Motivation for a Neutron-Antineutron Oscillation Search in the Sudbury Neutrino Observatory

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19 August 2003

Abstract

Grand unified theories generally predict the existence of baryon number nonconservation, manifesting itself in processes like proton decay and neutron-antineutron (n-$\overline{n}$) oscillation. Experimental nonobservation of proton decay has led to the development of several experimental searches for n-$\overline{n}$ oscillations. To date, this phenomenon has never been detected, setting an experimental lower limit on the n-$\overline{n}$ oscillation time ($\tau_{n\overline{n}}$) as $\tau_{n\overline{n}} > 1.3 \times 10^8$ s at 90 % C.L. (according to Soudan 2). The Sudbury Neutrino Observatory (SNO), a heavy water Čerenkov detector, can be used to increase the lower limit on $\tau_{n\overline{n}}$ if an experimental search is developed. It can be seen that such an experiment in SNO could result in $\tau_{n\overline{n}} > 2 \times 10^8$ s at 90 % C.L., a sizable improvement over the current limit. As more data becomes available from SNO, this limit can be increased.

1. Introduction

A consequence of most grand unified theories (GUTs) is the prediction of processes violating the conservation of baryon number (B), like proton decay ($p \to e^+\pi^0$ or $p \to \mu^+\pi^0$) and neutron-antineutron (n-$\overline{n}$) oscillations. In light of this prediction, many experiments have been built to search for proton decay. One method used in this search consists of collecting an enormous quantity of atoms and waiting for the proper decay signal to be seen. However, nonobservation of this decay signal in a vast number of experiments has led to doubts about the experimental observability of proton decay. In fact, the current lower limit on the lifetime of the proton is about $10^{31} - 10^{33}$ yrs [1]. This has led to the exclusion of certain GUTs such as the standard SU(5) model of grand unification. In order to further the progress of the GUT selection process, attempts have been made to develop experiments designed to observe n-$\overline{n}$ oscillations, the other possible B nonconserving process.

There are two methods that can be used in an n-$\overline{n}$ oscillation search: either detect antineutrons within a neutron beam or detect the signature from an antineutron-nucleon ($\overline{n}N$) annihilation caused by n-$\overline{n}$ oscillations within the nucleus of an atom. Experiments already designed to search for proton decay in the method described above can also be employed to detect n-$\overline{n}$ oscillations within the nucleus. However, these events occur at rates much lower than n-$\overline{n}$ oscillations in free space. Thus, a larger sample of neutrons must be collected to perform a search using neutrons contained within nuclei than neutrons in free space.

Among existing experiments that can be used to search for n-$\overline{n}$ oscillations using neutrons contained within nuclei is the Sudbury Neutrino Observatory (SNO) [2]. SNO, located near Sudbury, Ontario, is a heavy water (D_2O) Čerenkov detector built in a nickel mine 2070 m below ground designed primarily to detect solar neutrinos. It consists of an acrylic sphere measuring 12 m in diameter filled with heavy water surrounded by an array of 9456 photomultiplier tubes (PMTs) mounted on a stainless steel geodesic sphere 17.8 m in diameter. The acrylic sphere itself is immersed in ultra-pure water (regular H_2O) which is used to shield the heavy water from radioactivity both from the cavern walls and from the PMT array. The physical processes SNO is sensitive to are observed through the detection of Čerenkov photons generated by charged particles within the heavy water. The charged particles will emit a cone of Čerenkov light that will result in a ring pattern of PMT hits upon intersection with the PMT array. Fig. 1 shows an example from SNO data of the ring formed by this process.

