Energy Correlators for Jet Substructure

Rising Researchers Seminar
INT

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Los Alamos National Lab

Based on arXiv: 2210.09311, *Phys.Rev.D* 111 (2025) 1, L011502 and work on progress

June 10th, 2025

R2S2 – CODE OF CONDUCT

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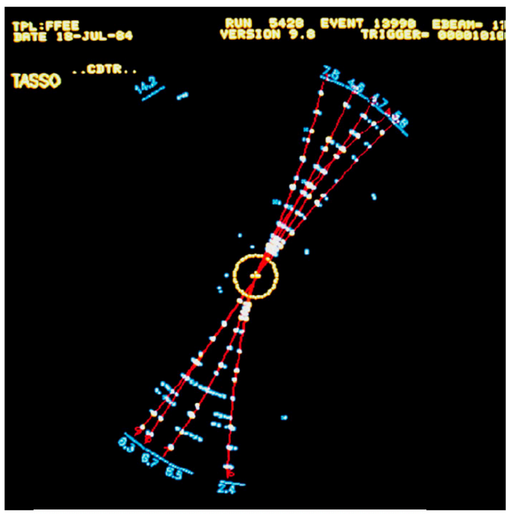
References: This code of conduct is based heavily on that of the INT and the APS. We are also grateful to Roxanne Springer for valuable discussion and guidance.

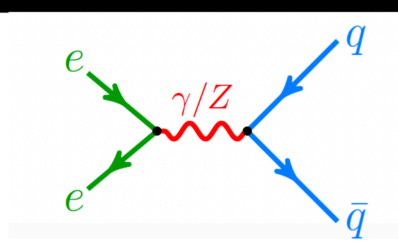
Jets

Emergent phenomena at colliders: a result of both dynamics and kinematics at high energies

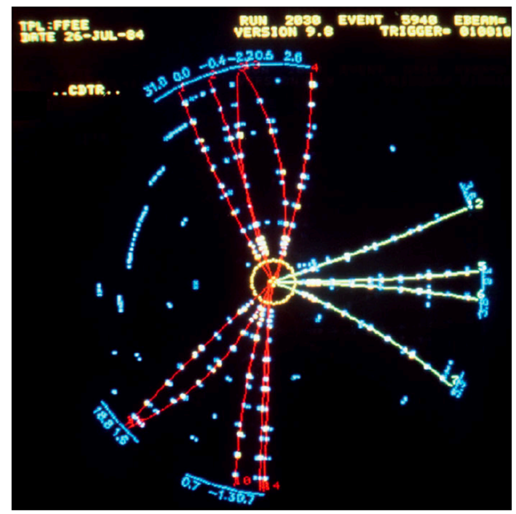
- Effective degrees of freedom at colliders → long distance manifestation of microscopic interactions of quarks and gluons

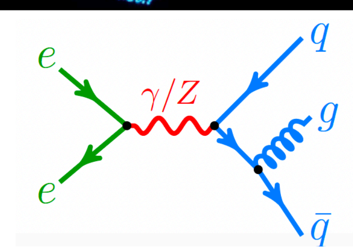
2 jet event



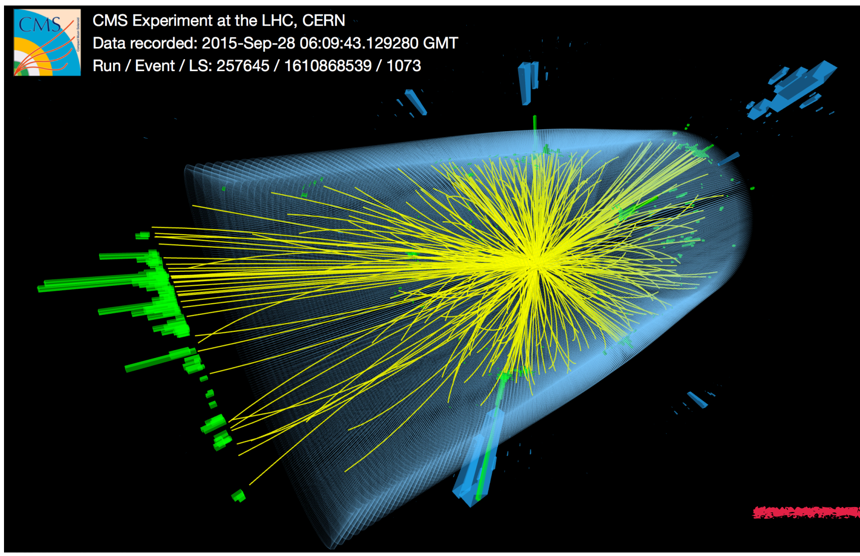


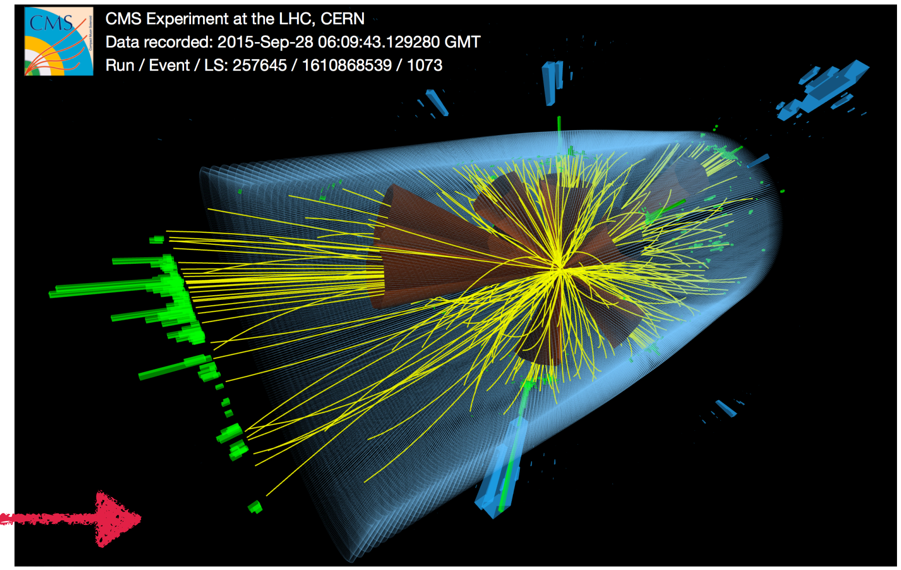
3 jet event





Jets: produced abundantly at particle colliders





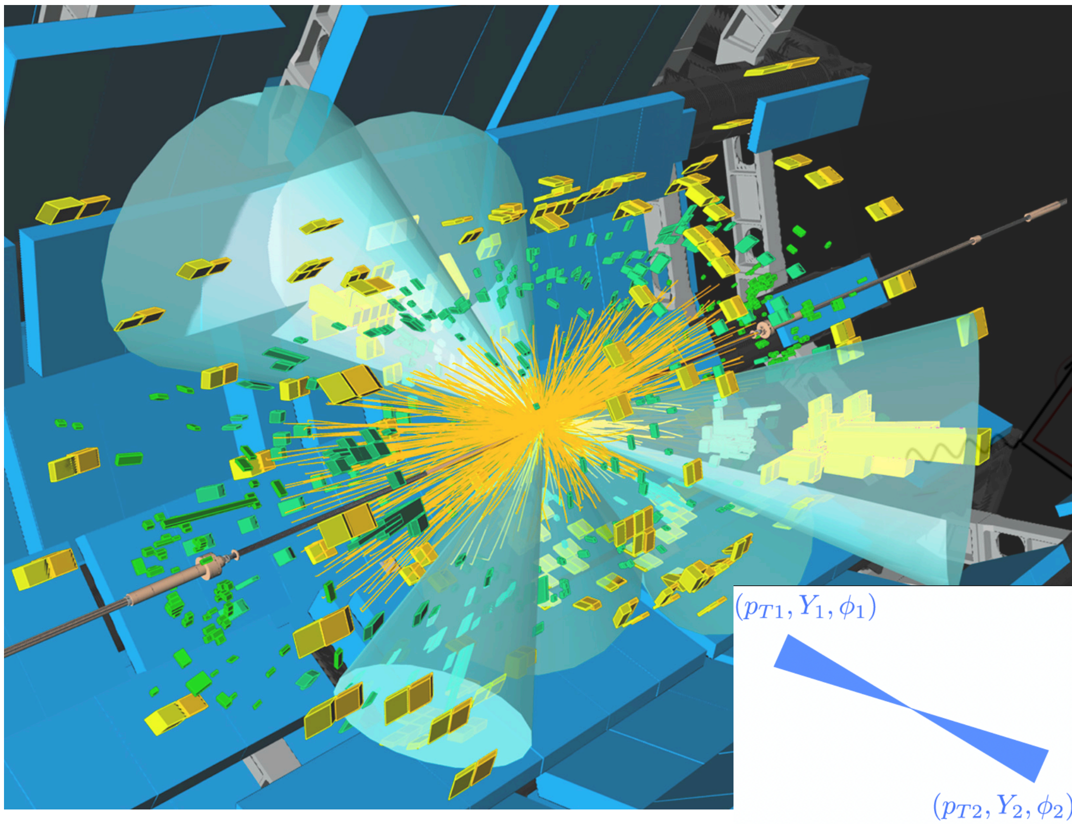
Jets are reconstructed using jet algorithms (anti-k_T)

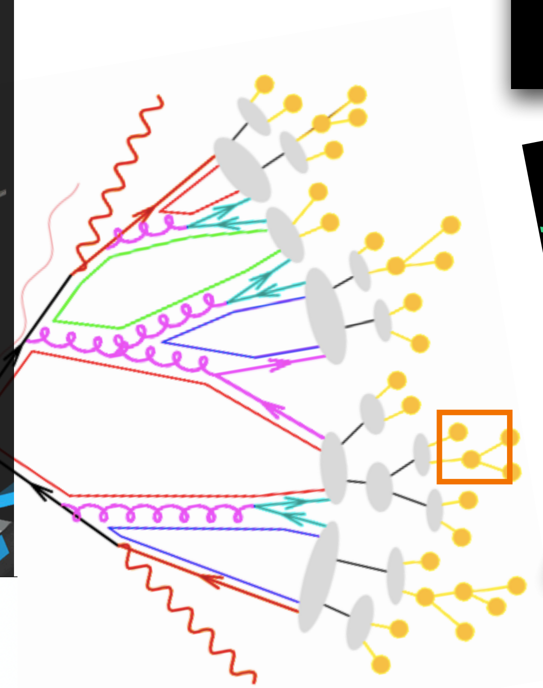
Cacciari, Salam 2006 Salam; Soyez 2007

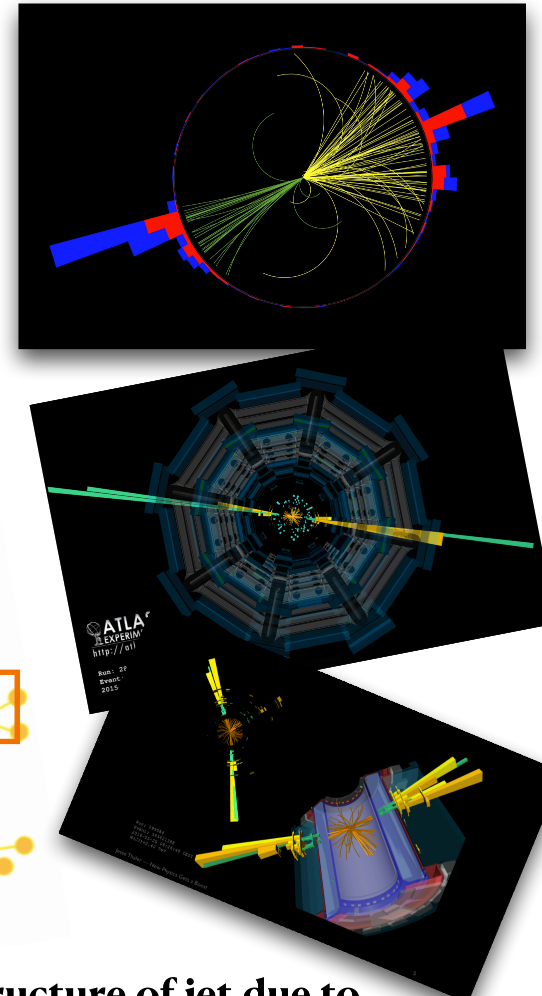
$\Rightarrow$ **Can systematically study jets and their substructure**

Jet Substructure

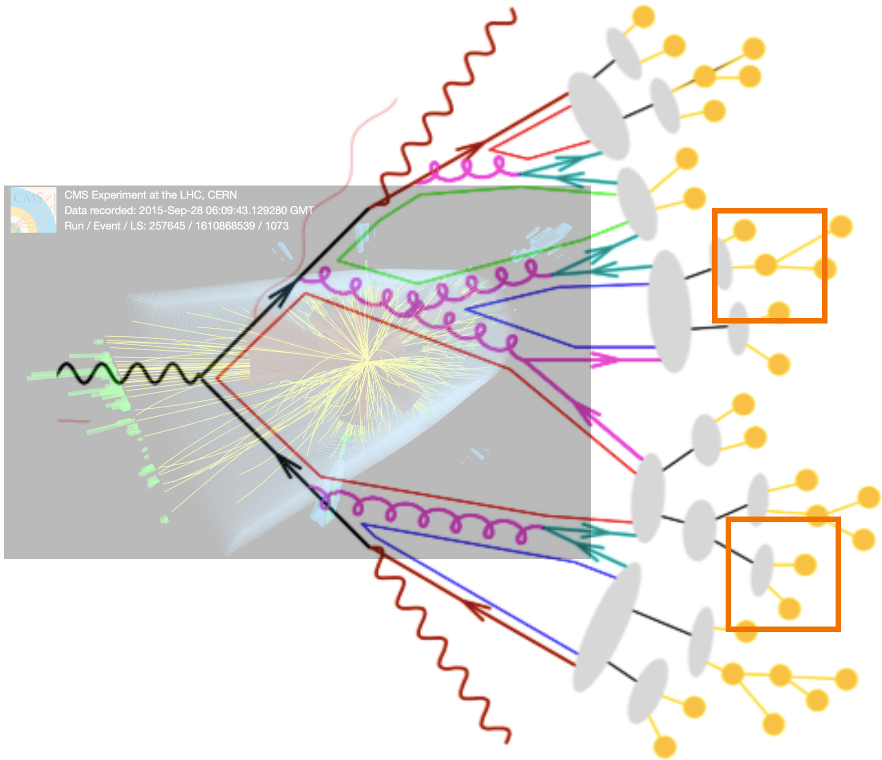
- Not all jets are the same.
- Jets have substructure: distinguished by kinematic properties



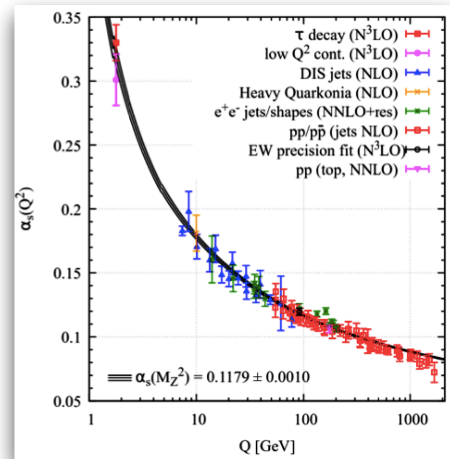




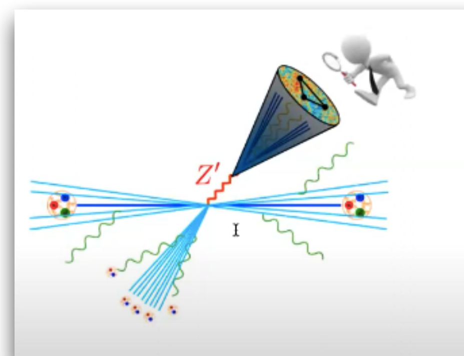
- Can resolve the substructure of jet due to excellent detector resolution



