Reconstructing PDFs from Mellin moments: the pion and kaon case

Martha Constantinou

Temple University

Institute for Nuclear Theory, University of Washington

S @ INT seminar

June 10, 2021

Reconstructing PDFs from Mellin moments: the pion and kaon case

from Lattice QCD

Martha Constantinou

Temple University

Institute for Nuclear Theory, University of Washington
S @ INT seminar

June 10, 2021

Collaborators

- **C. Alexandrou** (Univ. of Cyprus/Cyprus Institute)
- **S. Bacchio** (Cyprus Institute)
- **I. Cloet** (Argonne National Lab)
- **K. Hadjiyiannakou** (Univ. of Cyprus/Cyprus Institute)
- **G. Koutsou** (Cyprus Institute)
- **C. Lauer** (Temple University)

Collaborators

- **C. Alexandrou** (Univ. of Cyprus/Cyprus Institute)
- **S. Bacchio** (Cyprus Institute)
- **I. Cloet** (Argonne National Lab)
- **K. Hadjiyiannakou** (Univ. of Cyprus/Cyprus Institute)
- **G. Koutsou** (Cyprus Institute)
- **C. Lauer** (Temple University)

Extended Twisted Mass Collaboration

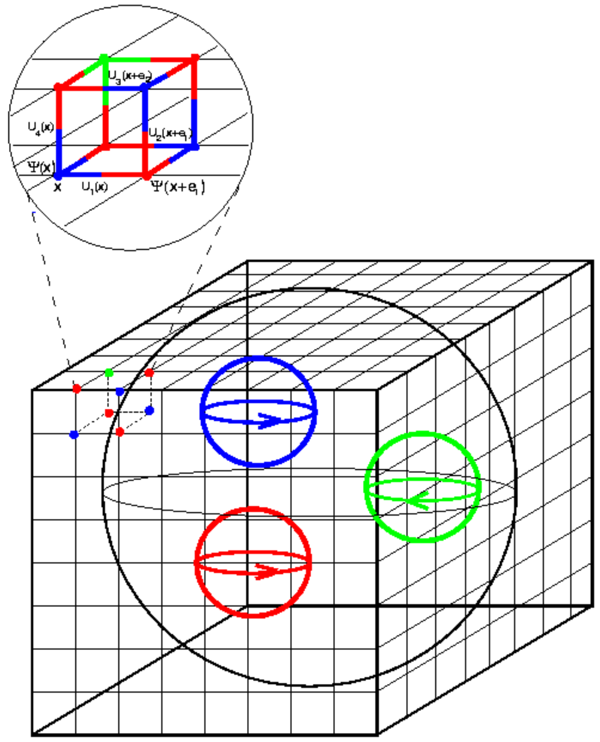
Relevant publications

- *The Mellin moments $\langle x \rangle$ and $\langle x^2 \rangle$ for the pion and kaon from lattice QCD,*
  C. Alexandrou, S. Bacchio, I. Cloet, M. Constantinou, K. Hadjiyiannakou, G. Koutsou, C. Lauer,
  [arXiv:2104.02247]

- *The pion and kaon $\langle x^3 \rangle$ from lattice QCD and PDF reconstruction from Mellin moments,*
  C. Alexandrou, S. Bacchio, I. Cloet, M. Constantinou, K. Hadjiyiannakou, G. Koutsou, C. Lauer,
  PRD 103, 014508 (2021), [arXiv:2010.03495]

Lattice formulation of QCD

Ideal first principle formulation of QCD
(simulations starting from original Lagrangian)



courtesy: USQCD

★ Space-time discretization on a finite-size 4-D lattice

★ Serves as a regulator:
UV cut-off: inverse lattice spacing
IR cut-off: inverse lattice size

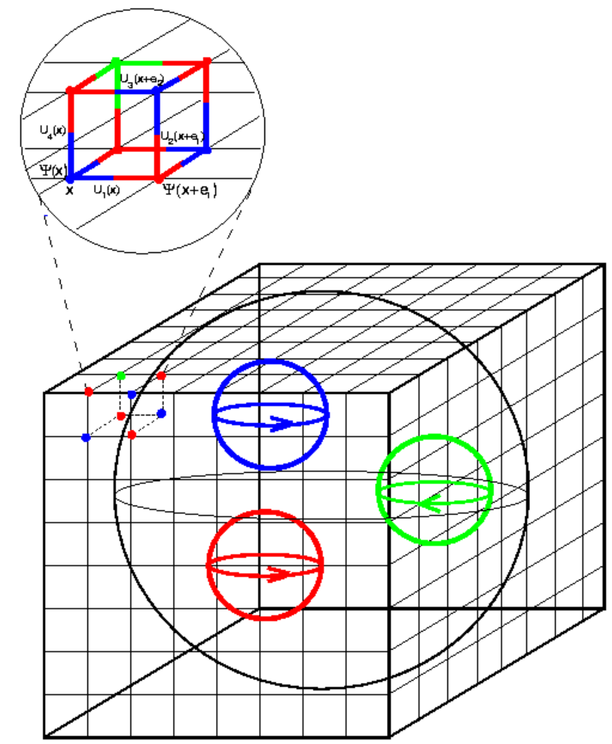
$$\int_{-\infty}^{\infty} dp \rightarrow \int_{-\pi/a}^{\pi/a} \frac{dp}{2\pi}$$

$$\int dp F(p) \rightarrow \sum_{n}^{N_{\max}} \frac{2\pi}{L} F(p_0 + \frac{2\pi n}{L})$$

★ Removal of regulator $L \rightarrow \infty\,,\ \ a \rightarrow 0$

Lattice formulation of QCD

Ideal first principle formulation of QCD
(simulations starting from original Lagrangian)



courtesy: USQCD

★ Space-time discretization on a finite-size 4-D lattice

★ Serves as a regulator:
UV cut-off: inverse lattice spacing
IR cut-off: inverse lattice size

$$\int_{-\infty}^{\infty} dp \;\rightarrow\; \int_{-\pi/a}^{\pi/a} \frac{dp}{2\pi}$$

$$\int dp F(p) \;\rightarrow\; \sum_{n}^{N_{\max}} \frac{2\pi}{L} F\left(p_0 + \frac{2\pi n}{L}\right)$$

★ Removal of regulator $L \rightarrow \infty\,,\;\; a \rightarrow 0$

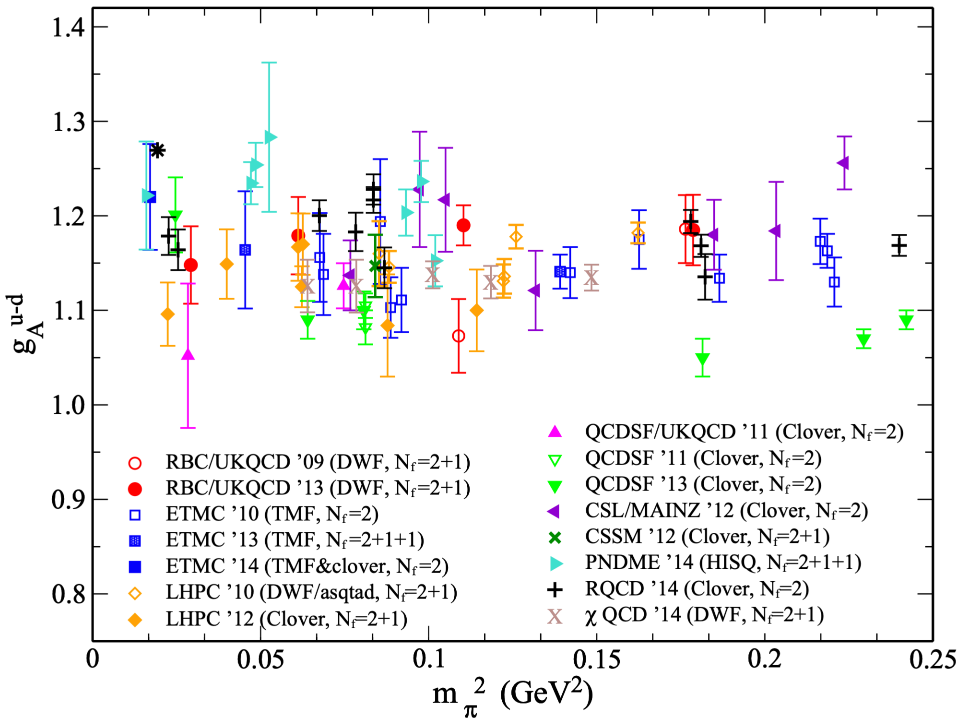
Technical Aspects

★ Parameters (define cost of simulations):
quark masses (aim at physical values)
lattice spacing (ideally fine lattices)
lattice size (need large volumes)

★ Discretization not unique:
Wilson, Clover, Twisted Mass, Staggered, Overlap, Domain Wall, Mixed actions

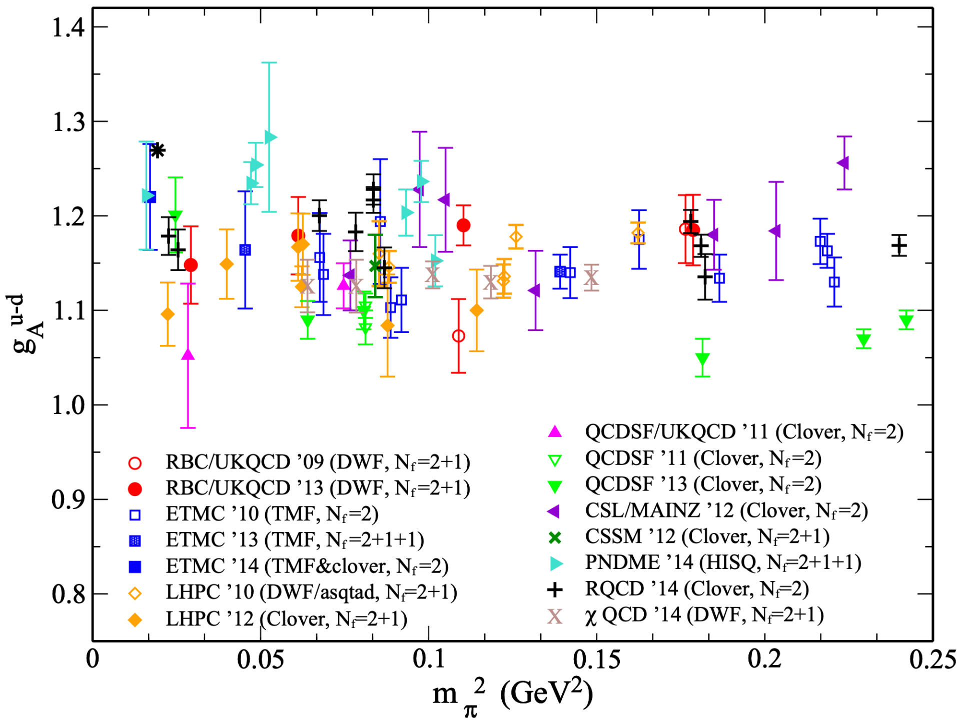
Proton structure from lattice QCD

Proton structure from lattice QCD

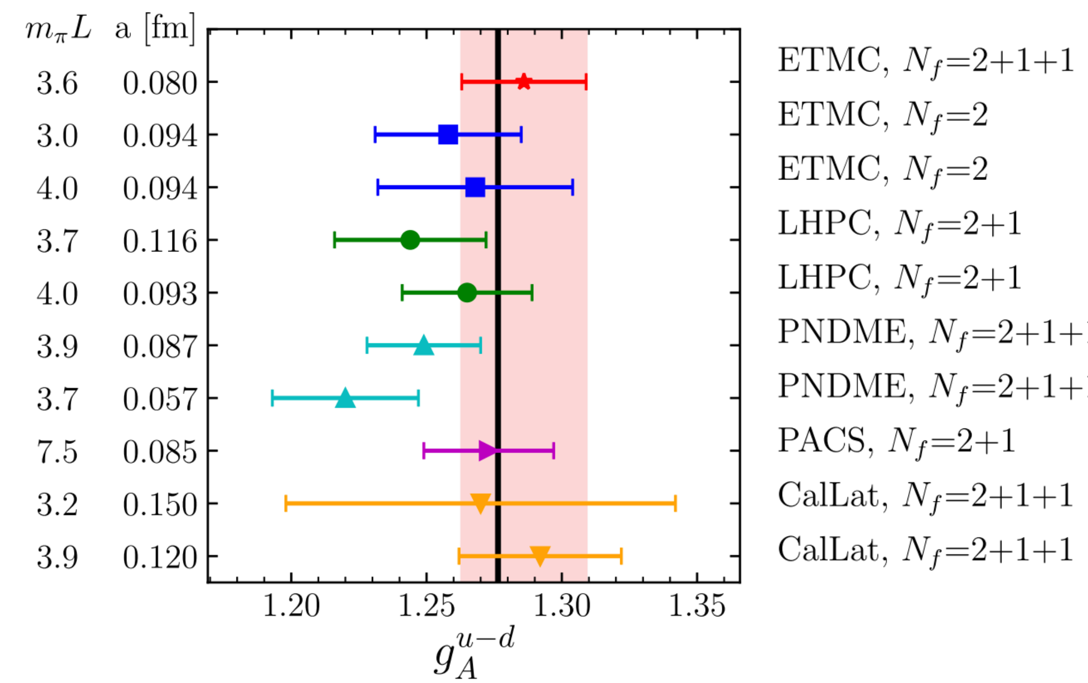


[M. Constantinou, Plenary Lattice 2014]

Proton structure from lattice QCD

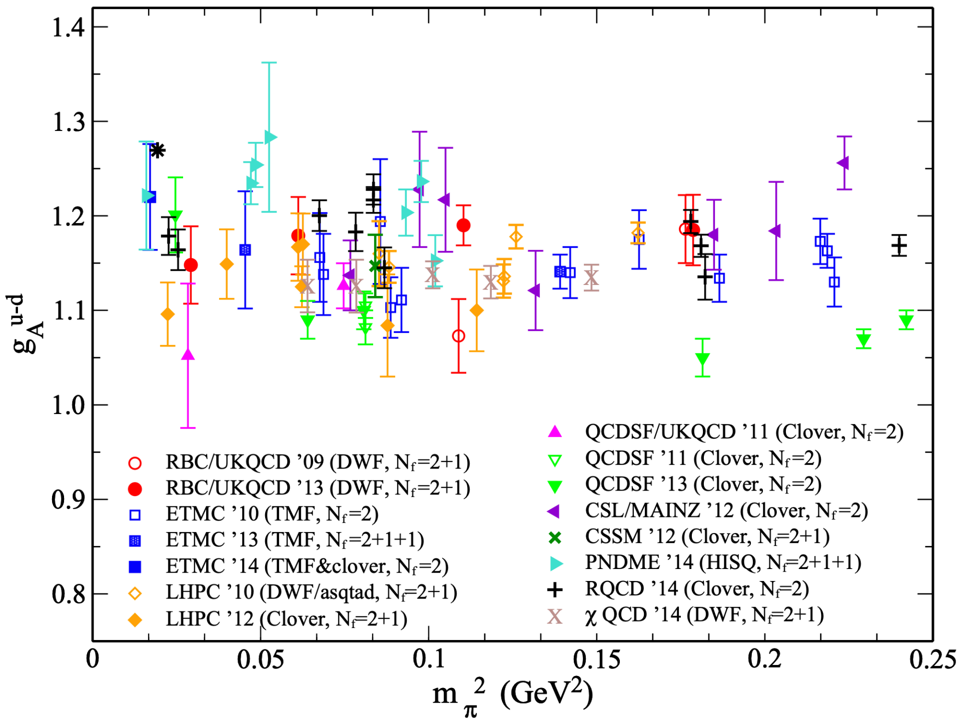


[M. Constantinou, Plenary Lattice 2014]

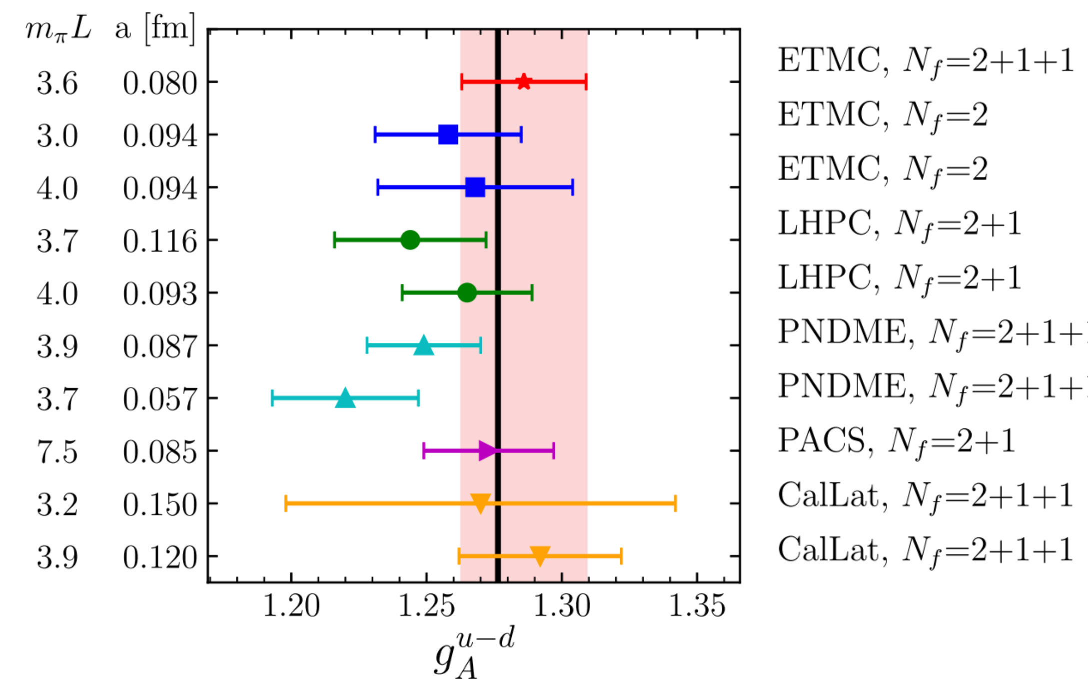


[C. Alexandrou et al., PRD 102 (2020) 5, 054517]

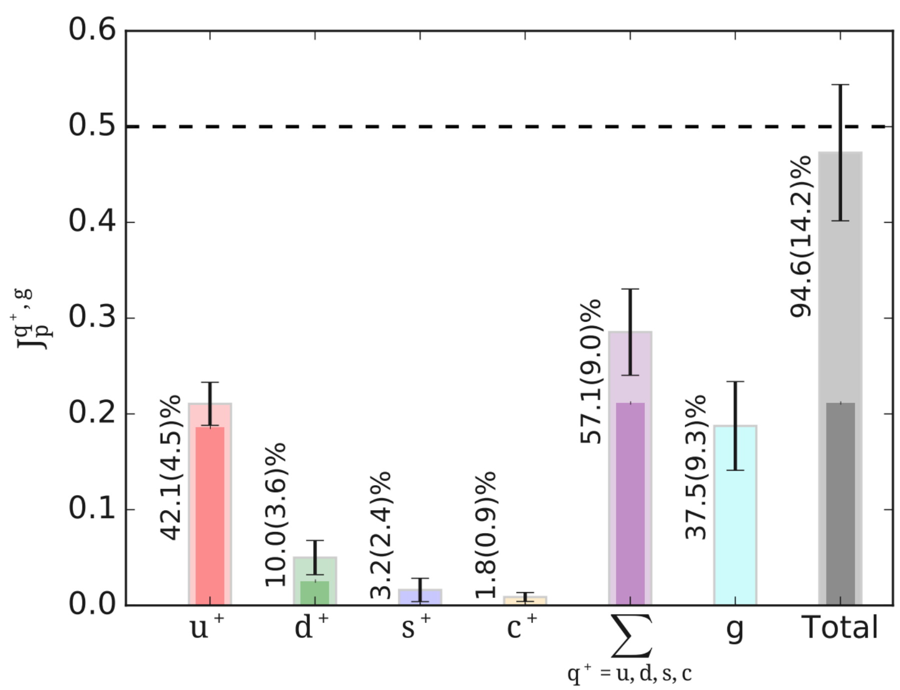
Proton structure from lattice QCD

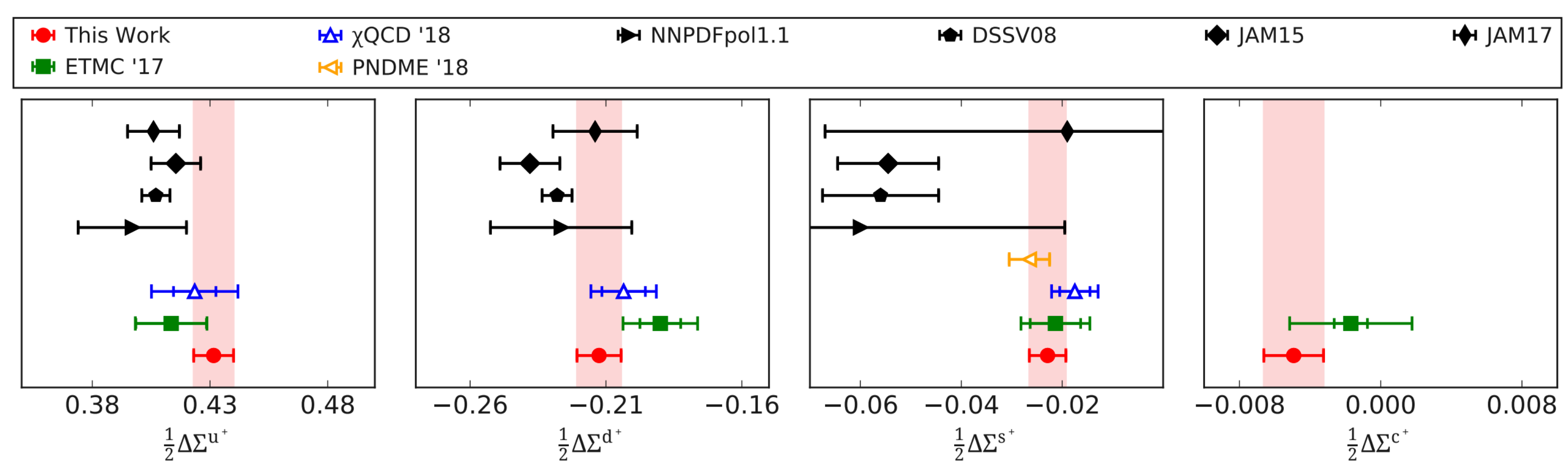


[M. Constantinou, Plenary Lattice 2014]



[C. Alexandrou et al., PRD 102 (2020) 5, 054517]





[C. Alexandrou et al., PRD 101 (2020) 9, 094513]

OUTLINE

A. Motivation

B. Mellin moments in lattice QCD

C. Reconstruction of PDFs

D. SU(3) flavor symmetry breaking

E. Summary

Pions and Kaons

★ **Pions and Kaons belong to the octet of Nambu-Goldstone bosons**
(dynamical chiral symmetry breaking (DCSB))

★ **Mass difference between pion and kaon can help understand the interplay between QCD dynamics and quark mass effects**

★ **Experimental data only for the pion (pion induced Drell-Yan reaction) and for the limited region $x \in [0.21 - 0.99]$** [J. S. Conway et al., PRD 39, 92 (1989)]

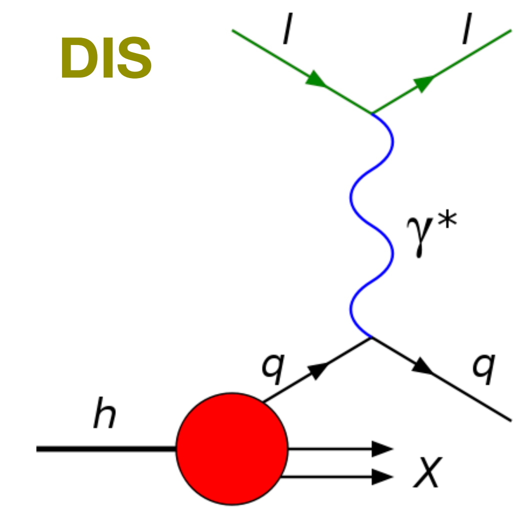
★ **Contradictory conclusions on the large-x behavior of pion PDF:**
- **initial E615 data show a $(1-x)^1$ behavior**
- **reanalysis of E615 data shows a $(1-x)^2$ fall** [R. Holt et al., RMP 82, 2991 (2010)], [M. Aicher et al., PRL 105, 252003 (2010)]
- **DSE predict $(1-x)^2$ fall** [K. Bednar et al. PRL 124, 042002 (2020)]

★ **Lattice QCD calculations using non-local operators do not reach to a consensus** [M. Constantinou, EPJA 57, 77 (2021), arXiv:2010.02445]

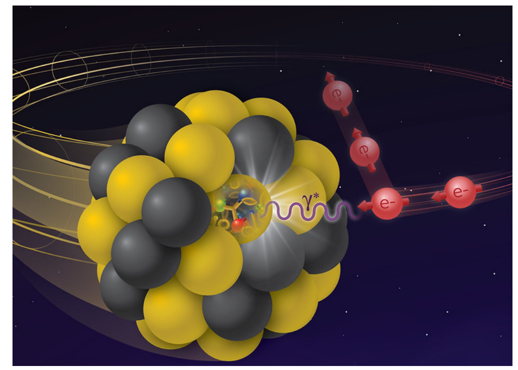
★ **EIC will address pion and kaon structure** [EIC Yellow Report, arXiv:2103.05419], [Aguilar et al., EPJA 55, 190 (2019)]

Hadron Structure

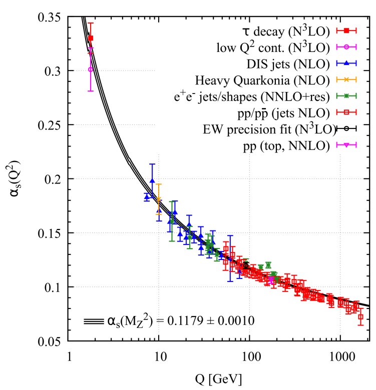
- **Structure of hadrons explored in high-energy scattering processes**



Collisions @ EIC



- **Due to asymptotic freedom, e.g.**



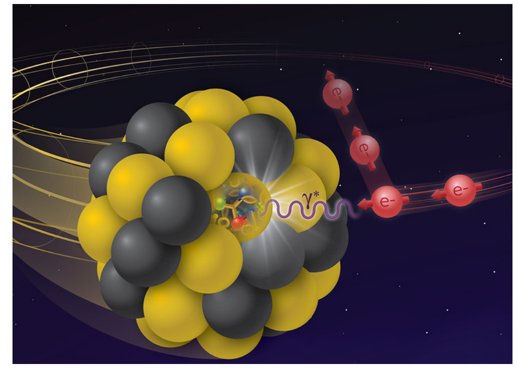
$$\sigma_{\rm DIS}(x,Q^2) = \sum_i \left[H^i_{\rm DIS} \otimes f_i\right](x,Q^2)$$

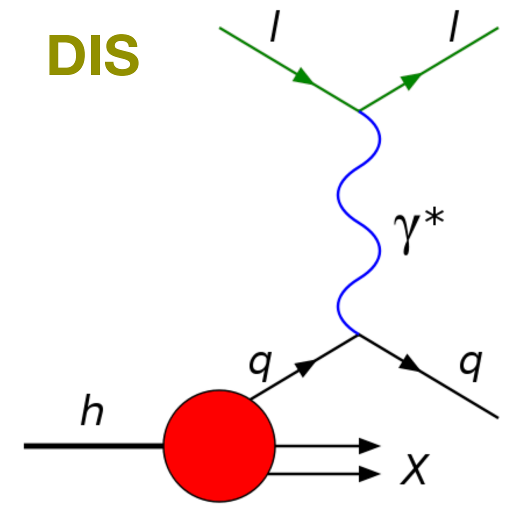
$$[a \otimes b](x) \equiv \int_x^1 \frac{d\xi}{\xi}\, a\left(\frac{x}{\xi}\right)\, b(\xi)$$

Hadron Structure

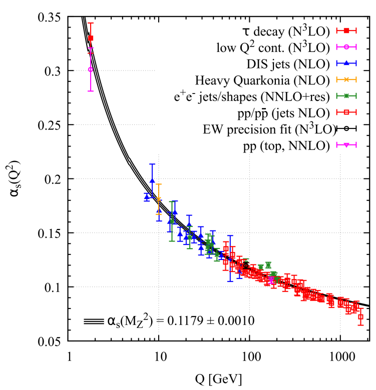
- **Structure of hadrons explored in high-energy scattering processes**

Collisions @ EIC





- **Due to asymptotic freedom, e.g.**



$$\sigma_{\rm DIS}(x,Q^2) = \sum_i \left[H^i_{\rm DIS} \otimes f_i\right](x,Q^2)$$

$$[a \otimes b](x) \equiv \int_x^1 \frac{d\xi}{\xi}\, a\left(\frac{x}{\xi}\right)\, b(\xi)$$

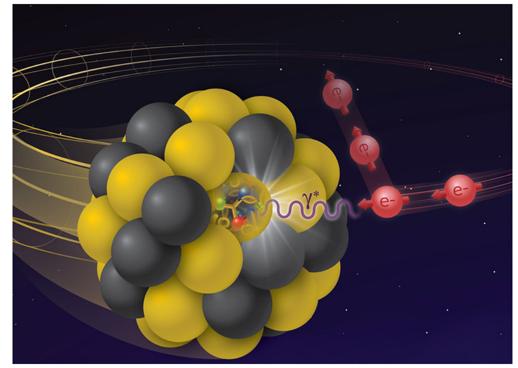
Perturb. part
(process dependent)

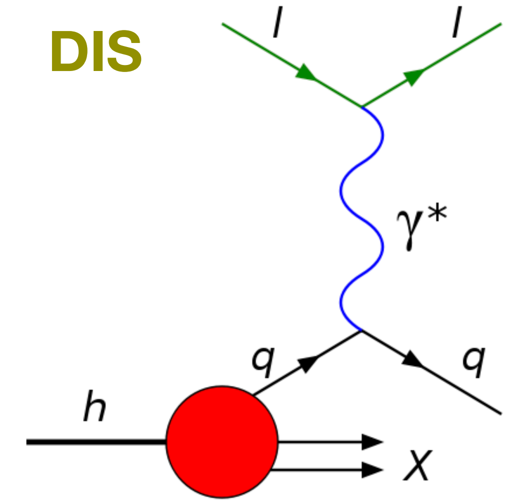
Non-Perturb. part
(process "independent")

Hadron Structure

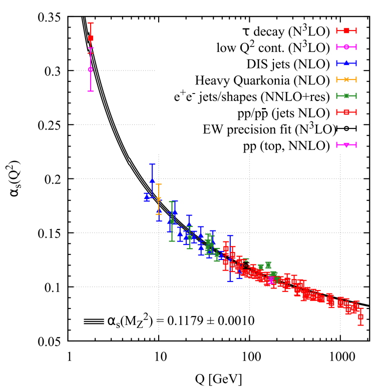
- **Structure of hadrons explored in high-energy scattering processes**

