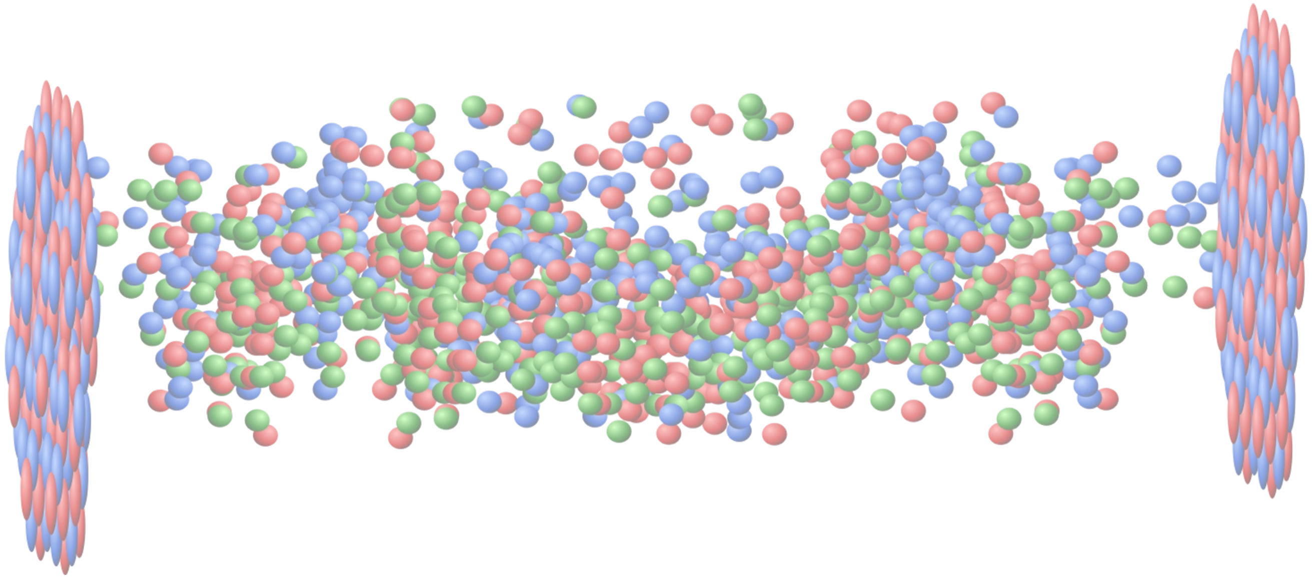
# Spatial Transverse Gradients in the Thermalisation of the Quark-Gluon-Plasma



Uri Sharell, Rising Researchers Seminar, 03/06/2025

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

UNIVERSITY OF OXFORD

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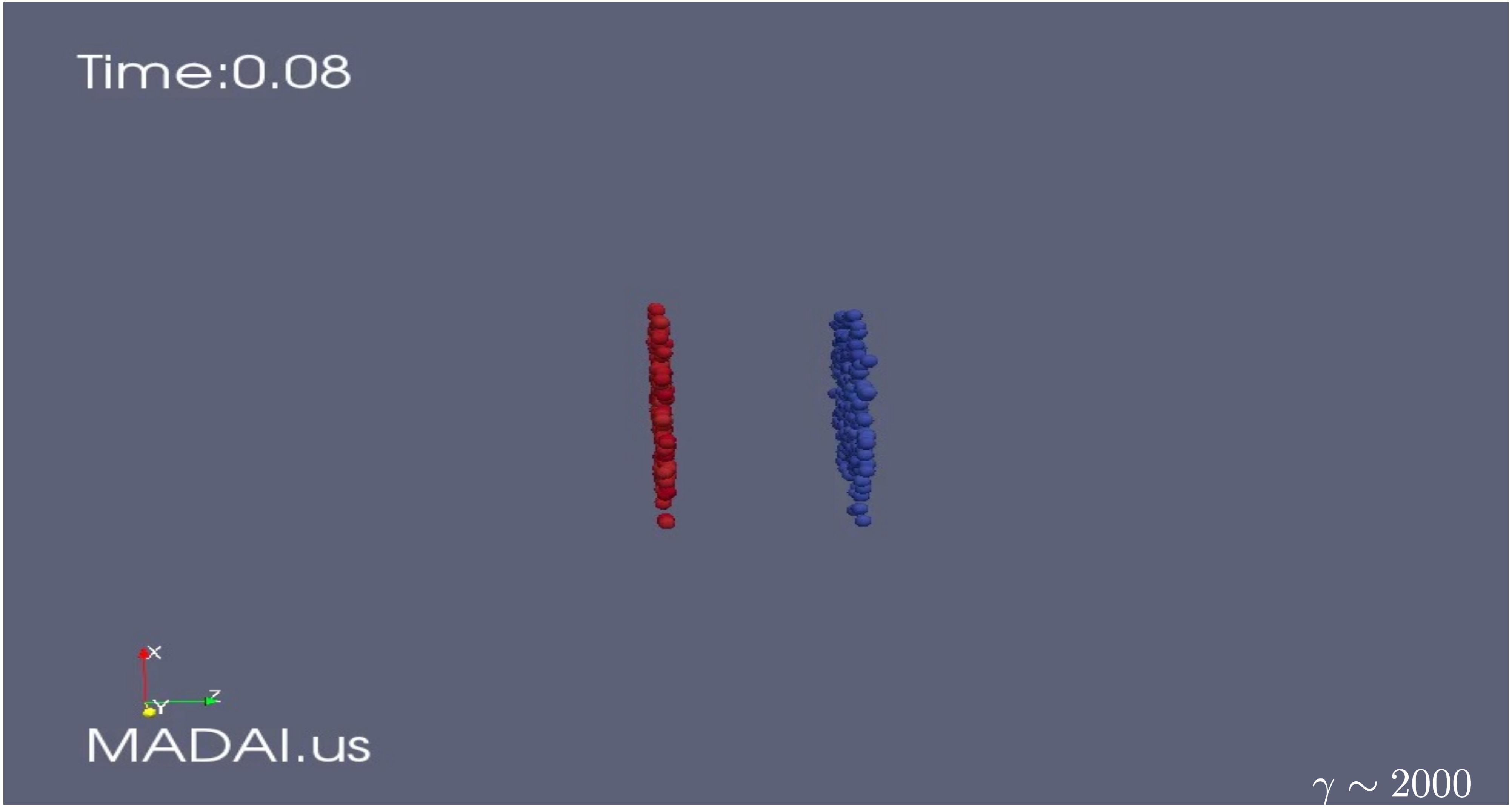
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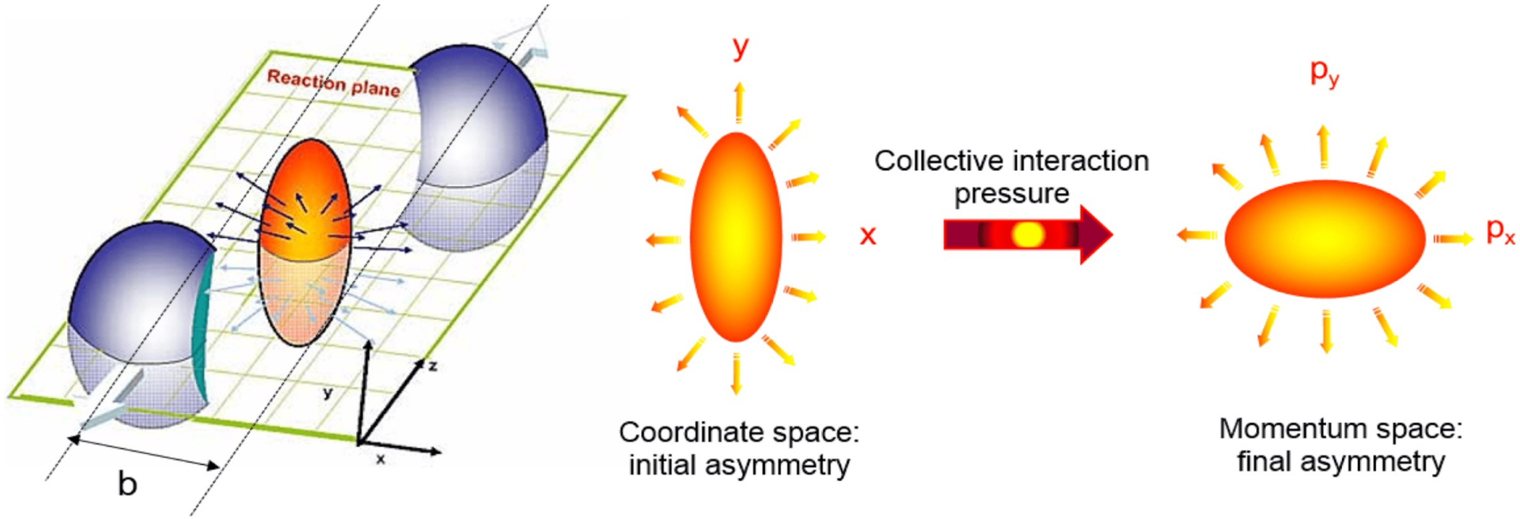
$\gamma \sim 2000$