QCD precision studies



New Physics searches

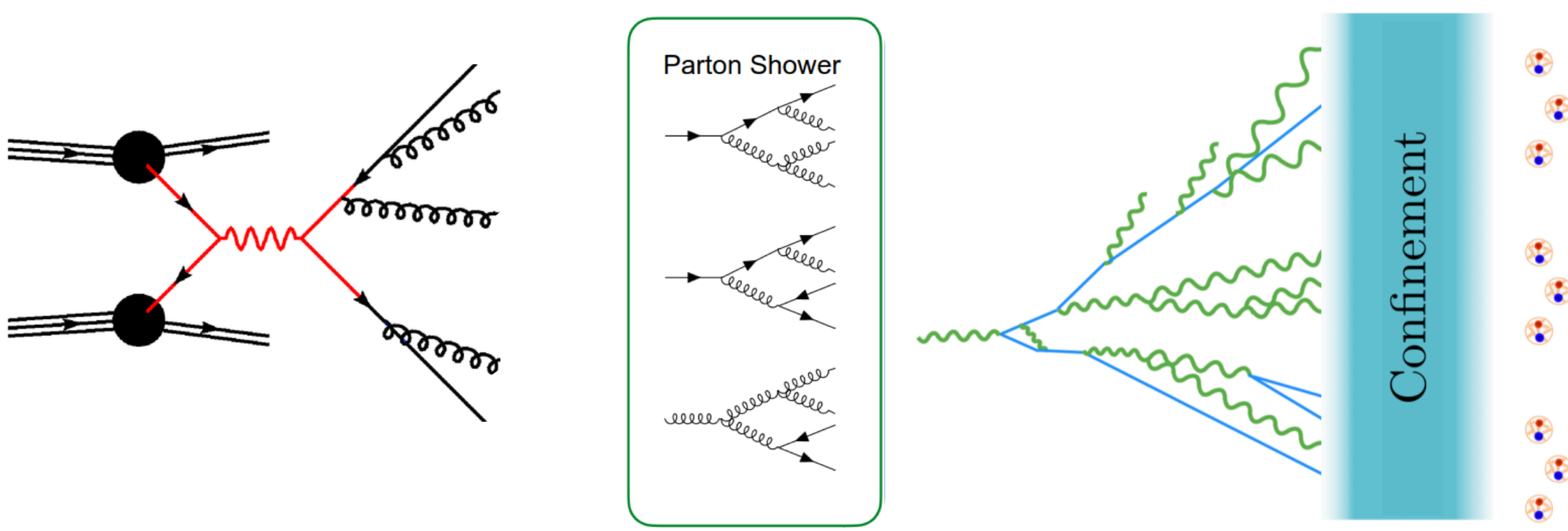


Jets: probes for precision QCD

The hard process:
Quarks and Gluons

Energy loss through a
cascade of radiation

Collimated final
state hadrons



Transition from weak to strong coupling

High Energy (Perturbative QCD) → Low Energy (Non-Perturbative QCD)

Jets: probes for precision QCD

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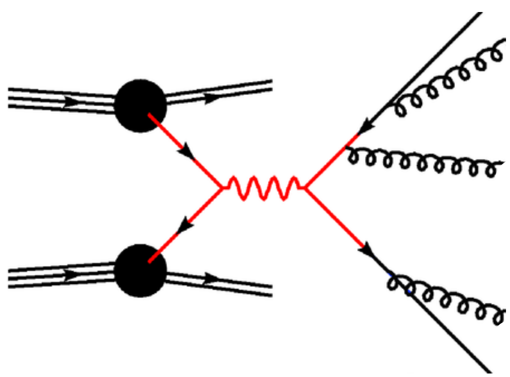

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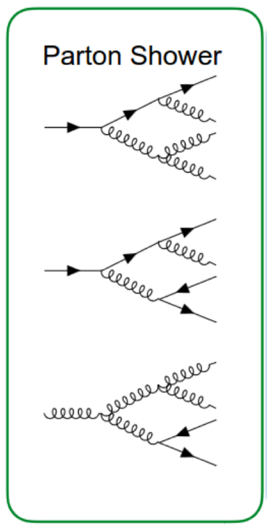

Collimated final
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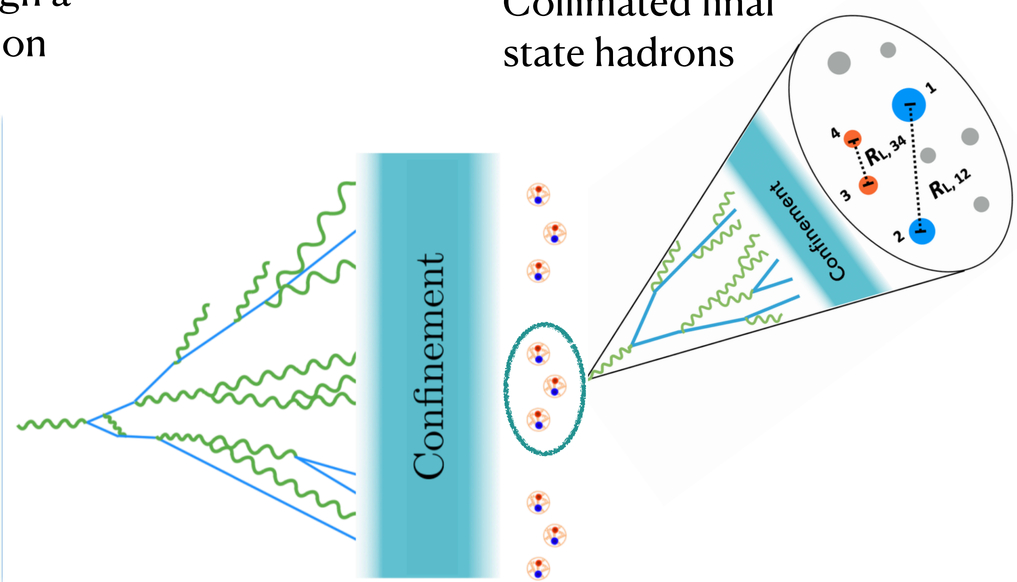


Jet substructure:

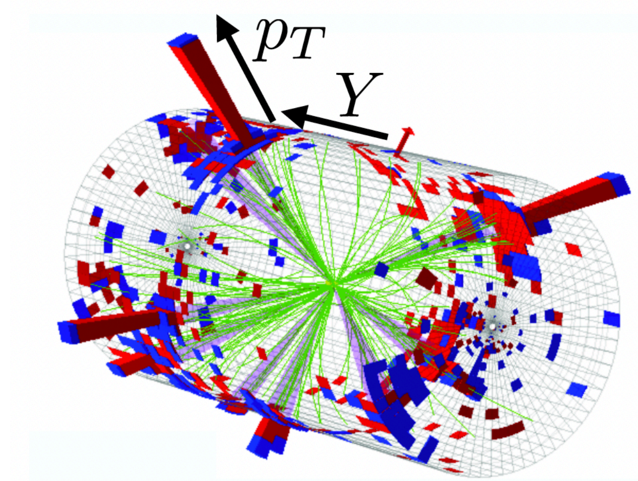
- ✦ Internal kinematic properties of the jet: distribution of energy inside the jet
- ✦ Encodes information about hadronization/confinement, energy loss patterns (vacuum and medium), QCD splitting functions,

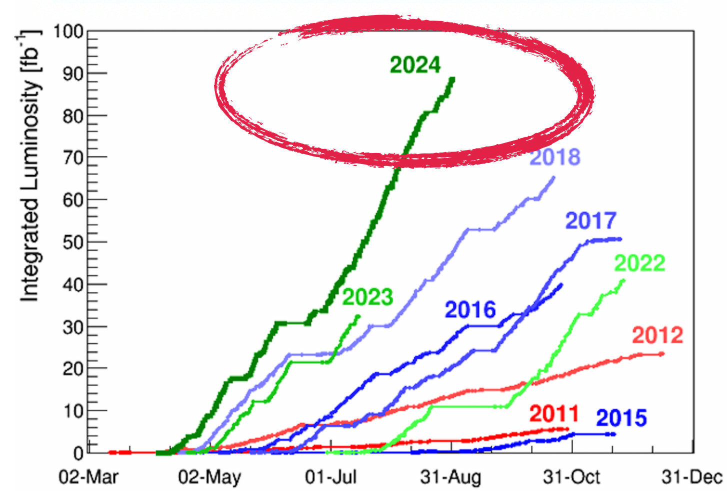
Formal Theory Meets Experiment

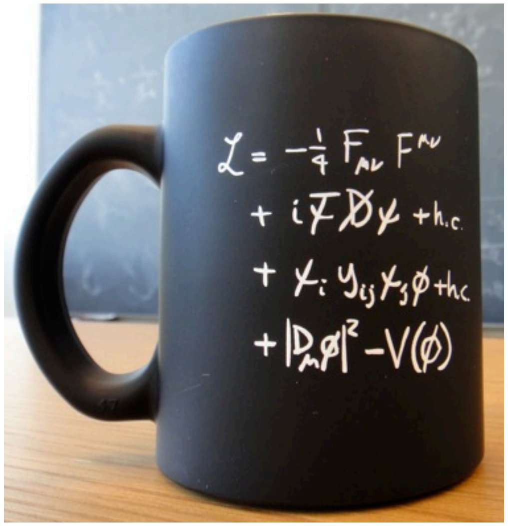
- A wealth of experimental data at high energies
- Lots of progress from the Quantum Field Theory (QFT) side on event shape-like observables.

$$\Downarrow$$

- **Revisit the way we define observables for studying jets and jet substructure.**
- **Combine Effective Theory methods with formal aspects from Quantum Field Theory.**

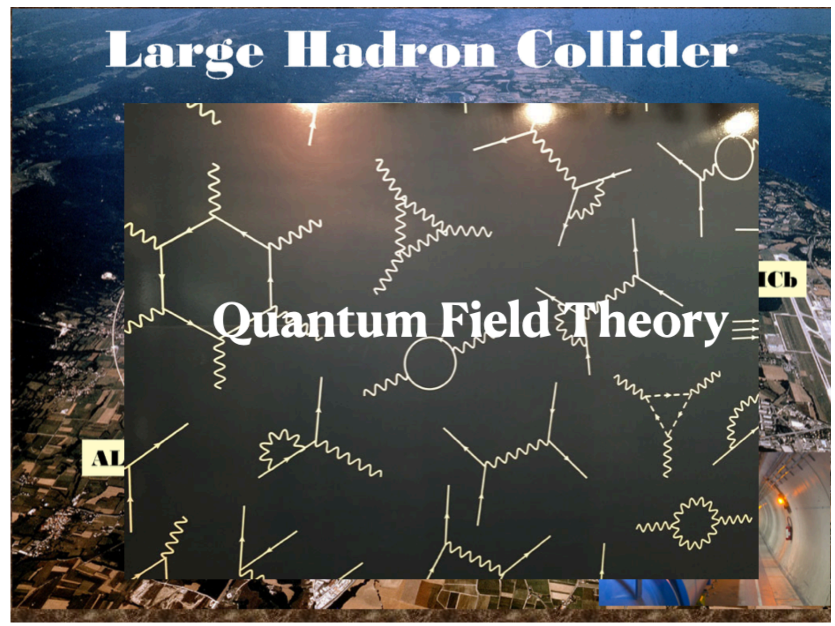






Two main Theory Questions

- **What is the underlying physics dynamics?**
- **What is the scale of the dynamics?**



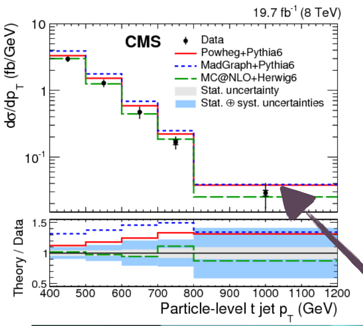
Effective description of the full dynamics at colliders:

- Relevant degrees of freedom in the experiment and in theory calculations
- Observables that capture this information

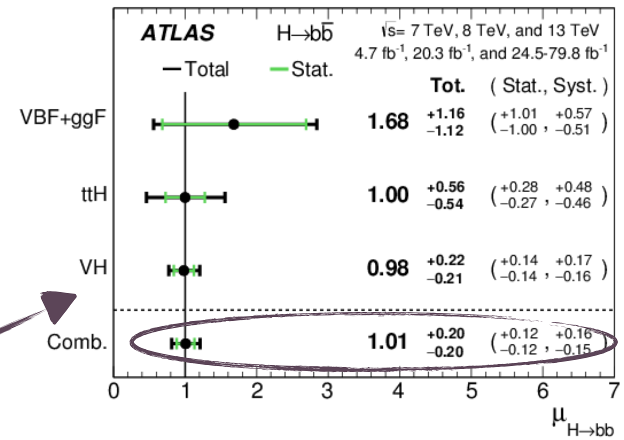
⇒ Can tackle the Standard Model open questions separately and independently

Jets: A Success Story of QCD

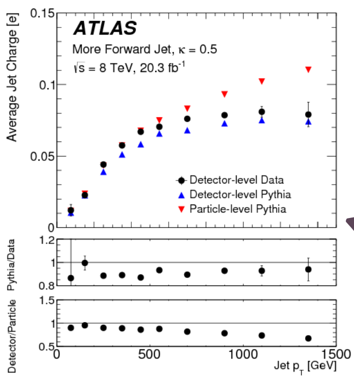
Many successful jet observable measurements and impressive calculations

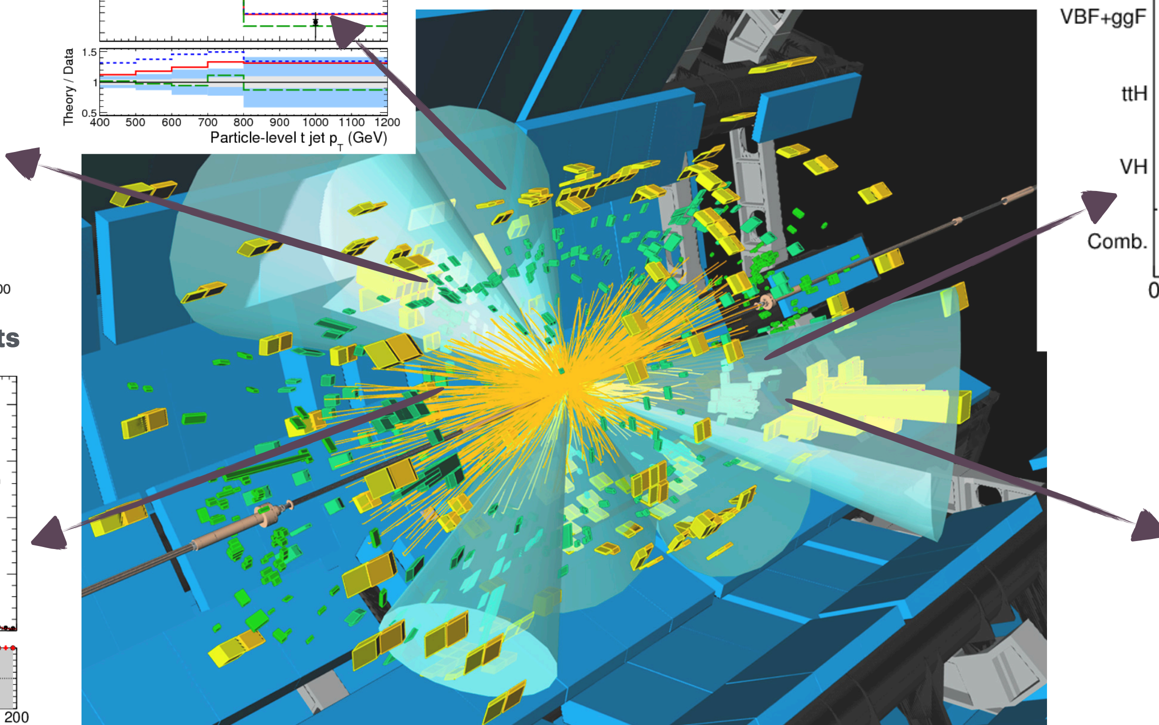


Higgs Boson coupling to third generation quarks

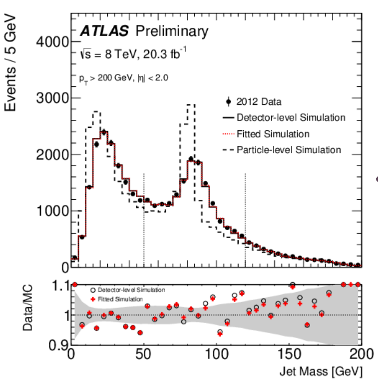


Jet Charge

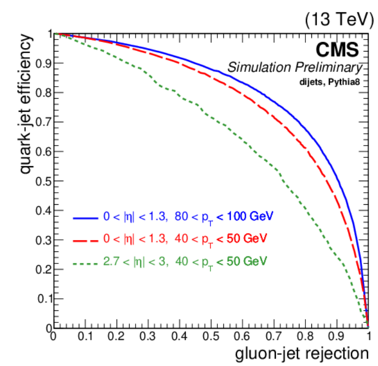




Jet mass measurements



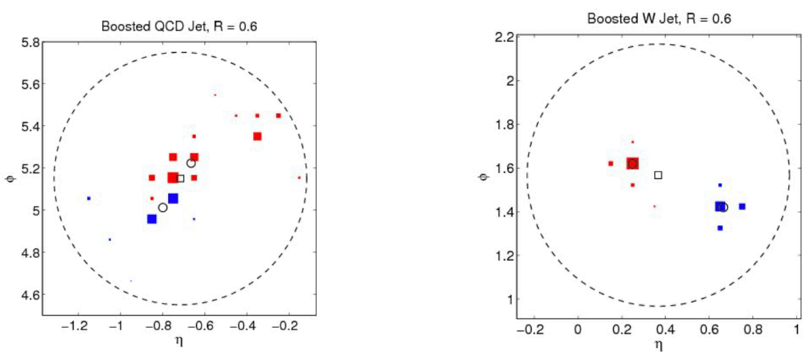
Quark\Gluon jet efficiency



Challenges with current jet substructure observables

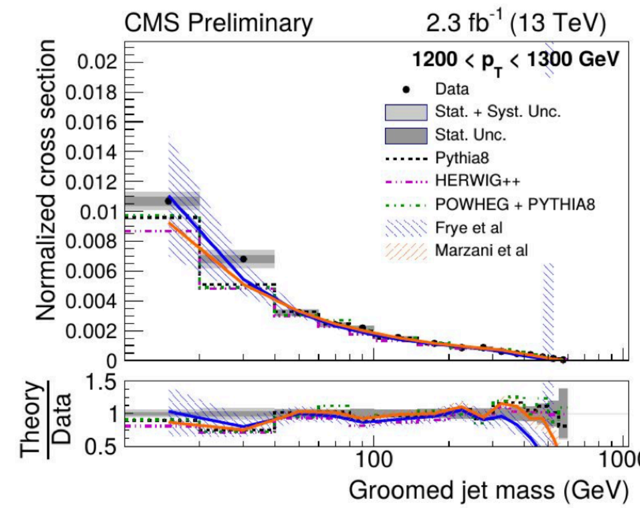
- **Difficulty to extend the calculations higher orders in pQCD ⇒ challenging to match experimental precision**

