Prospects for a Search for Sterile Neutrinos Using the Sudbury Neutrino Observatory

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August 2003

Abstract

I present the groundwork for an investigation into the question of whether the Sudbury Neutrino Observatory could be used to search for sterile neutrinos through oscillations from active neutrinos to sterile neutrinos. I calculate the day-night asymmetry as a function of the two parameters Δm^2 and $sin^2 2\theta$. I then present preliminary results which suggest it may in fact be possible to perform a search for sterile neutrinos using SNO, but also suggest it might only be possible to do this to 3σ for a very small range of the parameters. Finally, I detail future work which needs to be done before this investigation is complete.

1. Introduction

The purpose of this investigation is to examine the possibilities of using SNO to search for sterile neutrinos. The end result is to determine the sensitiviy of SNO to detecting sterile neutrinos (through detecting an absence of active neutrinos). This is accomplished by investigating whether or not it is possible to measure non-zero day-night asymmetries to 3σ over the entire SNO data set.

The Sudbury Neutrino Observatory (SNO) is a heavy water Čerenkov detector. It detects ^{8}B solar neutrinos through three different reactions, the charged current (CC) reaction, the neutral current (NC) reaction, and elastic scattering (ES):

$$\nu_e + d \to p + p + e^- \qquad (CC),$$

$$\nu_x + d \to p + n + \nu_x \qquad (NC),$$

$$\nu_x + e^- \to \nu_x + e^- \qquad (ES),$$

where $x = \{e, \mu, \tau\}$.

Unlike the CC, which is only sensitive to electron-type neutrinos, the NC is equally sensitive to all three active neutrinos. The ES is also sensitive to all three flavors, but the interactions involving ν_μ and ν_τ are surpressed with respect to those involving ν_e.

SNO is located in the INCO, Ltd. Creighton mine outside of Sudbury, Ontario, Canada, at a depth of 2070m. It consists of a 12 m diameter acrylic spherical shell, referred to as the acrylic vessel (AV), which is filled with heavy water. The AV is surrounded by an 18 m in diameter geodesic sphere, which serves as the support structure for the 9,456 photomultiplier tubes (PMTs) that detect the Čerenkov photons generated in the heavy water. The structure is immersed in H_2O filling the entire SNO cavity, to provide shielding from background radioactivity [1], [2]. SNO will operate in three phases. The first is the D_2O phase, in which the detector will run simply with pure heavy water, the second is the salt phase, in which purified salt will be dissolved in the heavy water, and

the third is the Neutral Current Detector (NCD) phase, in which discrete neutron detectors will be added to the heavy water as a direct measure of neutrons. The purpose of the second and third phases are to increase the sensitivity of SNO to the NC reactions.

The outline of the paper is as follows: In section 2, I present a brief introduction to and overview of neutrino oscillations. In section 3, I discuss neutrino masses, including Dirac and Majorana neutrinos and the see-saw mechanism. Section 4 provides a brief description of the Liquid Scintillator Neutrino Detector (LSND) experiment, and how its results motivate this investigation. In sections 5 and 6, I present the methodology used in this investigation and the results, respectively. Finally, section 7 contains the conclusions, as well as directions for further research.

2. Neutrino Oscillations

The standard model includes three types, or "flavors," of neutrinos - electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ). These three neutrinos are called the "weak states," because they are the three different neutrinos that are observed via the weak force. In the standard model, all of these neutrinos are massless.

Now, assume that neutrinos have mass, and further assume that the neutrinos of definite mass are not these three neutrinos, but instead three different neutrinos ν_1, ν_2, and ν_3, with masses m_1, m_2, and m_3. The neutrinos of definite mass are related to the weak neutrino states by [4]:

$$|\nu_x\rangle = \sum_m U_{xm}|\nu_m\rangle \qquad , \qquad (1)$$

where $x = \{e, \mu, \tau\}$, and $m = \{1, 2, 3\}$.

U is the neutrino mixing matrix, similar to the Cabibbo-Kobayashi-Maskawa (CKM) matrix for quark mixing. The existence of quark mixing has long been established experimentally, and only recently has it been

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shown experimentally that neutrinos can oscillate between the different flavors [1]. This implies that neutrinos obey a similar mixing principle, which in turn implies that neutrinos have mass [4], [1].

To illustrate the basic mechanics of neutrino oscillations, I assume that mixing only occurs between the first two flavors, neglecting completely ν_τ. In that case, the matrix U simplifies from a 3x3 matrix to a 2x2 matrix, and is given by [5]:

$$\mathbf{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad ,$$

and from (1) we have

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \qquad ; \qquad (2)$$

$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \qquad . \qquad (3)$$

To see how oscillations occur, we must understand the time evolution of a neutrino. From basic quantum mechanics,[2] we know that the time evolution of a state is determined by its energy, or more specifically the time evolution of each mass component v_m is found by multiplying that component by the phase factor $\exp[-i(E_m/\hbar)t]$. Thus, equation (2) becomes

$$|\nu_e(t)\rangle = \cos\theta exp[-i(E_1/\hbar)t]|\nu_1\rangle + \sin\theta exp[-i(E_2/\hbar)t]|\nu_2\rangle \qquad (4)$$