# Quark-gluon plasma in HIC

- High temperatures (>150 MeV) are produced in heavy ion collisions (little bang)
- Quarks and gluons (partons) deconfine and produce new state of matter
- Strong experimental evidence for collective flow




Graphic from (Pasechnik et al., 2017)

# What is a fluid ?

The Mathematicians answer:

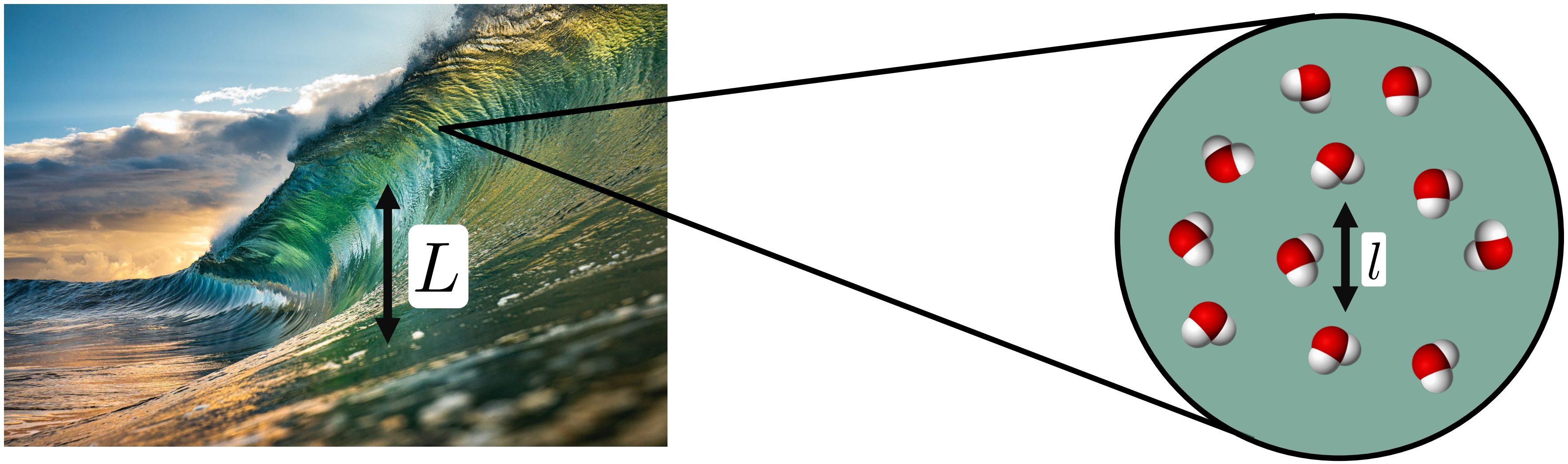
"A fluid is something that obeys fluid dynamics"

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot j = 0 \qquad \frac{\partial u}{\partial t} + u \cdot \boldsymbol{\nabla} u = -\frac{\boldsymbol{\nabla} P}{\rho} + \text{Viscous corrections}$$

$$(\boldsymbol{\nabla}_\mu T^{\mu\nu} = 0)$$

Macroscopic quantities: pressure, density, temperature, shear stresses, etc.

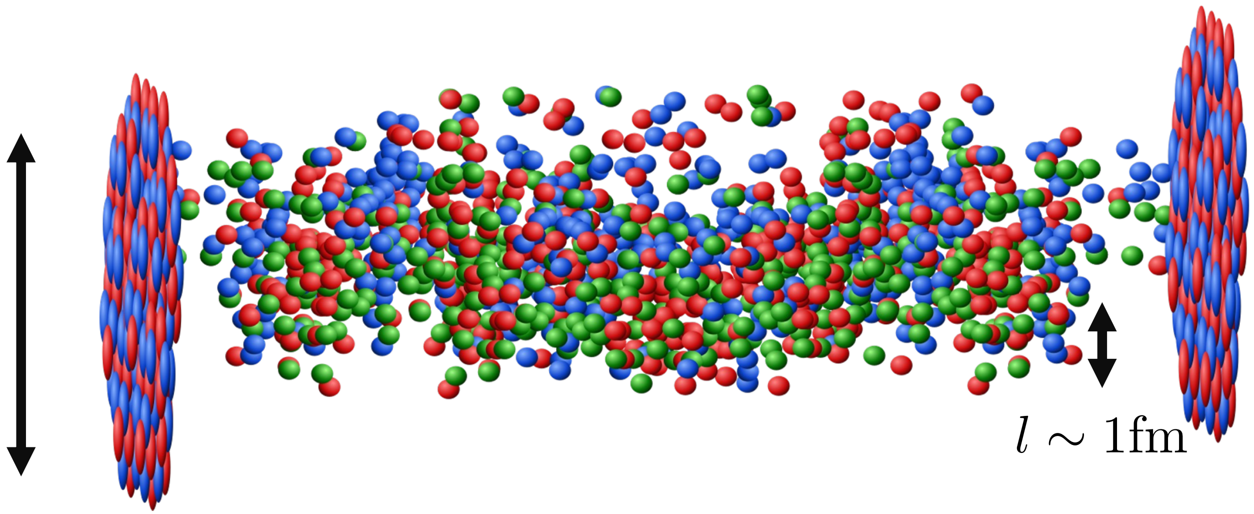
# ~~What~~ Why is a fluid ?




