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References: This code of conduct is based heavily on that of the INT and the APS. We are also grateful to Roxanne Springer for valuable discussion and guidance.

Kinematically Enhanced Lattice Interpolators for Boosted Hadrons

Rui Zhang

Argonne National Laboratory

Rising Researchers Seminar

5/27/2025

arxiv:2501.00729

In collaboration with A. Grebe, D. Hackett, M. Wagman, and Y. Zhao

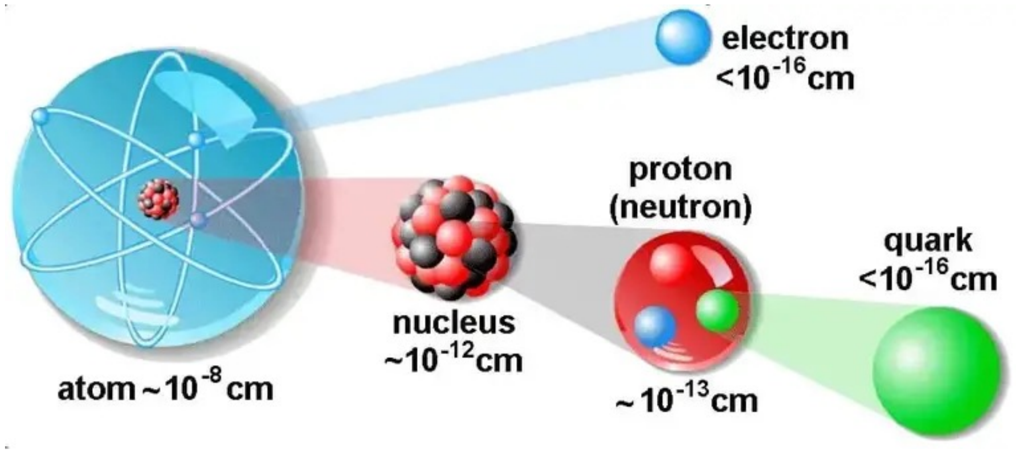
Outline

- Boosted Hadrons in Modern Physics
- Boosted Hadrons in Lattice QCD
- Kinematically Enhanced Interpolators
- Conclusion and Outlook

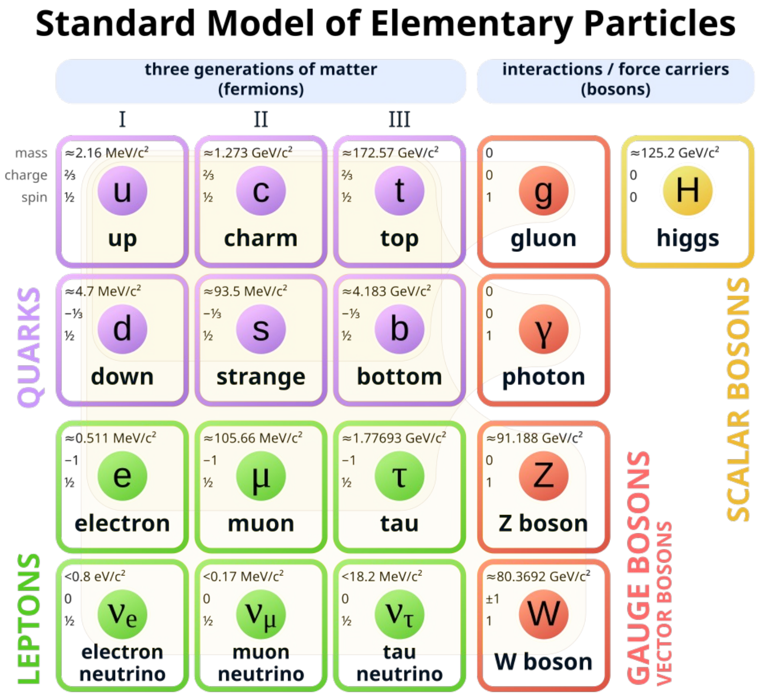
Boosted Hadrons in Modern Physics

Two biggest questions in physics

- WHAT and HOW is our world made up of?

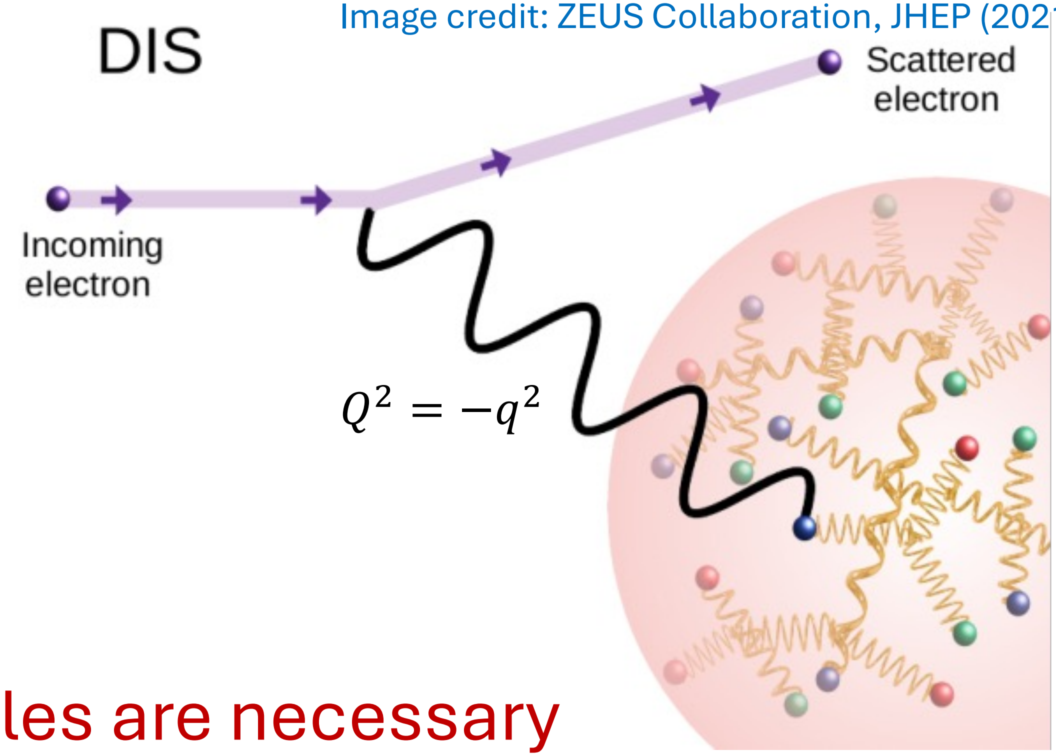


- Anything NEW we have not discovered?



Boosted hadrons are enssential

- Large $|Q|$ is essential
  - For higher resolution $|Q| \cdot \Delta x \geq \frac{\hbar}{4}$
  - For heavier particles $|Q| \geq M$



highly boosted particles are necessary

Hadron Structures in Boosted Systems

- Parton Physics
  - Momentum distribution of quarks and gluons inside the particles

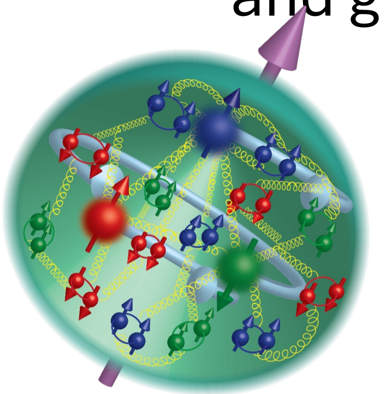
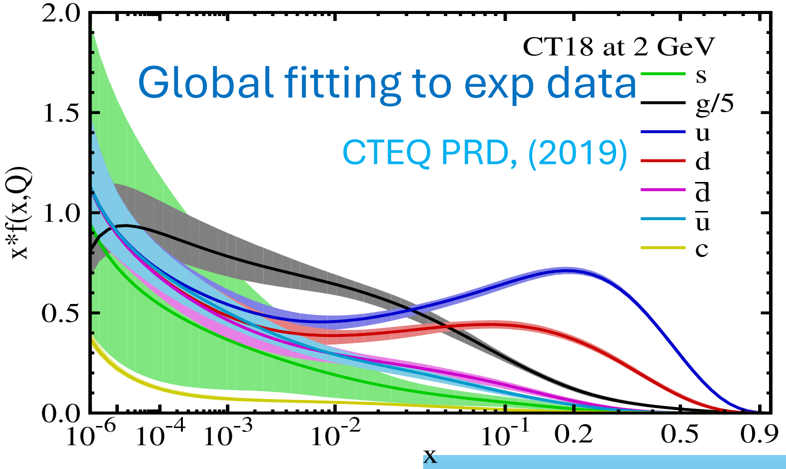
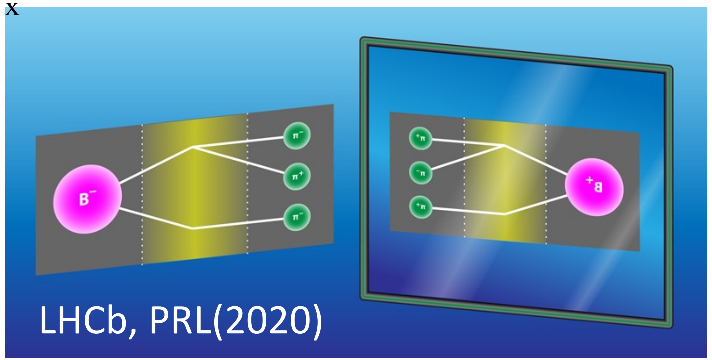


Image credit: ANL

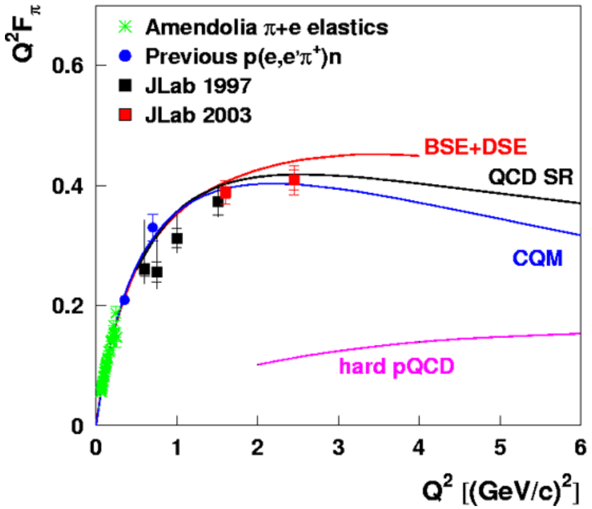


- Flavor Physics
  - Heavy meson decays



- Form Factors
  - “Shape” of the particle reacting to certain interactions

Horn, et.al., PRL(2006)



Creating Boosted Hadrons in Experiments

Expensive!