- **Dependance on kinematic cuts, predefined axis. Example: N-subjettiness**

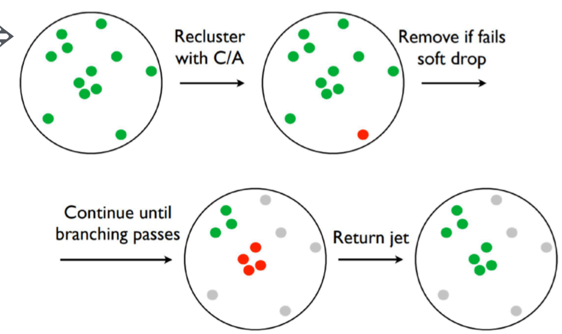


[Thaler, Van Tilburg, JHEP 1103 (2011) 015]

$$\tau_N^{(2)} = \frac{1}{p_{t,jet} R^2} \sum_{i \in jet} p_{t,i} \min_{a_i \dots a_N} (\theta_{ia_1}^2, \dots, \theta_{ia_N}^2).$$



SoftDrop



- **Traditional jet substructure observables are sensitive to soft physics ⇒ jet grooming: Pruning/Trimming\Soft Drop / mMDT**

- **Sensitive to non-global logarithms ⇒ Difficult to resum**

- **Limited calculable control over mass effects of heavy quarks**

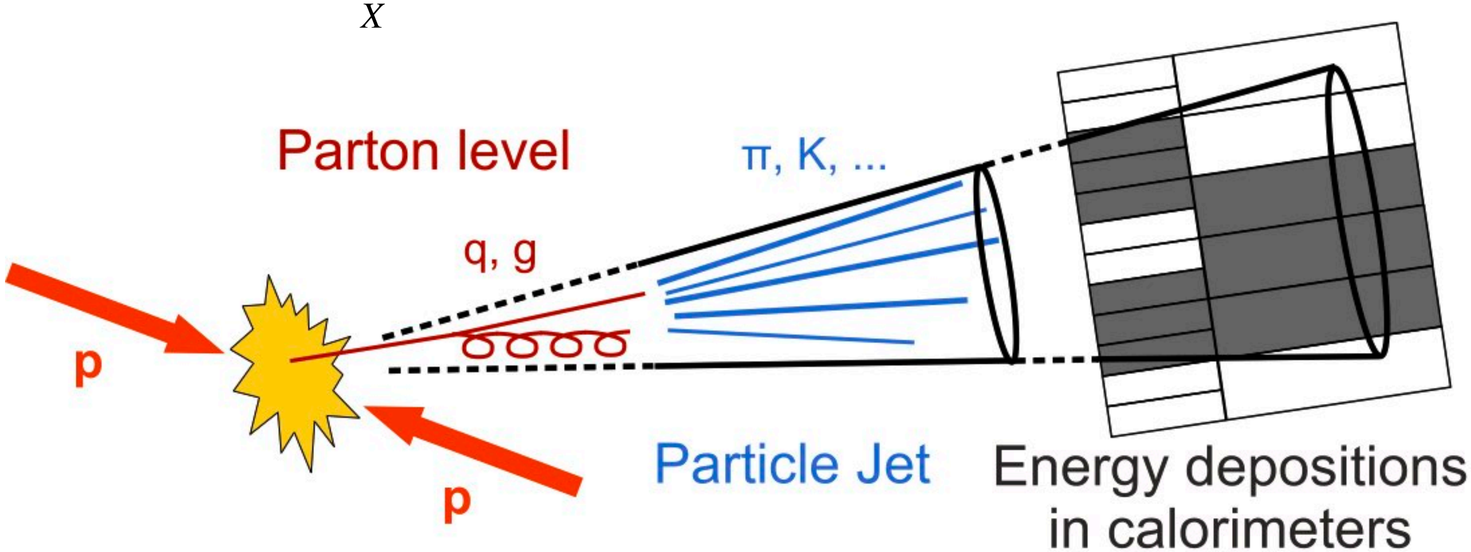
[JHEP 1309 (2013) 029, JHEP 1405 (2014) 146, JHEP 1806 (2018) 093]

Event Shapes and Energy Correlators

Rethinking of Event Shapes as Correlation Functions

Jet substructure observables\Event Shapes: weighted cross sections

$$\sigma_\omega(q) = \sum_X (2\pi)^4 \delta^{(4)}\left(q - k_X\right) \omega(X) \, | \langle X | O(0) | 0 \rangle |^2$$



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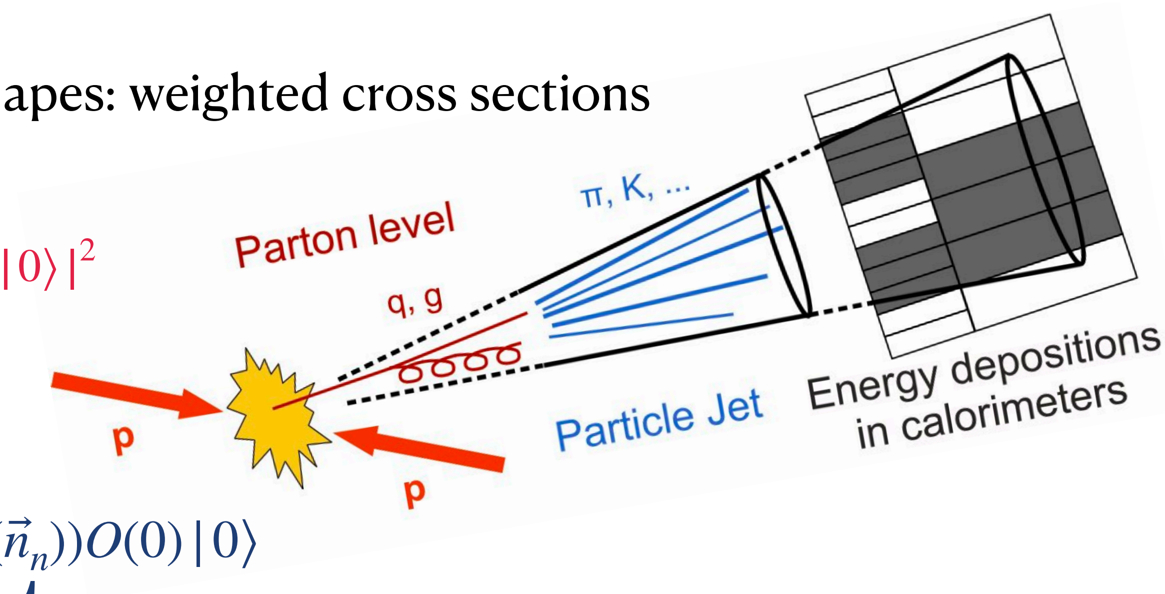

$$\sigma_w(q) = \int d^4x e^{iqx} \langle 0 \,|\, O^\dagger(x) \underbrace{(\varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n))}_{\text{Correlation Functions}} O(0) \,|\, 0 \rangle$$

Correlation Functions

Rethinking of Event Shapes as Correlation Functions

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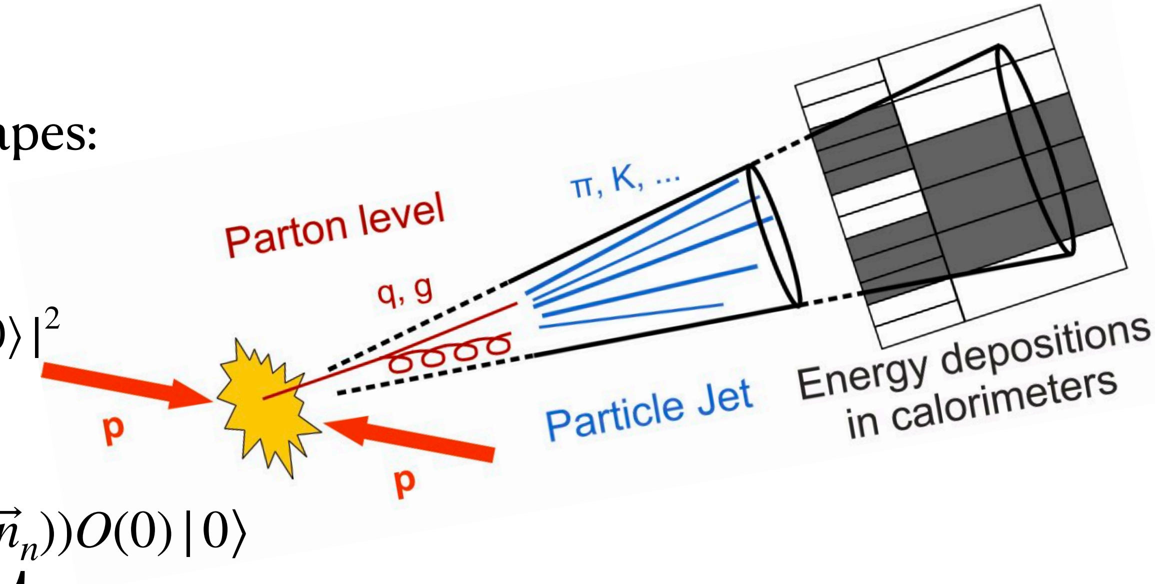

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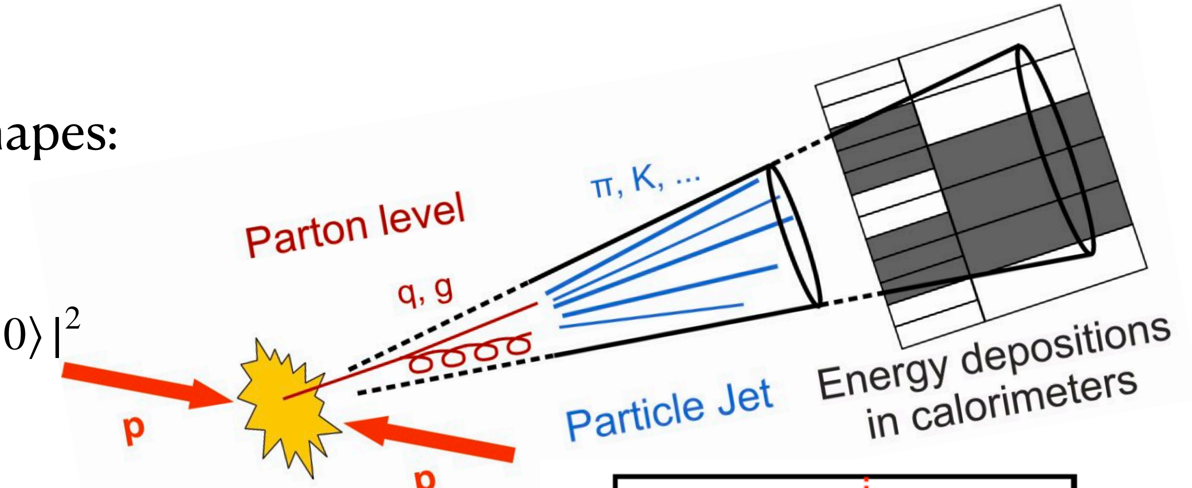
Correlation Functions

- **These are two equivalent pictures**
- **The correlation function language offers a more general QFT approach (both weak and strong coupling)**

Rethinking of Event Shapes as Correlation Functions

Jet substructure observables\Event Shapes:
weighted cross sections

$$\sigma_\omega(q) = \sum_X (2\pi)^4 \delta^{(4)}\left(q - k_X\right) \omega(X) \left| \langle X | O(0) | 0 \rangle \right|^2$$



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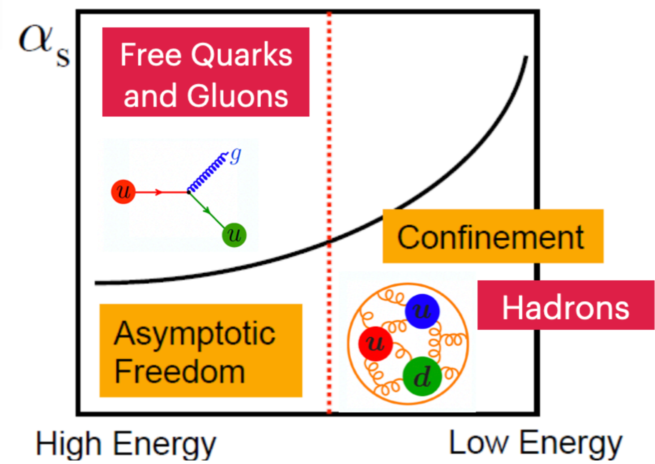


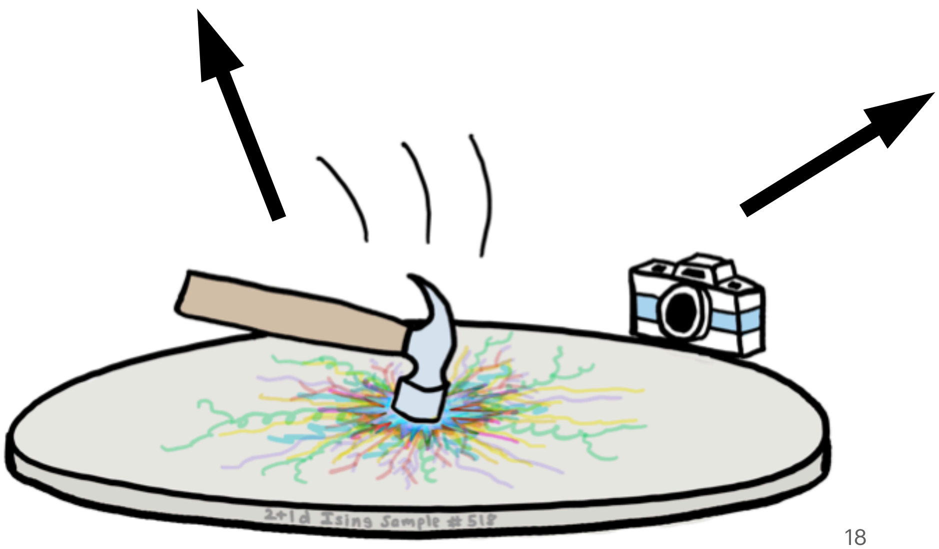
- **The correlation function language offers a more general QFT approach (both weak and strong coupling)**

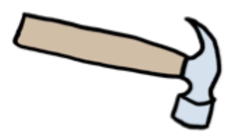
Detectors in Field Theory

Correlation Functions of operators at infinity behave as detectors of the theory.