$$L \sim 1\mathrm{m} \qquad \ggg \qquad l \sim 10^{-10}\mathrm{m}$$

- **Local** thermal equilibrium / coarse graining / system determined by few macroscopic (thermodynamic) quantities.
- Hydrodynamics: small deviations from thermal equilibrium

# Heavy ion collisions

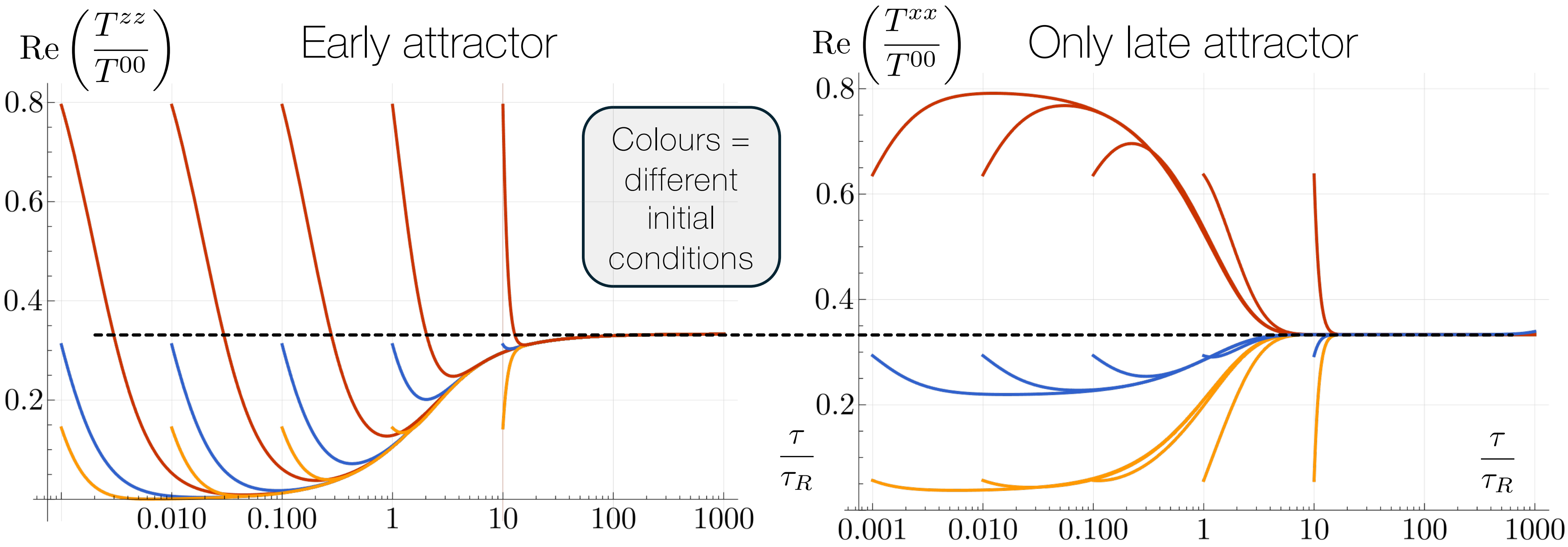



- No separation of scales (small system)
- Very far from thermal equilibrium !

So how and why is this a fluid ?

Spatial Transverse Gradients in the **Thermalisation** of the Quark-Gluon-Plasma

# Attractors




Dashed: Thermal equilibrium $(T^{ii}/T^{00} = 1/3)$

# The question

- Previous analyses had assumed rotational symmetry in the transverse direction for simplicity
- Spatial transverse gradients are relevant for small systems

Does breaking this symmetry change how the system thermalises ?

Spatial Transverse Gradients in the Thermalisation of the Quark-Gluon-Plasma

# Outline

1. Kinetic Theory
2. Eigenspectrum analysis
3. Thermalisation

# 1. Kinetic Theory

How do we compute things ? The theoretical setup

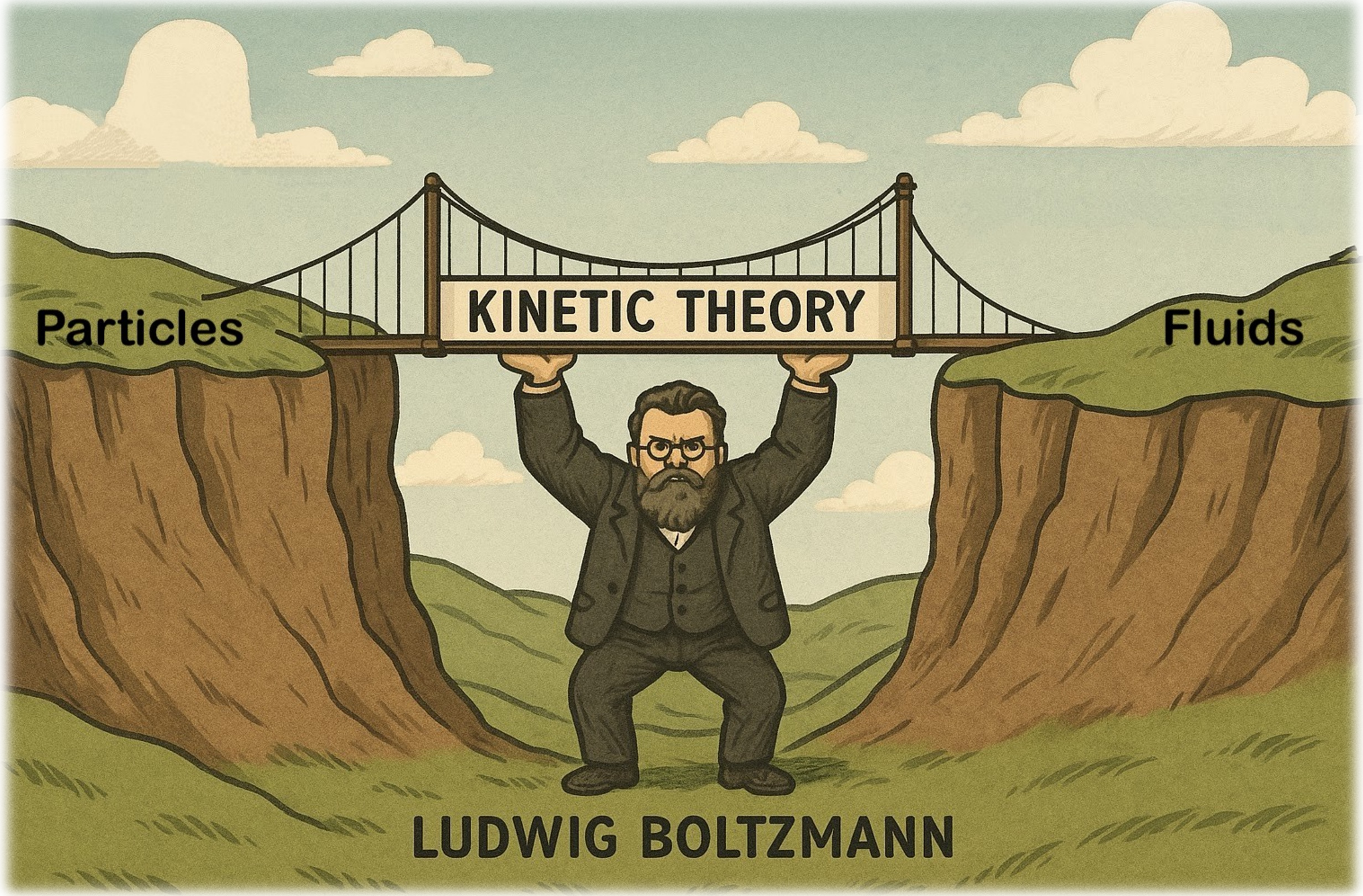



Image generated with DALL E 3

# A crash course in Kinetic Theory

Single-particle distribution function $f = f(t, x, y, z, p_x, p_y, p_z)$

Physical meaning, e.g. $n(t, \vec{r}) = \int f \mathrm{d}^3 p$ or $N(t) = \int f \mathrm{d}^3 p \, \mathrm{d}^3 x$

