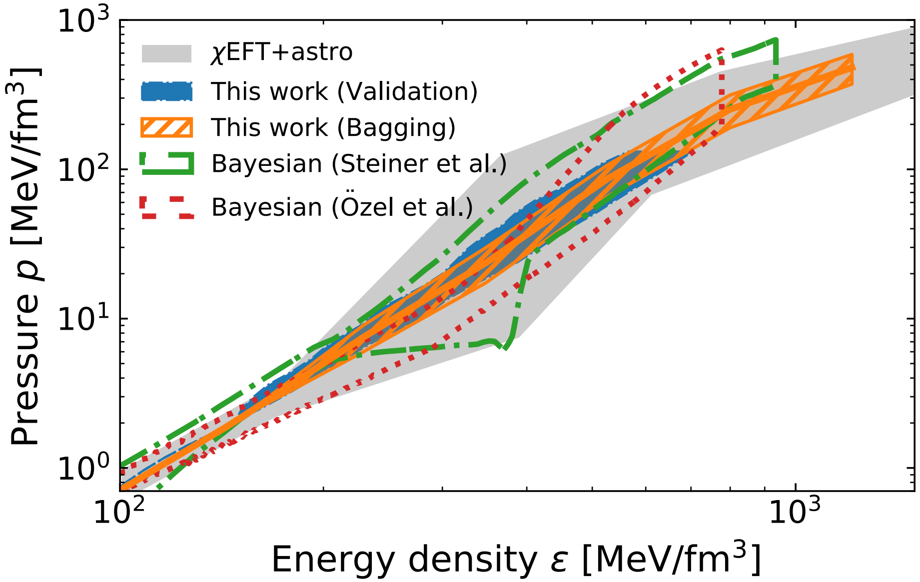
[Output Layer]

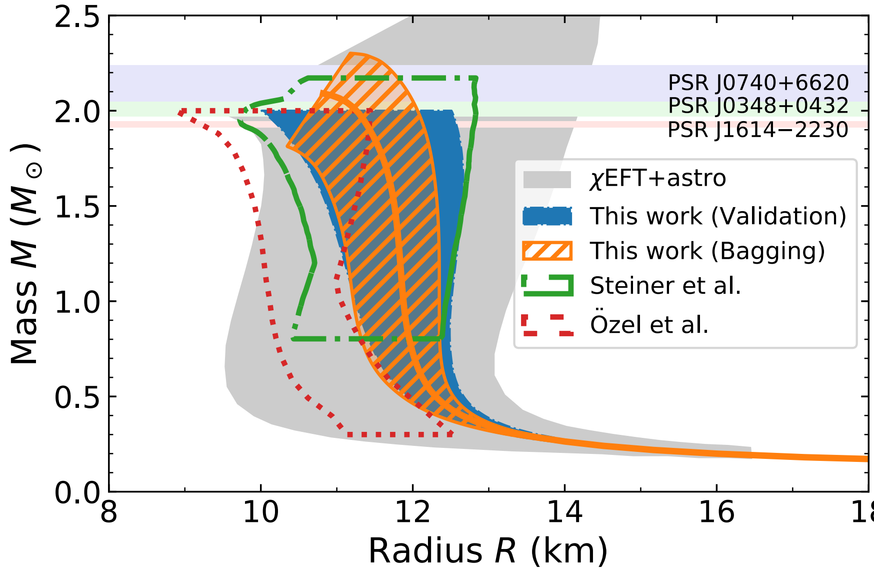
5 EoS Polytropic Indices

EoS Inference

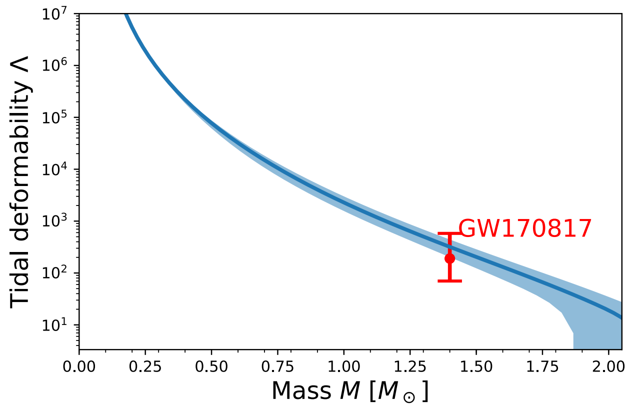
Most Likely EoS from ML

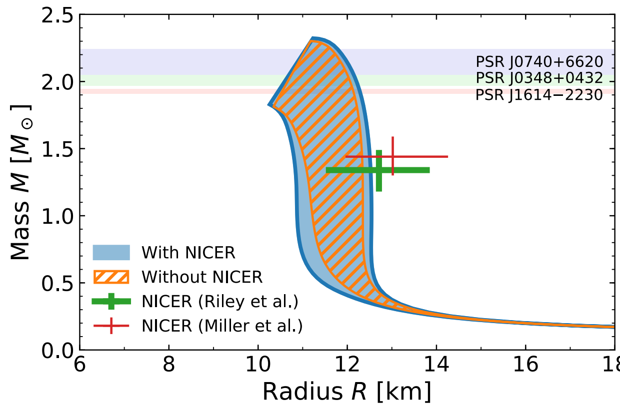
Fujimoto-Fukushima-Murase (2018,19,21)





