# Deeply Virtual Compton Scattering in the MIT bag model


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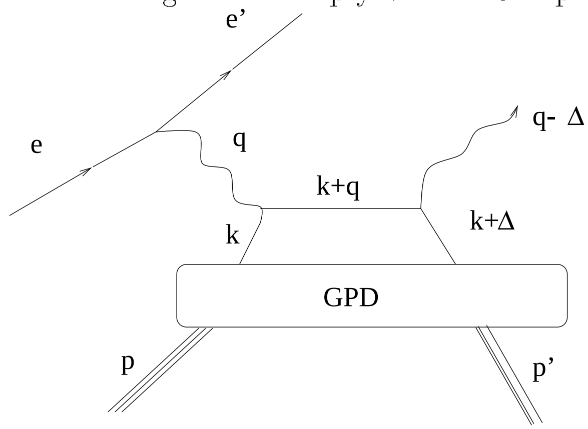
Figure 1: Schematic diagram of Deeply Virtual Compton Scattering




## 1 Introduction

The structure of the proton has intrigued scientists since the discovery that it was not a point-like Dirac particle by a SLAC-M.I.T team in the 1960s.[1] Current studies on proton structure include developing models that give the quark distribution within the nucleon. One method used to study quark distribution is Deeply Virtual Compton Scattering (DVCS). In DVCS, a high energy, high momentum electron emits a virtual photon which is absorbed by a quark within the proton as depicted in figure 1. At some time later, the quark emits a real photon and is absorbed back into the proton. This causes the proton to scatter with a final energy equal to its initial energy plus the energy gained by the quark.

The model of the proton that I have used in my studies of DVCS is the MIT bag model. In the MIT bag model, the proton is assumed to consist of three quarks where a binding pressure is introduced to constrain the quarks to a specified radius. The introduction of a binding pressure provides stability and conservation of energy and momentum.[2]

The MIT bag model provides a means of computing the quark distribution. This is valuable because knowledge about the quark distribution ultimately provides information about the structure of the proton. In the calculations that follow, information shall be provided leading to the quark distribution of various momentum and spin states. All calculations shall be made in the Bjorken limit where $Q^2 = -q^2 \to \infty$ and $x$ is fixed.[2]

## 2 MIT Bag Model

In this section, we will define quantities to be used in our calculations involving a MIT bag in its ground state. We begin by introducing the equation

$$m(x) \equiv \frac{1}{4\pi}\int d\xi^- e^{iq^+\xi^-}\langle N,\mathbf{p}',s'|\bar{\psi}(\xi)\gamma^+\psi(0)|N,\mathbf{p},s\rangle|_{\xi^+=\xi_\perp=0} \qquad (1)$$

where $\xi^\mu = (\xi^0, \xi_\perp, \xi^3)$ defines the usual light-cone coordinates and

$$\xi^\pm = \frac{\xi^0 \pm \xi^3}{\sqrt{2}},\ q^+ = \frac{-Mx}{\sqrt{2}},\ \gamma^+ = \frac{\gamma^0+\gamma^3}{\sqrt{2}}. \qquad (2)$$

A Peierls-Yoccoz projection of the MIT bag ground state is given by[3]

$$|N,\mathbf{p},s\rangle = \lambda\int d^3a\ e^{i\mathbf{p}\cdot\mathbf{a}}|\mathbf{R}=\mathbf{a}\rangle. \qquad (3)$$

In this form, the projected MIT bag is defined as

$$|\mathbf{R}=\mathbf{a}\rangle = b_0^\dagger(\mathbf{a})b_0^\dagger(\mathbf{a})b_0^\dagger(\mathbf{a})|EB;\mathbf{R}=\mathbf{a}\rangle, \qquad (4)$$

where $|EB;\mathbf{R}=\mathbf{a}\rangle$ is the empty bag centered at $\mathbf{R}=\mathbf{a}$ and $b_0^\dagger(\mathbf{a})$ is the quark creation operator at $\mathbf{a}$. The indices for quark flavor and color have been suppressed on $b_0^\dagger(\mathbf{a})$ to avoid confusion. A normalization constant $\lambda$ is defined for the MIT bag such that

$$\begin{aligned}\lambda^2 &= \frac{2M}{\int d^3a\langle\mathbf{R}=\mathbf{0}|\mathbf{R}=\mathbf{a}\rangle}\\ &= \frac{2M}{V}.\end{aligned} \qquad (5)$$

This quantity serves as an invariant in momentum-space.