Boltzmann equation

$$\left( \underbrace{p^\mu \frac{\partial}{\partial x^\mu}}_{\text{Streaming}} + \underbrace{F^\mu \frac{\partial}{\partial p^\mu}}_{\text{External forces}} \right) f = \underbrace{-\hat{C}[f]}_{\text{Collisions}}$$

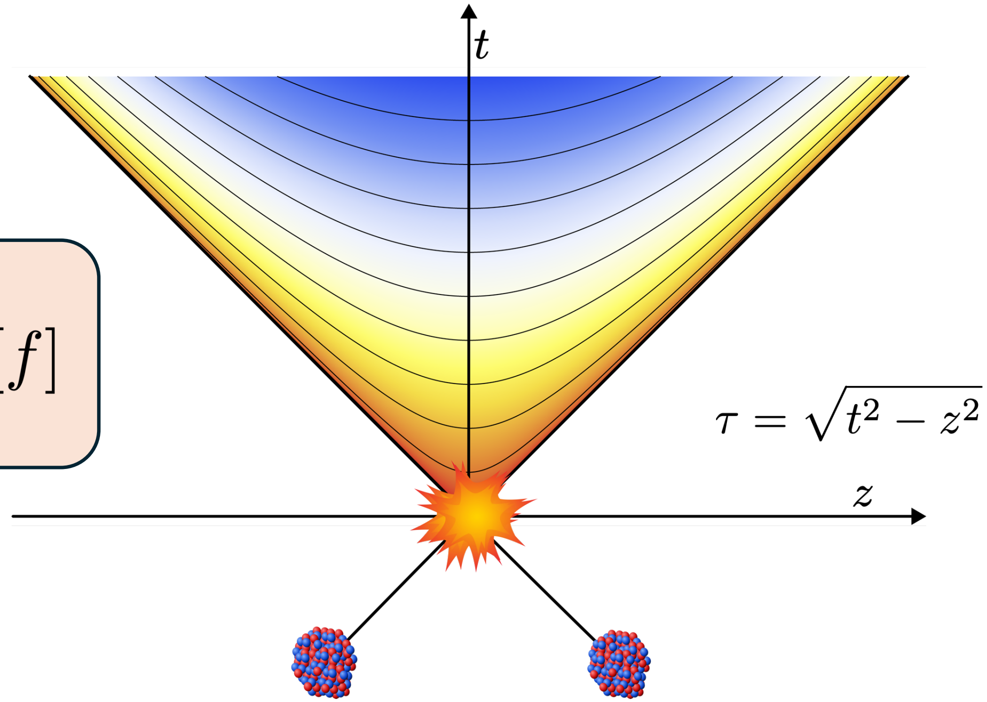
Relaxation-time approximation $\hat{C}[f] = \frac{f - f_{eq}}{\tau_R}$ $\tau_R = \text{const.} = 1$

# Boost-invariance

$$v_z = \frac{z}{t} \xrightarrow[\text{Boost}]{} v'_z = \frac{v+w}{1+vw} = \frac{\gamma_w(z+tw)}{\gamma_w(t+zw)} = \frac{z'}{t'}$$

$$(t, z) \to (\tau, 0) \qquad f = f(\tau, \vec{k}_\perp, \vec{p})$$

$$\boxed{\frac{\partial f}{\partial \tau} - \frac{p_z}{\tau}\frac{\partial f}{\partial p_z} + \frac{i\vec{k}_\perp \cdot \vec{p}_\perp}{p} f = -C[f]}$$



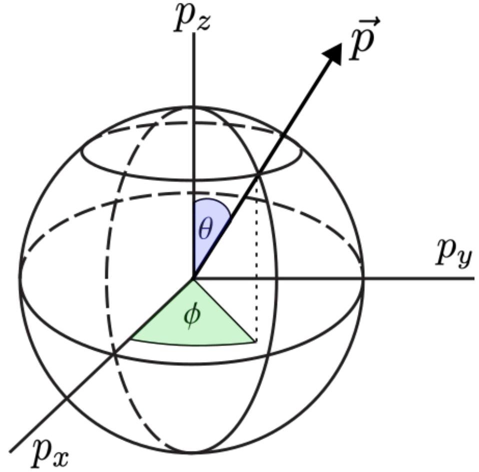

(Bjorken, 1983), (Baym, 1984)

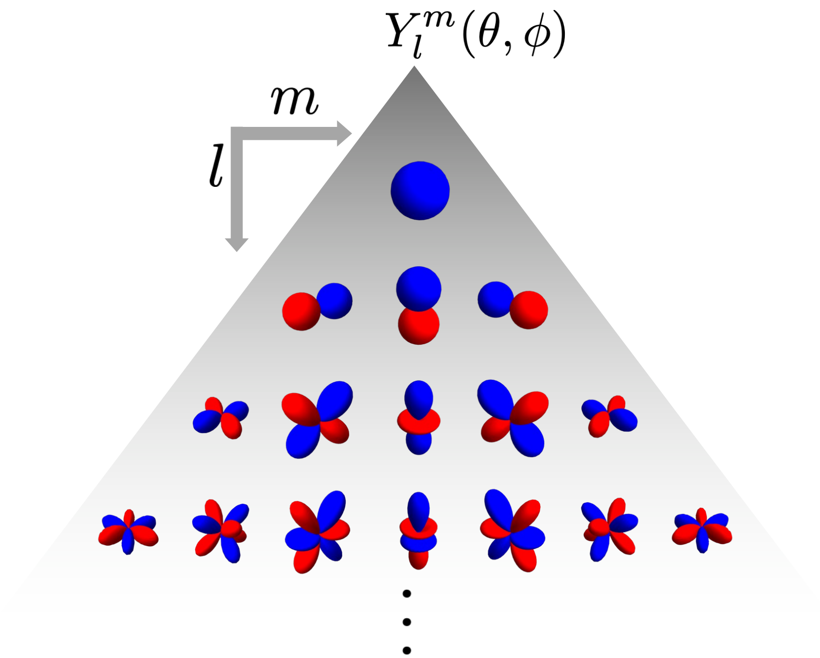
# Expansion

$$F \equiv \int \mathrm{d}p\, p^3 f \;\; = \sum L_{lm} Y_l^m$$

$$F = F(\tau, k_\perp, \theta, \phi)$$

$$L_{lm} = L_{lm}(\tau, k_\perp)$$



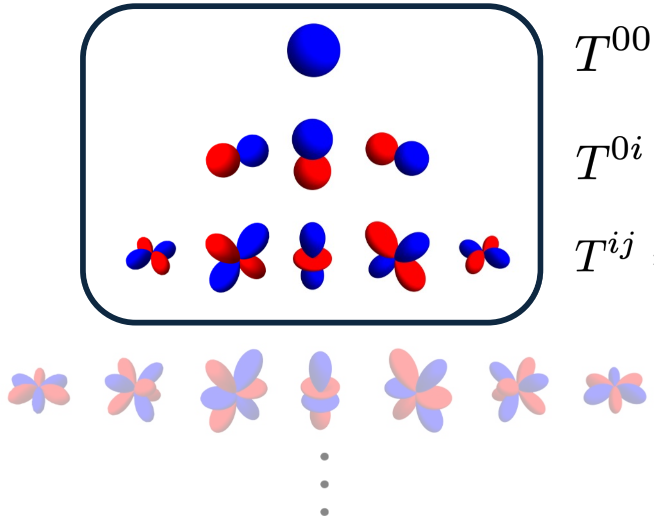




(Blaizot & Yan, 2018), (Brewer et al., 2021), (Brewer et al., 2022)