Now, the energy E_k is given by

$$E_k = \sqrt{p^2c^2 + m_k^2c^4} \qquad . \qquad (5)$$

If $m_1 = m_2$, then $E_1 = E_2$, and both mass states will evolve with the same phase and there will be no oscillations. However, if $m_1 \neq m_2$, then $E_1 \neq E_2$, and the two mass states will evolve with different phases, causing the neutrino to oscillate back and forth from one flavor to another along its length of travel. As described by Kayser in [4],

> Neutrino oscillation can be understood in a very simple way. What happens is that, at a given p_ν, the lighter mass states in the original ν_x travel faster than the heavier ones, and get ahead of the latter. Thus, the various ν_m components of the beam get out of phase with one another, and do not add up to a ν_x anymore. Thus, as it travels, the beam picks up components corresponding to other flavors.

Finally, define $P(\nu_e \to \nu_\mu)$ as the probability that an electron neutrino will oscillate into a muon neutrino, and $P(\nu_e \to \nu_e)$ as the probability that an electron neutrino will remain an electron neutrino. Quantum mechanics

states that the probability $P(\nu_e \to \nu_\mu)$ is the absolute square of the amplitude $\langle\nu_\mu|\nu_e(t)\rangle$. Then, we have (as derived in [5]),

$$P(\nu_e \to \nu_\mu) = sin^2 2\theta \, sin^2\left(\frac{1.27\Delta m^2 L}{E_\nu}\right) \qquad , \qquad (6)$$

and

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) \qquad . \qquad (7)$$

θ is the mixing angle between the two neutrinos, Δm^2 is the difference of the squares of the masses of the two neutrinos, L is the oscillation length (total length of travel of the neutrino from the point where it was created), and E_ν is the energy of the neutrino. $P(\nu_e \to \nu_e)$ is sometimes referred to as the "electron survival probability."

3. Neutrino Mass

Recently, SNO has provided very strong evidence that solar neutrino oscillations do occur by measuring both the CC and NC flux. The CC flux measures electron neutrinos, and since all of the neutrinos produced in the Sun are electron neutrinos, if no neutrino oscillations occured it would be identical to what the Standard Solar Model (SSM) predicts [6]. Measuring the CC flux, SNO found it to be much less than expected by the SSM [1]. However, this was no surprise, as solar neutrino experiments had been measuring this deficit in electron neutrinos, known as the "solar neutrino problem," for several decades.

The confirmation that neutrino oscillations explain the observed deficit came in the comparison of the CC measurement with the NC measurement. Since the NC measures the total neutrinos, and not simply the electron neutrinos, disagreement between the NC and CC (after being sure all backgrounds were subtracted out) would provide evidence that electron neutrinos are oscillating into other flavors while traveling from the sun to the earth. This is in fact what SNO measured - and not only was the NC flux higher than the CC flux, but it was also in nearly exact agreement with the SSM [3] [1], [6].

It should be noted that solar neutrino experiments are not the only ones to provide evidence of neutrino oscillations. Three other types of experiments, atmospheric neutrino experiments [7], particle-accelerator-produced neutrino experiments such as LSND [8], and reactor experiments have also provided evidence of oscillations. With so much strong evidence that neutrino oscillations do occur, it becomes apparent that at least one neutrino must have mass. If all of them had zero mass, then $\Delta m^2 = 0$, and as can be seen from the discussion in the previous section, there would be no oscillations. The question then arises of how neutrinos acquire mass.

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[1]See [1], and also see [3] for a list of papers quoting the results of Super Kamiokande.

[2]For a much more detailed description of neutrino oscillations, as well as a derivation of the expressions given here, see [5], [4].

[3]In units of $10^6 cm^{-2}s^{-1}$, SNO measured $\phi_{cc} = 1.76^{+0.06}_{-0.05}(stat.)^{0.09}_{-0.09}(syst.)$, and $\phi_{nc} = 5.09^{+0.44}_{-0.43}(stat.)^{0.46}_{-0.43}(syst.)$ [1].

According to the Standard Model, all particles acquire mass through interactions with the Higgs bosons.[4] According to Quantum field theory and Lortenz invariance [9], a particle changes its handedness when it interacts with a Higgs boson. The strength of these interactions is directly related to the mass of a particle - for example, muons collide with Higgs bosons much more frequently than electrons, making them more massive particles. Neutrinos can acquire mass in this same way, through interactions with the Higgs boson.

The problem with this is that experiments have shown that if neutrinos do in fact have mass, they are extremely light. Their masses are many orders of magnitude less than the other quarks and leptons in the standard model. One way to explain this is to simply assume that the interactions between neutrinos and the Higgs bosons is very weak. But to make this model work, the strength of these interactions would have to be at least 10^{12} times weaker than that of the top quark. [9] There is nothing physically wrong with this model, but it would require such a tiny fundamental constant (the constant which determines the strength of this interaction) that it is not a desirable explanation.

