

Interacting mesons: the missing piece of the puzzle to model dense and hot matter

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Outline

- ① Introduction
- ② CMF Framework
- ③ Update 1: Field Redefined CMF
- ④ Update 2: Interacting Thermal Mesons in CMF
- ⑤ Results
- ⑥ Summary

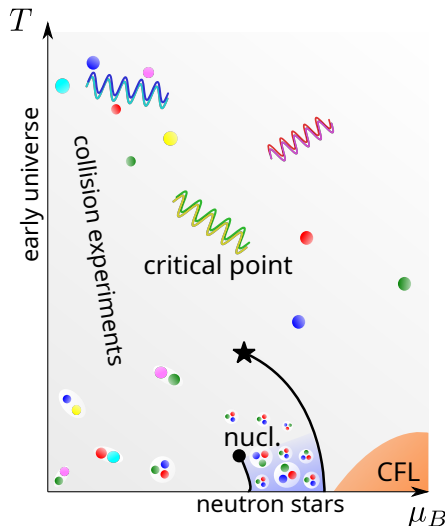
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Quantum Chromodynamics

QCD exhibits two important features:

- **Asymptotic freedom:** coupling constant becomes small at large energies/temperatures → Quark-Gluon Plasma (QGP).
- **Confinement:** quarks must be confined in bound states called hadrons at low energies/temperatures.
- QCD describes the interaction between quarks and gluons.



QCD and Chiral Symmetry

QCD is approximately invariant under chiral transformations
 $SU(3)_L \times SU(3)_R = SU(3)_V \times SU(3)_A$ flavor for the (u, d, s) quarks.

- Chiral symmetry is an exact symmetry of the QCD Lagrangian when $m_q = 0$.
- Chiral symmetry is slightly explicitly broken by the small finite quark mass.



However, states with different parity have very different mass.
Chiral symmetry is spontaneously broken

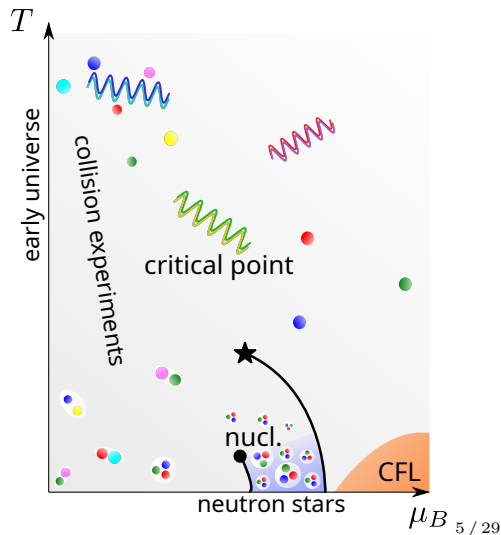
QCD Phase Diagram

We can probe the QCD phase diagram in relativistic heavy ion collisions and with multi-messenger observations.

Many models predict a first-order phase transition line with a critical point

Lattice QCD is the most reliable theoretical tool at $\mu_B = 0$, but at finite μ_B it is limited by the sign problem.

Extrapolated Lattice QCD using expansion schemes at $\mu_B \neq 0$.



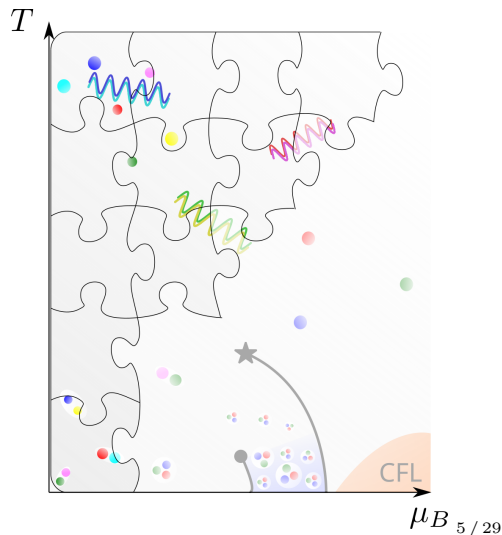
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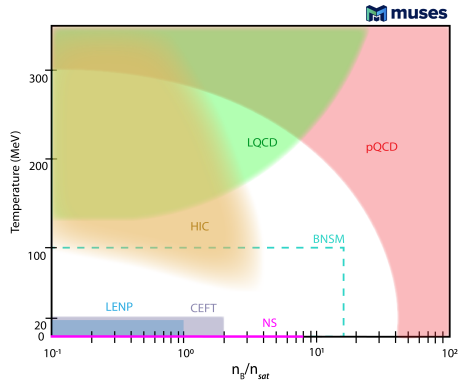
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QCD Phase Diagram as per Modern Theories and Experiments