# Physical meaning

Moments with $l = \{0, 1, 2\}$ correspond to the stress-energy tensor

$$T^{\mu\nu} = \int p^\mu p^\nu f \; = \text{Coefficients of}$$



$$T^{00} = \varepsilon \;\; = L_{00}$$

$$T^{0i} = p_i$$

$$T^{ij} = P_i, \Pi_{ij}$$

# Tower of ODEs

$$F \equiv \int \mathrm{d}p\, p^3 f \quad = \sum L_{lm} Y_l^m$$

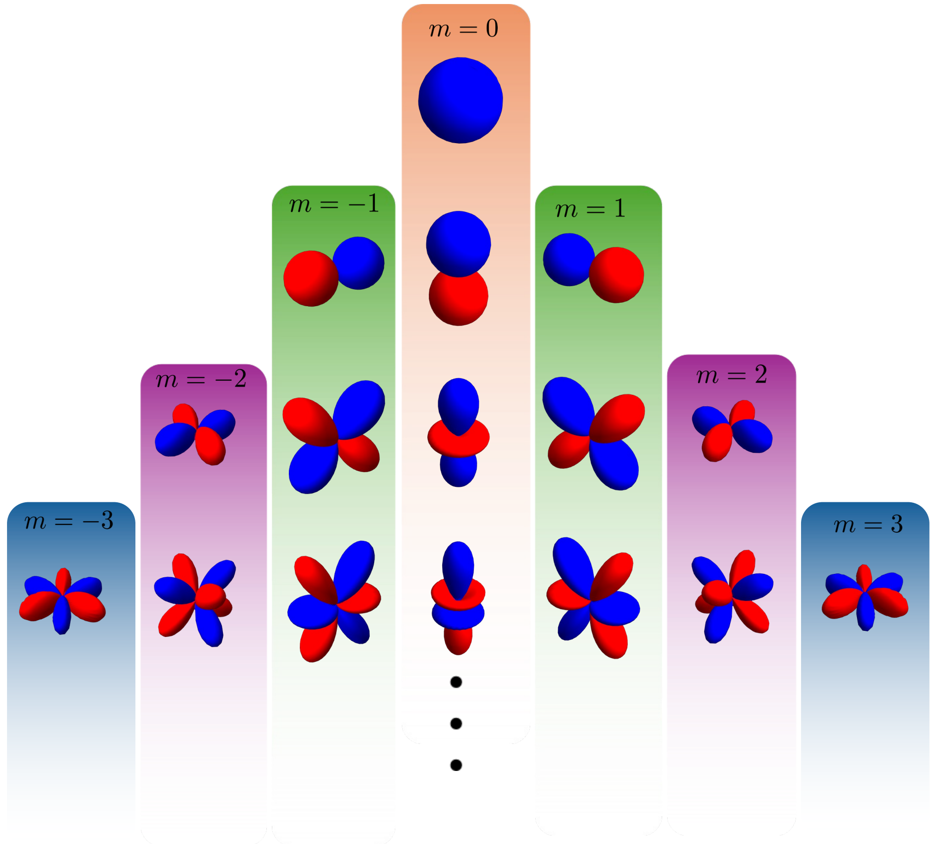
$$\partial_\tau L_{lm} = -\frac{1}{\tau}(A_l^m L_{l,m} + B_l^m L_{l-2,m} + C_l^m L_{l+2,m}) - (1 - \delta_{l,0} - \delta_{l,1})\frac{L_{lm}}{\tau_R}$$
$$- ik(D_{l+1}^{1-m} L_{l+1,m-1} + D_l^{-m} L_{l-1,m+1} - D_l^m L_{l-1,m-1} - D_{l+1}^{m+1} L_{l+1,m+1})$$

$$\partial_\tau \psi = -\mathcal{H}\psi \qquad \psi = (L_{00}, L_{1,-1}, L_{1,0}, L_{1,1}, L_{2,-2}, L_{2,-1}, \cdots)$$

Hydrodynamic (conserved) moments $\quad \nabla_\mu T^{\mu\nu} = 0$

# Sectors

- At zero (transverse) gradients, moments decouple into sectors, $(m, s)$, where $s$ is the parity given by $s = (-1)^{l+m}$
- m = 0,±1: “hydro sectors” (they contain all conserved quantities)



- Kinetic Theory is useful to connect the initial stage particle level description to the late time fluid description at thermal equilibrium.
- The spherical harmonic expansion creates a physical hierarchy of moments.
- Without gradients, expansion moments decouple into separate sectors due to system symmetries (SO(2) x Z2)

# 2. Eigenspectrum analysis

What insights can we gain before directly solving the ODEs ?

# Eigenvalues recap

Consider Time-independent hermitian Hamiltonian with $\mathcal{H}\phi_n = \lambda_n \phi_n$