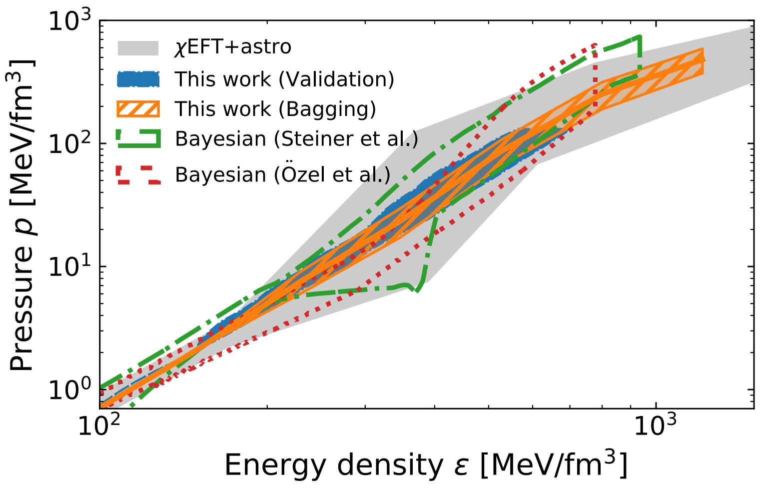
Consistency Check

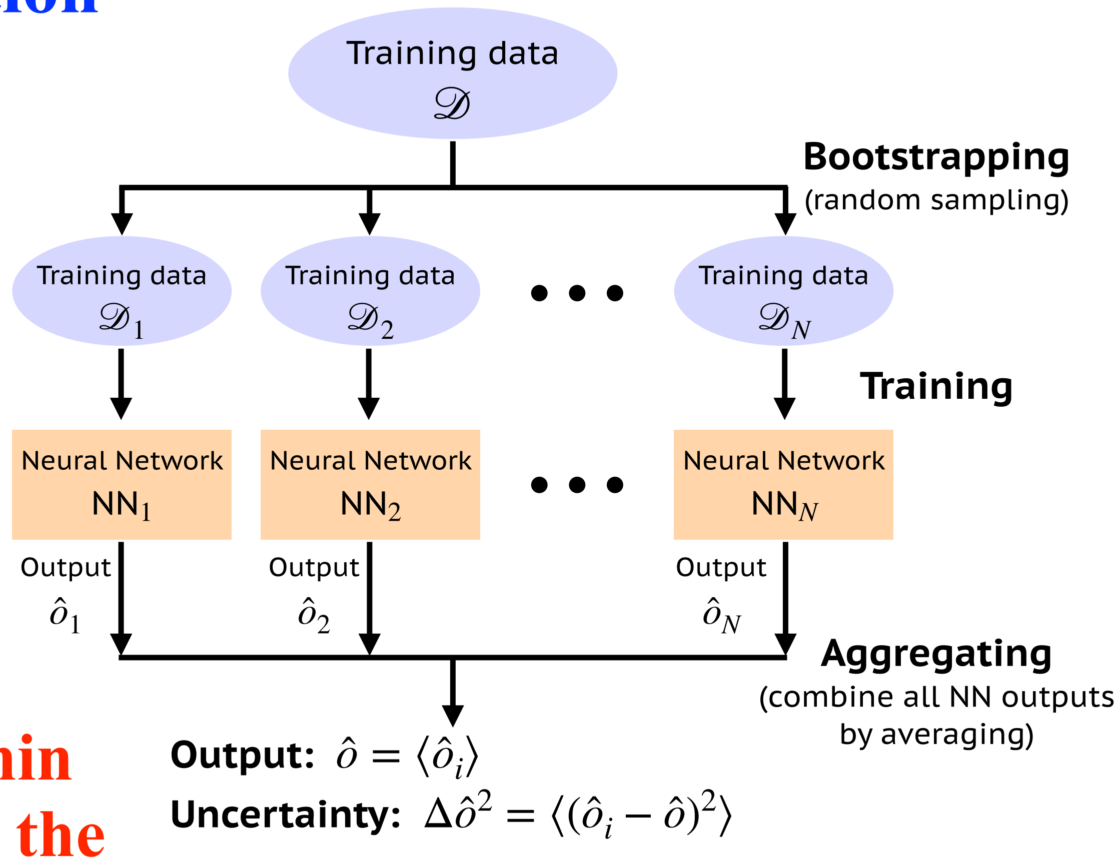




EoS Inference

Uncertainty Quantification





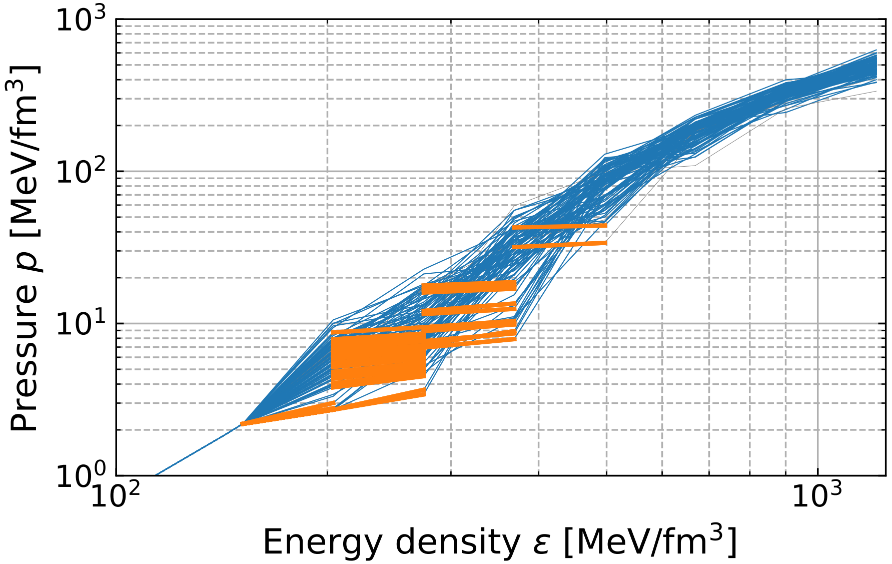
This is an uncertainty within this method, not including the uncertainty of this method.

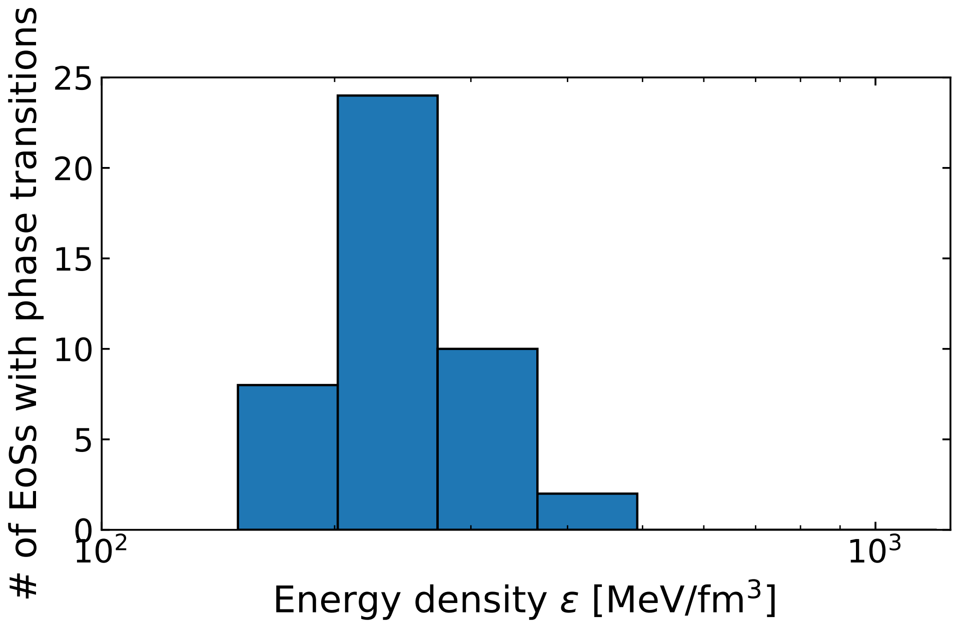
EoS Inference

Most Likely EoS from ML **Fujimoto-Fukushima-Murase (2018,19,21)**

Independently learned NNs lead to acceptable EoSs

Some among 100NNs contain a 1st-order transition



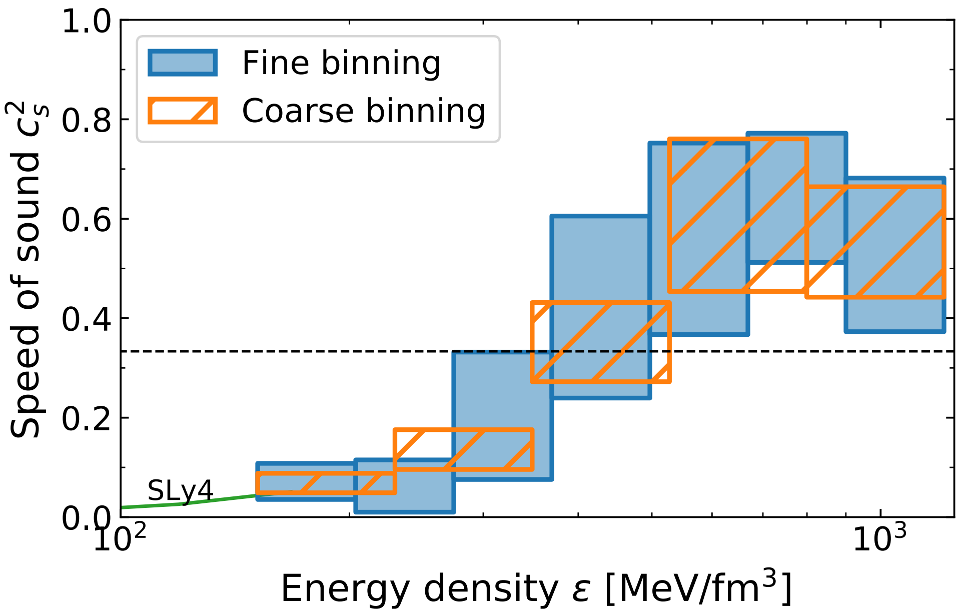


1st-order transition not necessarily excluded !

EoS Inference

Most Likely EoS from ML **Fujimoto-Fukushima-Murase (2018,19,21)**

Speed of sound may exceed the conformal limit (=1/3)



Is this a hint for the presence of quark matter ?