Collisions @ EIC





- **Due to asymptotic freedom, e.g.**



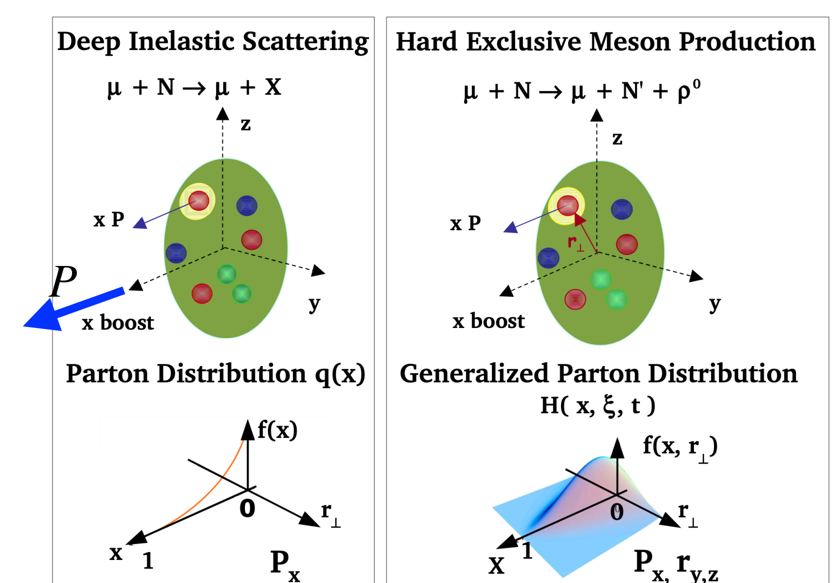
$$\sigma_{\text{DIS}}(x, Q^2) = \sum_i \left[H^i_{\text{DIS}} \otimes f_i\right](x, Q^2)$$

$$[a \otimes b](x) \equiv \int_x^1 \frac{d\xi}{\xi}\, a\left(\frac{x}{\xi}\right) b(\xi)$$

Perturb. part (process dependent) → H^i_{DIS}

Non-Perturb. part (process "independent") → f_i

- **Non perturb. part provides information on partonic structure of hadrons**



Distribution Functions

★ **DFs parameterized in terms of off-forward matrix elements of non-local light-cone operators (Not accessible on Euclidean lattice)**

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\,\mathcal{O}\,\underbrace{\mathcal{P}e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}}\psi(\lambda n/2)|p\rangle$$

$q = p' - p$, $\bar{P} = (p'+p)/2$, n: **light-cone vector (**$\bar{P}.n = 1$**)**, $\xi = -n\cdot\Delta/2$

Distribution Functions

★ **DFs parameterized in terms of off-forward matrix elements of non-local light-cone operators (Not accessible on Euclidean lattice)**

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar{\psi}(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2)|p\rangle$$

$q = p' - p$, $\bar{P} = (p'+p)/2$, n: light-cone vector ($\bar{P}.n = 1$), $\xi = -n\cdot\Delta/2$

★ **Their Mellin moments with respect to x can be accessed in LQCD, e.g.,**

$$\langle x^n \rangle = \int_{-1}^{+1} x^n f(x)\, dx$$

★ **Reconstruction of the light-cone counterpart via OPE, but not realistic:**

operator mixing

gauge noise

Distribution Functions

★ **DFs parameterized in terms of off-forward matrix elements of non-local light-cone operators (Not accessible on Euclidean lattice)**

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2)|p\rangle$$

$q = p' - p$, $\bar P = (p'+p)/2$, n: light-cone vector ($\bar P.n = 1$), $\xi = -n\cdot\Delta/2$

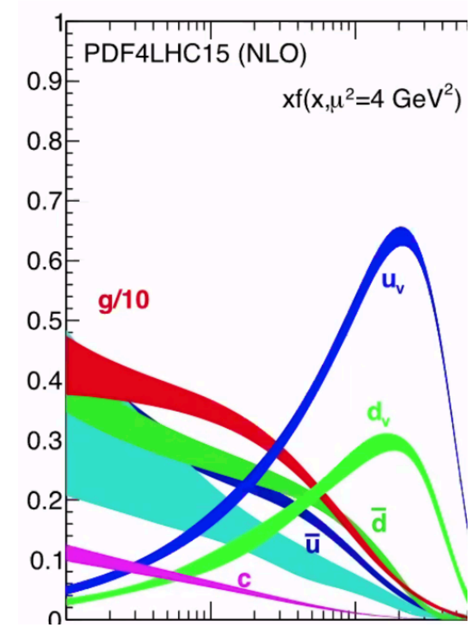
★ **Their Mellin moments with respect to x can be accessed in LQCD, e.g.,**

$$\langle x^n \rangle = \int_{-1}^{+1} x^n f(x)\, dx$$

★ **Reconstruction of the light-cone counterpart via OPE, but not realistic:**

operator mixing

gauge noise



Distribution Functions

★ **DFs parameterized in terms of off-forward matrix elements of non-local light-cone operators (Not accessible on Euclidean lattice)**

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2)|p\rangle$$

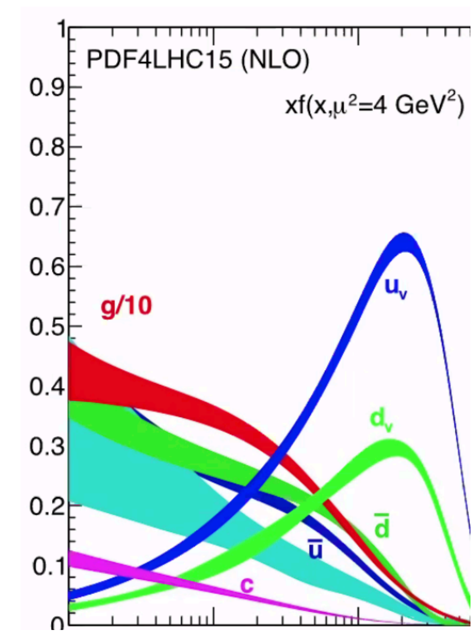
$q = p' - p$, $\bar P = (p'+p)/2$, n: light-cone vector ($\bar P.n = 1$), $\xi = -n\cdot\Delta/2$

★ **Their Mellin moments with respect to x can be accessed in LQCD, e.g.,**

$$\langle x^n \rangle = \int_{-1}^{+1} x^n f(x)\, dx$$

★ **Reconstruction of the light-cone counterpart via OPE, but not realistic:**

operator mixing
gauge noise



★ **Mellin moments have physical interpretation: spin, mass, …**

More on the PDF reconstruction

★ **Reconstruction of the light-cone PDFs** ***not realistic?***

- **increased statistical noise for high moments**
- **operator mixing**
- **need for boosted frame for $\langle x^2 \rangle$ and higher to avoid mixing**

More on the PDF reconstruction

★ **Reconstruction of the light-cone PDFs** *not realistic?*

- increased statistical noise for high moments
- operator mixing
- need for boosted frame for $\langle x^2 \rangle$ and higher to avoid mixing

★ **Indeed studies on "old" data have uncontrolled uncertainties (quenched, contain mixing, pert. renormalization …)**

★ **Early attempts for reconstruction inconclusive** [W. Detmold et al., EPJ direct 3 (2001) 1], [R. Holt et al., RMP 82, 2991 (2010)]

More on the PDF reconstruction

★ Reconstruction of the light-cone PDFs *not realistic?*

- increased statistical noise for high moments
- operator mixing
- need for boosted frame for $\langle x^2 \rangle$ and higher to avoid mixing

★ Indeed studies on "old" data have uncontrolled uncertainties (quenched, contain mixing, pert. renormalization …)

★ Early attempts for reconstruction inconclusive [W. Detmold et al., EPJ direct 3 (2001) 1], [R. Holt et al., RMP 82, 2991 (2010)]

★ No recent lattice QCD results for high moments using local operators

Reference	Method	Renorm.	mixing	m_π (MeV)	N_f	$\langle x^3 \rangle^u_\pi$ (2GeV)	initial scale
This work	local operator	non-perturb.	not present	260	2+1+1	0.024(18)	2 GeV
Ref. [5]	local operator	perturb.	present	chiral extrap.	0	0.051(21)	2.4 GeV
Ref. [41]	local operator	perturb.	present	chiral extrap.	0	0.046(16)	2.4 GeV
Ref. [7]	local operator	non-perturb.	present	chiral extrap.	2	0.074(10)	2 GeV

Reference	Method	Renorm.	mixing	m_π (MeV)	N_f	$\langle x^3 \rangle^u_K$ (2GeV)	$\langle x^3 \rangle^s_K$ (2GeV)	initial scale
This work	local operator	non-perturb.	not present	260	2+1+1	0.033(6)	0.073(5)	2 GeV

[5]. C. Best et al., PRD 56, 2743 (1997)

[41]. W. Detmold et al., PRD 68, 034025 (2003)

[7]. D. Brommel, Ph.D. thesis (2007)

Meson on the Lattice

★ Calculation of matrix elements with appropriate operators for the quantities under study (e.g., vector)

$$C^{2pt} = \langle M \,|\, M \rangle \qquad C_{\Gamma}^{3pt} = \langle M \,|\, \overline{q}\Gamma q \,|\, M \rangle$$

Meson on the Lattice

★ Calculation of matrix elements with appropriate operators for the quantities under study (e.g., vector)

$$C^{2pt} = \langle M \,|\, M \rangle \qquad C_\Gamma^{3pt} = \langle M \,|\, \overline{q}\Gamma q \,|\, M \rangle$$

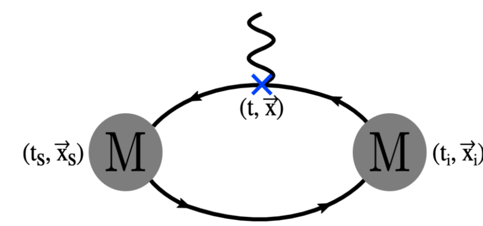
★ Construction of ratios and identification of ground state

forward limit: $$\frac{C_\Gamma^{3pt}(t,\tau)}{C^{2pt}(t)} \quad \xrightarrow{0 << \tau << t} \quad \Pi_\Gamma$$

Meson on the Lattice

★ Calculation of matrix elements with appropriate operators for the quantities under study (e.g., vector)

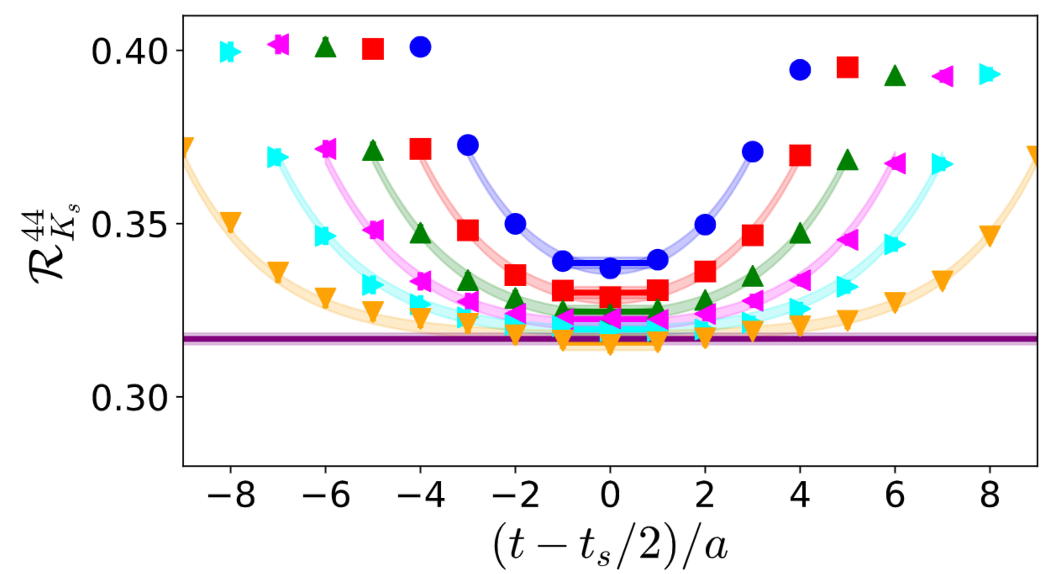
$$C^{2pt} = \langle M \,|\, M \rangle \qquad C_{\Gamma}^{3pt} = \langle M \,|\, \overline{q}\Gamma q \,|\, M \rangle$$



★ Construction of ratios and identification of ground state

forward limit:

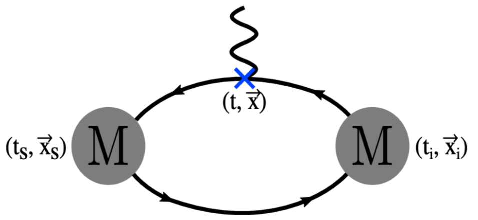
$$\frac{C_{\Gamma}^{3pt}(t,\tau)}{C^{2pt}(t)} \quad \xrightarrow{0 << \tau << t} \quad \Pi_{\Gamma}$$



Meson on the Lattice

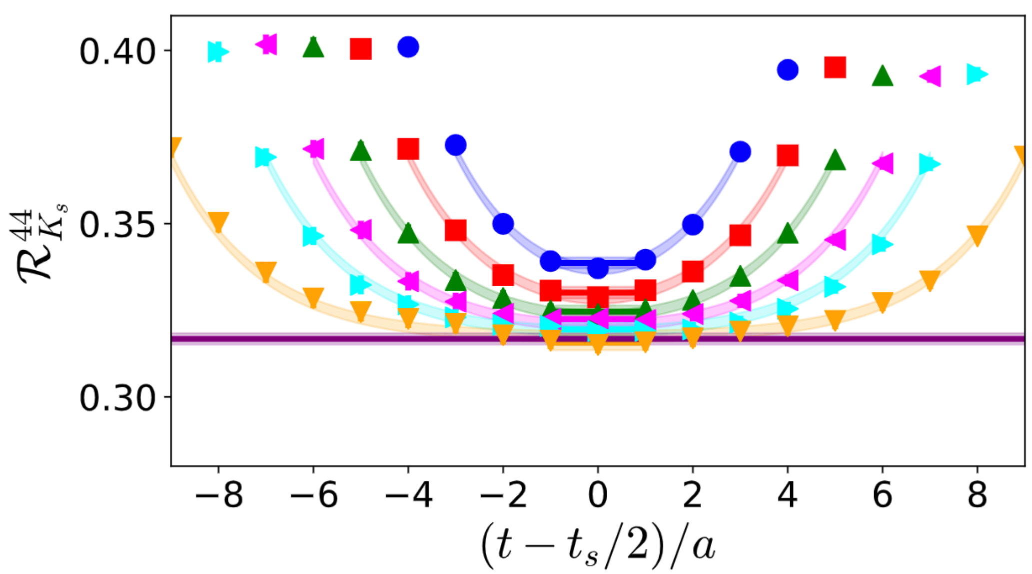
★ Calculation of matrix elements with appropriate operators for the quantities under study (e.g., vector)

$$C^{2pt} = \langle M | M \rangle \qquad C_\Gamma^{3pt} = \langle M | \bar{q}\Gamma q | M \rangle$$



★ Construction of ratios and identification of ground state

forward limit: $$\frac{C_\Gamma^{3pt}(t,\tau)}{C^{2pt}(t)} \xrightarrow{0 << \tau << t} \Pi_\Gamma$$



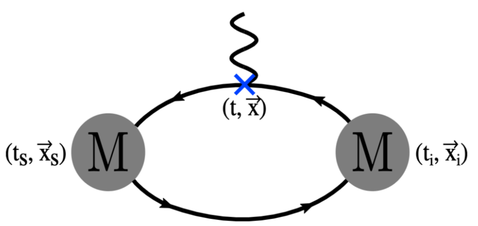
★ Renormalization

if multiplicative: $$\Pi_\Gamma^R = Z\Pi_\Gamma$$

Meson on the Lattice

★ Calculation of matrix elements with appropriate operators for the quantities under study (e.g., vector)

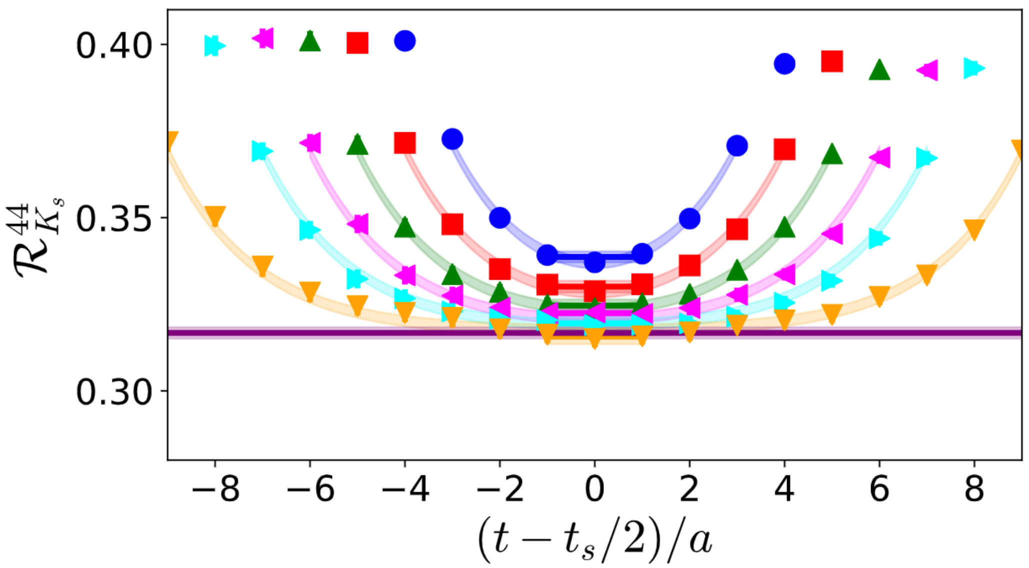
$$C^{2pt} = \langle M \,|\, M \rangle \qquad C^{3pt}_{\Gamma} = \langle M \,|\, \bar{q}\Gamma q \,|\, M \rangle$$



★ Construction of ratios and identification of ground state

forward limit:

$$\frac{C^{3pt}_{\Gamma}(t,\tau)}{C^{2pt}(t)} \quad \underset{\rightarrow}{0 << \tau << t} \quad \Pi_{\Gamma}$$



★ Renormalization

if multiplicative:

$$\Pi^{R}_{\Gamma} = Z\Pi_{\Gamma}$$

★ Kinematic factors based on symmetry properties

Decomposition of matrix elements

Euclidean space:

$$\langle M(p')\,|\,\overline{q}\gamma^{\{\mu}D^{\nu\}}q\,|\,M(p)\rangle = C\,\left[2P^{\{\mu}P^{\nu\}}A_{20} + 2\Delta^{\{\mu}\Delta^{\nu\}}B_{20}\right]$$

$$\langle M(p')\,|\,\overline{q}\gamma^{\{\mu}D^{\nu}D^{\rho\}}q\,|\,M(p)\rangle = C\,\left[2i\,P^{\{\mu}P^{\nu}P^{\rho\}}A_{30} + 2i\,\Delta^{\{\mu}\Delta^{\nu}P^{\rho\}}B_{30}\right]$$

$$\langle M(p')\,|\,\overline{q}\gamma^{\{\mu}D^{\nu}D^{\rho}D^{\sigma\}}q\,|\,M(p)\rangle = C\left[-2P^{\{\mu}P^{\nu}P^{\rho}P^{\sigma\}}A_{40} - 2\Delta^{\{\mu}\Delta^{\nu}P^{\rho}P^{\sigma\}}B_{40} - 2\Delta^{\{\mu}\Delta^{\nu}\Delta^{\rho}\Delta^{\sigma\}}C_{40}\right]$$

Decomposition of matrix elements

Euclidean space:

$$\langle M(p')\,|\,\bar{q}\gamma^{\{\mu}D^{\nu\}}q\,|\,M(p)\rangle = C\,\left[2P^{\{\mu}P^{\nu\}}A_{20} + 2\Delta^{\{\mu}\Delta^{\nu\}}B_{20}\right]$$

$$\langle M(p')\,|\,\bar{q}\gamma^{\{\mu}D^{\nu}D^{\rho\}}q\,|\,M(p)\rangle = C\,\left[2i\,P^{\{\mu}P^{\nu}P^{\rho\}}A_{30} + 2i\,\Delta^{\{\mu}\Delta^{\nu}P^{\rho\}}B_{30}\right]$$

$$\langle M(p')\,|\,\bar{q}\gamma^{\{\mu}D^{\nu}D^{\rho}D^{\sigma\}}q\,|\,M(p)\rangle = C\left[-2P^{\{\mu}P^{\nu}P^{\rho}P^{\sigma\}}A_{40} - 2\Delta^{\{\mu}\Delta^{\nu}P^{\rho}P^{\sigma\}}B_{40} - 2\Delta^{\{\mu}\Delta^{\nu}\Delta^{\rho}\Delta^{\sigma\}}C_{40}\right]$$

Forward limit (avoiding mixing)

$$\langle M(p)\,|\,\bar{q}\gamma^{\{0}D^{0\}}q\,|\,M(p)\rangle = \frac{1}{4E_M(p)}\left(m_M^2 - 4E_M^2(p)\right)\langle x\rangle_M^q$$

$$\langle M(p)\,|\,\bar{q}\gamma^{\mu}D^{\nu}D^{4}q\,|\,M(p)\rangle = -\,p_\mu\,p_\nu\,\langle x^2\rangle_M^q$$

$$\langle M(p)\,|\,\bar{q}\gamma^{\mu}D^{\nu}D^{\rho}D^{4}q\,|\,M(p)\rangle = -\,i\,p^\mu\,p^\nu\,p^\rho\,\langle x^3\rangle_M^q$$

$\mu \neq \nu \neq \rho \neq \mu$

$\mu, \nu, \rho : 1, 2, 3$

Decomposition of matrix elements

Euclidean space:

$$\langle M(p')|\overline{q}\gamma^{\{\mu}D^{\nu\}}q|M(p)\rangle = C\left[2P^{\{\mu}P^{\nu\}}A_{20} + 2\Delta^{\{\mu}\Delta^{\nu\}}B_{20}\right]$$

$$\langle M(p')|\overline{q}\gamma^{\{\mu}D^{\nu}D^{\rho\}}q|M(p)\rangle = C\left[2i\,P^{\{\mu}P^{\nu}P^{\rho\}}A_{30} + 2i\,\Delta^{\{\mu}\Delta^{\nu}P^{\rho\}}B_{30}\right]$$

$$\langle M(p')|\overline{q}\gamma^{\{\mu}D^{\nu}D^{\rho}D^{\sigma\}}q|M(p)\rangle = C\left[-2P^{\{\mu}P^{\nu}P^{\rho}P^{\sigma\}}A_{40} - 2\Delta^{\{\mu}\Delta^{\nu}P^{\rho}P^{\sigma\}}B_{40} - 2\Delta^{\{\mu}\Delta^{\nu}\Delta^{\rho}\Delta^{\sigma\}}C_{40}\right]$$

Forward limit (avoiding mixing)

$$\langle M(p)|\overline{q}\gamma^{\{0}D^{0\}}q|M(p)\rangle = \frac{1}{4E_M(p)}\left(m_M^2 - 4E_M^2(p)\right)\langle x\rangle_M^q$$

Boosted frame →

$$\langle M(p)|\overline{q}\gamma^{\mu}D^{\nu}D^{4}q|M(p)\rangle = -\,p_\mu\,p_\nu\langle x^2\rangle_M^q$$

$$\langle M(p)|\overline{q}\gamma^{\mu}D^{\nu}D^{\rho}D^{4}q|M(p)\rangle = -\,i\,p^\mu\,p^\nu\,p^\rho\langle x^3\rangle_M^q$$

$\mu \neq \nu \neq \rho \neq \mu$

$\mu, \nu, \rho : 1, 2, 3$

Decomposition of matrix elements

Euclidean space:

$$\langle M(p')|\bar{q}\gamma^{\{\mu}D^{\nu\}}q|M(p)\rangle = C\left[2P^{\{\mu}P^{\nu\}}A_{20} + 2\Delta^{\{\mu}\Delta^{\nu\}}B_{20}\right]$$

$$\langle M(p')|\bar{q}\gamma^{\{\mu}D^{\nu}D^{\rho\}}q|M(p)\rangle = C\left[2i\,P^{\{\mu}P^{\nu}P^{\rho\}}A_{30} + 2i\,\Delta^{\{\mu}\Delta^{\nu}P^{\rho\}}B_{30}\right]$$

$$\langle M(p')|\bar{q}\gamma^{\{\mu}D^{\nu}D^{\rho}D^{\sigma\}}q|M(p)\rangle = C\left[-2P^{\{\mu}P^{\nu}P^{\rho}P^{\sigma\}}A_{40} - 2\Delta^{\{\mu}\Delta^{\nu}P^{\rho}P^{\sigma\}}B_{40} - 2\Delta^{\{\mu}\Delta^{\nu}\Delta^{\rho}\Delta^{\sigma\}}C_{40}\right]$$

Forward limit (avoiding mixing)

$$\langle M(p)|\bar{q}\gamma^{\{0}D^{0\}}q|M(p)\rangle = \frac{1}{4E_M(p)}\left(m_M^2 - 4E_M^2(p)\right)\langle x\rangle_M^q$$

Boosted frame →

$$\langle M(p)|\bar{q}\gamma^{\mu}D^{\nu}D^{4}q|M(p)\rangle = -\,p_\mu\,p_\nu\langle x^2\rangle_M^q$$

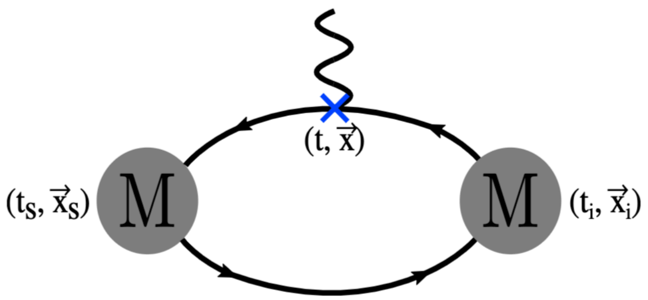
$$\langle M(p)|\bar{q}\gamma^{\mu}D^{\nu}D^{\rho}D^{4}q|M(p)\rangle = -\,i\,p^\mu\,p^\nu\,p^\rho\langle x^3\rangle_M^q$$

$\mu \neq \nu \neq \rho \neq \mu$

$\mu, \nu, \rho : 1, 2, 3$

★ Avoiding mixing increases the computational cost!