As an alternative, the see-saw mechanism has been developed.[5] To understand the see-saw mechanism, first an understanding of Dirac and Majorana neutrinos is required.[6]

A Majorana particle is one in which the particle and anti-particle are actually the same particle. The left-handed particle is just the left-handed state of the particle, while the right-handed anti-particle is the right-handed state of the particle. The same holds true for the right-handed particle and left-handed anti-particle, making a total of two states for Majorana particles.

Dirac particles have distinct particle and anti-particle states. In this case, the left-handed particle and right-handed anti-particle are in fact two different particles, just as are the left-handed particle and the right-handed anti-particle. Thus, Dirac particles have a total of four states.

Now, according to the see-saw mechanism, if the neutrino is a Majorana particle, the product of the two states of the particle is of the same order of magnitude of the mass of a quark of a lepton. This mechanism can be used to make the neutrino very light in the following way: Call

one of the states the light state, ν_l, and call the other state the heavy state, ν_r (the reason for these labels will become apparent in a moment). According to the mechanism, we then have:

$$M_{\nu_l} M_{\nu_r} = M_{q\,or\,l}^2 \qquad . \qquad (8)$$

Experiments have shown that if the neutrino does in fact have mass, it is extremely small [7]. Also, the only way that neutrinos can be detected is through the weak force, a left-handed force. Thus only one of the two Majorana neutrinos can be detected, the one that is a left-handed neutrino and a right-handed anti-neutrino (the one I have called ν_l). The other one, the one which is a right-handed neutrino and a left-handed anti-neutrino, can't actually be detected. Thus this second Majorana neutrino can be made as heavy as necessary in order to make the first Majorana neutrino, ν_l, light enough to fit experimental constraints. It should be noted that it is not yet known whether the neutrino is a Dirac or Majorana particle, but even if it does turn out to be Dirac the see-saw mechanism can still work. All that would be required is for the neutrino, which starts out as a Dirac neutrino with four states, to split into two Majorana neutrinos, each with two states [4].

In a sense, the ν_r discussed in the above section can be thought of as a sterile neutrino, because it can never be detected. However, if this were the case, active neutrinos would never oscillate into a sterile state. This is because those oscillations would be governed by (6), and there is such a large mass difference between ν_l and ν_r that Δm^2 would be much too large for oscillations to occur, and there would be no sense in ever searching for sterile neutrinos using neutrino oscillations. In order to justify searching for sterile neutrinos in SNO, there should be reason to believe that another sterile neutrino, called a light sterile neutrino because it must be much lighter than the ν_r from the see-saw mechanism, exists. It turns out that there is evidence supporting the existence of such a neutrino, evidence which was provided by LSND.

4. LSND - Light Sterile Neutrino?

LSND (**L**iquid **S**cintillator **N**eutrino **D**etector) was a neutrino experiment performed at the Los Alamos National Laboratory. A high-energy proton beam from the LANSCE accelerator struck a water target, producing pions. Those pions then travel a short distance in air and come to rest in a copper beam stop, where they decay into muons, positrons, and neutrinos. The neutrinos are emitted isotropically, and some of them will enter the LSND detector, which is a large tank filed with 52,000 gallons of mineral oil [11].

The neutrinos produced by the decay of the pions are muon anti-neutrinos. LSND works by searching for the

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[4]Higgs bosons are particles of zero spin, and therefore are neither left- or right-helicity particles [9]. The helicity of a particle describes the direction of its spin along the direction of its motion. A left-helicity particle has its spin aligned with its momentum, while a right-helicity particle has its spin anti-aligned with its momentum. Helicity is not an invariant quantity, but the handedness of a particle is an invariant quantity. For massless particles, helicity and handedness are the same thing, but this is not always the case for massive particles. For a more detailed discussion of handedness, see [9], [5].

[5]Only the basic idea of the see-saw mechanism is presented here. See [9] and [4] for a much more detailed explanation.

[6]See [4] for a more detailed description.

[7]See [10] for a list of references quoting the results of tritium beta decay experiments.

presence of electron anti-neutrinos in the detector. Since there were no electron anti-neutrinos to start with, if they are detected it is a sign that muon anti-neutrinos have oscillated into electron anti-neutrinos.

As reported in [8], LSND did in fact detect electron anti-neutrinos. Assume that electron neutrinos and anti-neutrinos are made up almost entirely of ν_1, muon neutrinos and anti-neutrinos are made up almost entirely of ν_2, and tau neutrinos and anti-neutrinos are made up almost entirely of ν_3. Since LSND measures oscillation between muon and electron anti-neutrinos, it appears that the relevant mass scale here is Δm_{12}^2. However, as described in [12], this is not the case. LSND is actually measuring "indirect" oscillations between muon and electron anti-neutrinos, and in reality the relevant mass scale is Δm_{13}^2.