Not strongly constrained in the high density region…

EoS from Deep Neural Network

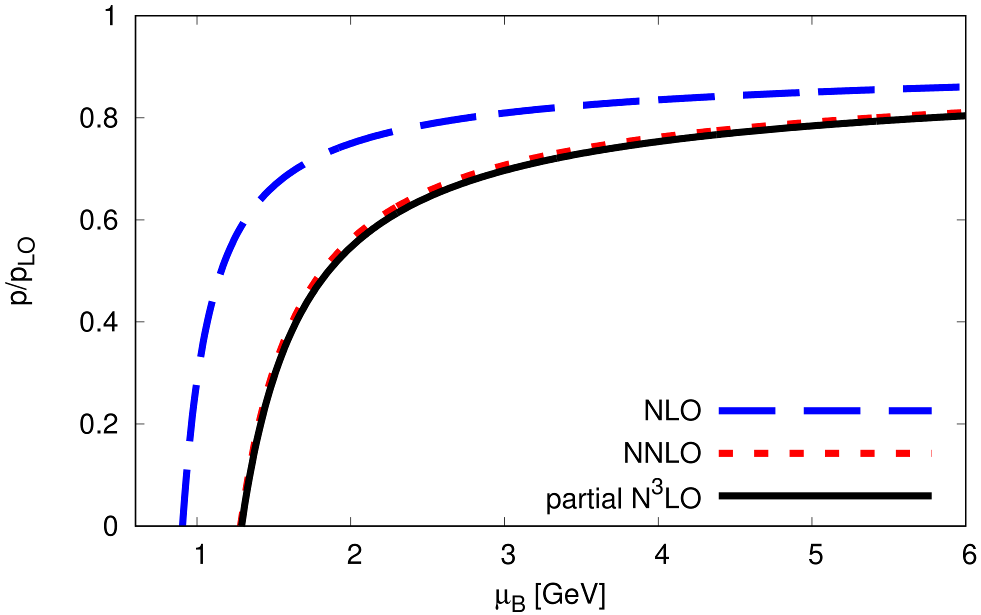
EoS from resummed pQCD

EoS and gravitational waves

QCD Approach

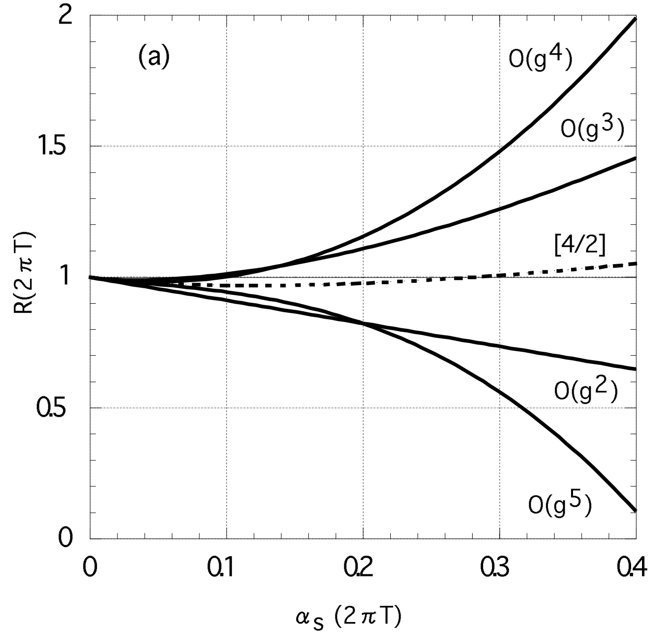
At high density perturbation theory should work.

Gorda-Kurkela-Romatschke-Sappi-Vuorinen: 1807.04120



Convergence is much better than high-*T* perturbation.

cf. high-T perturbation



Hatsuda: hep-ph/9708257

QCD Approach

Perturbations at zero quark mass

[LO] $$P_{(0)} = \frac{N_{\mathrm{c}}}{12\pi^2} \sum_f \mu_f^4$$

[NLO] $$P_{(2)} = -\frac{\alpha_s}{4\pi} \frac{d_A}{4\pi^2} \sum_f \mu_f^4 \qquad (d_A = N_{\mathrm{c}}^2 - 1)$$

[NNLO]
$$P_{(4)} = \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{d_A}{4\pi^2} \sum_f \mu_f^4 \left[2\beta_0 \ln \frac{\mu_i^2}{\mu_0^2} + \cdots \right]$$

Breakdown of the perturbation

QCD Approach

Improved perturbations at zero quark mass

[NNLO]

$$P_{(4)} = \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{d_A}{4\pi^2} \sum_f \mu_f^4 \left[2\beta_0 \ln \frac{\mu_i^2}{\mu_0^2} + \cdots\right]$$

Running (Screened) Coupling

$$\alpha_s \;\rightarrow\; \alpha_s(\bar{\Lambda}/\mu_0)$$

$$\ln \frac{\mu_i^2}{\mu_0^2} \;\rightarrow\; \ln \frac{\mu_i^2}{\bar{\Lambda}^2}$$

Singular log terms can be reduced by a choice of $\bar{\Lambda}^2 \sim \mu_i^2$

But… $\mu?\quad 2\mu?\quad 4\mu?$

QCD Approach

Scale uncertainty appears from NNLO

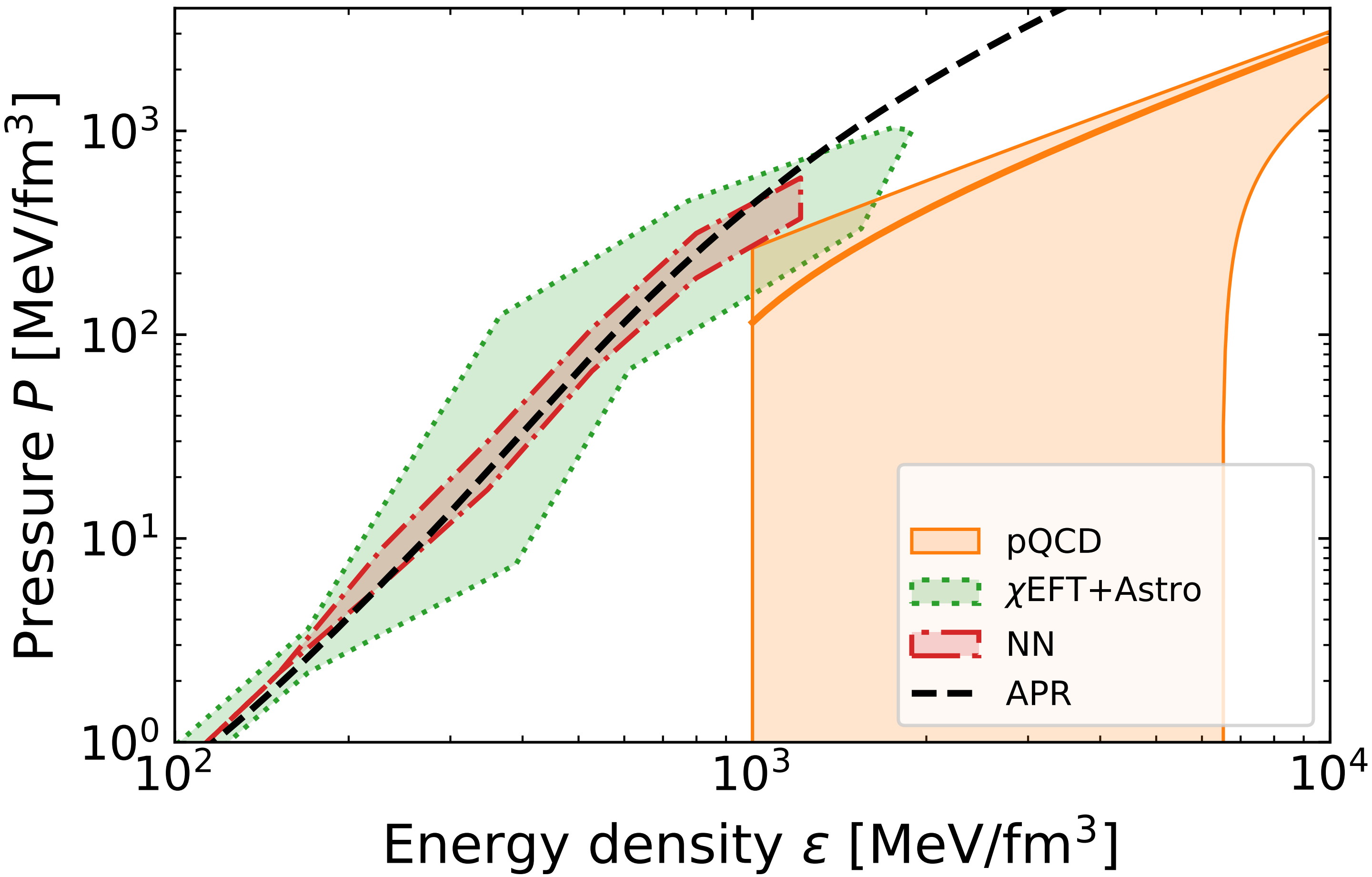
[LO] $$P_{(0)} = \frac{N_c}{12\pi^2} \sum_f \mu_f^4$$

[NLO] $$P_{(2)} = -\frac{\alpha_s}{4\pi} \frac{d_A}{4\pi^2} \sum_f \mu_f^4$$

As long as the chemical potential dependence is such simple, the EoS has no uncertainty even though α_s changes. Uncertainty appears from the NNLO only.

