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Participants shall **treat each other professionally and respectfully**. Participants shall refrain from any inappropriate behaviour, including discriminatory actions and comments based on individual characteristics such as age, race, ethnicity, sexual orientation, gender identity, gender expression, marital status, nationality, immigration status, political affiliation, ability status, or educational background. Disruptive or harassing behavior will not be tolerated including but not limited to inappropriate or intimidating language and unwelcome jokes or comments.

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References: This code of conduct is based heavily on that of the [INT](#) and the [APS](#). We are also grateful to Roxanne Springer for valuable discussion and guidance.

Updated constraints on the neutron star equation of state

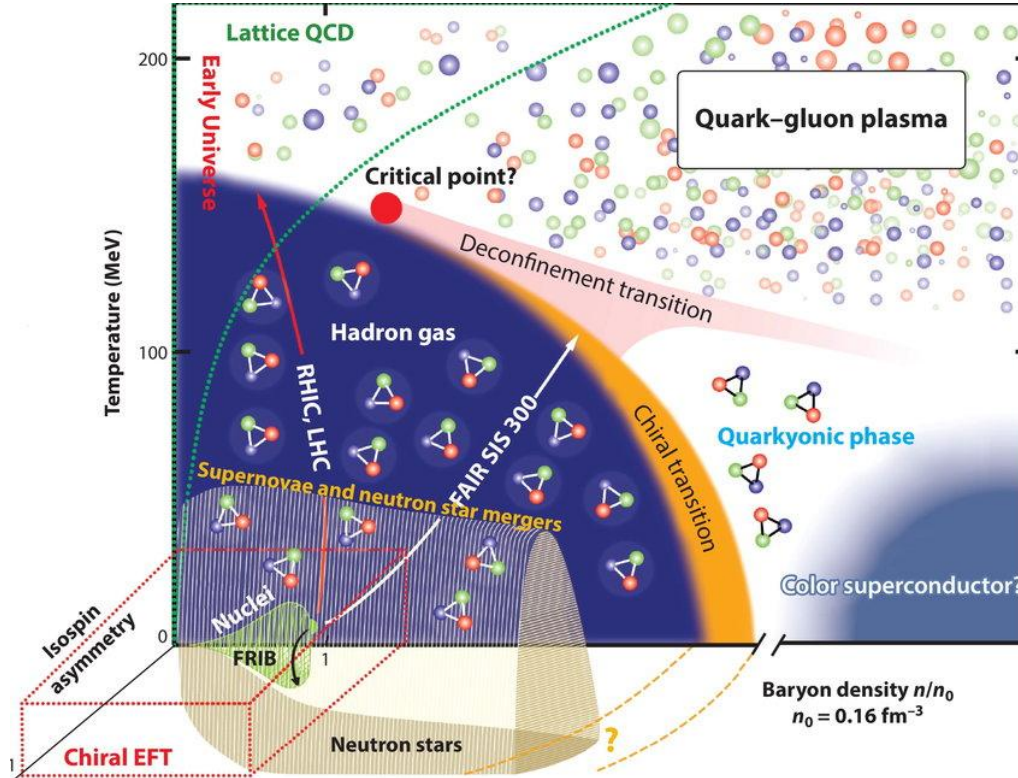
Melissa Mendes - TU Darmstadt - 09/12

Updated constraints on the neutron star equation of state

Melissa Mendes
TU Darmstadt



QCD phase diagram

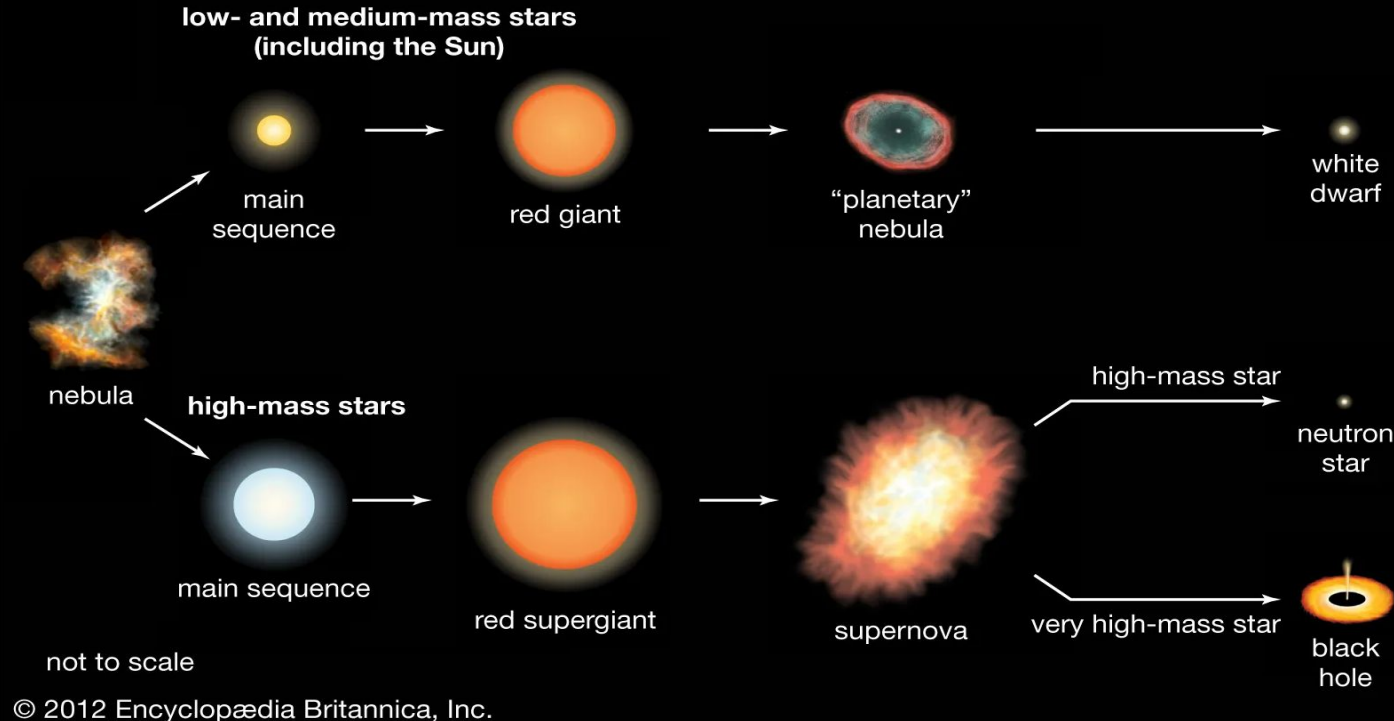


Neutron stars are interesting systems to investigate the nuclear matter equation of state

Figure from Drischler et al, Annu. Rev. Nucl. Part. Sci. (2021), arxiv:2101.01709

Neutron stars (NS)

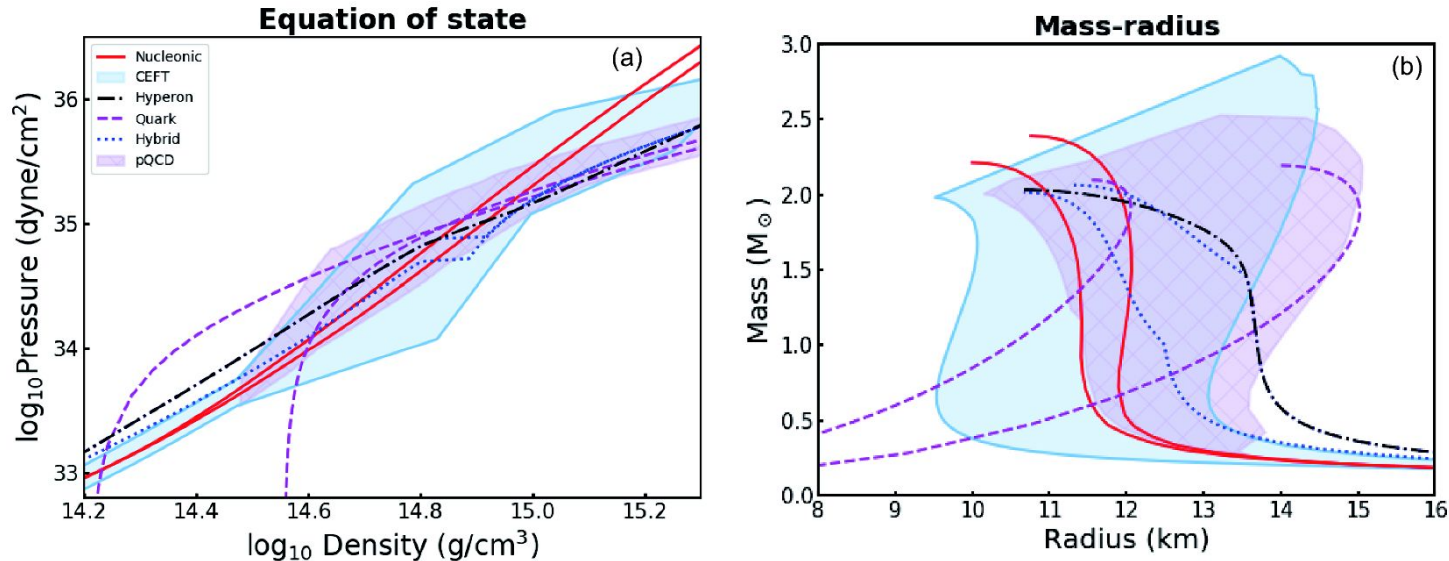
Stellar evolution



The final stage of
evolution of
high-mass stars

Figure from Encyclopedia
Britannica (2012)
<https://www.britannica.com/science/star-astronomy/Subsequent-development-on-the-main-sequence>

Constraining equation of state (EOS) with NS observables



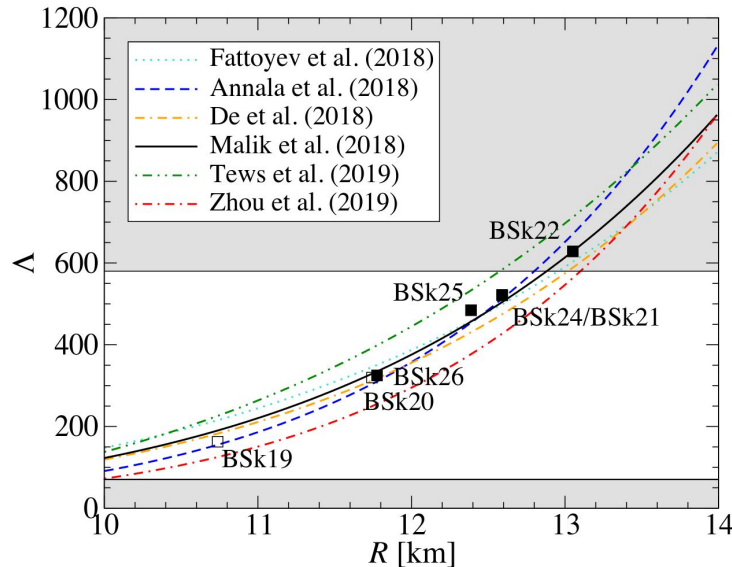
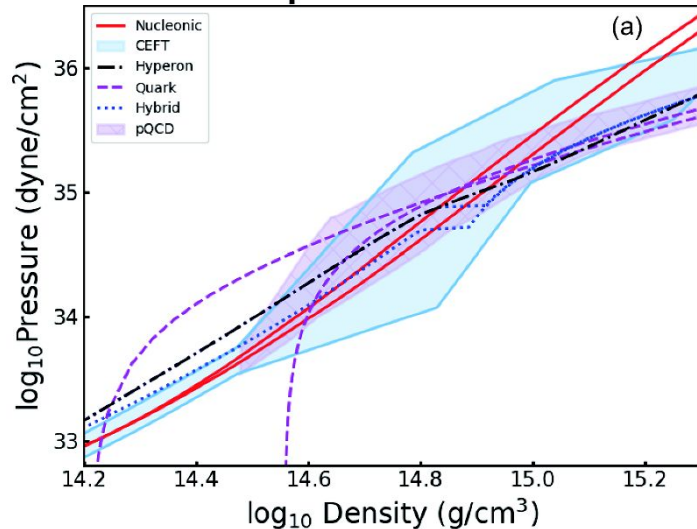
Mass-Radius
observations can
constrain the
nuclear matter
EOS (in
beta-equilibrium)

Figure from Watts et al,
Sci China Phys Mech
Astron (2019),
arxiv:1812.04021

Constraining equation of state (EOS) with NS observables



Equation of state



Tidal deformability
-mass-radius
observations can
constrain the nuclear
matter EOS (in
beta-equilibrium)

Modified figure from Watts
et al, Sci China Phys Mech
Astron (2019),
arxiv:1812.04021;

Perot et al, Phys. Rev. C
(2019), arxiv: 1910.13202

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Raaijmakers, ..., **MM**, ... et al, 2025 (JOSS), <https://doi.org/10.21105/joss.06003>

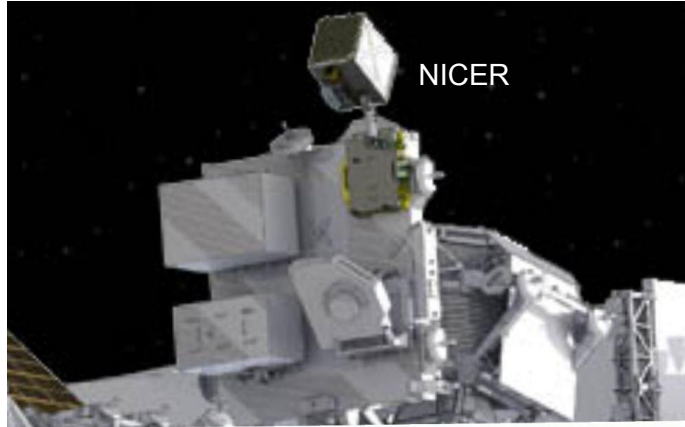
Mauviard, ..., **MM**, ... et al, 2025 (ApJ), arxiv: 2506.14883