Solar neutrino experiments measure the oscillation of ν_e into ν_μ and ν_τ, mostly ν_μ, while atmospheric neutrino experiments measure the oscillation of ν_μ into ν_τ. Thus, the relevant mass scale for solar neutrino experiments is Δm_{12}^2, while the relevant mass scale for atmospheric neutrino experiments is Δm_{23}^2. Using the fact that $\Delta m_{ij}^2 = m_j^2 - m_i^2$, we see that:

$$\Delta m_{12}^2 + \Delta m_{23}^2 = \Delta m_{13}^2 \qquad (9)$$

As reported in [12], the experimentally established upper limit for Δm_{12}^2 is $10^{-5}eV^2$, and for Δm_{23}^2 it is $10^{-3}eV^2$. The experimentally established lower limit for Δm_{13}^2, however, is $10^{-1}eV^2$. Equation (9) is not satisfied, therefore there must be at least one more mass scale in order to make the equation work, which means there must be at least one more neutrino. Furthermore, that mass scale must be on the order of $10^{-1}eV^2$.

If LSND is correct, this establishes that there must be at least one more neutrino, but says nothing about whether the neutrino is active or sterile. However, measurements of the Z decay width at LEP[8] have shown that the Z boson can decay into exactly three flavors of active neutrinos. If there is indeed a fourth neutrino, it must be sterile (meaning that it does not interact with the Z, and thus there is no way for us to detect it directly). And since (9) states that its mass must be on the order of $10^{-1}eV^2$, it is natural to assume that oscillation from an active state into the sterile state is possible, and evidence for it could be able to be found using neutrino detectors such as SNO.

A final note before moving on to the methodology of this investigation. This section has said nothing of what this sterile neutrino might actually be. It could simply be a light version of the ν_r that was discussed above in the context of the see-saw mechanism, although reconciling this with the fact that the see-saw mechanism demands ν_r be heavy would pose a new problem. Other possibilities for what the sterile neutrino may be exist as well, and are discussed in [15].

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[8]See the list of references in [14] for more information on the LEP experiments.

5. Methodology

This section describes the method followed in this investigation. The end result is to calculate both N and D, where N represents the number of events recorded by SNO during the night (when the Sun is below the horizon), and D represents the number of events recorded by SNO during the day (when the Sun is above the horizon)[9].

Once N and D are known, a quantity known as the day-night asymmetry can be calculated as follows:

$$A = 2\frac{N-D}{N+D} \qquad (10)$$

As can be seen, the day-night asymmetry is simply the difference between the night and day fluxes divided by the average of the two. A non-zero asymmetry would mean that different fluxes are recorded during the night and the day, meaning that the extra oscillation length that the neutrinos travel during the night (they must travel through the Earth as well as the Earth-Sun distance) could be causing some of the neutrinos to oscillate either into or out of a sterile state. It should be noted that it is important to normalize both the day and night fluxes with respect to their livetimes.

A is a function of N and D, which in turn are both dependent on the oscillation probability $P(\nu_a \to \nu_s)$. As can be seen from (6), this probability depends on two parameters: $sin^2 2\theta$ and Δm^2. Thus, A is also a function of these two parameters, $A = A(sin^2 2\theta, \Delta m^2)$. The asymmetry was calculated over a large range of these two parameters, and the results are presented in the next section.

The error in A, σ_A, can be calculated once A, N, and D are known. From basic error analysis, we know that:

$$\sigma_A^2 = (\frac{\delta A}{\delta N}\sigma_N)^2 + (\frac{\delta A}{\delta D}\sigma_D)^2 + 2(\frac{\delta^2 A}{\delta N \delta D}\sigma_N \sigma_D) \quad (11)$$

Under the assumption that the errors are uncorrleated, the last term can be neglected, and working out equation (11) yields:

$$\sigma_A = (1 - \frac{A^2}{4})\sqrt{\frac{\sigma_N^2}{N^2} + \frac{\sigma_D^2}{D^2}}$$

σ_N and σ_D can be replaced with $\sqrt{N}$ and $\sqrt{D}$, respectively, leading to

$$\sigma_A = (1 - \frac{A^2}{4})\sqrt{\frac{\sqrt{N}^2}{N^2} + \frac{\sqrt{D}^2}{D^2}} \qquad (12)$$

Both the asymmetry and σ_A were calculated over a large range of these two parameters, and the results are presented in the next section. But first, we need to know how to calculate N and D.

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[9]A more mathematical way of expressing this is by saying it is night when $-1 \leq cos\,\theta_z < 0$, and it is day when $0 \leq cos\,\theta_z \leq 1$.
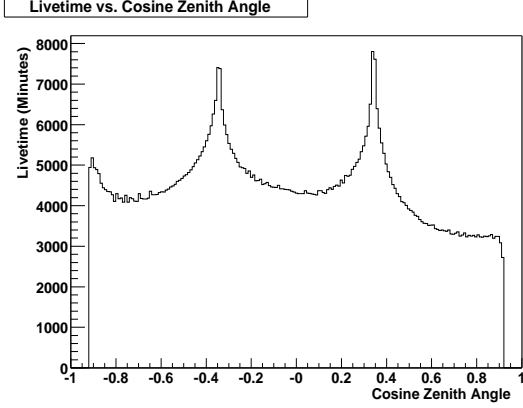
Figure 1 - Livetime (in minutes) vs. $cos\,\theta_z$.

