The Fractional Quantum Hall Effect

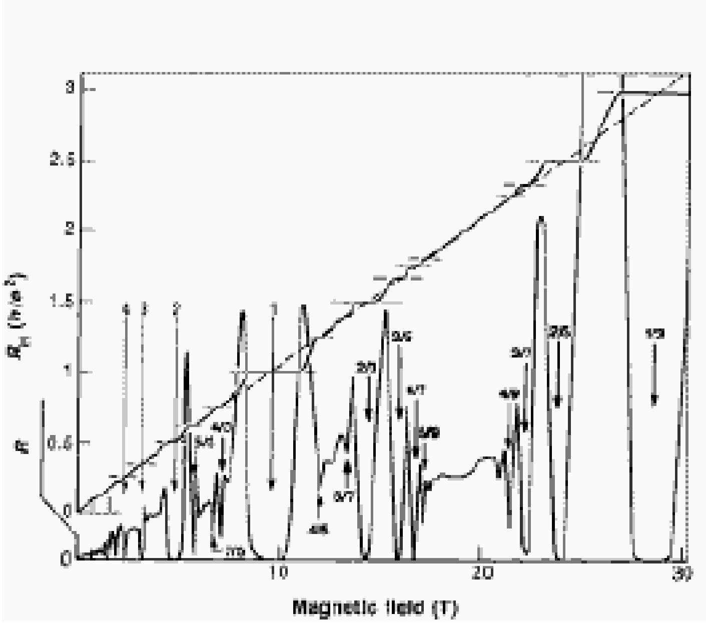
Natalia Toro
under the direction of Wick Haxton

August 28, 2002

Overview

- Integer and Fractional QHE Observations
- Integer QHE Theory and Landau Levels
- Fractional QHE Theory—toy model
- Intuition through model Hamiltonians

Quantum Hall Effects:
Cold, Thin Semiconductors at Large B



Resistances along and across the direction of applied voltage show detailed structure at specified magnetic fields.

Behavior at integer “fillings” first observed by Klitzing, 1978. Fractional effect observed by Stui, Störmer, 1982.

(Figure from Eisenstein, Störmer, Science **248**, 1510 (1990))

Integer QHE and Landau Levels

- Magnetic field splits allowed electron energies into Landau Levels, which hold more electrons for larger B

- Bizarre behavior occurs when a Landau level is exactly full

- Theoretical explanation of the observations involves considering effect of impurities in the system

- Skip that: What matters to us is that at these points there is one preferred state–if you moved an electron, energy would rise dramatically.

Fractional QHE Background

- Plateaus and minima observed at fractional fillings of 1/3, 2/5, 3/7,...;1/5, 2/9, 3/13...; many other fractions

- Laughlin devised theory to explain $1/m$ fillings, with m odd

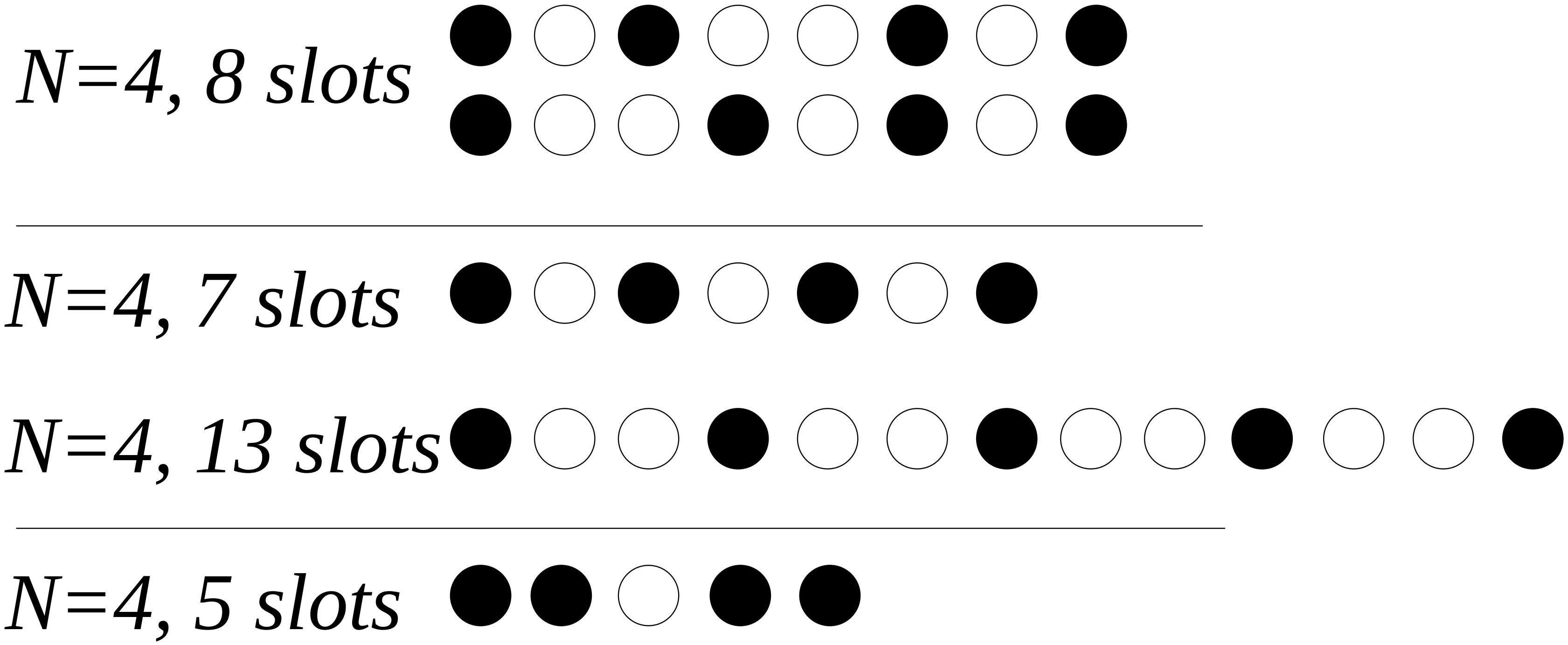
- Jain extended this theory to include fractional fillings with other numerators