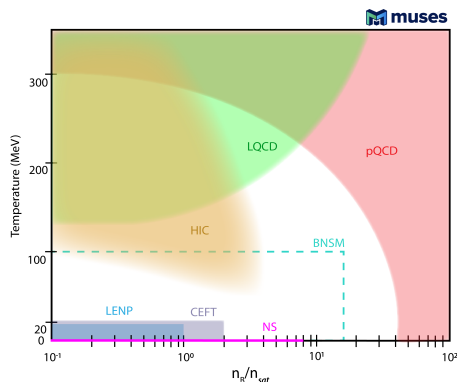
- Theoretical insights include:
 - Lattice QCD results
 - Perturbative QCD results
 - Chiral effective field theory results
- Experimental insights include:
 - Heavy-ion collision results
 - Low-energy nuclear physics results
 - Observations from neutron stars and their mergers
- White Region: Effective models.
- Living Review (to be updated regularly).



R.K., et al., Living Rev. Rel. 27, 3 (2024)

Role of Interacting Mesons in the QCD Equation of State

- Interacting mesons are important at finite temperature because of their large thermal population.
- Medium-dependent meson properties are crucial for a realistic QCD equation of state.
- Meson condense at finite isospin fraction, therefore important for neutron star physics.
- Key question: how can interacting mesons be modeled consistently at finite μ_B , and μ_Q ?



R.K., et al., Living Rev. Rel. 27, 3 (2024)

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Effective model for Nuclear Matter

We need a simplified theoretical framework that describes QCD in the desired energy range

Interpret data \iff make predictions

Requirements:

- QCD Chiral symmetry.
- Broken scale invariance.
- Nuclear matter degrees of freedom and interactions.
- Constrained by first-principles results and/or experiments/observations.

Chiral Mean-field (CMF) Model

- Quarks and hadrons interactions are mediated via the exchange of scalar (σ , ζ and δ) and vector (ω , ϕ and ρ) mesons.
- A comprehensive equation of state encompassing the baryon octet, decuplet, mesons, and quarks.
- Pseudoscalar and Vector mesons are added as a non-interacting gas.
- CMF is a non-linear extension of the sigma model and is fitted to agree with low- and high-energy physics data.
- CMF uses a Polyakov loop-inspired deconfinement potential to describe the deconfinement phase transition.
- Uses the mean field approximation, neglecting thermal fluctuations.

V. A. Dexheimer and S. Schramm, Phys. Rev. C 81, 045201 (2010)

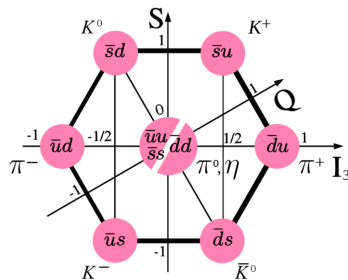
The mean field approximation (MFA)

- Mesons are not dynamical degrees of freedom.
- We assume a dense, spatially isotropic, and rotationally invariant system.
- Negative parity states (π, K, η) are neglected.
- Only mesons with $I_z^3 = 0$ survive.