Hoogkamer, ..., **MM**, ... et al, 2025 (Editor's suggestion PRD), arxiv: 2510.27619

MM et al, 2025 (in preparation)



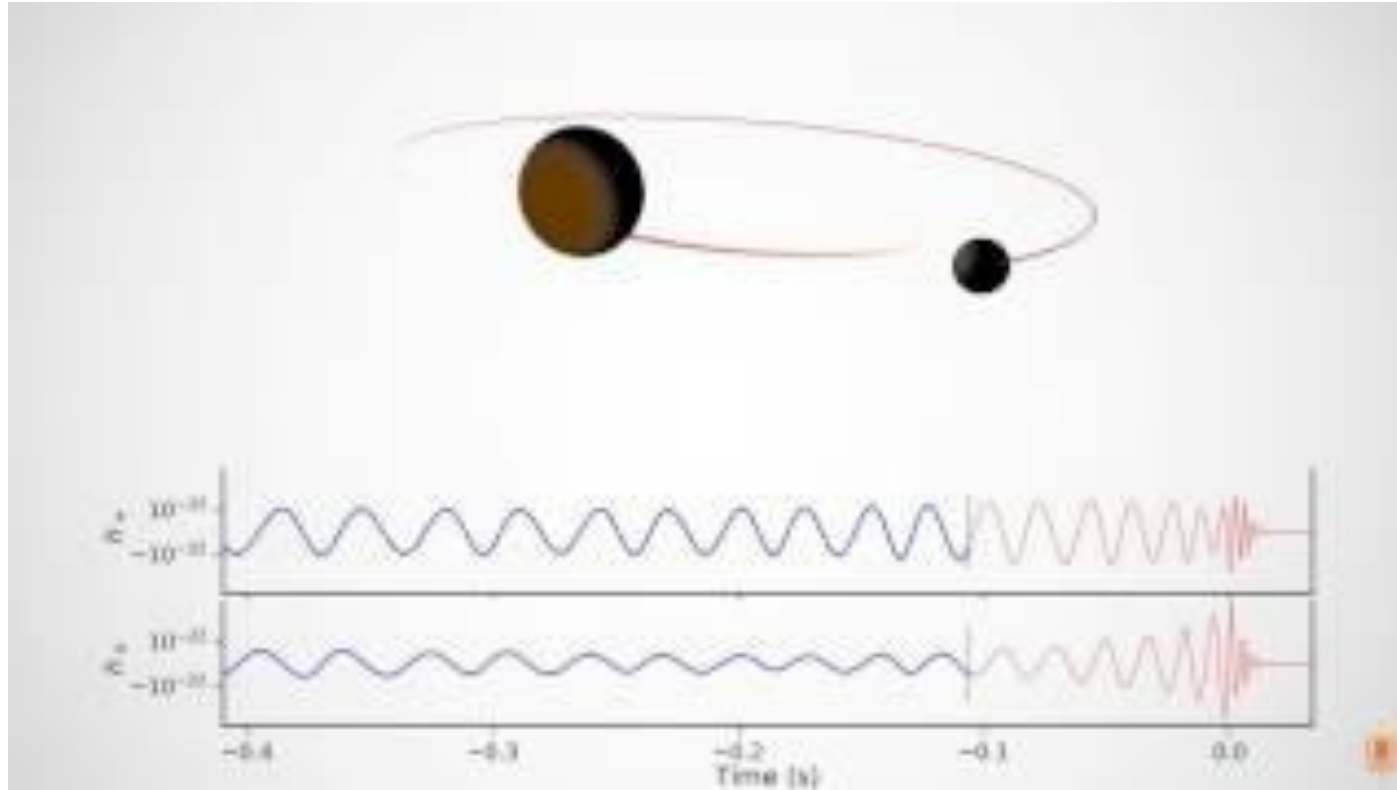
Data collection facilities



Recent (and future) observations provide unprecedentedly precise neutron star and nuclei data

Modified figure from
<https://www.ligo.org/science/Publication-GWHEN-IceCube/index.php>

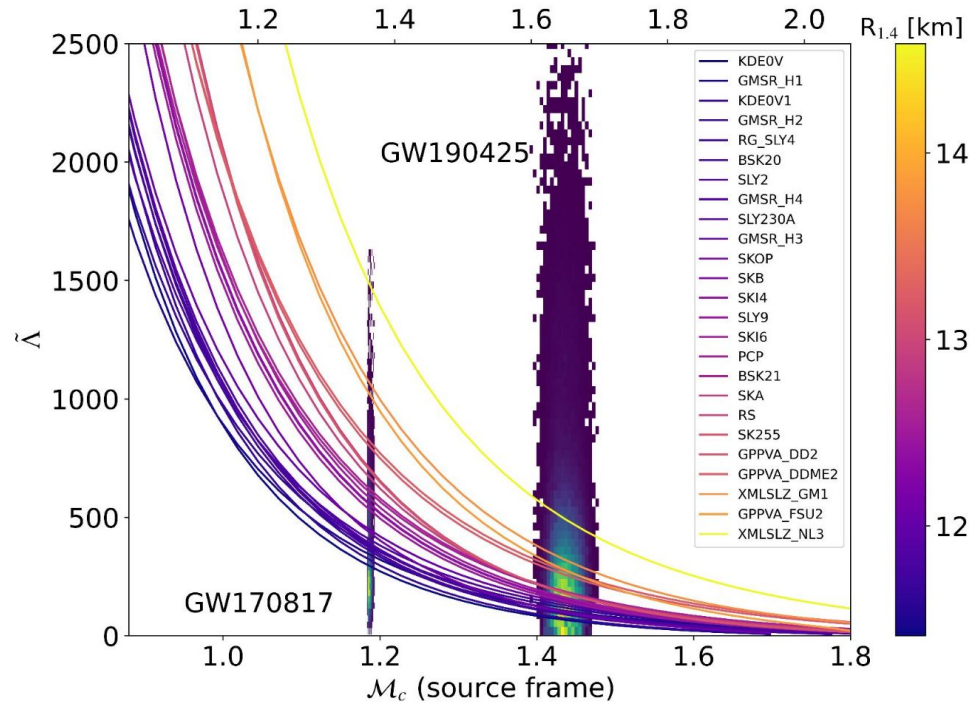
Gravitational waves



Waveform
signals feature
information
about neutron
star properties

Animation from
<https://www.ligo.caltech.edu/video/ligo20170601v2>

Two binary NS merger events



GW170817 much more
constraining

$$\tilde{\Lambda} = 300^{+420}_{-230}$$

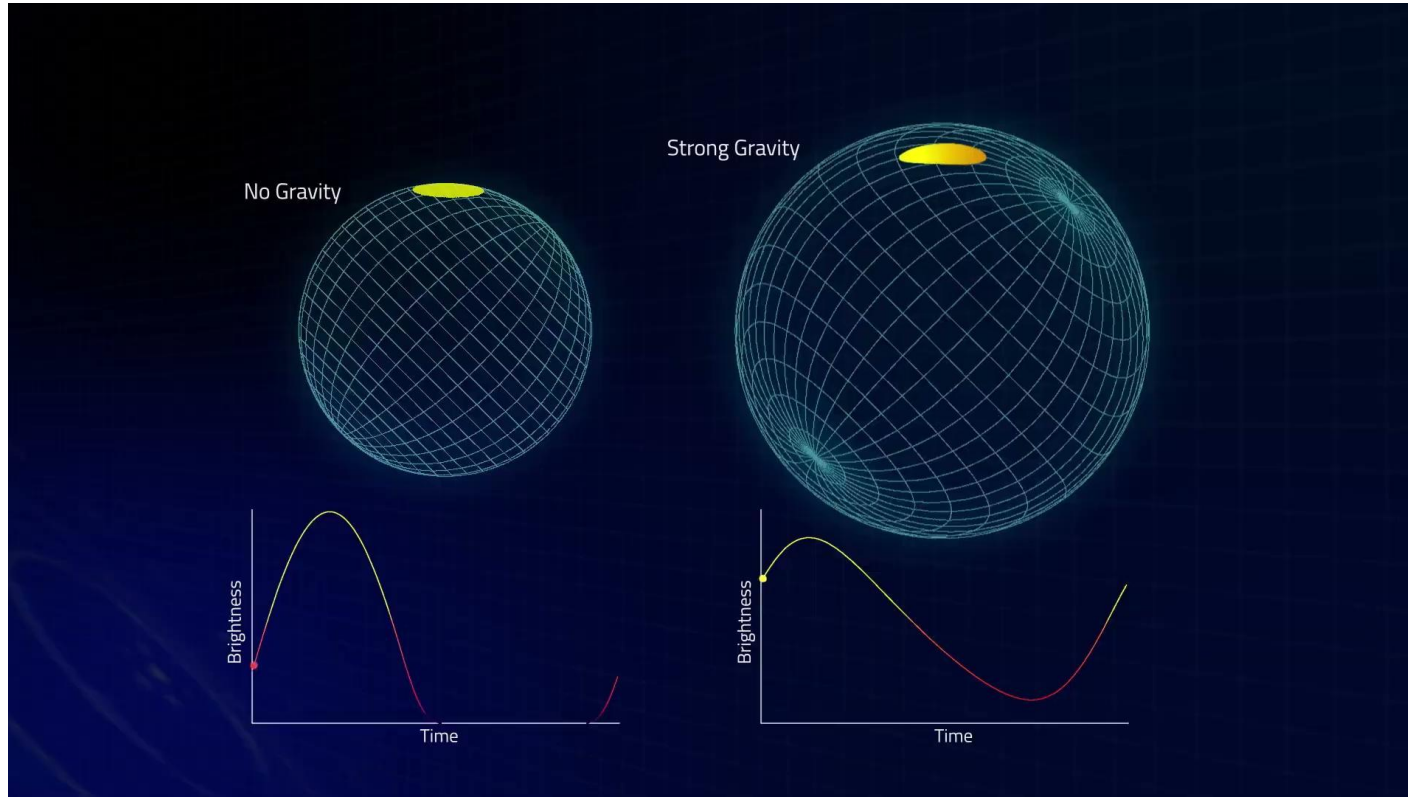
GW190425 binary NS or
NS-black hole merger still
open question

$$\tilde{\Lambda} \leq 600$$

Figure from J. Read, L. Suleiman's slides
on behalf of LVK collaboration at Nuclei
in the Cosmos 2025 -

<https://indico.icc.ub.edu/event/341>

Pulse profile modeling

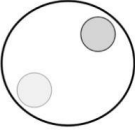
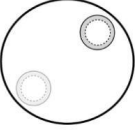
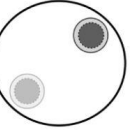
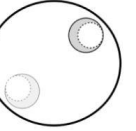
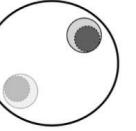
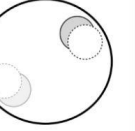
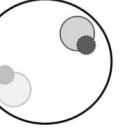
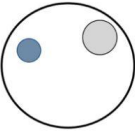
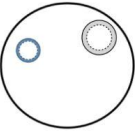
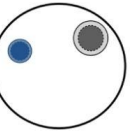
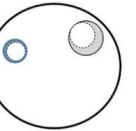
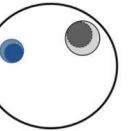
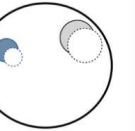
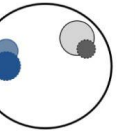
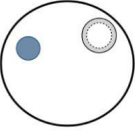
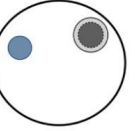
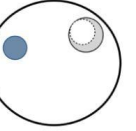
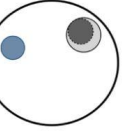
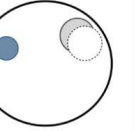
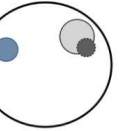


Proportional effect to
the stars'
compactness (M/R)

Simultaneous
constraint on neutron
star mass and radius

Animation from
<https://svs.gsfc.nasa.gov/vis/a020000/a020200/a020268/Lensing.1080.mp4>

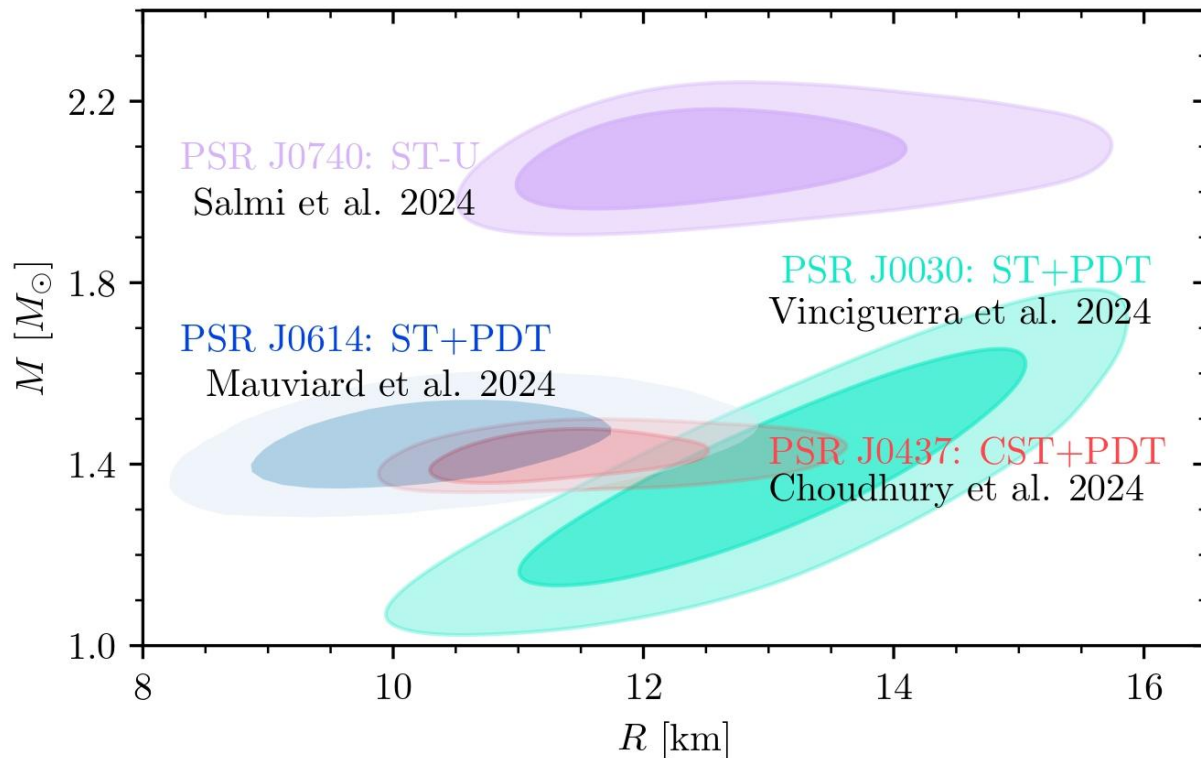
Non-trivial hot-spot modeling

		ST	CST	CDT	EST	EDT	PST	PDT
		Single Temperature	Concentric Single Temperature	Concentric Double Temperature	Eccentric Single Temperature	Eccentric Double Temperature	Protruding Single Temperature	Protruding double Temperature
-S	Antipodal Symmetry							
-U	Unshared parameters							
ST		ST-U/ST-S						
...								