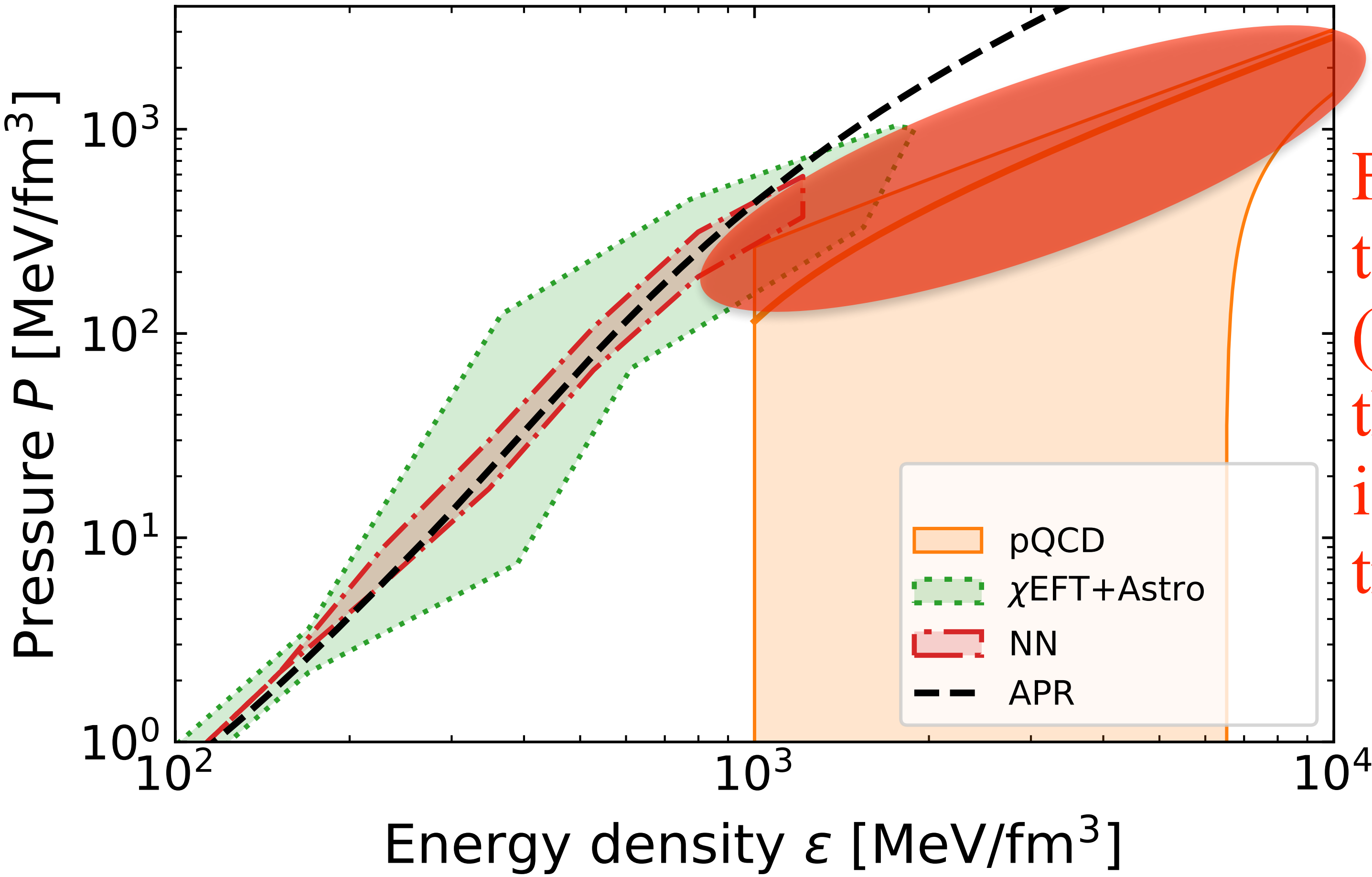
QCD Approach

pQCD not quite predictable???



QCD Approach

pQCD not quite predictable???



For continuation to the low density (empirical) EoS, the upper bound is likely to be the true EoS…?

QCD Approach

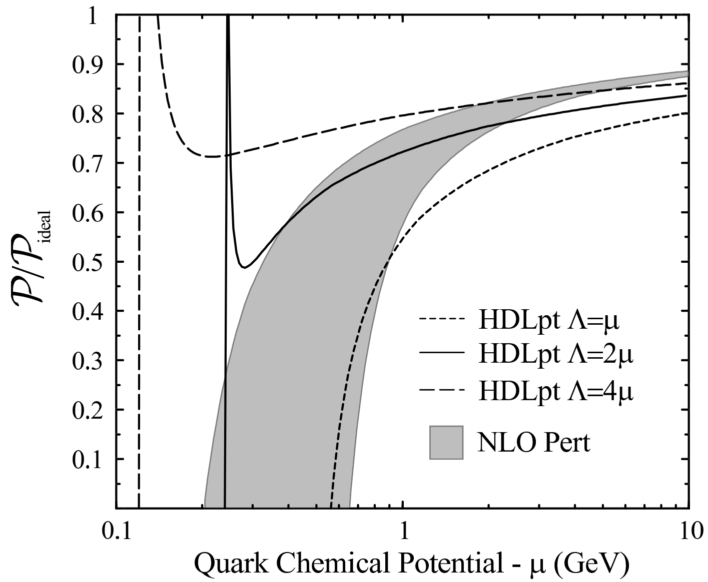
Approach with resummation (one-loop of HTL/HDL prop.)

Baier-Redlich: hep-ph/9908372

Zero temperature limit of HTL resummation (HDL)

Andersen-Strickland: hep-ph/0206196

HDL EoS to solve dense quark stars



If the coupling constant runs, the results depends on the scale. Besides, nontrivial scale dependence appears from the Debye mass $\alpha_s \mu_f^2$. (Log terms also appear!)

QCD Approach

Approach with resummation (one-loop of HTL/HDL prop.)

In terms of the 2PI language:

$$\Gamma = \boxed{\frac{1}{2}\mathrm{tr}\ln G^{-1}} - \frac{1}{2}\mathrm{tr}\ln(1 - G_0^{-1}G) + \Gamma_2[G]$$

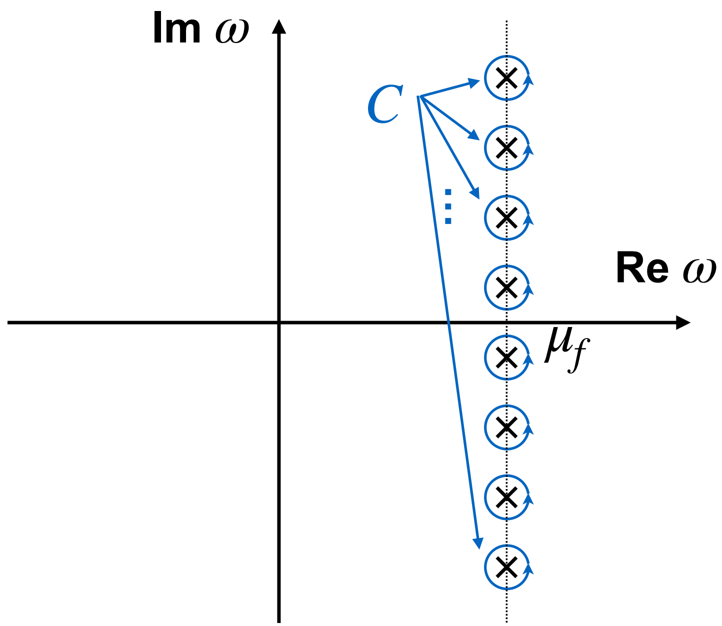
using the propagator with the self-energy insertion

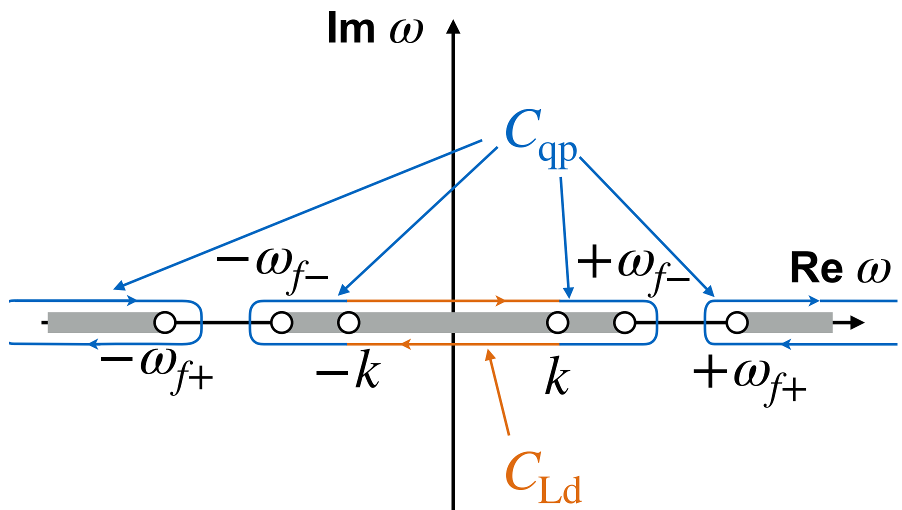
* Regarded as a "**quasi-particle**" approximation
* Usually the thermodynamics is dominated by this alone
* Not really a systematic expansion in the coupling

Conceptually straightforward but technically complicated especially with finite strange quark mass…

QCD Approach

$$P_{\mathrm{q},f}(T,\mu_f) = \mathrm{tr}\ln G_f^{-1}$$
$$= \sum\!\!\!\!\!\!\int_{\{K\}} \ln\det\left[\not{k} - M_f - \Sigma(i\tilde{\omega}_n + \mu_f, k)\right]$$
$$= 2\sum\!\!\!\!\!\!\int_{\{K\}} \ln\left[A_S^2(i\tilde{\omega}_n + \mu_f, k) + M_f^2 - A_0^2(i\tilde{\omega}_n + \mu_f, k)\right]$$