The hammer represents the quantum states that excite the vacuum. They are the QCD operators that act locally to create the jets

The camera is the set of calorimeter cells that measures the energy



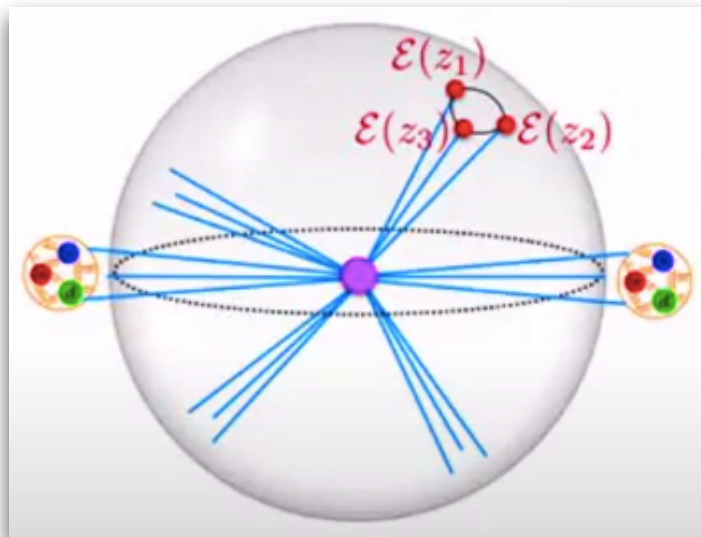
 $= \sum_i h_i \mathcal{O}_i$

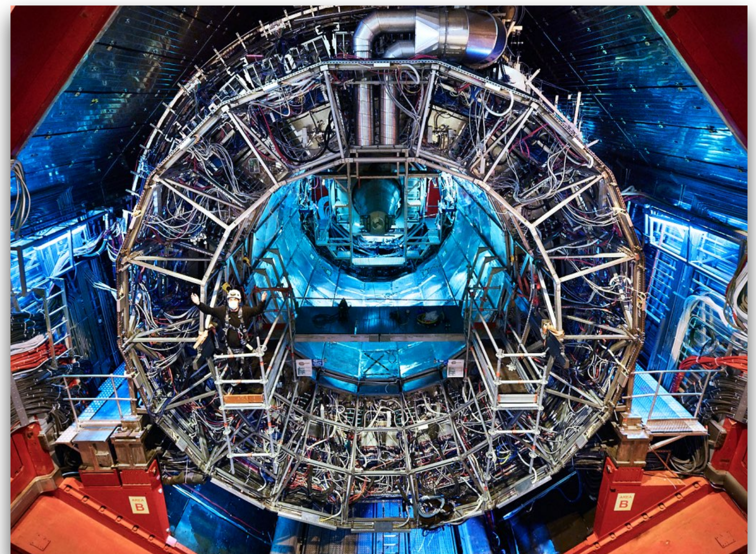
 $= \sum_j c_j \mathcal{D}_j$

An operator description of BOTH the observable and the states

Detectors in QFT and the Experiment

Correlation Functions describe the detector cells at infinity on the celestial sphere





$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n) \mid \Psi \rangle$$

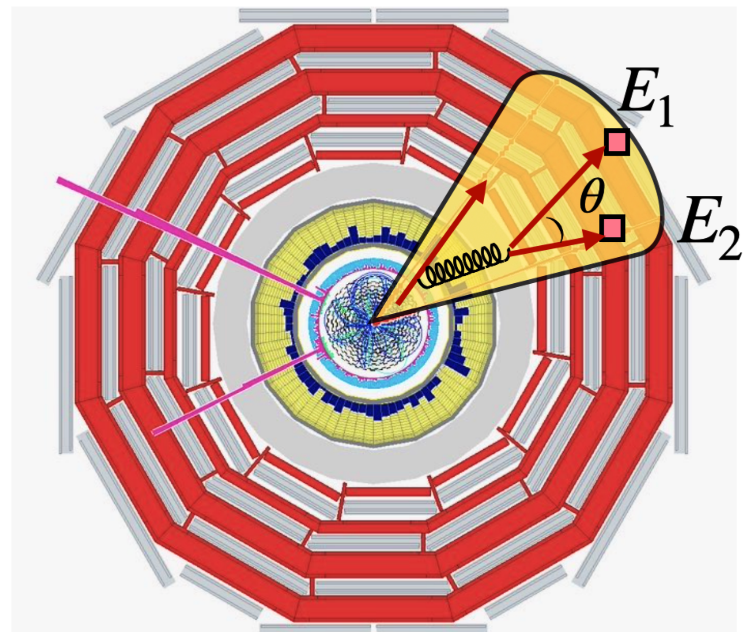
New Observable: Energy-Energy Correlator (EEC)

- Weighted cross section by the energy of particles inside the jet

Energy weight

$$\frac{d\sigma_{EEC}}{d\theta} = \sum_{i,j} d\sigma \frac{2E_iE_j}{Q^2\sigma_{\text{tot}}}\delta(\cos\theta - \cos\theta_{ij})$$

Angular distance between particles i and j inside the jet



- Generally can study EEC for any angular distance θ_{ij}
- For jet substructure take the limit $\theta_{ij} \to 0$

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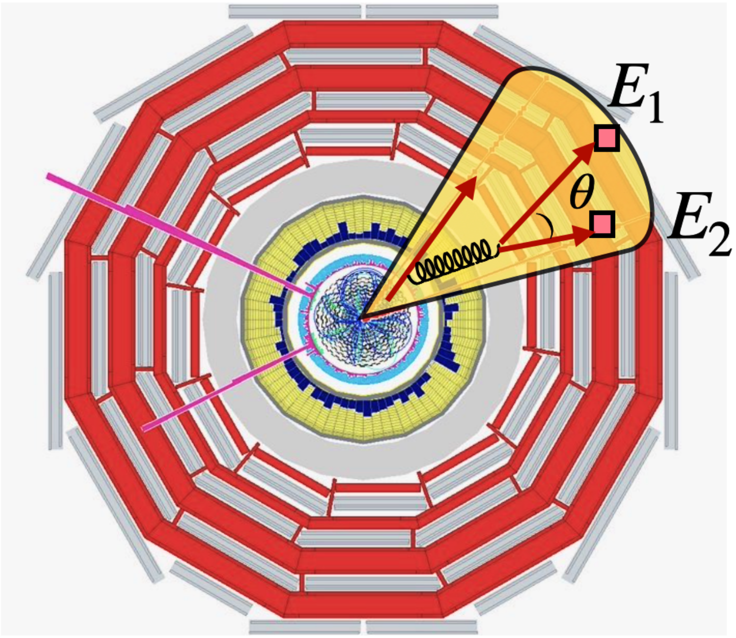

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Energy Flow Inside the Jet

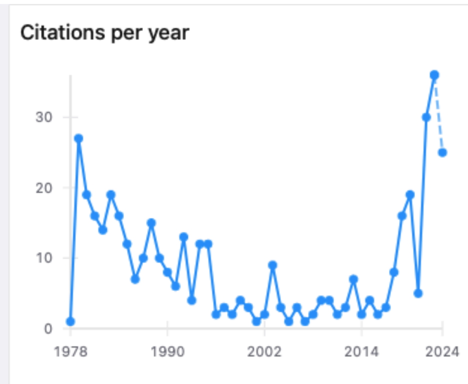
Correlation functions of the energy flow operators $\left\langle \varepsilon\left(\vec{n}_1\right)\cdots\varepsilon\left(\vec{n}_n\right)\right\rangle$ characterize the final state hadrons in QCD

Energy Correlations in electron - Positron Annihilation: Testing QCD

C.Louis Basham (Washington U., Seattle), Lowell S. Brown (Washington U., Seattle), Stephen D. Ellis (Washington U., Seattle), Sherwin T. Love (Washington U., Seattle)

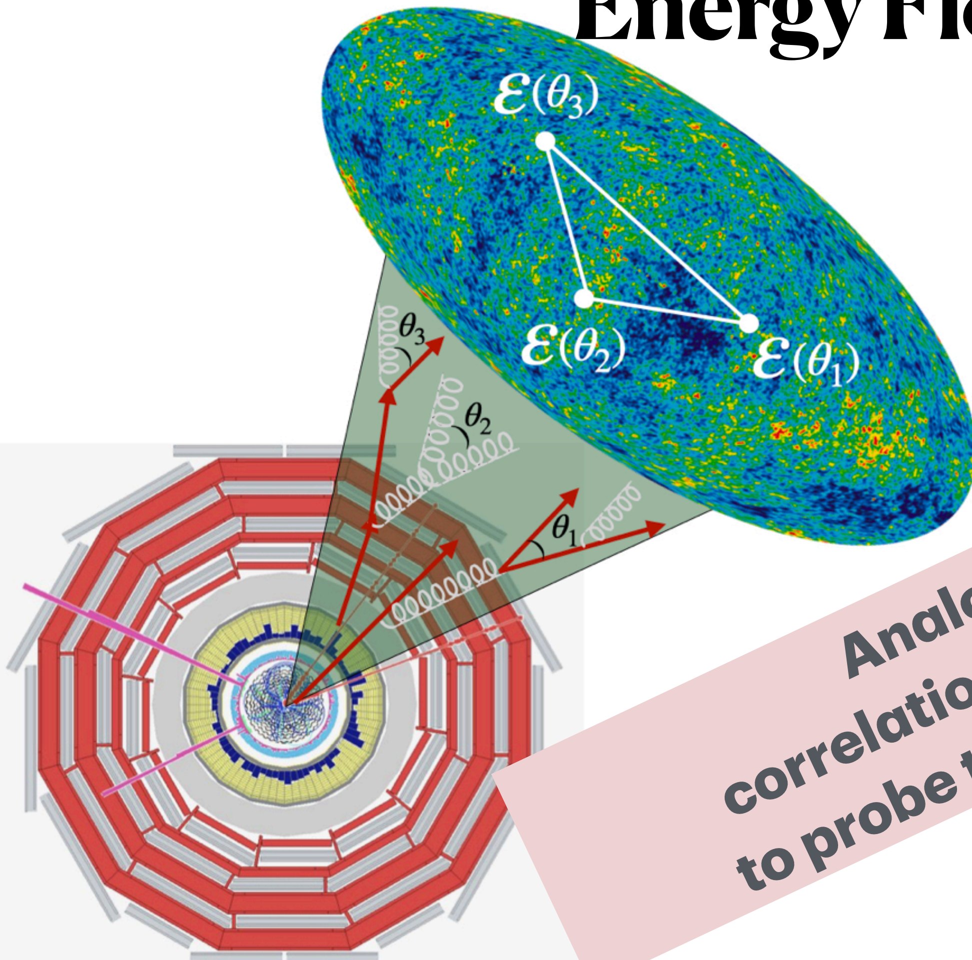
Aug, 1978

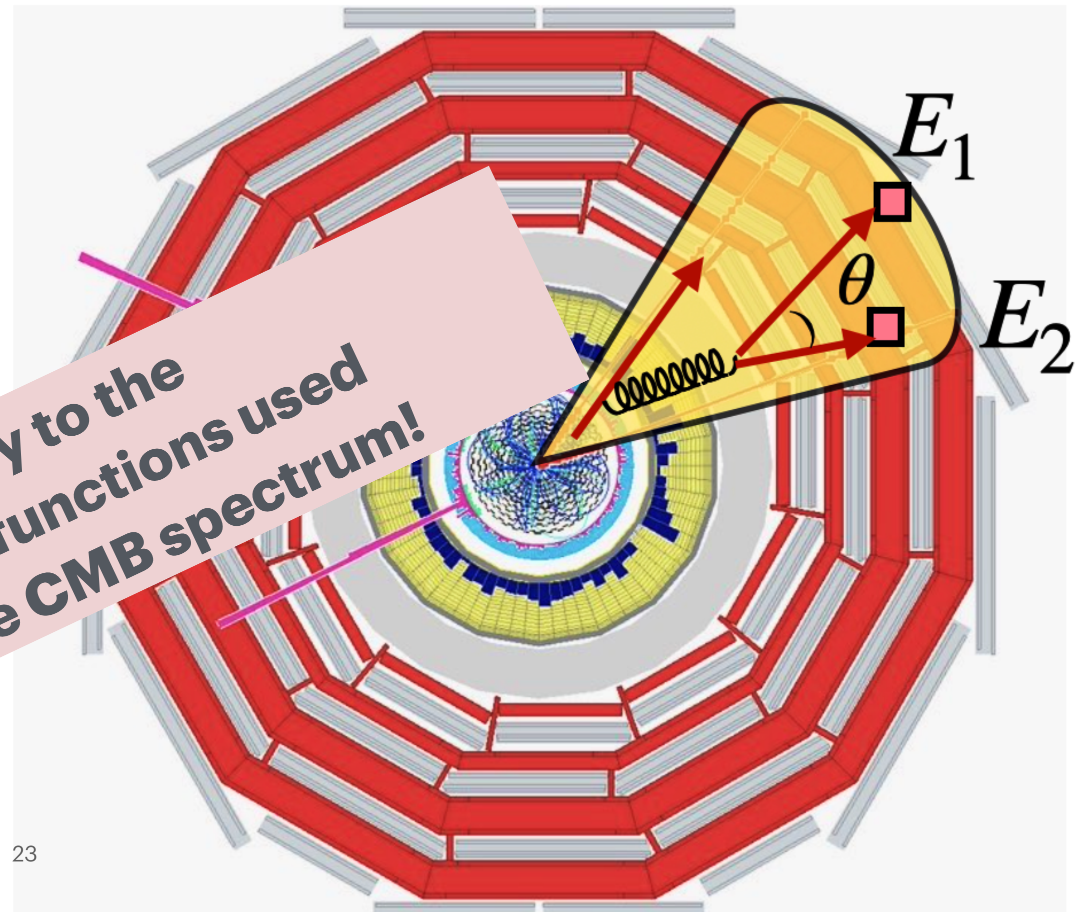
13 pages
Published in: *Phys.Rev.Lett.* 41 (1978) 1585
DOI: 10.1103/PhysRevLett.41.1585
Report number: RLO-1388-759
View in: OSTI Information Bridge Server, ADS Abstract Service



Abstract: (APS)
An experimental measure is presented for a precise test of quantum chromodynamics. This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process e+e−→hadrons at energy W. It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order 1W2) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

Energy Flow inside a Jet



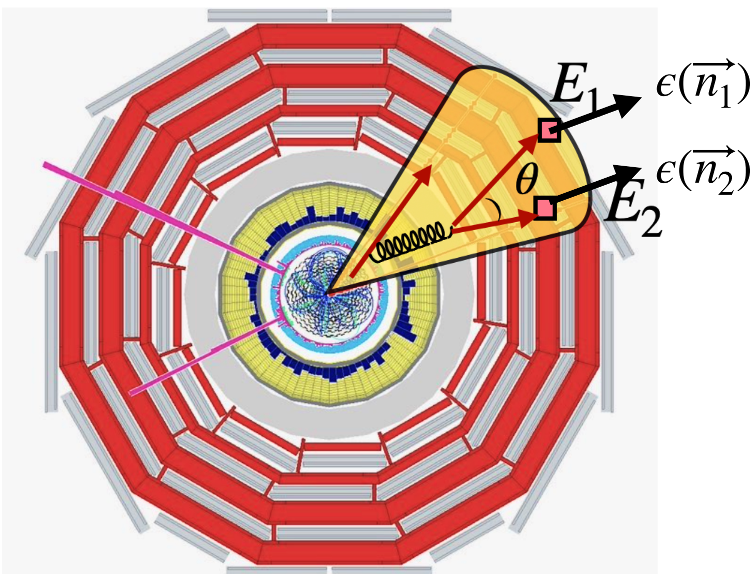


Analogy to the correlation functions used to probe the CMB spectrum!

