Modeling binary neutron star merger remnants

Giovanni Camelio
with Tim Dietrich, Miguel Marques, Brynmor Haskell, and Stephan Rosswog

Camelio+2019: PRD 100:123001 (2019), arXiv:1908.11258
Camelio+2021: PRD 103:063014 (2021), arXiv:2011.10557

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Motivation/1: why binary neutron star merger remnants?

- are the metastable result of a binary neutron star merger
- are neutron rich, compact, hot, and differentially rotating
- may collapse to **black holes** or become **neutron stars**
- if they don't collapse, **quasi-stationary** after ~ 15 ms
- emit light, gravitational waves, and neutrinos
- information on high-density, high-temperature equation of state, fundamental theory of gravity, cosmological constant, nucleosynthesis, ...

Motivation/2: why stationary codes?

hydrodynamical code	stationary code
more accurate ✔	more approximate ✘
complex ✘	simpler ✔
slow ✘	fast ✔
supercomputer ✘	any computer ✔

We can use stationary codes for:

- model the output of hydrodynamical simulations
- perform extensive parametric explorations
- create initial conditions for hydrodynamical simulations

Motivation/3: what should be improved?

Stationary codes used the strong, unnecessary, and wrong "effectively barotropic" approximation:

$$p = p(\rho) \qquad T = T(\rho) \qquad s = s(\rho) \qquad Y = Y(\rho)$$

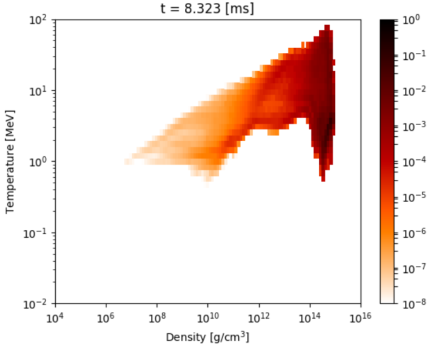

Figure: Thermodynamical conditions in a BNSM remnant (Perego+2019, EPJA 55:124)

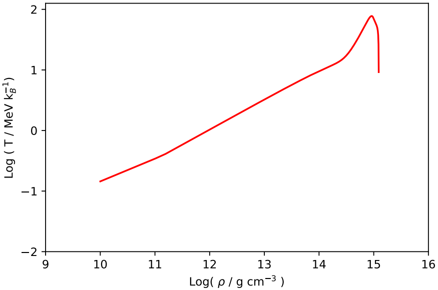

Figure: Best approximation with an effective barotrope

Introduction/1: how does a stationary code work?

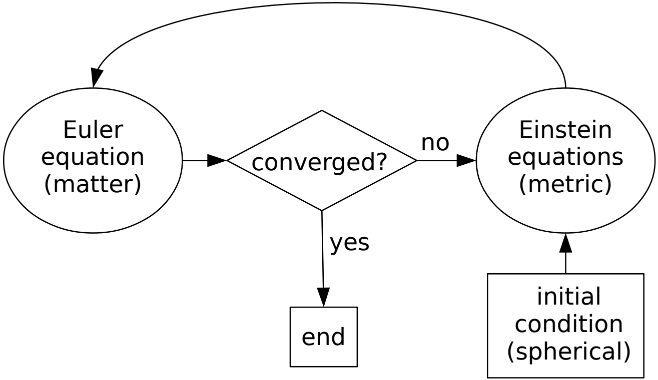

Figure: the self-consistent field method

our code is based on XNS-v2 (Bucciantini+2011, Pili+2014)

Introduction/2: how was the Euler equation solved?

$$\nabla\Big(\overbrace{\log\alpha}^{\text{gravity}} - \overbrace{\log\Gamma_L}^{\text{rotation}}\Big) + \overbrace{\frac{\nabla p}{\hbar(p,T)}}^{\text{bouyancy}} + \overbrace{F(\ell,\Omega)\nabla\Omega}^{\text{rotation}} = 0$$

effective barotrope: $\hbar = \hbar(p)$ $F = F(\Omega)$

$$\nabla\Big(\overbrace{\log\alpha}^{\text{gravity}} - \overbrace{\log\Gamma_L}^{\text{rotation}} + \underbrace{\overbrace{H(p)}^{\text{bouyancy}}}_{\int\frac{\mathrm{d}p}{\hbar}} + \underbrace{\overbrace{\mathcal{F}(\Omega)}^{\text{rotation}}}_{\int F\mathrm{d}\Omega} = \text{const}\Big)$$

$$-\log\alpha(r,\theta) + \log\Gamma_L(r,\theta,\Omega) \equiv \text{const} + H(p) + \mathcal{F}(\Omega)$$