An n-$\overline{n}$ oscillation search in SNO can be developed by creating search techniques designed to detect $\overline{n}N$ annihilation signatures. Since the major contribution to $\overline{n}N$ annihilation signatures is from multiple pion channels,

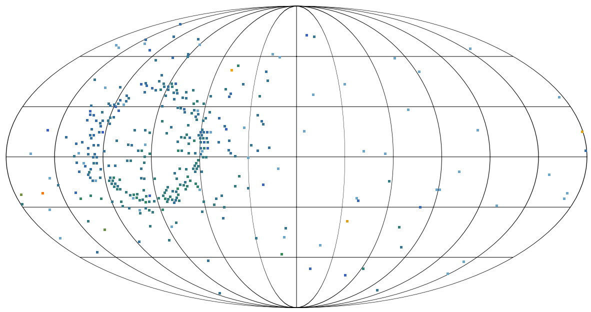

Fig. 1: An example of the ring pattern formed by the intersection of a cone of Čerenkov light with the sphere formed by the PMT array.

SNO must be equipped with the capability of detecting multi-ring events. Ideally, SNO can set the lower limit on the free space n-$\overline{n}$ oscillation time ($\tau_{n\overline{n}}$) at around 2×10^8 s at 90 % C.L. This can be compared with the current lower limit on $\tau_{n\overline{n}}$ of 1.3×10^8 s at 90 % C.L. set by Soudan 2 [3].

2. Motivation from GUTs

The simplest SU(5) model of grand unification[1] predicts the existence of B nonconserving processes that still conserve the quantity $B - L$ (where L is the lepton number) [4]. Consequently, proton decay is the favored B nonconserving process[2]. However, experimental nonobservation of proton decay [1] has fostered interest in GUTs which suppress $\Delta(B - L) = 0$ processes and favor $\Delta(B - L) \neq 0$ processes [6, 7].

One such model is based on the left-right symmetric electroweak group $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{B-L}}$ embedded within the partial unification group $\mathrm{SU(2)_L \times SU(2)_R \times SU(4)_c}$ (where $\mathrm{SU(4)_c}$ contains $\mathrm{U(1)_{B-L} \times SU(3)_c}$ and adopts $B - L$ as the fourth color charge) [6]. Models of this type predict $(B - L)$-symmetry breaking by two units, leading directly to the existence of Majorana neutrinos and n-$\overline{n}$ oscillations while suppressing the existence of proton decay. Unfortunately, in standard $\mathrm{SU(2)_L \times SU(2)_R \times SU(4)_c}$ unification models, for n-$\overline{n}$ oscillations to be observable, the mass hierarchy for the neutrinos must be of the type eV-keV-MeV, in conflict with current experimental results [8]. In order to correct this problem, the supersymmetric left-right (SUSYLR) model with the seesaw mechanism for neutrino masses is embedded within the $\mathrm{SU(2)_L \times SU(2)_R \times SU(4)_c}$ unification group [9]. This model allows for the existence of n-$\overline{n}$ oscillations at observable rates while still yielding neutrino masses within the current experimental boundaries.

A large class of supersymmetric $\mathrm{SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}}$ models with the seesaw mechanism for neutrino masses[3] can be used to predict $\tau_{n\overline{n}} < 10^9 - 10^{10}$ s [10]. Compared with results from experiments using neutron beams such as Grenoble ($\tau_{n\overline{n}} > 0.86 \times 10^8$ s at 90 % C.L.) [11] and from experiments using neutrons bound in nuclei such as Soudan 2 ($\tau_{n\overline{n}} > 1.3 \times 10^8$ s at 90 % C.L.) [3], this corresponds to an n-$\overline{n}$ oscillation timescale only a few orders of magnitude above the current experimental lower limits. Therefore, an effort to develop an n-$\overline{n}$ oscillation search in SNO is highly recommended not only because it would aid in the selection of a proper GUT but also because some GUTs predict that there is a realistic possibility of detecting an n-$\overline{n}$ oscillation signal by only slightly improving upon previous experimental results.

3. Nuclear Suppression of n-$\overline{n}$ Oscillations

Experimental searches for nuclear instability caused by n-$\overline{n}$ oscillations, such as the proposed search in SNO, must account for the nuclear suppression of n-$\overline{n}$ oscillations arising from the fact that neutrons and antineutrons feel vastly different nuclear potentials. This difference removes the mass degeneracy that would allow for pure n-$\overline{n}$ oscillations in free space. In fact, the nuclear annihilation lifetime T caused by n-$\overline{n}$ oscillations is related to the free space oscillation time $\tau_{n\overline{n}}$ by

$$T = {\tau_{n\overline{n}}}^2 T_R \tag{1}$$

for a given nuclear suppression factor T_R.