Time evolution: $\partial_\tau \psi = -\mathcal{H}\psi$

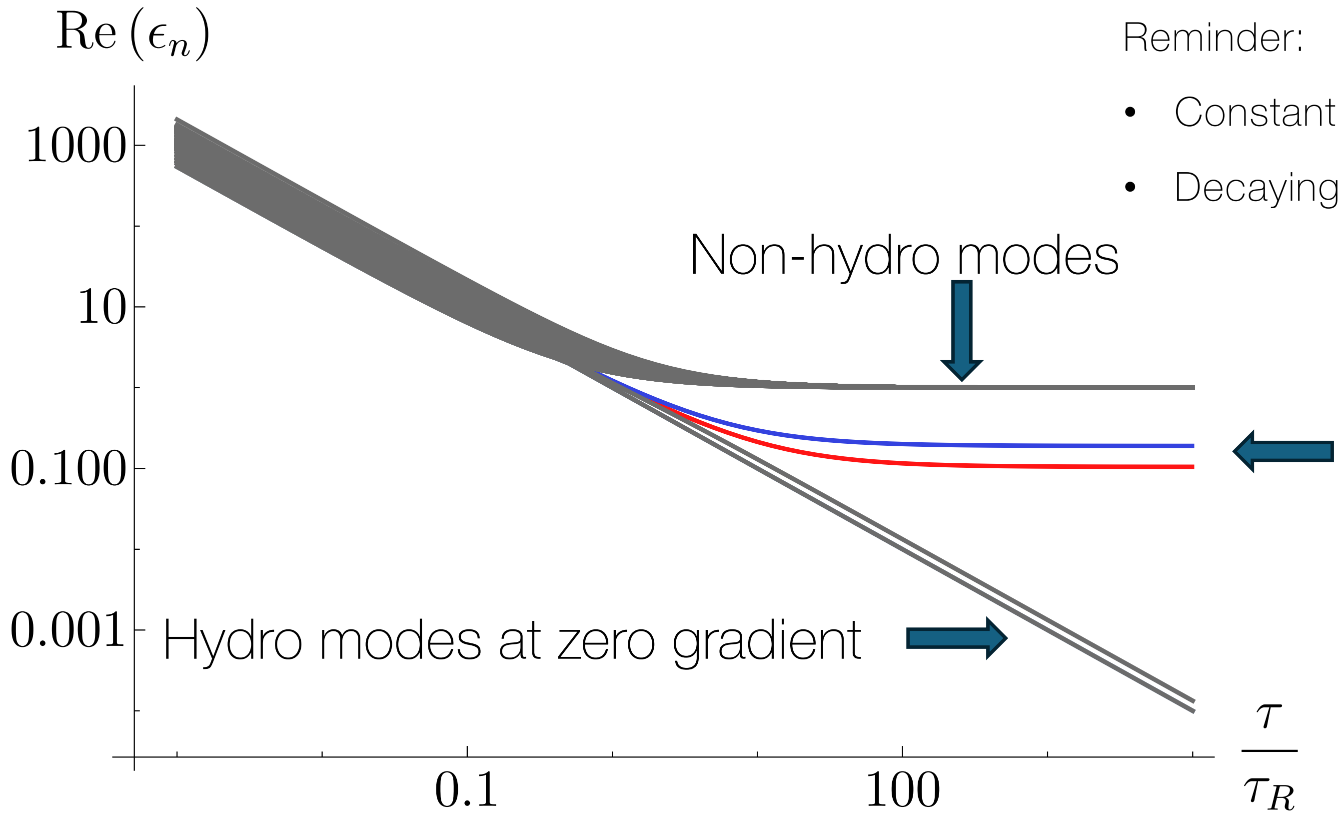
Decomposing: $\psi(\tau) = \sum_n a_n(\tau)\phi_n$

$$\partial_\tau a_n = -\lambda_n a_n \quad \Longleftrightarrow \quad a_n(\tau) \propto e^{-\lambda_n \tau}$$

- Real part of eigenvalue ~ decay rate of the eigenmode
- Gap in spectrum $\Rightarrow$ all modes except dominant modes decay quickly

$\Rightarrow$ **Attractors!**

# Eigenvalues numerical




Reminder:

- Constant eigenvalue: exponential decay
- Decaying eigenvalue $\sim 1/\tau$ : power law decay

Hydro modes (shear and sound) at finite gradient

# Dispersion relations

Can extract at late times analytically, power series in gradients

$$\epsilon_{0,1} = \pm i\frac{k}{\sqrt{3}} + \frac{2k^2}{15} \pm i\frac{8k^3}{75\sqrt{3}} - \frac{68k^4}{1575} + \mathcal{O}\left(k^5\right)$$ Sound mode

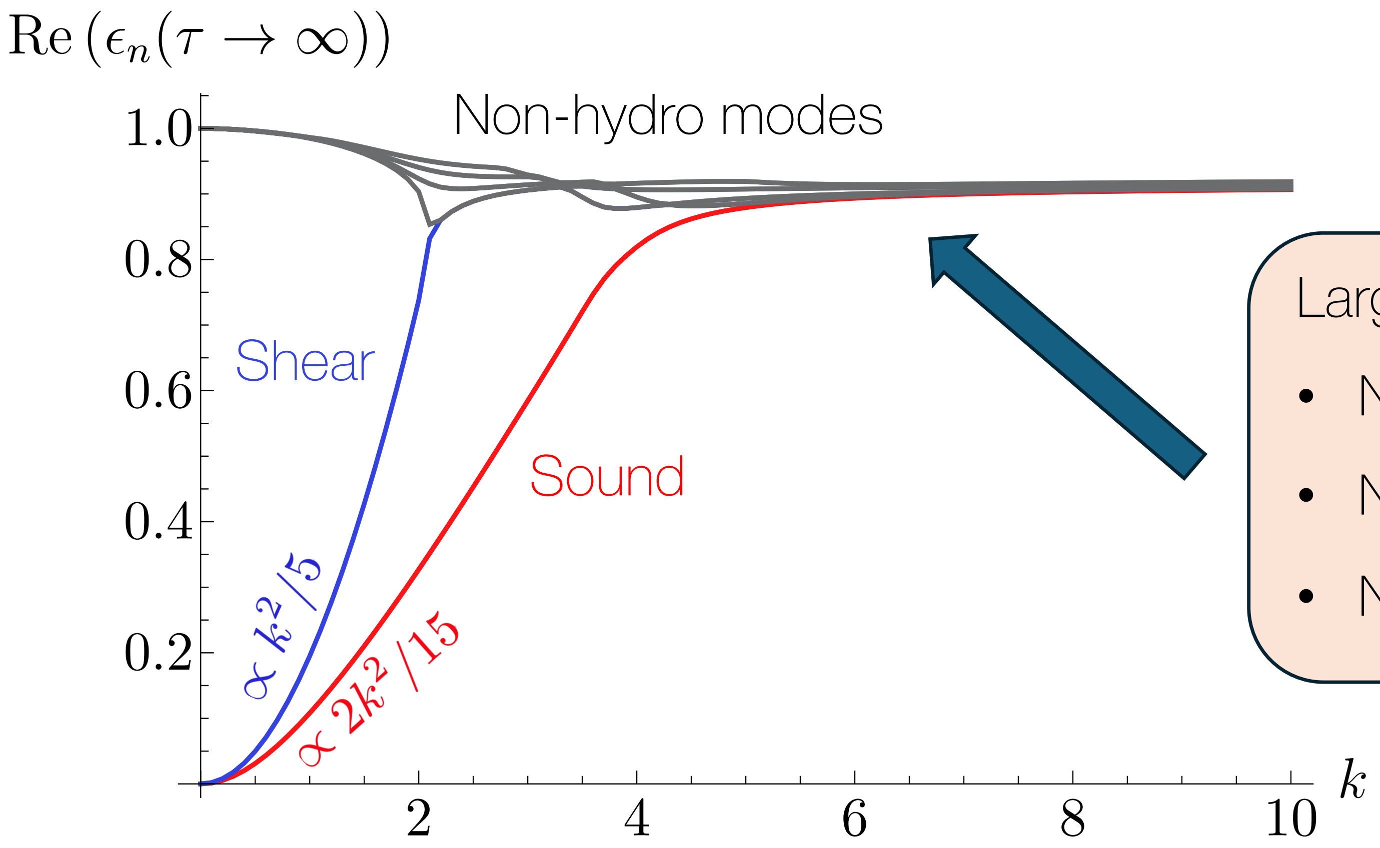
$$\epsilon_{2,3} = \frac{k^2}{5} - \frac{k^4}{175} + \mathcal{O}\left(k^5\right)$$ Shear mode (2-degenerate)

= dispersion relation obtained in relativistic hydrodynamic gradient expansion (see, e.g. Baier et al, 2008)

Sanity ☑

But – very easy to extract in this framework !