Energy Frontier: Large Hadron Collider

Image credit: CERN

Intensity Frontier: Electron-Ion Collider (upcoming)

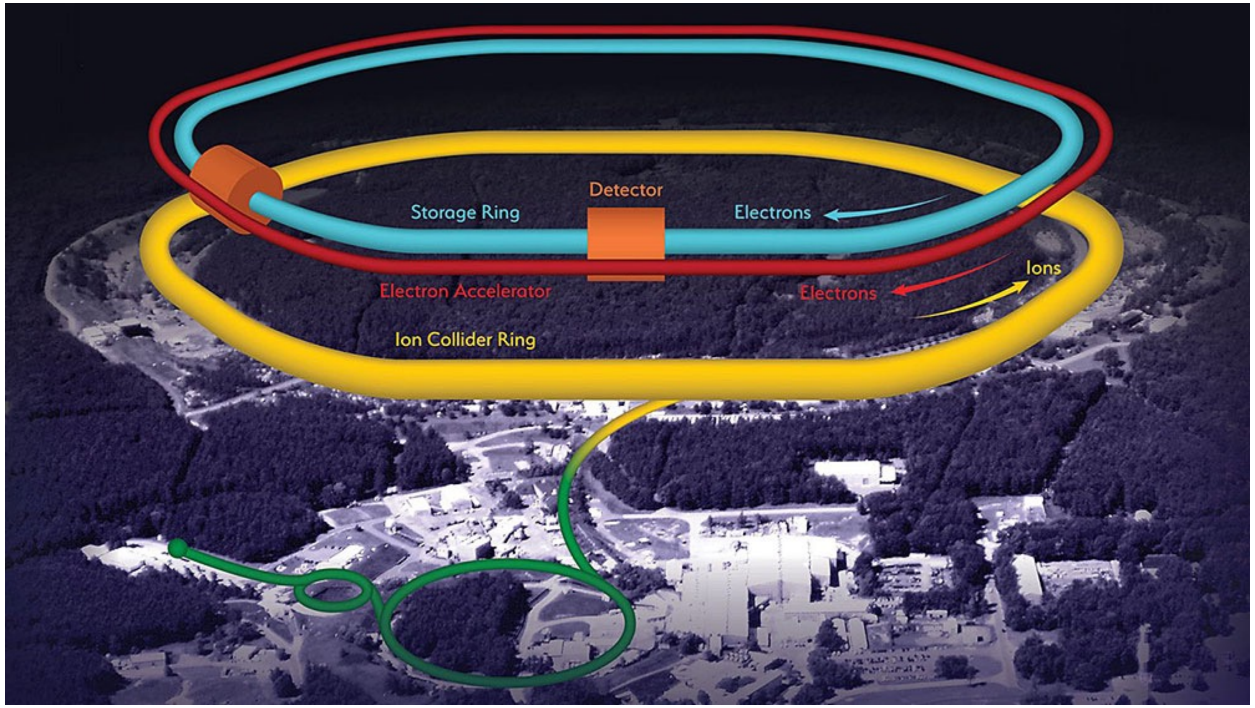


Image credit: BNL

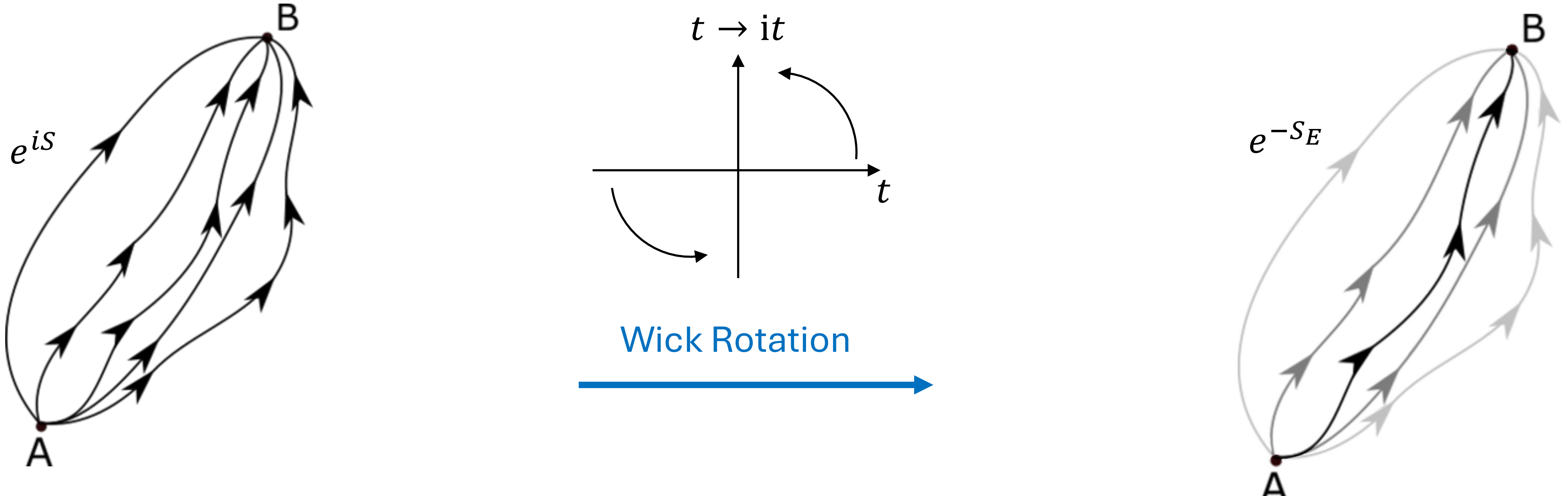
Boosted Hadrons in Lattice QCD

Theoretical prediction of hadron structures

- Hadron structures are governed by non-perturbative QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^{a,\mu\nu} G^{a}_{\mu\nu} + \sum_{i} \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi$$

- Lattice QCD: First-principle prediction from QCD



$\langle O \rangle = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ O\ e^{iS_{\text{QCD}}(A,\psi,\bar{\psi})}$

$\langle O_{\text{lat}} \rangle = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ O\ e^{-S_{\text{E}}(A,\psi,\bar{\psi})}$

Theoretical prediction of hadron structures

- Original formalism: Impossible to exhaustively explore all field configurations
- Wick-rotated path integral: Statistically estimable

Lattice QCD is all about Monte Carlo sampling:

$$P(A,\psi,\bar{\psi}) = \frac{e^{-S^E_{\mathrm{QCD}}}}{\int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{-S^E_{\mathrm{QCD}}}}$$

$$\langle O_{\mathrm{lat}}\rangle = \sum_N O_N / N$$

Statistical Uncertainty $\sim 1/\sqrt{N}$

Image credit: ANL

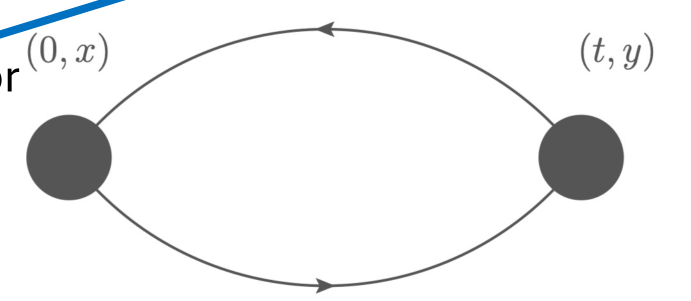
Numerical Simulation of QCD

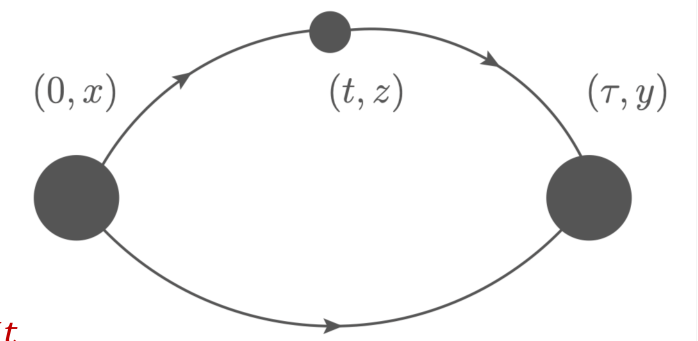
K.G. Wilson, Nobel Prize Winner (1982)

- Discretization of QCD action:
- Construction of correlators:

$C_{2pt}(t) = \langle \chi_{snk}(t) | \chi_{src}(0) \rangle$ $\qquad$ $C_{3pt}(t) = \langle \chi_{snk}(t) | O(t) | \chi_{src}(0) \rangle$

Interpolator





e^{-Ht} → t

Euclidean 4D spacetime

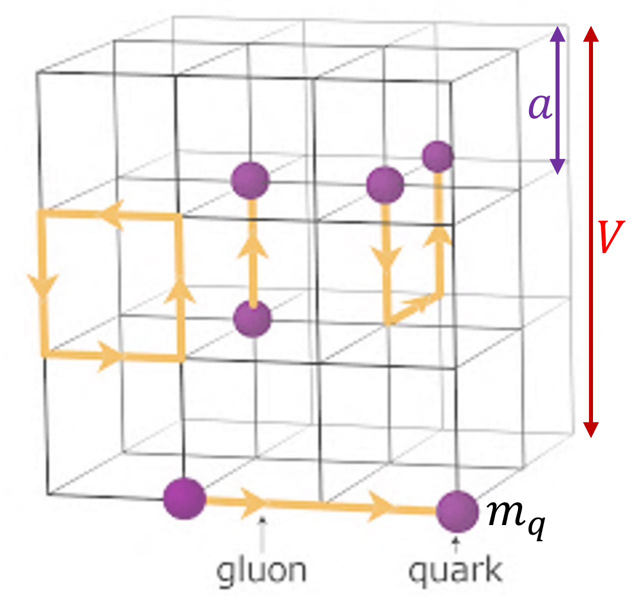
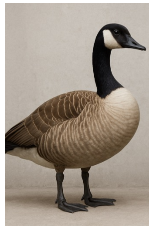
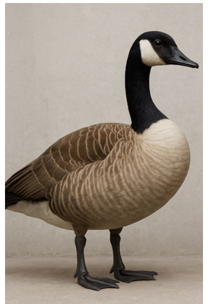
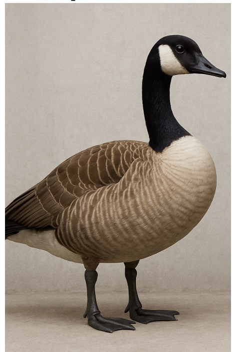