$$\sigma \rightarrow \langle \sigma \rangle \equiv \sigma_0,$$

$$V^\mu \rightarrow \langle V^\mu \rangle \equiv \langle V_0, 0 \rangle,$$

$$\langle \pi_i \rangle = 0$$



Chiral Mean-Field Model

The chiral mean-field Lagrangian is written as

$$\mathcal{L}_{\text{CMF}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{\text{esb}} - U_{\Phi}.$$

where \mathcal{L}_{kin} stands for kinetic, \mathcal{L}_{int} for meson-baryon interactions, $\mathcal{L}_{\text{scal}}$ for scalar self interactions, \mathcal{L}_{vec} for vector self interactions (C1-C4), \mathcal{L}_{esb} for explicit symmetry breaking, and U_{Φ} is a Polyakov loop inspired potential may be written as

$$U_{\Phi} = \left(a_0 T^4 + a_1 \mu_B^4 + a_2 T^2 \mu_B^2 \right) \Phi^2 + a_3 T_0^4 \ln \left(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4 \right).$$

R.K., et al., Phys. Rev. D 109, 074008 (2024)

Motivation

Including Interacting Thermal Mesons in CMF

- Old CMF
 - Degenerate vector mesons masses
 - Non-interacting pseudoscalar and vector mesons
- For consistent treatment of mesons, we need to break the mass degeneracy of vector mesons.
- To match thermodynamics from extrapolated Lattice QCD data at finite T and μ_B , we need an EoS with interacting mesons.

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Field Redefined Chiral Mean-Field Model

The field redefined chiral mean-field Lagrangian is written as

$$\mathcal{L}_{\text{CMF}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{\text{esb}} - U_{\Phi}.$$

where \mathcal{L}_{kin} stands for kinetic, \mathcal{L}_{int} for meson-baryon interactions, $\mathcal{L}_{\text{scal}}$ for scalar self interactions, \mathcal{L}_{vec} for vector self interactions (**RC1-RC4**), \mathcal{L}_{esb} for explicit symmetry breaking, and U_{Φ} is a Polyakov loop inspired potential may be written as

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Breaking the vector mass degeneracy in CMF

The mass of the vector meson nonet is degenerate in the previous CMF. To break the vector nonet mass degeneracy, a chiral invariant term is added in \mathcal{L}_{vec}

$$\tilde{\mathcal{L}}_{\text{vec}}^{\text{CI}} = \frac{1}{4} \mu \text{Tr} \left[\tilde{V}_{\mu\nu} \tilde{V}^{\mu\nu} \langle X \rangle^2 \right].$$

which gives

$$m_{K^*}^2 = Z_{K^*} m_V^2, \quad m_{\omega/\rho}^2 = Z_{\omega/\rho} m_V^2, \quad m_\phi^2 = Z_\phi m_V^2, \\ \tilde{\xi} = Z_\xi^{1/2} \xi, \quad \xi = \rho, \omega, K^*, \phi$$

Meson	ω	ρ	K^*	ϕ
Old Mass (MeV)	687.33	687.33	687.33	687.33
New Mass (MeV)	770.87	770.87	865.89	1007.76

Lagrangian terms and corresponding constraints

Lagrangian Term	Used to constrain
$\mathcal{L}_{\text{int}} + \mathcal{L}_{\text{vec}}^{\text{SI}}$	Nuclear saturation properties (BE/A , E_{sym} , L , K)
$\mathcal{L}_{\text{vec}}^{\text{m}} + \mathcal{L}_{\text{vec}}^{\text{CI}}$	Vacuum mass of vector mesons
\mathcal{L}_{esb}	Hyperon-nucleon potential
	Deconfinement transition temperature (pure glue Lattice QCD)
U_{Φ}	Pseudocritical transition temperature (Lattice QCD)
	Crossover region (Lattice QCD)