New Observable: Energy-Energy Correlator (EEC)

- **The EEC can be written in terms of QFT operators as the expectation value of the correlation functions of the energy flux operator $\epsilon(\vec{n})$ inside the jet.**

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^{\infty} dt\ r^2 n^i T_{0i}(t, r\vec{n})$$



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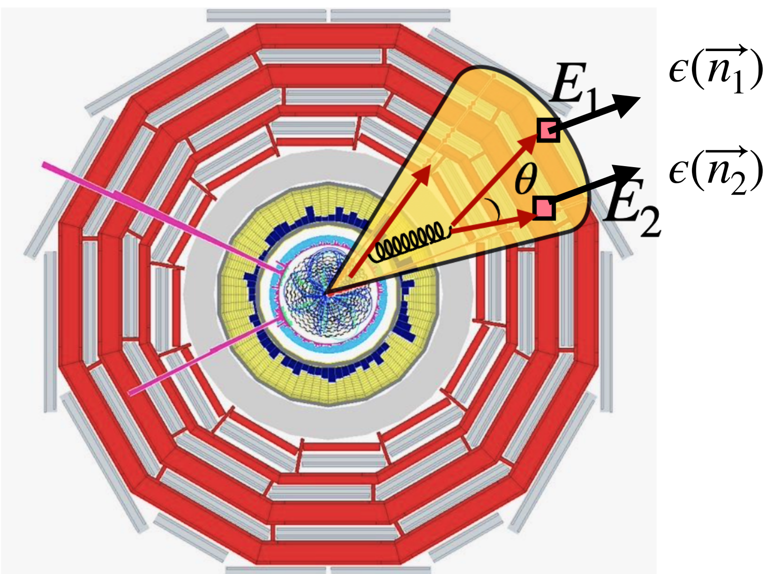
- **Energy weighted ⇒ IR safe**
- **Most importantly: calculable in pQCD even for small angle θ (as long as $p_t\theta$ is still a perturbative scale)**

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\; r^2 n^i T_{0i}(t, r\vec{n})$$

Energy momentum tensor



Weighted cross section

$$\text{EEC} = \sum_{i,j} d\sigma \frac{2E_iE_j}{Q^2\sigma_{\text{tot}}}\delta(\cos\theta_{ij} - \cos\theta)$$

Correlation functions

$$\text{EEC} = \langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle$$

[Hofman, Maldacena *JHEP* 05 (2008) 012]

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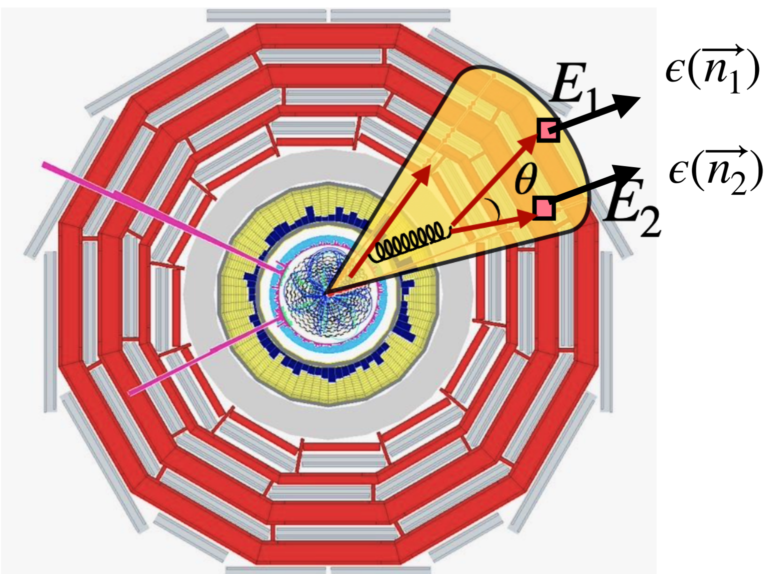

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Correlation functions

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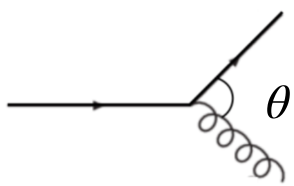
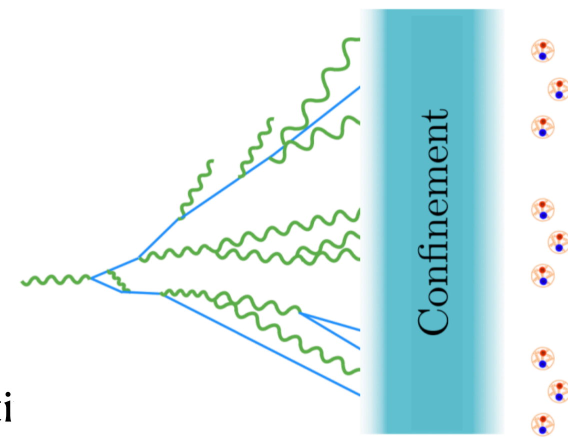
Jet state

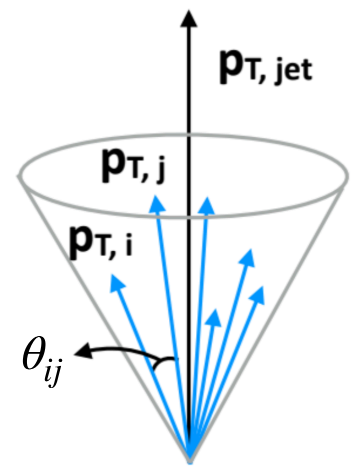
Scan the Physics in different scale regimes

- The observable preserves the angular correlation between particles in the jet
- Probe fixed scale $p_T\theta$ for fixed angle θ

virtuality $\sim p_T\theta$



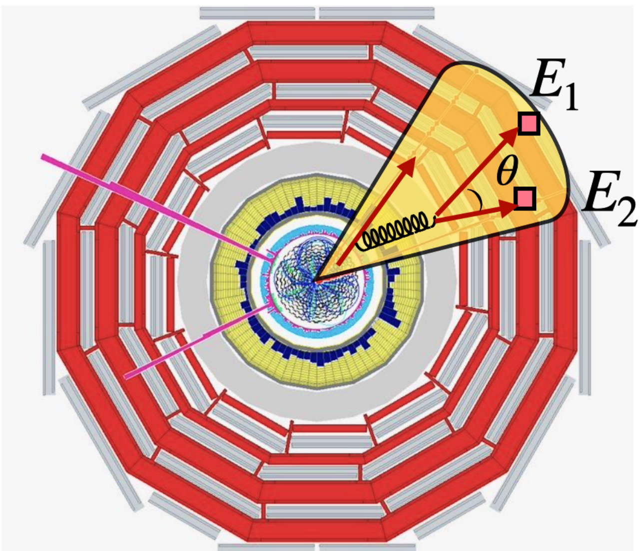




- Scan for different scales both in the perturbati perturbative regime
- At colliders $\sqrt{s} \gg m \rightarrow$ Conformal Field Theory (CFT)
-

The Small Angle Limit

- **Energy correlators inside high energy jets at the LHC $\Rightarrow$small angle limit**



$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \overset{\theta \ll}{=} \sum \theta^{\gamma_i} \mathcal{O}_i$$

- **Energy correlators admit a simplified Operator Product Expansion (OPE)**

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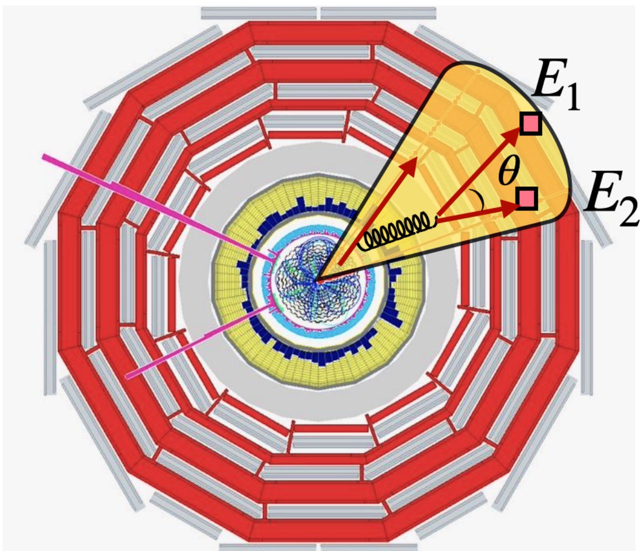

$$\stackrel{\theta \ll}{=} \sum \theta^{\gamma_i} \mathcal{O}_i$$

QCD operators

$\Rightarrow$ Use LHC jets to test the leading QCD operators in this expansion

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle$$

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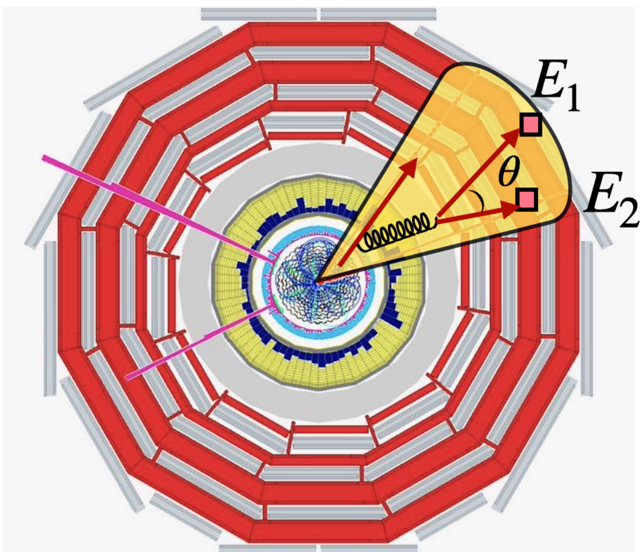


$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \overset{\theta \ll}{=} \sum \theta^{\gamma_i} \mathcal{O}_i$$

Scaling parameter (γ_i)

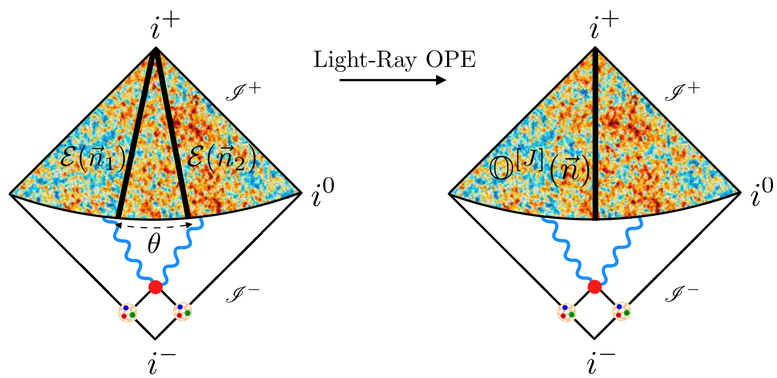
QCD operators ($\mathcal{O}_i$)

⇒ Use LHC jets to test the leading QCD operators in this expansion

- **Energy correlators admit a simplified Operator Product Expansion (OPE)**

Scaling Behavior

- **Energy correlators inside high energy jets at the LHC $\Rightarrow$small angle limit**



- **Energy correlators admit an Operator Product Expansion (OPE):**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

$\Rightarrow$ Use LHC jets to test the leading QCD operators in this expansion

On the more formal aspect…

Observables in CFT are used to describe data at hadron colliders

→take full advantage of the progress in formal field theory

Conformal collider physics:
Energy and charge correlations

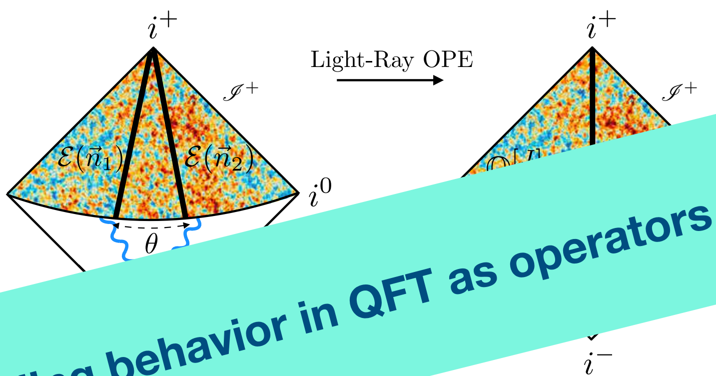
Diego M. Hofman[a] and Juan Maldacena[b]

[a] *Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA*
[b] *School of Natural Sciences, Institute for Advanced Study*
Princeton, NJ 08540, USA

- Can we relate the asymptotic data at colliders to underlying properties of the theory
  - Coupling constants, transport coefficients, particle spectrum….?
  - What is the space of observables at null infinity?

Scaling Behavior

- **Energy correlators inside high energy jets at the LHC $\Rightarrow$small angle limit**



Universal scaling behavior in QFT as operators are brought together!

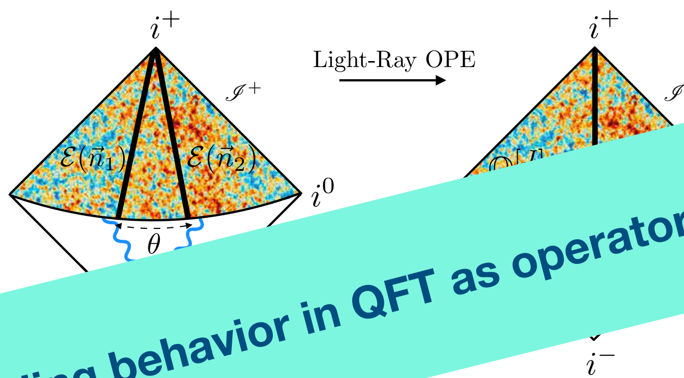
- **Energy ... an Operator Product Expansion (OPE):**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

$\Rightarrow$ Use LHC jets to test the leading QCD operators in this expansion

Scaling Behavior

- **Energy correlators inside high energy jets at the LHC ⇒small angle limit**

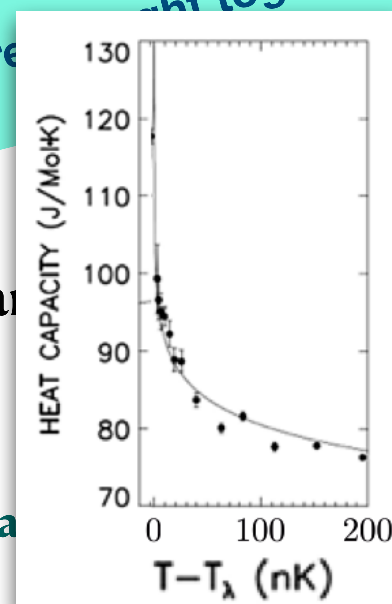


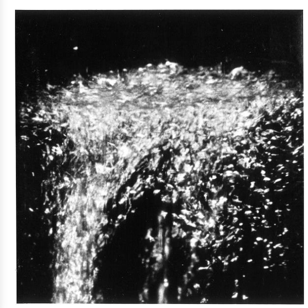
Universal scaling behavior in QFT as operators are ...ght together!

- **Energy ... an Operator Product Expan...**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$

⇒ Use LHC jets to test the leading QCD opera... expansion

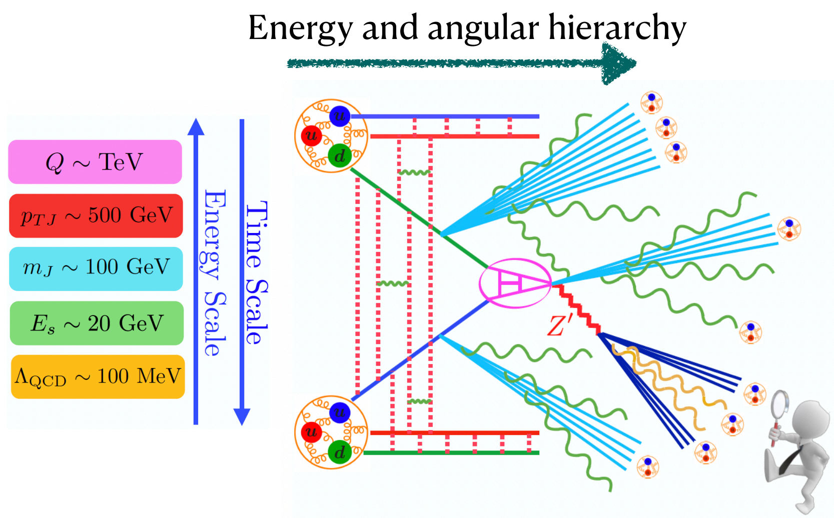




Energy Correlators for Hadronic Final States at the LHC

Perturbative calculations

- Collider physics processes involve multiple scales $\Rightarrow$ theory calculation separates the scales through factorization theorems
- Use effective field theory methods to compute the EEC correlation function



Phys.Rev.D 63 (2001) 114020,
Phys.Rev.D 66 (2002) 014017,
Nucl.Phys.B 594 (2001) 371-419
JHEP 10 (2016) 125
JHEP 03 (2017) 146

Perturbative calculations: factorization

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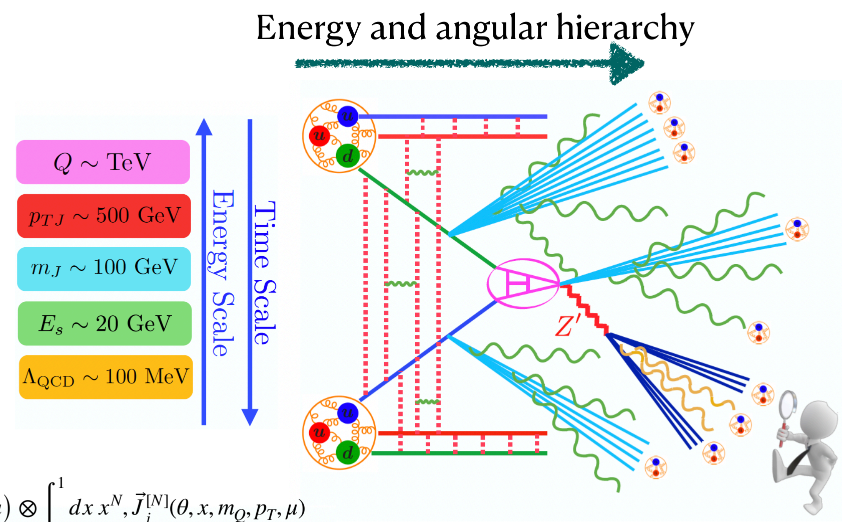
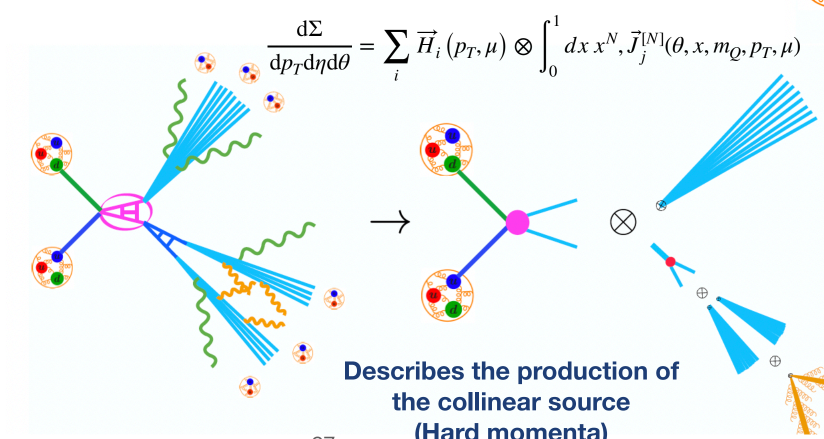