Technical Aspects



★ Nf=2+1+1 twisted mass fermions & clover term

★ Ensemble parameters:

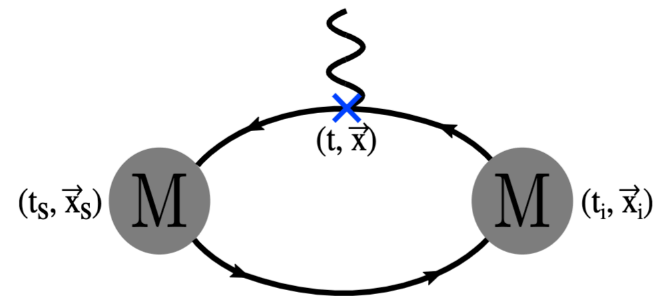
Pion mass:	260 MeV
Kaon mass:	530 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm

★ Kinematical setup:

$\vec{p}$	$T_{\rm sink}/a$	$N_{\rm confs}$	$N_{\rm src}$	Total statistics
(0,0,0)	12, 14, 16, 18, 20, 24	122	16	1,952
$(\pm1,\pm1,\pm1)$	12	122	16	15,616
$(\pm1,\pm1,\pm1)$	14, 16, 18	122	72	70,272

★ Excited states: single-state & two-state fits

Technical Aspects



★ Nf=2+1+1 twisted mass fermions & clover term

★ Ensemble parameters:

Pion mass:	260 MeV
Kaon mass:	530 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm

★ Kinematical setup:

$\vec{p}$	$T_{\rm sink}/a$	$N_{\rm confs}$	$N_{\rm src}$	Total statistics
(0,0,0)	12, 14, 16, 18, 20, 24	122	16	1,952
$(\pm1,\pm1,\pm1)$	12	122	16	15,616
$(\pm1,\pm1,\pm1)$	14, 16, 18	122	72	70,272

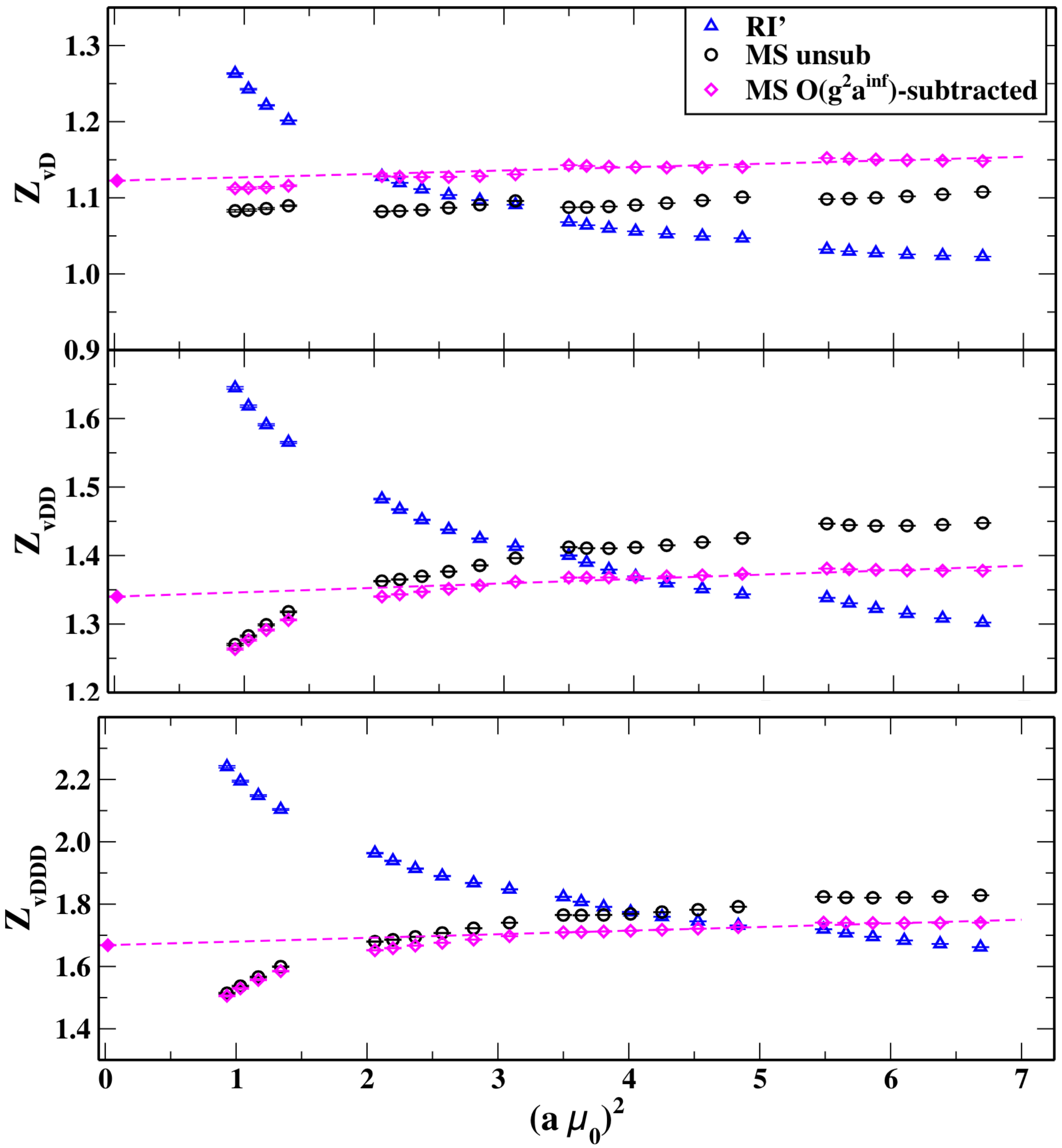
Rest frame: signal constant with Tsink increase

[Lepage, "*The Analysis of Algorithms for Lattice Field Theory*" (1989)]

Boosted frame: signal decays with Tsink increase

★ Excited states: single-state & two-state fits

Non-perturbative Renormalization



$$Z_{\rm vD}^{\overline{\rm MS}}(2\,{\rm GeV}) = 1.123(1)(5)$$
$$Z_{\rm vDD}^{\overline{\rm MS}}(2\,{\rm GeV}) = 1.340(1)(15)$$
$$Z_{\rm vDDD}^{\overline{\rm MS}}(2\,{\rm GeV}) = 1.668(1)(26)$$

★ RI' scheme (democratic momenta)

$$Z_q^{-1}\, Z_{\mathcal{O}}\, \frac{1}{12}{\rm Tr}\left[\Gamma^L_{\mathcal{O}}(p)\, \left(\Gamma^{\rm Born}_{\mathcal{O}}(p)\right)^{-1}\right]\Big|_{p^2=\mu_0^2} = 1$$

$$Z_q = \frac{1}{12}{\rm Tr}\left[(S^L(p))^{-1}\, S^{\rm Born}(p)\right]\Big|_{p^2=\mu_0^2}$$

$$(ap) \equiv 2\pi\left(\frac{n_t}{L_t} + \frac{1}{2\,L_t}, \frac{n_x}{L_s}, \frac{n_x}{L_s}, \frac{n_x}{L_s}\right) \qquad \sum_i p_i^4/(\sum_i p_i^2)^2 < 0.3$$

[M. Constantinou et al., JHEP 08, 068 (2010), arXiv:1004.1115]

★ Chiral extrapolation (negligible)

$\beta = 1.726,\quad a = 0.093$ fm		
$a\mu$	am_{PS}	lattice size
0.0060	0.1680	$24^3 \times 48$
0.0080	0.1916	$24^3 \times 48$
0.0100	0.2129	$24^3 \times 48$
0.0115	0.2293	$24^3 \times 48$
0.0130	0.2432	$24^3 \times 48$

★ Subtraction of $\mathcal{O}(g^2\, a^\infty)$

[M. Constantinou et al., PRD 91, 014502 (2015), arXiv:1408.6047]

★ Conversion & evolution to $\overline{\rm MS}(2\,{\rm GeV})$

$$Z_{\mathcal{O}}^{\overline{\rm MS}}(a\mu_0) = Z_{\mathcal{O}}^{\overline{\rm MS}}(2\,{\rm GeV}) + Z_{\mathcal{O}}^{(1)} \cdot (a\,\mu_0)^2$$

Recapitulation

★ Matrix elements of pion and kaon coupled with local operators

★ Isolation of ground state

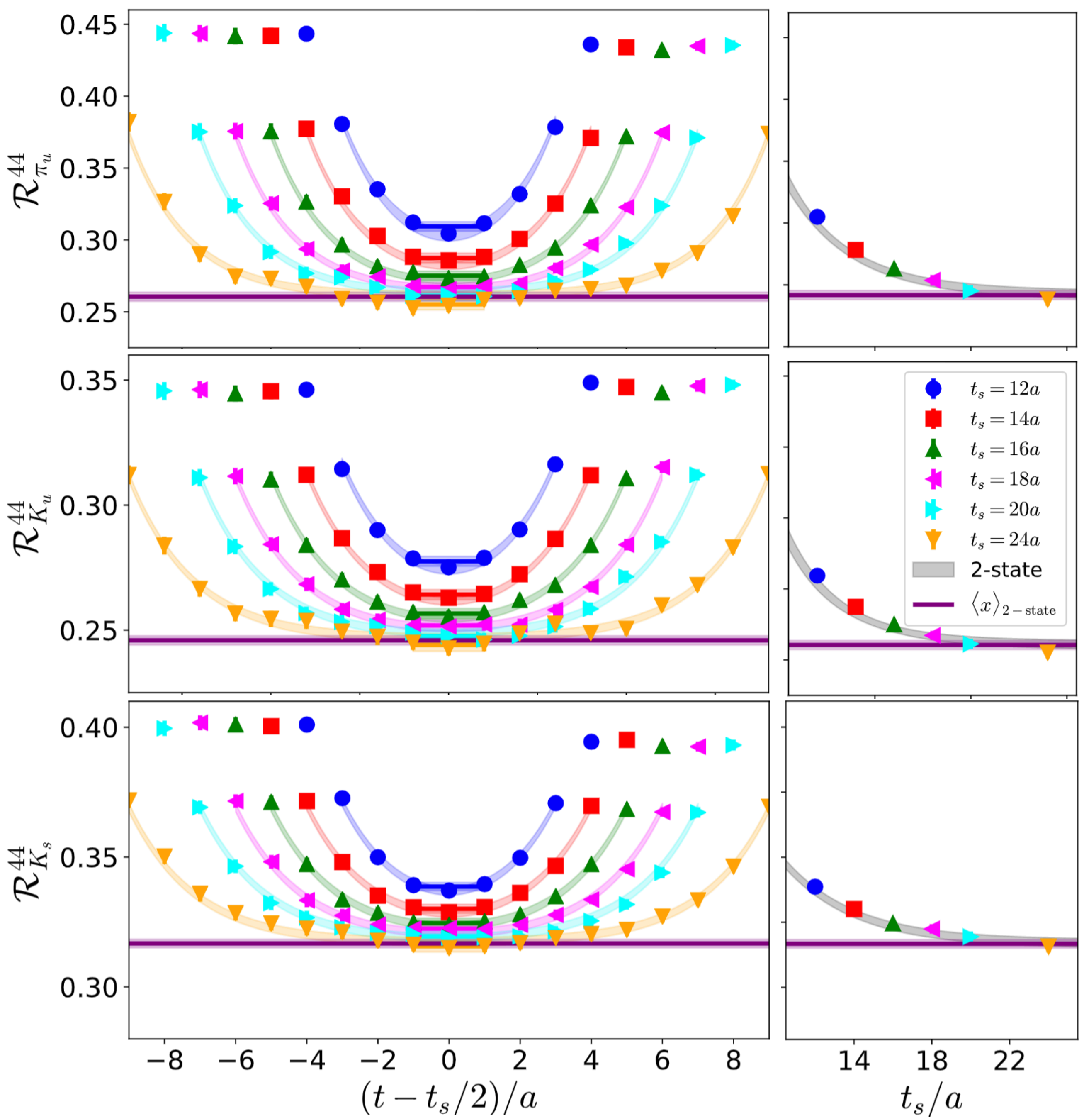
★ Renormalization

★ Extraction of Mellin moments

Mellin Moments

Excited-states contamination

Rest frame



★ **Signal does not decay with Tsink increase in rest frame**

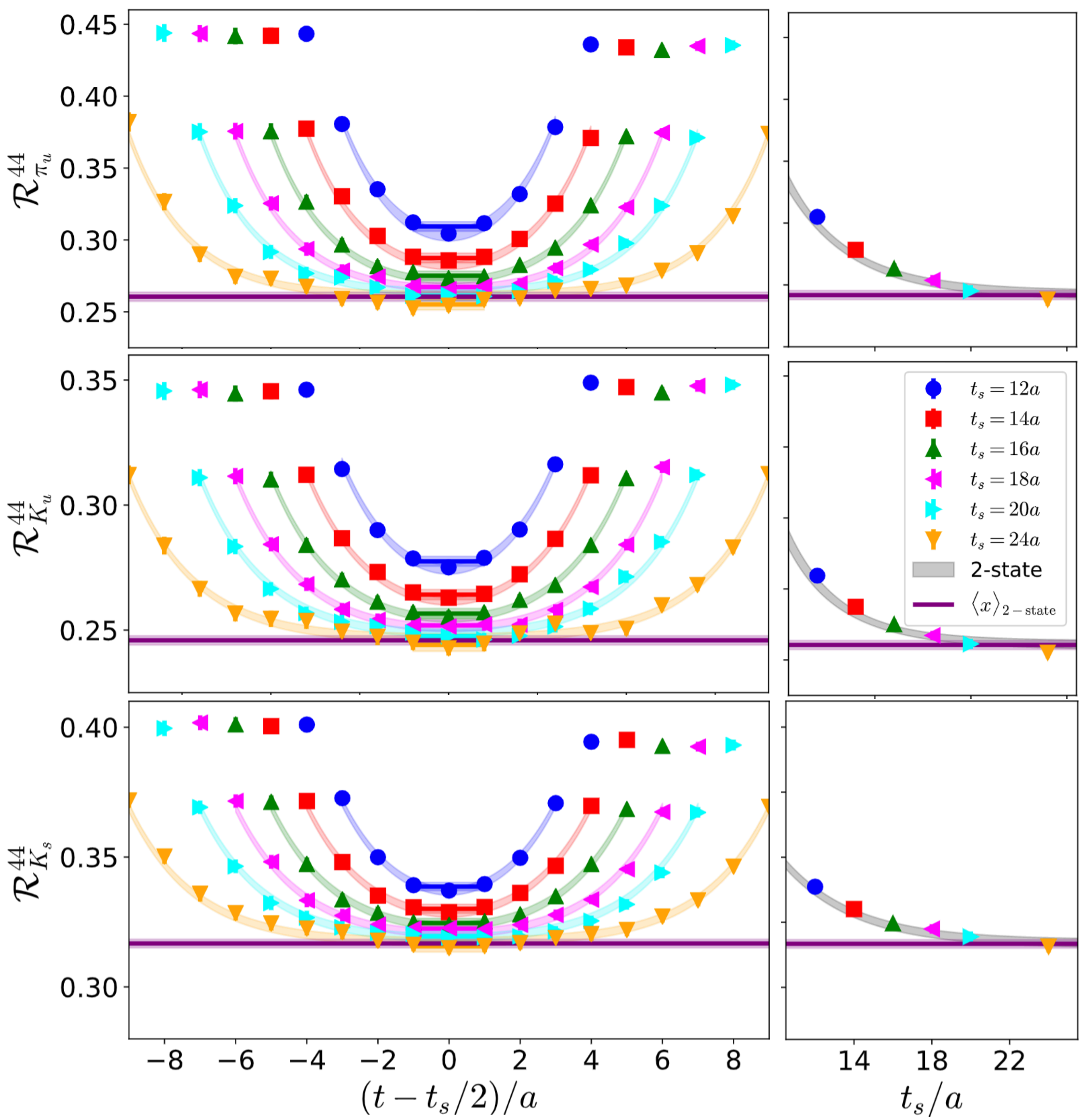
[G. P. Lepage, "*The Analysis of Algorithms for Lattice Field Theory*" (1989)]

★ **Excited-states contamination sizable in $\langle x \rangle$**

★ **Convergence found for Tsink > 1.65 fm**

Excited-states contamination

Rest frame



- ★ **Signal does not decay with Tsink increase in rest frame**

  [G. P. Lepage, "*The Analysis of Algorithms for Lattice Field Theory*" (1989)]

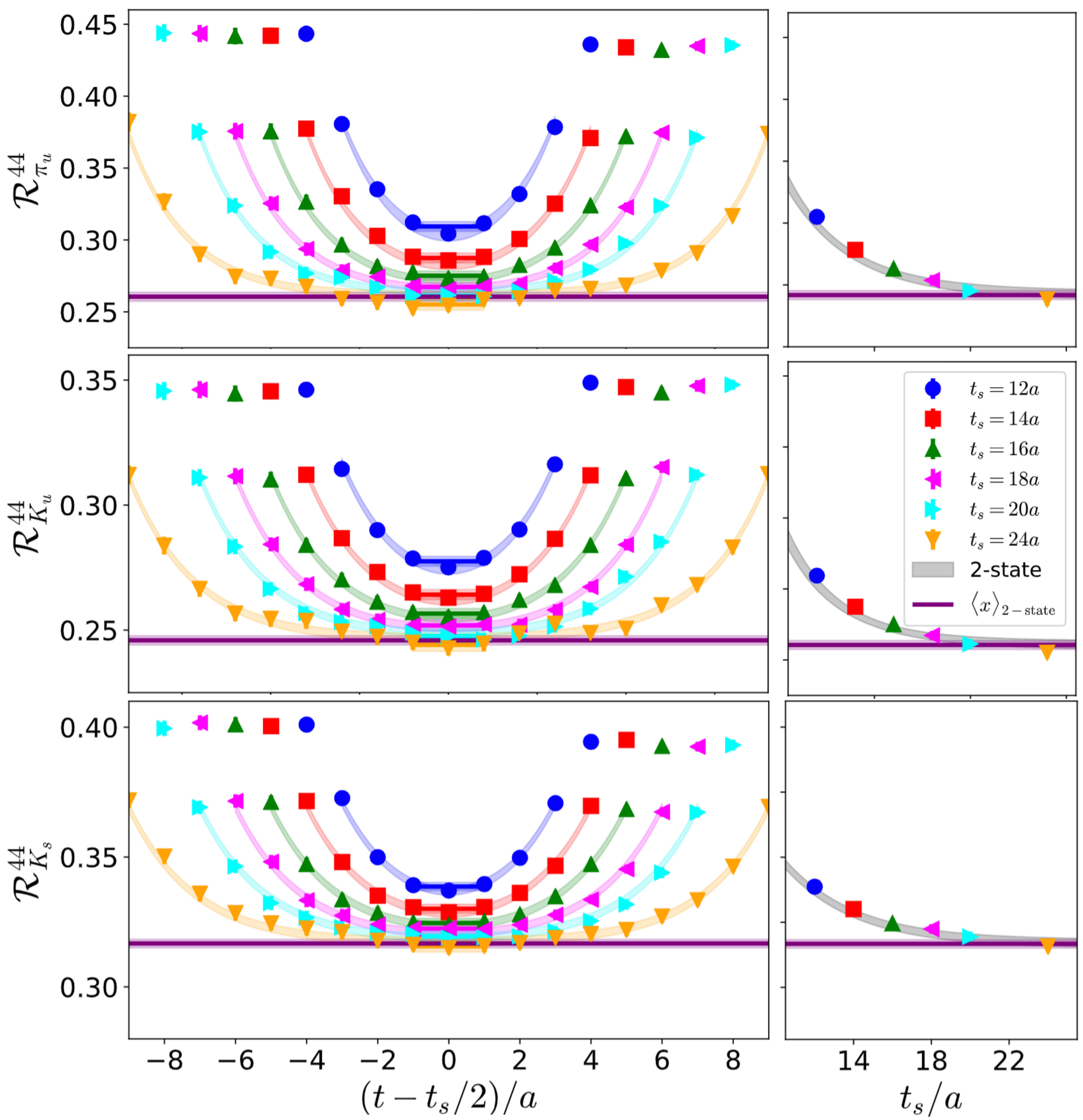
- ★ **Excited-states contamination sizable in $\langle x \rangle$**

- ★ **Convergence found for Tsink > 1.65 fm**

t_s/a	$\langle x \rangle^{\pi}_{u^+}$	$\langle x \rangle^{k}_{u}$	$\langle x \rangle^{k}_{s}$
12	0.309(3)	0.278(2)	0.339(2)
14	0.287(3)	0.264(2)	0.330(2)
16	0.275(3)	0.257(2)	0.325(2)
18	0.267(3)	0.252(2)	0.322(2)
20	0.261(4)	0.248(2)	0.319(2)
24	0.255(4)	0.244(3)	0.316(2)
2-state (a)	0.261(3)	0.246(2)	0.317(2)
2-state (b)	0.262(4)	0.246(2)	0.317(2)

Excited-states contamination

Rest frame



- ★ **Signal does not decay with Tsink increase in rest frame**

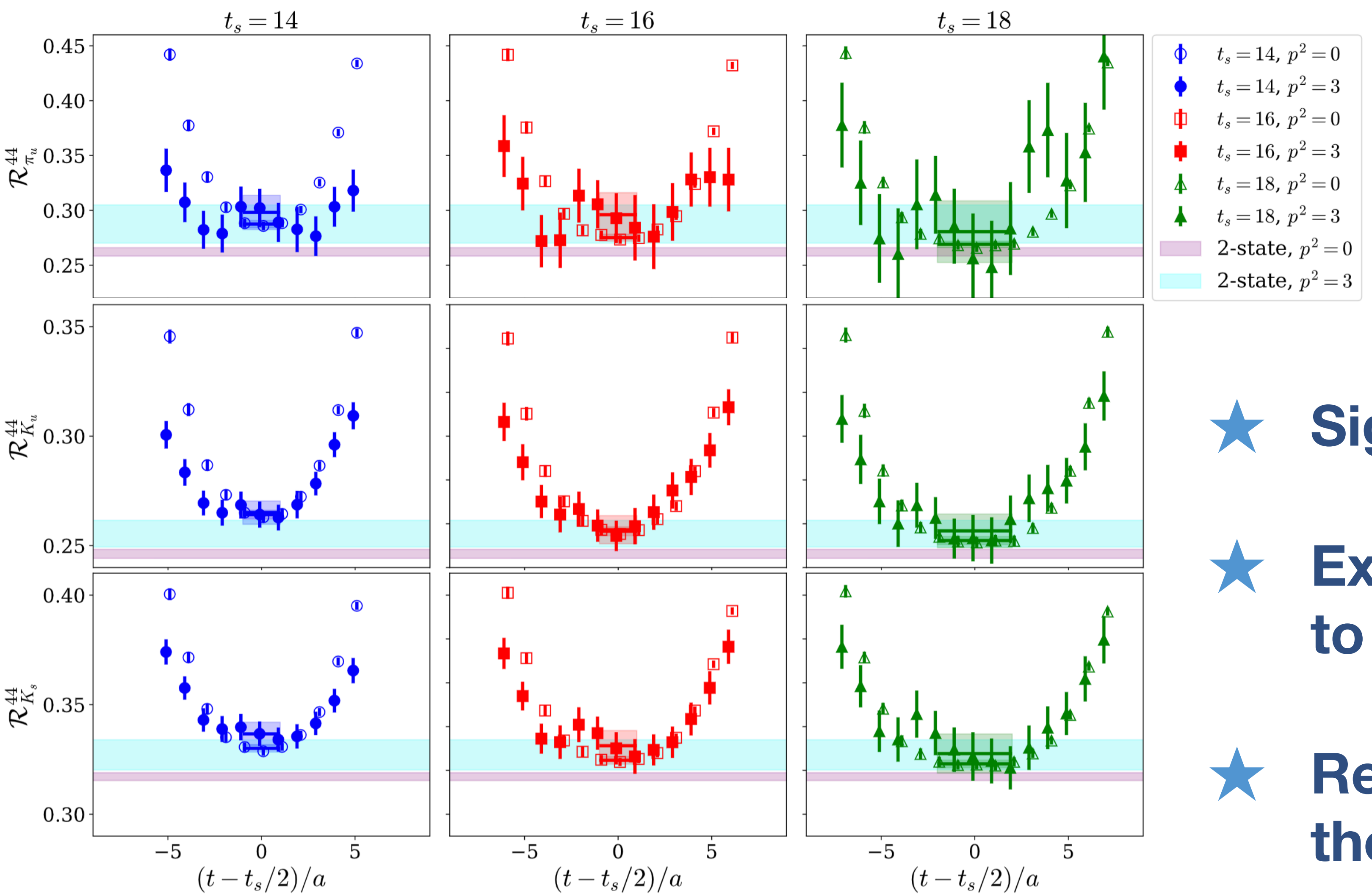
  [G. P. Lepage, "*The Analysis of Algorithms for Lattice Field Theory*" (1989)]

- ★ **Excited-states contamination sizable in $\langle x \rangle$**

- ★ **Convergence found for Tsink > 1.65 fm**

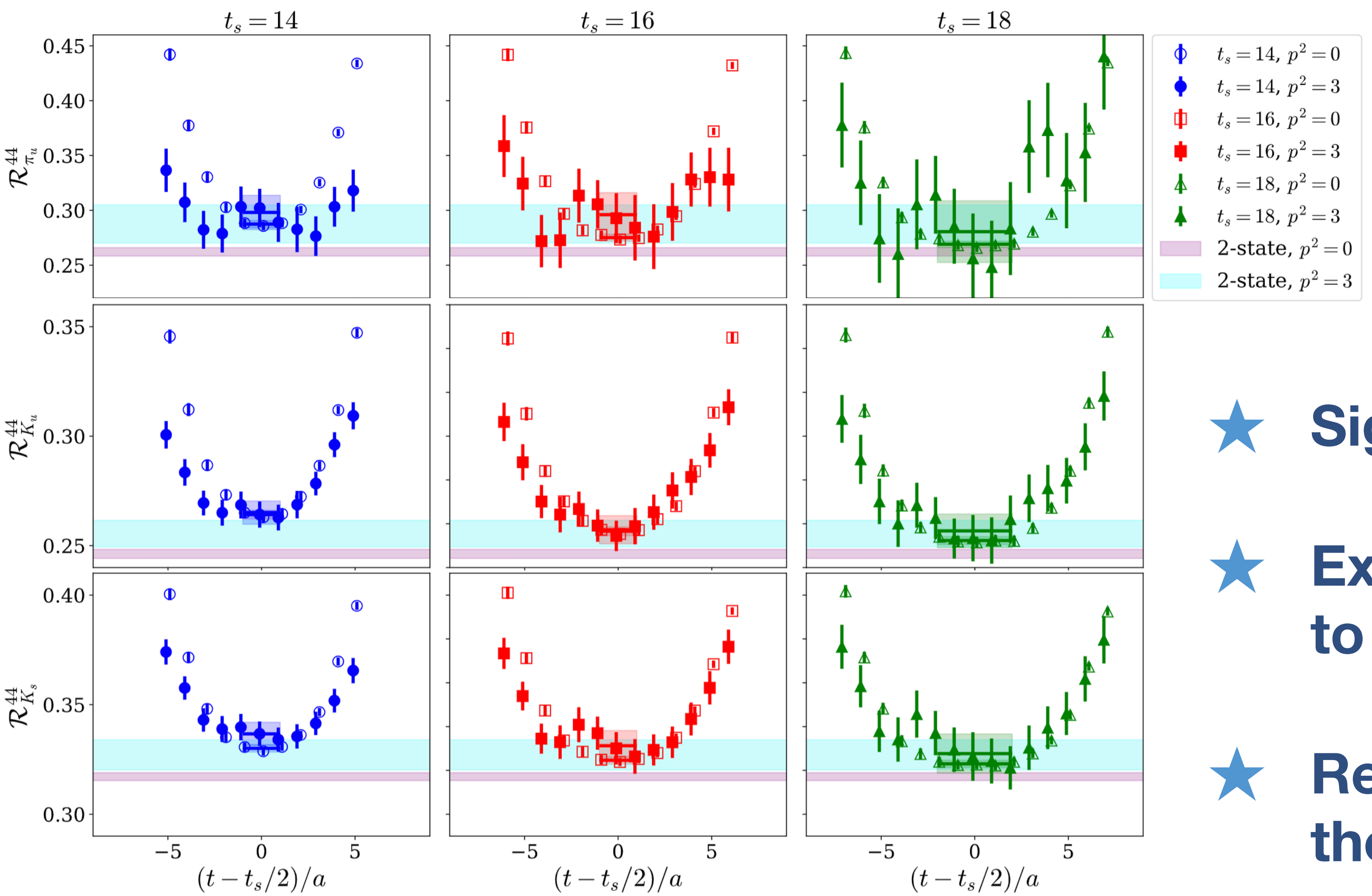
t_s/a	$\langle x \rangle^{\pi}_{u^+}$	$\langle x \rangle^{k}_{u}$	$\langle x \rangle^{k}_{s}$
12	0.309(3)	0.278(2)	0.339(2)
14	0.287(3)	0.264(2)	0.330(2)
16	0.275(3)	0.257(2)	0.325(2)
18	0.267(3)	0.252(2)	0.322(2)
20	0.261(4)	0.248(2)	0.319(2)
24	0.255(4)	0.244(3)	0.316(2)
2-state (a)	0.261(3)	0.246(2)	0.317(2)
2-state (b)	0.262(4)	0.246(2)	0.317(2)

Rest frame vs boosted frame



- ★ **Signal decays with Tsink increase**
- ★ **Excited-states effects comparable to statistical uncertainties**
- ★ **Results compatible between the two frames**

Rest frame vs boosted frame



★ **Signal decays with Tsink increase**

★ **Excited-states effects comparable to statistical uncertainties**

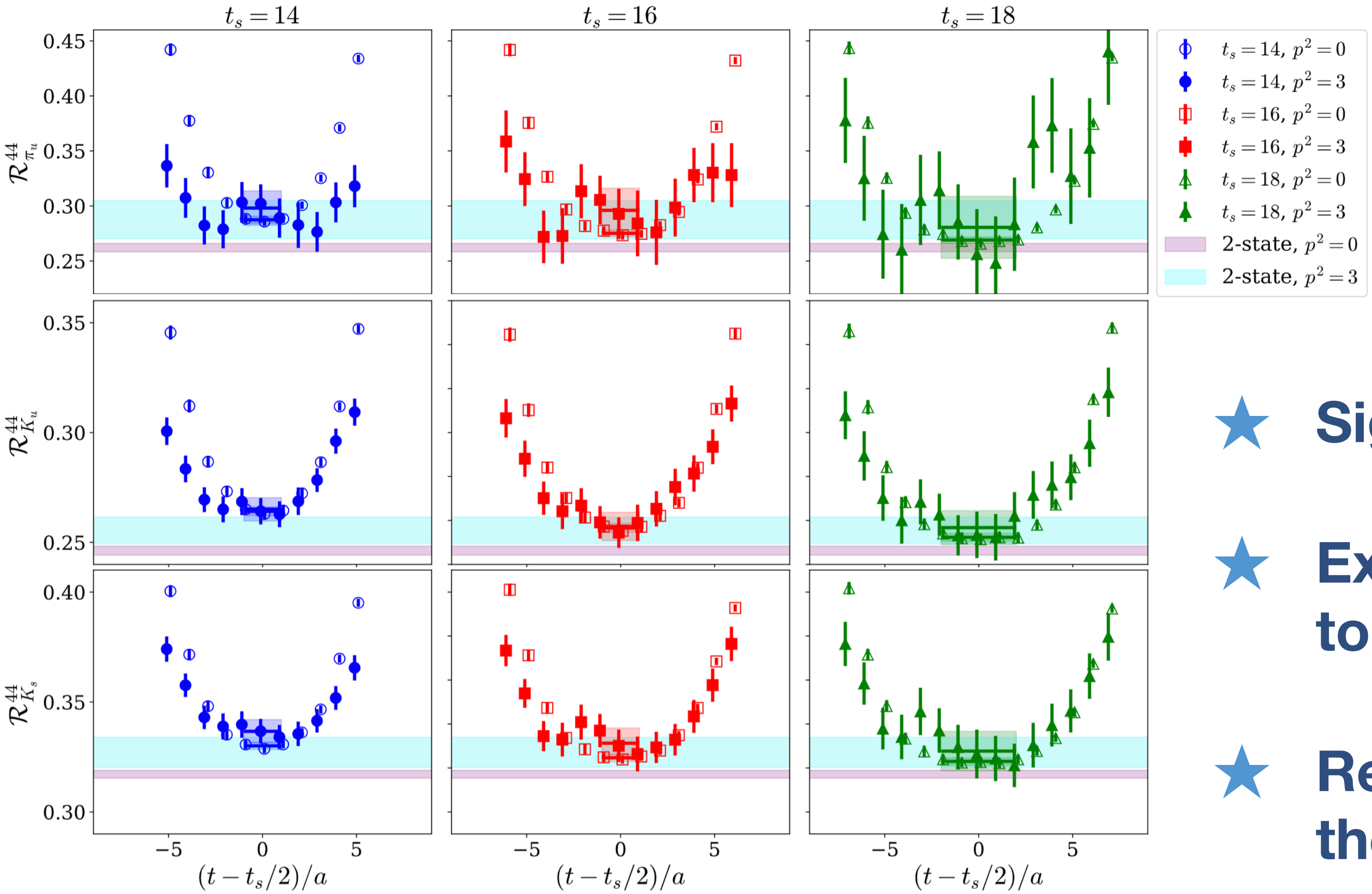
★ **Results compatible between the two frames**

Conclusions:

★ **Tsink between** $1.3 - 1.7\,fm$ **sufficient to capture excited-states effects**

★ **Momentum boost** $\overrightarrow{p} = 2\pi/L\,(\pm 1, \pm 1, \pm 1)$ **gives reasonable signal**