QCD Approach

For massless pressure:

$$\frac{P_{\text{HDLpt}}}{P_{\text{ideal}}} \approx 1 - 6\frac{m_{\text{q}f}^2}{\mu_f^2} + \mathcal{O}\left(\frac{m_{\text{q}f}^4}{\mu_f^4}\right) = 1 - 4\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)$$

$$\frac{P_{\text{pQCD}}}{P_{\text{ideal}}} = 1 - 2\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)$$

This discrepancy was known since Baier-Redlich, and a correction term is necessary for order-by-order matching:

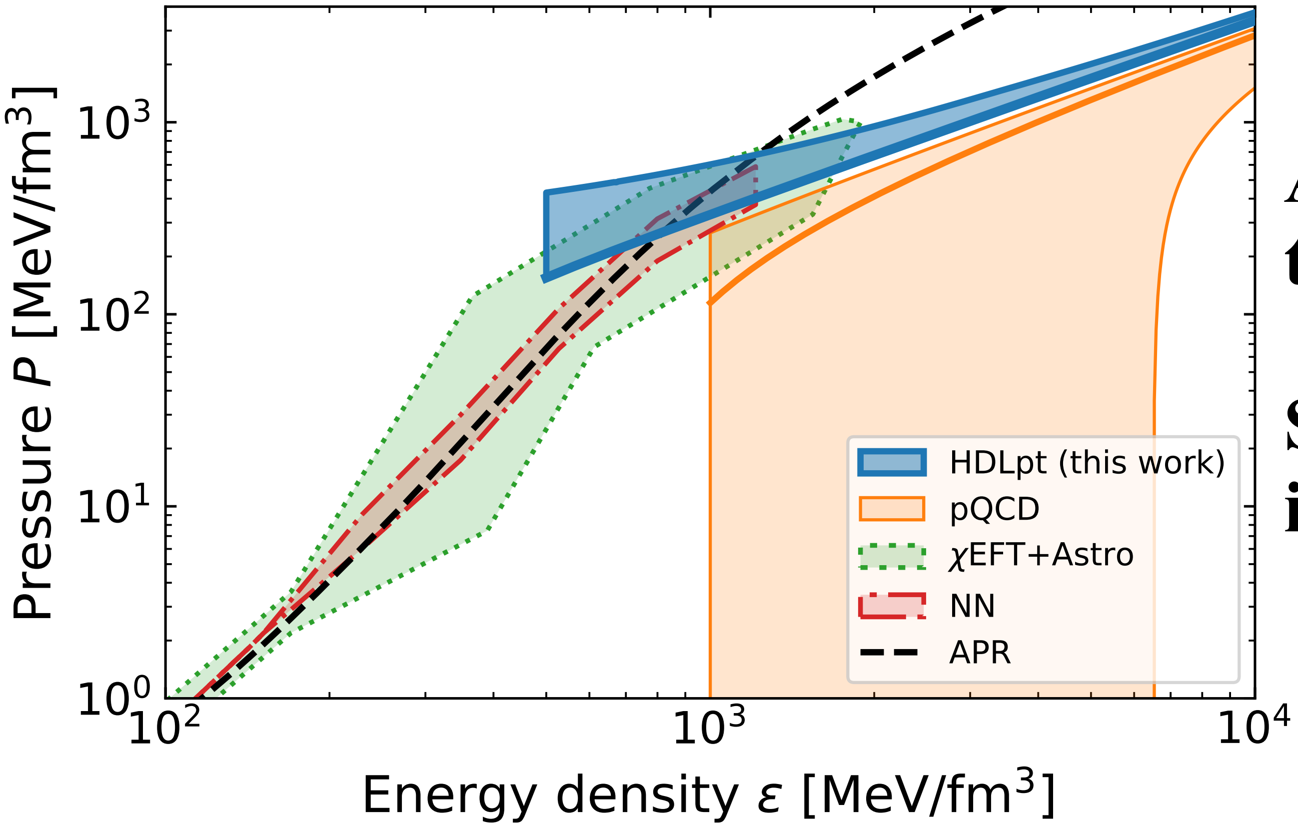
$$P_{\text{corr}} = 2\frac{\alpha_s}{\pi} P_{\text{ideal}}$$

We have checked that this is negligible at high density.

QCD Approach

Fujimoto-Fukushima: 2011.10891

Smooth continuation from the nuclear side to the quark side could be possible now! Slope change?

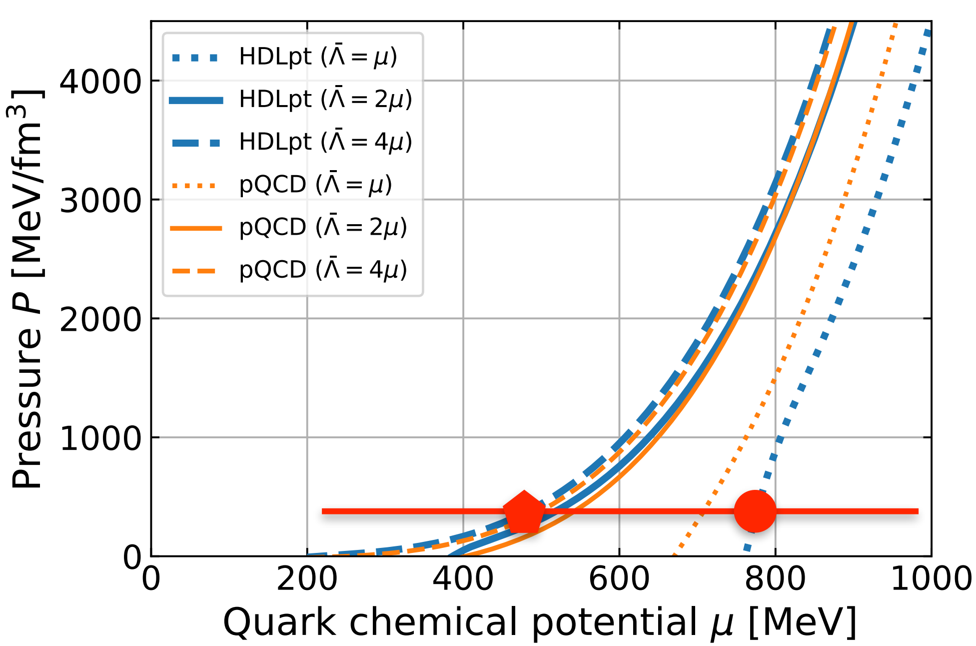


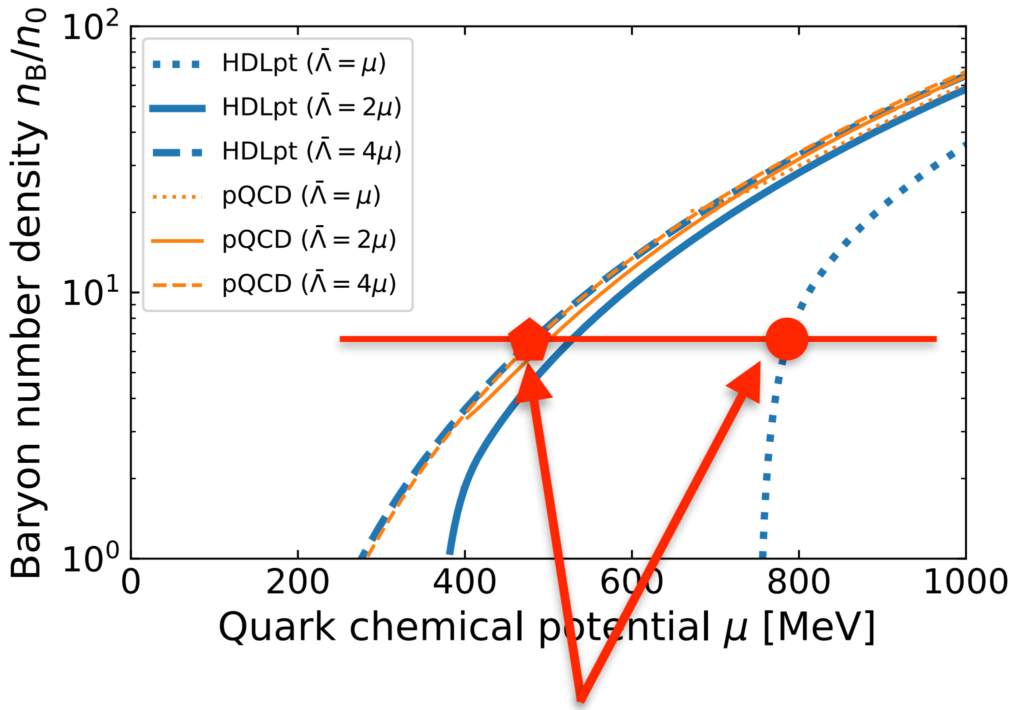
As expected from the continuation!

Slope changes to imply crossover?