Large number of parameters needed to describe hot spots

Figure from Vinciguerra et al, APJ (2023), arxiv:2308.08409

Recent data from NICER



Already including mass constraints from radio observations and background constraints (when available)

Modified figure from Mauviard, ..., **MM**, ... et al, ApJ (2025), arxiv: 2506.14883

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MM et al, 2025 (in preparation)



Chiral effective field theory

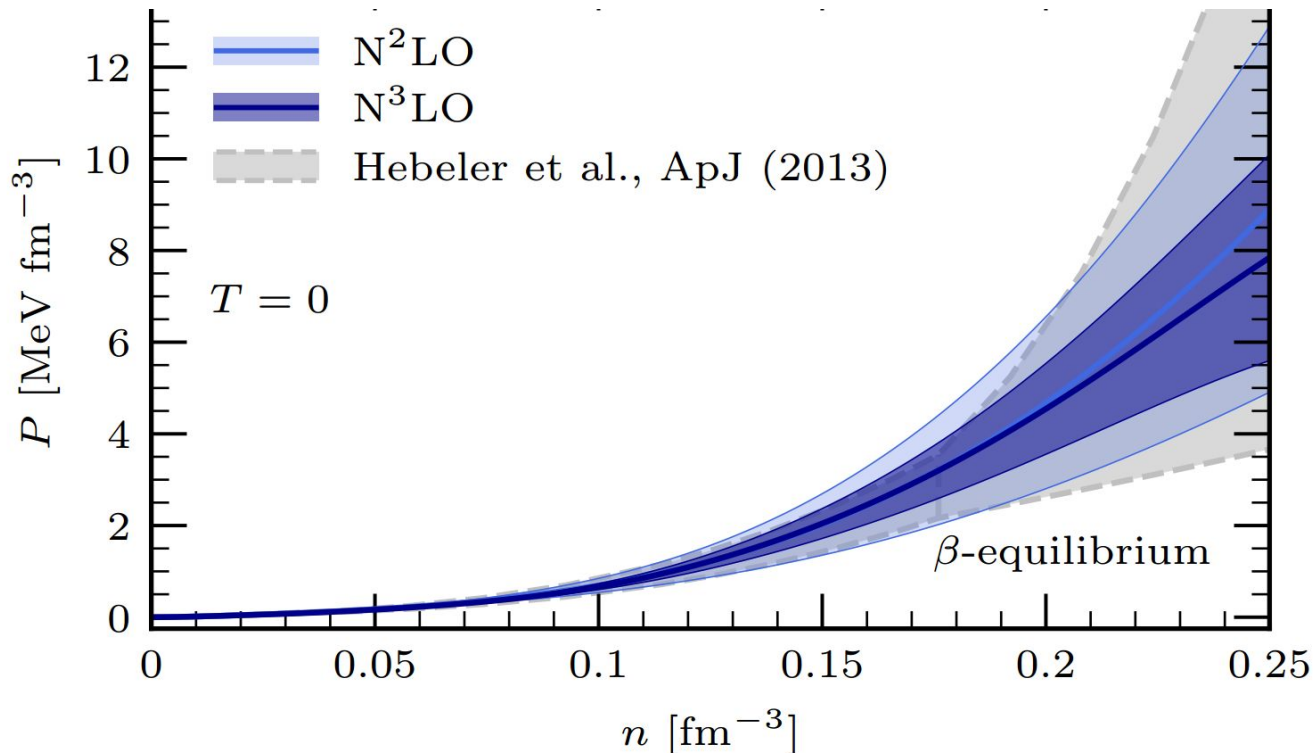


	NN	3N	4N
LO $\mathcal{O}(Q^0/\Lambda^0)$	1990 [151,152] 2 	—	—
NLO $\mathcal{O}(Q^2/\Lambda^2)$	1992 [164,165] 7 	1992,1994 [166-169] —	—
N ² LO $\mathcal{O}(Q^3/\Lambda^3)$	1992 [164,165] 0 	1994 [167,170] 2 	—
N ³ LO $\mathcal{O}(Q^4/\Lambda^4)$	2000–2002 [179-182] 12 	2008–2011 [183-185] 0 	2006 [186] 0
N ⁴ LO $\mathcal{O}(Q^5/\Lambda^5)$	2015 [188,189] 0 	2011– [190-192] ? 	?

Ab initio
calculations can
determine the
EOS at low
densities

Figure from Hebeler,
Phys.Rept. (2021),
arxiv:2002.09548

Recent χ EFT EOS calculations



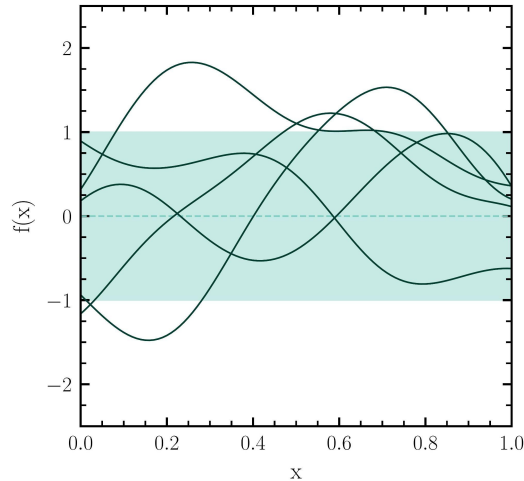
Based on asymmetric matter calculations and improved uncertainty quantification, for both next-to-next-to ($N^2\text{LO}$) and next-to-next-to-next-to ($N^3\text{LO}$) leading order

Figure from Keller et al, PRL (2023), arxiv:2204.14016

Gaussian processes (GP)

$$f(x) \sim GP[m(x), \kappa(x, x'; \theta)]$$

$$\kappa(x, x'; \theta) = \bar{c}^2 \exp \left[-\frac{1}{2} (x - x')^T L (x - x') \right]$$

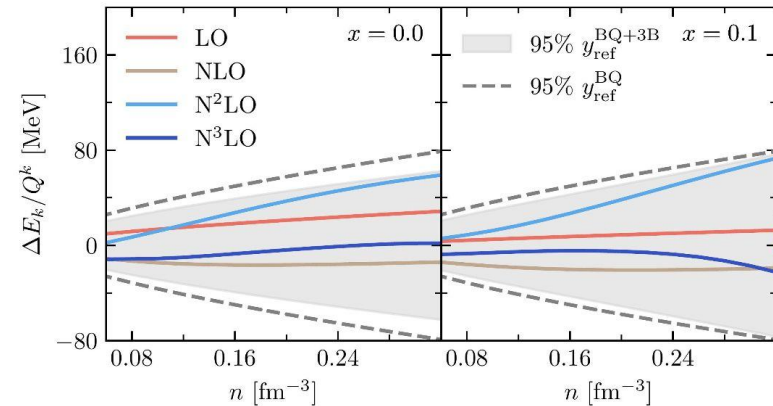


where

$$L = \begin{pmatrix} \ell_n^{-2} & 0 \\ 0 & \ell_x^{-2} \end{pmatrix}$$

hyperparameters $\bar{c}^2$, ℓ determined from expansion coefficients

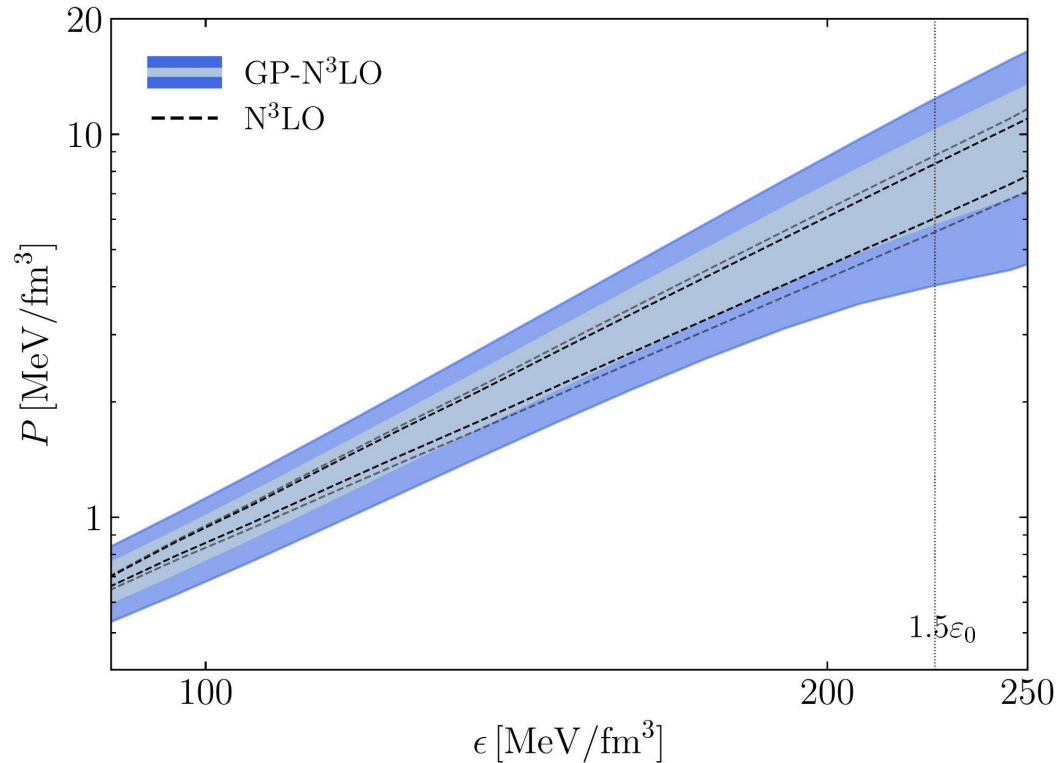
Calculations performed by H. Götting



Modified figure from Götting et al (2025), in preparation

GP for uncertainty quantification following Drischler et al., PRC (2020); Melendez et al., PRC (2019)

Recent χ EFT EOS implementation



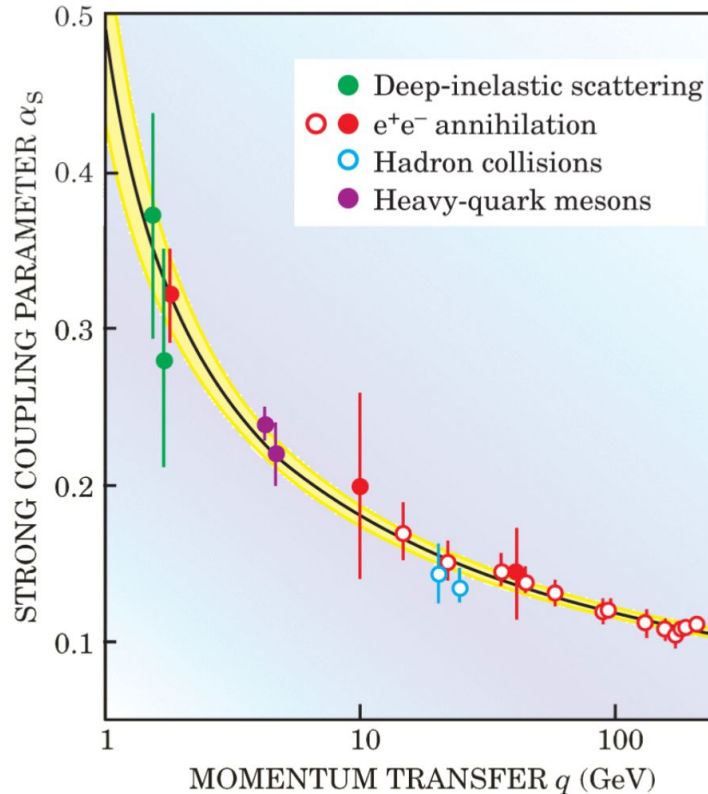
Broader pressure range covered with χ EFT EOS with uncertainty quantification (GP-N³LO)