Rest frame vs boosted frame



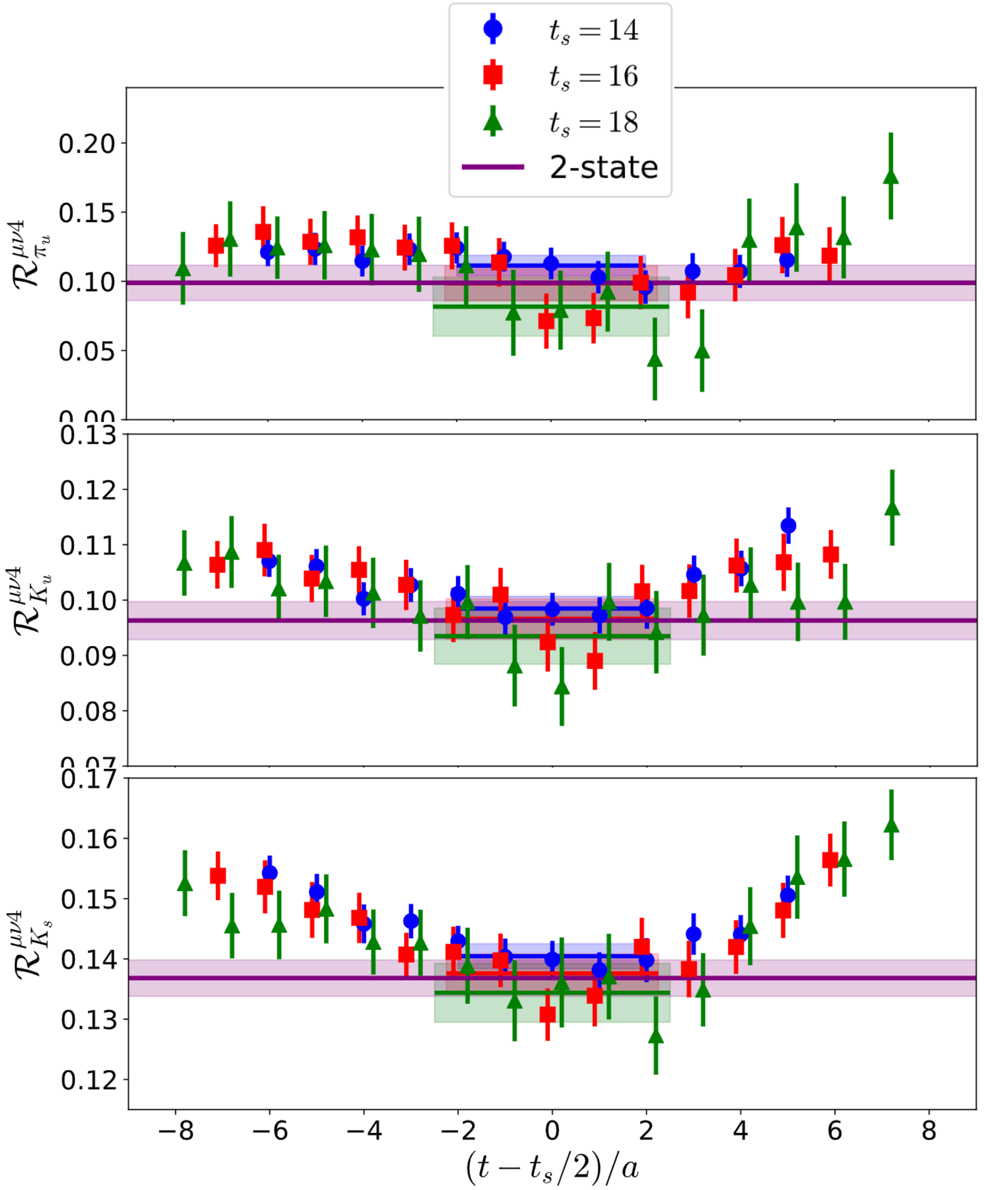
- ★ **Signal decays with Tsink increase**
- ★ **Excited-states effects comparable to statistical uncertainties**
- ★ **Results compatible between the two frames**

Conclusions:

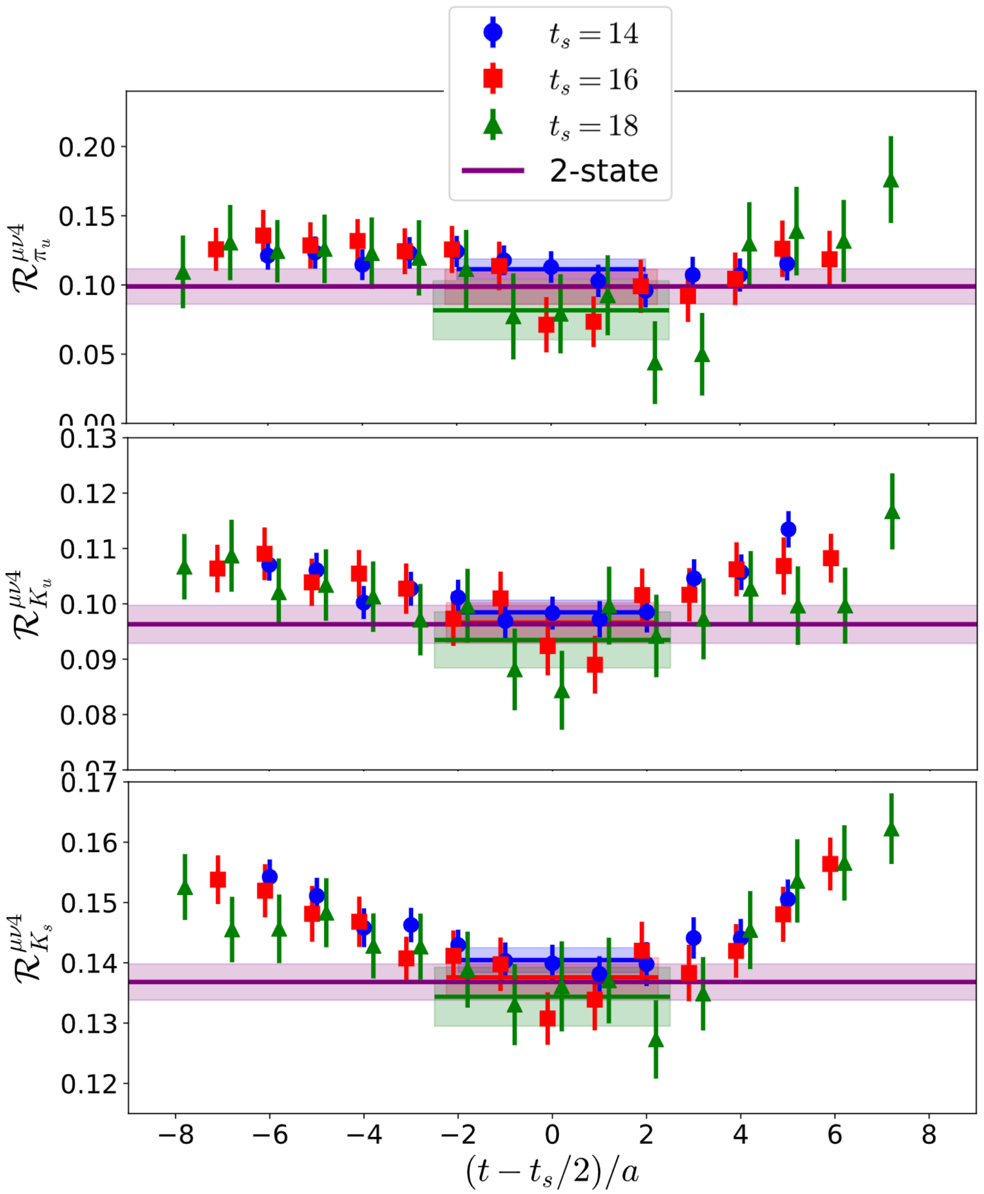
- ★ **Tsink between $1.3 - 1.7\, fm$ sufficient to capture excited-states effects**
- ★ **Momentum boost $\overrightarrow{p} = 2\pi/L\,(\pm 1, \pm 1, \pm 1)$ gives reasonable signal**

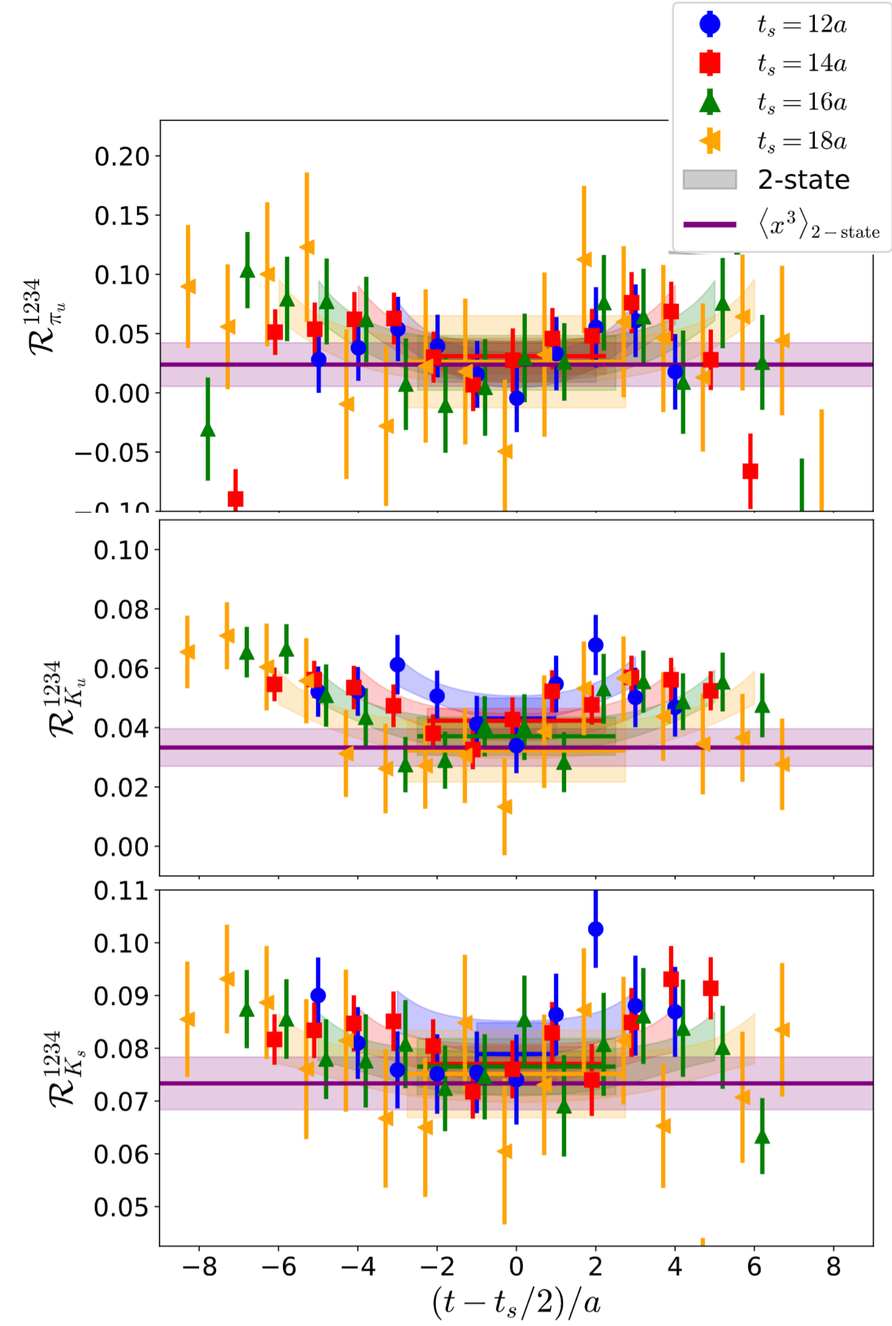
Calculations of $\langle x^2 \rangle$ and $\langle x^3 \rangle$ can be combined without increase in computational cost

Higher moments

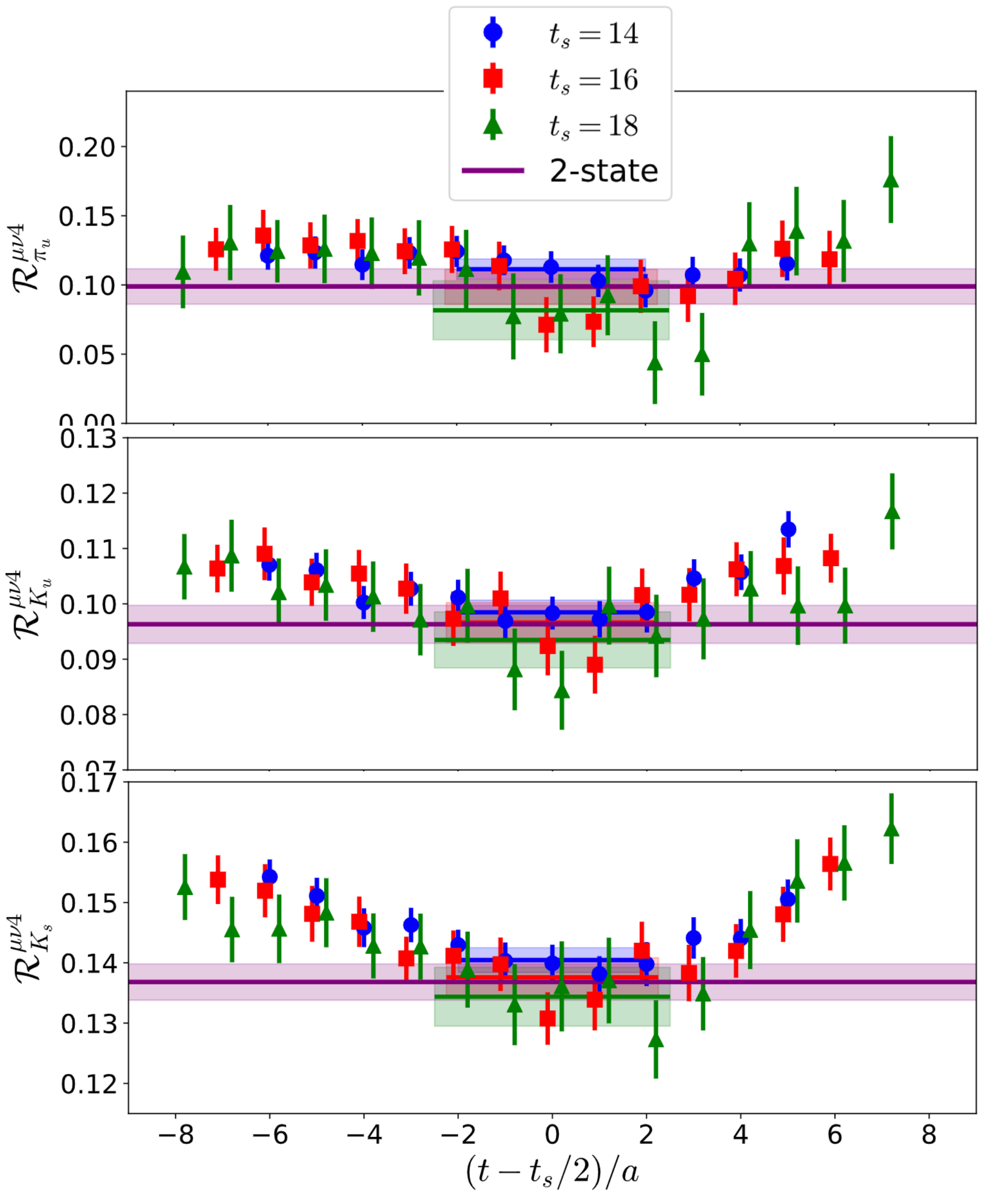


Higher moments





Higher moments

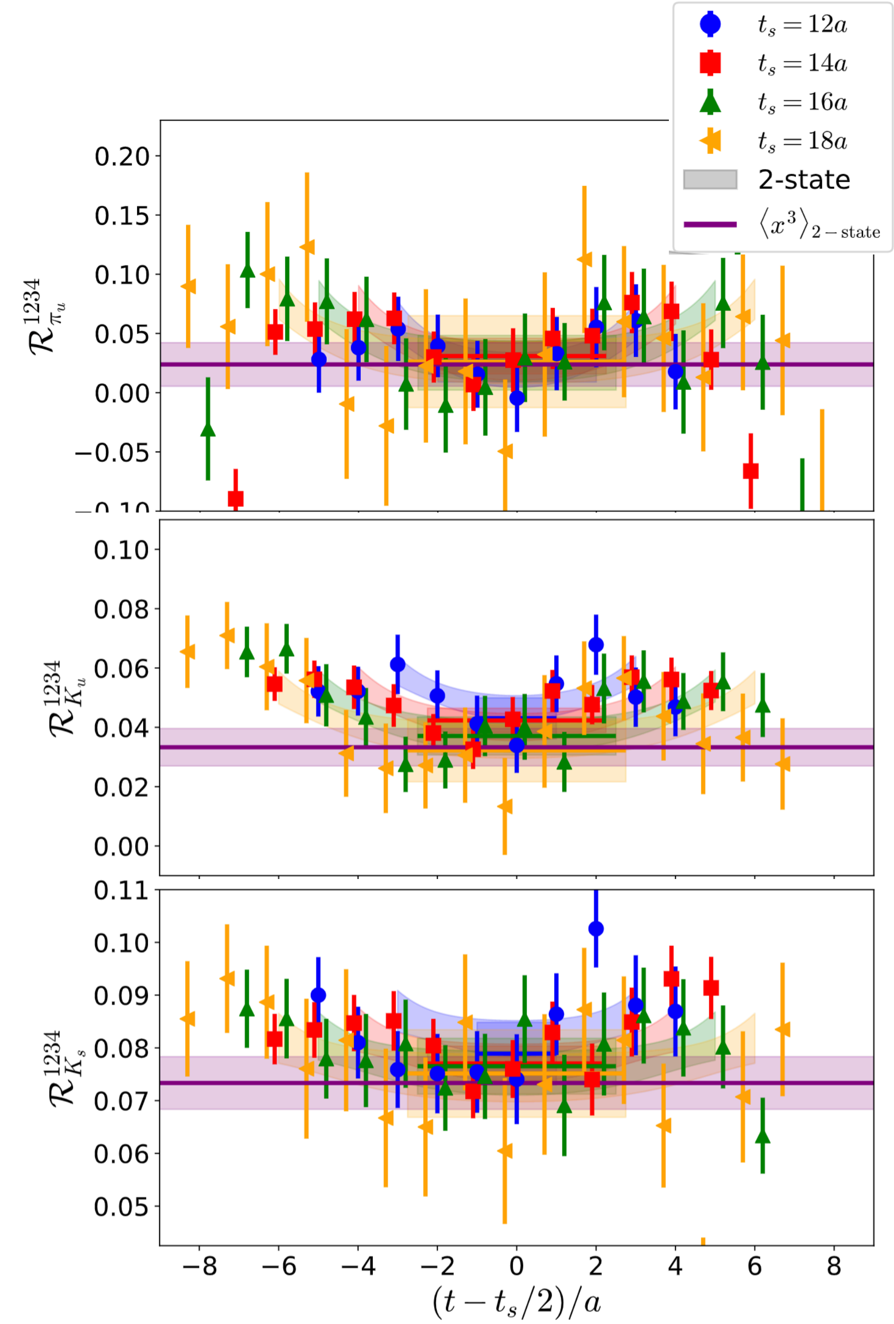


Pion

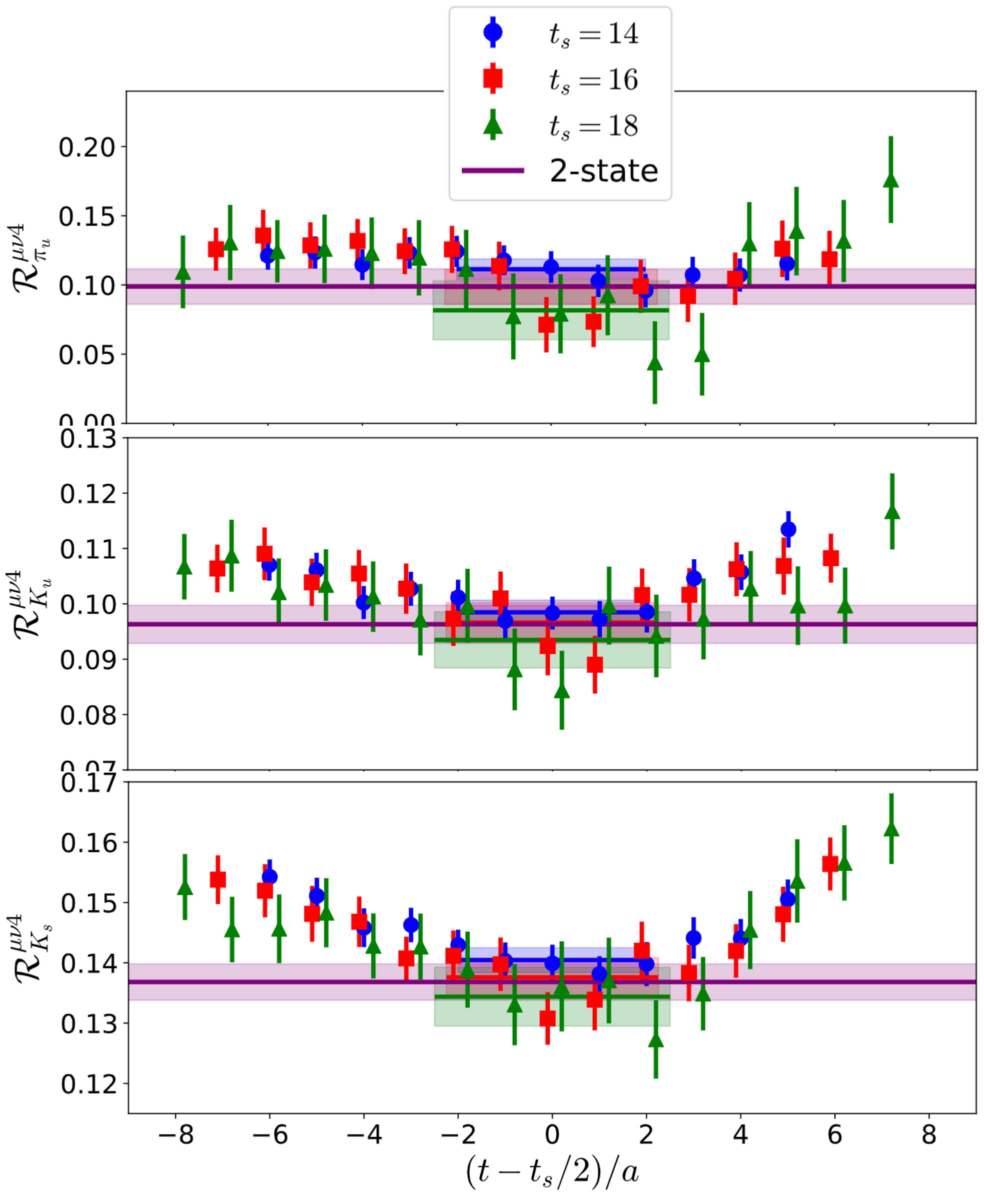
t_s/a	$\langle x^2\rangle^u_\pi$	$\langle x^3\rangle^u_\pi$
12	0.110(6)	0.026(17)
14	0.114(5)	0.031(15)
16	0.105(9)	0.025(23)
18	0.099(15)	0.026(39)
2-state	0.110(7)	0.024(18)

Kaon

t_s/a	$\langle x^2\rangle^u_K$	$\langle x^2\rangle^s_K$	$\langle x^3\rangle^u_K$	$\langle x^3\rangle^s_K$
12	0.101(2)	0.146(2)	0.043(7)	00.079(6)
14	0.099(2)	0.142(2)	0.042(4)	0.077(3)
16	0.096(2)	0.139(2)	0.037(6)	0.077(5)
18	0.095(3)	0.138(3)	0.032(11)	0.075(8)
2-state	0.096(2)	0.139(2)	0.033(6)	0.073(5)



Higher moments

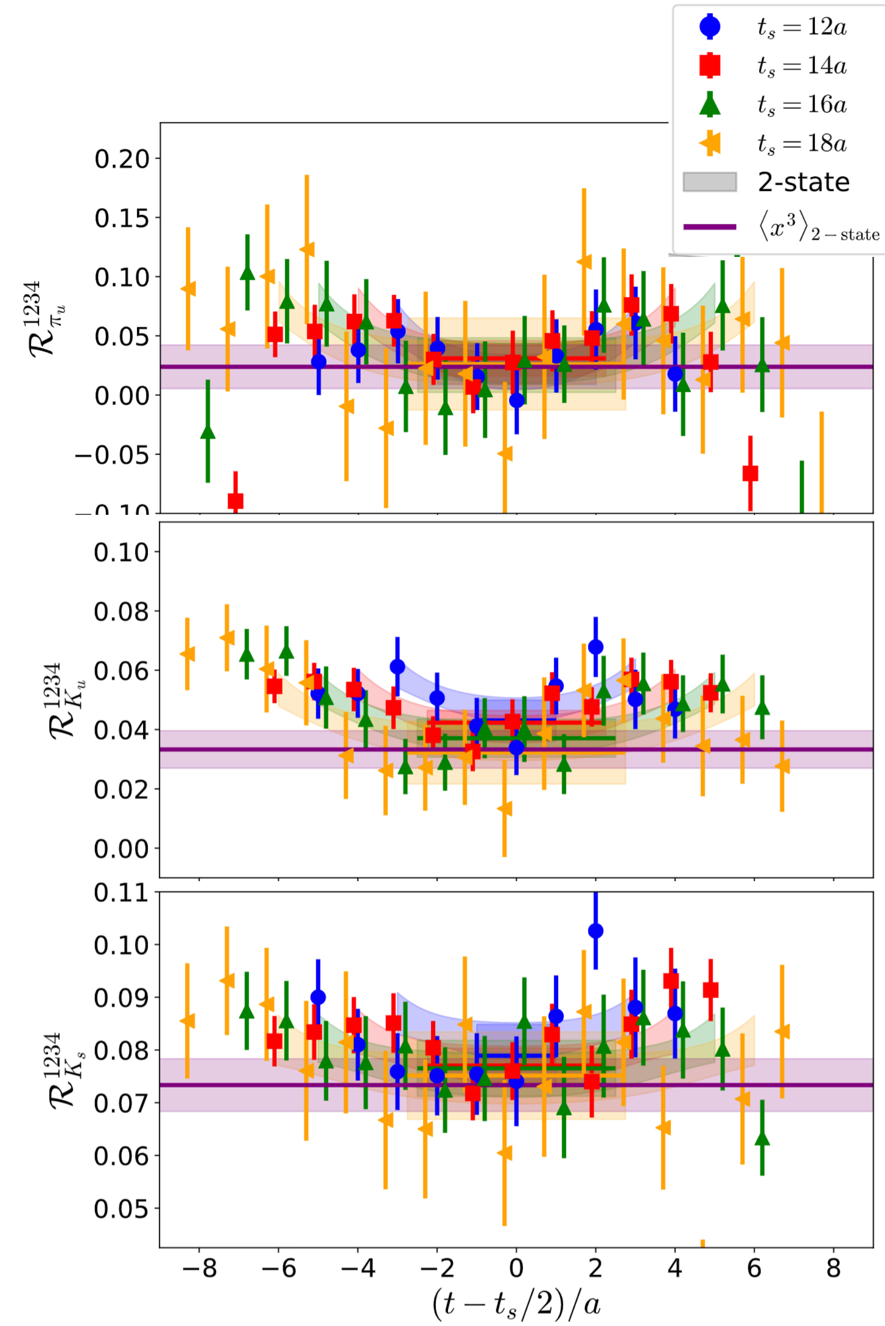


Pion

t_s/a	$\langle x^2\rangle^u_\pi$	$\langle x^3\rangle^u_\pi$
12	0.110(6)	0.026(17)
14	0.114(5)	0.031(15)
16	0.105(9)	0.025(23)
18	0.099(15)	0.026(39)
2-state	0.110(7)	0.024(18)

Kaon

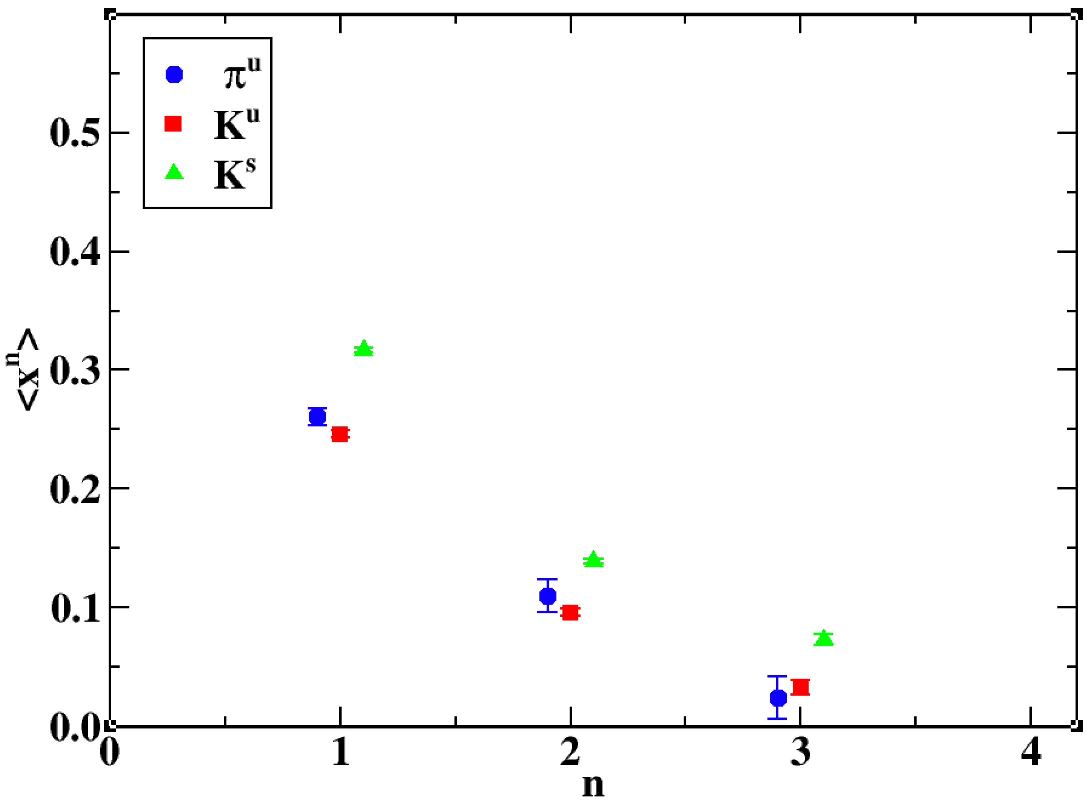
t_s/a	$\langle x^2\rangle^u_K$	$\langle x^2\rangle^s_K$	$\langle x^3\rangle^u_K$	$\langle x^3\rangle^s_K$
12	0.101(2)	0.146(2)	0.043(7)	00.079(6)
14	0.099(2)	0.142(2)	0.042(4)	0.077(3)
16	0.096(2)	0.139(2)	0.037(6)	0.077(5)
18	0.095(3)	0.138(3)	0.032(11)	0.075(8)
2-state	0.096(2)	0.139(2)	0.033(6)	0.073(5)