Results: Field Redefined CMF Model

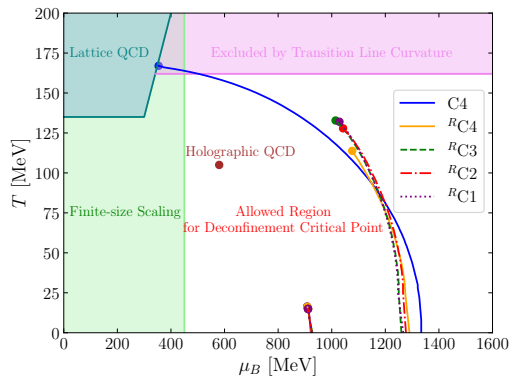


Figure: QCD Phase Diagram

R.K., et al., Phys. Rev. D 109, 074008 (2024)

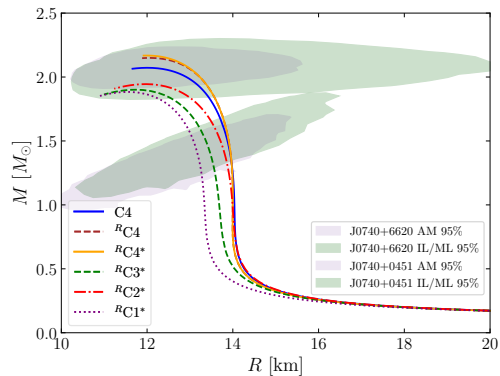


Figure: Mass-Radius Curve

R.K., et al., Phys. Rev. D 109, 074008 (2024)

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Interacting Thermal Mesons in CMF (mCMF)

Motivation

- In the field redefined CMF, mesons were also added as a non-interacting gas.
- In-medium masses of baryons and quarks in the CMF

$$M_B^* = g_{B\sigma}\sigma + g_{B\delta}\tau_3\delta + g_{B\zeta}\zeta + M_{0_B} + g_{B\Phi}\Phi^2.$$

$$M_q^* = g_{q\sigma}\sigma + g_{q\delta}\tau_3\delta + g_{q\zeta}\zeta + M_{0_q} + g_{q\Phi}(1 - \Phi).$$

- In-medium masses of mesons

$$M_M^* = M_M^*(\sigma, \zeta, \delta, \omega, \rho, \phi) + g_{M\Phi}f(\Phi).$$

- Comparison of EoS with interacting mesons from CMF with extrapolated lattice QCD, as well as the HRG model.

Interacting Thermal Mesons in Hadronic CMF

Starting with the hadronic CMF

- Add in-medium baryon masses at finite temperature and density.

$$M_B^* = g_{B\sigma}\sigma + g_{B\delta}\tau_3\delta + g_{B\zeta}\zeta + M_{0_B}.$$

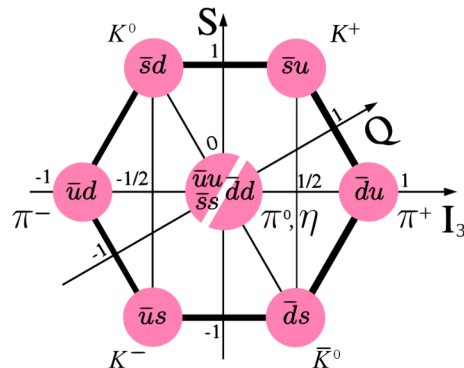
- Add in-medium meson masses at finite temperature and density

$$M_M^* = M_M^*(\sigma, \zeta, \delta, \omega, \rho, \phi).$$

- Comparison of EoS with interacting mesons from CMF with extrapolated lattice QCD, as well as the HRG model up to 160 MeV in temperature.

Including pseudoscalar and vector mesons as dynamical degrees of freedom

- Computed in-medium mass of pseudoscalar mesons from an explicit symmetry-breaking term in the CMF lagrangian.
- Before applying the MFA.
- Computed in-medium mass of vector mesons from the vector meson self-interaction term.