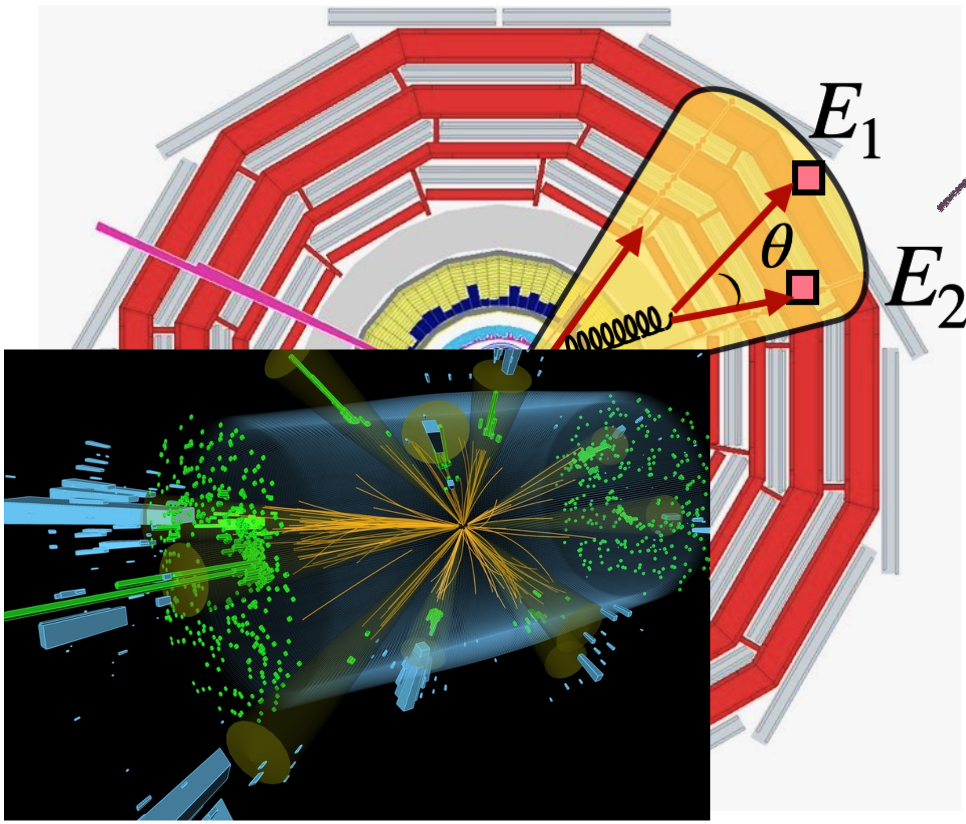
$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle =$$



Describes the dependance on the observable (Collinear kinematics)

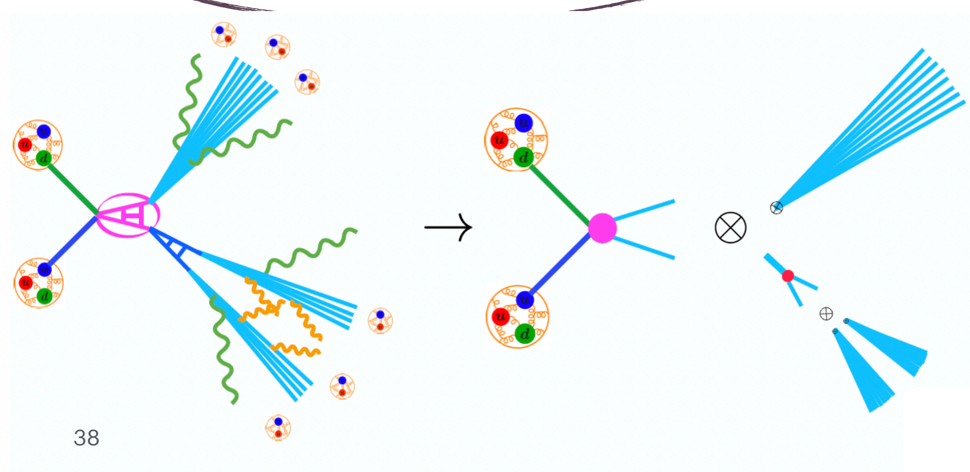
The EFT of Jet Substructure

- The **EFT of Jet Substructure** is an operator based formalism that allows for **systematically improvable** calculations exploiting separation of scales.

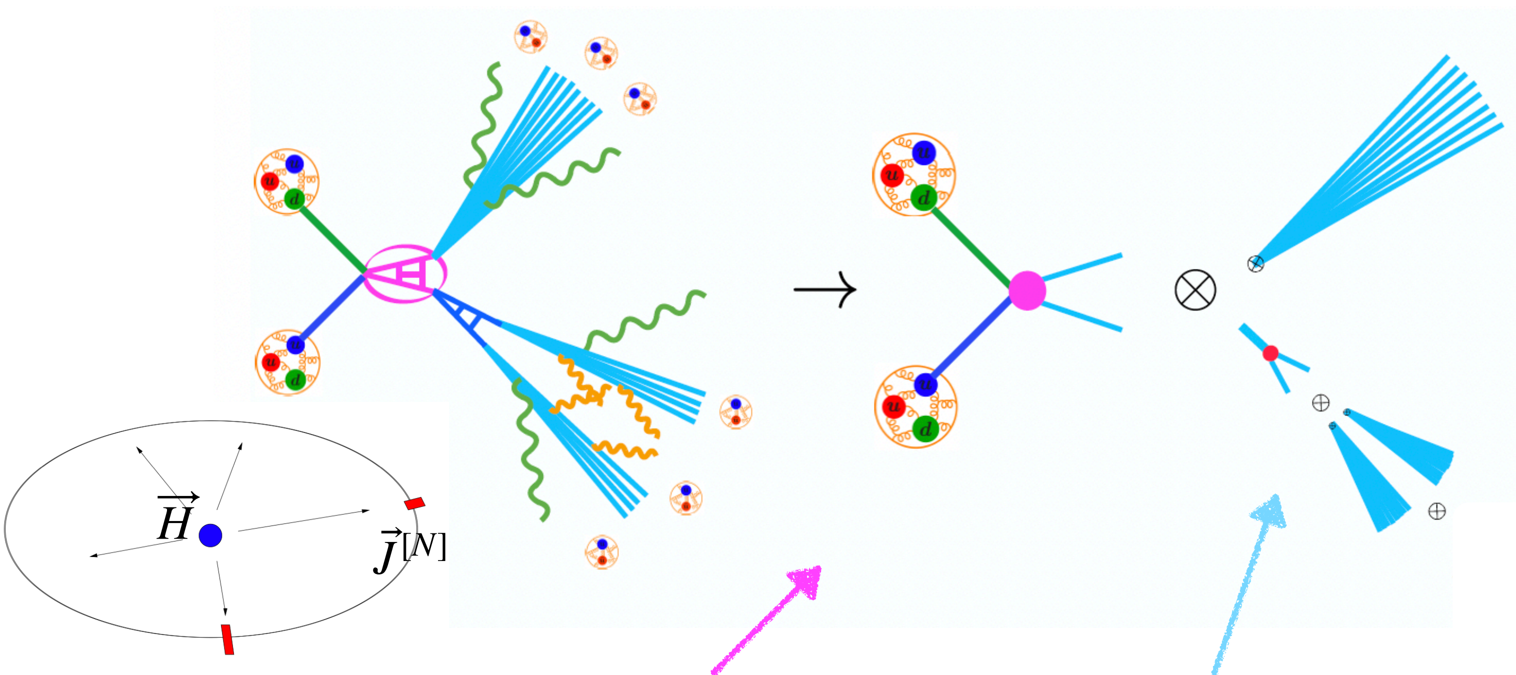


$$\text{EEC} = \sum_{i,j} d\sigma \frac{2E_iE_j}{Q^2\sigma_{\text{tot}}}\delta(\cos\theta - \cos\chi)$$

$$\text{EEC} = \langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle$$



Redefining Jet Substructure with Energy Correlators



$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}z} = \sum_i \vec{H}_i \left(p_T, \eta, \mu\right) \otimes \int_0^1 dx\, x^N, \vec{J}_j^{[N]}(z, x, m_Q, p_T, \mu)$$

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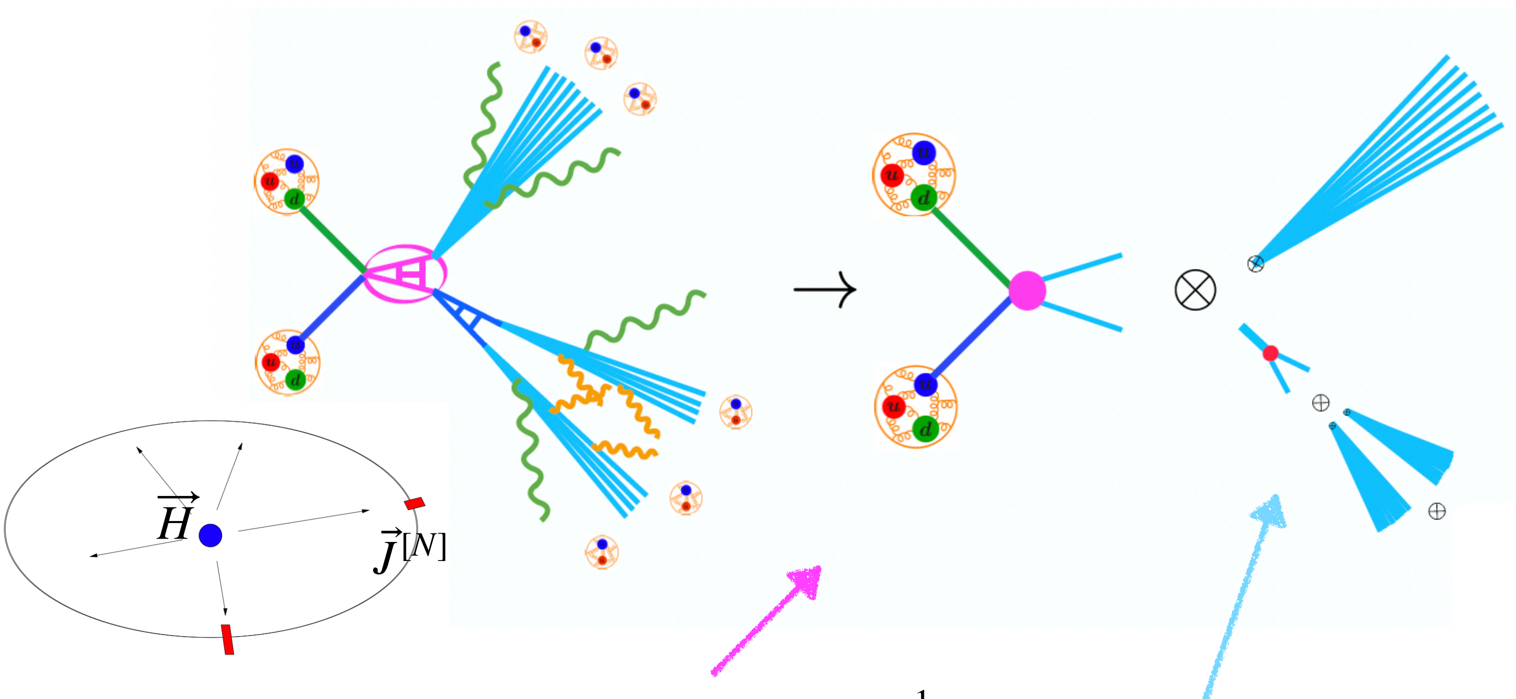
Hard Function **Jet Function**

Redefining Jet Substructure with Energy Correlators



Three scales:

$\mu_H \sim p_T$

$\mu_J \sim p_T R$

m_Q

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}z} = \sum_i \vec{H}_i\left(p_T, \eta, \mu\right) \otimes \int_0^1 dx\, x^N, \vec{J}_j^{[N]}(z, x, m_Q, p_T, \mu)$$

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Hard Function **Jet Function**

Factorization theorem

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T\mathrm{d}\eta\mathrm{d}z} = \sum_i \vec{H}_i\left(p_T, \eta, \mu\right) \otimes \int_0^1 dx\, x^N, \vec{J}_j^{[N]}(z, x, m_Q, p_T, \mu)$$

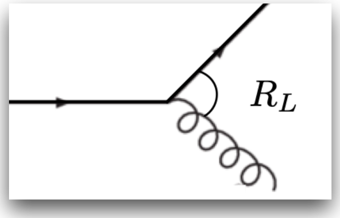
$$\vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) = \left\{\begin{array}{l} \vec{J}_g^{[N]}\left(R_L, x, m_Q, \mu\right) \\ \vec{J}_q^{[N]}\left(R_L, x, m_Q, \mu\right) \\ \vec{J}_Q^{[N]}\left(R_L, x, m_Q, \mu\right) \end{array}\right\}$$

$$\vec{H}\left(x, p_T^2, \mu\right) = \left\{\begin{array}{l} H_g\left(x, p_T^2, \mu\right) \\ H_q\left(x, p_T^2, \mu\right) \\ H_Q\left(x, p_T^2, \mu\right) = H_q\left(x, p_T^2, \mu\right) \end{array}\right\}$$

Factorization theorem

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}z} = \sum_i \vec{H}_i\left(p_T, \eta, \mu\right) \otimes \int_0^1 dx\, x^N, \vec{J}_j^{[N]}(z, x, m_Q, p_T, \mu)$$

$$\vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) = \begin{Bmatrix} \vec{J}_g^{[N]}\left(R_L, x, m_Q, \mu\right) \\ \vec{J}_q^{[N]}\left(R_L, x, m_Q, \mu\right) \\ \vec{J}_Q^{[N]}\left(R_L, x, m_Q, \mu\right) \end{Bmatrix}$$



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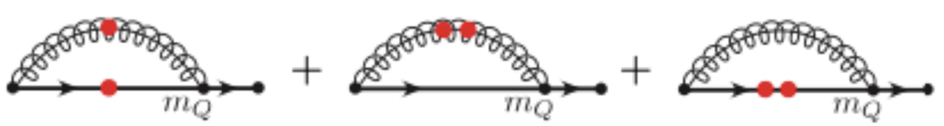
Heavy Quark Jet Function

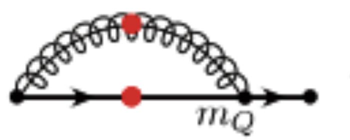
$$J_Q^{[N]}\left(R_L, m_Q\right) = \sum_X \sum_{i_1,i_2,\ldots,i_N\in X} \left\langle 0 \left|\bar{\chi}_n\right| X\right\rangle \frac{E_{i_1}E_{i_2}\cdots E_{i_N}}{p_T^N}\Theta\left(\max\left\{\theta_{ij}\right\} < R_L\right)\left\langle X\left|\chi_n\right| 0\right\rangle$$

$$W_{n_i}^{(A)}(x) = P\,\exp\left[ig_A\, t_A^a \int_{-\infty}^{0} ds\, \bar{n}_i\cdot A_{n_i}^a(x+s\bar{n}_i)\right] \qquad \chi_{n_i}(x) = \frac{\not{n}_i\bar{\not{n}}_i}{4}\, W_{n_i}^{\dagger}(x)\,\psi(x)$$

At $\mathcal{O}(\alpha_s)$ the jet function is describes by the one-loop $1\to 2$ splitting of a quark weighted by the energy of each particle in the loop



Heavy Quark Jet Function



$$\delta = \frac{im_Q}{p_T R_L}$$

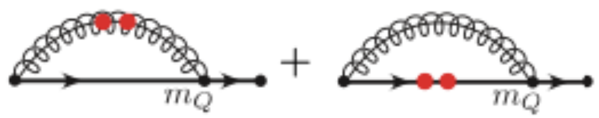
$$J_Q^{[N]}(R_L, m_Q)|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\int dx \frac{2(1-(1-x)^N - x^N)\left[2x^3 + (1+x^2)(x+\delta)(x+\bar{\delta})\ln\frac{\delta\bar{\delta}}{(x+\delta)(x+\bar{\delta})}\right]}{(-1+x)(x+\delta)(x+\bar{\delta})}$$

$$J_Q^{[2]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\delta^4 - 4\delta^3 + 2\delta^2 - 3\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(9\delta^2 + \frac{31}{6}\right)\right\} + c.c\,,$$

$$J_Q^{[3]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\frac{3}{2}\delta^4 - 6\delta^3 + 3\delta^2 - \frac{9}{2}\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(\frac{27}{2}\delta^2 + \frac{31}{4}\right)\right\} + c.c\,,$$

$$J_Q^{[4]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\frac{2}{3}\delta^6 - \frac{16}{5}\delta^5 - \delta^4 - \frac{20}{3}\delta^3 + 4\delta^2 - \frac{83}{15}\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(\frac{106}{15}\delta^4 + \frac{74}{5}\delta^2 + \frac{1417}{150}\right)\right\} + c.c\,,$$

Heavy Quark Jet Function



$$J_Q^{[2]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-3\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{M^2}\right]-\frac{49}{6}\right\},$$
$$J_Q^{[3]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-\frac{9}{2}\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{M^2}\right]-\frac{47}{4}\right\},$$
$$J_Q^{[4]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-\frac{83}{15}\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{M^2}\right]-\frac{6611}{450}\right\},$$

$$\delta = \frac{i m_Q}{p_T R_L}$$

- **Mass regulates IR divergences!**
- **The remaining** $\frac{1}{\epsilon}$ **poles are UV poles regulated by renormalization.**

Comparison with massless jet functions

Two-point energy-energy correlator (EEC)

The mass should not affect the UV behavior of the jet function.