Introduction/3: consequences of the approximation

... for a binary neutron star merger remnant:

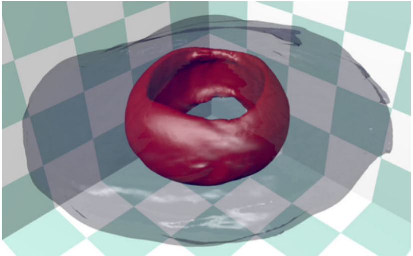

Figure: Entropy distribution from dynamical simulation (Kastaun+2016, PRD 94:044060)

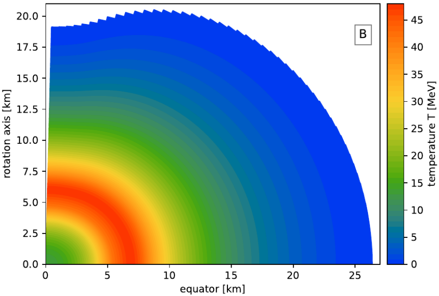

Figure: Effectively barotropic temperature distribution from stationary code (Camelio+2021)

Idea/1: comparison with the thermodynamic potential

$$\nabla\Big(\underbrace{\overbrace{-\log\alpha}^{\text{gravity}}+\overbrace{\log\Gamma_L}^{\text{rotation}}}_{Q}\Big) = \frac{\overbrace{\nabla p}^{\text{bouyancy}}}{\mathscr{h}} + \overbrace{F\nabla\Omega}^{\text{rotation}}$$

1st law of thermodynamics:

$$\mathrm{d}E = T\mathrm{d}S - p\mathrm{d}V$$

$$E = E(S,V)$$

$$T(S,V) = \left.\frac{\partial E}{\partial S}\right|_V$$

$$p(S,V) = -\left.\frac{\partial E}{\partial V}\right|_S$$

Euler equation:

$$\mathrm{d}Q = \frac{\mathrm{d}p}{\mathscr{h}} + F\mathrm{d}\Omega$$

$$Q = Q_0 + H(p) + \mathcal{F}(\Omega)$$

$$\mathscr{h}^{-1}(\boldsymbol{p}) = \boldsymbol{H'(p)} = \left.\frac{\partial Q}{\partial p}\right|_\Omega$$

$$F(\boldsymbol{\Omega}) = \boldsymbol{\mathcal{F}'(\Omega)} = \left.\frac{\partial Q}{\partial \Omega}\right|_p$$

Idea/1: comparison with the thermodynamic potential

$$\nabla\Big(\underbrace{\overbrace{-\log\alpha}^{\text{gravity}}+\overbrace{\log\Gamma_L}^{\text{rotation}}}_{Q}\Big) = \frac{\overbrace{\nabla p}^{\text{bouyancy}}}{\hbar} + \overbrace{F\nabla\Omega}^{\text{rotation}}$$

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$$\mathrm{d}Q = \frac{\mathrm{d}p}{\hbar} + F\mathrm{d}\Omega$$

$$Q = Q_0 + H(p) + \mathcal{F}(\Omega) + \boldsymbol{bH(p)\mathcal{F}(\Omega)}$$

$$\hbar^{-1}(p,\boldsymbol{\Omega}) = \left.\frac{\partial Q}{\partial p}\right|_\Omega$$

$$F(\boldsymbol{p},\Omega) = \left.\frac{\partial Q}{\partial \Omega}\right|_p$$

Result/1: numerical check

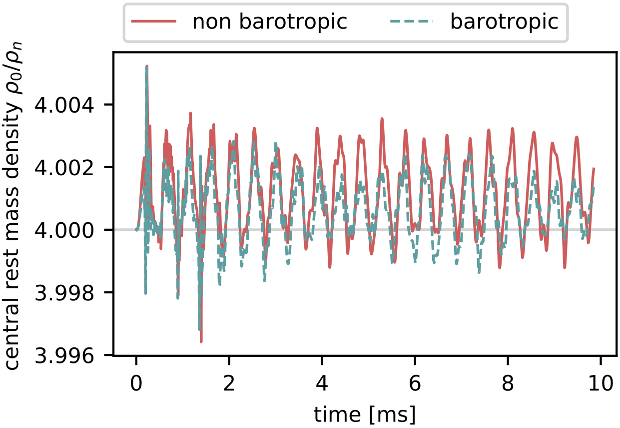

Figure: We checked the stationary model with the BAM general relativistic hydrodynamical code (Camelio+2019)