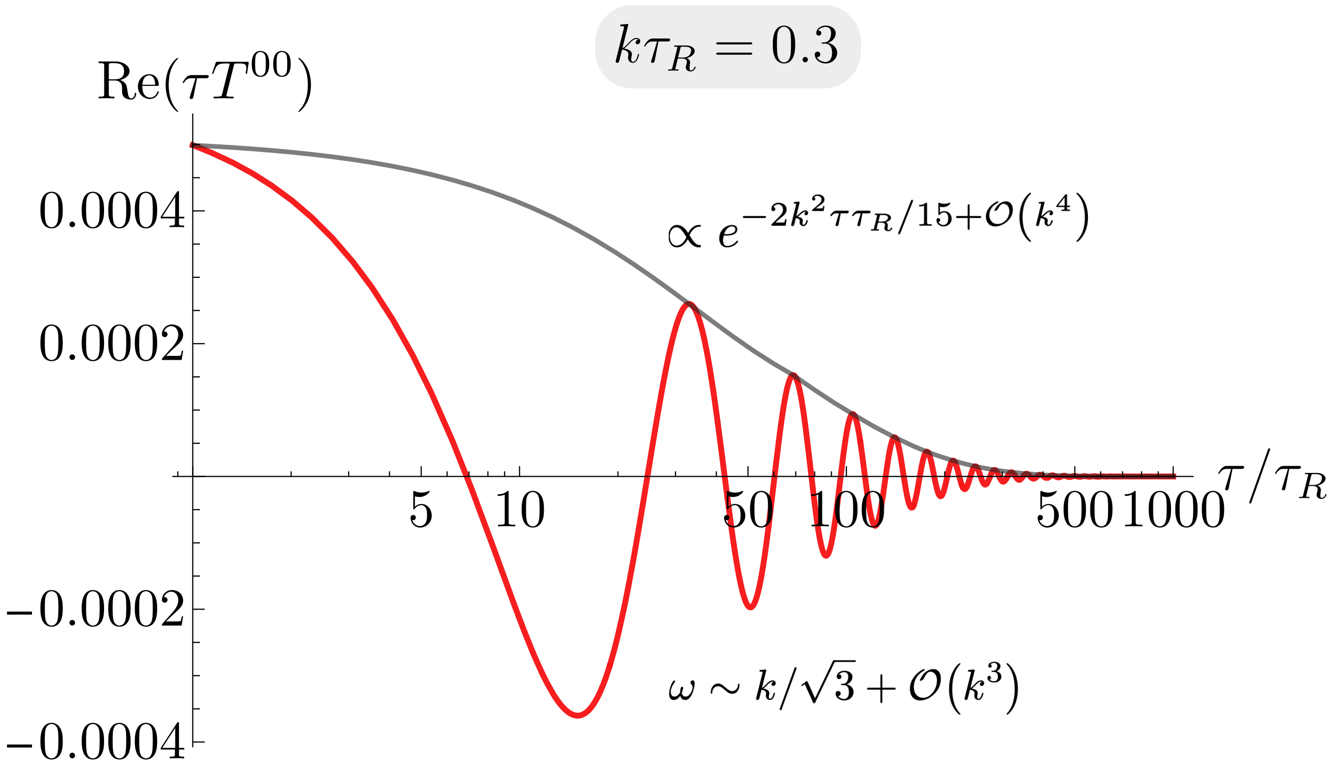
# Dispersion (numerical)




Large gradients:

- No gap
- No attractor
- No hydrodynamic modes

# Sound mode (numerical)



- Gradients allow description of shear and sound modes

At late times:

- Equivalence to the hydrodynamic gradient expansion
- Gradients increase decay rate of hydro modes
- Very strong gradients destroy hydro modes, and the system **no longer thermalises**

# 3. Thermalisation

Do transverse gradients change if and how the QGP thermalises ?

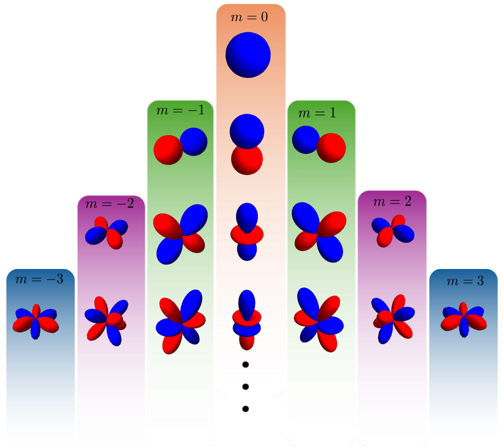
# Contribution of sectors

- Decompose moment vector into different sectors

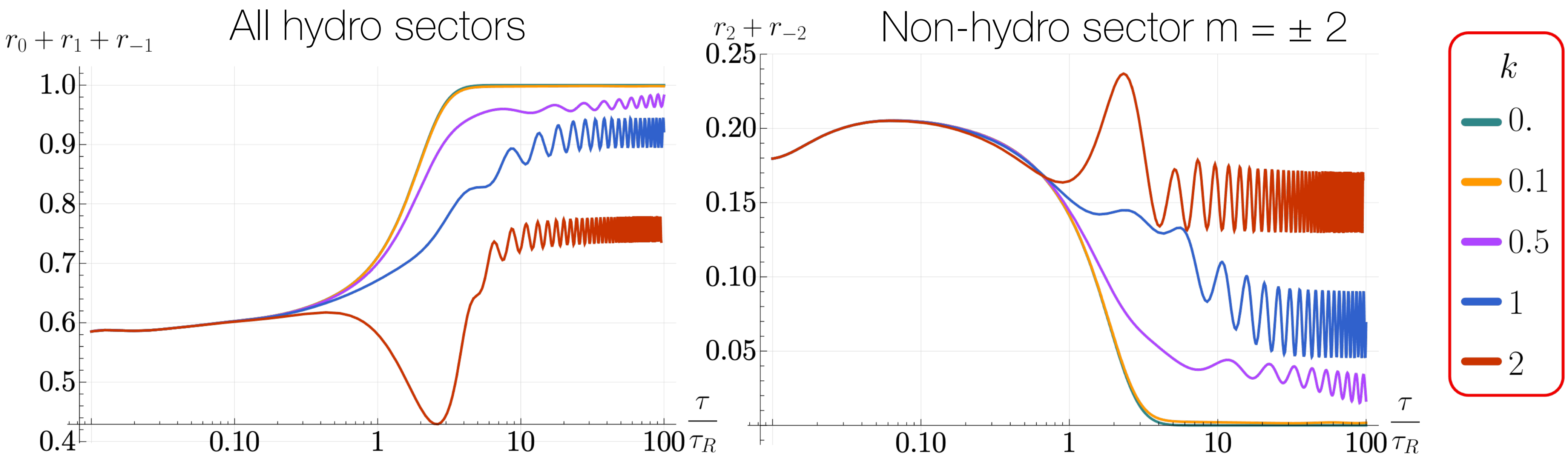
$$\psi = \sum_m \psi_m \qquad \psi_m^\dagger \psi_n \propto \delta_{mn} \qquad \text{e.g. } \psi_2 = (0, \ldots, 0, L_{22}, 0, \ldots, 0, L_{42}, 0, \ldots)$$