Calculations of T_R have been carried out in two different manners: one using a multi-body approximation of nuclear matter with two-body nucleon-nucleon or antinucleon-nucleon interactions [12] and another using two-body interactions between a nucleon or antinucleon and the rest of the nucleus using optical potentials [13, 14]. The latter method makes the assumption

[1] In this model, SU(5) contains as subgroups the electroweak group $\mathrm{SU(2)_L \times U(1)}$ (where $\mathrm{SU(2)_L}$ is the left-handed weak isospin group and U(1) is the weak hypercharge group) and the strong color group $\mathrm{SU(3)_c}$

[2] Although this is the prediction of the simplest SU(5) model of grand unification, theories have been developed to allow for the violation of $(B - L)$-symmetry by two units in other SU(5) models. However, in these models proton decay is still a predicted process [5].

[3] One should be aware that this class does not include supersymmetric models based on $\mathrm{SU(2)_L \times SU(2)_R \times SU(4)_c}$. If the $\mathrm{SU(3)_c \times U(1)_{B-L}}$ group is embedded within the $\mathrm{SU(4)_c}$ group, this argument collapses. Thus, this class of models proposes $\mathrm{SU(3)_c \times U(1)_{B-L}}$ symmetry up to the Planck scale.

Nucleus	Model 1	Model 2	Average
^{2}H	0.256	0.240	0.248
^{16}O	1.2	0.8	1.0

Table 1: Nuclear suppression factor T_R (in units of 10^{23} s^{-1}) for the neutrons in deuterium and oxygen [13].

that any n-$\overline{n}$ oscillations within the nucleus will occur at the nuclear perimeter due to lower binding energies. However, the validity of this argument has been questioned, and a detailed calculation taking the entire nuclear volume into account has been completed [15]. This calculation leads to corrections to the method used in Refs. [13, 14] that can be taken into account when determining $\tau_{n\overline{n}}$. For the purposes of the current argument, the difference between the methods of Refs. [13, 14] and Ref. [15] will be assumed to be negligible.

The current estimates of T_R for deuterium and oxygen are of particular importance to an n-$\overline{n}$ oscillation search in SNO for obvious reasons. For deuterium, the optical potential method provides very reliable results for T_R. For oxygen, the two methods described above give slightly different results. However, for simplicity and consistency, the value of T_R for oxygen calculated using the optical potential method will be used within this document[4]. Table 1 shows the values of T_R from different optical potentials for both deuterium and oxygen. The value of T_R averaged over the different optical potentials will be used.

4. $\overline{N}N$ Annihilation Signature

In order to experimentally determine the nuclear annihilation lifetime T using SNO, the products of $\overline{n}N$ annihilations must be determined. The significant annihilation products will define the n-$\overline{n}$ oscillation search by establishing the total number of particles that will be detected by the PMT array in an n-$\overline{n}$ oscillation event. Further, the identity of each annihilation product determines certain quantities that will affect particle detection such as the particle's momentum and Čerenkov light distribution.

Data from $\overline{p}p$ annihilations [16, 17] and from $\overline{p}n$ annihilations [18] at rest provide the only reliable data for $\overline{N}N$ annihilations at rest since data from $\overline{n}N$ annihilations at rest are meager[5]. Therefore, previous experimental searches have made assumptions that $\overline{n}n$ interactions have the same cross section as $\overline{p}p$ interactions and that $\overline{n}p$ interactions occur at the same rate as $\overline{p}p$ interactions and have a cross section derived from $\overline{p}p$ interactions [3, 21]. The percent contribution of each annihilation channel from $\overline{p}p$ annihilations can be found in Table 2. From this table, it can be inferred that pure pion channels make up the vast majority of $\overline{N}N$ annihilation channels while smaller contributions are made by ρ, ω, η, and K channels.