Modified figure from **MM** et al (2025), in preparation

Asymptotic freedom



Confinement
at low q



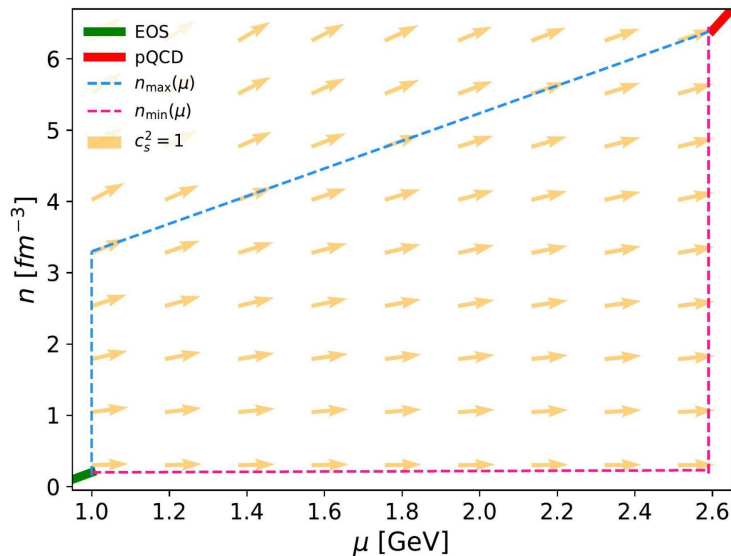
Freedom at
large q

Perturbative QCD
(pQCD) explores the
region of smaller strong
coupling constant

Modified figure from Fritzsche, H.
(2018), Murray Gell-Mann and
the Physics of Quarks, Springer

pQCD implementation

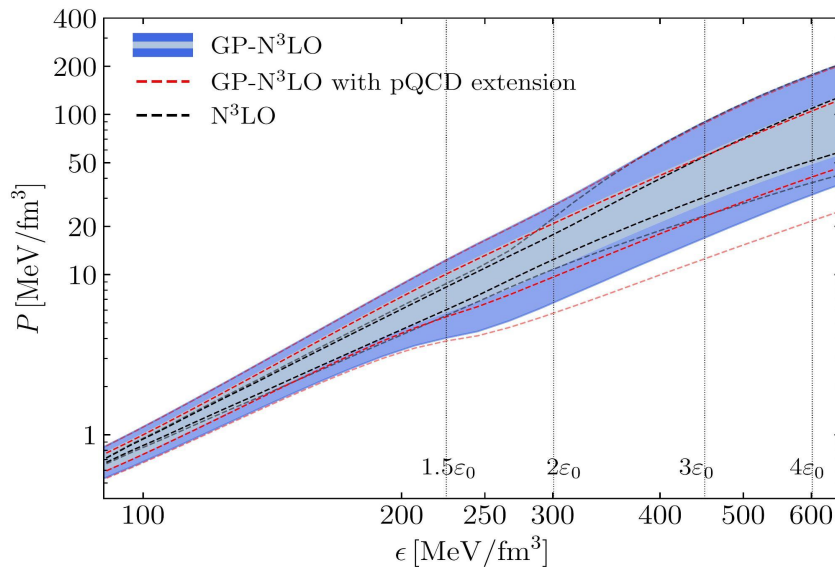
Extension procedure built by A. Hensel



Modified figure from Komoltsev and Kurkela, PRL (2022),
arxiv:2111.05350

EOS extensions built to respect pQCD constraints

Further expands prior space, especially at high densities



Modified figure from **MM** et al (2025), in preparation

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MM et al, 2025 (in preparation)



Bayesian framework

Following previous works, (Raaijmakers et al, 2020; 2019), posterior distributions of all EOS parameters (θ) and central energy density (ε):

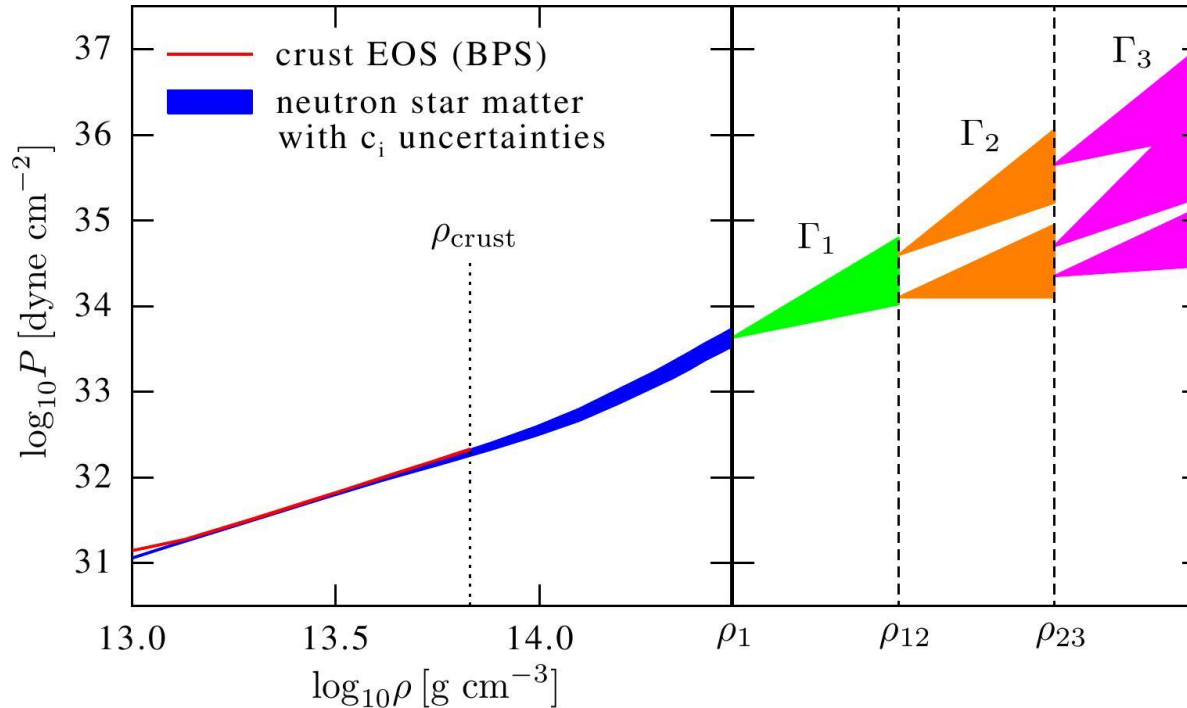
$$\overbrace{p(\theta, \varepsilon \mid d, \mathbb{M})}^{\text{Posterior}} \propto \overbrace{p(\theta \mid \mathbb{M}) p(\varepsilon \mid \theta, \mathbb{M})}^{\text{Prior}} \times \prod_i p(\Lambda_{1,i}, \Lambda_{2,i}, q_i \mid \mathcal{M}_c, d_{\text{GW},i}) \times \\ \times \prod_l p_{\text{new}}(M_l, R_l \mid d_{\text{NICER}(+\text{radio}),l}),$$

with radio mass measurements included through NICER M-R likelihoods when available

See NEST: <https://xpsi-group.github.io/neost/overview.html>



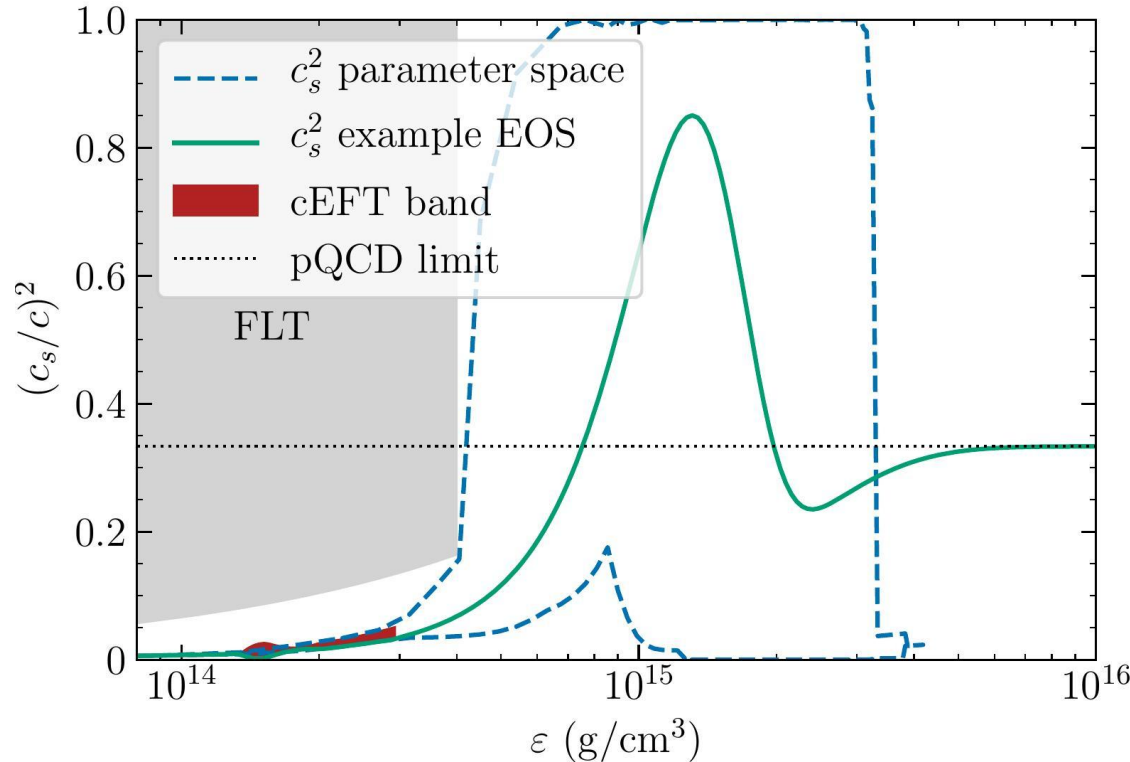
Piecewise polytropic parametrization (PP)



3 independent polytropes,
respecting causality,
following Read et al, PRD
(2009), arxiv: 0812.2163

Figure from Hebeler et al, ApJ (2013)
arxiv:1303.4662

Speed of sound parametrization (CS)

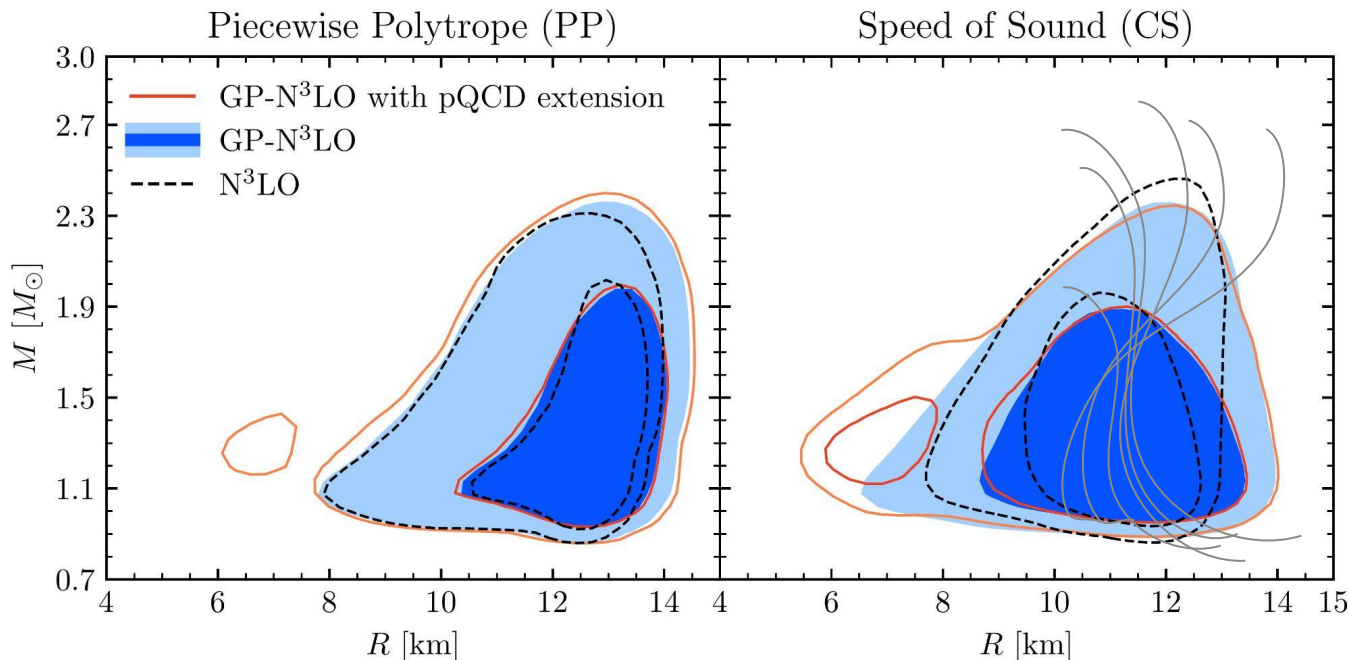


Analytical expression to speed of sound, limited by

- Fermi liquid theory
- causality
- $\lim_{n \geq 50} c_s^2 \rightarrow 1/3$

From Greif et al, MNRAS (2019)
arxiv: 1812.08188

Mass-Radius (MR) priors



Radii around 10-14 km

Both the new treatment
of the cEFT EOS and
pQCD extensions
expand MR priors

From **MM** et al, in preparation
(2025)