Image credit: M. J. Savage

χ overlaps with all physical states carrying the same quantum number

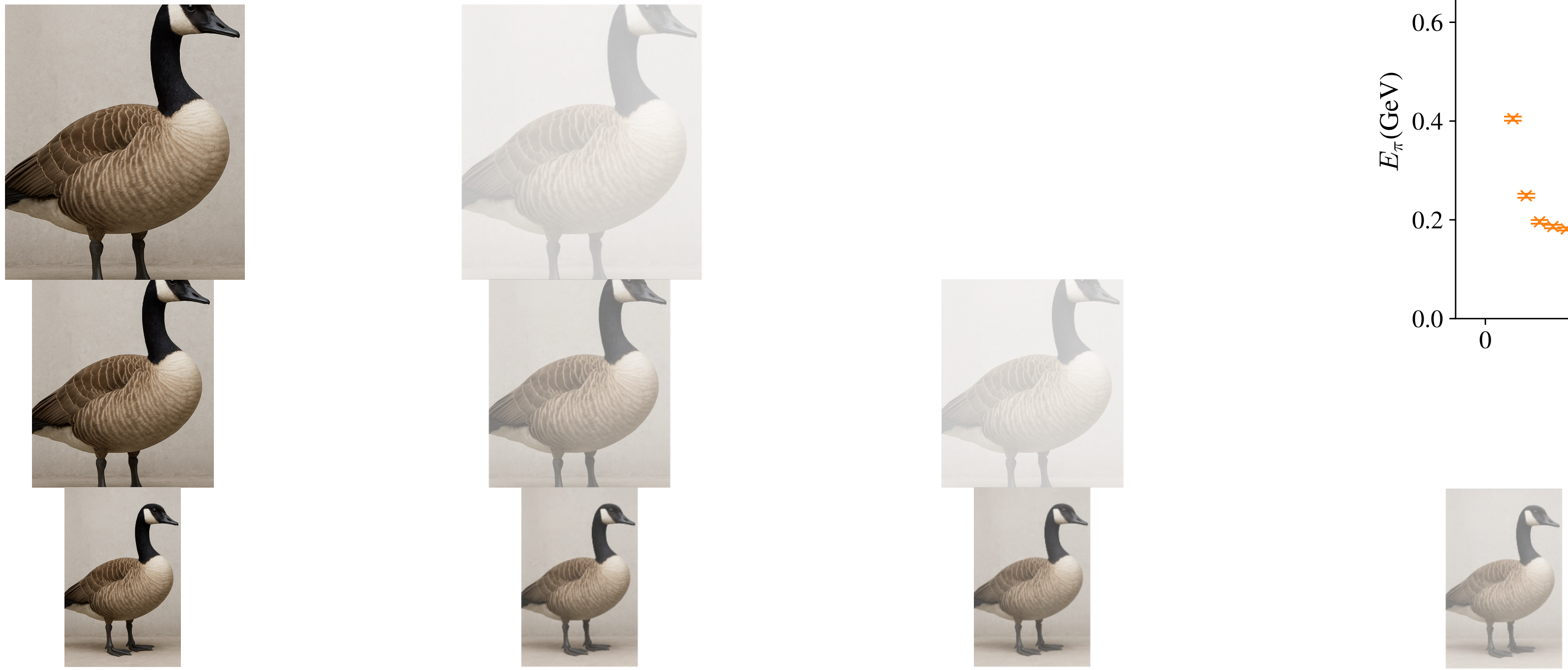
For a goose: $\chi = c_0$  $+ c_1$  $+ c_2$  +...

Extraction of ground-state information

Spectrum expansion:

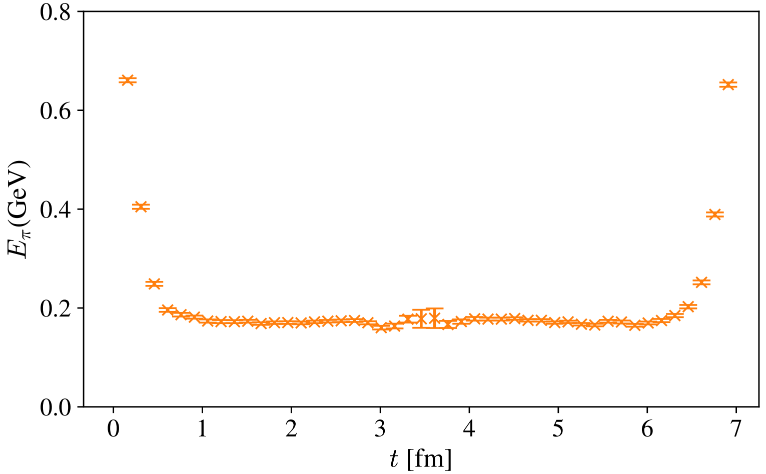
$$C_{2\text{pt}}(t) = \sum |c_n|^2 e^{-E_n t}$$

$$C_{3\text{pt}}(t,\tau) = \sum c_m^* c_n \langle m|O|n\rangle e^{-E_m(\tau-t)} e^{-E_n t}$$

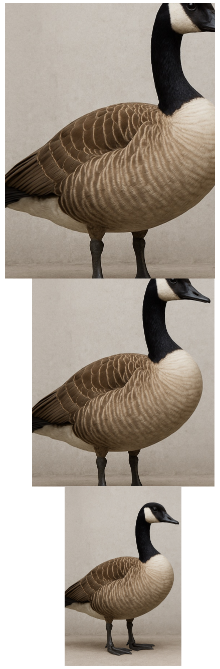


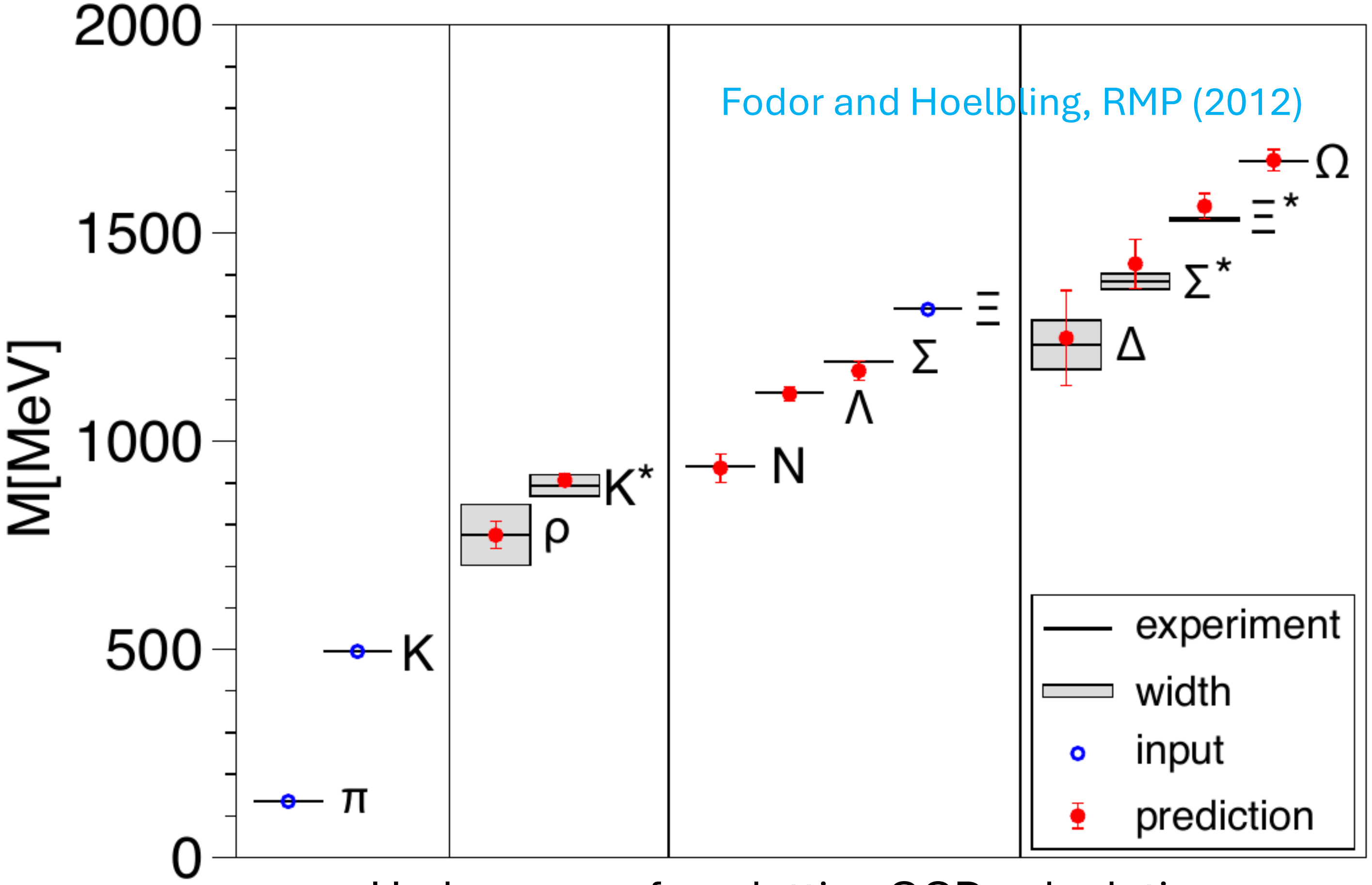
$e^{-E_n t}$ → t

Effective mass $\mathrm{E_{eff}(t)} = \frac{1}{a}\ln\frac{C_{2\text{pt}}(t)}{C_{2\text{pt}}(t+a)} \xrightarrow{t\to\infty} E_0$



Extraction of ground-state information





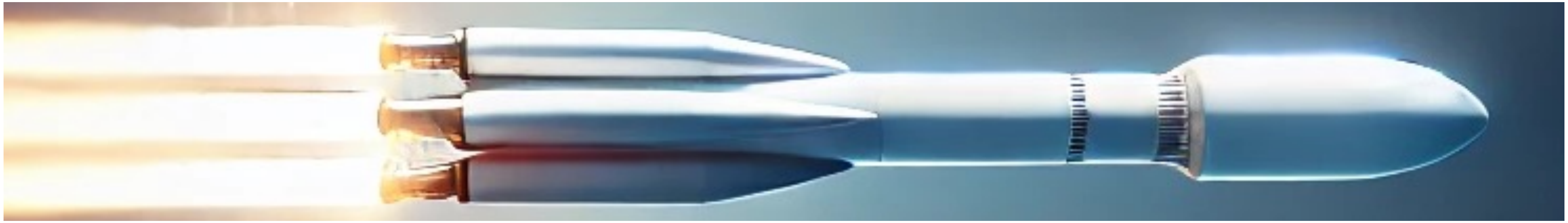
Hadron mass from lattice QCD calculation