QCD Approach

Fujimoto-Fukushima: 2011.10891





For a given density the corresponding μ is pushed up by the resummation

Not so trivial because of the Debye and *s*-quark masses

QCD Approach

Fujimoto-Fukushima: 2011.10891

In principle we should require:

$P(\mu, \alpha_s(\bar{\Lambda}), m(\bar{\Lambda}); \bar{\Lambda})$ **is independent of the scale.**

It seems to be very hard in the resummed theory… but…

The scale insensitivity of the EoS is good enough!

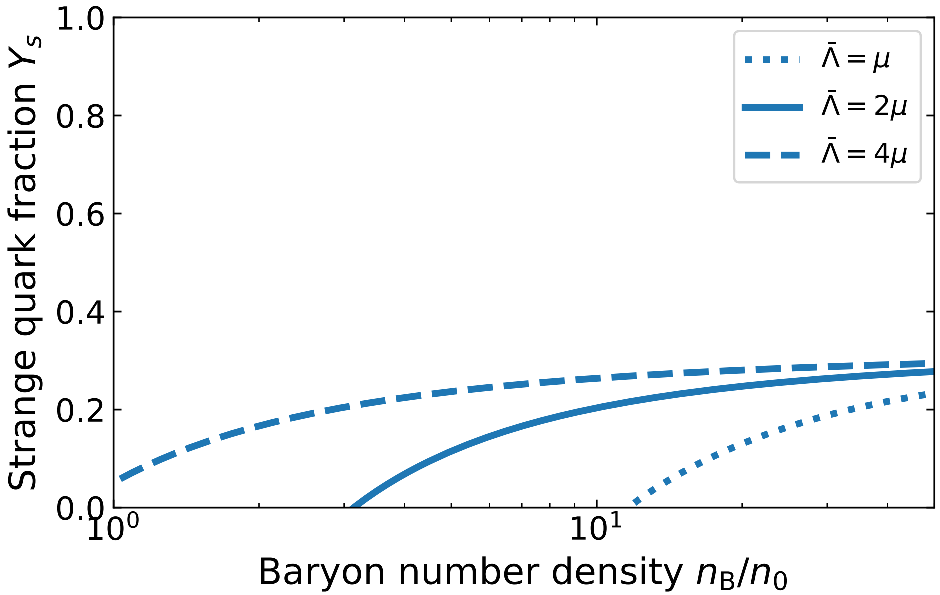
$$\frac{\partial P(\mu_{\mathrm{B}}; \bar{\Lambda})}{\partial \bar{\Lambda}} - c_s^2 \frac{\partial \varepsilon(\mu_{\mathrm{B}}; \bar{\Lambda})}{\partial \bar{\Lambda}} = 0$$

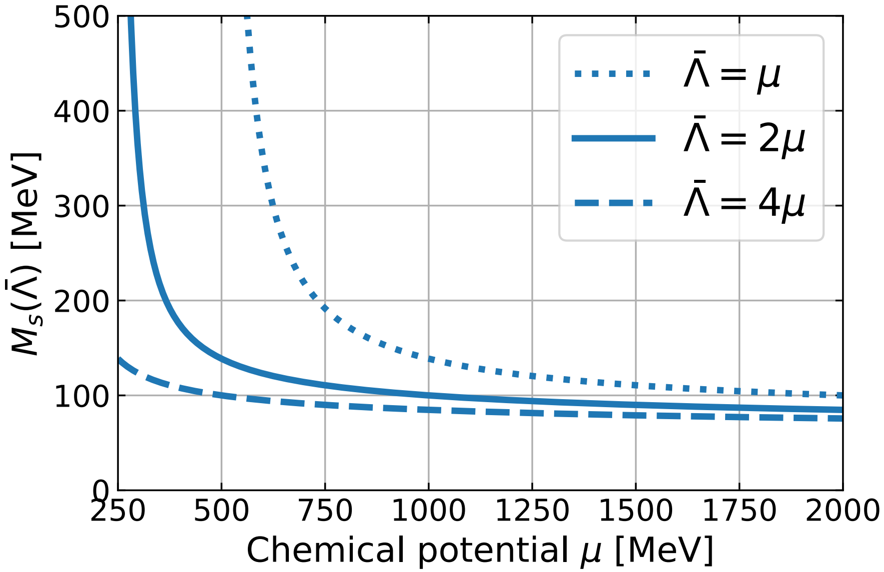
Surprisingly, this approximately holds in the resummation.

QCD Approach

Strangeness could be quantified:

Fujimoto-Fukushima: 2011.10891





Strange quark mass runs (like α_s) and the threshold strongly depends on the scale choice…

EoS from Deep Neural Network

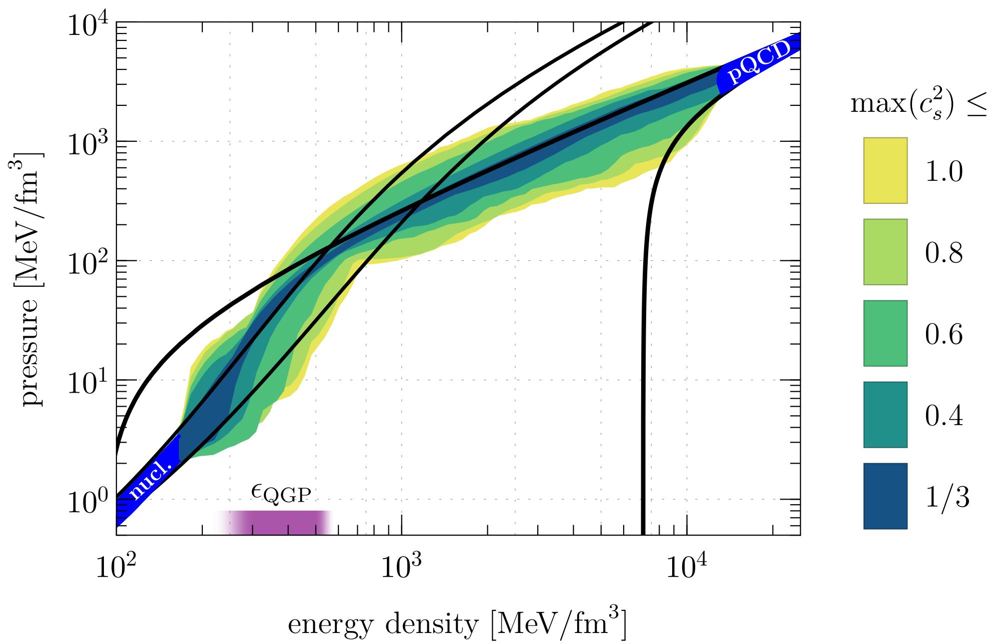
EoS from resummed pQCD

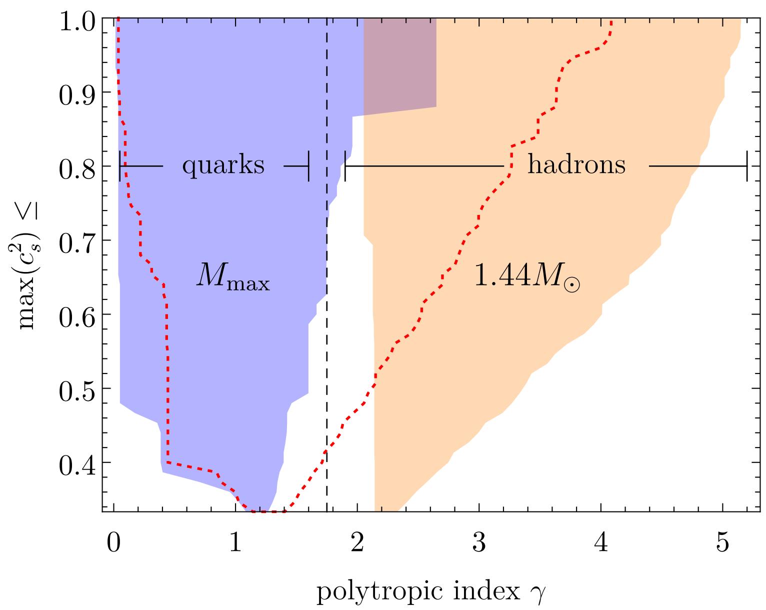
EoS and gravitational waves

Quarkyonic Crossover

Phase transition (qualitative change) to Quark Matter?