- Although Jain's proposed wavefunctions work, they rely on projections from higher Landau Levels, composite fermions

- Ginocchio and Haxton have proposed an alternative generalization of Laughlin's wavefunction, which leads to interpretation in terms of multi-electron clusters

Laughlin Wavefunctions and Generalization: Toy Model

Idea: Just as the magnetic field produces an energy gap between the ground state and 1st excited state in a full Landau level, the Coulomb interaction between electrons produces an energy gap for a level with fractional filling.



Research Goals

Understand physical properties of these states by using simple effective operators

- What are the proper operators?

- What information can we extract from the operators?
  - Energy spectrum for a given number of particles, slots
  - Lowest eigenvalue of operator for given number of particles, varying number of slots

Choice of Operators

- Effective operators measure the unfavorable interaction producing the gap

- Want to keep particles far apart—corresponds to large coupling of angular momenta

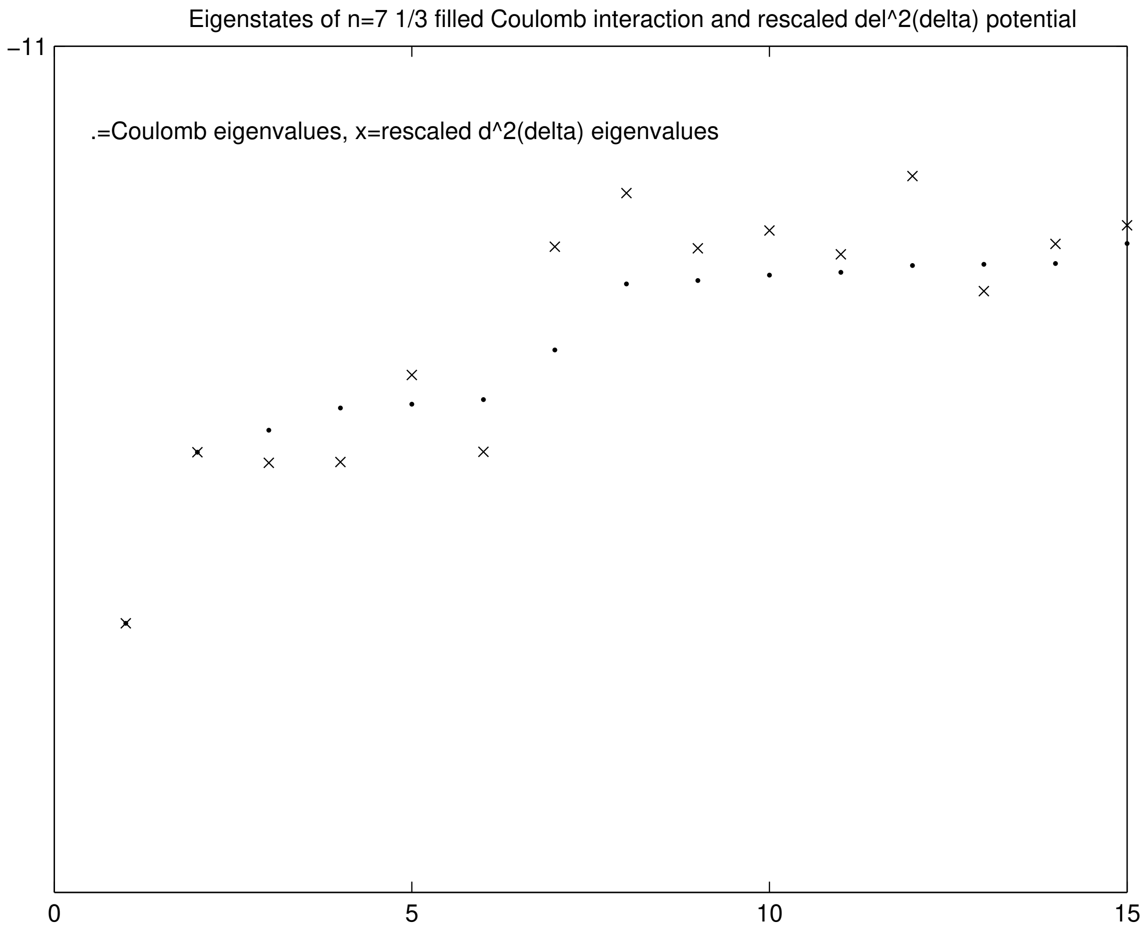
- Any two angular momenta must couple to at least 1.