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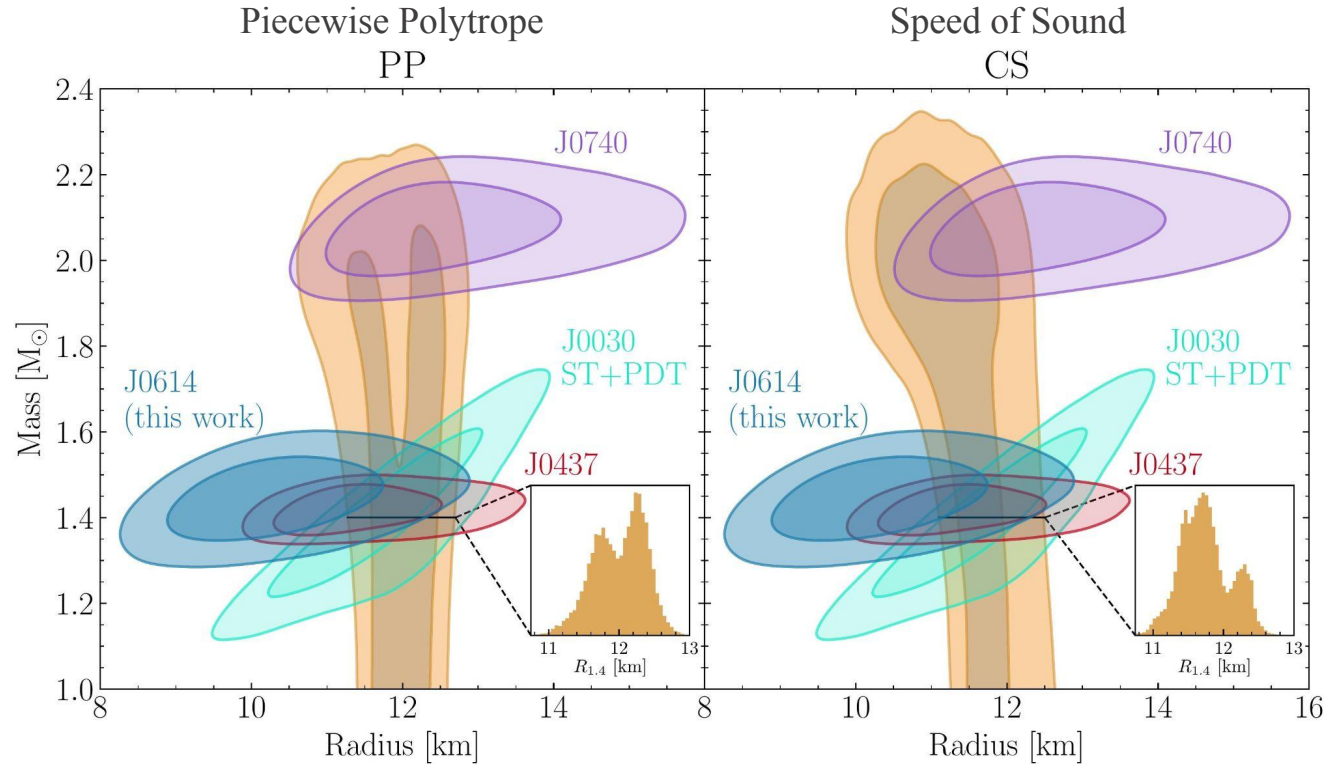
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MM et al, 2025 (in preparation)



Most likely M-R, N³LO only



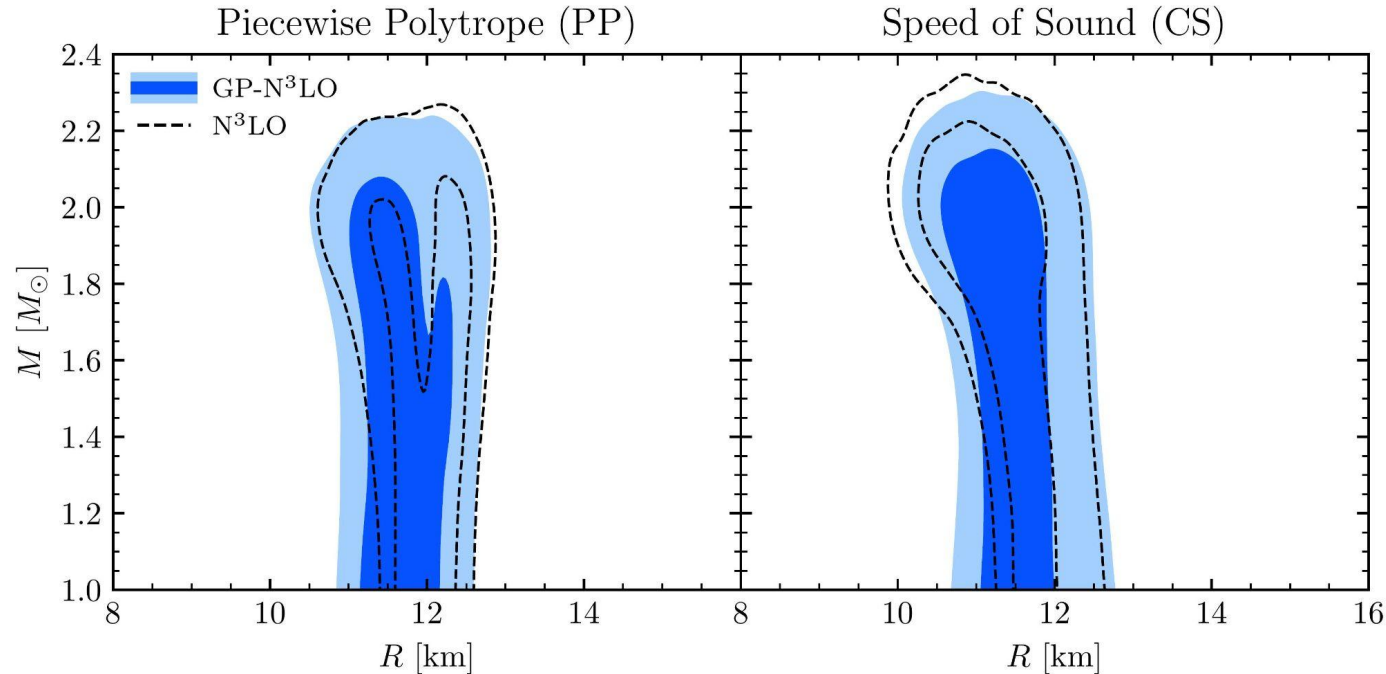
Consistent results for PP and CS parameterizations

Slight EOS softening compared to previous posteriors

Minimal impact on maximum mass

From Mauviard, ..., **MM**, ... et al, ApJ (2025), arxiv:2506.14883

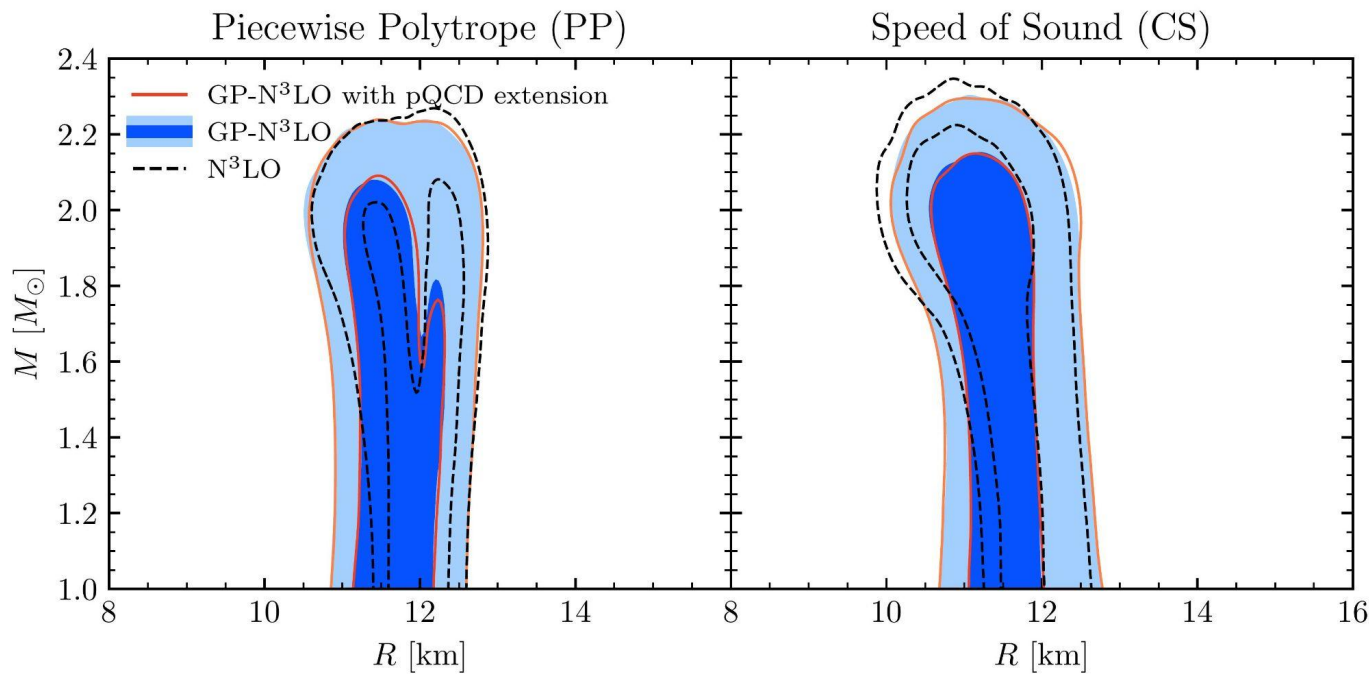
Most likely M-R with GP-N3LO



New cEFT
implementation
broadens the radius,
especially for low mass
stars

From **MM** et al, in preparation
(2025)

With pQCD constraints

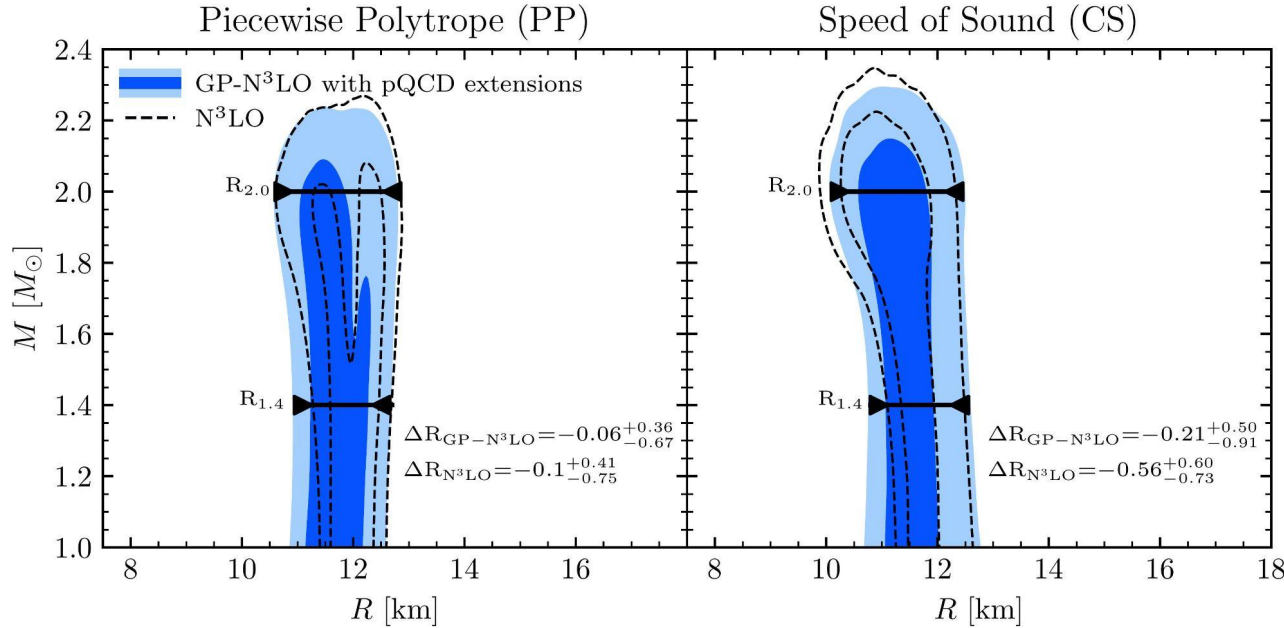


Minor differences to the
posterior without pQCD
constraints

PSR J0740
measurement more
constraining for
 $P_{\max}$ extension

From **MM** et al, in preparation
(2025)