$e^{-E_n t}$

t

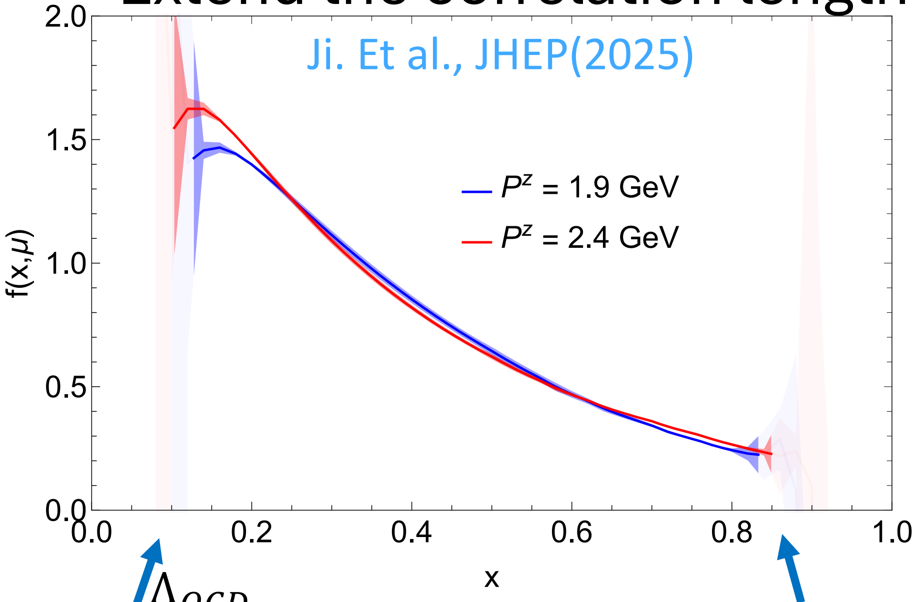
Hadron Structures from Lattice QCD

- Static
  - Spectroscopy
  - Charges
  - Lowest PDF Moments
- Low-Energy
  - Decay constants
  - Higher PDF Moments
  - Form Factors
- Relativistic
  - x-dependent PDF
  - Form Factors
  - Heavy meson decay



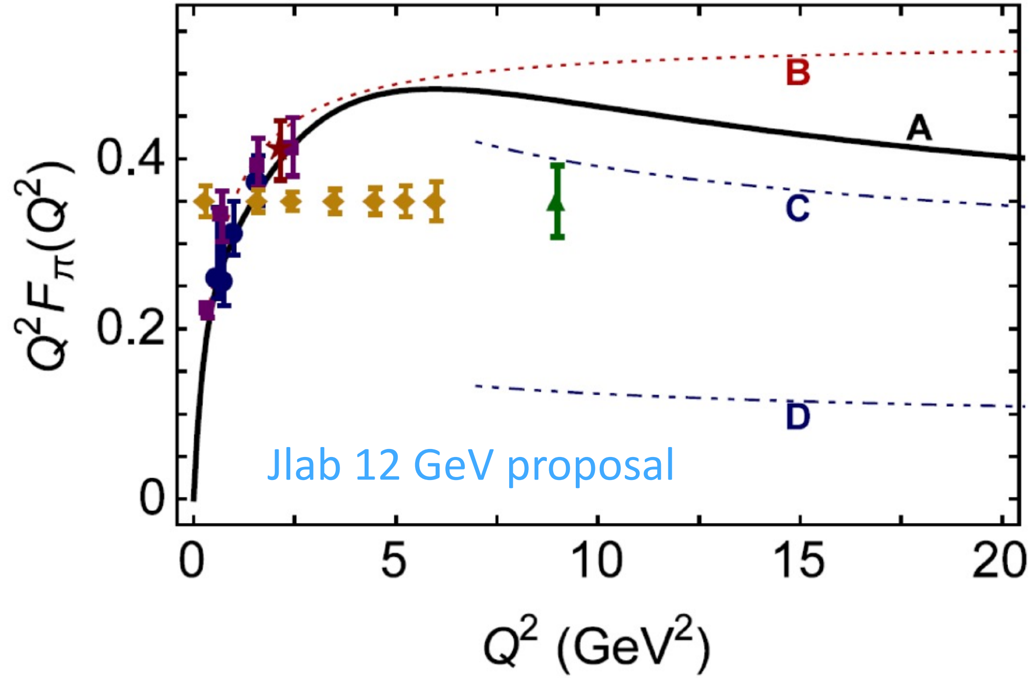
Needs for boosted hadrons on lattice

- Parton Physics:
  - Extend the x-range prediction
  - Reduce power correction $\sim \frac{\Lambda_{QCD}^2}{(xP_z)^2}$
  - Extend the correlation length $z \cdot P$



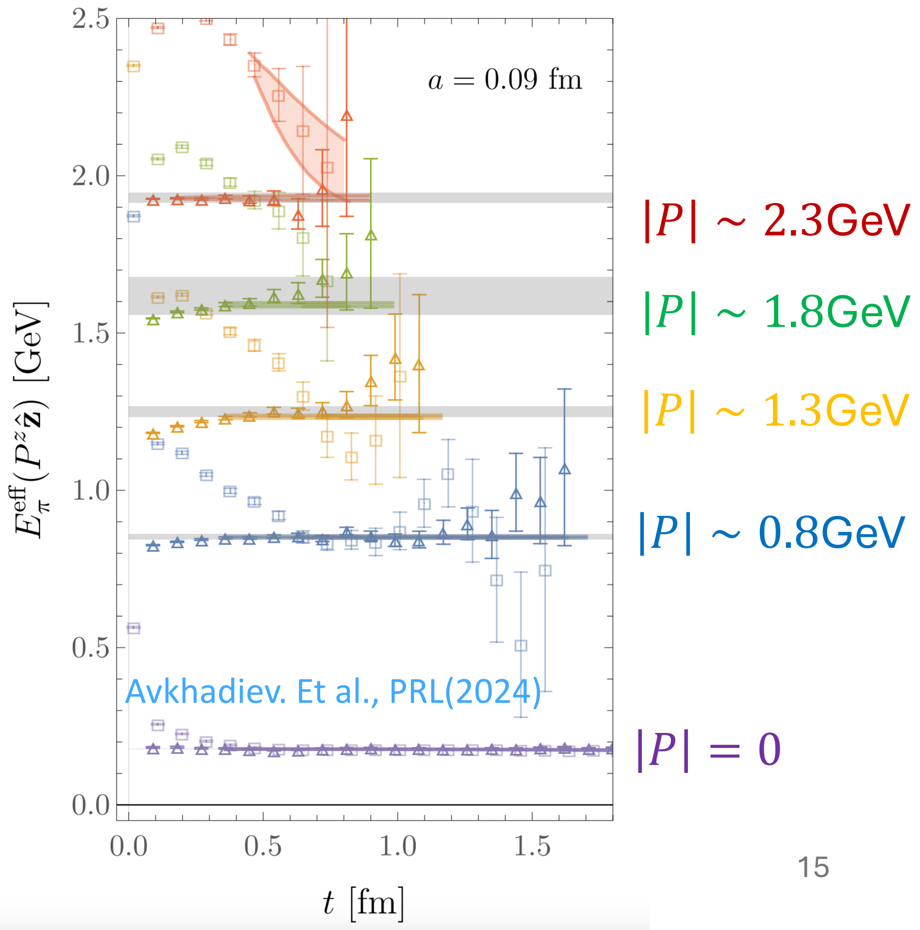
$x_0 \sim \frac{\Lambda_{QCD}}{2P_z}$ $1 - x_0$

- Form Factors:
  - Push to perturbative region
  - Provide guidance for experiments
  - FT to get radial distribution



Unfortunately…

- Boosted hadrons are also expensive to simulate on lattice!
  - The data quality is always good for static pions
  - After a large boost, the data becomes very noisy with the same statistics

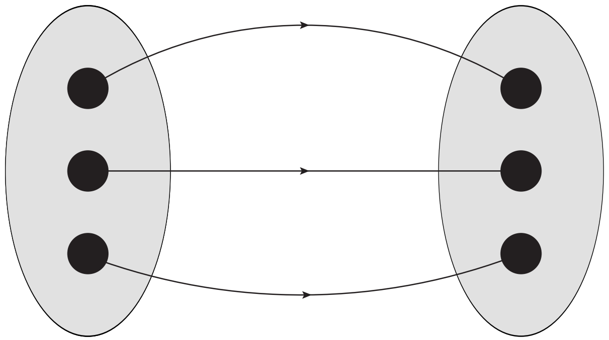


Why is it so difficult?

- Data quality depends on signal and fluctuation of the correlators

- Signal of data on lattice

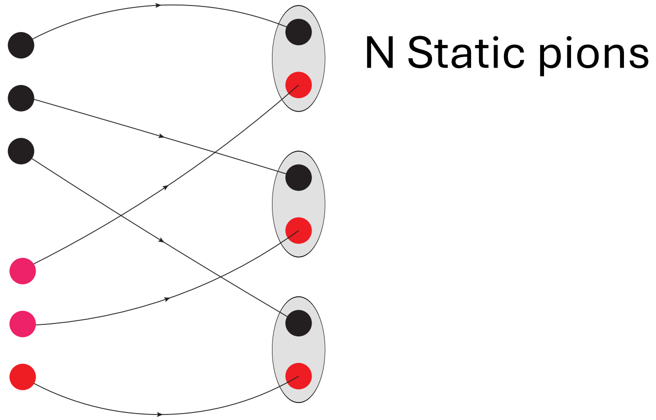
$\langle C_2(t)\rangle \quad \sim e^{-E_0 t}$



Momentum Projection $\sum_x e^{iPx}$

- Statistical Fluctuations (Noise)

$\sigma\langle C_2(t)\rangle = \sqrt{\langle |C_2(t)|^2\rangle - \langle C_2(t)\rangle^{2}} \quad \sim e^{-Nm_\pi t}$



$\sqrt{\sum_x e^{iPx} \sum_y e^{-iPy}}$

Data quality $\equiv$ Signal-to-Noise ratio (SNR) $\sim e^{-(E_0 - Nm_\pi)t}$ decays exponentially

The first leap: momentum smearing

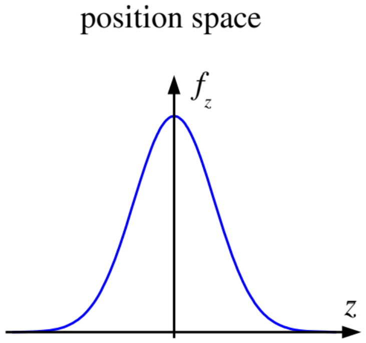
Bali, et al., PRD(2016)

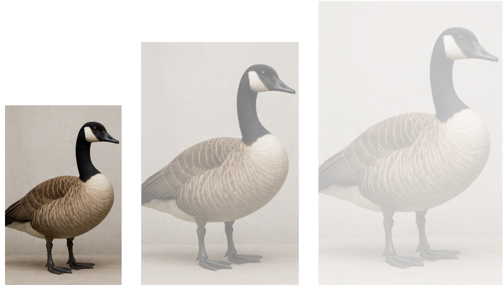
- Projecting hadron state: Fock-State interpolators

$$\chi_\pi = \bar{u}\gamma_5 d, \qquad \chi_N = \epsilon_{abc}(d_a^T C\gamma_5 u_b)u_c$$