Figure 2 - Livetime (normalized to one) vs. $cos\,\theta_z$.

The first task was to calculate the cosine of the zenith angle, $cos\,\theta_z$, as a function of time. As reported in [1], for the D_2O phase SNO was recording data between Nov. 2, 1999 and May 28, 2001. Using the method described in [16], $cos\,\theta_z$ was calculated over this time period assuming perfect livetime. A plot of detector livetime vs. $cos\,\theta_z$ is shown in figure 1.

Once $cos\,\theta_z$ is known, the total length of travel of the neutrino through the Earth can be calculated quite easily. Define R as the radius of the Earth, $\wp$ as the distance from the surface to SNO, λ as the length of travel of neutrino through the Earth, and **v** as the unit vector pointing in the direction of the sun. In that case, it can be shown quite easily that:

$$|(R - \wp) - \lambda v|^2 = R^2 \qquad . \qquad (13)$$

Working this out will result in a quadratic equation for λ, and taking the positive root (so that the length is not negative) results in the total length of travel of the neutrino through the Earth begin given by:

$$\lambda = -(R - \wp)cos\,\phi + \sqrt{(R - \wp)^2 cos^2\phi - (R - \wp)^2 + R^2} \qquad (14)$$

The total length of travel of the neutrino from the Sun to SNO, then, is simply $L = 1AU + \wp$. For the purpose of this investigation, $\wp$ was set to 0, as it is negligible when compared both to the Earth-Sun distance of 1 AU and to the diameter of the Earth.

To calculate the yield, or the number of neutrinos detected by SNO, first define U as the sum of deuteron targets. In that case,

$$U = \frac{\rho_{D_2O} N_A n_d}{A_{D_2O}} \qquad ,$$

where ρ_{D_2O} is the density of the heavy water, $\rho_{D_2O} = 1.1053$, N_A is Avogadro's Number, $N_A = 6.022 \times 10^{23}$, n_d is the number of deuterons per D_2O molecule, $n_d = 2$,
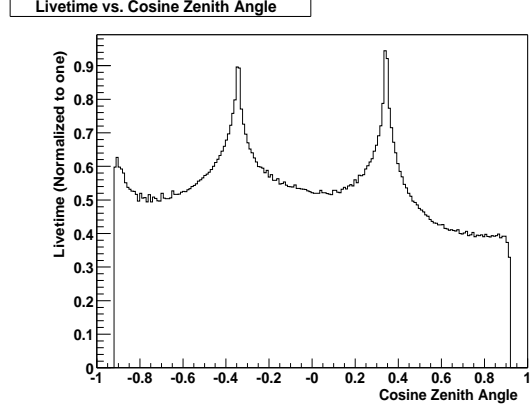
and A_{D_2O} is the atomic mass of the heavy water, $A_{D_2O} = 20.0265$.

Define $R_{NC}(T)$ as the detector response function, where T is the energy of the particle emitting the Čerenkov light. As illustrated in [17], it is described by a Gaussian:

$$R_{NC}(T) = \frac{1}{\sqrt{2\pi}\sigma_{NC}} exp[\frac{-(T_{NC} - T)^2}{2\sigma_{NC}^2}] \qquad , \qquad (15)$$

with $E_{NC} = 5.59 MeV$, and $\sigma_{NC} = 1.11$. Also, ϵ_n is defined as the mean neutron capture efficiency. As reported in [1], $\epsilon_n = 29\%$, and

$$\epsilon_n \int_0^V dV \int_{T_{th}}^\infty R_{NC}(T)\,dT = 14.4\% \qquad .$$

If we define Ω as the number of targets multipled by $\epsilon_n \int_0^V dV \int_{T_{th}}^\infty R_{NC}(T)\,dT$, the total detector yield is given by [17]:

$$Y = \Omega \int dt \int_0^\infty \int_a^b W(cos\,\theta_z)\Phi_{SSM}(E_\nu)\sigma_{\nu d}^{NC}(E_\nu)P(\nu_a \rightarrow \nu_s)dcos\,\theta_z dE_\nu \qquad (16)$$

$W(cos\,\theta_z)$ represents the weighting factor to properly account for how often $cos\,\theta_z$ is at a certain value. It is just the amount of time spent at that value, and can be read directly off Figure 2, which is simply Figure 1 normalized to one.

$P(\nu_a \rightarrow \nu_s)$ is the probability that the neutrino produced in the sun, an active neutrino, will have oscillated into a sterile state when it reaches SNO. The probability is described by (6), except now θ represents the mixing angle between active and sterile neutrinos, and Δm^2 represents the mass difference between the two, $m_a^2 - m_s^2$.

$\Phi_{SSM}(E_\nu)$ is the ^8B neutrino flux from the sun[10]. It has been accurately calculated [6], and is seen in figure 3.