Most likely M-R with GP-N3LO



We calculate

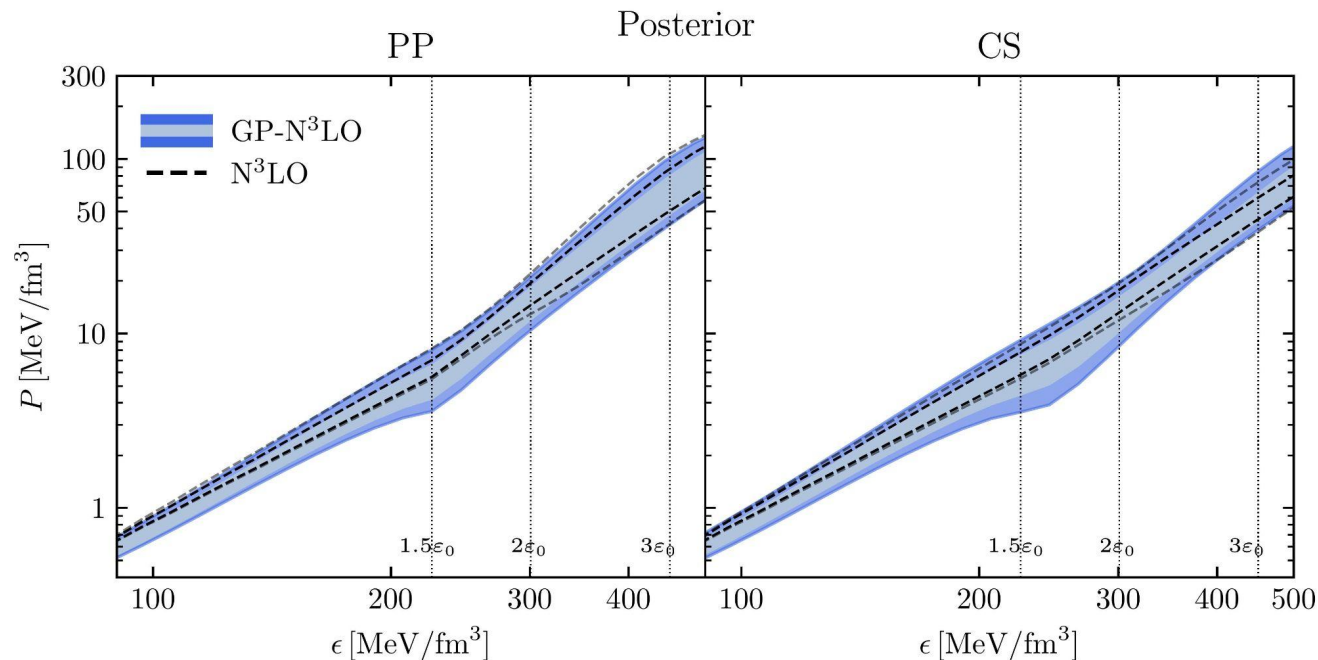
$$\Delta R = R_{2.0} - R_{1.4}$$

$\Delta R > 0$ suggests a stiffening of the EOS

Small dependence on high-density extension

From **MM** et al, in preparation (2025)

Corresponding P-E curves



Broadening of
pressure posterior
beyond cEFT range

Minimal impact of
pQCD extension

From **MM** et al, in preparation
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MM et al, 2025 (in preparation)



EOS parametrization

$$E(n, \delta) = E_{\text{SNM}}(n) + E_{\text{sym}}(n) \cdot \delta^2 + \mathcal{O}(\delta^4), \quad \delta = (n_n - n_p)/n$$

with symmetry energy:

$$E_{\text{sym}}(n) = J + L \left(\frac{n - n_0}{3n_0} \right) + \frac{1}{2} K_{\text{sym}} \left(\frac{n - n_0}{3n_0} \right)^2 + \dots$$

such that $L = 3P_{\text{PNM}}/n_0$

We explore the correlation between β -equilibrated matter and pure neutron matter to infer L

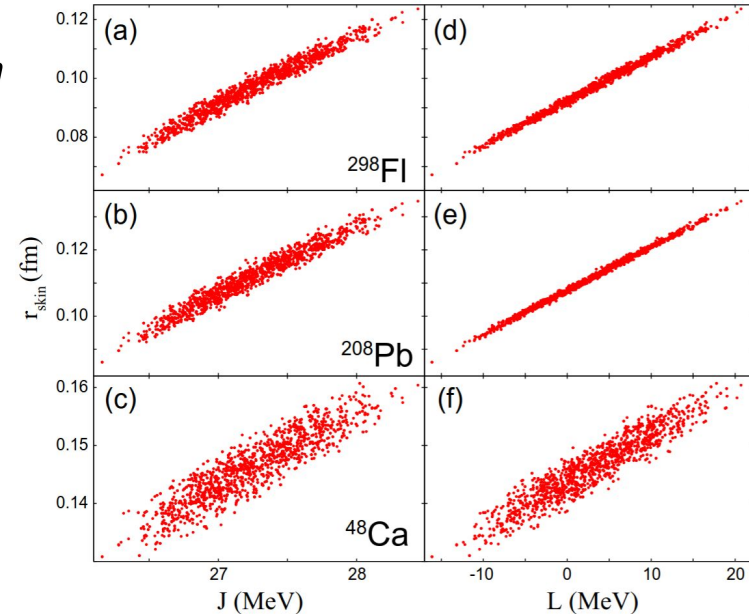
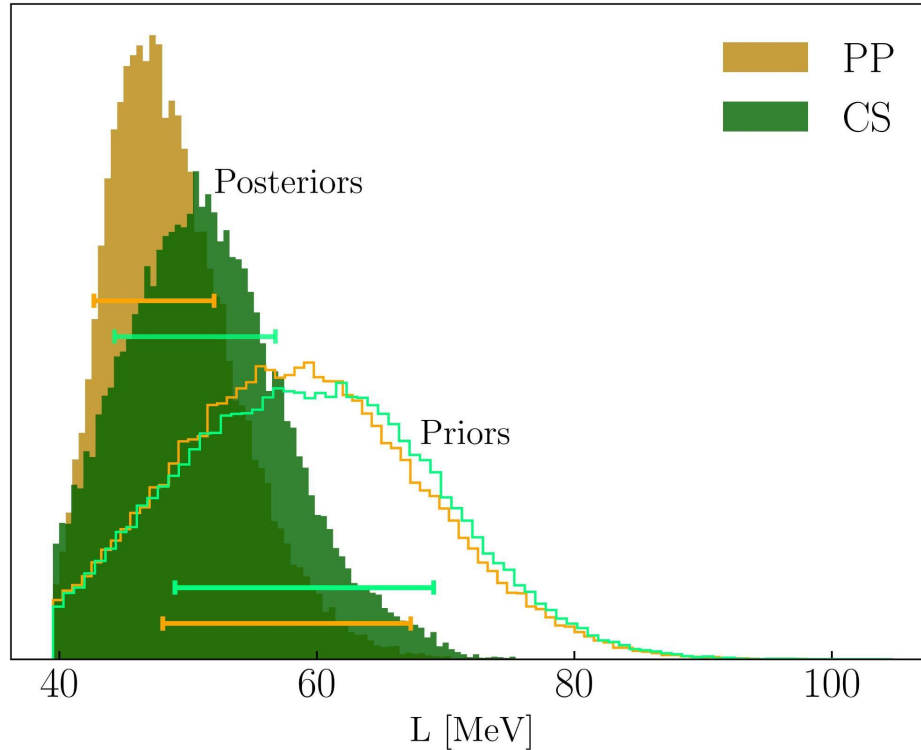


Figure from Reinhard and Nazarewicz, PRC (2016),
arxiv:1601.06324

L parameter inference



Astrophysical data informs the value of L, the slope of symmetry energy

$$L_{68\% \text{ PP}} = 42.7 - 52.0 \text{ MeV}$$

$$L_{68\% \text{ CS}} = 44.3 - 56.8 \text{ MeV}$$

From **MM** et al, in preparation (2025)

Summary



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- ❑ **Astrophysical data significantly constrains the EOS in high density, low temperature region**
- ❑ **Ab initio calculation updates can easily be integrated into EOS inference frameworks, including uncertainty quantification of cEFT EOS**
- ❑ **Inference of EOS parameters, such as L , statistically meaningful and astro informed**
- ❑ **Precision of observations only increases! More astrophysical and laboratory data expected soon**

Special thanks to A. Hensel, H.
Göttling, I. Svensson, K. Hebeler,
A. Schwenk

Thank you!

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Additional slides

NICER telescope primer

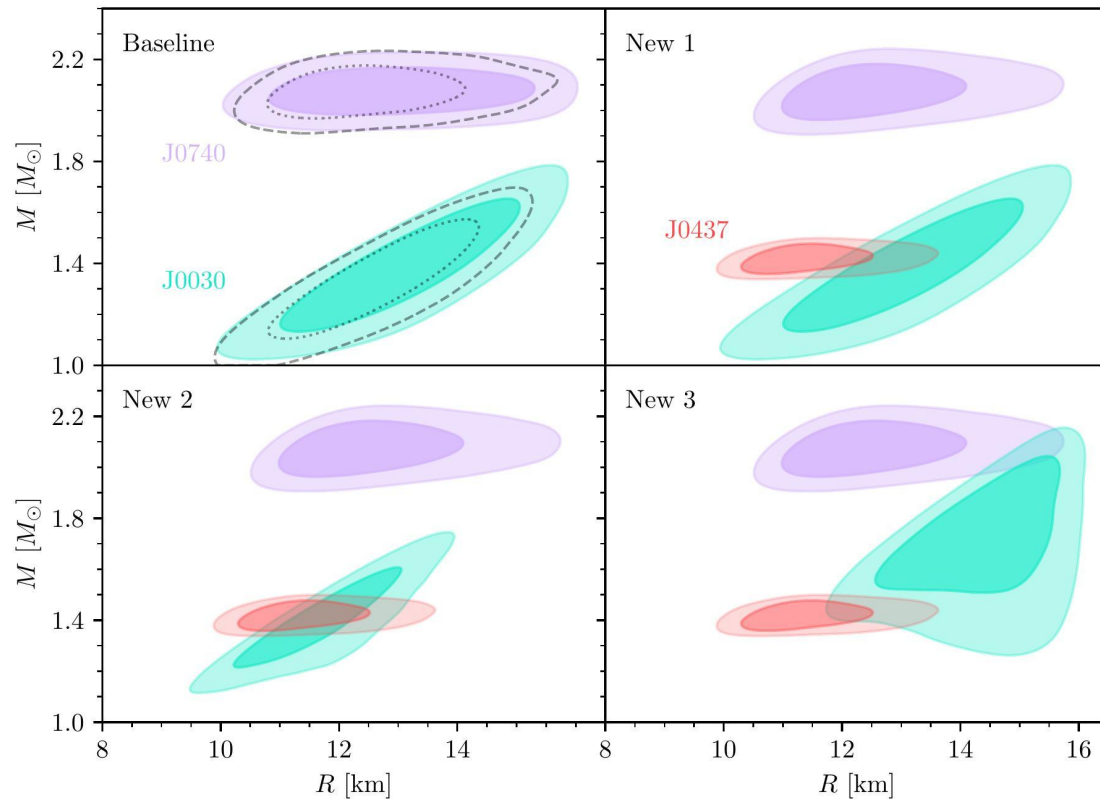
Neutron star Interior Composition ExploreR (NICER)

- X-ray telescope, aboard the International Space Station since 2017
- Needs rotating sources with $f > 100$ Hz
- Inferences depend on model for the surface emission, with free parameters:
 - mass, radius, spin rate, temperature distribution, beaming function or atmosphere model, observer inclination, distance, interstellar hydrogen column density and a background model

Targets:

- 5-10% precision level for PSR J0437-4715, PSR J0030+0451, PSR J1231+1411 and PSR J2124-3358
- Plus PSR J0614-3329, PSR J1614-2230, PSR J2124-3358, and others

Rutherford et al NICER data

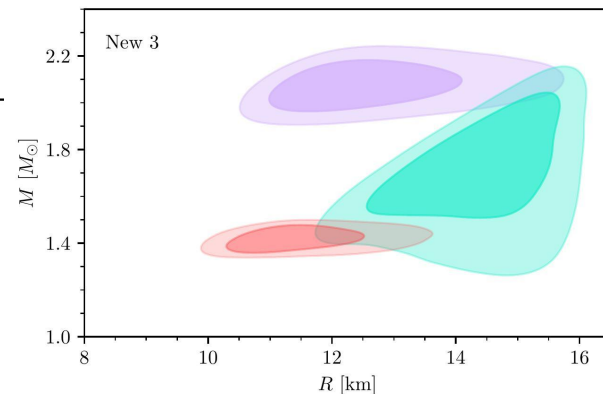
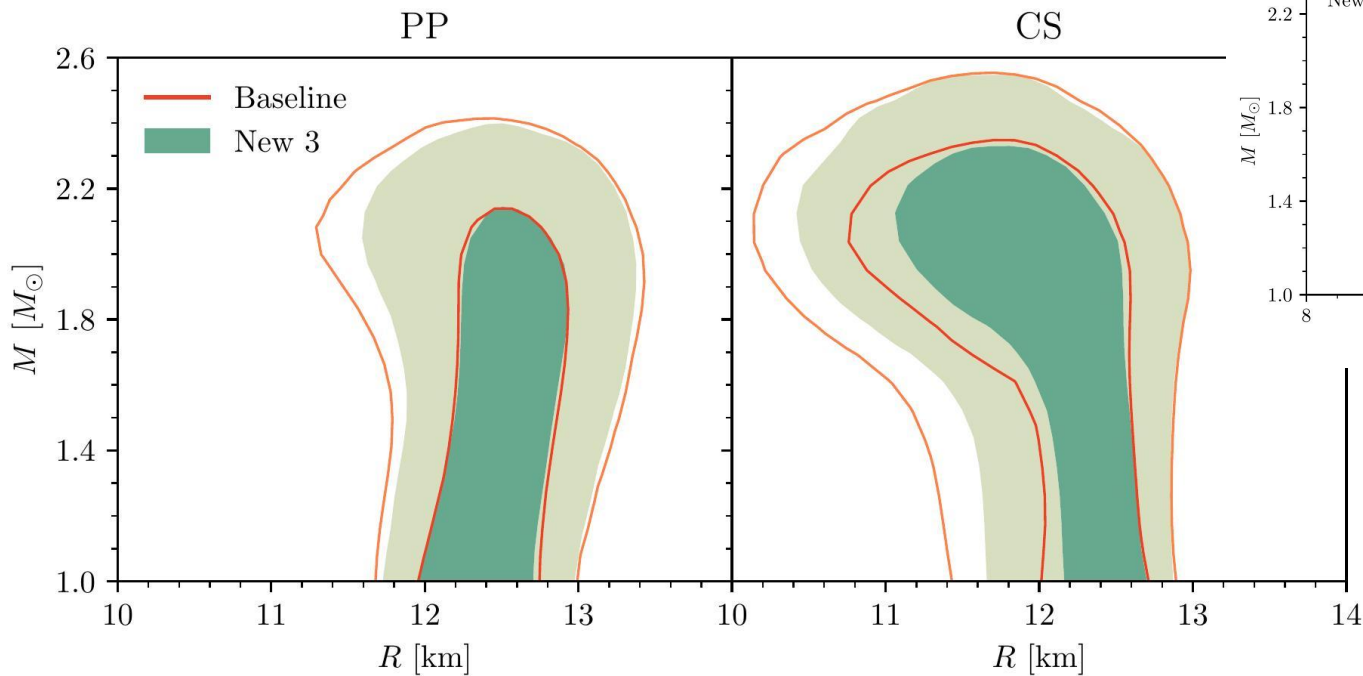