- In the 1/3-filled ground state, all couple to 3. We want an operator that counts pairs coupling to 1: $\nabla^2_{12}(\delta(\vec{r}_1 - \vec{r}_2))$

- We have found operators for the 1/m-filled states, as well as for 2/5

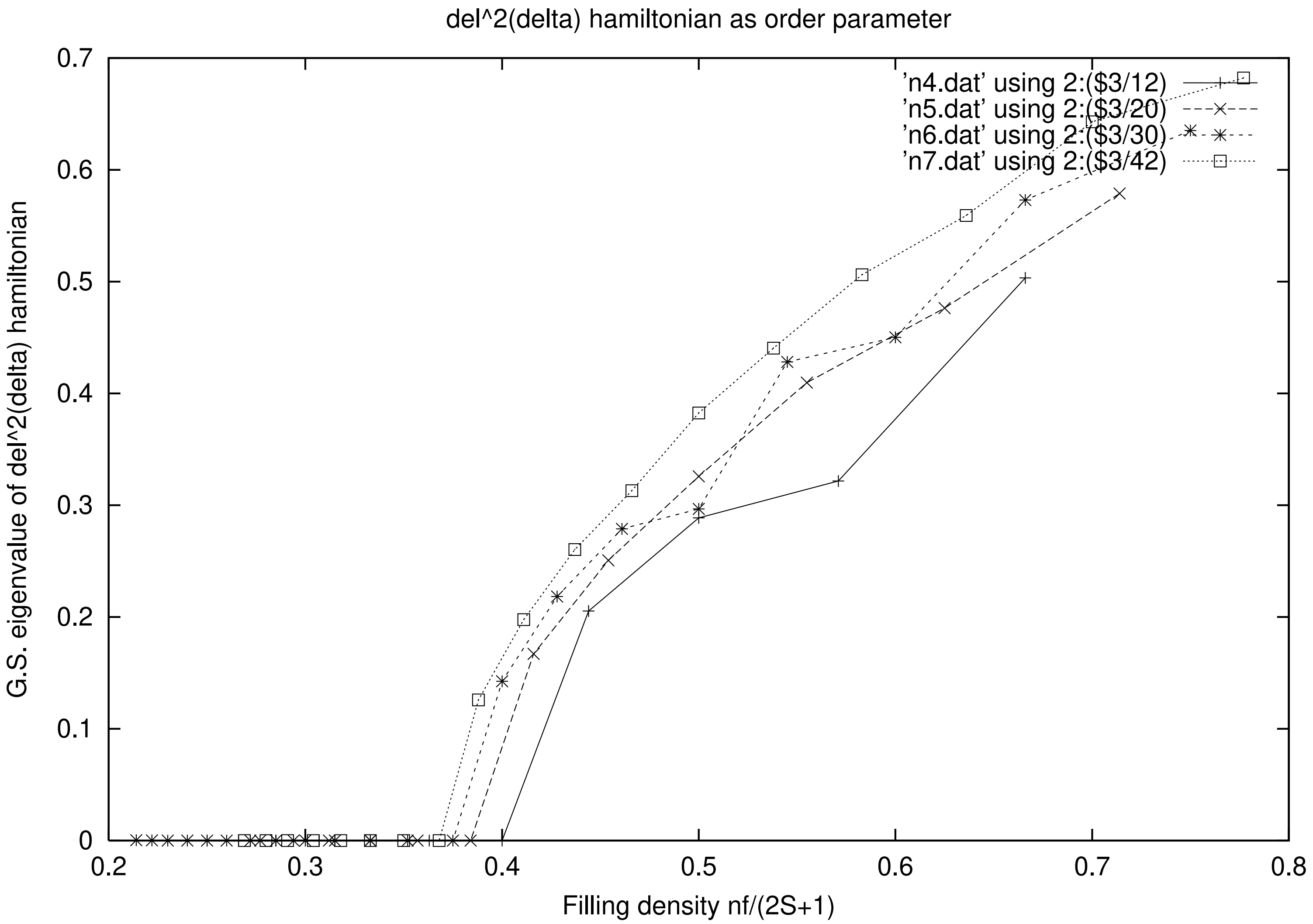
Energy Spectra

If the interaction we guess is indeed responsible for the observed energy gap, then its spectrum should look like the spectrum for the Coulomb interaction.



Phase Transition

The ground state eigenvalue as a function of filling behaves like an order parameter in a 2nd-order phase transition.



Further Directions

- General description of operator associated with each FQHE state

- Understanding phase transition better, gaining more information from behavior at phase transition?

- Generalize to states involving electrons in higher Landau levels

Acknowledgements

- Wick Haxton
- Joe Ginocchio
- Lesley Reece
- INT, NSF
- Fellow REU students