Idea/2: Legendre transformation

Thermodynamics:

$$\text{FreeEnergy} = E - TS$$

$$\mathrm{d}\text{FreeEnergy} = -S\mathrm{d}T - p\mathrm{d}V$$

$$\text{FreeEnergy} = \text{FreeEnergy}(T, V)$$

$$S(T,V) = -\left.\frac{\partial \text{FreeEnergy}}{\partial T}\right|_V$$

$$p(T,V) = -\left.\frac{\partial \text{FreeEnergy}}{\partial V}\right|_T$$

Euler equation:

$$\tilde{Q} = Q - \Omega F$$

$$\mathrm{d}\tilde{Q} = \frac{\mathrm{d}p}{\hbar} - \Omega \mathrm{d}F$$

$$\tilde{Q} = \tilde{Q}_0 + H(p) + \boldsymbol{G(F)} + bH(p)\boldsymbol{G(F)}$$

$$\hbar^{-1}(p, \boldsymbol{F}) = \left.\frac{\partial \tilde{Q}}{\partial p}\right|_F$$

$$\Omega(\boldsymbol{p}, F) = -\left.\frac{\partial \tilde{Q}}{\partial F}\right|_p$$

- in a merger remnant the angular velocity Ω is not monotonic with the distance from the axis, while F does
- equivalent to Uryū+2017 for a non-barotropic star

Idea/3: apply to the merger remnant

non-barotropic: $\tilde{Q}(p, F) = \tilde{Q}_0 + H(p) + \boldsymbol{G}(F) + bH(p)\boldsymbol{G}(F)$

GRHD simulation:

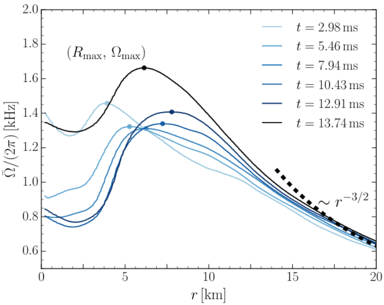

Figure: Hanauske+2017 (PRD 96:043004)

effectively barotropic approx.:

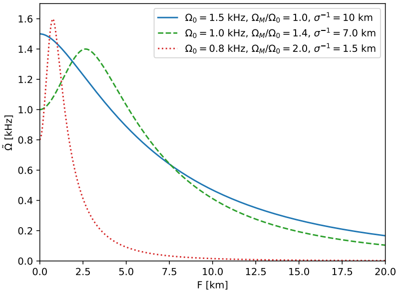

Figure: piecewise polytrope, parameters: Ω_0, Ω_M, radial scale (Camelio+2021)

Idea/3: apply to the merger remnant

non-barotropic: $\tilde{Q}(p,F) = \tilde{Q}_0 + \boldsymbol{H}(p) + G(F) + b\boldsymbol{H}(p)G(F)$

GRHD simulation:

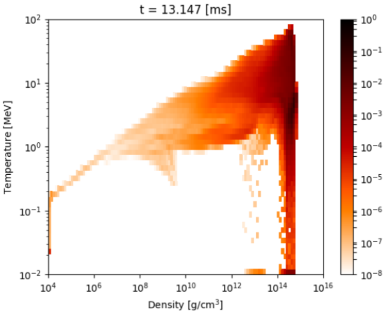

Figure: Perego+2019 (EPJA 55:124)

effectively barotropic approx.:

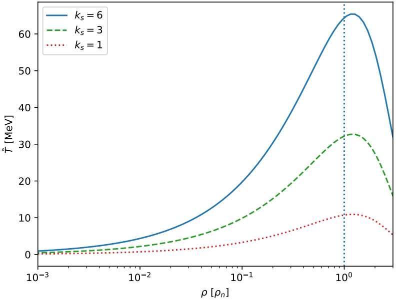

Figure: $T \propto k_s \frac{\rho^{\Gamma_T}}{1+\exp(\rho/\rho_n - 1)}$ (Camelio+2021)