★ **Excited-states contamination not as prominent as for $\langle x\rangle$**

★ **Effect of excited states non negligible for PDF analysis**

Moments summary

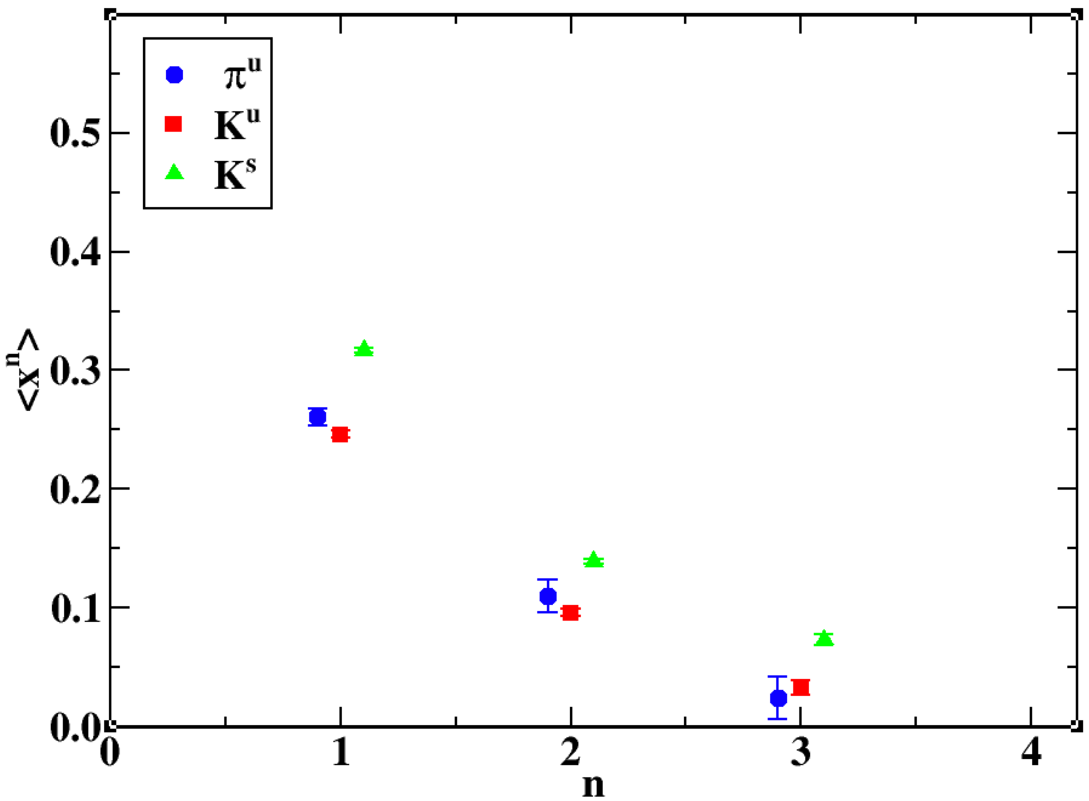


★ **Expected decay as Mellin moment increases**

★ **Up contribution to pion and kaon is similar**

★ **Strange contribution to kaon dominant**

Moments summary



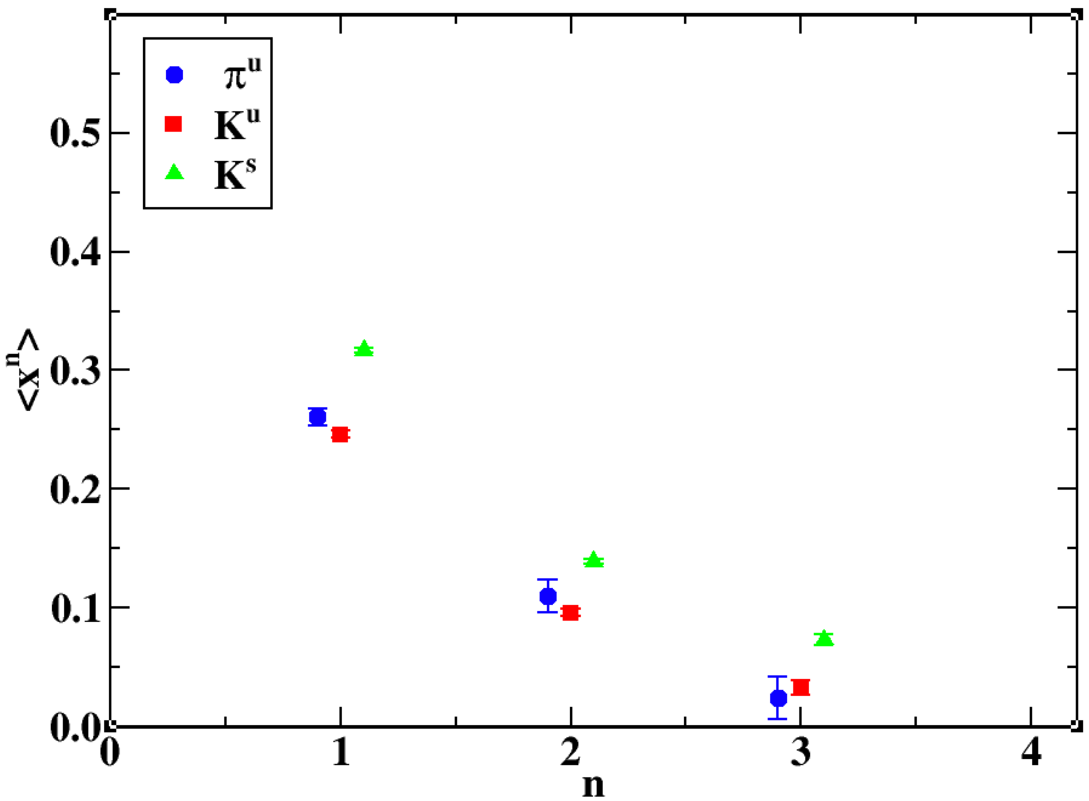
★ **Expected decay as Mellin moment increases**

★ **Up contribution to pion and kaon is similar**

★ **Strange contribution to kaon dominant**

What can we learn for PDFs from their moments

Moments summary

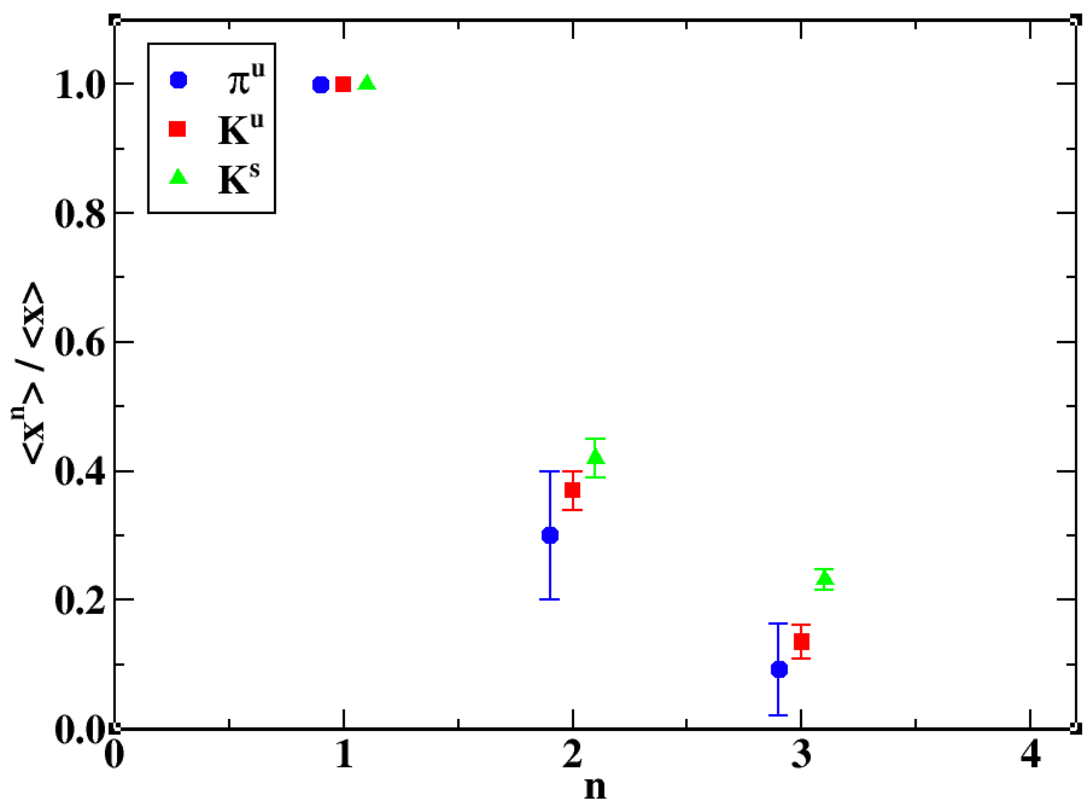


★ **Expected decay as Mellin moment increases**

★ **Up contribution to pion and kaon is similar**

★ **Strange contribution to kaon dominant**

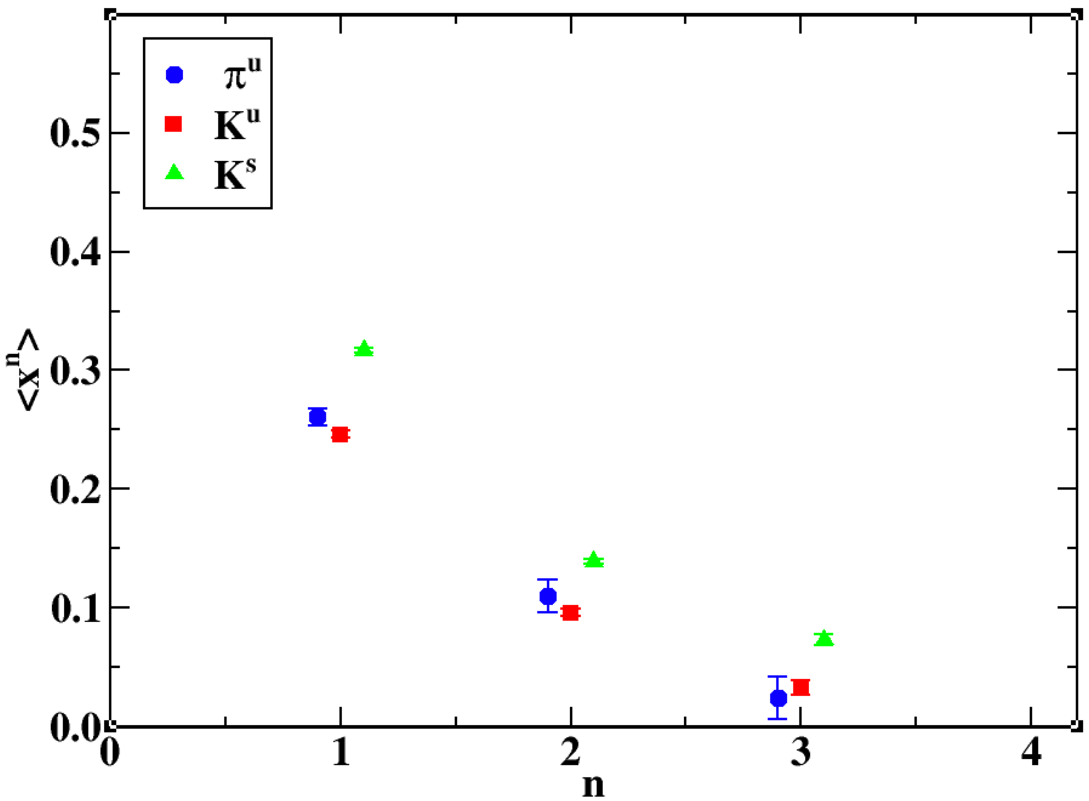
What can we learn for PDFs from their moments



★ **Larger moments have support at higher x**

- $\langle x^2\rangle^u_\pi \sim 20-40\,\%\,\langle x\rangle^u_\pi$ $\quad\langle x^3\rangle^u_\pi \sim 5-20\,\%\,\langle x\rangle^u_\pi$
- $\langle x^2\rangle^u_K \sim 35-40\,\%\,\langle x\rangle^u_K$ $\quad\langle x^3\rangle^u_K \sim 10-15\,\%\,\langle x\rangle^u_K$
- $\langle x^2\rangle^s_K \sim 40-45\,\%\,\langle x\rangle^s_K$ $\quad\langle x^3\rangle^s_K \sim 20-25\,\%\,\langle x\rangle^s_K$

Moments summary

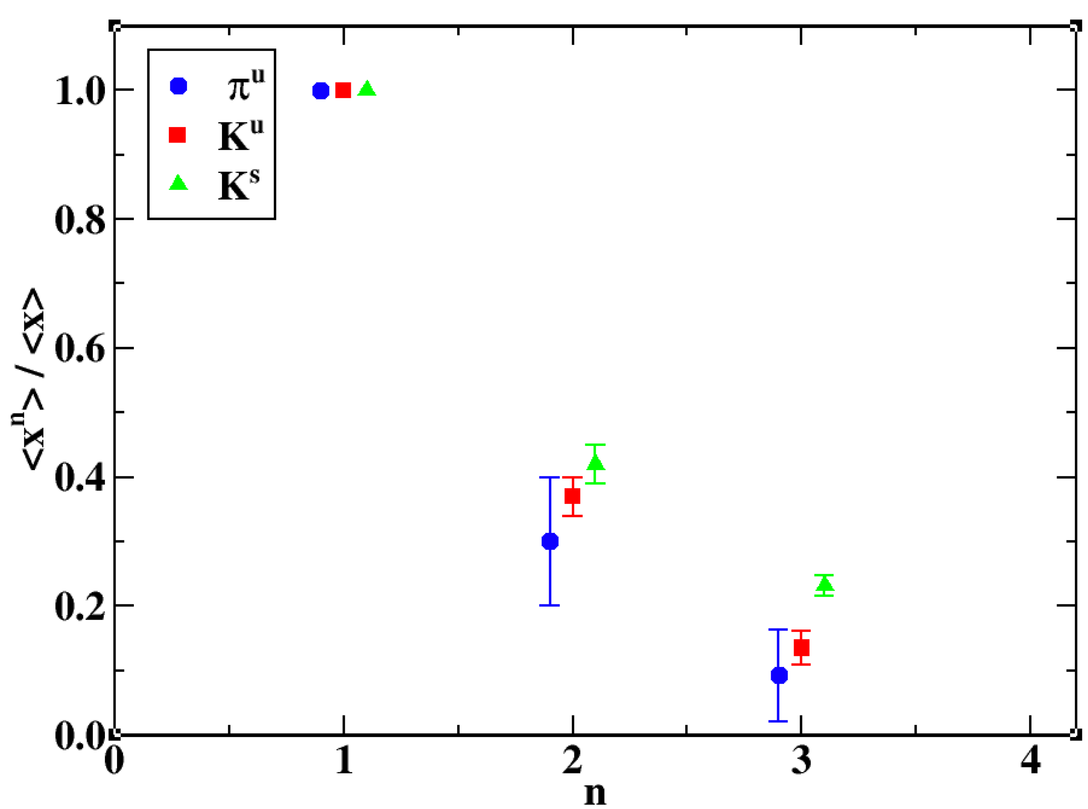


★ Expected decay as Mellin moment increases

★ Up contribution to pion and kaon is similar

★ Strange contribution to kaon dominant

What can we learn for
PDFs from their moments



★ Larger moments have support at higher x

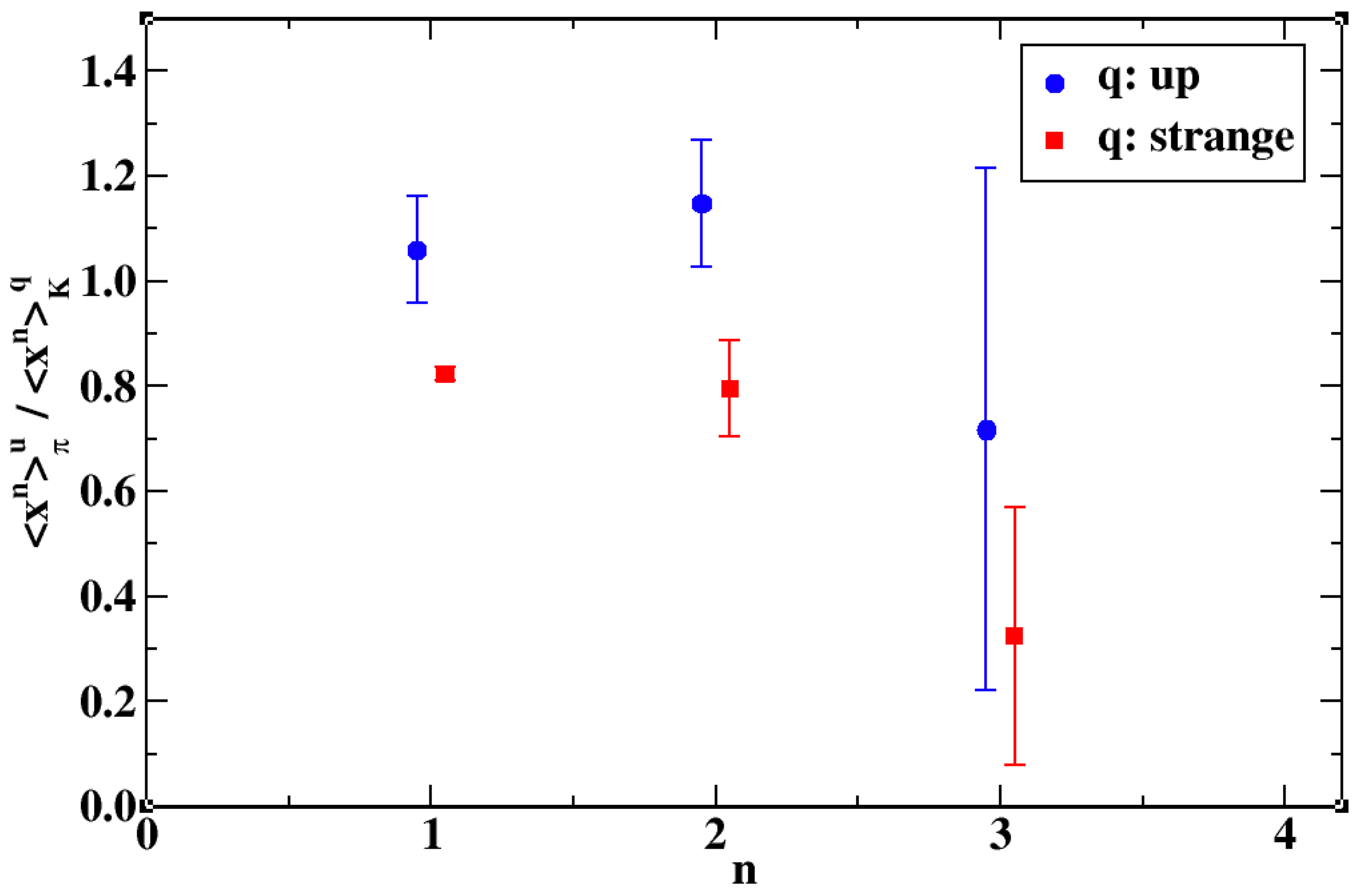
- $\langle x^2 \rangle^u_\pi \sim 20-40\,\%\,\langle x \rangle^u_\pi$ $\quad \langle x^3 \rangle^u_\pi \sim 5-20\,\%\,\langle x \rangle^u_\pi$
- $\langle x^2 \rangle^u_K \sim 35-40\,\%\,\langle x \rangle^u_K$ $\quad \langle x^3 \rangle^u_K \sim 10-15\,\%\,\langle x \rangle^u_K$
- $\langle x^2 \rangle^s_K \sim 40-45\,\%\,\langle x \rangle^s_K$ $\quad \langle x^3 \rangle^s_K \sim 20-25\,\%\,\langle x \rangle^s_K$

What can we learn for
SU(3) flavor symmetry breaking

SU(3) flavor symmetry breaking

★ Shape of up-quark pion and kaon PDFs expected to be similar

★ Strange-quark kaon expected to have support at higher-x than up-quark

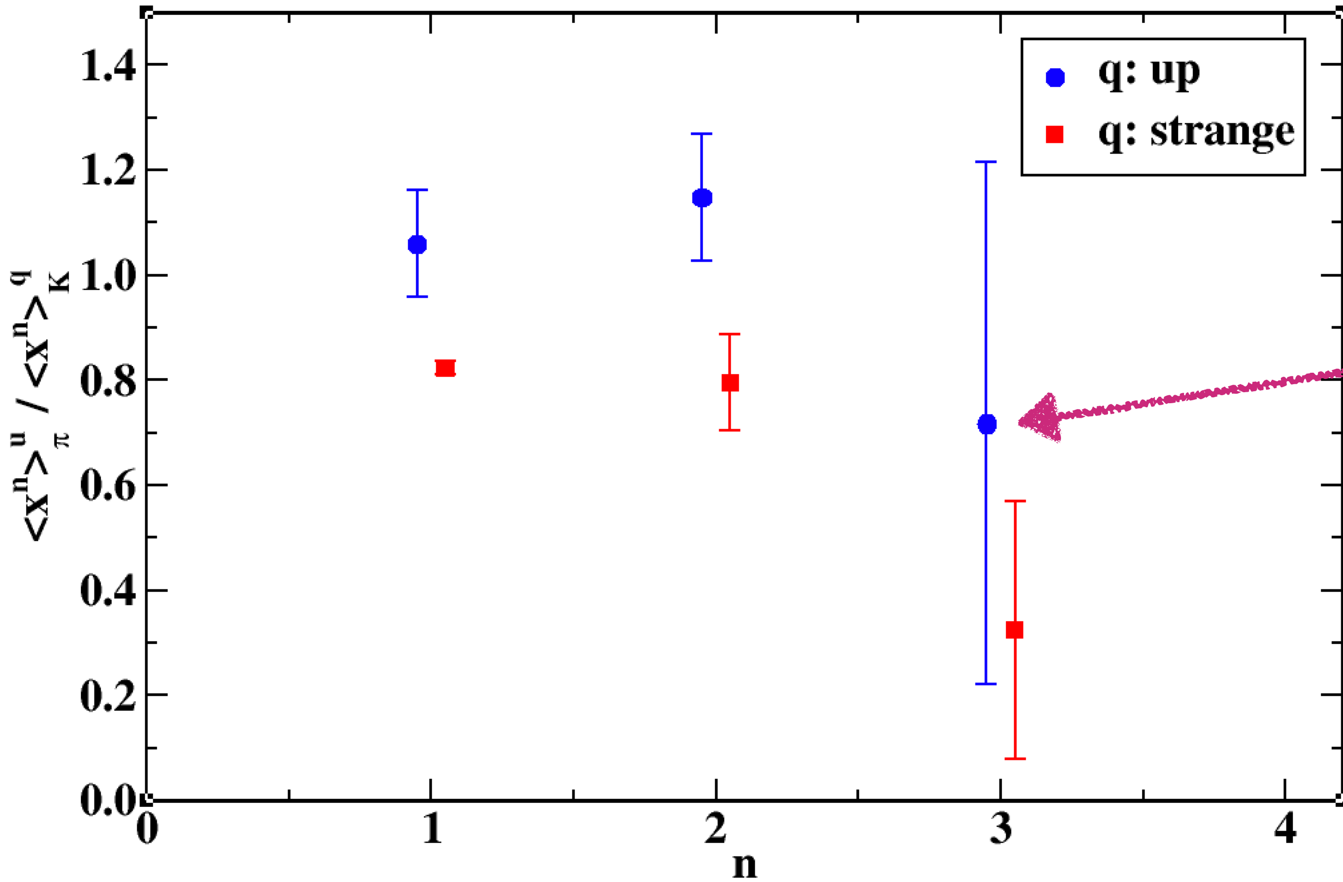


★ Qualitative picture confirms expectations from quark mass effects

SU(3) flavor symmetry breaking

★ Shape of up-quark pion and kaon PDFs expected to be similar

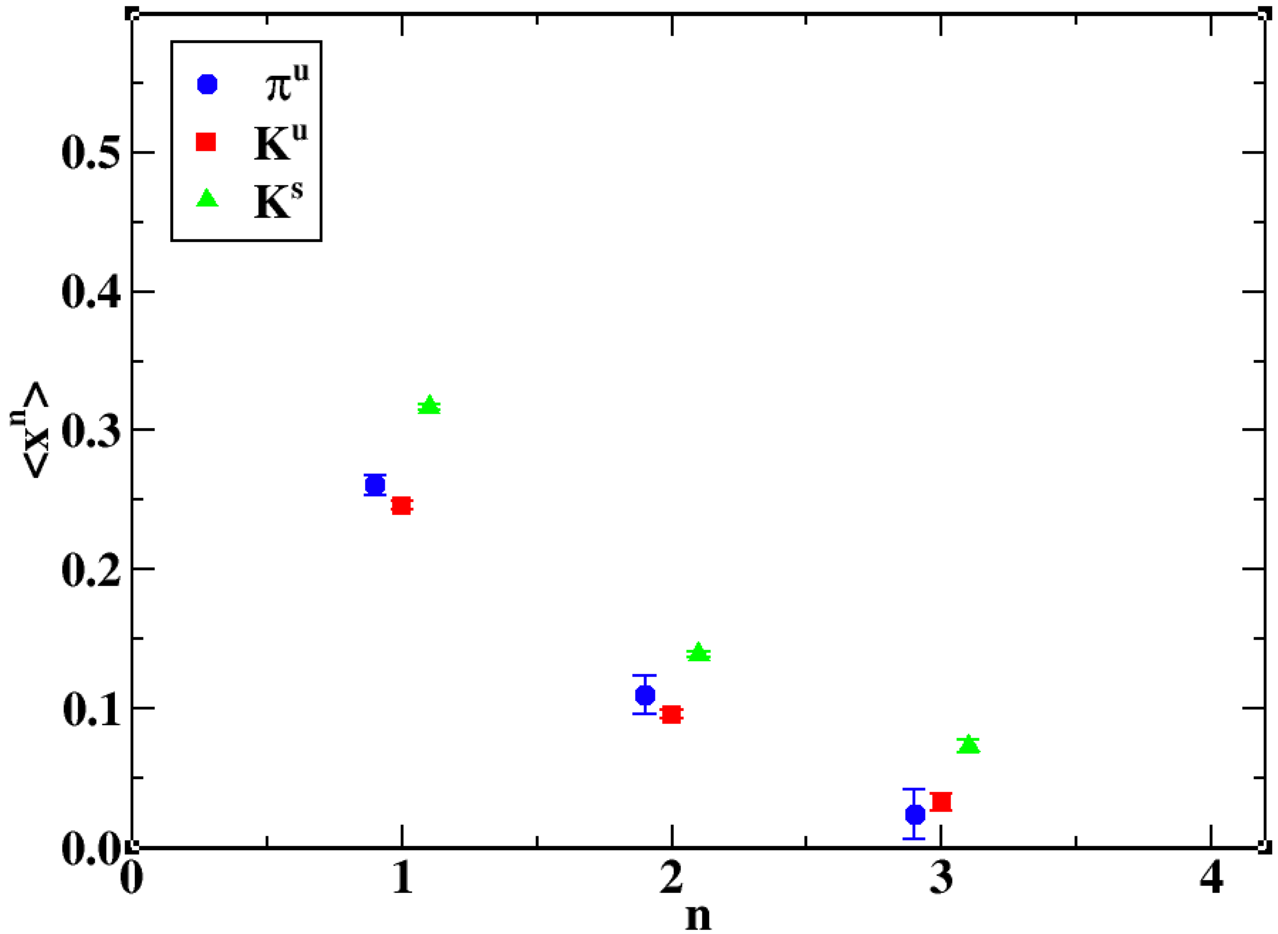
★ Strange-quark kaon expected to have support at higher-x than up-quark



Inconclusive
(small numbers)

★ Qualitative picture confirms expectations from quark mass effects

Recapitulation

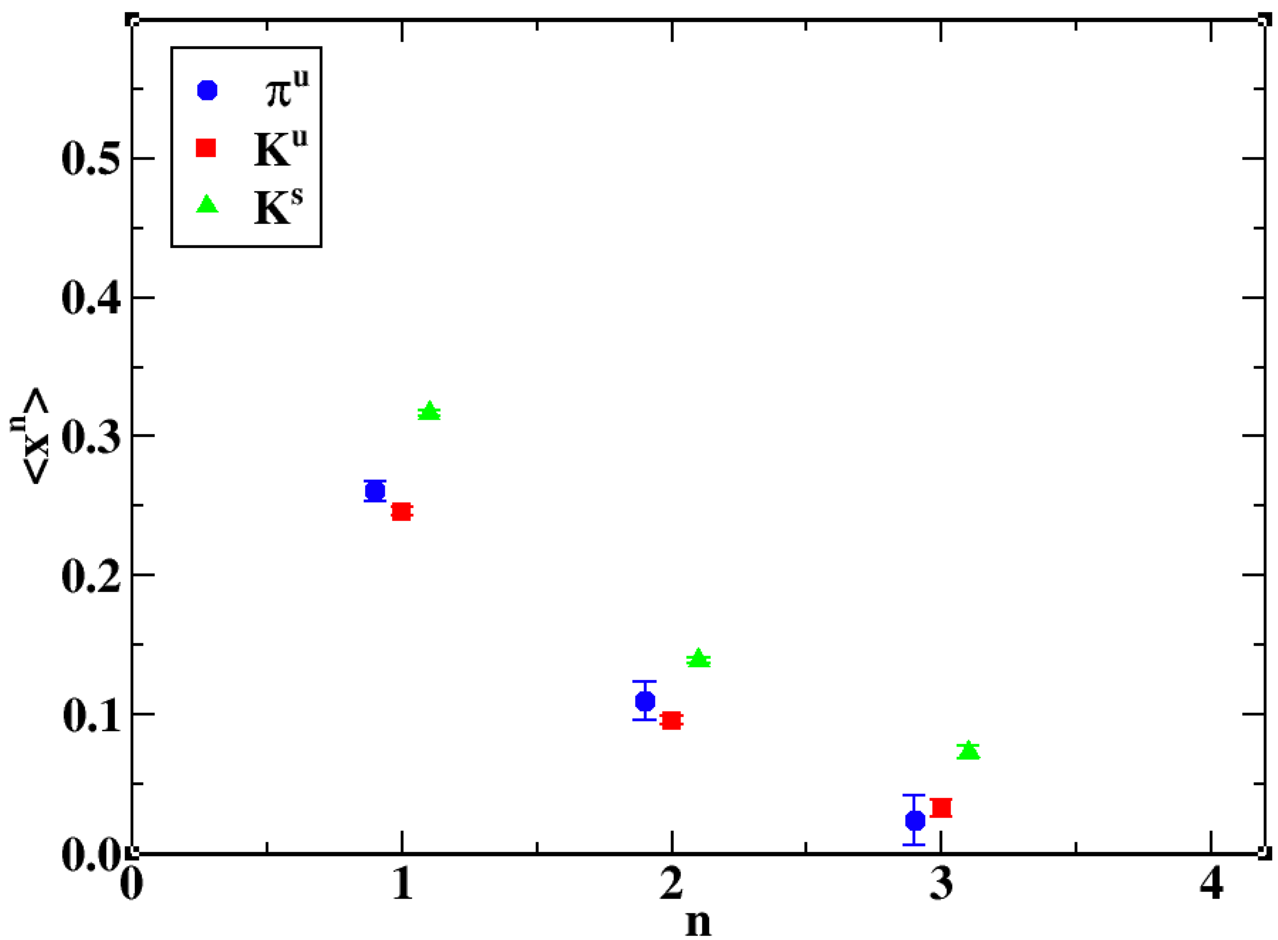


$$\langle x\rangle^u_{\pi^+} = 0.261(3)(6) \qquad \langle x^2\rangle^u_{\pi^+} = 0.110(7)(12) \qquad \langle x^3\rangle^u_{\pi^+} = 0.024(18)(2)$$

$$\langle x\rangle^u_{K^+} = 0.246(2)(2) \qquad \langle x^2\rangle^u_{K^+} = 0.096(2)(2) \qquad \langle x^3\rangle^u_{K^+} = 0.033(6)(1)$$

$$\langle x\rangle^s_{K^+} = 0.317(2)(1) \qquad \langle x^2\rangle^s_{K^+} = 0.139(2)(1) \qquad \langle x^3\rangle^s_{K^+} = 0.073(5)(2)$$