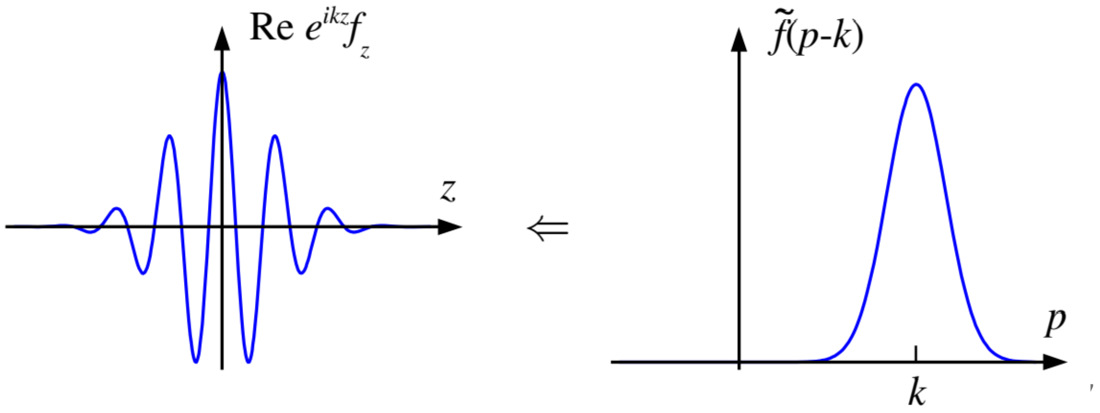
The overlap with hadron states ∝ the wave functions of hadrons

- Conventional Smearing Gusken, NPB(1990)
  - Mimic the size of the ground state



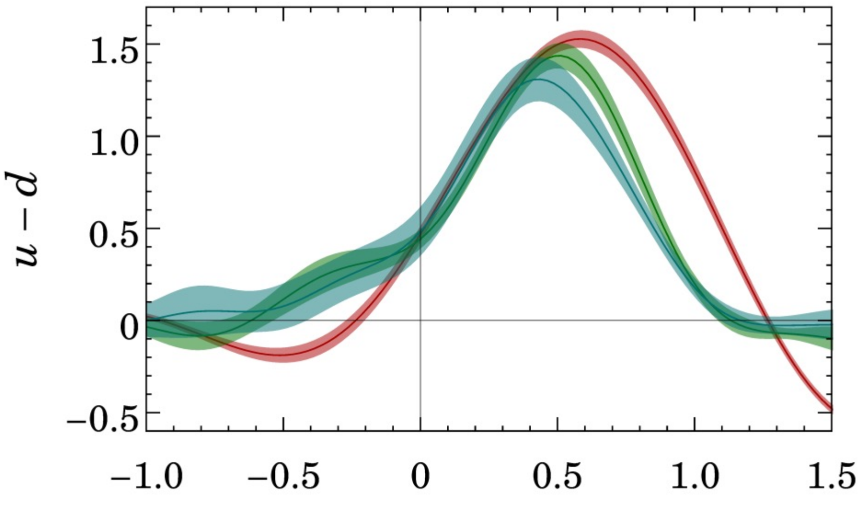


- Momentum Smearing
  - Using moving valence quarks

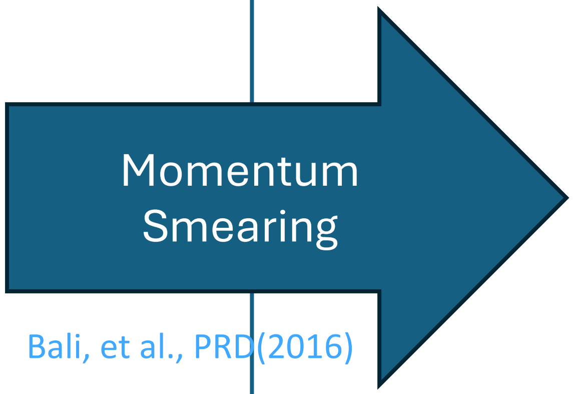


Progresses in boosted hadrons in LQCD

- Early attempts (before 2016)
  - $P_z \leq 1$ GeV

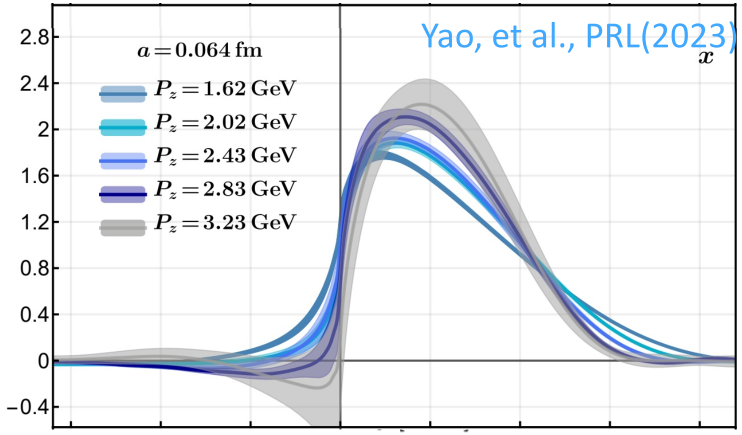


Lin, et al., PRD(2015)

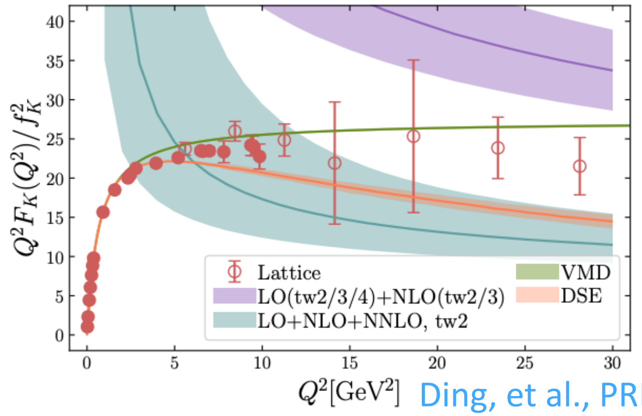


Bali, et al., PRD(2016)

- Recent calculations
  - P_z up to 3 GeV



Yao, et al., PRL(2023)

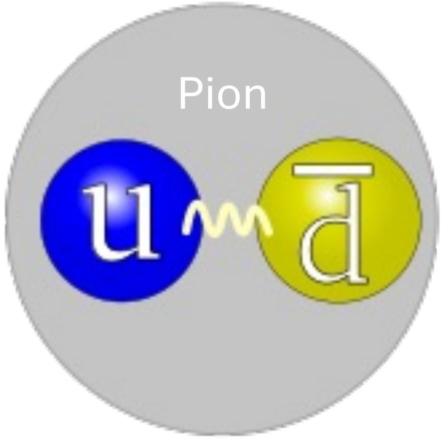


Ding, et al., PRL(2024)

Kinematically Enhanced Interpolators

What's been ignored: spin structure

Largest overlap on lattice

⟨ 

| Pseudo-scalar Interpolator

$\bar{d}\gamma_5 u$ ⟩

Boost

Still true?

No!

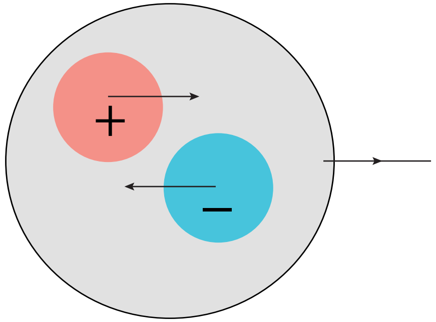
Near the lightcone, the plus-component of spinors are most important!

To get a fast-moving goose

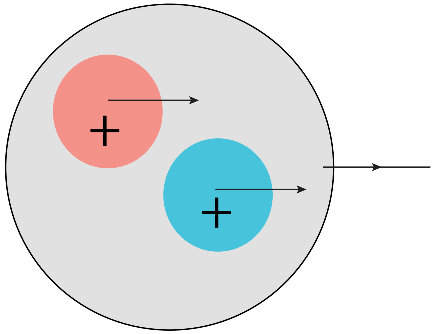
Improved gesture

The spin structure is the “gesture” of hadrons.

Spinors on the lightcone



Traditional



Leading-twist

$\gamma_\pm \equiv (\gamma_t \pm i\gamma_z)/\sqrt{2}, \qquad \psi_\pm = \gamma_\mp\gamma_\pm\psi/\sqrt{2}$

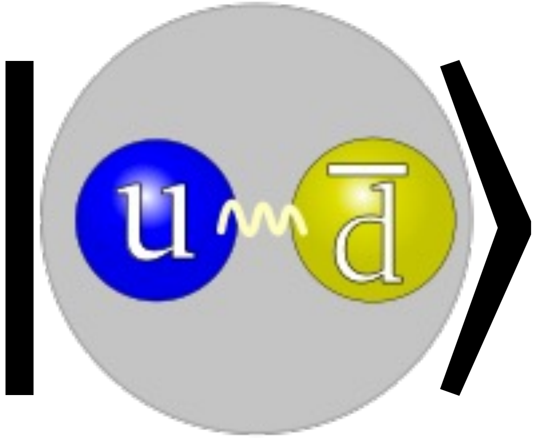
- $\psi = \begin{pmatrix} \uparrow_L \\ \downarrow_L \\ \uparrow_R \\ \downarrow_R \end{pmatrix} = \psi_+ + \psi_- = \begin{pmatrix} 0 \\ \downarrow_L \\ \uparrow_R \\ 0 \end{pmatrix} + \begin{pmatrix} \uparrow_L \\ 0 \\ 0 \\ \downarrow_R \end{pmatrix}$ in chiral basis
- In moving frame, $\psi_+ \propto \sqrt{E + P_z}$, $\psi_- \propto \sqrt{E - P_z}$
- Constructing the interpolator with $\psi_+^\dagger$ and ψ_+ provides the largest amplitude

Ji, Ma, Yuan, EPJC(2004)

- Traditional interpolator: $\bar{\psi}\gamma_5\psi = \frac{(\psi_+^\dagger\gamma_t\gamma_5\psi_- + \psi_-^\dagger\gamma_t\gamma_5\psi_+)}{2} \propto M$
- Pion leading-twist wave function: $\psi_+^\dagger\gamma_5\psi_+ = \sqrt{2}\bar{\psi}\gamma_+\gamma_5\psi \propto E + P_z$

Lepage, Brodsky, PLB(1979)
Efremov, Radyushkin, PLB(1980)

Better spin structures for boosted hadrons



$$= |\bar{d}\gamma_+\gamma_5 u\rangle + O\left(\frac{1}{P_+}\right)|\ldots\rangle$$