This can be seen from comparing the UV poles with the light quark jet function.

$$J_Q^{EEC}(z, M, \mu) = \delta(z)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\left(\frac{1}{\epsilon_{\mathrm{UV}}} + \ln\frac{\mu^2}{M^2}\right) - \frac{19}{6}\right]\right) + \text{finite terms}$$

$$z = \frac{1 - \cos\theta_{ij}}{2}$$

$$J_q^{EEC} = \delta(z) + \frac{\alpha_s C_F}{4\pi}\left[\delta(z)\left(-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{37}{3}\right) + 3\frac{Q^2}{\mu^2}\mathscr{L}_0\left(\frac{Q^2}{\mu^2}z\right)\right]$$

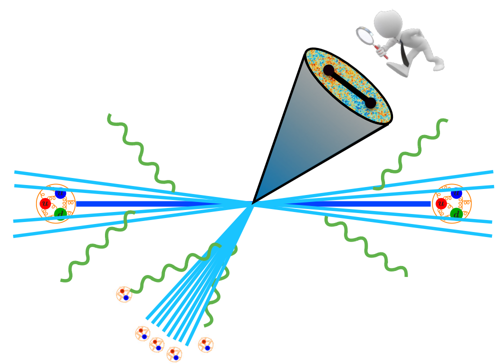
$$\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} = \begin{pmatrix} \frac{25}{6}C_F & -\frac{7}{15}n_f \\ -\frac{7}{6}C_F & \frac{14}{5}C_A + \frac{2}{3}n_f \end{pmatrix}$$

Heavy quark jet function

Result

$$J_Q^{EEC}(z,M,\mu) = \delta(z)\left(1+\frac{\alpha_s C_F}{4\pi}\left[-\left(\gamma_{qq}^{(0)}(3)+\gamma_{gq}^{(0)}(3)\right)\left(\frac{1}{\epsilon_{\rm UV}}+\ln\frac{\mu^2}{M^2}\right)-\frac{19}{6}\right]\right)$$

$$+\frac{\alpha_s C_F}{\pi}\frac{1}{z}\left[\frac{3}{4}-\frac{5}{2}\delta^2-\frac{\delta^4}{1+\delta^2}+3\delta^3\arctan\left(\frac{1}{\delta}\right)+\frac{1}{2}\delta^2\left(1-\delta^2\right)\ln\frac{\delta^2}{1+\delta^2}\right]$$



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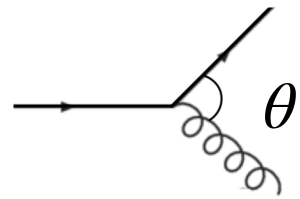
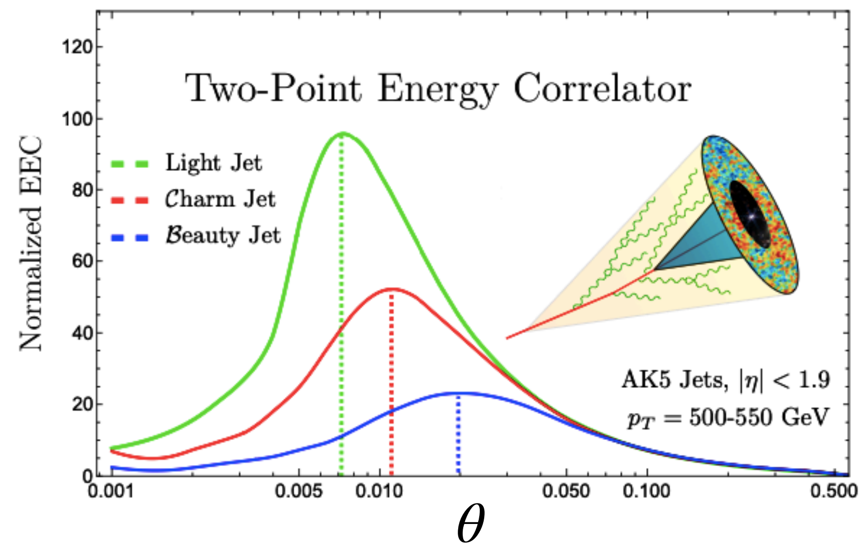
Sensitivity to intrinsic dynamical scales



Virtuality $\sim p_T\theta + m_Q^2$

arXiv 2210.09311

- **Formation time changes with the mass of the quark.**
- **The turnover dependent on the quark mass**
- **At large angles the mass effects become irrelevant: larger effective scale so particles are effectively massless**



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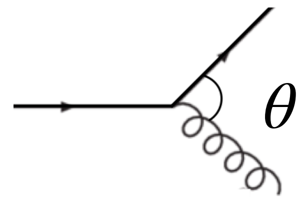

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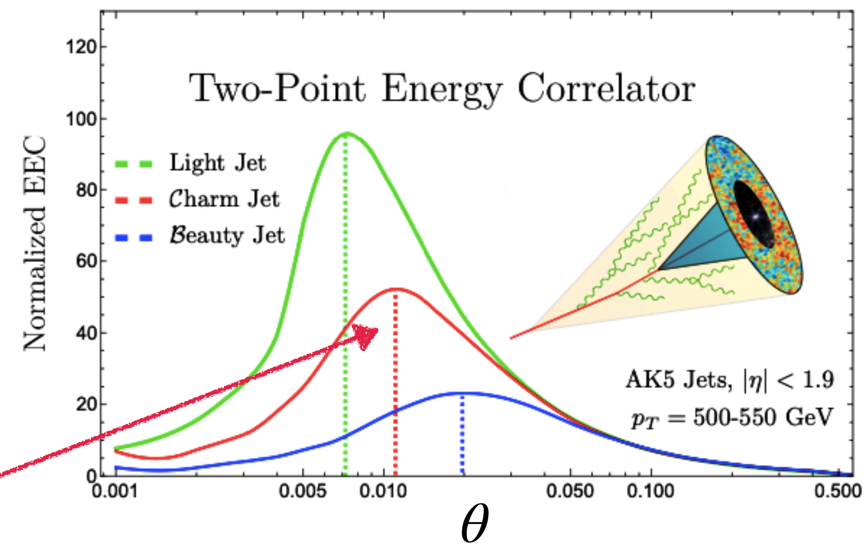
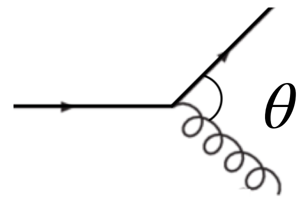
- Turn-over scale $\theta \sim m_Q/P_T$ (pQCD calculable for massive quarks)



$\theta \sim m_Q/P_T$

Sensitivity to intrinsic dynamical scales

- **Formation time changes with the mass of the quark.**
- **The turnover dependent on the quark mass**
- **At large angles the mass effects become irrelevant: larger effective scale so particles are effectively massless**
- **Turn-over scale $\theta \sim m_Q/P_T$ (pQCD calculable for massive quarks)**
- **For light quarks the run-over is related to the non-perturbative scale**



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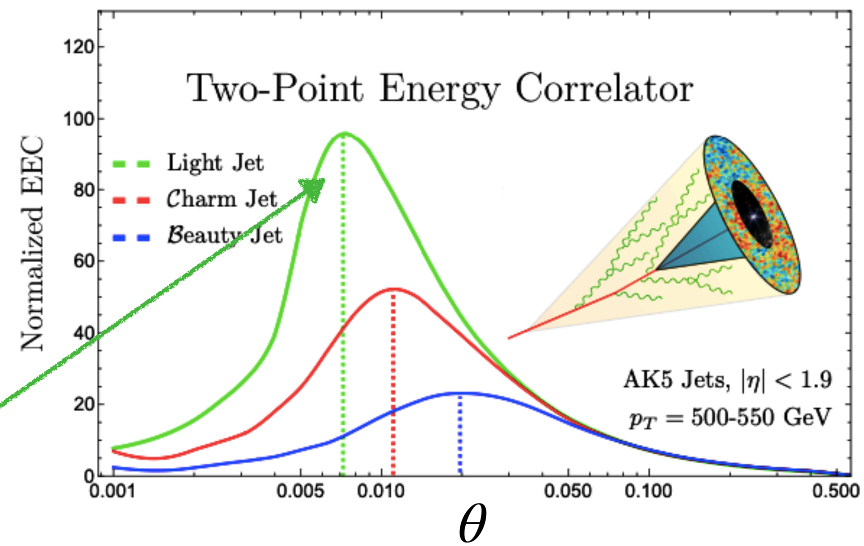
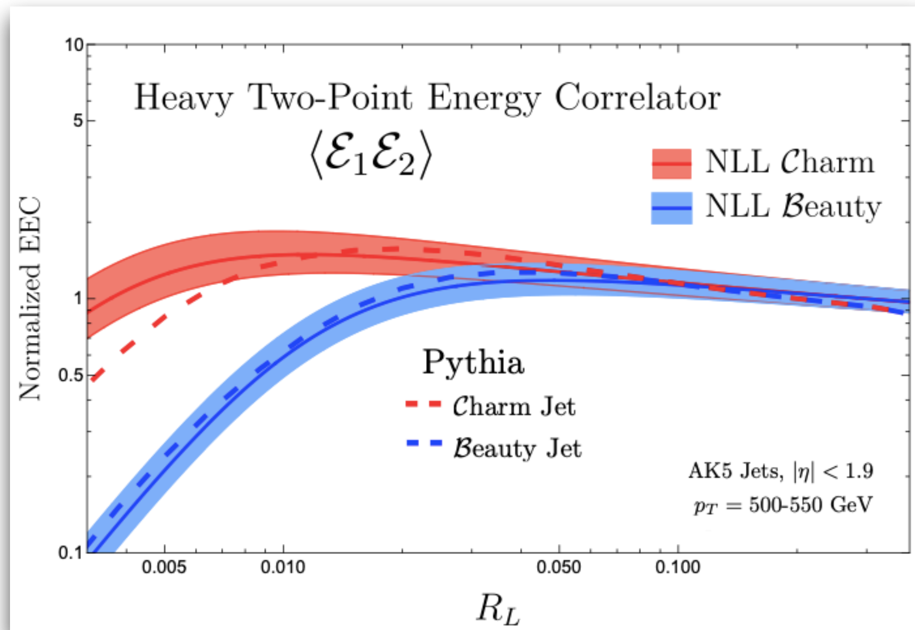


$\theta \sim \Lambda_{QCD}/P_T$

Massive two point correlator

A massive jet substructure observable at NLL

- **Scaling behaviour identical to massless case for larger scales.**
- **A turn-over for $R_L \to m_Q/p_T$**
- **The change in the slope is perturbative effect contrary to massless jets:** $R_L \to \Lambda_{QCD}/p_T$
- **The turn-over region is of interest for improving heavy quark description in parton shower.**



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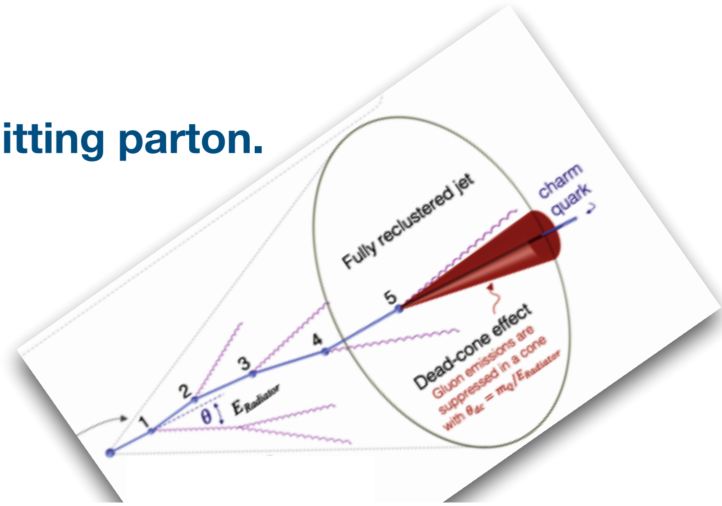
arXiv 2210.09311

Dead-cone effect in QCD

Fundamental phenomena

- **Parton-shower pattern depends on the mass of the emitting parton.**
- **Angular suppression** $\propto \frac{M}{E}$.



Observable used for the observation of the dead-cone effect in LHC data

$$R(\theta) = \frac{1}{N^{\mathrm{D^0\,jets}}} \frac{\mathrm{d}n^{\mathrm{D^0\,jets}}}{\mathrm{d}\ln(1/\theta)} \Big/ \frac{1}{N^{\mathrm{inclusive\,jets}}} \frac{\mathrm{d}n^{\mathrm{inclusive\,jets}}}{\mathrm{d}\ln(1/\theta)} \Bigg|_{k_\mathrm{T}, E_\mathrm{Radiator}}$$

- **Can we observe the dead-cone with EEC?**

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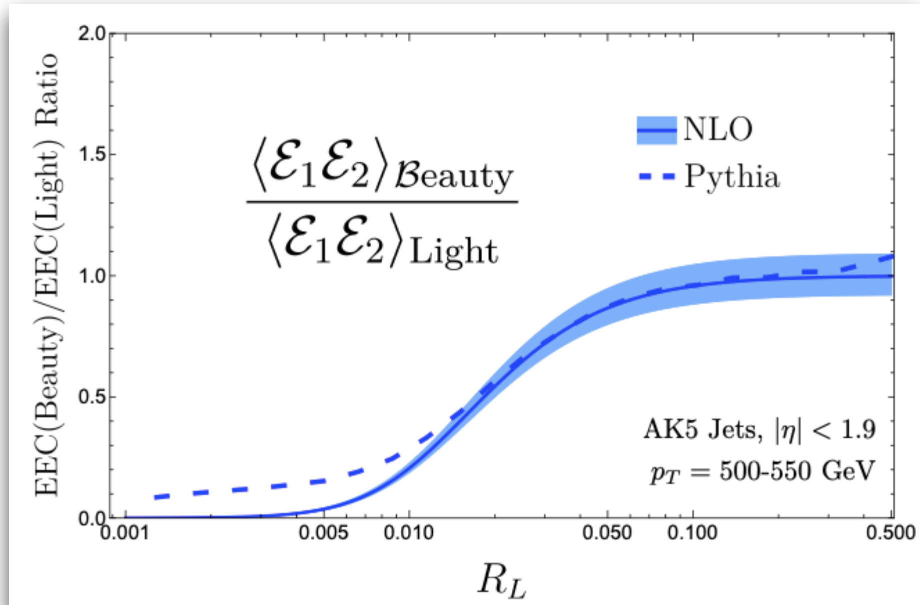
Direct observation of the dead-cone effect in quantum chromodynamics