Recapitulation



Increase of moment →

Pion (u)	$\langle x\rangle^u_{\pi^+} = 0.261(3)(6)$	$\langle x^2\rangle^u_{\pi^+} = 0.110(7)(12)$	$\langle x^3\rangle^u_{\pi^+} = 0.024(18)(2)$
Kaon (u)	$\langle x\rangle^u_{K^+} = 0.246(2)(2)$	$\langle x^2\rangle^u_{K^+} = 0.096(2)(2)$	$\langle x^3\rangle^u_{K^+} = 0.033(6)(1)$
Kaon (s)	$\langle x\rangle^s_{K^+} = 0.317(2)(1)$	$\langle x^2\rangle^s_{K^+} = 0.139(2)(1)$	$\langle x^3\rangle^s_{K^+} = 0.073(5)(2)$

PDF reconstruction

Fit functions for PDFs

$$q_M^f(x) = N x^{\alpha}(1-x)^{\beta}(1+\rho\sqrt{x}+\gamma x)$$

$$N = \frac{1}{B(\alpha+1,\beta+1)+\gamma B(2+\alpha,\beta+1)}$$

$$\langle x^n \rangle = \frac{\left(\prod_{i=1}^{n}(i+\alpha)\right)\left(n+2+\alpha+\beta+(i+1+\alpha)\gamma\right)}{\left(\prod_{i=1}^{n}(i+2+\alpha+\beta)\right)\left(2+\alpha+\beta+(1+\alpha)\gamma\right)}, \quad n > 0$$

Fit functions for PDFs

$$q_M^f(x) = N x^\alpha (1-x)^\beta (1 + \rho\sqrt{x} + \gamma x)$$

$$N = \frac{1}{B(\alpha+1, \beta+1) + \gamma B(2+\alpha, \beta+1)}$$

$$\langle x^n \rangle = \frac{\left(\prod_{i=1}^{n} (i+\alpha)\right)\left(n+2+\alpha+\beta+(i+1+\alpha)\gamma\right)}{\left(\prod_{i=1}^{n} (i+2+\alpha+\beta)\right)\left(2+\alpha+\beta+(1+\alpha)\gamma\right)}, \quad n > 0$$

Lattice data

Fit functions for PDFs

$$q_M^f(x) = N x^{\alpha}(1-x)^{\beta}(1+\rho\sqrt{x}+\gamma x)$$

$$N = \frac{1}{B(\alpha+1,\beta+1)+\gamma B(2+\alpha,\beta+1)}$$

$$\langle x^n \rangle = \frac{\left(\prod_{i=1}^{n}(i+\alpha)\right)\left(n+2+\alpha+\beta+(i+1+\alpha)\gamma\right)}{\left(\prod_{i=1}^{n}(i+2+\alpha+\beta)\right)\left(2+\alpha+\beta+(1+\alpha)\gamma\right)}, \quad n>0$$

Lattice data

$\overline{\text{MS}}(5.2\,\text{GeV})$

fit type	α_π^u	β_π^u	γ_π^u
2-parameter	-0.04(20)	2.23(65)	0
3-parameter	-0.54(22)	2.76(64)	22.17(17.87)
fit type	α_K^u	β_K^u	γ_K^u
2-parameter	-0.05(7)	2.42(24)	0
3-parameter	-0.56(72)	3.01(23)	25.11(5.23)
fit type	α_K^s	β_K^s	γ_K^s
2-parameter	0.21(8)	2.13(20)	0
3-parameter	0.18(95)	2.051(3.46)	0.347(16.10)

Fit functions for PDFs

$$q_M^f(x) = N x^\alpha (1-x)^\beta (1 + \rho\sqrt{x} + \gamma x)$$

$$N = \frac{1}{B(\alpha+1, \beta+1) + \gamma B(2+\alpha, \beta+1)}$$

$$\langle x^n \rangle = \frac{\left(\prod_{i=1}^{n} (i+\alpha)\right)\left(n+2+\alpha+\beta+(i+1+\alpha)\gamma\right)}{\left(\prod_{i=1}^{n} (i+2+\alpha+\beta)\right)\left(2+\alpha+\beta+(1+\alpha)\gamma\right)}, \quad n > 0$$

Lattice data

$\overline{\mathrm{MS}}(5.2\,\mathrm{GeV})$

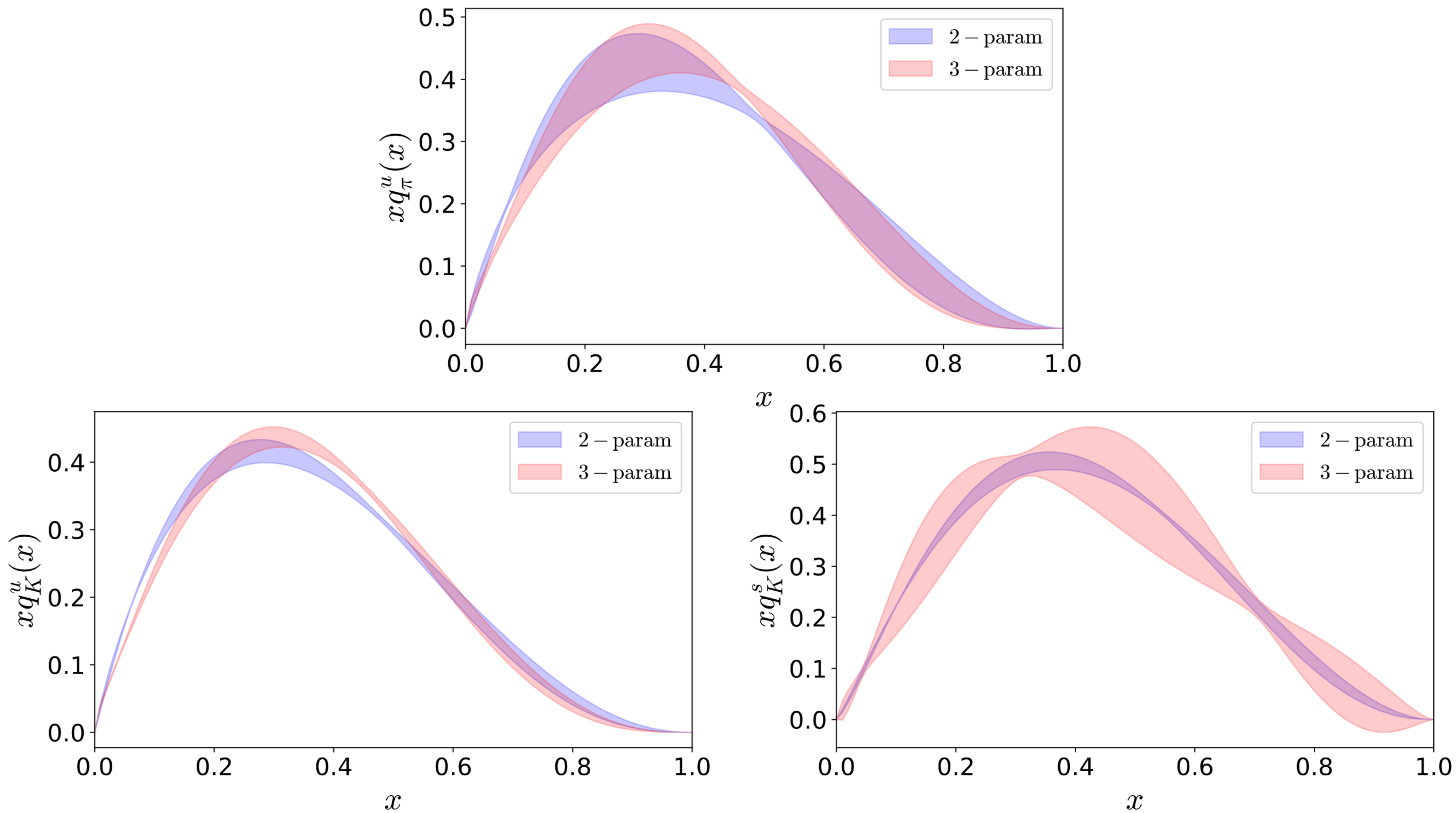
fit type	α_π^u	β_π^u	γ_π^u
2-parameter	-0.04(20)	2.23(65)	0
3-parameter	-0.54(22)	2.76(64)	22.17(17.87)

fit type	α_K^u	β_K^u	γ_K^u
2-parameter	-0.05(7)	2.42(24)	0
3-parameter	-0.56(72)	3.01(23)	25.11(5.23)

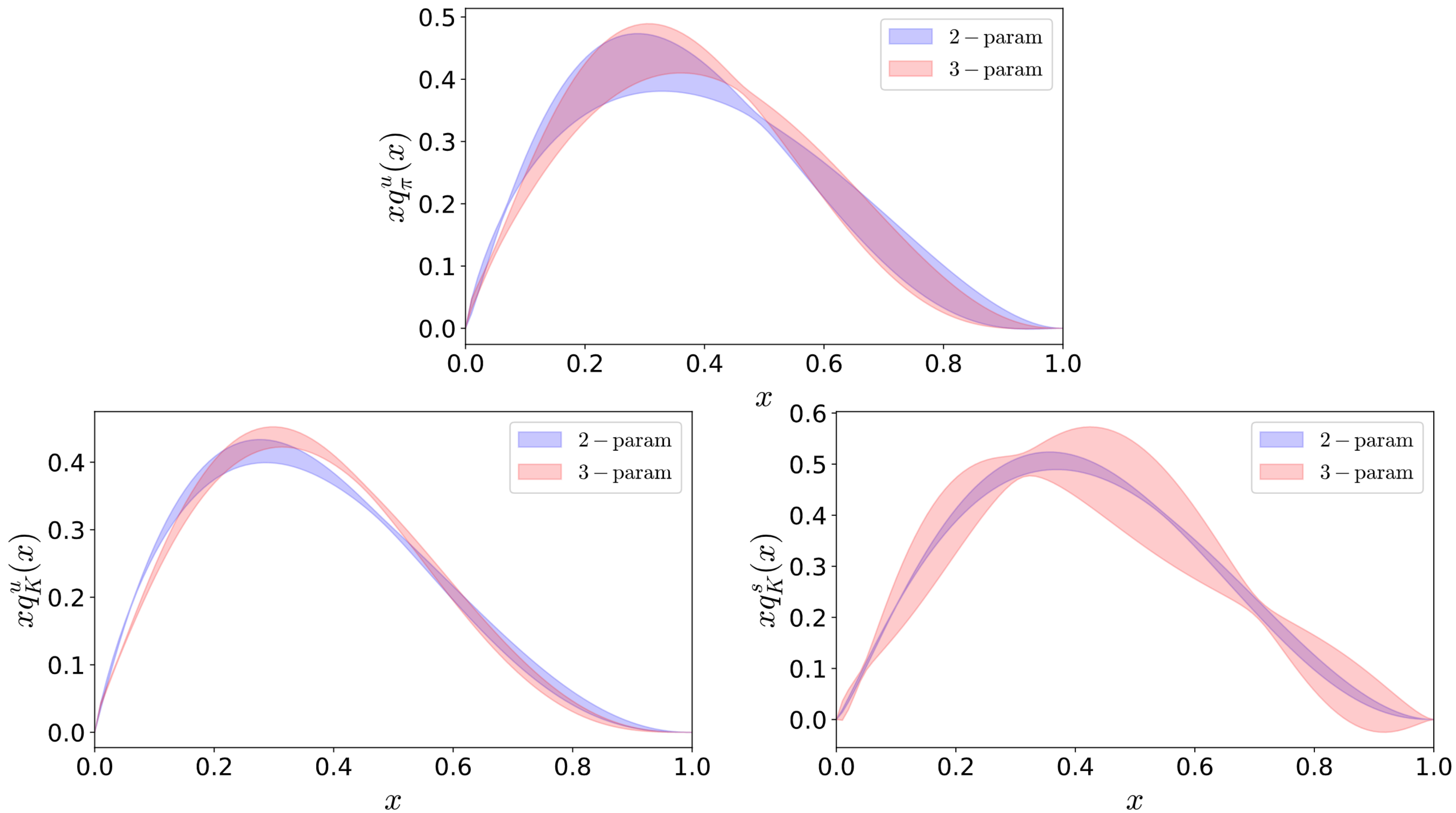
fit type	α_K^s	β_K^s	γ_K^s
2-parameter	0.21(8)	2.13(20)	0
3-parameter	0.18(95)	2.051(3.46)	0.347(16.10)

- ★ **3-parameter fit not very stable**
- ★ **β governs large-x behavior**
- ★ **Lattice data favor $(1-x)^2$ decay**

PDFs dependence on fits



PDFs dependence on fits



★ **Estimating γ is competing with other parameters (information up to $\langle x^3 \rangle$)**

★ **PDFs shape compatible for both fits**

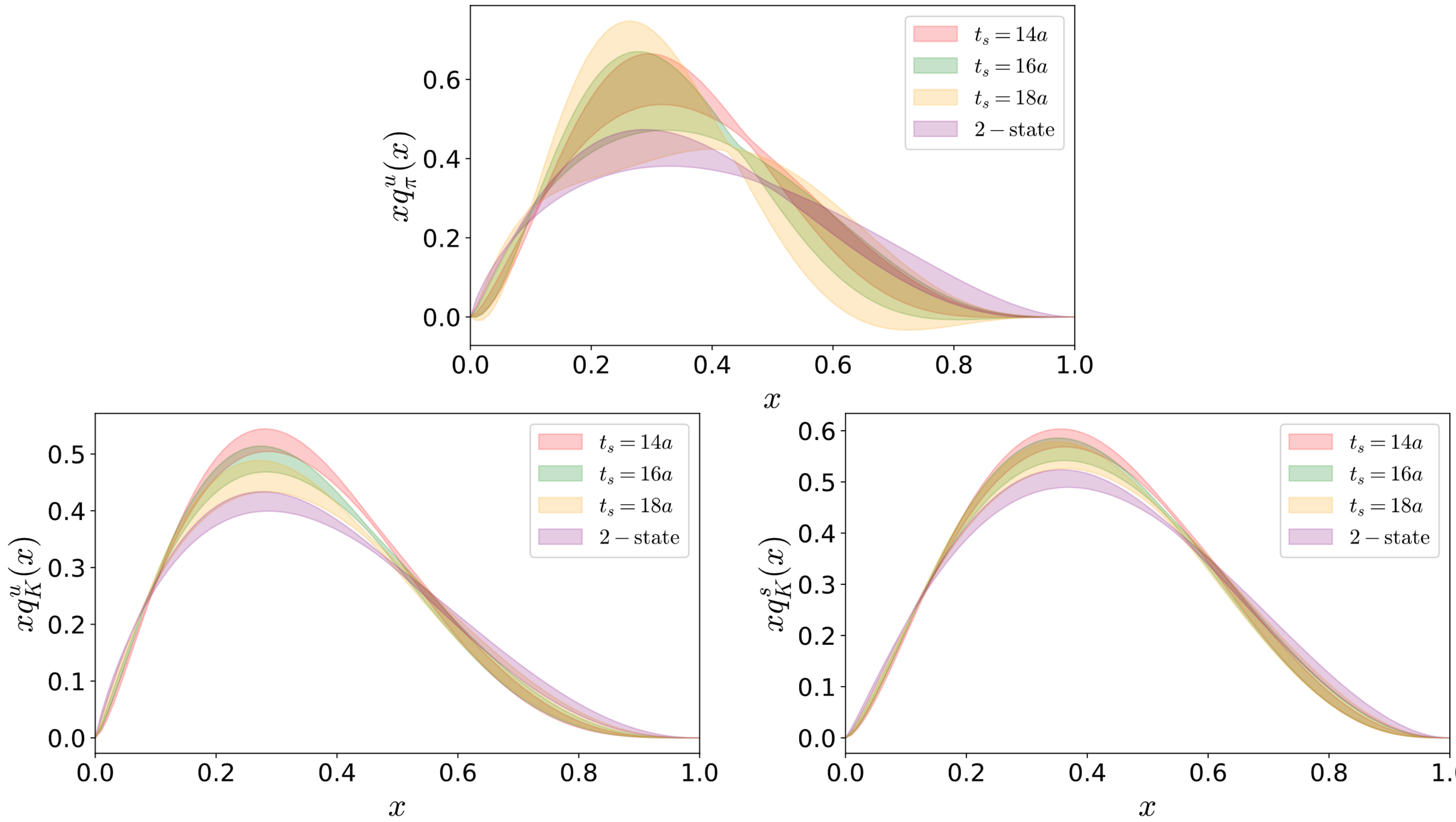
★ **2-parameter fit has smaller uncertainties**

Excited-states effects

★ **Excited-states effect more prominent for $\langle x \rangle$**

Excited-states effects

- ★ **Excited-states effect more prominent for $\langle x \rangle$**



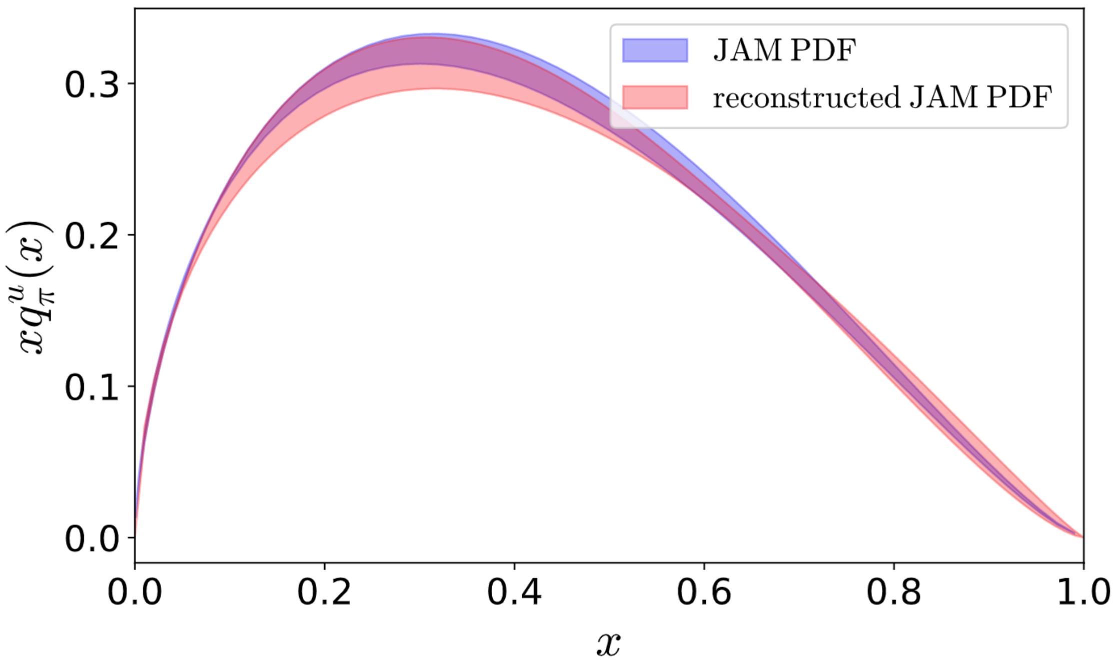
- ★ **Small-x region insensitive to excited-states effects**
- ★ **Large-x region: 2-state fit higher than small Tsink values**
- ★ **Peak: susceptible to excited-states effect**
  (Elimination of excited states bring the peak to the expected value)

Quality of reconstruction

★ **How much information do higher moments contain?**

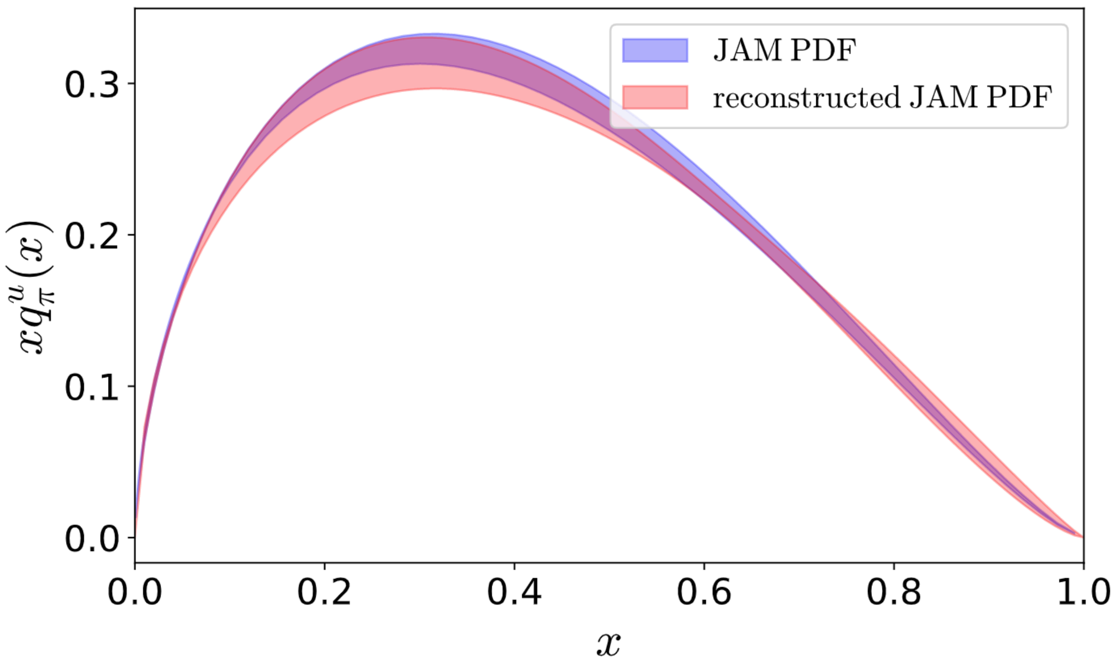
Quality of reconstruction

★ **How much information do higher moments contain?**



Quality of reconstruction

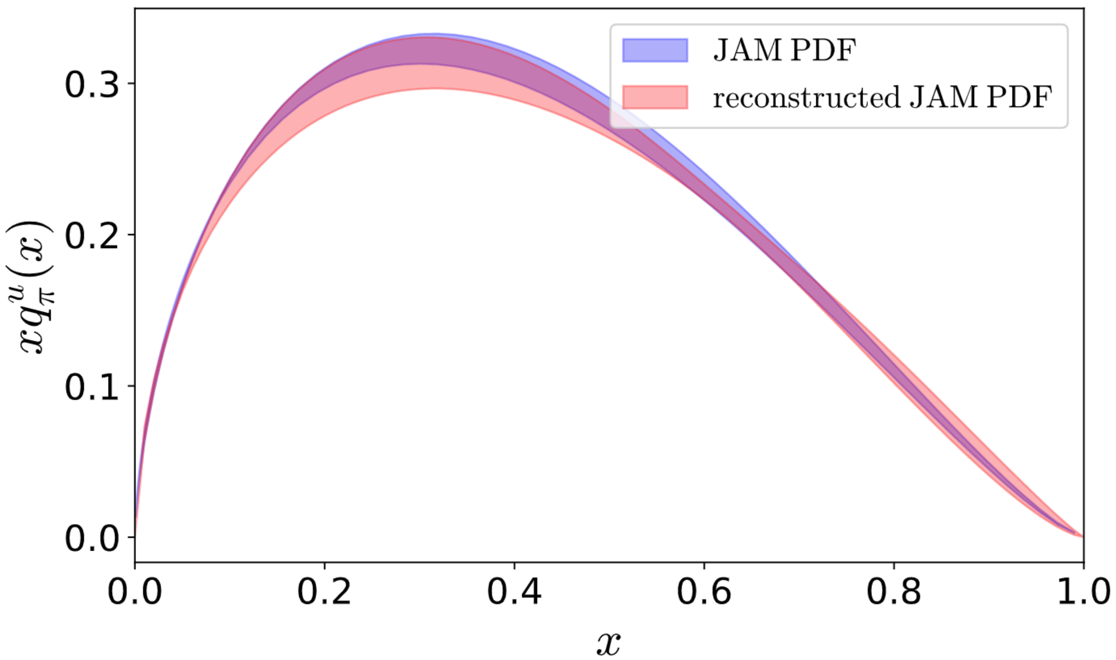
★ How much information do higher moments contain?



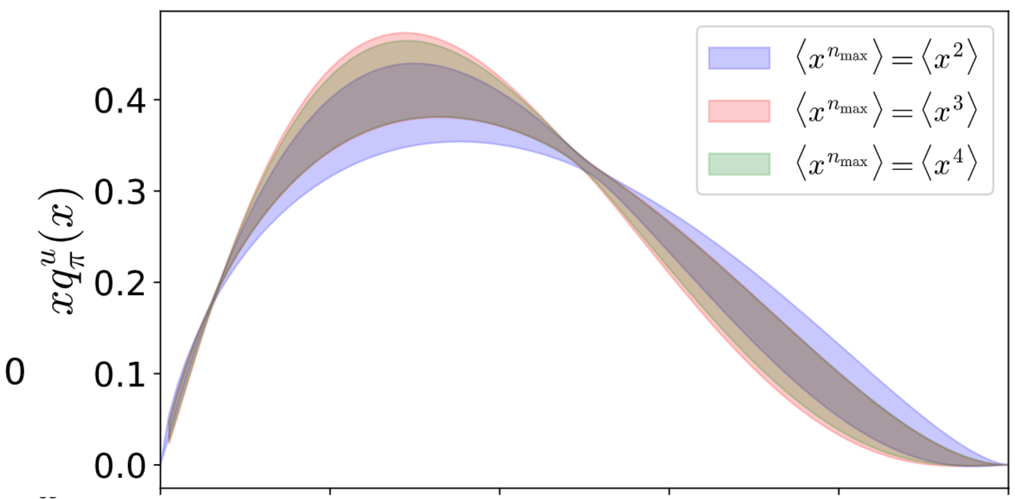
JAM PDF reconstructed correctly using the first 3 nontrivial moments

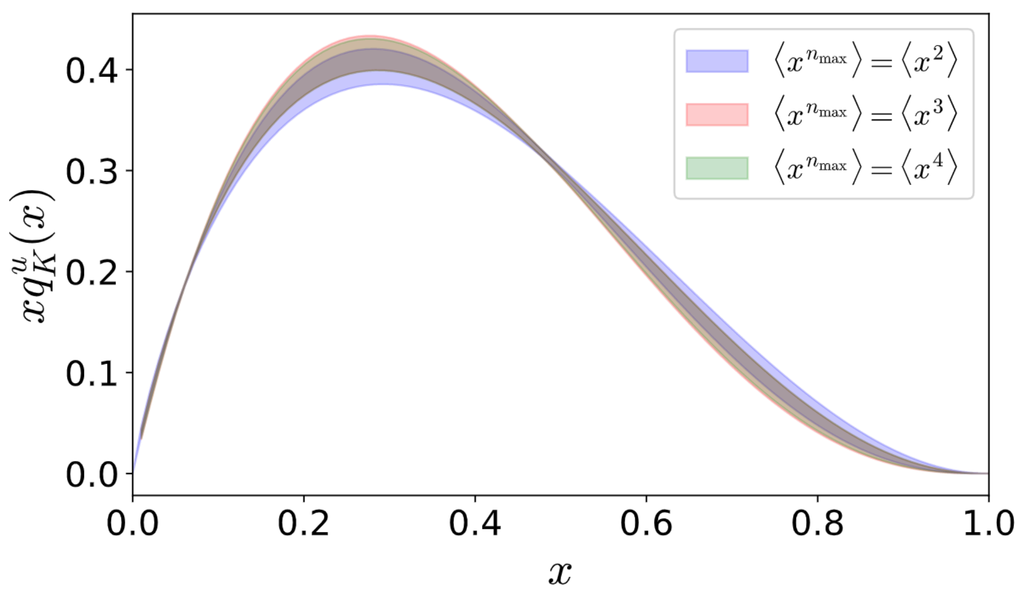
Quality of reconstruction

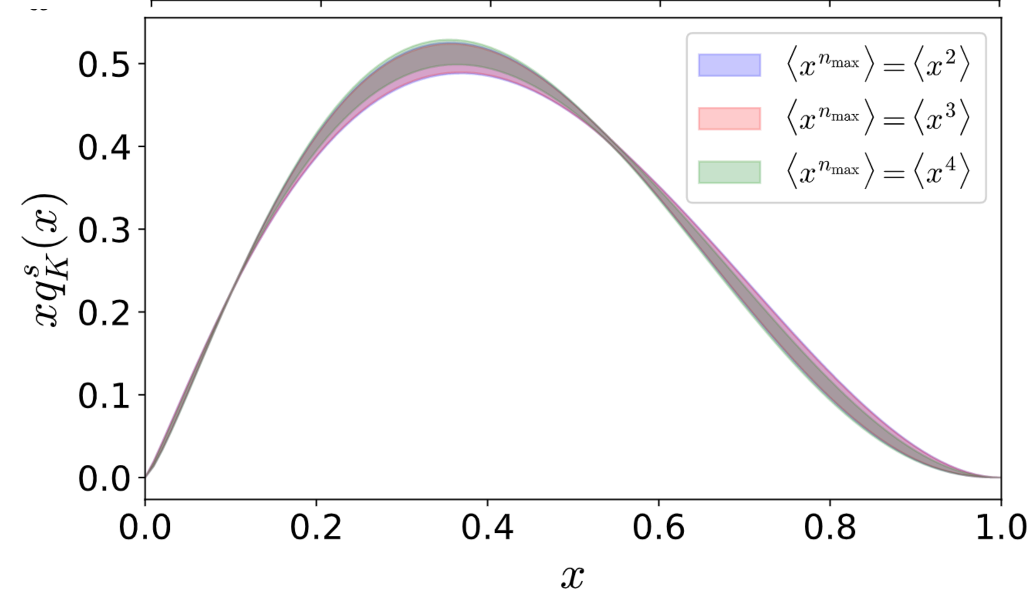
★ **How much information do higher moments contain?**