Kinematical enhancement at finite momentum:

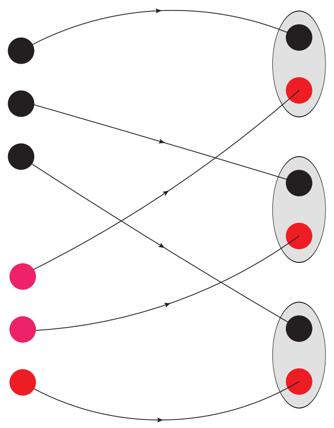
$\langle\pi|\bar{d}\gamma_\mu\gamma_5 u\rangle = if_\pi P_\mu,$ increases with P_μ

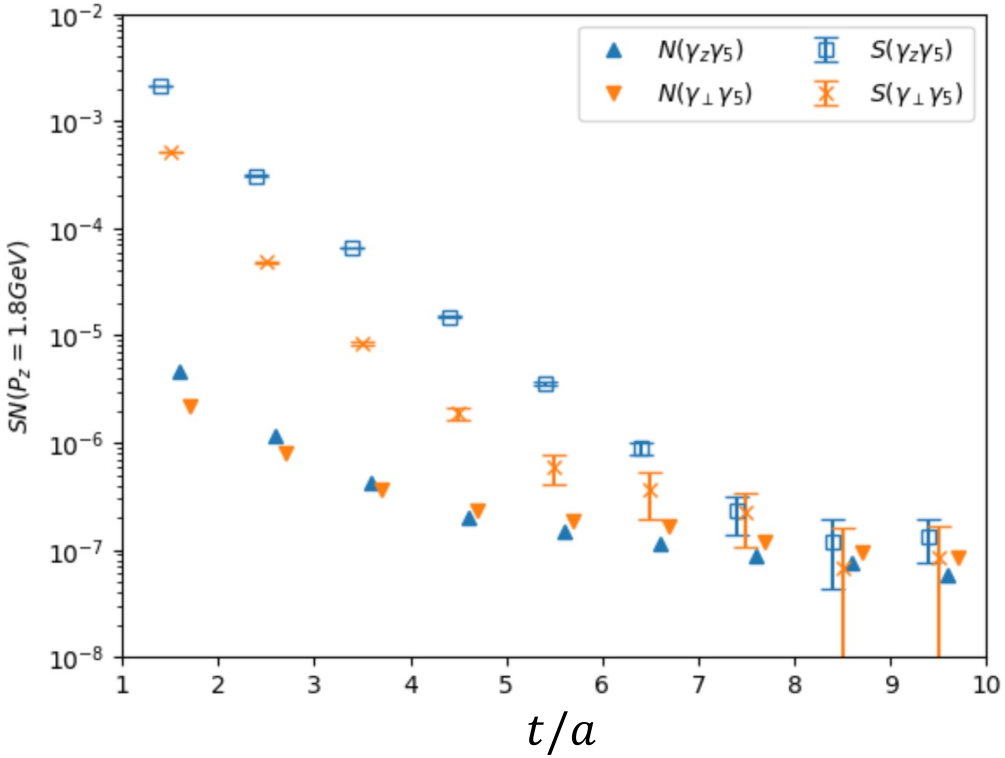
Larger momentum = Larger overlap!

Actual precision: Signal-to-Noise Ratio (SNR)

- What about noise? $\mathrm{Var}(C_{2\mathrm{pt}}) = \left\langle \Re[C_{2\mathrm{pt}}]^2 \right\rangle - \langle \Re[C_{2\mathrm{pt}}] \rangle^2 = \frac{1}{2}\langle C_{2\mathrm{pt}}^\dagger C_{2\mathrm{pt}} \rangle + \cdots$
- The asymptotic contribution is from static multi-pion states
  - No kinematic enhancement in noise

N static pions

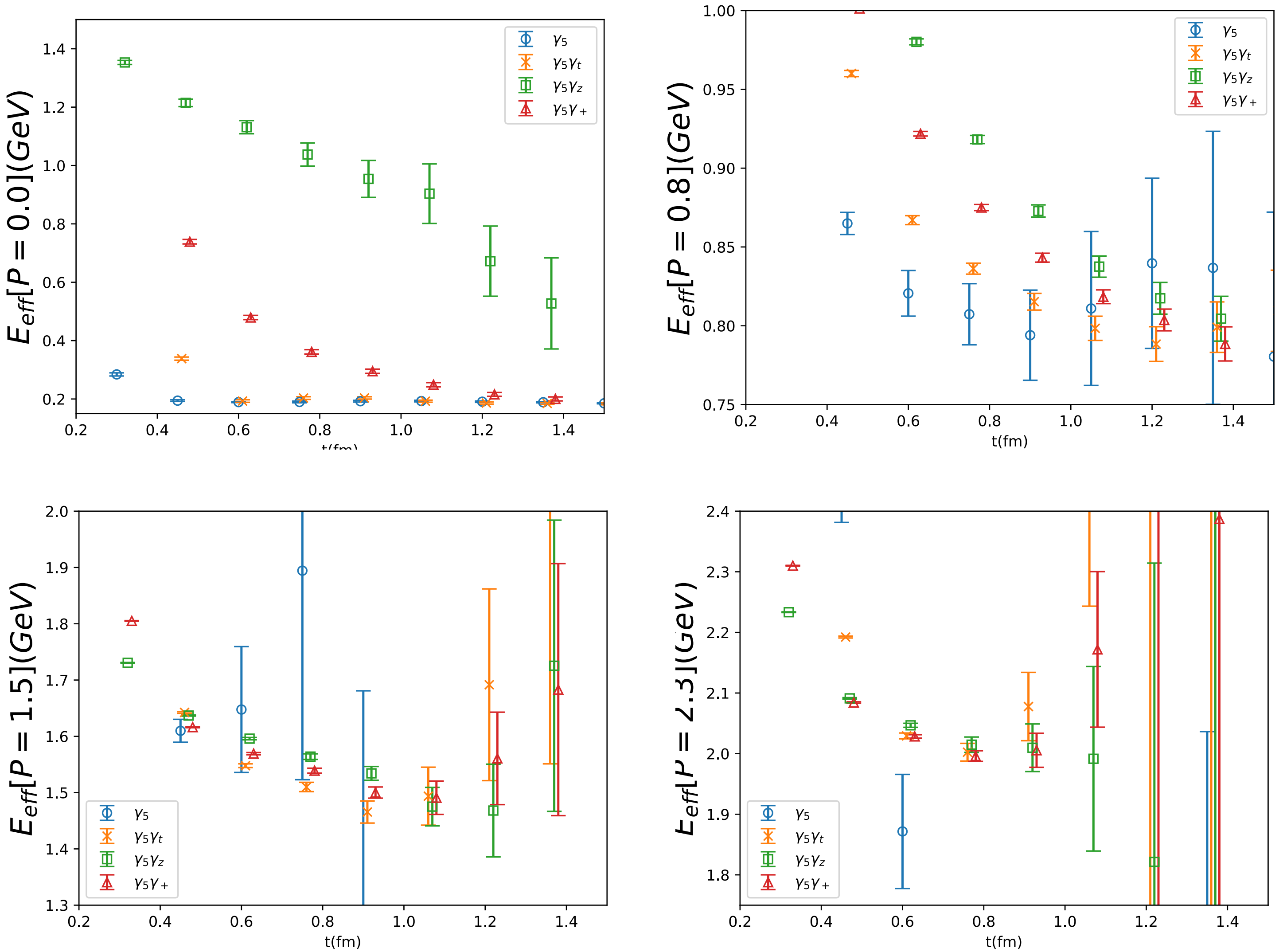




The SNR indeed gets enhanced!