Some possible MR
data contours
depending on hot spot
modelling

Plus GW170817 and
GW190425

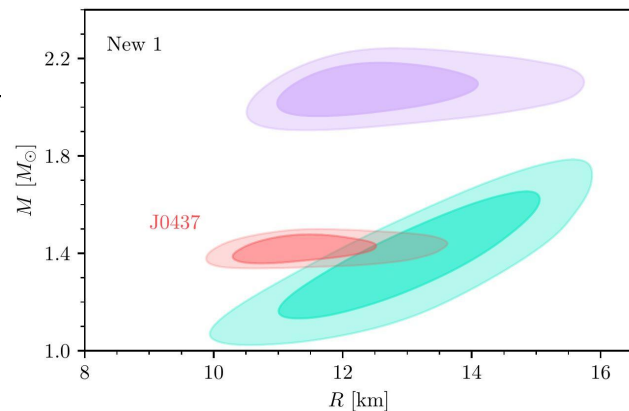
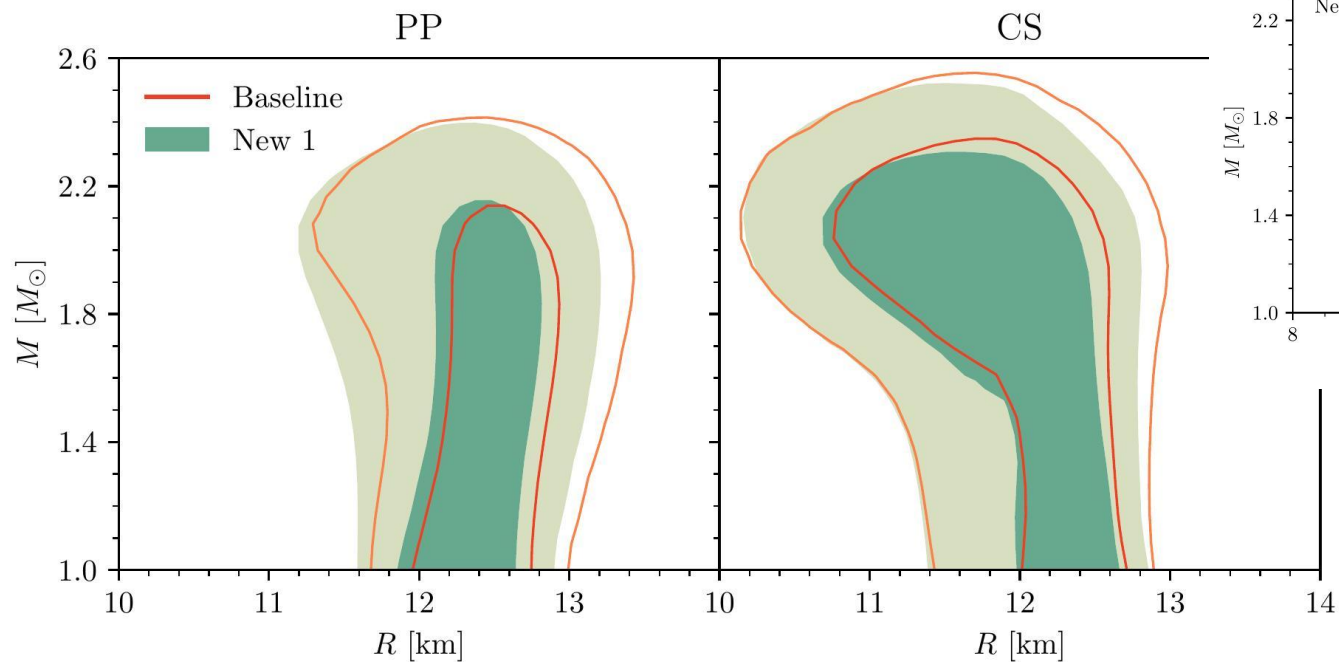
From Rutherford, **MM**,
Svensson et al, ApJL (2024),
arxiv:2407.06790

New 3 contours



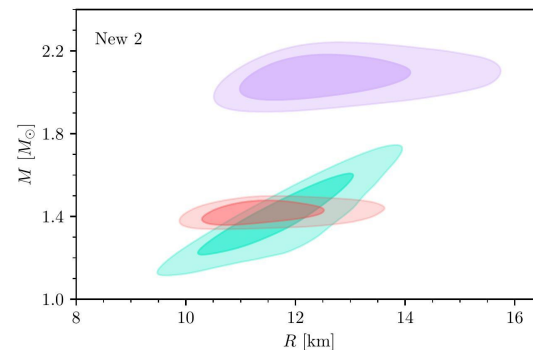
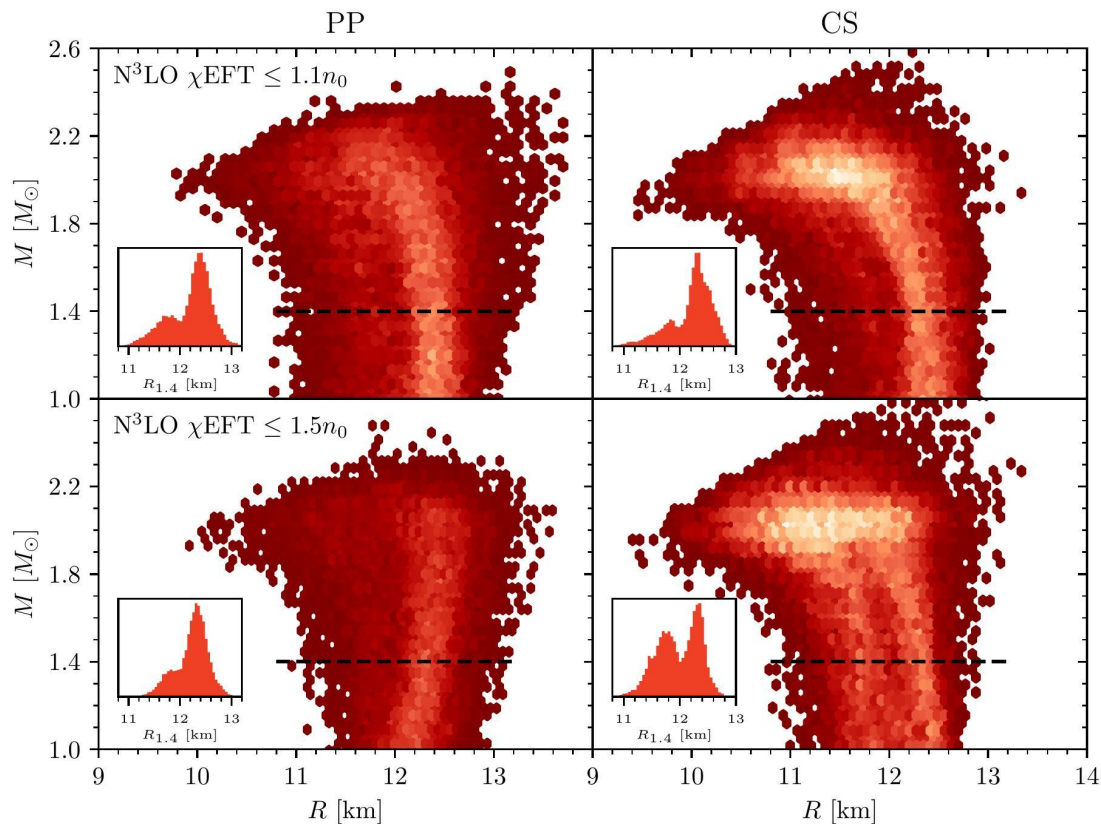
From Rutherford, **MM**,
Svensson et al, ApJL (2024)
arxiv:2407.06790

Other hot spot scenarios



From Rutherford, **MM**,
Svensson et al, ApJL (2024),
arxiv:2407.06790

Most likely M-R

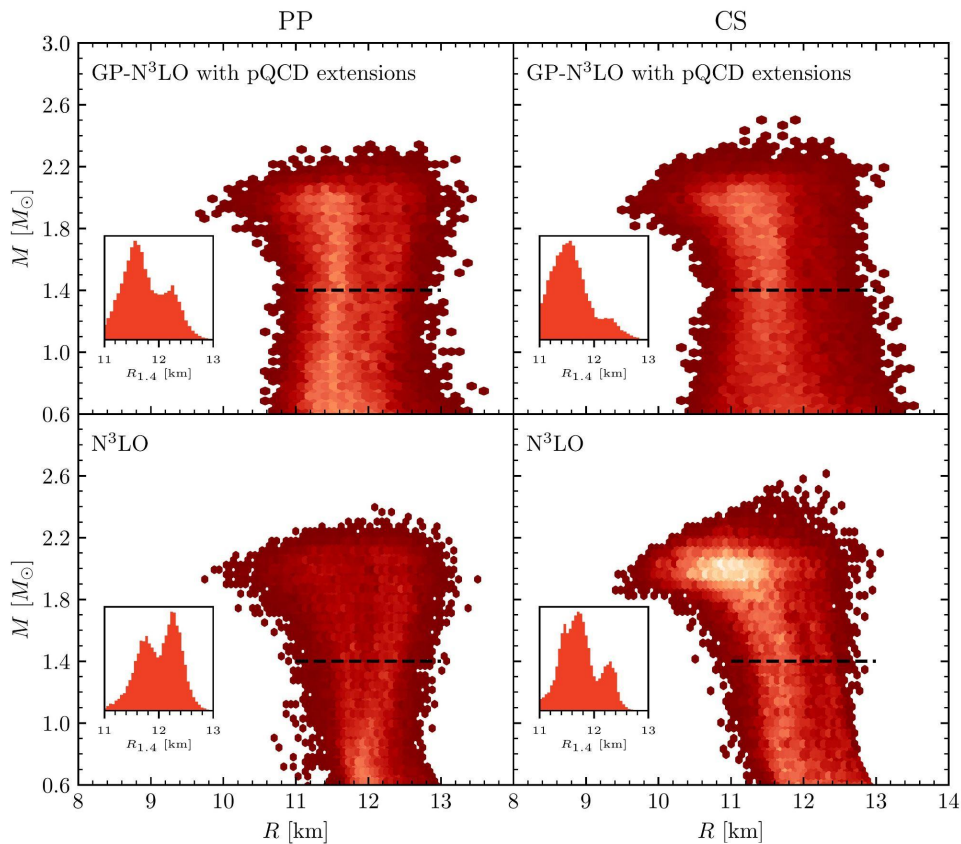


Consistent results for PP and
CS parameterizations

Bimodal-like tendency for all
posteriors

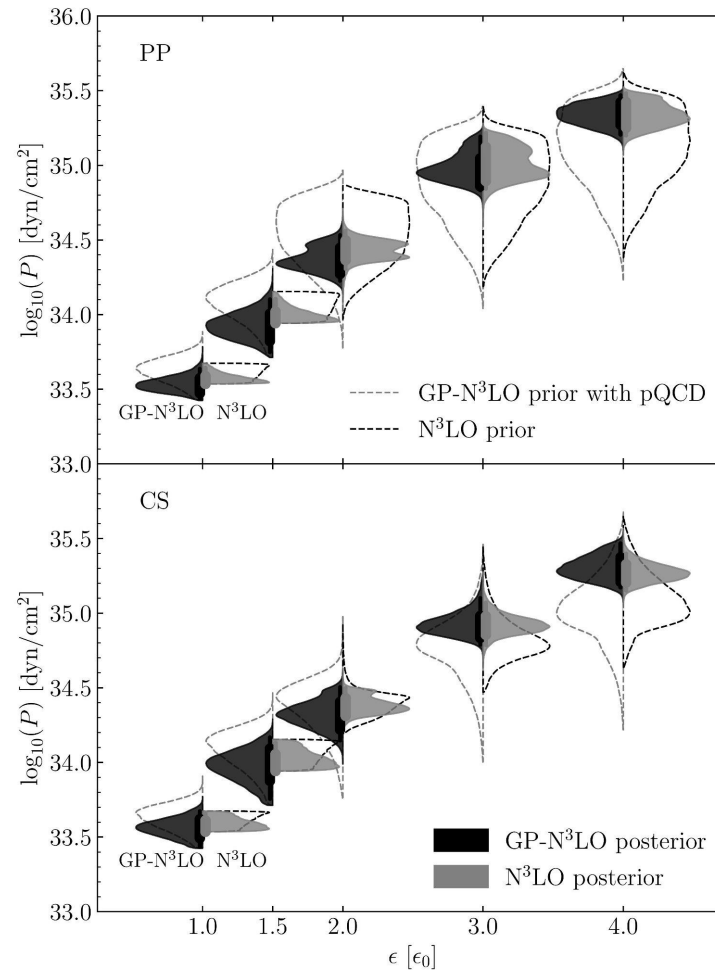
From Rutherford, **MM**, Svensson et al,
ApJL (2024), arxiv:2407.06790

Most likely M-R



Bimodal-like tendency still
present with new cEFT
implementation and pQCD
extensions