JAM PDF reconstructed correctly using the first 3 nontrivial moments

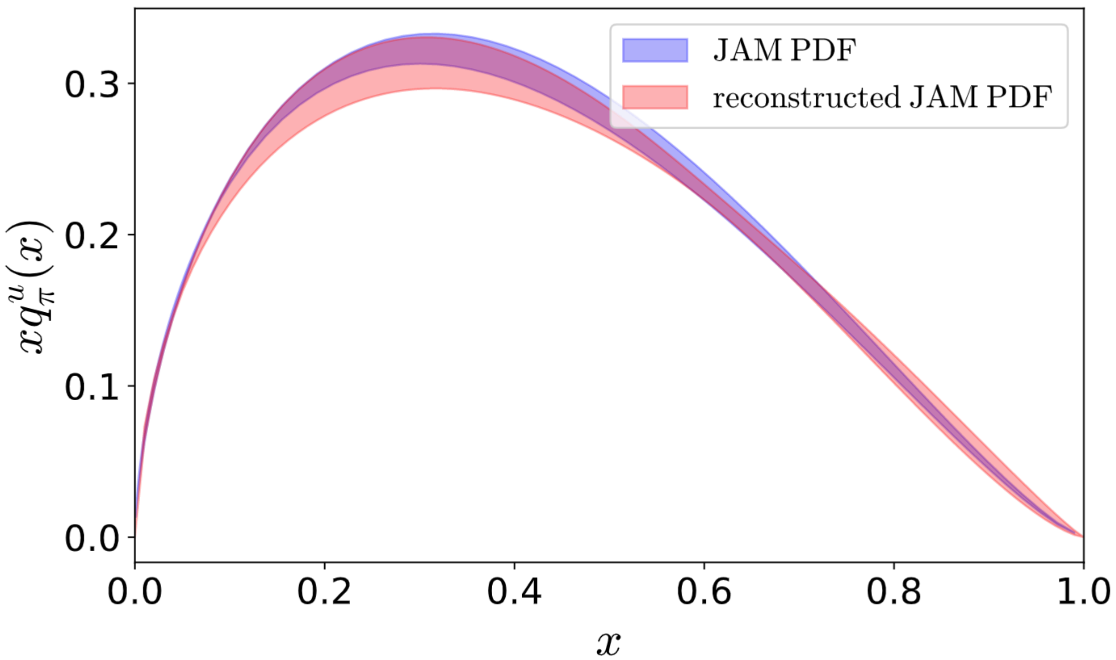




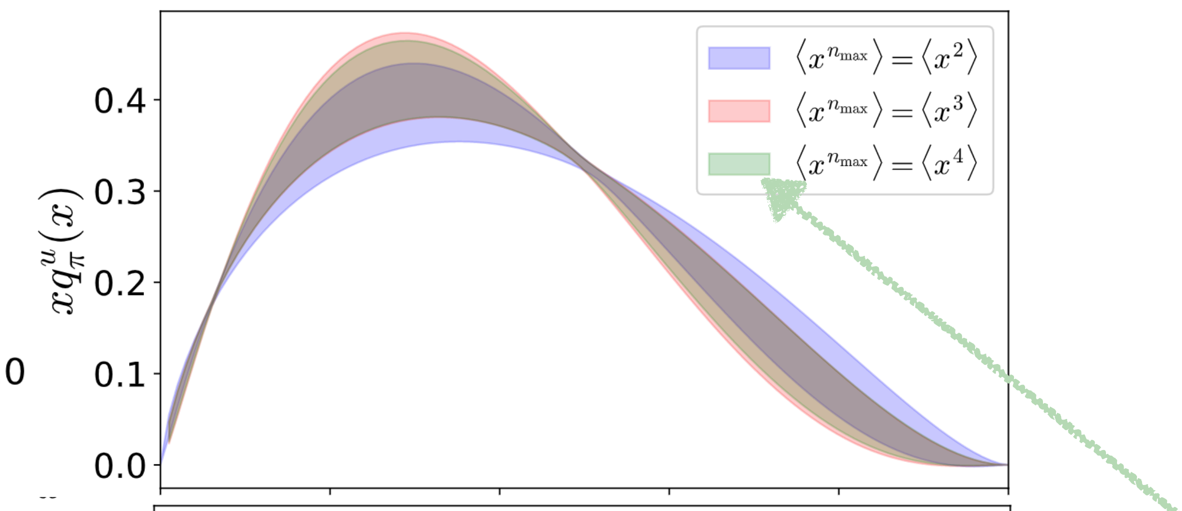


Quality of reconstruction

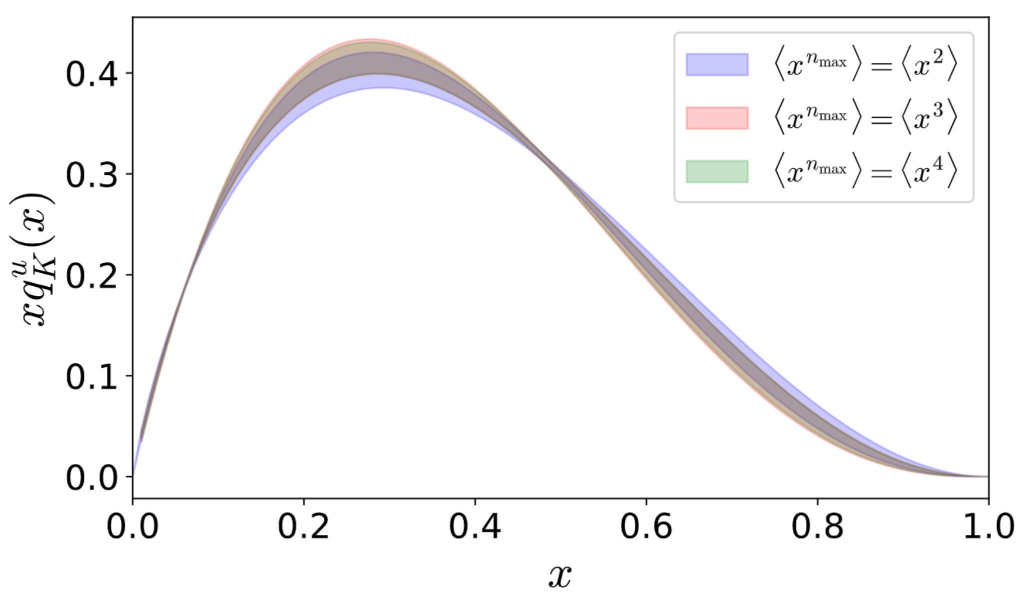
★ How much information do higher moments contain?

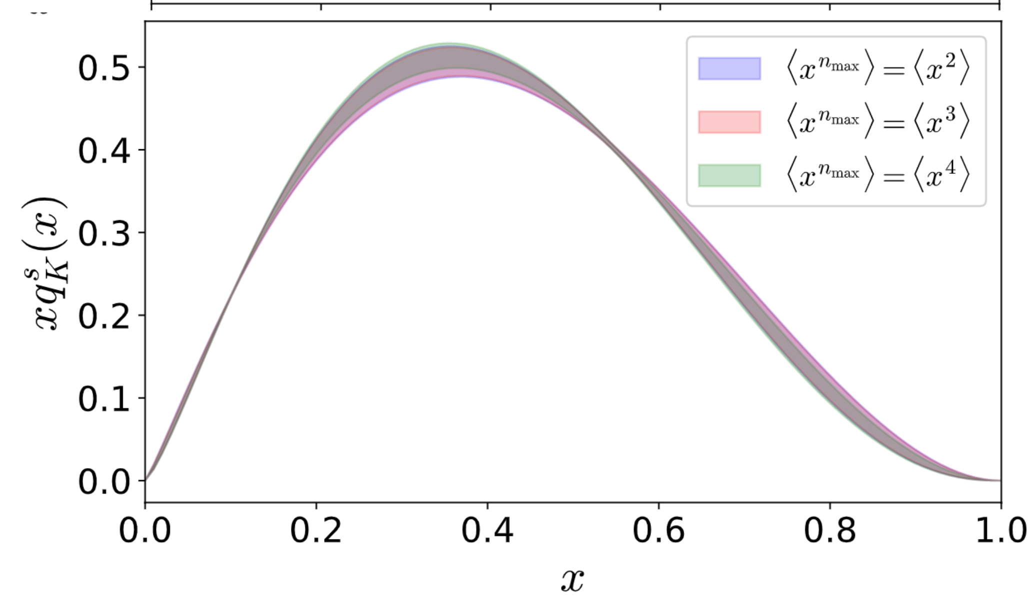


JAM PDF reconstructed correctly using the first 3 nontrivial moments



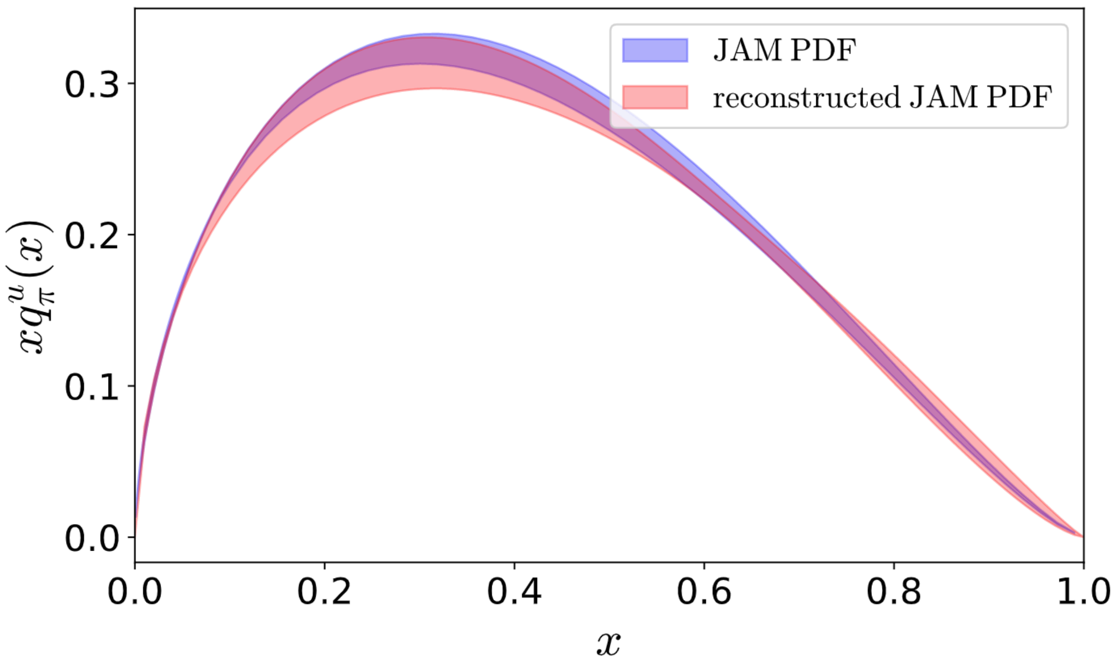
Using JAM $\langle x^4 \rangle$



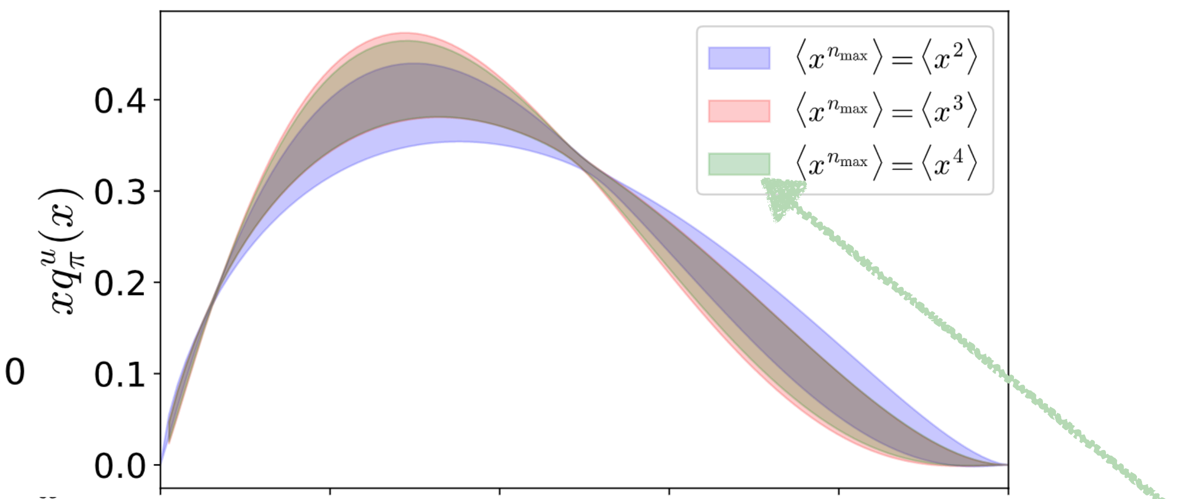


Quality of reconstruction

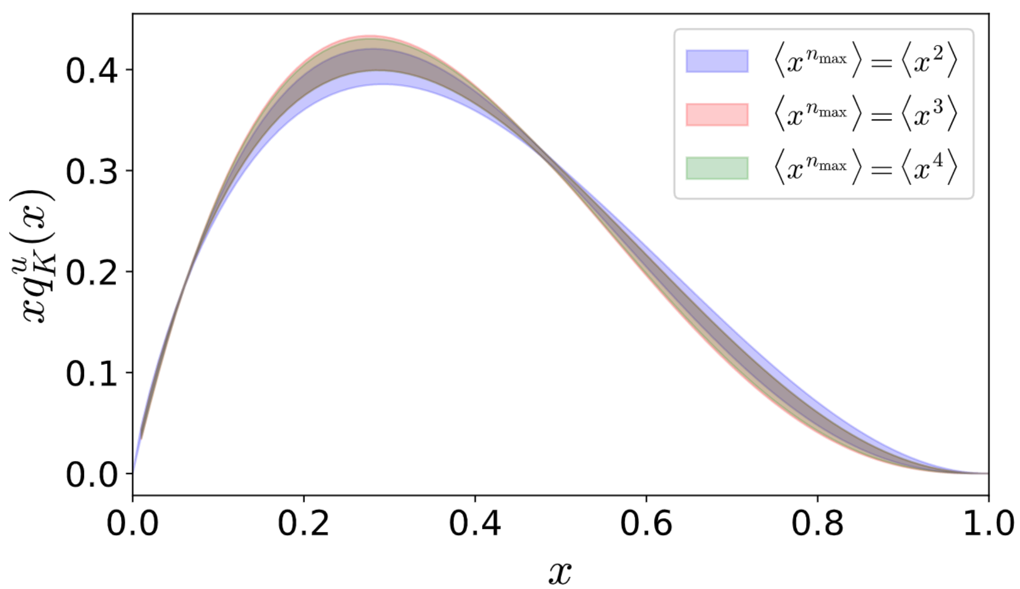
★ **How much information do higher moments contain?**

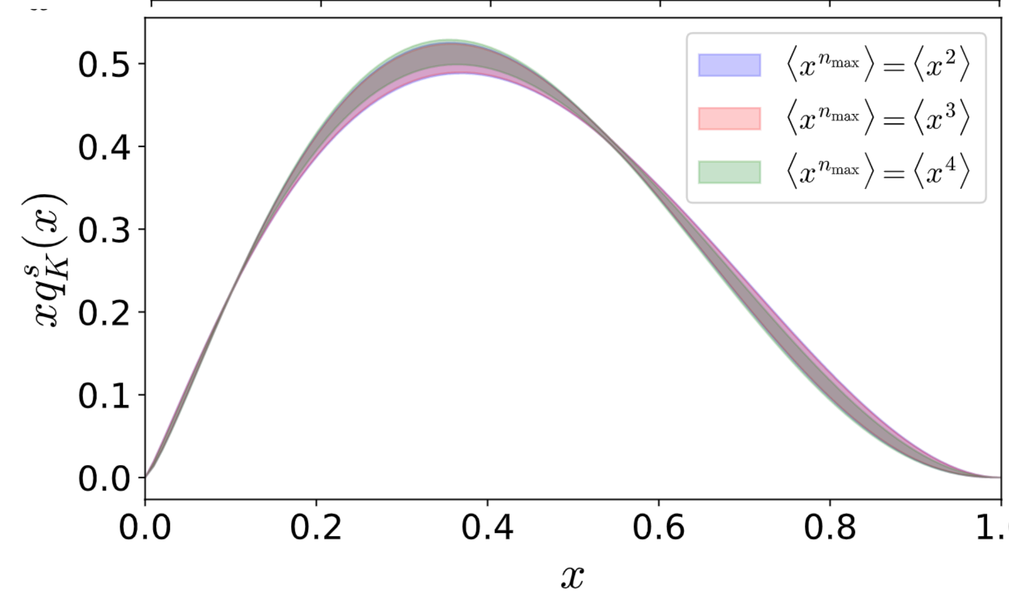


JAM PDF reconstructed correctly using the first 3 nontrivial moments



Using JAM $\langle x^4\rangle$

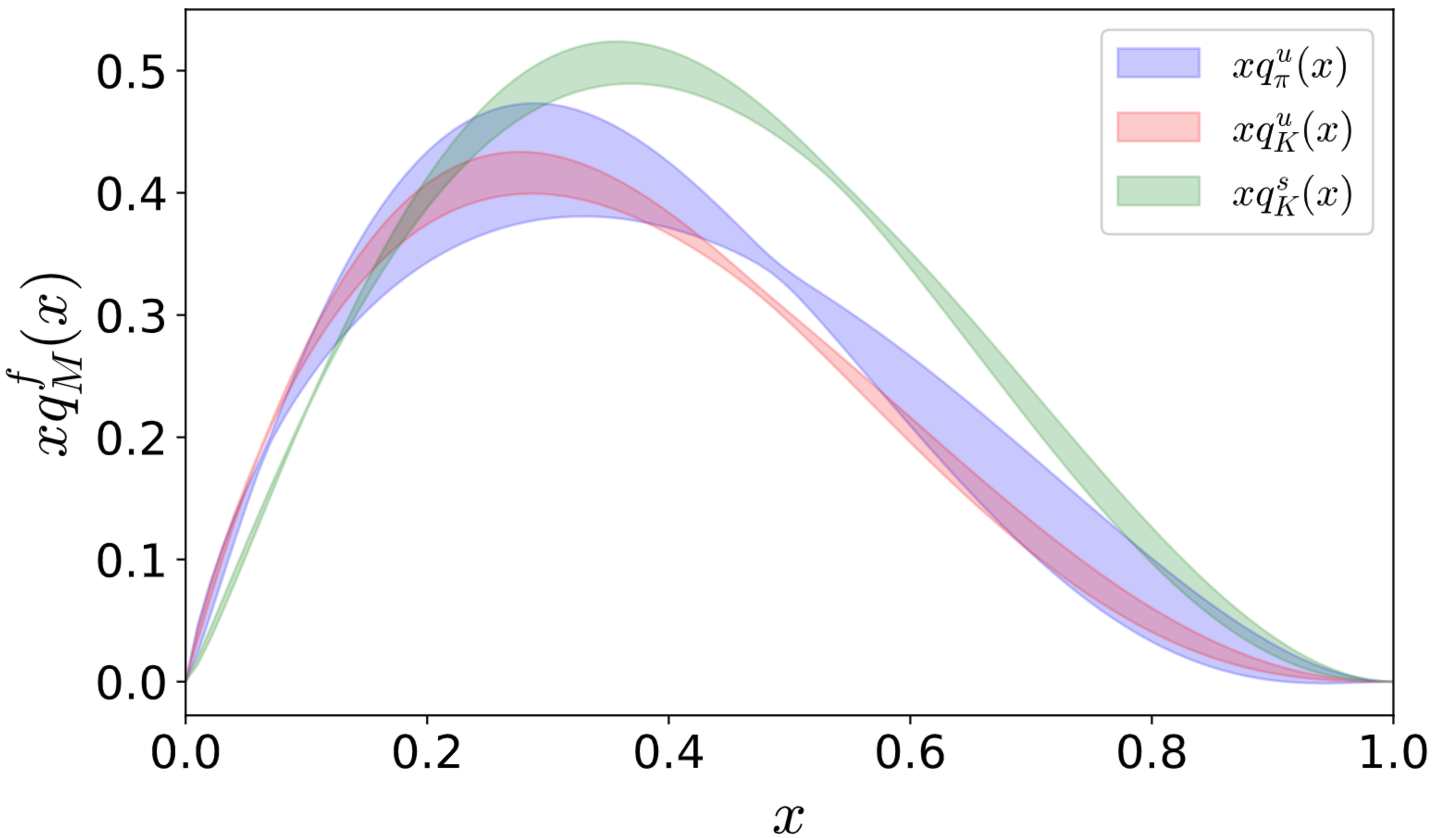




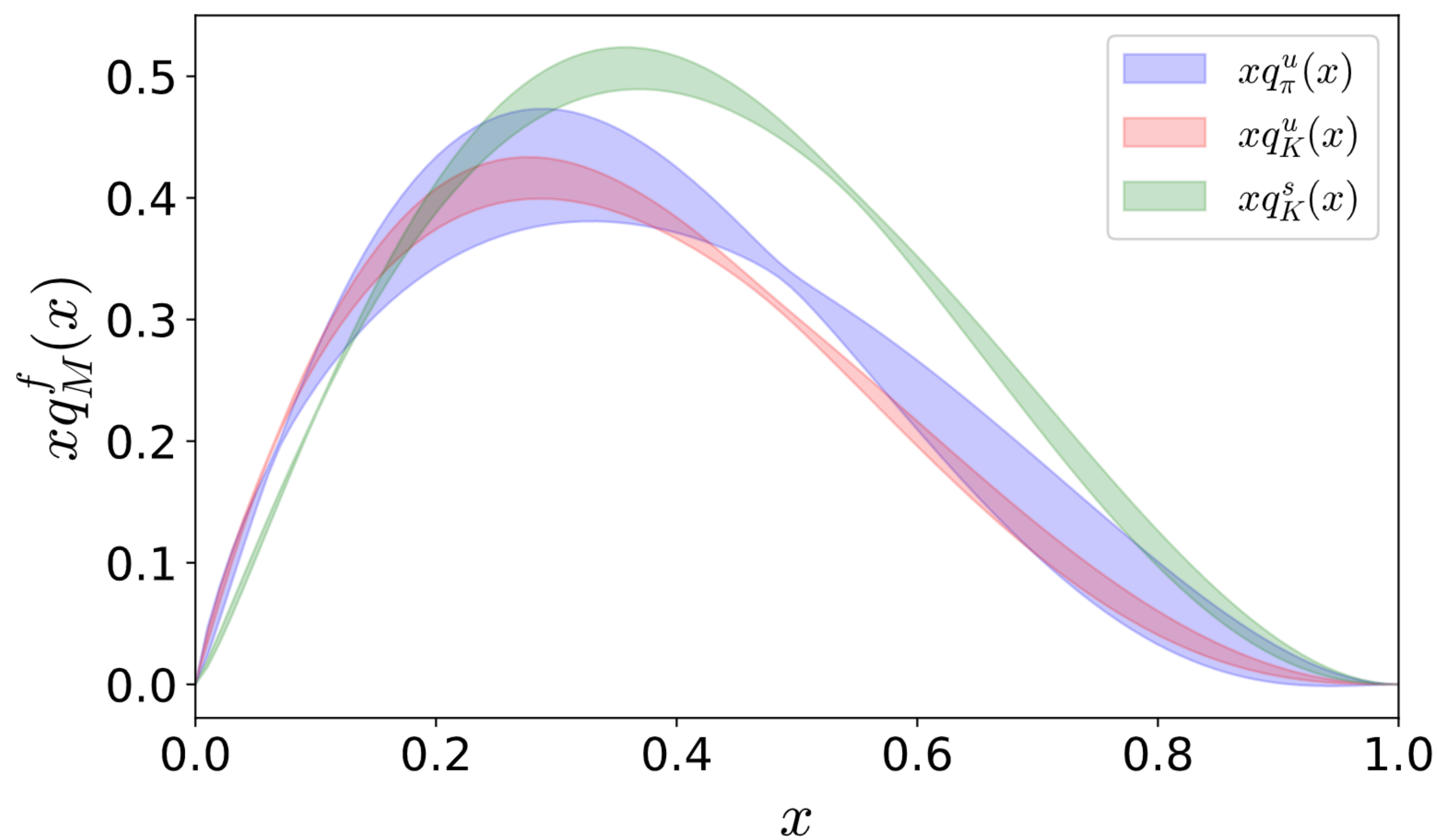
★ **Most of the information is contained in the moments up to $\langle x^3\rangle$**

$\langle x^3\rangle_{\max}$ **fully compatible with** $\langle x^4\rangle_{\max}$

SU(3) flavor symmetry breaking



SU(3) flavor symmetry breaking



★ **Up-quark seems to have a similar role in pion and kaon.**
$xq^u_\pi(x)$ **compatible with** $xq^u_K(x)$ **(small difference in** $x \in [0.45 - 0.55]$**)**

★ **Up-quark contribution support at small and intermediate x.**
Peak of $xq^u_\pi(x)$ **and** $xq^u_K(x)$ **around** $x = 0.3$

★ **Strange-quark contribution support at intermediate and large x.**
Peak of $xq^s_K(x)$ **around** $x = 0.36$

x-dependent PDFs from lattice QCD

★ **Alternative approaches proposed, e.g.:**

Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
Fictitious heavy quark	[W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
Auxiliary scalar quark	[V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
Higher moments	[Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
Quasi-distributions (LaMET)	[X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
Compton amplitude and OPE	[A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
Pseudo-distributions	[A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
Good lattice cross sections	[Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]

x-dependent PDFs from lattice QCD

★ **Alternative approaches proposed, e.g.:**

Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
Fictitious heavy quark	[W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
Auxiliary scalar quark	[V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
Higher moments	[Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
Quasi-distributions (LaMET)	[X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
Compton amplitude and OPE	[A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
Pseudo-distributions	[A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
Good lattice cross sections	[Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]

Eur. Phys. J. A (2021) 57:77
https://doi.org/10.1140/epja/s10050-021-00353-7
THE EUROPEAN PHYSICAL JOURNAL A

Review

The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD

Martha Constantinou[a]

Temple University, Philadelphia, USA

x-dependent PDFs from lattice QCD

★ **Alternative approaches proposed, e.g.:**

Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
Fictitious heavy quark	[W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
Auxiliary scalar quark	[V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
Higher moments	[Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
Quasi-distributions (LaMET)	[X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
Compton amplitude and OPE	[A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
Pseudo-distributions	[A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
Good lattice cross sections	[Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]

Eur. Phys. J. A (2021) 57:77
https://doi.org/10.1140/epja/s10050-021-00353-7

THE EUROPEAN PHYSICAL JOURNAL A

Check for updates

Review

The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD

Martha Constantinou[a]

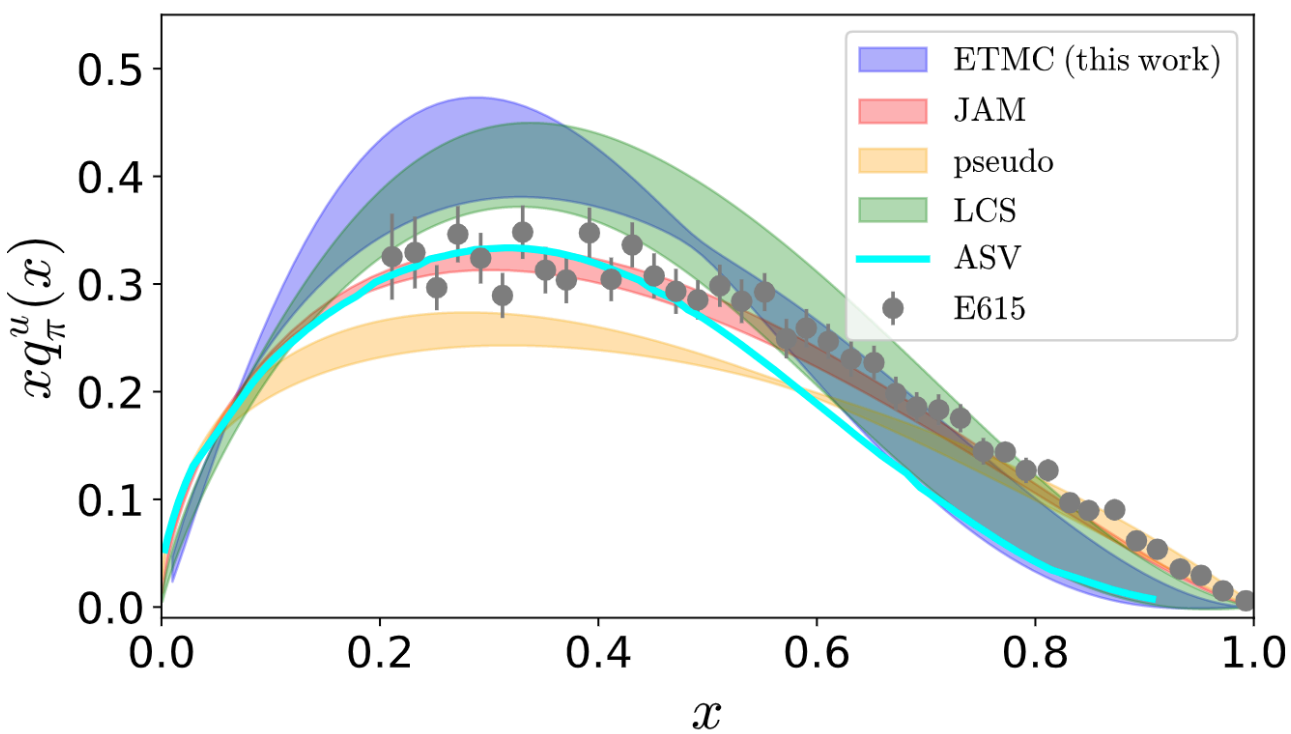
Temple University, Philadelphia, USA

Other Reviews:

[K. Cichy, M. Constantinou, Adv. in HEP, Volume 2019, 3036904, arXiv:1811.07248]

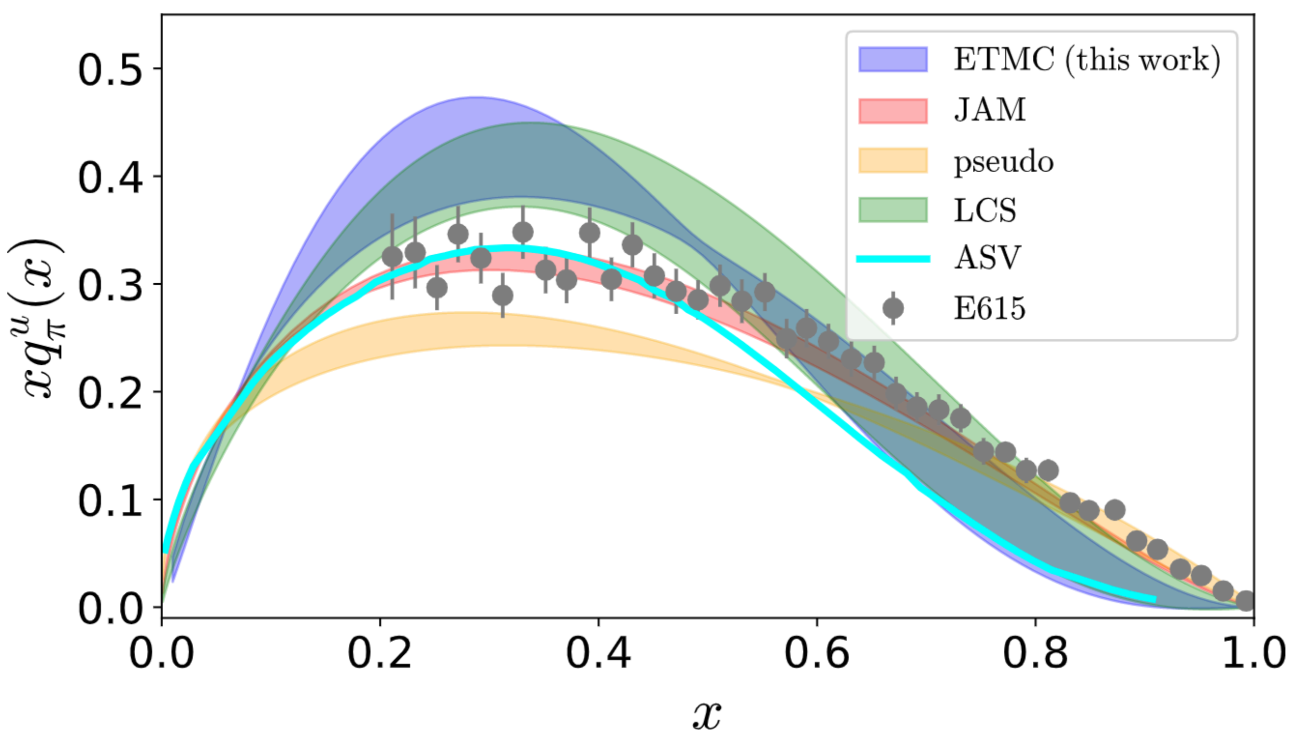
[X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543]