Channel	Resonant Intermediate State	Percent Contribution
$n\pi^0$ ($n \geq 0$)	—	3.2
$\pi^+\pi^-$	—	0.4
$2\pi^+2\pi^-$	—	6.9
$3\pi^+3\pi^-$	—	2.1
$\pi^+\pi^-\pi^0$	—	3.7
	$\rho^\mp\pi^\pm$	2.7
	$\rho^0\pi^0$	1.4
$\pi^+\pi^-2\pi^0$	—	9.3
$\pi^+\pi^-3\pi^0$	—	23.3
$\pi^+\pi^-4\pi^0$	—	2.8
$2\pi^+2\pi^-\pi^0$	$\omega\pi^+\pi^-$	3.8
	$\rho^0\pi^+\pi^-\pi^0$	7.3
	$\rho^\pm\pi^\mp\pi^+\pi^-$	6.4
	$\eta\pi^+\pi^-$	1.2
	$\eta\rho^0$	0.22
$2\pi^+2\pi^-2\pi^0$	—	16.6
$2\pi^+2\pi^-3\pi^0$	—	4.2
$3\pi^+3\pi^-\pi^0$	—	1.3
	$\eta 2\pi^+2\pi^-$	0.6
$K\overline{K}n\pi^0$ ($n \geq 0$)	—	6.82

Table 2: The percent contribution of each annihilation channel for $\overline{p}p$ annihilations at rest [16].

It can be calculated that the average charged pion ($\pi^\pm$) multiplicity is approximately 3 while the average neutral pion (π^0) multiplicity is approximately 2. The $\pi^\pm$ can be observed directly through the detection of Čerenkov radiation by the PMT array. Although the π^0 cannot be directly observed through the emission of Čerenkov radiation (due to the fact that it is chargeless), it quickly decay into two photons that Compton scatter

[4]Using Ref. [13] to provide T_R for oxygen is particularly fitting because previous n-$\overline{n}$ oscillation search results are based on this value.

[5]The OBELIX Collaboration at LEAR has performed experiments in which $\overline{n}p$ annihilations take place in-flight [19]. These experiments can be used together with annihilations at rest to deduce more appropriate decay signals for $\overline{n}p$ annihilations at rest.

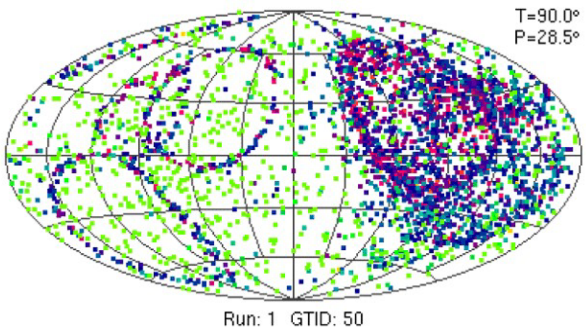

Fig. 2: An example of a multi-ring event in SNO from a Monte Carlo simulation of an n-$\overline{n}$ oscillation [20].

off of electrons which can be detected. Therefore, the n-$\overline{n}$ oscillation events in SNO will result in a multi-ring signature on the PMT array. An image from a Monte Carlo simulation of an n-$\overline{n}$ oscillation event in SNO [20] can be seen in Fig. 2.

The width of the ring produced by the intersection of the Čerenkov cone with the PMT array is a measure of the amount of time the particle spent above the speed of light in the medium. Hence, by knowing the rate of energy loss of a propagating particle in the medium, the original energy of the particle can be ascertained. In other words, the width of the ring (and equivalently, the number of PMTs hit) is a measure of the initial energy of the particle. Therefore, by measuring the total number of PMTs hit by all of the rings in an event, the total energy of the event can be approximately determined. For n-$\overline{n}$ oscillation events and subsequent $\overline{n}N$ annihilations, the total energy of the event should be approximately 2 GeV, twice the rest mass of the nucleon.