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[10]The ^8B neutrinos are one of several types of neutrinos produced in the sun through fusion. See [6] for more information.
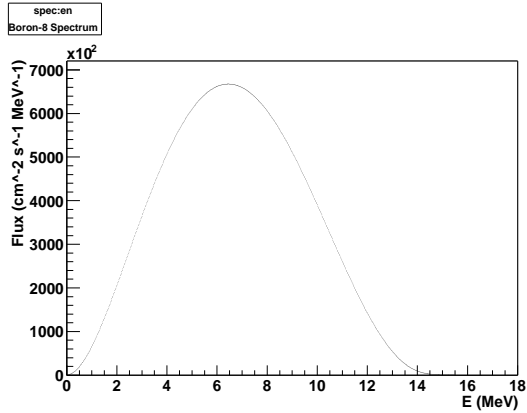
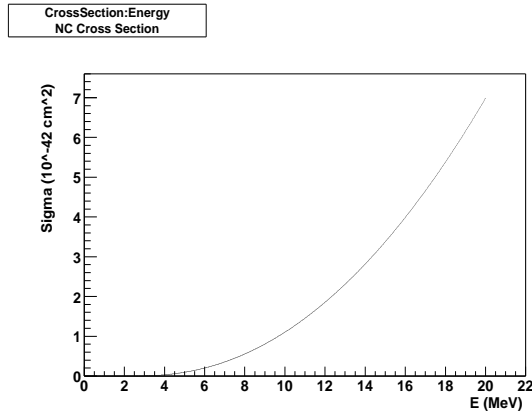Figure 3 - The ^8B spectrum, as calculated by [6].

Figure 4 - The Neutral Current neutrino cross section on deuterium, as calculated by [6].

$\sigma_{\nu d}^{NC}(E_\nu)$ is the NC neutrino cross section on deuterium. It has also been calculated by [6], and is seen in figure 4.

The integral in (16) was actually calculated twice. For N (the detector yield at night), $a = -1$, $b = 0$. For D (the detector yield during the day), $a = 0$, $b = 1$. Once N and D are known, the day - night asymmetry can be calculated according to (10). The total number of neutrinos detected can also be calculated by simply adding N and D.

6. Results

Figure 5 shows a contour plot of A, as a function of the two parameters.

Inspecting this figure, we see the expected general trends. For most of the smaller values of $sin^2 2\theta$, the asymmetry is zero. Even though the neutrinos are travelling a greater distance at night, the probability for them to oscillate into a sterile state remains basically zero because the first term, the $sin^2 2\theta$, is too small for any oscillations to occur. Also, the same is true for small values of Δm^2. By inspection of the figure, we see that no asymmetry can be detected for $sin^2 2\theta \leq 10^{-2}$, and
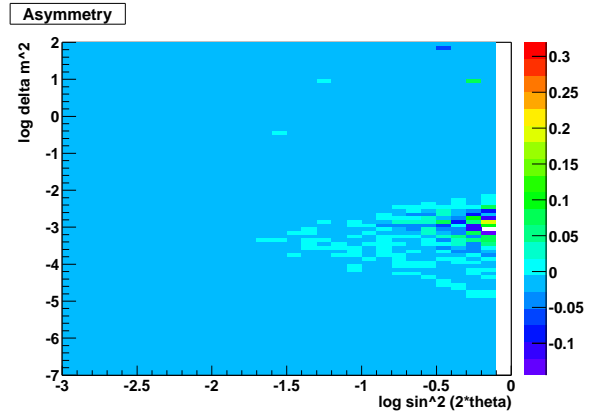

Figure 5 - Contour Plot of the day-night asymmetry, as a function of $sin^2 2\theta$ and Δm^2.

for $\Delta m^2 \leq 10^{-5}$. Finally, no asymmetry is seen for $\Delta m^2 \geq 10^{-2}$, because at this point the second term in (6) is oscillating much too quickly for the oscillations to actually be observed. Summarizing this paragraph, the following limits can be set: $10^{-2} \leq \Delta m^2 \leq 10^{-5}$ and $sin^2 2\theta \geq 10^{-2}$.

Figure 6 presents another contour plot of the asymmetry, using a different method of drawing contours. A few of the contours have been labeled to emphasize the oscillations in the asymmetry.

Figure 7 presents a contour plot of the total number of neutrinos detected, $N + D$, for a livetime approximately equal to the D_2O livetime of 306.4 days [1]. For much of the same excluded region concluded above, we see approximately 560 neutrinos detected. This is consistent with the SSM predictions and the fact that oscillations into sterile neutrinos should not be observed here (for reasons discussed in the above paragraph) [6].

Finally, Table 1 contains the asymmetry and uncertainty σ_A for a sampling of the values of the parameters. The parameters chosen to include in the table were those in the region of greatest oscillation, as seen by examining the figures. While the calculation of A is independent of the number of data points, the calculation of σ_A is not. The third column in Table 1 presents σ_A assuming a detector livetime of 306.4 days, which is approximately the livetime of the D_2O phase of SNO. The fourth column in Table 1 presents σ_A assuming a detector livetime of four times as long. This is approximately the total livetime that is expected over the three phases - D_2O, salt, and NCD, without taking into account the fact that the efficiencies will be different in the other phases.