From **MM** et al (2025), in preparation



Consistent results for PP and CS parameterizations

Bimodal-like tendency for all posteriors

From **MM** et al (2025), in preparation

TOV equations I

$$\frac{dP}{dr} = -\frac{\mathcal{E}(r)}{c^2} \frac{Gm(r)}{r^2} \left[1 + \frac{P(r)}{\mathcal{E}(r)} \right] \left[1 + \frac{4\pi r^3 P(r)}{m(r)c^2} \right] \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1},$$

$$\frac{dm}{dr} = 4\pi r^2 \frac{\mathcal{E}(r)}{c^2},$$

$$\frac{dy(r)}{dr} = -\frac{y(r)^2}{r} - \frac{y(r)}{r} F(r) - rQ(r), \text{ where}$$

$$F(r) = \left[1 - 4\pi r^2 (\mathcal{E}(r) - P(r)) \right] \left[1 - \frac{2m(r)}{r} \right]^{-1},$$

$$Q(r) = 4\pi \left[5\mathcal{E}(r) + 9P(r) + \frac{\mathcal{E}(r) + P(r)}{c_s^2(r)} - \frac{6}{r^2} \right] \left[1 - \frac{2m(r)}{r} \right]^{-1} \\ - \frac{4m^2(r)}{r^4} \left[1 + \frac{4\pi r^3 P(r)}{m(r)} \right]^2 \left[1 - \frac{2m(r)}{r} \right]^{-2}$$

TOV equations II



With $y(R) = y_R$, calculate the tidal Love number $k_2(\xi, y_R)$,

$$\begin{aligned} k_2(\xi, y_R) = & \frac{1}{20} \xi^5 (1 - \xi)^2 [(2 - y_R) + (y_R - 1) \xi] \left\{ \left[(6 - 3y_R) + \frac{3}{2} (5y_R - 8) \xi \right] \xi + \right. \\ & + \frac{1}{2} \left[(13 - 11y_R) + \frac{1}{2} (3y_R - 2) \xi + \frac{1}{2} (1 + y_R) \xi^2 \right] \xi^3 + \\ & \left. + 3 [(2 - y_R) + (y_R - 1) \xi] (1 - \xi)^2 \ln(1 - \xi) \right\}^{-1}, \end{aligned}$$

where $\xi = 2GM/Rc^2$. The dimensionless tidal deformability is

$$\Lambda = \frac{2}{3} k_2 \xi^{-5} = \frac{2}{3} k_2 \left(\frac{Rc^2}{2GM} \right)^5$$

but the actual observable is the mass-weighted tidal deformability:

$$\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2) M_1^4 \Lambda_1 + (M_2 + 12M_1) M_2^4 \Lambda_2}{(M_1 + M_2)^5}.$$

PP parametrization equation

Each polytrope given by $p(\rho) = K \rho^\Gamma$
such that $\varepsilon(\rho) = (1 + a)(p/K)^{1/\Gamma} + p/(\Gamma - 1)$

Parameter ranges are:

For χ_{EFT} up to $1.1n_0$,
 $\Gamma_1: [1, 4.5]$, $\Gamma_2: [0, 8]$, $\Gamma_3: [0.5, 8]$, $\rho_{12}: [1.5, 8.3]$, $\rho_{23}: [1.5, 8.3]$

For χ_{EFT} up to $1.5n_0$,
 $\Gamma_1: [0, 8]$, $\Gamma_2: [0, 8]$, $\Gamma_3: [0.5, 8]$, $\rho_{12}: [2, 8.3]$, $\rho_{23}: [2, 8.3]$

CS parametrization equation

High density EOS given by

$$c_s^2(x)/c^2 = a_1 e^{-\frac{1}{2}(x-a_2)^2/a_3^2} + a_6 + \frac{\frac{1}{3} - a_6}{1 + e^{-a_5(x-a_4)}},$$

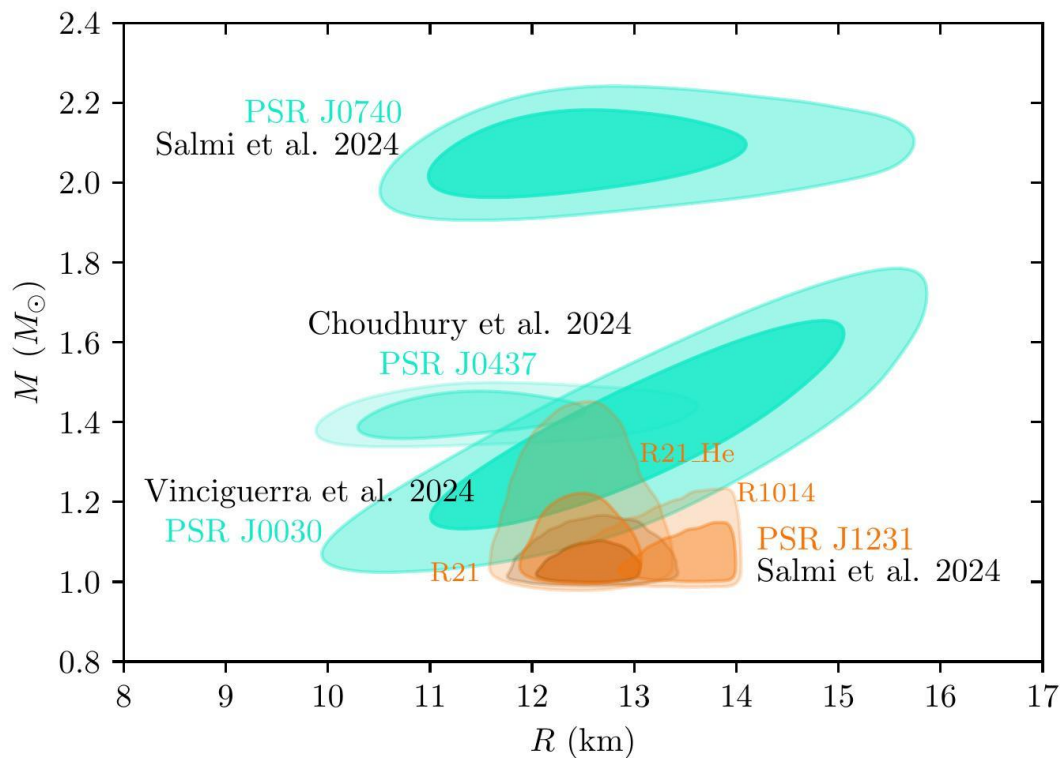
a_6 to match χ_{EFT} band.

Fermi liquid theory (FLT) limit is $c_s^2 \text{FLT}(1.5 n_0)/c^2 \leq 1/m_N^2 (3\pi^2 n)^{2/3}$

With $a_1:[0.1, 1.5]$, $a_2:[1.5, 12]$, $a_3:[0.05, 2]$, $a_4:[1.5, 37]$, $a_5:[0.1, 1]$

a_1 outside this range fails maximum mass constraint or causality

Other NICER released data



Other hot spot models for J0614

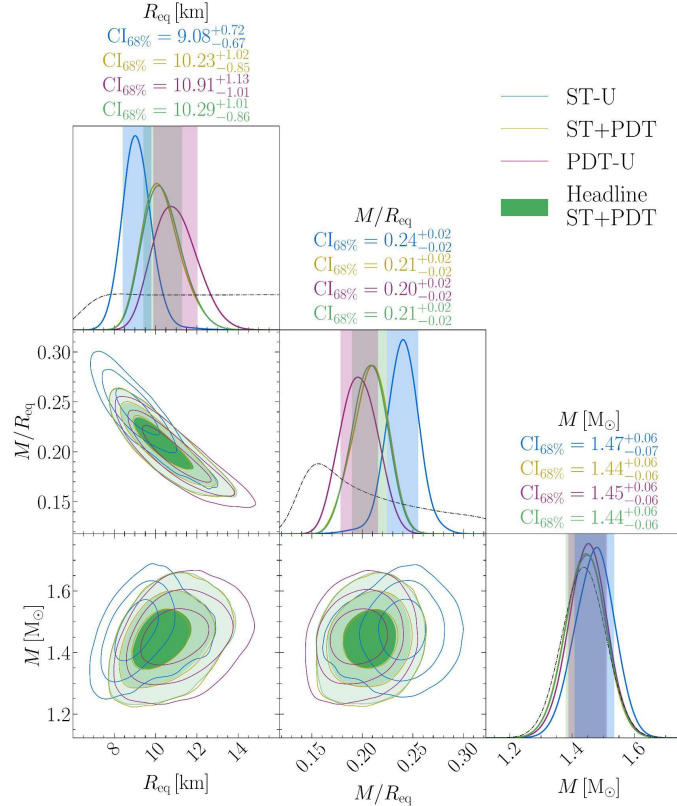
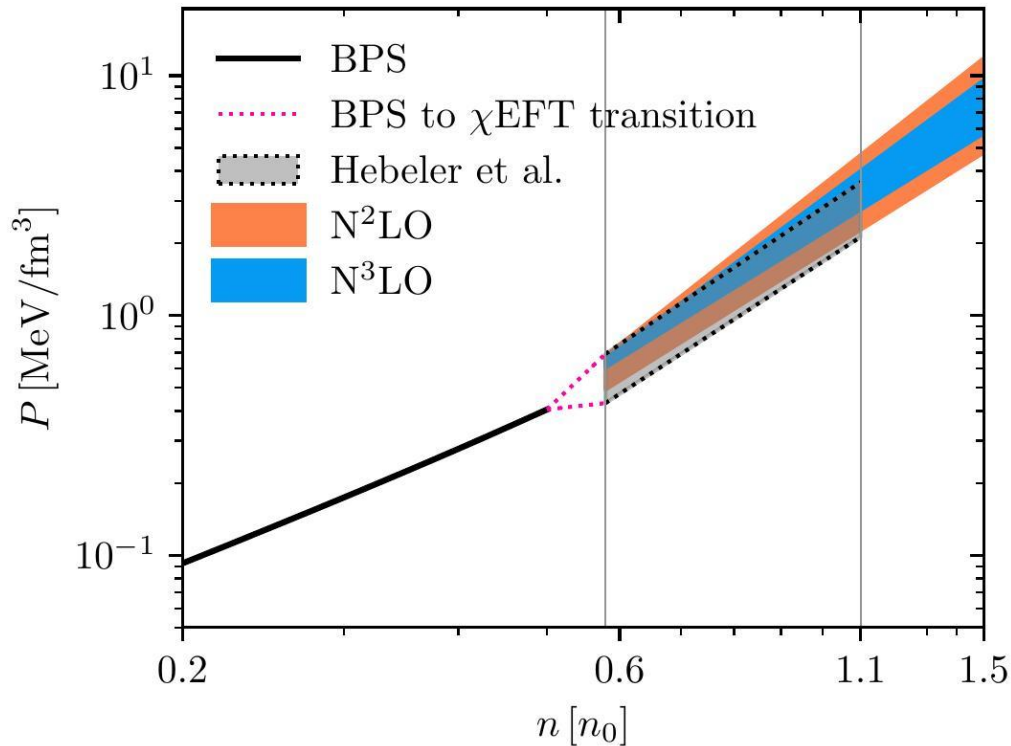


Figure from Mauviard, ... **MM**, ... et al, ApJ (2025), arxiv:2506.14883

Crust and outer core

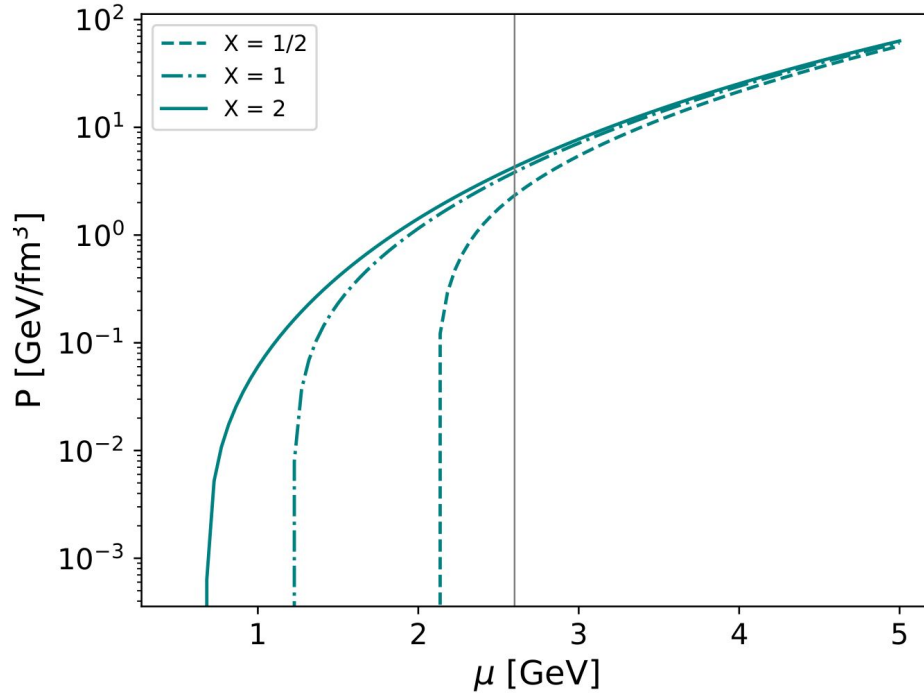


BPS EOS to model crust and
polytropic fit ($P = K n^\Gamma$) for χ EFT

Extended to both $1.1n_0$
and $1.5 n_0$

From Rutherford, **MM**, Svensson et al, ApJL
(2024), arxiv:2407.06790

pQCD implementation details



where $X \equiv 3\Lambda/(2\mu_B)$

with Λ renormalization scale

In NEST, $\text{Log}X$ uniformly sampled
from $[0.5, 2]$

Figure from A. Hensel Master's thesis