The momentum of each annihilation product can be deduced from the total energy of the particle and the direction in which the particle propagated. The direction can be found by applying a vertex fitter to the ring to determine the location of the Čerenkov cone's vertex. By comparing the location of the vertex with the location of the center of the ring, the direction vector along which the particle propagated can be ascertained. Once each particle's momentum has been calculated, the total momentum of the system can be found. Assuming that $\overline{n}N$ annihilations occur approximately at rest, the total momentum of all annihilation products in an n-$\overline{n}$ oscillation event should be zero.

Ultimately, an n-$\overline{n}$ oscillation event in SNO will result in a multi-ring signature with total energy of 2 GeV and total momentum of zero. By applying these constraints to the events to be analyzed, a better background rejection can be realized. This will improve the results of a SNO analysis of n-$\overline{n}$ oscillations.

5. Detection of $\overline{n}N$ Annihilations in SNO

The development of a multi-ring fitting algorithm for SNO is the major obstacle that must be surmounted before an n-$\overline{n}$ oscillation search can be performed. All of the ring fitting algorithms currently employed by SNO were originally designed for the detection of single-ring events only. In order to develop a multi-ring fitting algorithm, one must either alter one of the many currently used ring fitting algorithms to be able to detect multiple rings or develop a new algorithm designed specifically to fit multi-ring events.

Of all of the currently used algorithms, two were looked at specifically to determine their potential application as a multi-ring fitter: FTU, a C++-based timing fitter [22], and FTP, a Fortran-based path fitter[6] [23]. Both algorithms fit to a probability distribution function (PDF) that is used to determine the most likely vertex position and direction for each event, assuming a single Čerenkov-emitting electron. Either fitter could be modified so that it fits the hit PMTs to a PDF for multiple Čerenkov cones rather that for a single one. In this way, they could be employed as multi-ring fitters.

Alternatively, one can develop a new ring fitter designed specifically to fit multi-ring events. Rather than using a PDF to determine a best fit vertex position and direction, pattern recognition can be utilized to fit circles to an image consisting of PMT hits. A suitable method for accomplishing this feat is a technique for pattern recognition known as the Hough transform [24]. This method has already been employed in proton decay searches, such as the one carried out at Super-Kamiokande (Super-K) [25].

An example of the Hough transform process is shown in Fig. 3. Suppose a number of PMTs lie on an unknown circle of radius r that is to be fit by the algorithm (represented by the dashed circle in Fig. 3). Circles of radius r centered on each PMT will intersect at the center of the dashed circle. A transformation space, called Hough space, can then be created which is congruent to the image space. Each point in Hough space will receive a "vote" for each circle centered on a PMT that passes through it. Therefore, the point in Hough space with the most overall votes (referred to as a peak) corresponds to the point in image space that is the intersection of the most circles, namely the center of the circle to be fit. The circle of radius r plotted in image space around a

[6] The names FTU and FTP are internally designated names for the algorithms.

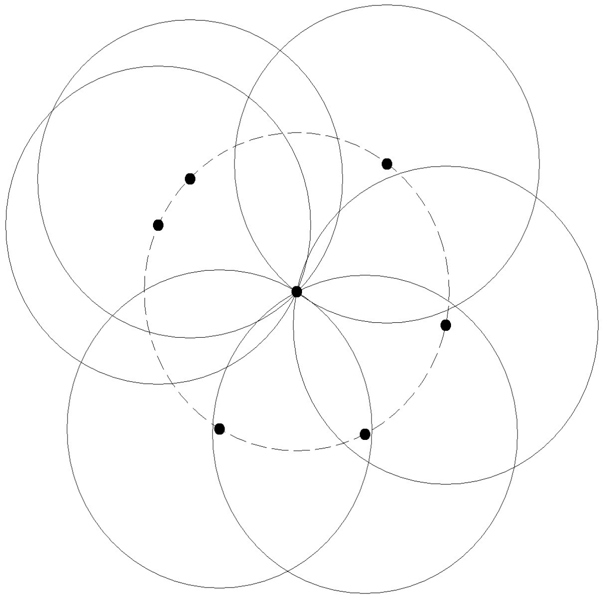

Fig. 3: An illustration of the Hough transform technique. The six points on the dashed circle represent PMTs hit in an event. Circles of a given radius centered at each of these points will intersect at the center of the dashed circle of the same radius. This point will appear as a peak in Hough space that can be understood as the center of a circle with the specific radius that fits each of the six points.