As the third column shows, even in the region where the greatest asymmetries exist, it is impossible to measure an asymmetry when error is taken into account except for a select few points. The fourth column suggests it will be possible to measure non-zero asymmetries, at least for certain values of the parameters, once the entire data set

has been compiled. However, even once this occurs it is questionable whether or not one will be able to measure non-zero asymmetries to 3σ.

7. Conclusions

In section 4, it was stated that the mass scale of Δm^2_{as} must be approximtely of the order 10^{-2}. However, it was found above that SNO would only measure a day-night asymmetry if $10^{-2} \leq \Delta m^2 \leq 10^{-5}$. It appears that there is no reason to continue this investigation, as the necessary value of the parameter is not in the region of sensitivity. However, there is reason to continue, because these limits set on Δm^2 are based on a model which is not yet complete.

This work has only laid the groundwork for a continued investigation. To begin with, there are two factors which may increase the day-night asymmetry due to sterile neutrino oscillation, but have not yet been taken into account. One of them is the fact that the Earth-Sun distance does not remain constant at 1 AU over the course of a year. The second is the MSW effect[11], an effect which can enhance the oscillation probability of neutrinos as they travel through matter instead of through a vacuum. As is described in detail in [14], neutrino oscillations when matter effects are taken into account are given by:

$$P^{Sun}_{\nu_e \to \nu_s} = cos^2\theta_{23}cos^2\theta_{24}(1 - P^{Sun}_{\nu_e \to \nu_e}) \quad ,$$

$$P^{Sun}_{\nu_e \to \nu_a} = (1 - cos^2\theta_{23}cos^2\theta_{24})(1 - P^{Sun}_{\nu_e \to \nu_e}) \quad ,$$

where

$$P^{Sun}_{\nu_e \to \nu_e} = \frac{1}{2} + (\frac{1}{2} - P_c)cos2\theta_{12}cos2\theta^M_{12} \quad .$$

θ_{12}, θ_{23}, and θ_{24} represent the mixing angles between the various mass states. θ^M_{12} is the effective mixing angle between the first two mass states, and depends on the matter oscillations [14], [18]. P_c represents the crossing probability [19].

In addition to these factors which may affect the assymetry, the extension of σ_A to cover the entire SNO data set was only an approximation. It won't be possible to do this with any accuracy until it is known just how large the data set is. And going even further, the number of events measured in the second and third phases are expected to be higher than the first phase. This will lead to an overall decrease in σ_A which has not yet been taken into account.

There remains one other part of this investigation that has not been pursued. In this investigation, it is assumed that only one sterile neutrino exists. However, the only constraints on the neutrinos is that there be only three active flavors, and that all of their mass differences add up in such a way that (9) is satisfied. There is nothing which says there can only be one sterile neutrino, it was only

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[11]See [18] for a good description of the MSW effect

assumed in this investigation for simplicity. It would be worthwhile to examine the effects of what other models which have more than one sterile neutrino would have on the results presented here. As is explained in detail in [20], the probability of neutrino oscillations given n neutrino generations is given by:

$$P(\nu_\alpha \to \nu_\beta) =$$

$$\delta_{\alpha\beta} - 4 \sum_{j>1}^{n} \sum_{i=1}^{n} U_{\alpha,j}U_{\beta,j}U_{\alpha,i}U_{\beta,i}sin^2(\frac{1.27\Delta m^2_{ji}L}{E})$$
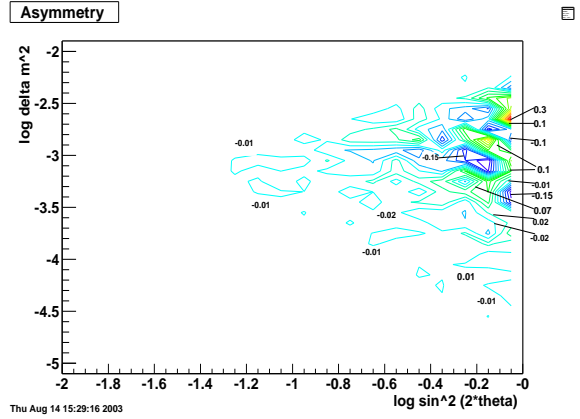

Figure 6 - A contour plot of the day-night asymmetry as a function of $sin^2 2\theta$ and Δm^2, using a second method of plotting contours. A few of the contours have been labeled.
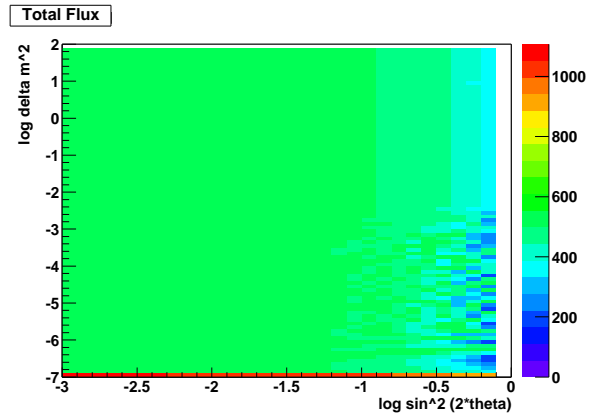

Figure 7 - Contour Plot of the total neutrinos detected during a livetime approimately equal to the livetime of the D_2O phase, as a function of $sin^2 2\theta$ and Δm^2.