Annala-Gorda-Kurkela-Nattila-Vuorinen: 1903.09121



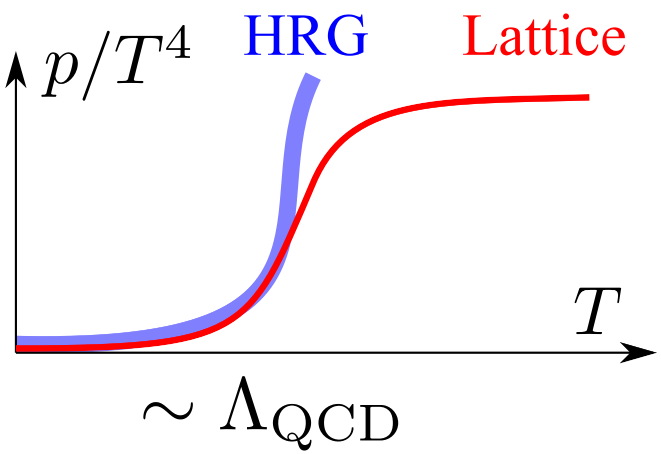


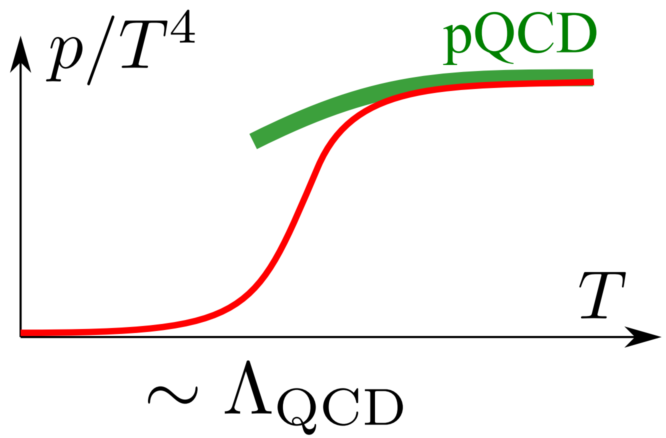
Phenomenological characterization of crossover to quark matter

$$\gamma = \frac{d \ln p}{d \ln \epsilon}$$

Quarkyonic Crossover

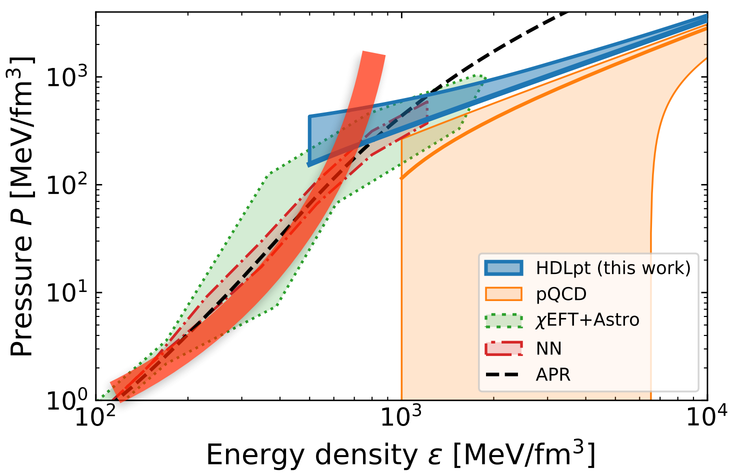
High-T has been understood by HRG + pQCD





High-Density

Maybe a duality region where the hadrons and quarks coexist (quarkyonic).



pQCD

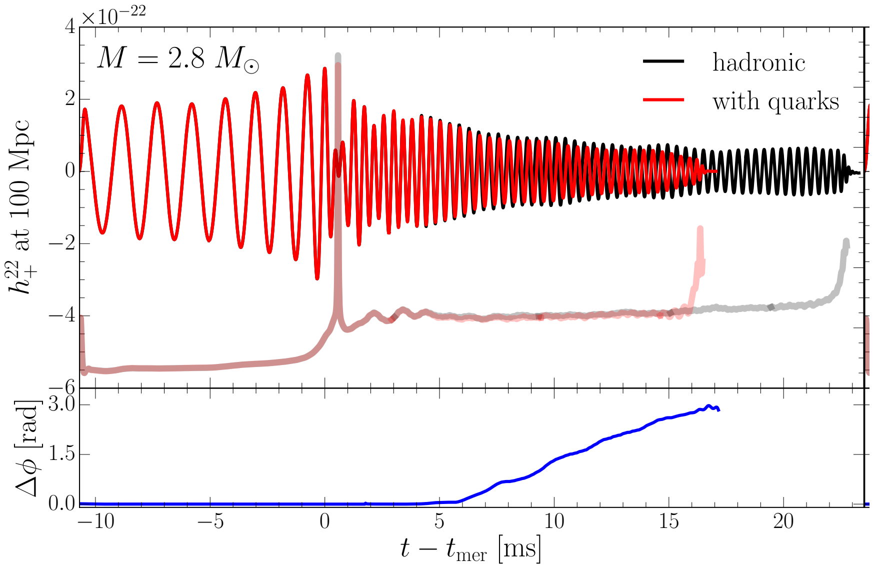
December 9, 2021 @ online talk at S@INT seminar

Quarkyonic Crossover

How to find such crossover, if any???
1-st order phase transition might be straightforward???

Most-Papenfort-Dexheimer-Hanauske-Schramm-Stocker-Rezzolla (2018)

$\mathrm{CMF_Q}$: EOS with a strong-1st PT to Quark Matter (3~4 times n_0)
$\mathrm{CMF_H}$: EOS without quarks



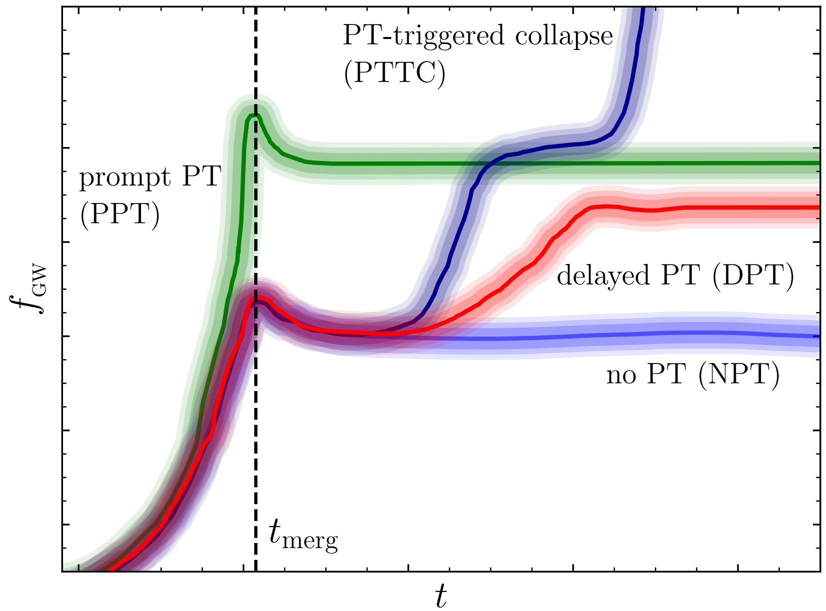
Quark matter shortens the lifetime of post-merger hypermassive neutron star. (Easier to collapse into BH.)

Quarkyonic Crossover

How to find such crossover, if any???
1-st order phase transition might be straightforward???

Weih-Hanauske-Rezzola (2019)

Several different scenarios (but still a 1st-order assumed)



Finite-*T* treatment:

$$P = P_{\rm cold} + P_{\rm thermal}$$

$$\varepsilon = \varepsilon_{\rm cold} + \varepsilon_{\rm thermal}$$

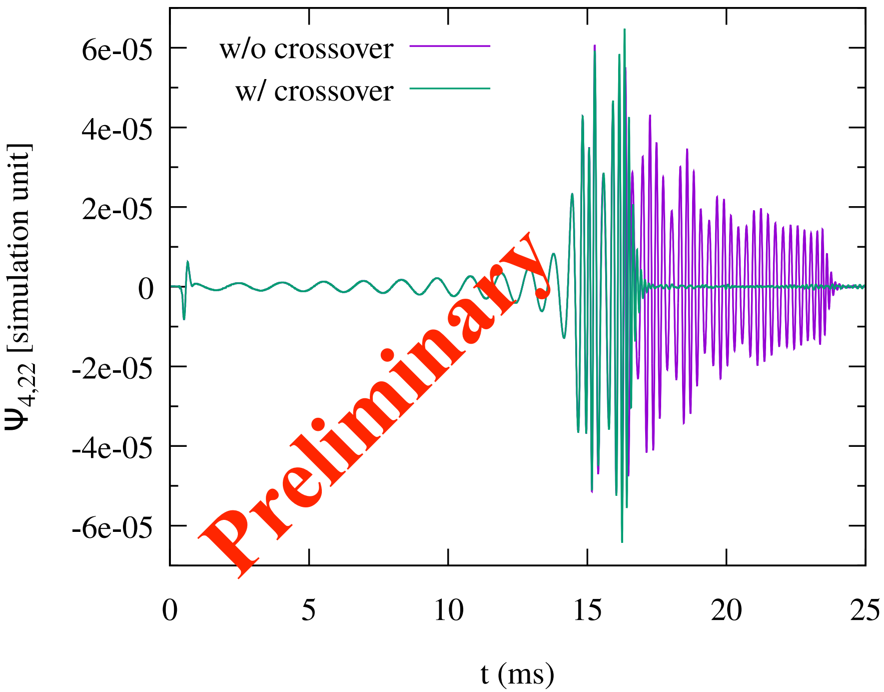
$$P_{\rm thermal} \approx \rho\varepsilon_{\rm thermal}(\Gamma_{\rm th} - 1)$$

$\Gamma_{\rm th}$ = 1.75 is fixed by hand

Quarkyonic Crossover

Crossover also detectable? ← YES!
Thermal effect??