peak in Hough space will be the best fit circle for the hit PMTs.

For each PMT hit in an event, an array of circles of different radii can be created. For each radius, peaks can be searched for in Hough space that would correspond to the center of a best fit circle with that radius. Once every PMT has been assigned an array of circles, the entire image will have a number of peaks in Hough space equal to the number of rings in image space. Each ring will have a specific radius defined by the radius of the circles used to find the peak in Hough space. In this way, a multi-ring fitting algorithm can be created independent of the vertex positions of the rings. A vertex fitter can then be applied to each ring to determine the origin of the Čerenkov cone that defines it. From this, the physics of the interaction can be recreated, and the search for a proper n-$\overline{n}$ oscillation signal can be performed.

When using this technique, several parameters must be defined to minimize misidentifications while maximizing detection efficiency. The total number of votes that corresponds to a valid ring will be different given the energy of each interaction (since the energy is proportional to the total number of PMT hits). Therefore, a minimum number of votes must be set that will allow for a majority of rings to be found while still limiting the number of misidentified rings. Further, given that a ring will have a width associated with it, several different sized rings will be associated with each peak in Hough space. The step size of the radius can be set to minimize the number of multiply counted peaks. However, this will decrease the accuracy to which the radius of the fit ring can be known. Therefore, the proper radius step size that maximizes the efficiency should also be determined. Finally, rings can be produced that have a high eccentricity[7], resulting in a span of peaks in Hough space. This can be compared with the case in which multiple circular rings are found with centers very close to one another. In other words, an event that results in one elliptical ring can appear as an event in which multiple rings with centers very close together are found. A minimum distance between ring centers should be set to minimize the occurance of this complication.

6. Major Backgrounds

In order to accurately analyze the data obtained from an n-$\overline{n}$ oscillation search in SNO, the major backgrounds must be taken into account. Most processes in SNO can be eliminated based on the selection criteria set for an n-$\overline{n}$ oscillation (see Sec. 4). Only two common processes can result in a multi-ring signature with energy greater than 2 GeV: deep inelastic scattering of muons off of nuclei and atmospheric neutrino events. By rejecting events occuring directly after muon events (which have a very unique signature), the deep inelastic scattering background can be eliminated. Therefore, the only background that will contribute to an n-$\overline{n}$ oscillation search in SNO will be from atmospheric neutrinos.

Of the total number of atmospheric neutrino events observed by SNO, only a small fraction will have a high ring multiplicity and energy greater than 2 GeV. However, without a multi-ring fitter, the only selection criterium that can be applied to the total atmospheric neutrino population is an energy cutoff of 2 GeV. This will result in a large background that can be reduced if multi-ring events could be detected. Therefore, before a complete investigation of n-$\overline{n}$ oscillation backgrounds can be carried out, a multi-ring fitter must be developed.

[7]This is a consequence of the fact that the geometry of the detector is such that the intersection of a cone of Čerenkov light with the PMT array will rarely result in a perfectly circular ring.

7. Expected Results

Initial estimates of the expected results of an n-$\overline{n}$ oscillation search in SNO can be calculated [20] assuming certain parameters such as number of background events and total detection efficiency. Further, assumptions must be made as to the percent contribution of deuterium and oxygen to the overall n-$\overline{n}$ oscillation signal. This contribution will affect the T_R used in determining the free-space n-$\overline{n}$ oscillation time.