This investigation has explored whether or not it will be possible to use SNO to search for sterile neutrinos. Whether or not it will be possible to measure non-zero values of the day-night asymmetry to 3σ is not yet determined. The preliminary results presented here suggest it may be at least possible, if difficult, for a small range

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of the parameters. But much work is still required before the true sensitivity of SNO to a search for sterile neutrinos can be determined.

Acknowledgements

References

[1] Q.R. Ahmad *et al.,* Phys. Rev. Lett. **89**, 011301 (2002)

[2] See http://www.sno.phy.queensu.ca/sno/sno2.html

[3] http://arxiv.org/find/hep-ex/1/au:+super_kamiokande/0/1/0/all/0/1?skip=0&query_id=2deeea5b06966f99

[4] B. Kayser *et al.,* "The Physics of Massive Neutrinos", World Scientific Publishing Co. Pte. Ltd., Singapore (1989)

[5] R. Slansky *et al.,* "The Oscillating Neutrino", Los Alamos Science - Celebrating the Neutrino, No. 25 (1997)

[6] See http://www.sns.ias.edu/ jnb/
http://www.sns.ias.edu/ jnb/Papers/Preprints/neutrinoenergy.html
http://www.sns.ias.edu/ jnb/SNdata/sndata.html
http://www.sns.ias.edu/ jnb/SNdata/b8spectrum.html
http://www.sns.ias.edu/ jnb/SNdata/deuteriumcross.html

[7] T.K. Gaisser and M. Goodman, "Proceedings of 1994 Snowmass Summer Study, Particle and Nuclear Astrophysics in the Next Millennium," World Scientific Publishing Co. Pte. Ltd., Singapore (1994)

[8] C. Athanassopoulos *et al.,* Phys. Rev. C, **54**, 2685 (1996)
C. Athanassopoulos *et al.,* Phys. Rev. C, LA-UR-97-1998 (1997)

[9] H. Murayama, "The Origin of Neutrino Mass", Physics World. **15**, 5, 35-39 (2002)

[10] T.J. Bowles *et al.,* "Tritium Beta Decay and the Search for Neutrino Mass", Los Alamos Science - Celebrating the Neutrino, No. 25 (1997)

[11] B. Louis *et al.,* "A Thousand Eyes - The Story of LSND", Los Alamos Science - Celebrating the Neutrino, No. 25 (1997)

[12] B. Louis *et al.,* "The Evidence for Oscillations", Los Alamos Science - Celebrating the Neutrino, No. 25 (1997)

[13] S.P. Rosen, "A Possible Solution to the Solar-Neutrino Problem", Los Alamos Science - Celebrating the Neutrino, No. 25 (1997)

[14] C. Giunti *et al.,* Phys. Rev. D, **62**, 013005 (2000)

[15] P.C. de Holanda and A.Y. Smirnov, hep-ph/0211264

[16] J.N. Bahcall and P.I. Krastev, Phys. Rev. **C56**, 2839-2857 (1997)

[17] SNO Internal Documentation for *Q.R. Ahmad* et al., *Phys. Rev. Lett.* **89**, 011302 (2002)

[18] S. Rosen, "A Possible Solution to the Solar-Neutrino Problem", Los Alamos Science - Celebrating the Neutrino, No. 25 (1997)

[19] S.M. Bilenky *et al.,* Prog. Part. Nucl. Phys. **43**, 1 (1999).

[20] M. Sorel, hep-ph/0305255

$logsin^2 2\theta, log\Delta m^2$	Asymmetry	$\sigma_A(D_2O)$	σ_A (all phases)
-0.1, -2.2	-0.7%	10.9%	5.5%
-0.1, -2.4	14.3%	10.9%	5.4%
-0.1, -2.6	32.0%	11.3%	5.6%
-0.1, -2.8	-8.1%	9.0%	4.5%
-0.1, -3.0	13.7%	9.7%	4.9%
-0.1, -3.2	-0.6%	8.6%	4.3%
-0.1, -3.4	-11.2%	13.9%	7.0%
-0.1, -3.6	2.5%	9.0%	4.5%
-0.1, -3.8	-0.4%	8.6%	4.3%
-0.1, -4.0	-2.7%	12.4%	6.2%
-0.1, -4.2	-1.8%	13.3%	6.6%
-0.1, -4.4	-0.5%	9.2%	4.6%
-0.1, -4.6	0.4%	9.7%	4.8%
-0.1, -4.8	0.0%	8.5%	4.3%
-0.1, -5.0	0.1%	9.0%	4.5%

Table 1 - Values of A, $\sigma_A(D_2O)$, and σ_A (all phases) for a sampling of parameters in the region of highest oscillation in A.