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022)



Neutrino processes are not included (like Hanauske's)

Similar trend can be confirmed. We also checked the thermal index dependence:

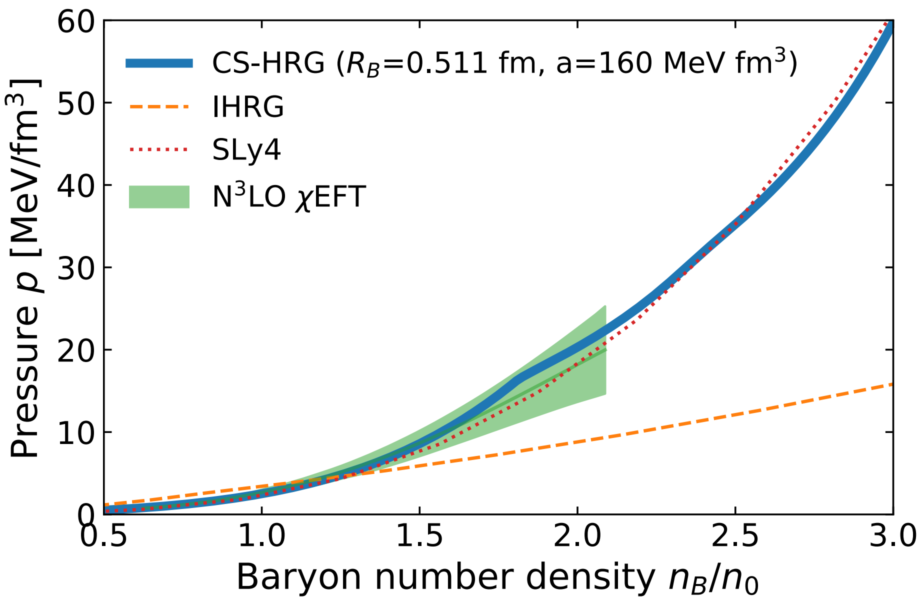
For small Γ_{th} thermal pressure is not enough and the lifetime is shortened to go to BH.

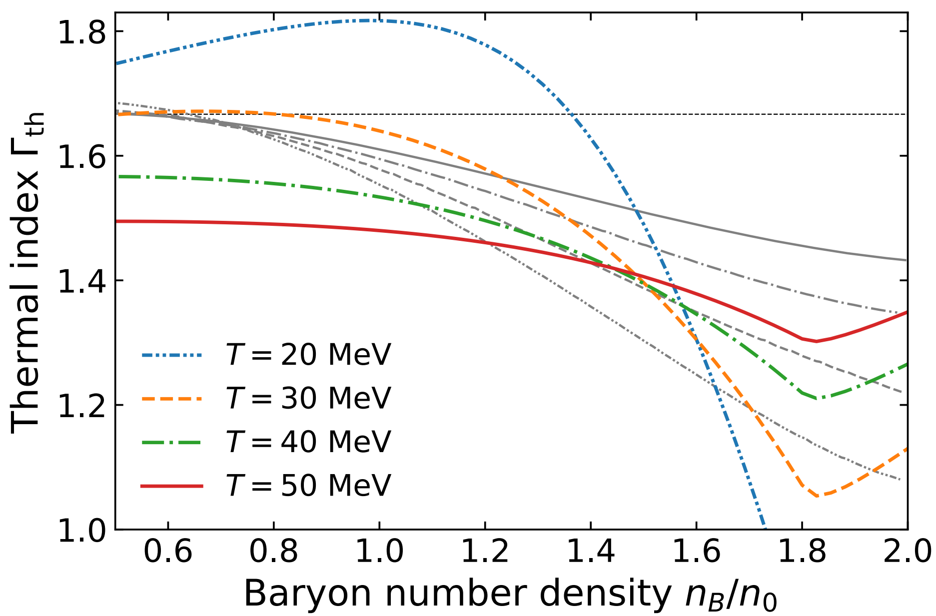
For $\Gamma_{th} \sim 2$ the thermal pressure can sustain the hypermassive NS.

Quarkyonic Crossover

Estimation of the (T-dependent) thermal index
Thermal effect??

Fujimoto-Fukushima-Hidaka-Hiraguchi-Iida (2021)



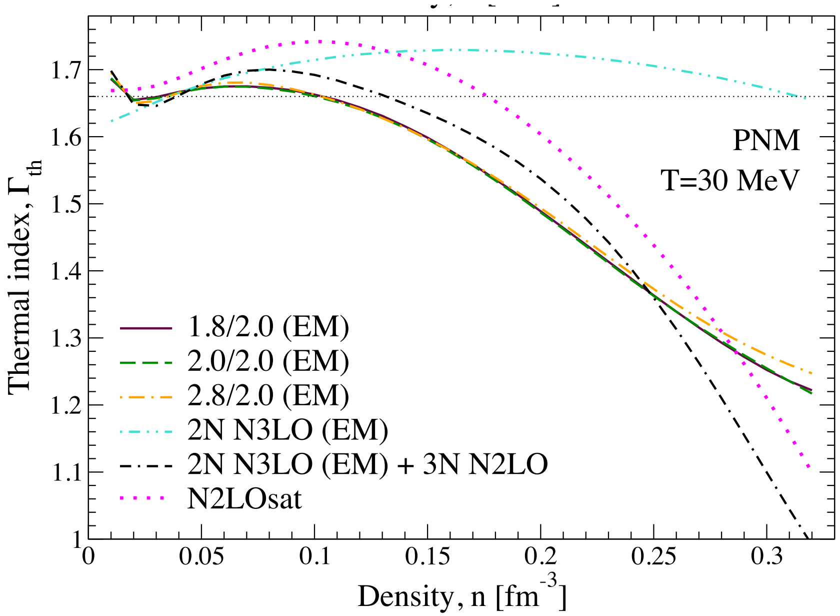


Van der Waals type EoS with interacting hadrons.
Thermal index looks consistent with other approaches…?

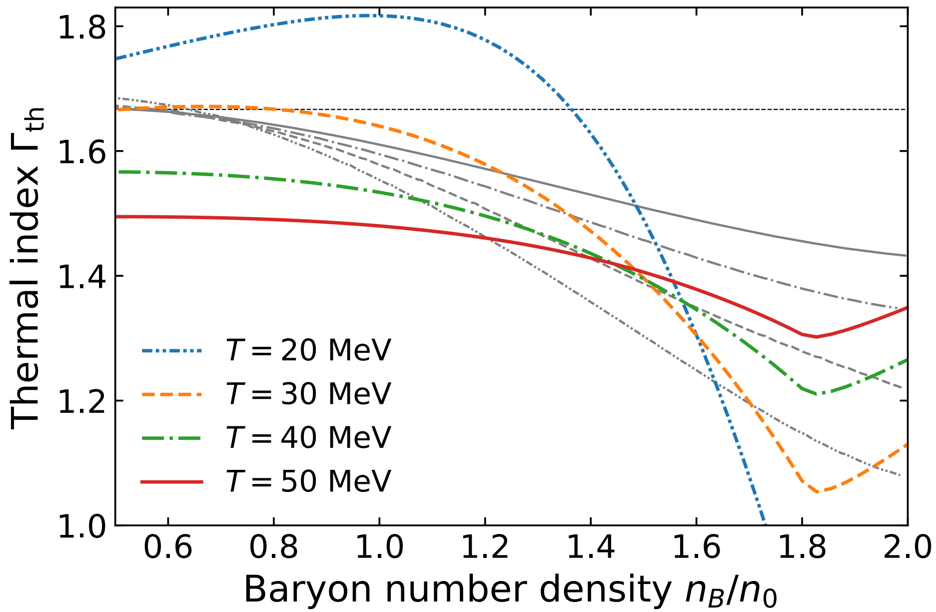
Quarkyonic Crossover

Crossover also detectable?
Thermal effect??

Fujimoto-Fukushima-Hidaka-Hiraguchi-Iida (2021)



Carbone-Schwenk (2019)



Most probably the thermal pressure is not large (that is good!)

Summary

Deep Neural Network (DENSE Collaboration)

- □ Predicts the most likely EoS that stays close to the empirical nuclear EoS (SLy4).
- □ Codes publicized very soon…

pQCD Resummation

- □ Many underestimate the predictability of pQCD at high density — resummation cures the problem better.

Gravitational wave signals?

- □ Crossover causes detectable effects (EoS softening is important), but the thermal pressure should be carefully considered (maybe not large, fortunately).