ALICE Collaboration

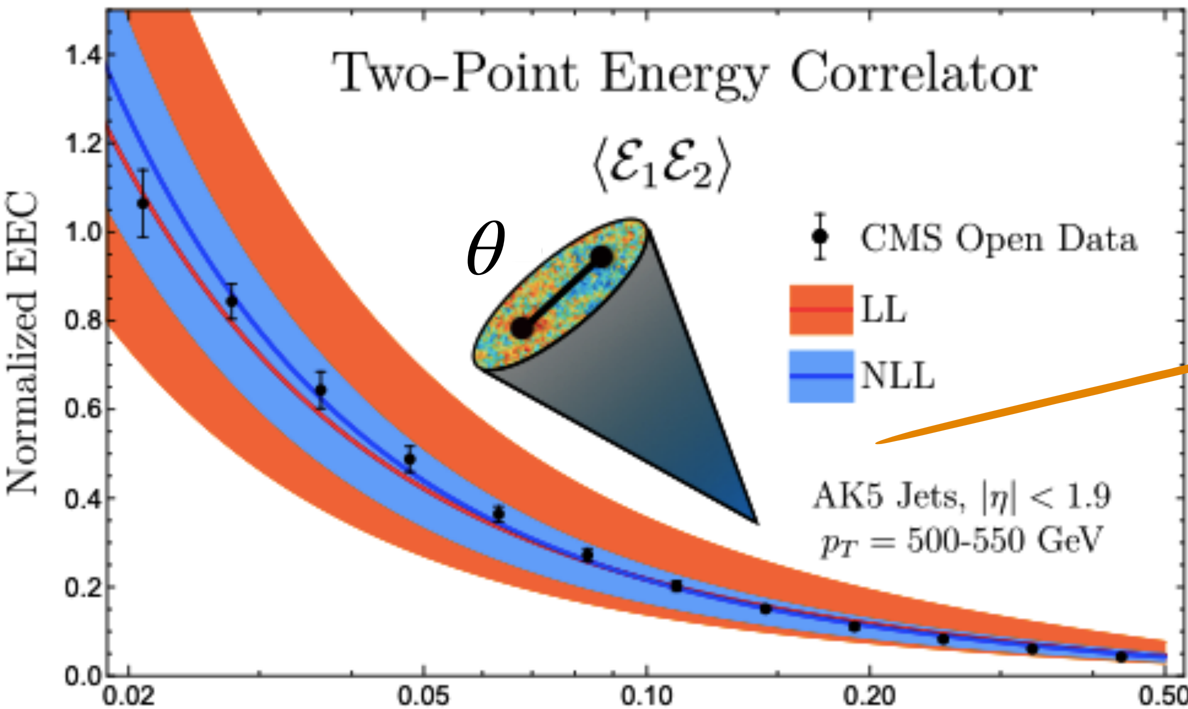
Intrinsic mass effects



arXiv 2210.09311

- Ratios of the massive and massless EEC isolate mass (IR) effects.
- A transition region related to the quark mass: perturbatively calculable.
- Excellent agreement with MC.
- Small angle suppression can be interpreted as a dead-cone effect.

Data supports the theory calculation



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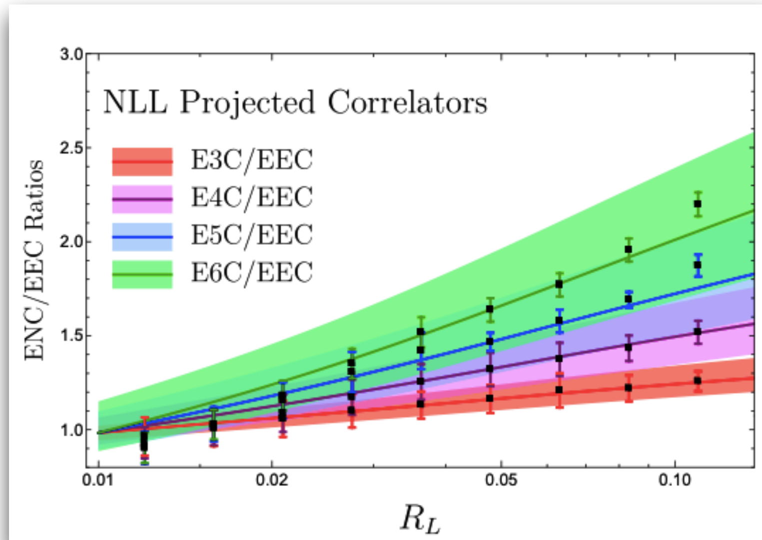
$$\lim_{\theta \to 0} \frac{d\Sigma}{dp_T d\theta} \sim \sum \theta^{\gamma_i} \mathcal{O}_i$$

Probe the anomalous dimensions of the operators ⇒ Firsthand test of the theory from collider data!

The angle between any two particles inside the jet

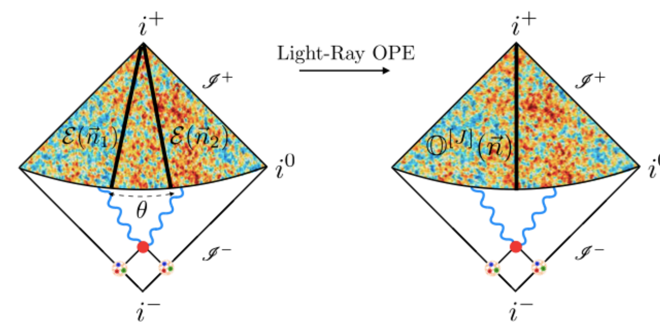
Higher point correlators



The light-ray OPE

- The leading scaling behavior at the LHC is described by the leading terms in the OPE: **twist two light-ray operators**.
- Light-ray OPE is a rigorous and convergent expansion in CFT.

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle = \sum c_i \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$



$$\langle \varepsilon\left(\vec{n}_1\right) \varepsilon\left(\vec{n}_2\right) \cdots \varepsilon\left(\vec{n}_k\right) \rangle = \frac{1}{R_L^2} \left\{ f_q^{[k]}\left(u_i, v_i\right) \mathbb{O}_q^{[k+1]}\left(\vec{n}_1\right) + f_g^{[k]}\left(u_i, v_i\right) \mathbb{O}_g^{[k+1]}\left(\vec{n}_1\right) \right\} + \mathcal{O}\left(R_L^0\right)$$

$$u_i = \left(\frac{x_{i_1 i_2} x_{i_3 i_3}}{x_{i_1 i_3} x_{i_2 i_4}}\right)^2 \qquad v_i = \left(\frac{x_{i_1 i_2} x_{i_3 i_4}}{x_{i_1 i_4} x_{i_2 i_3}}\right)^2$$

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ \left(iD^+\right)^{J-1} \psi,$$

$$\vec{\mathbb{O}}^{[J]} = \left(\mathbb{O}_q^{[J]}, \mathbb{O}_g^{[J]}\right)^T = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{\mathcal{O}}^{[J]}(t, r\vec{n})$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} \left(iD^+\right)^{J-2} F_a^{\mu+}$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

Leading twist light-ray OPE

Control scaling at leading power

- Twist-2 operators in QCD are characterized by a spin J and transverse spin j=0,2.
- They can be transformed to a twist-2 light-ray operator vector parametrized by J

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$$\mathcal{O}_{\bar{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}\left(iD^+\right)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

$$\xrightarrow{\lim_{r\to\infty} r^2\int_0^\infty dt}$$

$$\vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{bmatrix} \mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n}) \\ \mathbb{O}_{\bar{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\bar{g},-}^{[J]}(\vec{n}) \end{bmatrix}$$

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Unpolarized

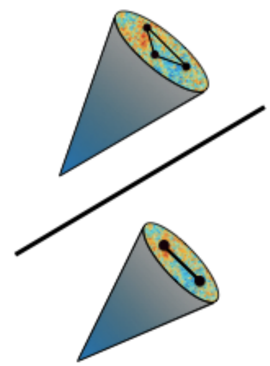
Polarized

Unpolarized Scaling

LHC scenario

- **Probe the unpolarized spin $j = 0$ operators**
- **The leading scaling behavior is determined by the anomalous dimension $\gamma(N+1)$ for an operator of spin $N+1$.**

$\rightarrow$ can isolate the anomalous dimensions!

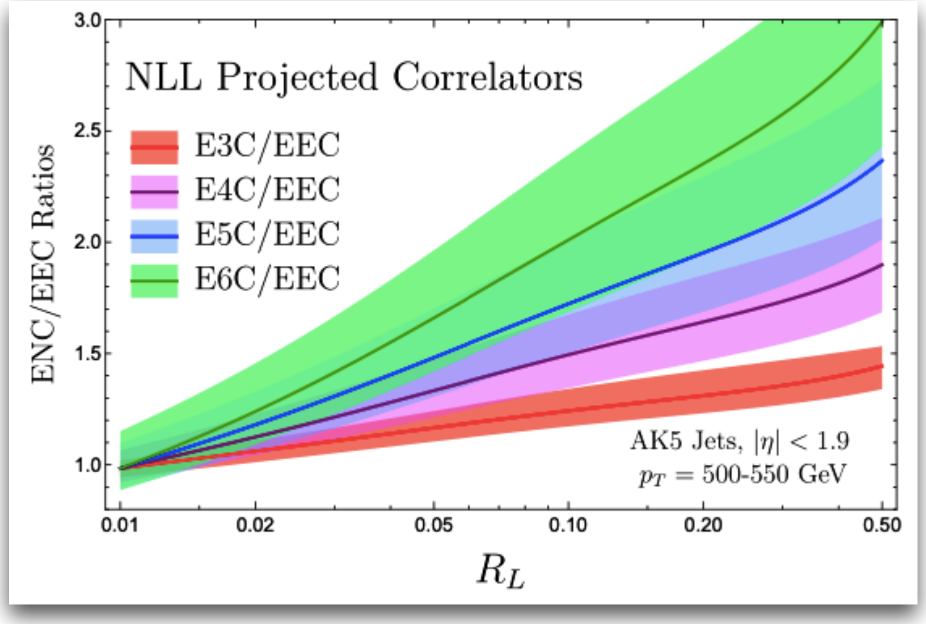


$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim \frac{\langle \mathbb{O}^{[J]} \rangle}{\langle \mathbb{O}^{[3]} \rangle}$$

The jet spectrum

Higher-point correlators

- **Asymptotic energy flux directly probes the spectrum of (twist-2) lightray operators at the quantum level**
- **Ratio of the higher-point correlators with the two-point isolates anomalous scaling!**
- **The anomalous scaling behavior depends on N (slope increases with N)**
- **First hand probe of the anomalous dimensions of QCD operators.**



[Lee, BM, Moult]
[Chen, Moult, Zhang, Zhu]

Measurement of α_s with EEC

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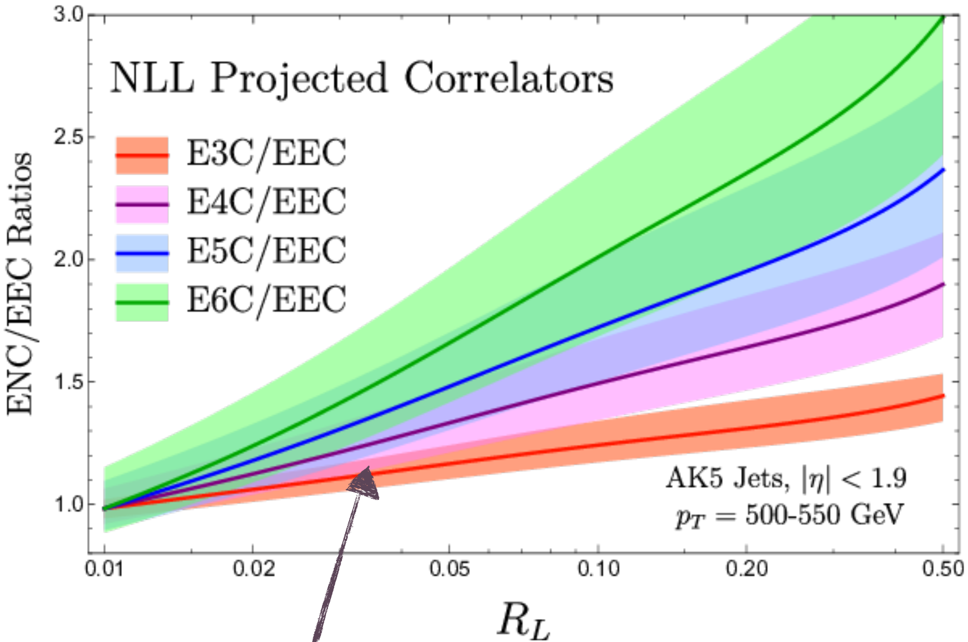
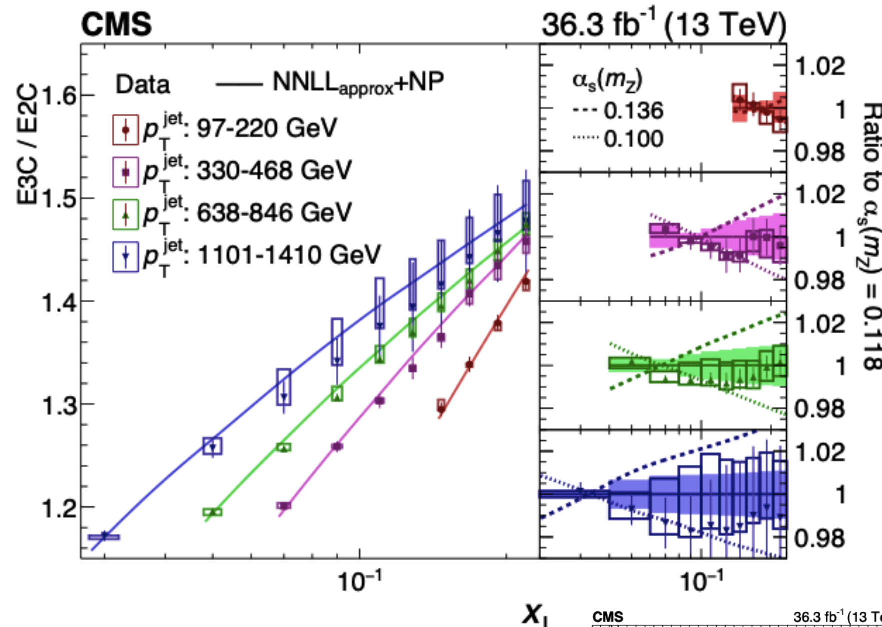
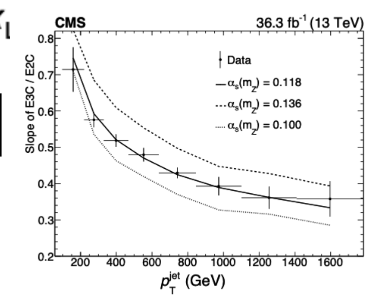


$$\theta^{\gamma} \to \exp\left(\frac{\hat{\gamma}}{2\beta_0} \ln \frac{\alpha_s(\theta Q)}{\alpha_s(Q)}\right)$$

Phys.Rev.Lett. 133 (2024) 7, 071903

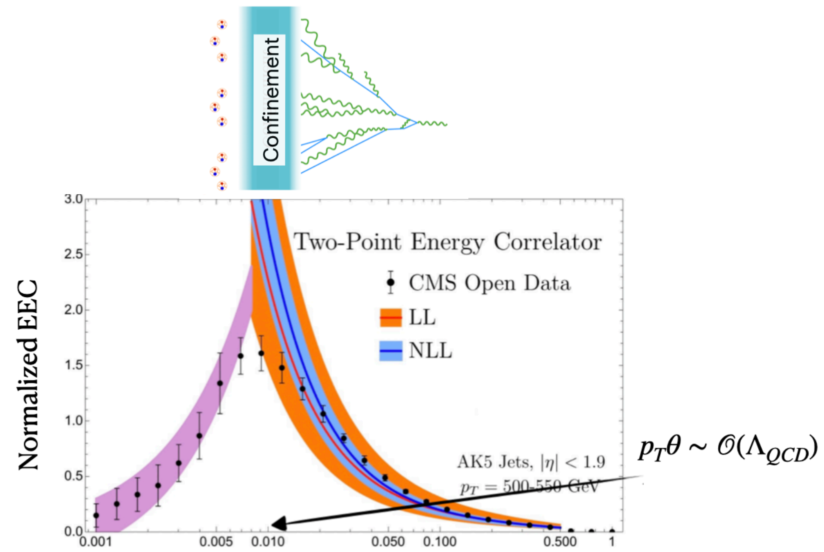


$$\alpha_S(m_Z) = 0.1229^{+0.0014}_{-0.0012}\,(\text{stat})^{+0.0030}_{-0.0033}\,(\text{theo})^{+0.0023}_{-0.0036}(\text{exp})$$



Conclusions and Outlook

- Energy Energy Correlators are robust jet substructure observables
    - Sensitive to dynamical scales: probes both pQCD and NP QCD
    - pQCD calculations available beyond LL
    - Opportunity for precise measurements in QCD
- Compare theory and experiment in different regimes (angular and energy)
- Medium effects of EEC are not much explored
    - Comparison with the vacuum
    - Comparison with other existing observables\models



Thank you!