Result/2: parametric study of the remnant

- 100'000 configurations varying 6 parameters $\times$ 4 EOSs
- cold piecewise polytrope with SLy crust + thermal $\Gamma_{\rm th} = 1.75$

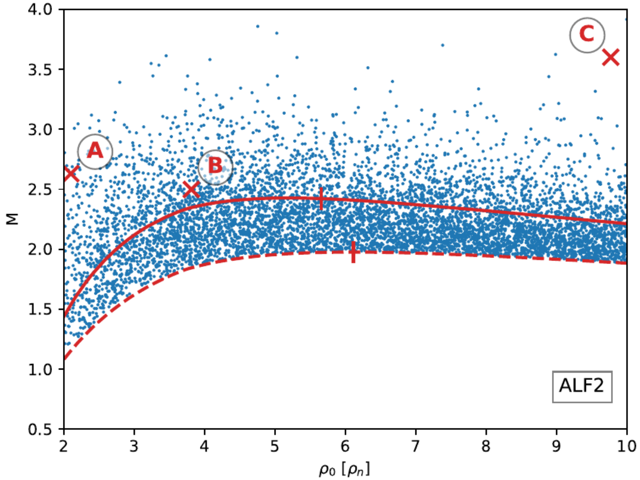

Figure: Camelio+2021

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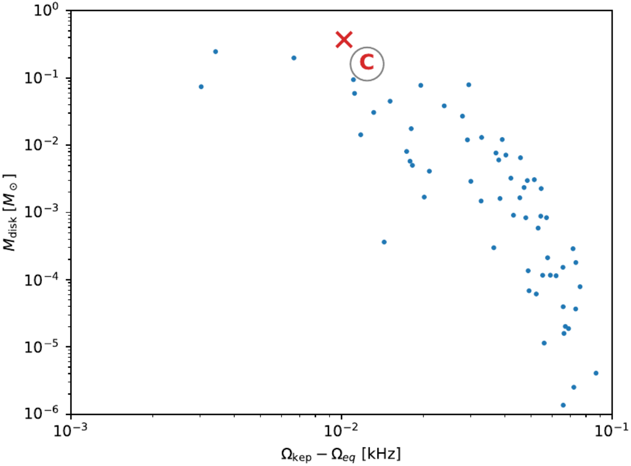
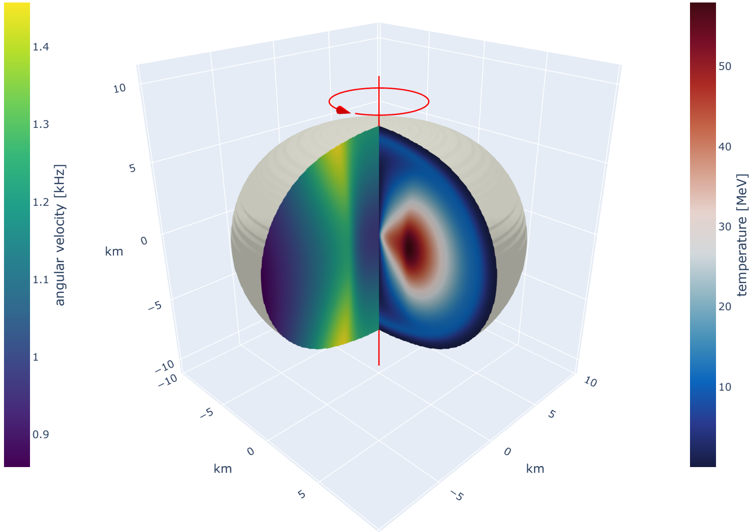