The field operator in Eq.(1) is given by[3]

$$\psi(\mathbf{x},t) = \sum_{n,\kappa}[b_{n,\kappa}(\mathbf{a})\phi^{(s)}_{n,\kappa}(\mathbf{x}-\mathbf{a})e^{-i\omega_{n\kappa}t/R_0} + d^\dagger_{n,\kappa}(\mathbf{a})\tilde{\phi}^{(s)}_{n,\kappa}(\mathbf{x}-\mathbf{a})e^{i\omega_{n\kappa}t/R_0}] \quad (6)$$

where the wavefunction is

$$\phi^{(s)}_{n\kappa}(\mathbf{x}) = \frac{N_{n\kappa}}{\sqrt{4\pi}}\left[\begin{array}{c} j_0(\omega_{n\kappa}|\mathbf{x}|/R_0)U_M \\ i\sigma\cdot\hat{\mathbf{x}}j_1(\omega_{n\kappa}|\mathbf{x}|/R_0)U_M\end{array}\right]\Theta(R_0-x), \qquad (7)$$

the quark creation operator is given by $b_{n,\kappa}(\mathbf{a})$, and the antiquark creation operator is $d^\dagger_{n,\kappa}(\mathbf{a})$. Since we are only considering the ground state of the nucleon, we will simplify the variables used such that

$$\omega_{0,-1} \equiv \epsilon = 2.04,\ \phi_{0,-1}(\mathbf{x}) = \phi_0(\mathbf{x}),\ N_{0,-1}\equiv N \qquad (8)$$

## 3 Calculations

In this section, we will use the definitions provided in Section 2 to calculate quantities relating to the quark distribution in a proton. Given the definition for $m(x)$ in Eq. (1), consider the matrix element

$$\mathcal{M} = \langle N, \mathbf{p}', s'|\bar{\psi}(\xi)\gamma^+\psi(0)|N, \mathbf{p} = \mathbf{0}, s\rangle. \tag{9}$$

This quantity is related to the probability of finding a proton in an initial state $\mathbf{p} = 0$, spin $= s$ and a final state $\mathbf{p} = \mathbf{p}'$, spin$= s'$.

In the following calculations, choose $s = s' = +\frac{1}{2}$. Expanding $\mathcal{M}$ using the definitions above gives

$$\begin{aligned} \mathcal{M} &= \lambda^2 \int d^3a d^3b \ \langle \mathbf{R} = \mathbf{a}|e^{-i\mathbf{p}'\cdot\mathbf{a}}\bar{\psi}(\xi)\gamma^+\psi(0)|\mathbf{R} = \mathbf{b}\rangle \\ &= 18\lambda^2 \int d^3a d^3b \ \Delta^2(\mathbf{a} - \mathbf{b})e^{-i(\mathbf{p}'\cdot\mathbf{a} - \epsilon\xi_0/R_0)}\bar{\phi}_0(\xi - \mathbf{a})\gamma^+\phi_0(-\mathbf{b})\langle EB; \mathbf{R} = \mathbf{a}|EB; \mathbf{R} = \mathbf{b}\rangle \end{aligned} \tag{10}$$

where the function $\Delta(\mathbf{a} - \mathbf{b})$ comes from the anticommutation relation[3]

$$\begin{aligned} \{b_n^\dagger(\mathbf{a}), b_m(\mathbf{b})\} &= \int d^3r \ \phi_n^\dagger(\mathbf{r} - \mathbf{a})\phi_m(\mathbf{r} - \mathbf{b}) \\ &= \Delta_{nm}(\mathbf{a} - \mathbf{b}). \end{aligned} \tag{11}$$

It can be shown that

$$\mathcal{M} = 18\lambda^2 \int \frac{d^3k}{(2\pi)^3}\tilde{F}(\mathbf{k})\bar{\tilde{\phi}}_0(\mathbf{p}' - \mathbf{k})\gamma^+\tilde{\phi}_0(\mathbf{k})e^{-i(\mathbf{p}'\cdot\mathbf{a} - \mathbf{k}\cdot\xi - \epsilon\xi_0/R_0)} \tag{12}$$

where[3]

$$\tilde{F}(\mathbf{k}) = \int d^3z \ e^{-i\mathbf{k}\cdot\mathbf{z}}\Delta^2(\mathbf{z})\langle EB; \mathbf{R} = \mathbf{a}|EB; \mathbf{R} = \mathbf{b}\rangle, \tag{13}$$

$$\tilde{\phi}_0(\mathbf{k}) = \int d^3z \ e^{-i\mathbf{k}\cdot\mathbf{z}}\phi_0(\mathbf{z}). \tag{14}$$