The nuclear annihilation lifetime T can be calculated using

$$T = \frac{\epsilon N_0 t}{N} \tag{2}$$

where ϵ is the efficiency of detection, N_0 is the total number of neutrons taken into account, t is the length of time during which the experiment was run, and N is the number of n-$\overline{n}$ oscillation events. For this calculation, the efficiency is assumed to be 100%[8]. The total live time of SNO during only the pure D_2O phase is used for t, giving $t = 306.4$ days $= 2.647 \times 10^7$ s. Further, it is assumed that no events will be seen and no background will be expected (corresponding to $N < 2.44$ events at 90 % C.L. [26]).

If only deuterium is considered in the calculation, $N_0 = 6 \times 10^{31}$. Therefore, $T > 6.5 \times 10^{38}$ s which, according to (1), corresponds to

$$\tau_{n\overline{n}} > 1.6 \times 10^8 \text{ s} \tag{3}$$

using the value of T_R found in Table 1. If oxygen is taken into account as well, then $N_0 = 3 \times 10^{32}$, which implies $T > 3.3 \times 10^{39}$ s. In order to determine $\tau_{n\overline{n}}$ for both deuterium and oxygen, T_R for heavy water must be known. An approximation of this is computed as a weighted average of T_R for deuterium and T_R for oxygen where the weights are the number of neutrons each atom contributes. This implies $T_R = 8.5 \times 10^{22}$ s^{-1}. Using this value and $T > 3.3 \times 10^{39}$ s, it can be found that the free-space n-$\overline{n}$ oscillation time using both deuterium and oxygen is

$$\tau_{n\overline{n}} > 2 \times 10^8 \text{ s}. \tag{4}$$

Expected results (3) and (4) can be improved upon if the time t is increased. Data from the salt phase of SNO[9] can be used with only a few further assumptions. Since the salt makes up only approximately 2% of the total detector volume, its contribution to the n-$\overline{n}$ oscillation search can be neglected. However, in the salt phase, the number of atmospheric neutrino events will be higher than in the pure D_2O phase. Therefore, the backgrounds to an n-$\overline{n}$ oscillation search must be recalculated for the salt phase.

[8] This assumption is made simply due to the lack of a better assumption. It can be estimated that the detection efficiency can be more realistically approximated as about 50%. However, this factor of two can be compensated for by increasing the overall live time used in the calculation.

[9] NaCl was added to the heavy water in SNO to increase the efficiency of solar neutrino detection.

8. Conclusion

Developing an n-$\overline{n}$ oscillation search in SNO could improve the current lower limit on $\tau_{n\overline{n}}$ and provide another experimental affirmation to previous and future results. This lower limit can be improved by increasing the total live time used in the calculation. The lower limit on $\tau_{n\overline{n}}$ can also be improved by other experiments such as Super-K (which uses regular H_2O instead of D_2O). Even though Super-K has more nuclei to base their results on, SNO benefits in that the deuterium nucleus is rather simple and has a lower value of T_R than does oxygen. Overall, experimental searches for n-$\overline{n}$ oscillations in both SNO and Super-K should result in improved lower limits on $\tau_{n\overline{n}}$ that are very similar. Completing n-$\overline{n}$ oscillation searches in both experiments is valuable because each would provide confirmation to the other's results.

Ultimately, the development of an n-$\overline{n}$ oscillation search in SNO is a worthwhile endeavour. Before this can be accomplished, a multi-ring fitting algorithm must be created that can be used to not only fit rings to PMT hits but also eliminate some of the atmospheric neutrino background. Further, the overall efficiency of the detection of n-$\overline{n}$ oscillation events must be calculted. Once these issues are addressed, analysis of SNO data in a search for n-$\overline{n}$ oscillations can be undertaken.

Acknowledgements

This work was supported by the National Science Foundation's Research Experience for Undergraduates at the University of Washington. The author is particularly grateful to Dr. Peter Doe, Dr. Joseph Formaggio, and Kathryn Miknaitis for their continued support and infinite patience in answering a multitude of questions. Their educational guidance will forever be appreciated.

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