Figure: Camelio+2021

Result/3: remnant structure



Result/3: remnant structure

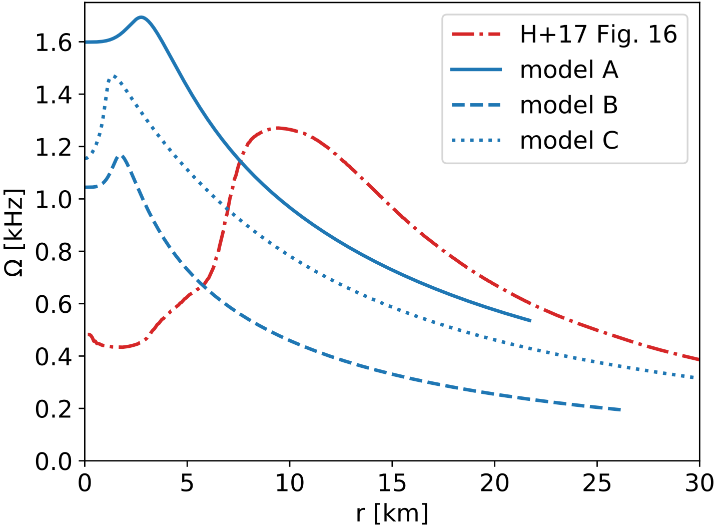


Figure: Camelio+2021

Result/3: remnant structure

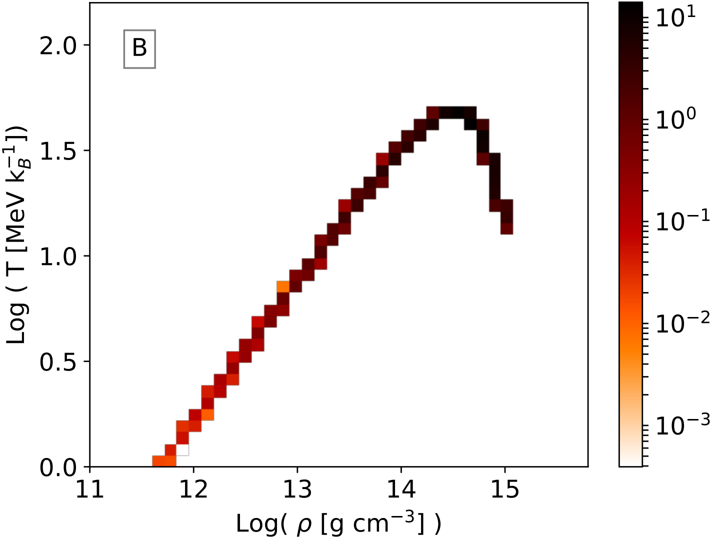


Figure: Camelio+2021 ($b \approx 0$)

Result/3: remnant structure

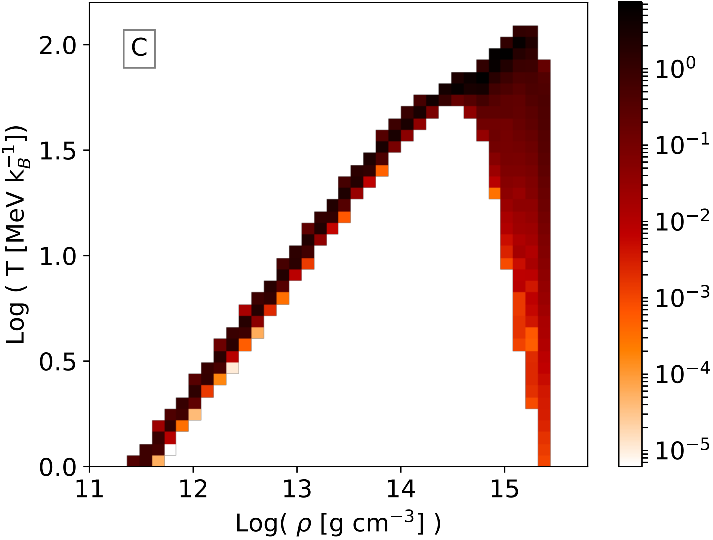


Figure: Camelio+2021 ($b \approx -1.6$)

Outlook/1: arbitrary thermal & rotational profiles

our procedure:

$$Q(p, F|\text{pars}) \quad \rightarrow \quad T(r,\theta), \quad \Omega(r,\theta), \quad \rho(r,\theta)$$

desiderata:

$$T_{ij}, \quad \Omega_{ij} \quad \rightarrow \quad \rho(r_i, \theta_j)$$

two possible approaches:

1. iterate on ‘pars’ till
   $Q(p, F|\text{pars}) \quad \rightarrow \quad T(r_i,\theta_j) \equiv T_{ij}, \quad \Omega(r_i,\theta_j) \equiv \Omega_{ij}$
   problem: we should iterate on ∼1000 parameters...
2. solve the Euler equation directly (see Roxburgh, Rieutord & Espinoza Lara, Fujisawa)
   problem: difficult to implement.