The simplest case to consider is when $\mathbf{p}' = \mathbf{0}$. This reduces $m(x)$ to the quark distribution density

$$\begin{aligned} q(x) &= \frac{1}{4\pi}\int d\xi^- e^{iq^+\xi^-}\mathcal{M}|_{\mathbf{p}'=\mathbf{0}} \\ &= 18\lambda^2\frac{1}{4\pi}\int d\xi^- \frac{d^3k}{(2\pi)^3}\tilde{F}(\mathbf{k})\bar{\tilde{\phi}}_0(\mathbf{k})\gamma^+\tilde{\phi}_0(\mathbf{k})e^{i(q^+\xi^- + \mathbf{k}\cdot\xi + \epsilon\xi_0/R_0)} \\ &= 9\sqrt{2}\lambda^2\int_{k_-}^{\infty}\frac{kdk}{(2\pi)^2}\tilde{F}(\mathbf{k})\bar{\tilde{\phi}}_0(\mathbf{k})\gamma^+\tilde{\phi}_0(\mathbf{k}) \end{aligned} \tag{15}$$

where $k_- = |Mx - \epsilon/R_0|$. Plugging in the wavefunctions given in Eq. (7) yields

$$q(x) = C\int_{\beta_-}^{\infty} d\beta\ \tilde{G}(\beta)[t_0^2(\epsilon,\beta) + t_1^2(\epsilon,\beta) + \frac{2\beta_-}{\beta} t_0(\epsilon,\beta)t_1(\epsilon,\beta)] \tag{16}$$

where $C$ is a normalization constant and[3]

$$\begin{aligned}
\tilde{G}(\beta) &= \int_0^1 ydy\ \sin(2\beta \mathrm{y})\Delta^2(2\mathrm{R_0 y})\langle \mathrm{EB}; \mathbf{R} = 2\mathrm{R_0 y}\hat{\mathbf{z}}|\mathrm{EB}; \mathbf{R} = \mathbf{0}\rangle, \\
t_0(\epsilon,\beta) &= \int_0^1 y^2 dy\ j_0(\epsilon y) j_0(\beta y), \\
t_1(\epsilon,\beta) &= \int_0^1 y^2 dy\ j_1(\epsilon y) j_1(\beta y), \\
\beta &= \mathbf{k}R_0 \\
\beta_- &= |MR_0 x - \epsilon|.
\end{aligned} \tag{17}$$

Assuming that the empty bag matrix elements are approximately constant in the region of integration[3] it is possible to numerically evaluate the quark distribution density q(x).

Now consider the case when $\mathbf{p}' \neq \mathbf{0}$. Using the methods above we see that

$$m(x) = \frac{18\lambda^2}{\sqrt{2}(2\pi)^3}\int d^2k_\perp\ \tilde{F}(\mathbf{k})\bar{\tilde{\phi}}_0(\mathbf{p}' - \mathbf{k})\gamma^+\tilde{\phi}_0(\mathbf{k})|_{k_3 = -(Mx - p_3' - \epsilon/R_0)}. \tag{18}$$

Let us redefine $t_0(\epsilon,\beta)$ and $t_1(\epsilon,\beta)$ as

$$\begin{aligned}
t_0(\epsilon,\beta,\mathbf{p}) &= \int_0^1 y^2 dy\ j_0(\epsilon y) j_0(y|\mathbf{p}R_0 - \beta|), \\
t_1(\epsilon,\beta,\mathbf{p}) &= \int_0^1 y^2 dy\ j_1(\epsilon y) j_1(y|\mathbf{p}R_0 - \beta|).
\end{aligned} \tag{19}$$

Putting in the wavefunctions in Eq.(7) and using the definition for $\tilde{G}(\beta)$ given in Eq.(17),

$$\begin{aligned}
m(x) &= C'\int \frac{d^2\beta_\perp}{\beta}\ \tilde{G}(\beta)[t_0(\epsilon,\beta,\mathbf{p}')t_0(\epsilon,\beta,0) + t_1(\epsilon,\beta,\mathbf{p}')t_1(\epsilon,\beta,0) \qquad (20)\\
&+ \frac{\beta_-}{\beta} t_0(\epsilon,\beta,\mathbf{p}')t_1(\epsilon,\beta,0) + \frac{\beta_-}{\beta} t_0(\epsilon,\beta,0)t_1(\epsilon,\beta,\mathbf{p}')]|_{\beta_3 = -(MR_0 x - p_3 R_0 - \epsilon)}.
\end{aligned}$$

Clearly, this reduces to Eq.(16) when $\mathbf{p}' = \mathbf{0}$.

# 4 Discussion and Conclusion

The calculations given in Section 3 provide a starting point for calculating the quark distribution within a proton. I have shown that the quark distribution when the initial and final state of the proton are $\mathbf{p} = \mathbf{0}$ can be extended to the case when the initial state is $\mathbf{p} = \mathbf{0}$ and the final state is $\mathbf{p} = \mathbf{p}'$. It has yet to be determined whether this process can be extended to calculating antiquark distribution within the proton. In the future, I plan to calculate $m(x)$ when $s \neq s'$ and also compare my results to existing theory.

# References


[1] Rajat K. Bhaduri. Models of the Nucleon: From Quarks to Soliton. Addison-Wesley Publishing Company, Inc. New York. 1988.

[2] Ulrich Mosel. Fields, Symmetries, and Quarks. Springer. New York. 1979.

[3] C. J. Benesh and G. A. Miller, Phys. Rev. D **36**, 1344 (1987).