- Population fraction

$$r_m \equiv \frac{|\psi_m|^2}{|\psi|^2} \text{ such that } \sum_m r_m = 1$$



# Contribution of hydro sectors

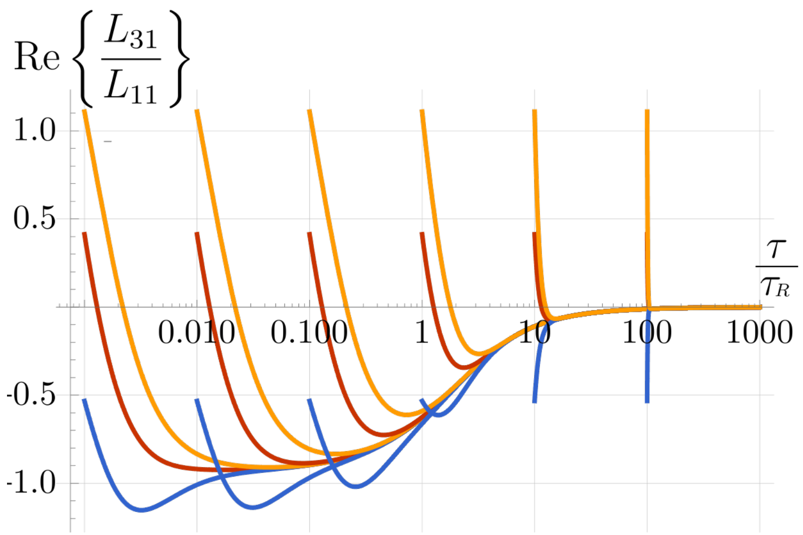



Gradients ↗ ⟶ Hydro ↘ Non-hydro ↗

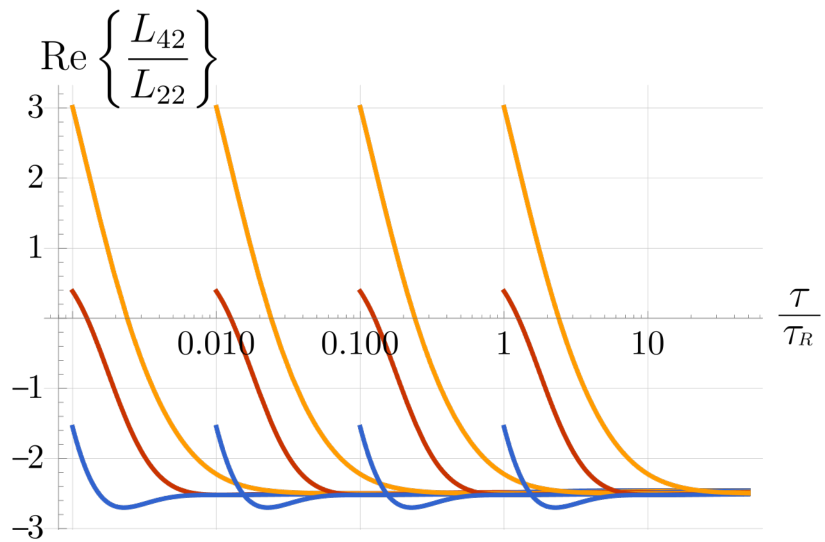
No full thermalisation!

# Attractors – hydro vs non-hydro sectors

Hydro sector

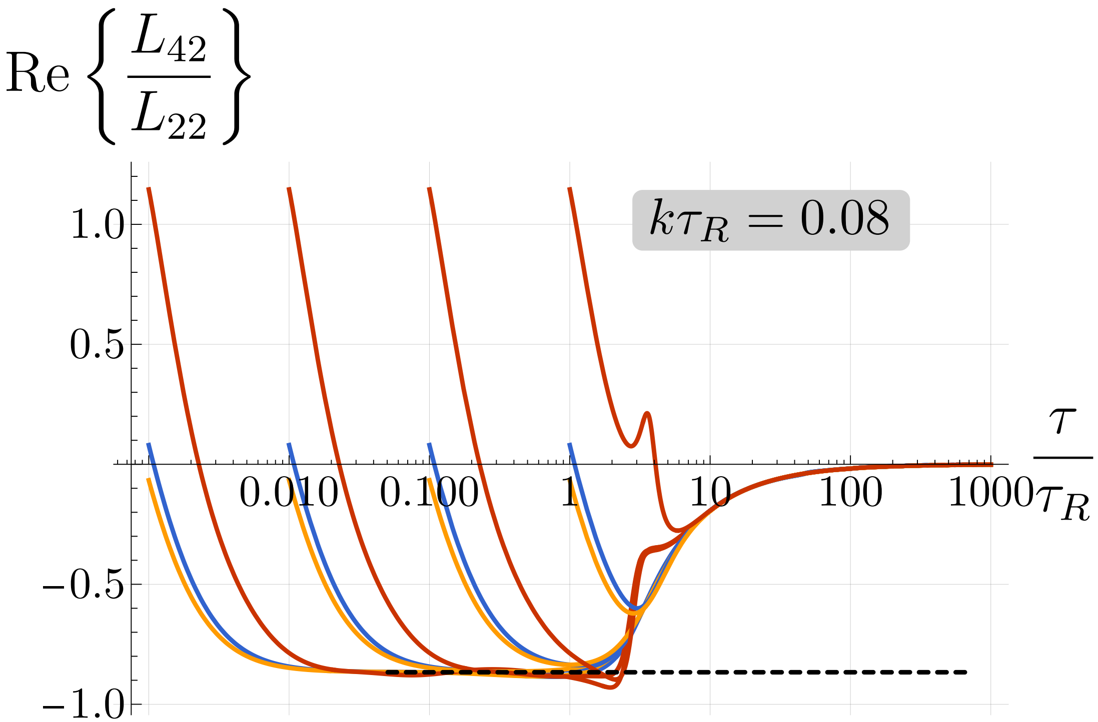



Non-hydro sector




- Hydro sectors: strong moment hierarchy due to conserved moments
- Non-hydro sectors: all moments decay exponentially ➡ constant attractor

# Attractor modification with gradients




Dashed: Attractor for zero gradients

Attractor is modified and now looks like hydro attractor:

- Sectors are now coupled
- Hierarchy of hydro sectors is imprinted on non-hydro moments.

- Gradients increase contribution of non-hydrodynamic modes, preventing full thermalisation of the system
- Small gradients imprint hierarchy of hydro modes onto non-hydro sectors.
- Attractor is modified: All attractors look like the hydrodynamic ones.

# What now ?

Caveats, current work, and summary

# Caveats

- Fourier space, keeping gradients constant, only then full thermalisation is "prevented". In HIC, gradients decay in time.
- Relaxation time is temperature-dependent, which itself is time-dependent. However, dynamics don't change up to time rescaling
- Analysis assumes perfect boost–invariance, additional gradients in rapidity could further change dynamics.
- Conformality is broken by nonzero quark masses (e.g. speed of sound$^2 \neq 1/3$)

# Current work

- Attractor deviation time – analytic expression.
- Hierarchy imprinting should improve accuracy of low truncation – currently checking this
- Writing up paper (stay tuned!)

# Summary

- QGP hydro can be understood through studying attractors in kinetic theory.
- Transverse spatial gradients are relevant in small systems and:
  - Allow for the description of sound and shear modes in the kinetic theory framework
  - Modify the attractor in non-hydrodynamic sectors by imprinting the hydro hierarchy onto previously decoupled sectors.
  - Excite non-hydrodynamic modes and prevent the full thermalisation of the system.

Thank you! Questions?

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