Outlook/2

- add neutrino diffusion and evolve quasi-stationarily the remnant (requirement: arbitrary T, Ω profiles)
- use the stationary model as background for quasi-periodic stellar perturbation (GW), see Krüger & Kokkotas (2020)
- include the magnetic field and meridional currents (convection) in the Euler equations. Equivalent to consider other species in thermodynamics: $+\mu \mathrm{d}N$
- apply the same non-barotropic modelling for other systems: Newtonian stars, accretion disks, proto-neutron stars, quark-hadron phase transition remnants...

Conclusions

Camelio, Dietrich, Marques & Rosswog (2019), *Rotating neutron stars with nonbarotropic thermal profile*, PRD 100:123001 (arXiv:1908.11258), editor's suggestion.

Easy, fast, and general model of hot and differentially rotating neutron stars without the effectively barotropic approximation

Camelio, Dietrich, Haskell & Rosswog (2021), *Axisymmetric models for neutron star merger remnants with realistic thermal and rotational profiles*, PRD 103:063014 (arXiv:2011.10557), dataset on zenodo.

Application to the new method to the binary neutron star merger remnant and extensive parameter space exploration

Backup/1: re-derivation of the von Zeipel theorem

hypothesis: axisymmetry, stationarity, no magnetic field, no meridional flows (e.g. convection).

thesis: a star is effectively barotropic iff the angular velocity Ω depends only on the specific angular momentum ℓ.

demonstration:

$$F \equiv \frac{\ell}{1 - \Omega\ell} \quad \Rightarrow \quad F(\Omega) \leftrightarrow \ell(\Omega)$$

$$0 = \partial_\Omega \mathscr{h}^{-1}(p) = \partial_{\Omega,p} Q = \partial_{p,\Omega} Q = \partial_p F(\Omega) = 0 \qquad \square$$

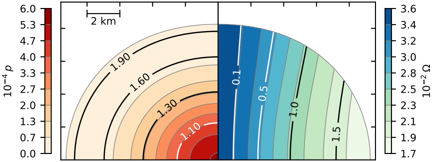


Figure: Effectively barotropic (Camelio+2019)

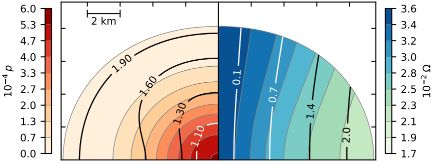


Figure: Non-barotropic (Camelio+2019)

Backup/2: EOS inversion

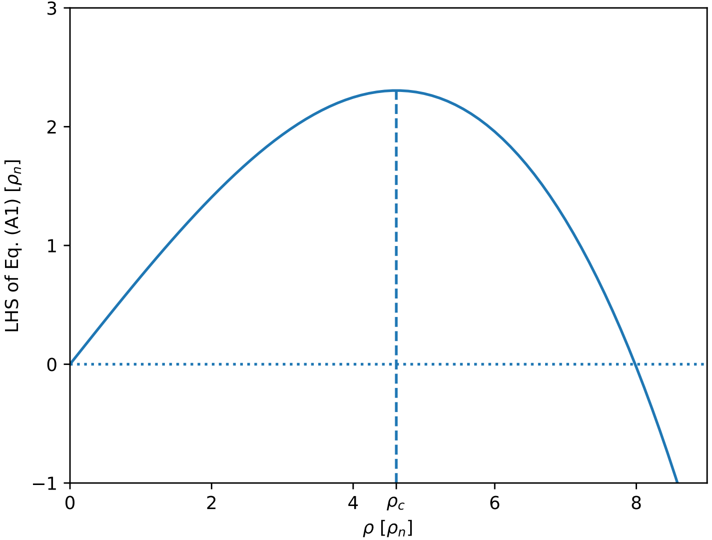


Figure: LHS $= (\Gamma_{\rm th} - 1)\mathscr{h} - \Gamma_{\rm th} p$ (Camelio+2021)

Backup/3: accretion disk (Camelio+2021)