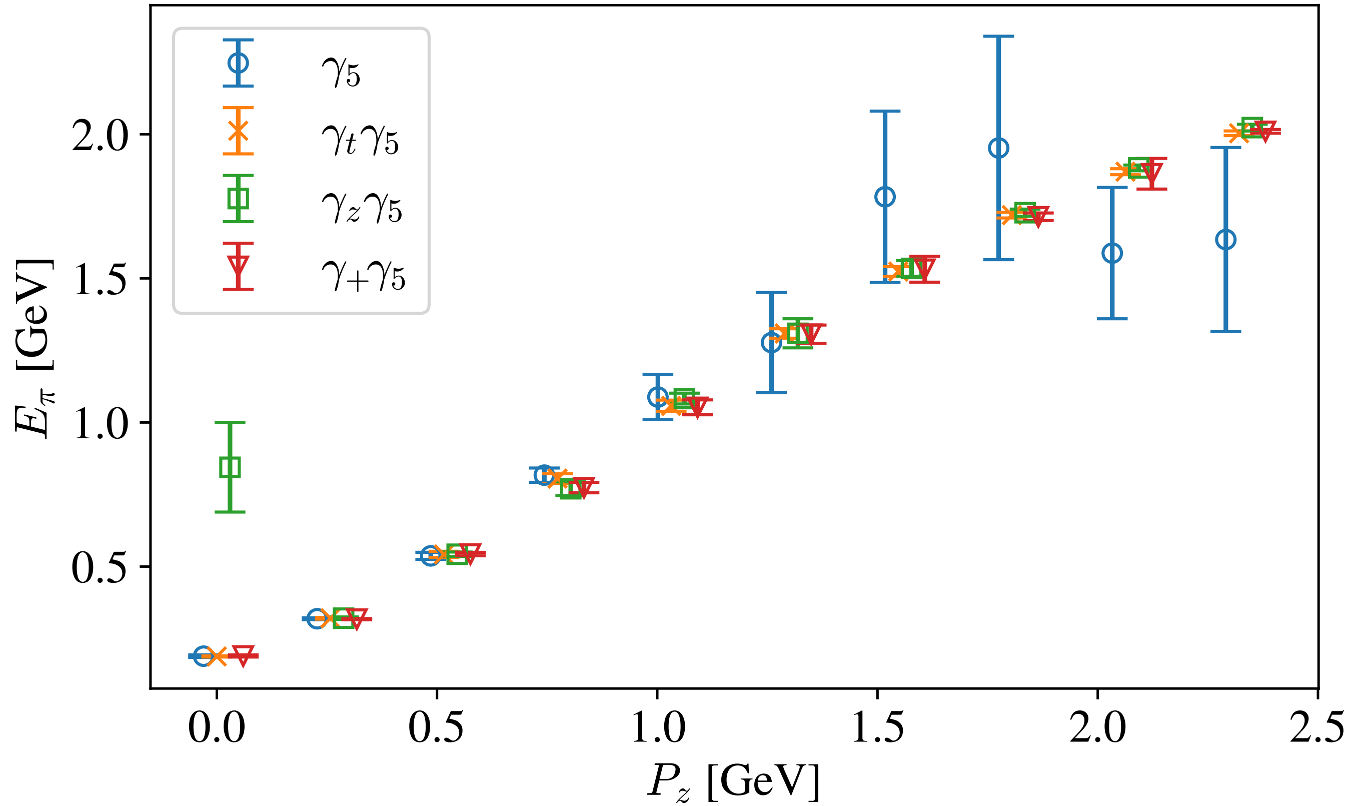
Numerical Tests

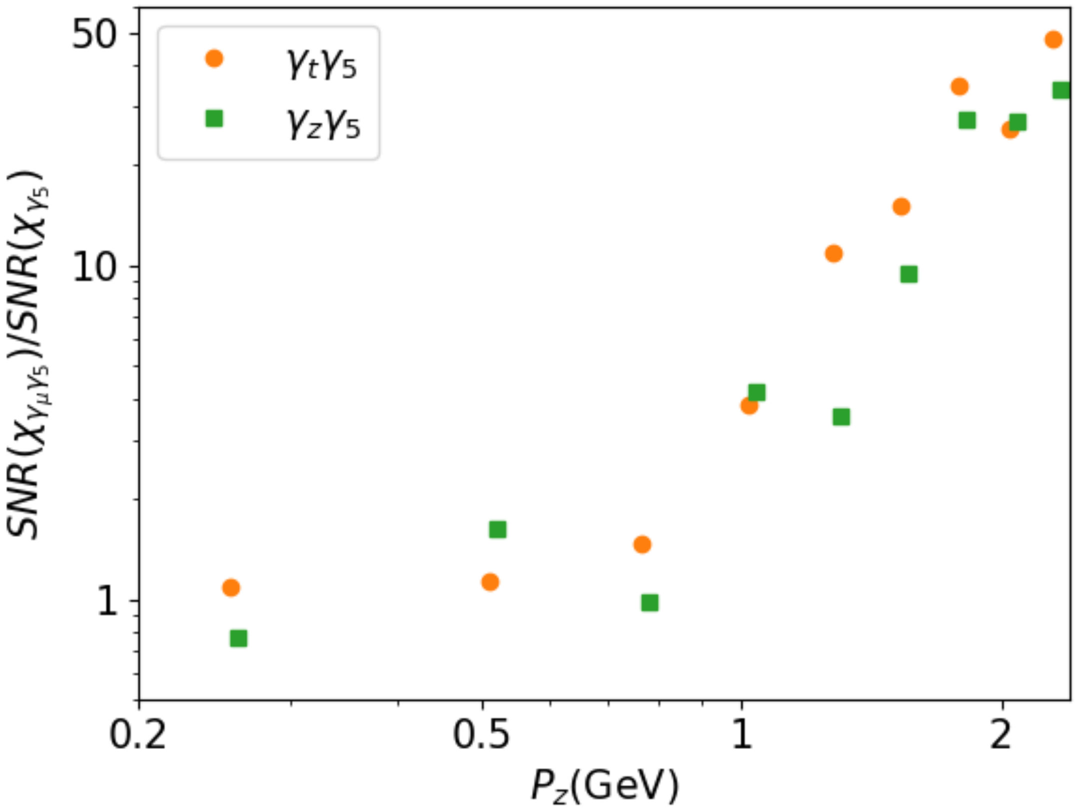
- Lattice Spacing: $a = 0.15$ fm
- Volume: $L^3 \times T = 32^3 \times 48$
- Action: clover-on-HISQ
- Pion Masses:
  - $m_\pi^{sea} = 135$ MeV
  - $m_\pi^{val} = 190$ MeV
- Smearing:
  - 2 Steps HYP
  - $k = 1.55$ GeV
- Momentum: $P = [0,2.3]GeV$
- Stat: 334 cfgs
- Sources: 64 on each cfg



E_0 extraction and the enhancement

- Extract the energy spectrum:



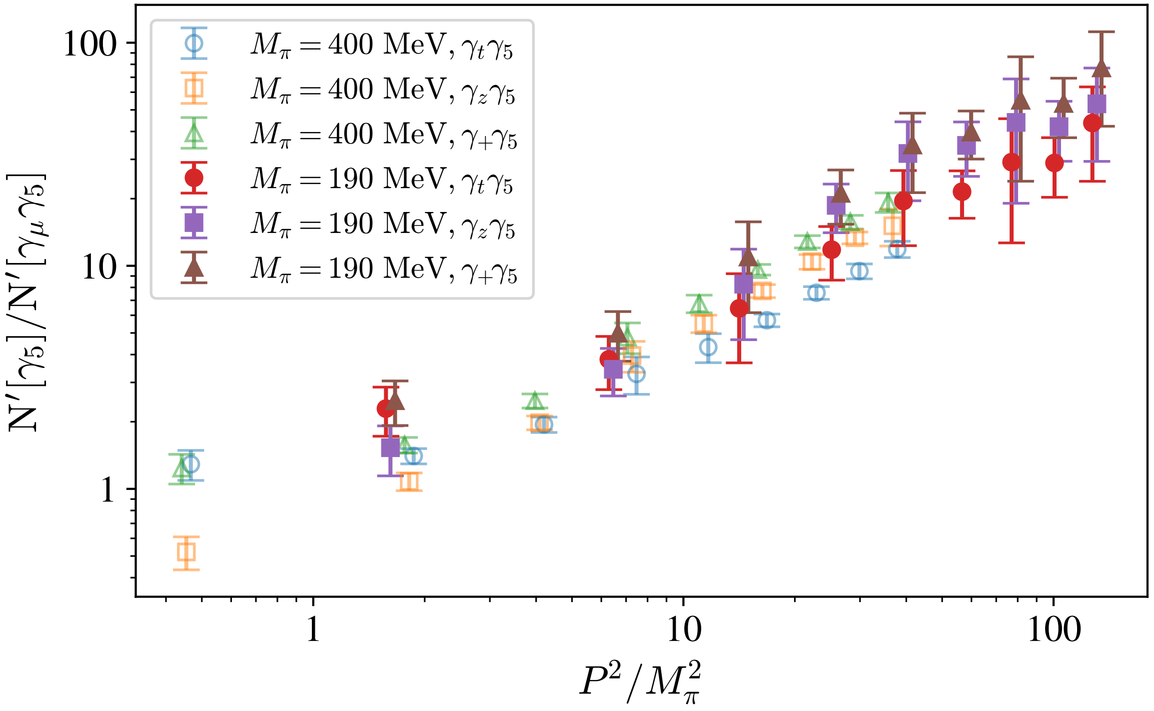


Larger momentum = Larger improvement!

Dependence on hadron masses

- Enhancement factor $\propto \frac{P_\mu^2}{m_H^2}$
- Lighter pion masses = larger improvements!
- Ground-state hadron gets the largest enhancement

Tests with different pion masses

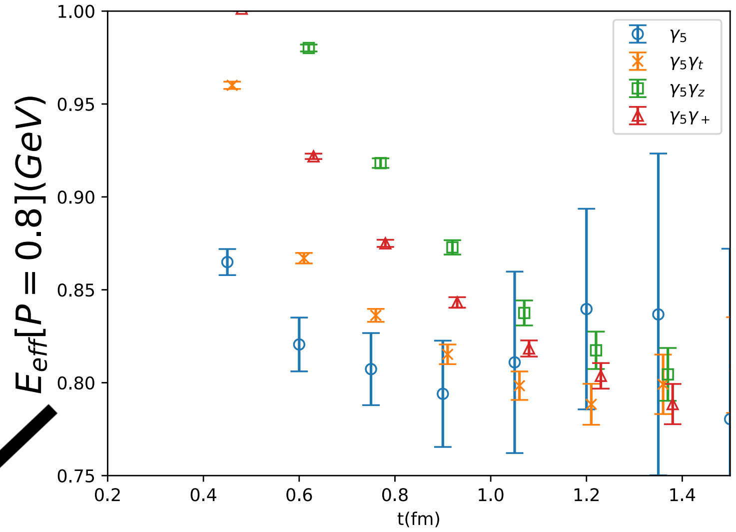


Excited State Contamination

- The same interpolator overlaps with multiple hadron states

$$\langle[\bar{u}\gamma_\mu\gamma_5 d]^\dagger[\bar{u}\gamma_\nu\gamma_5 d]\rangle = \sum_{n,S=1}\left(\frac{P_\mu P_\nu}{M_{n,1}^2} - g_{\mu\nu}\right)C_{n,1} + \sum_{n,S=0}\frac{P_\mu P_\nu}{M_{n,0}^2}C_{n,0}$$

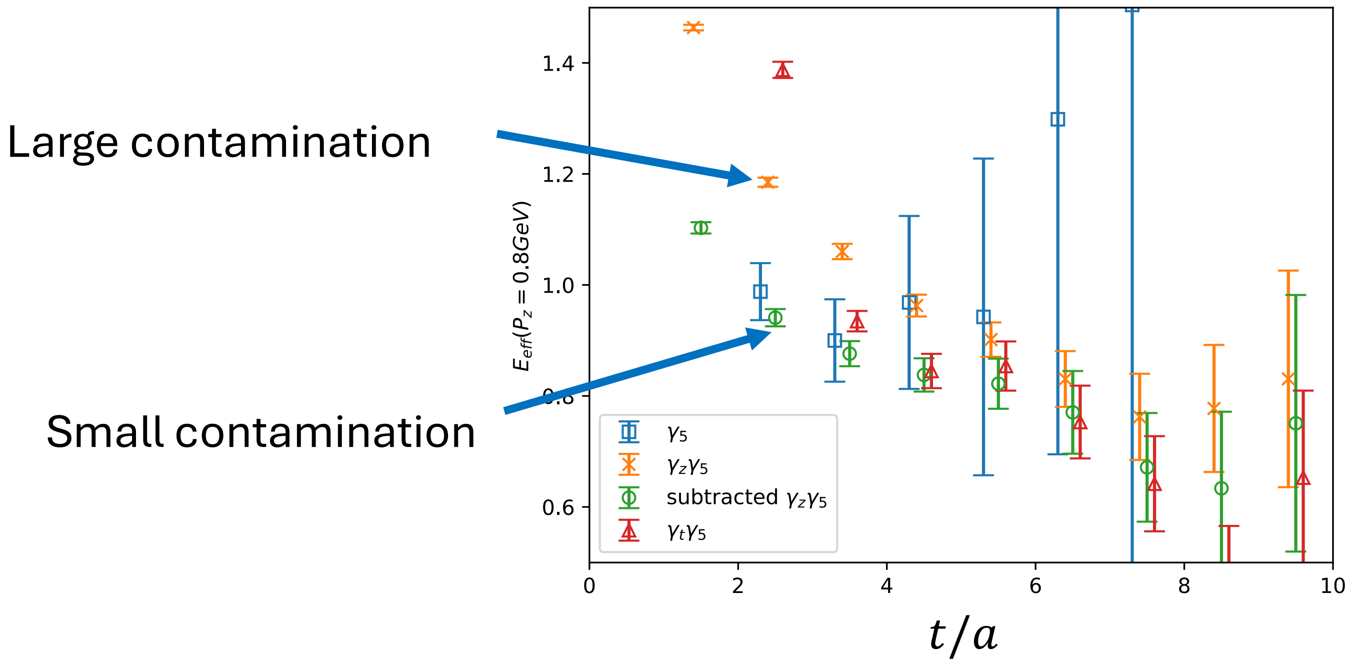
- States with same quantum number
  - γ_t: $\frac{m_0^2 E_n^2}{m_n^2 E_0^2}$ suppression, no suppression at small P_z
  - γ_z: $\frac{m_0^2}{m_n^2}$ suppression, suppressed at all P_z ✓
- States with higher spin
  - γ_t: $\frac{m_0^2 P_z^2}{m_n^2 E_0^2}$ suppression, increases with P_z until $\frac{m_0^2}{m_n^2}$ ✓
  - γ_z: $\frac{m_0^2 E_n^2}{m_n^2 P_z^2}$ suppression, decrease with P_z until $\frac{m_0^2}{m_n^2}$