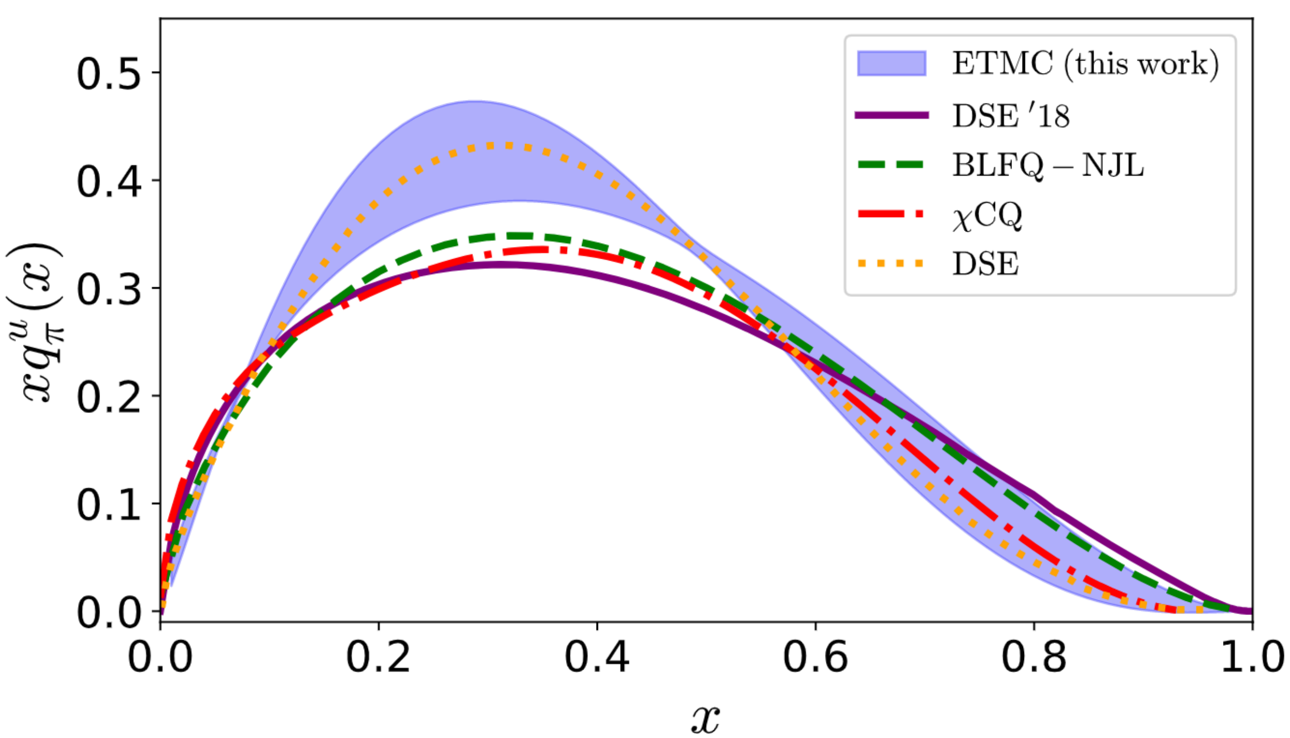
Pion: Comparison with other studies



- ★ **Lattice calculations of pseudo-PDFs and current-current correlators (LCS) use non-local operators**
- ★ **Very good agreement with PDF from LCS**
- ★ **Tension with E615 data in region** $x \in [0.2 - 0.55]$
- ★ **Large-x behavior compatible with rescaled ASV**

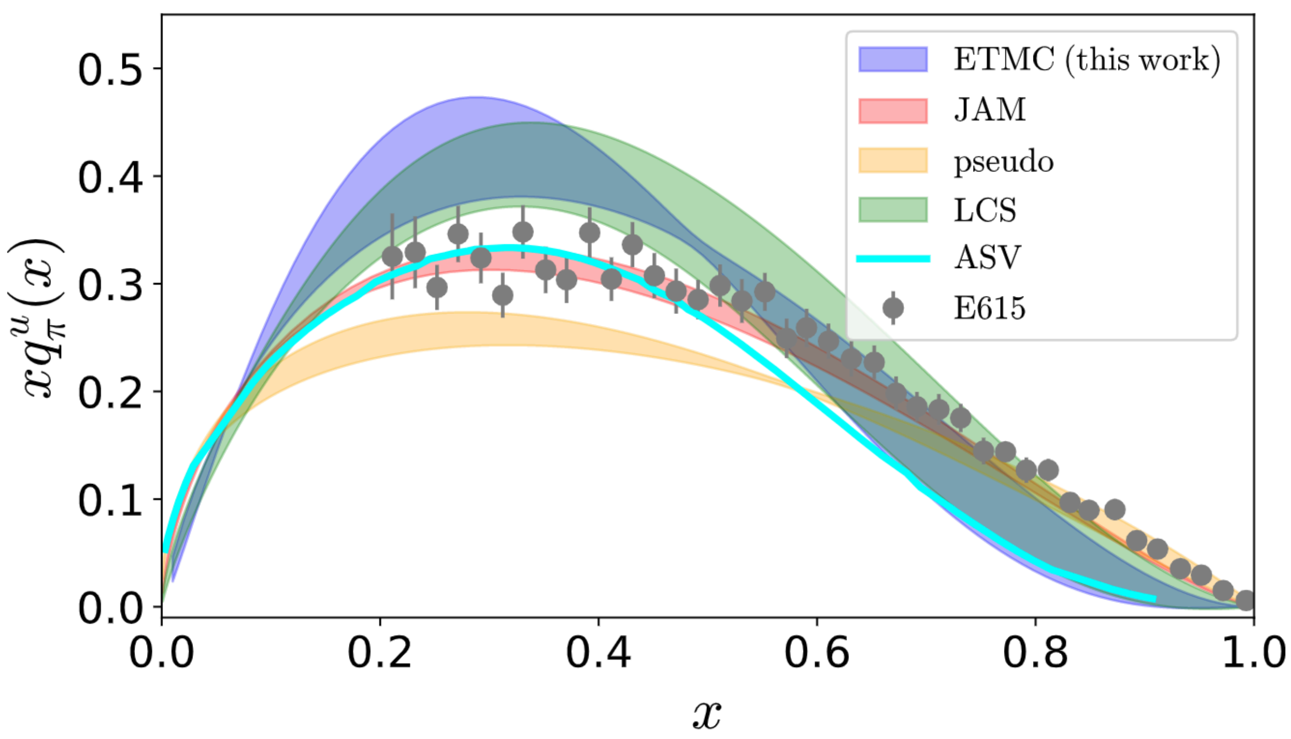
Pion: Comparison with other studies

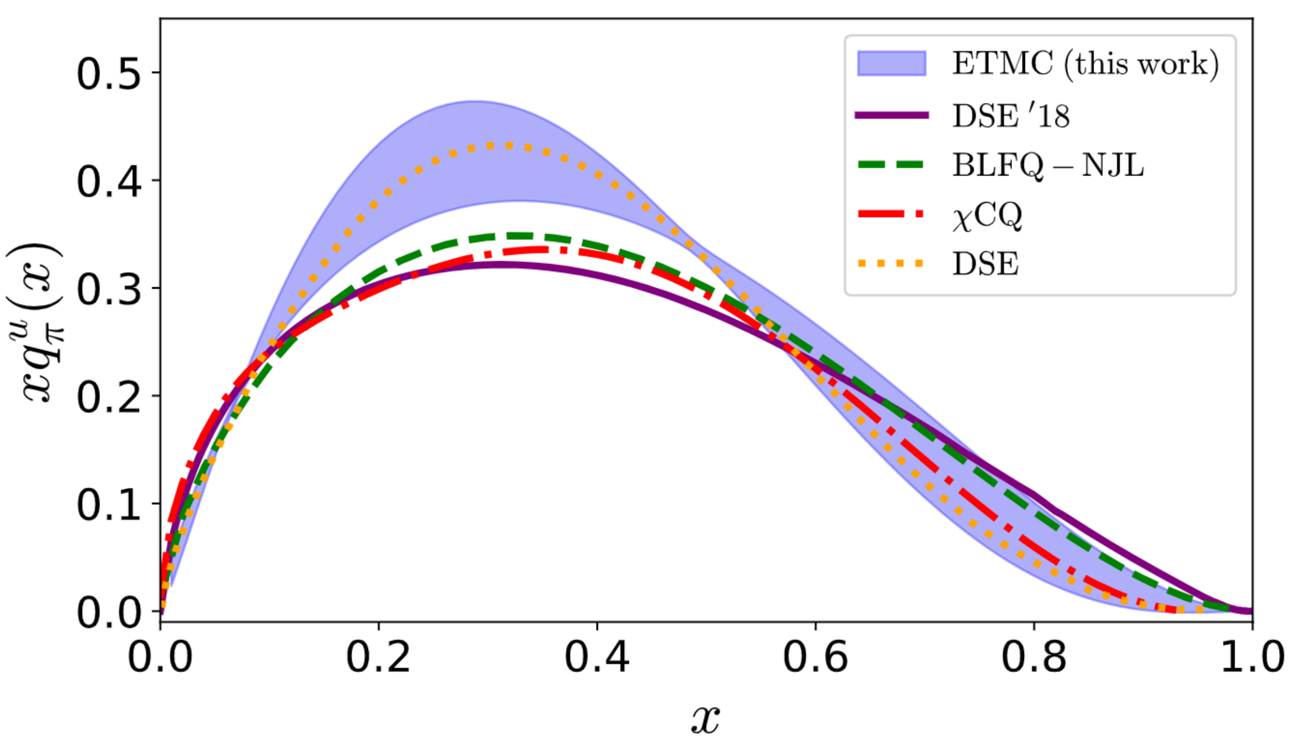




- ★ **Lattice calculations of pseudo-PDFs and current-current correlators (LCS) use non-local operators**
- ★ **Very good agreement with PDF from LCS**
- ★ **Tension with E615 data in region** $x \in [0.2 - 0.55]$
- ★ **Large-x behavior compatible with rescaled ASV**
- ★ **Peak of lattice data compatible with DSE 2016**
- ★ **Small- and large-x regions compatible with models**

Pion: Comparison with other studies

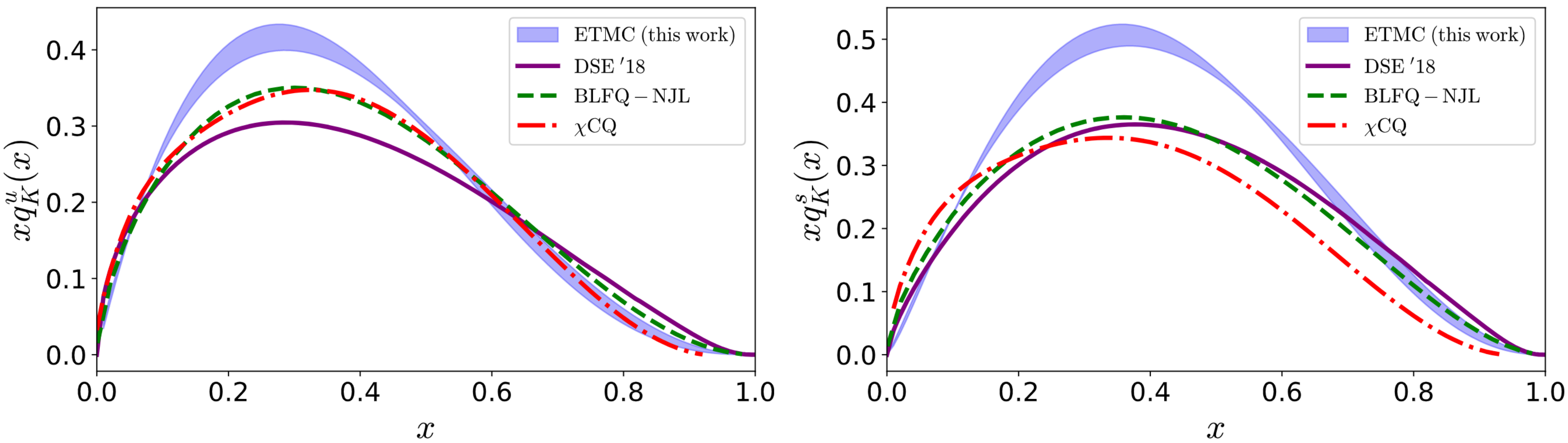




- ★ **Lattice calculations of pseudo-PDFs and current-current correlators (LCS) use non-local operators**
- ★ **Very good agreement with PDF from LCS**
- ★ **Tension with E615 data in region** $x \in [0.2 - 0.55]$
- ★ **Large-x behavior compatible with rescaled ASV**
- ★ **Peak of lattice data compatible with DSE 2016**
- ★ **Small- and large-x regions compatible with models**

Comparison qualitative!

Kaon: Comparison with other studies



- ★ **Very limited studies**
- ★ **Peak of lattice data higher than models**
- ★ **Mellin moment $\langle x^4 \rangle_K^{u,s}$ compatible with lattice data**

Mellin moments from PDFs

$$\langle x^n \rangle = \int x^n f(x)\, dx$$

Mellin moments from PDFs

$$\langle x^n \rangle = \int x^n f(x)\, dx$$

q_M^f	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$	$\langle x^5 \rangle$	$\langle x^6 \rangle$
q_π^u	0.230(3)(7)	0.087(5)(8)	0.041(5)(9)	0.023(5)(6)	0.014(4)(5)	0.009(3)(3)
q_K^u	0.217(2)(5)	0.079(2)(1)	0.036(2)(2)	0.019(1)(2)	0.011(1)(2)	0.007(1)(1)
q_K^s	0.279(1)(5)	0.115(2)(6)	0.058(2)(2)	0.033(2)(2)	0.021(1)(2)	0.014(1)(2)

Mellin moments from PDFs

$$\langle x^n \rangle = \int x^n f(x)\, dx$$

q_M^f	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$	$\langle x^5 \rangle$	$\langle x^6 \rangle$
q_π^u	0.230(3)(7)	0.087(5)(8)	0.041(5)(9)	0.023(5)(6)	0.014(4)(5)	0.009(3)(3)
q_K^u	0.217(2)(5)	0.079(2)(1)	0.036(2)(2)	0.019(1)(2)	0.011(1)(2)	0.007(1)(1)
q_K^s	0.279(1)(5)	0.115(2)(6)	0.058(2)(2)	0.033(2)(2)	0.021(1)(2)	0.014(1)(2)

For comparison

JAM: $\langle x^4 \rangle_\pi^u = 0.027(2)$

BLFQ-NJL

$\langle x^4 \rangle_K^u = 0.021(3)$

$\langle x^4 \rangle_K^s = 0.029(5)$

Mellin moments from PDFs

$$\langle x^n \rangle = \int x^n f(x)\, dx$$

q_M^f	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$	$\langle x^5 \rangle$	$\langle x^6 \rangle$
q_π^u	0.230(3)(7)	0.087(5)(8)	0.041(5)(9)	0.023(5)(6)	0.014(4)(5)	0.009(3)(3)
q_K^u	0.217(2)(5)	0.079(2)(1)	0.036(2)(2)	0.019(1)(2)	0.011(1)(2)	0.007(1)(1)
q_K^s	0.279(1)(5)	0.115(2)(6)	0.058(2)(2)	0.033(2)(2)	0.021(1)(2)	0.014(1)(2)

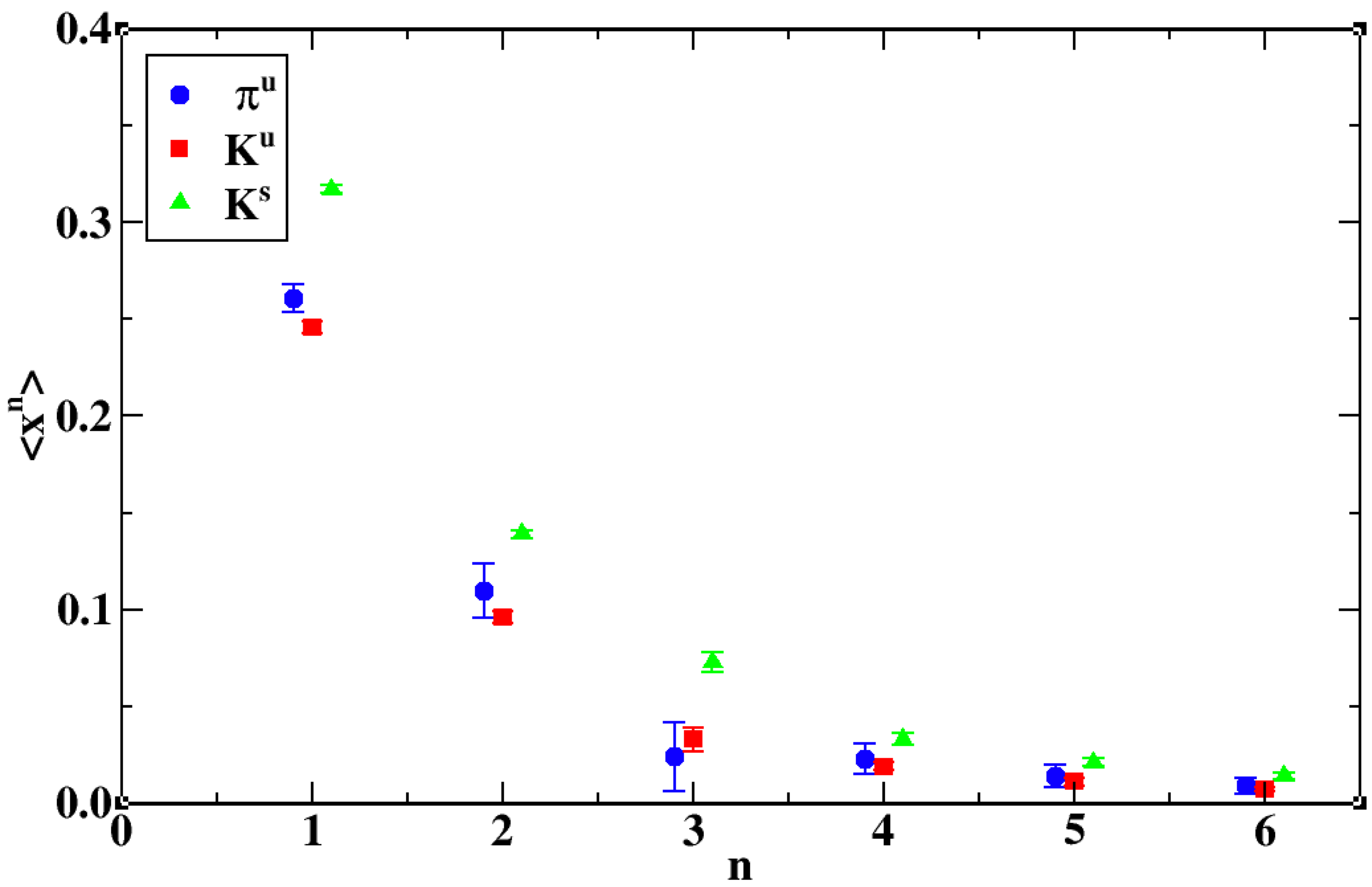
For comparison

JAM: $\langle x^4 \rangle_\pi^u = 0.027(2)$

BLFQ-NJL

$\langle x^4 \rangle_K^u = 0.021(3)$

$\langle x^4 \rangle_K^s = 0.029(5)$



What's next?

★ **Pion and kaon form factors**

★ **SU(3) flavor symmetry breaking**

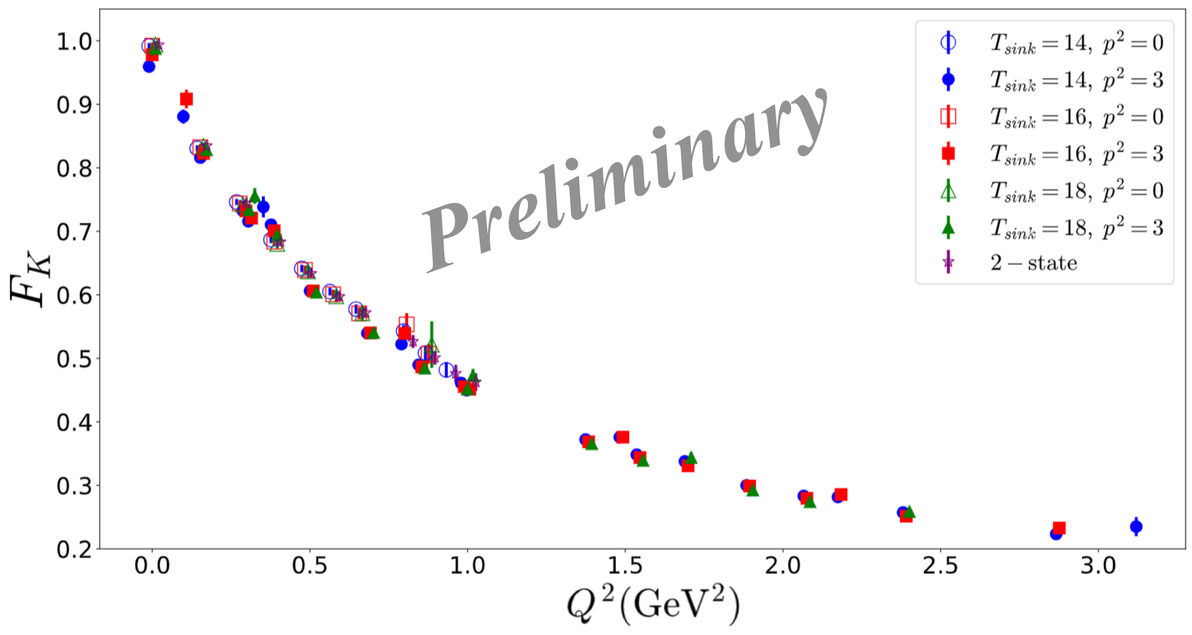
★ **Transverse spin**
(quark probability density in impact parameter space)

What's next?

★ **Pion and kaon form factors**

★ **SU(3) flavor symmetry breaking**

★ **Transverse spin**
(quark probability density in impact parameter space)

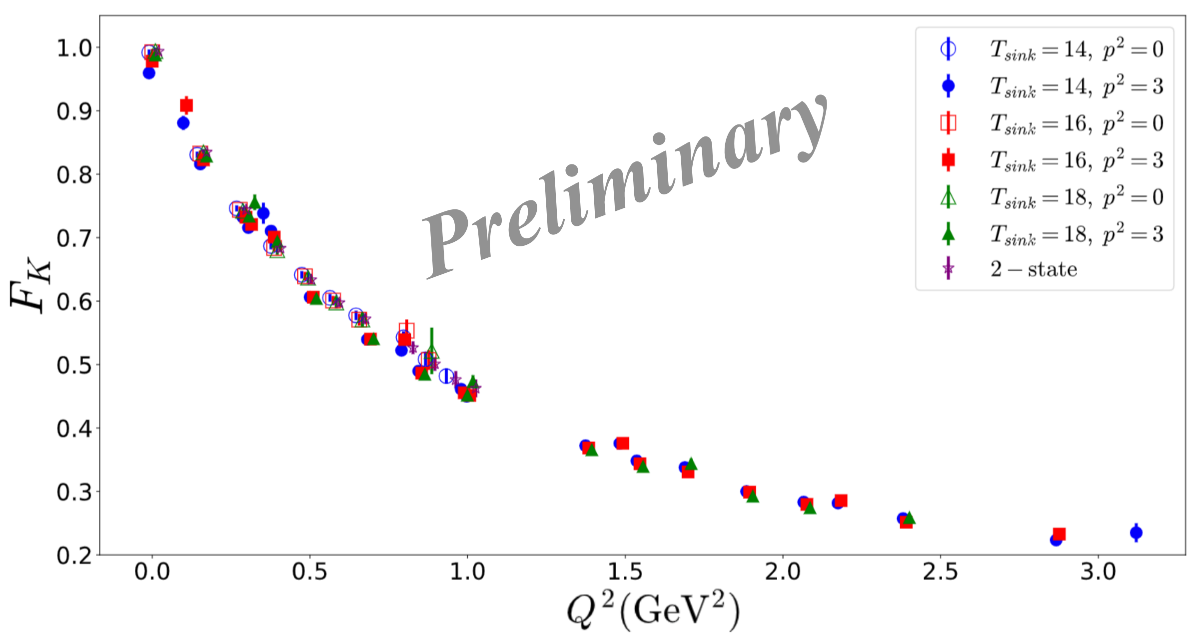


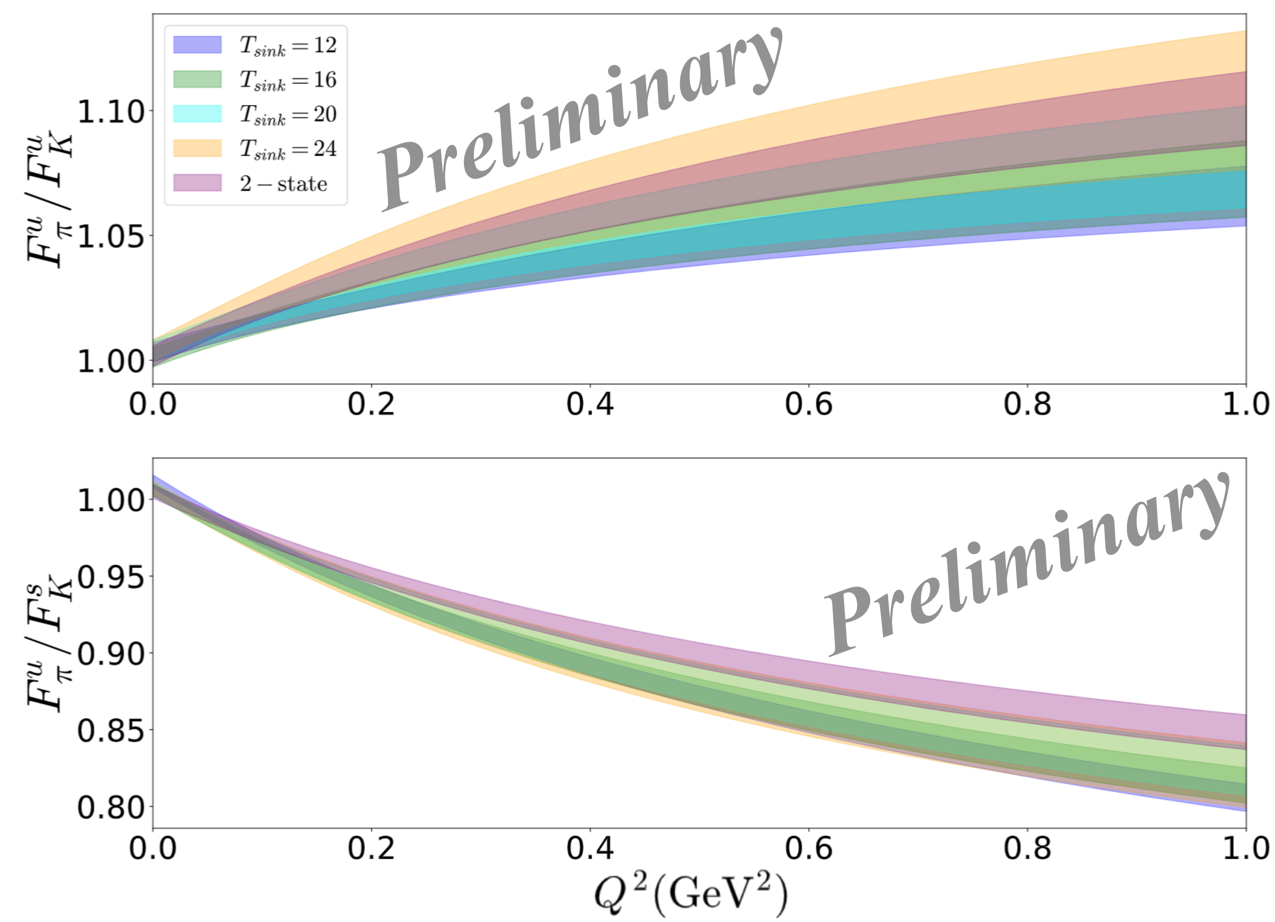
What's next?

★ **Pion and kaon form factors**

★ **SU(3) flavor symmetry breaking**

★ **Transverse spin**
(quark probability density in impact parameter space)





Concluding Remarks

Concluding Remarks

★ Accessing Mellin moments up to $\langle x^3 \rangle$ without mixing is feasible

★ SU(3) flavor symmetry breaking non-negligible

★ Reconstruction of PDFs using up to $\langle x^3 \rangle$ is possible

★ Our lattice data propose a $(1-x)^2$ decay for both pion and kaon PDFs

Concluding Remarks

- ★ Accessing Mellin moments up to $\langle x^3 \rangle$ without mixing is feasible
- ★ SU(3) flavor symmetry breaking non-negligible
- ★ Reconstruction of PDFs using up to $\langle x^3 \rangle$ is possible
- ★ Our lattice data propose a $(1-x)^2$ decay for both pion and kaon PDFs
- ★ A lot of work needed to quantify systematic uncertainties
- ★ Disconnected contributions to be evaluated
- ★ Future direction: form factors and generalized form factors

Concluding Remarks

- ★ Accessing Mellin moments up to $\langle x^3 \rangle$ without mixing is feasible
- ★ SU(3) flavor symmetry breaking non-negligible
- ★ Reconstruction of PDFs using up to $\langle x^3 \rangle$ is possible
- ★ Our lattice data propose a $(1-x)^2$ decay for both pion and kaon PDFs
- ★ A lot of work needed to quantify systematic uncertainties
- ★ Disconnected contributions to be evaluated
- ★ Future direction: form factors and generalized form factors

Thank you

U.S. DEPARTMENT OF ENERGY | Office of Science

DOE Early Career Award (NP)
Grant No. DE-SC0020405

Argonne NATIONAL LABORATORY

NSF

Grant No. PHY-1714407