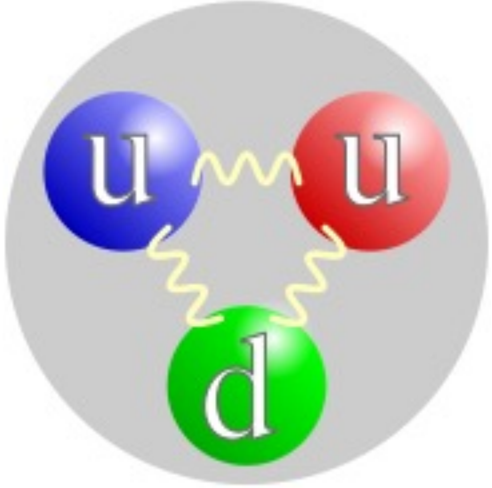
Both interpolators have non-negligible excited-state contamination at small P_z

An approach to suppress both excited states

- $\bar{q}\gamma_z\gamma_5 q$ at small P_z: smaller contamination from spin-0 states, but larger contamination from spin-1 states
- The same spin-1 states exist in $\bar{q}\gamma_\perp\gamma_5 q$ with smaller strength
- $\langle \bar{q}\gamma_z\gamma_5 q | \bar{q}\gamma_z\gamma_5 q\rangle - \langle \bar{q}\gamma_\perp\gamma_5 q | \bar{q}\gamma_\perp\gamma_5 q\rangle$ suppresses the spin-1 states



Interpolators for nucleons



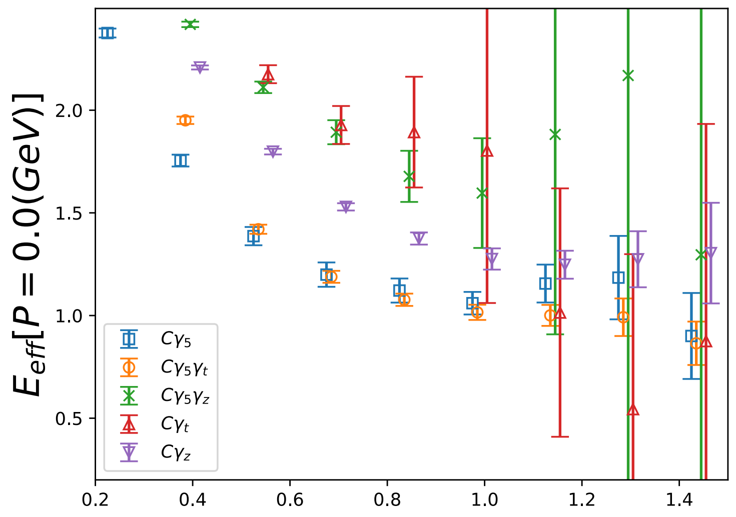
- Traditional interpolator: $\epsilon_{abc}(d_a^T C\gamma_5 u_b)u_c$
- Constructing the interpolator with only plus components
- Diquark: $\epsilon_{abc}d_a^T C\gamma_5 u_b \rightarrow \epsilon_{abc}d_a^T C\gamma_5\gamma_\mu u_b$ and $\epsilon_{abc}d_a^T C\gamma_\mu u_b$
- Free quark: $u_c \rightarrow \gamma_\mu u_c$ (already been used in the parity projection $1 \pm \gamma_t$)
- $\epsilon_{abc}(d_a^T C\gamma_5 u_b)u_c$ provides largest component at rest, but when boosted, the following two are largest:
  - $\epsilon_{abc}\left(d_a^T C\gamma_5\gamma_\mu u_b\right)\gamma_t u_c$
  - $\epsilon_{abc}\left(d_a^T C\gamma_\mu u_b\right)\gamma_t u_c$

Nucleon Test

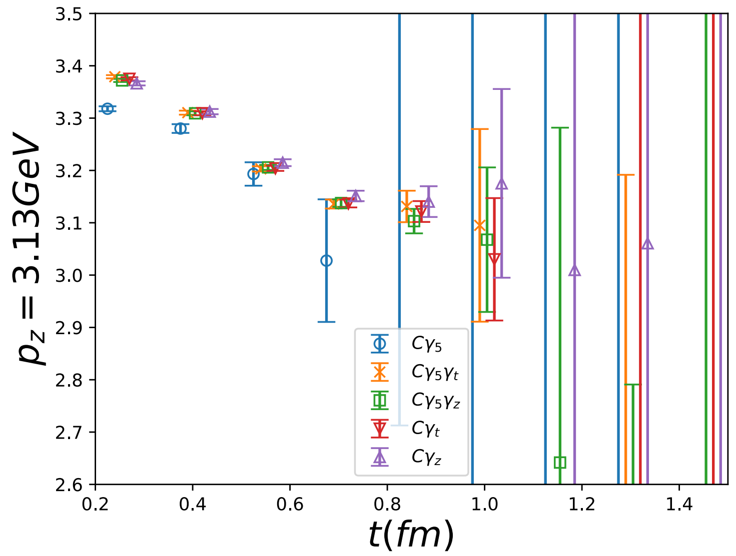
- Lattice Spacing: $a = 0.15$ fm
- Volume: $L^3 \times T = 32^3 \times 48$
- Action: clover-on-HISQ
- Pion Masses:
    - $m_\pi^{sea} = 135$ MeV
    - $m_\pi^{val} = 190$ MeV
- Smearing:
    - 2 Steps HYP
    - $k = 1.55$ GeV
- Momentum: $P = [0,4] GeV$
- Stat: 208 cfgs
- Sources: 16 on each cfg

For $P_z > 3$ GeV SNR is enhanced by a factor of 10

Hitting heavier states without boost



All converge to ground-state nucleon, large enhancements



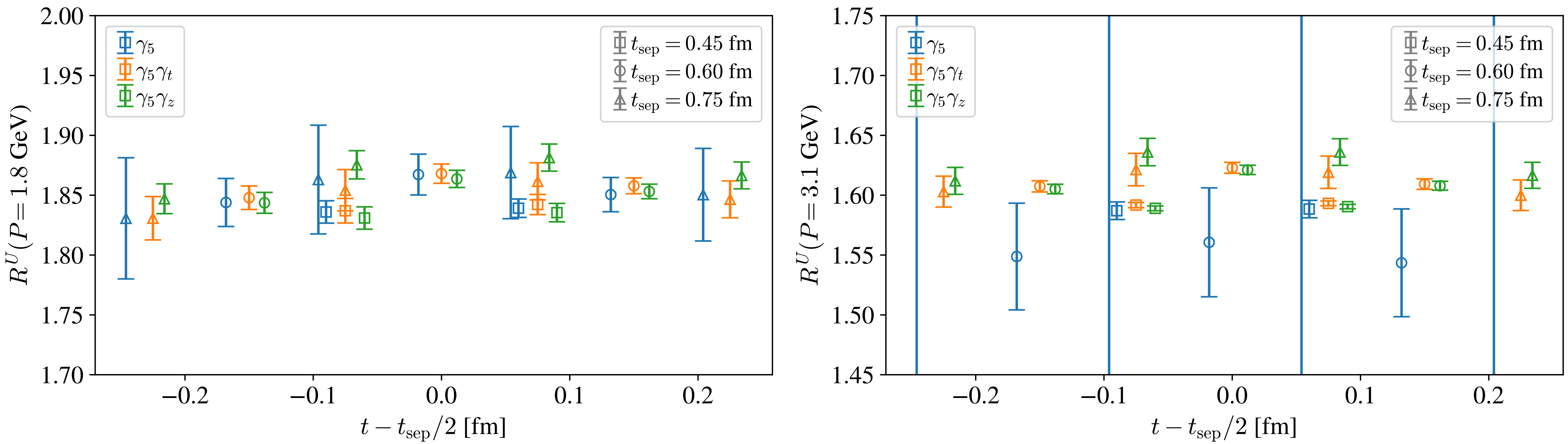
Same improvement in 3pt correlators

Extraction of matrix elements:

$$C_2(t) = \sum |c_n|^2 e^{-E_n t}, \qquad C_3(t, t_{sep}) = \sum c_m^* c_n \langle m|O|n\rangle e^{-E_m(t_{sep}-t)} e^{-E_n t}$$

$$\langle 0|O|0\rangle \approx R(t, t_{sep}) \equiv \frac{C_3(t, t_{sep})}{C_2(t_{sep})}$$

- SNR is enhanced by a factor of 10 at $P_z > 3GeV$ for $t_{sep} \geq 0.6$ fm



Conclusion and Outlook

Summary

- Measuring boosted hadrons is important but expensive in lattice QCD
- We propose a set of kinematically interpolators for boosted hadrons
- The enhancement exists in signal but not in noise, thus improves the precision of the measurement by a large factor
- The improvement of pion can reach 40~50 for $\frac{P_z}{m_\pi} > 10$
- The improvement of nucleon 2pt and 3pt can reach ~10 for $\frac{P_z}{m_N}$ >3
- Enhancement is larger for lighter states, thus suppresses the excited state contamination for pion
- Can be easily combined with all existing techniques (momentum smearing, GEVP, distillation, etc.)

Outlook

- The new interpolators open a new door to calculations of hadron structure at very large momentum.
- Improve the precision of calculating parton physics
- Enable us to access form factors at very large Q^2
- Improve the the calculation of $\pi - \pi$ scattering with large invariant mass by a factor of P_z^4/m_π^4
- Improve the study of heavy meson decay to pion

- More studies on excited state contaminations are needed especially for nucleons

Thank you for listening!

Kinematically Enhanced Lattice Interpolators

This work was developed shortly after I found out I was expecting my son,

Zheng, KELI （郑愘理）.

“Keli” (愘理) means ‘to revere and uphold principle and reason with integrity and respect’.