In-medium mass of thermal mesons in CMF

The explicit chiral symmetry-breaking Lagrangian term in the CMF model is written as

$$\mathcal{L}_{\text{esb}}^u = \left(-\frac{1}{2}m_{\eta^0}^2 \text{Tr} Y^2 - \frac{1}{2} \text{Tr} \left[A_p \left(u(X + iY)u + u^\dagger(X - iY)u^\dagger \right) \right] \right)$$

The second derivative of the $\mathcal{L}_{\text{esb}}^u$ at its minimum with respect to the respective mesons φ_i , gives the in-medium mass

$$m_{\varphi_{ij}}^{*2} = - \lim_{\varphi \rightarrow \langle \varphi \rangle} \frac{\partial^2}{\partial \varphi_i \partial \varphi_j} \mathcal{L}_{\text{esb}}^u$$

with $\varphi_i = \pi, \eta, \eta', K$, and for the vacuum expectation for the mesons we consider $\langle \varphi \rangle = 0$. Similarly, the in-medium mass can be obtained for vector mesons using \mathcal{L}_{vec} .

In-medium mass of thermal mesons in CMF

The explicit chiral symmetry-breaking Lagrangian term in the CMF model is written as

$$\mathcal{L}_{\text{esb}}^u = \left(-\frac{1}{2}m_{\eta^0}^2 \text{Tr} Y^2 - \frac{1}{2} \text{Tr} \left[A_p \left(u(X + iY)u + u^\dagger(X - iY)u^\dagger \right) \right] \right)$$

The feedback in CMF equation of motion

$$\frac{\partial (\Omega^H/V)}{\partial \vartheta} = \frac{\partial (\Omega^{\text{orig}}/V)}{\partial \vartheta} + \sum_{i \in M} n_{s_i}^M \frac{\partial m_i^*}{\partial \vartheta}.$$

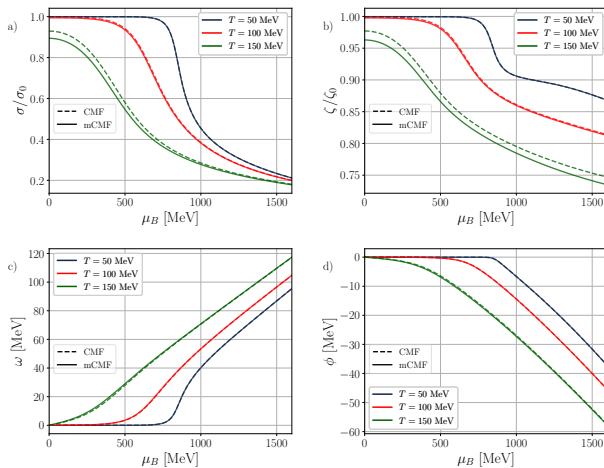
where $\vartheta = \sigma, \zeta, \delta, \omega_0, \rho_0, \phi_0$. This generates a backreaction in the equations of motion, which in turn contributes directly to the thermodynamic quantities, including the pressure, energy density, and entropy density.

R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

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Mean fields in isospin symmetric matter

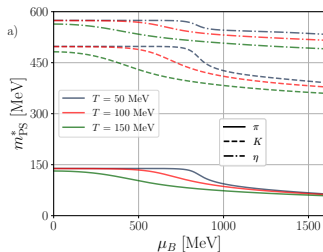


R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

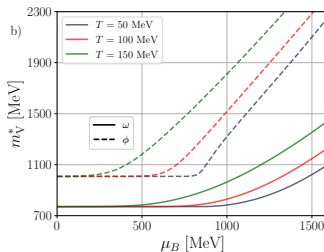
- pion masses depend directly on the σ meson (proxy for the chiral condensate).
- meson masses reveal the onset of chiral symmetry restoration.
- Their inflection point moves to a smaller μ_B as T increases.
- The particle population can be obtained from the model.

Outlook: inclusion of quark degrees of freedom and deconfinement potential

In-medium meson mass in isospin symmetric matter

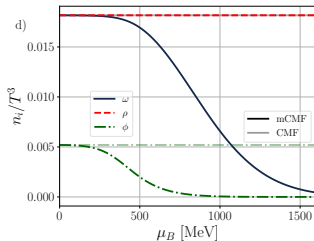
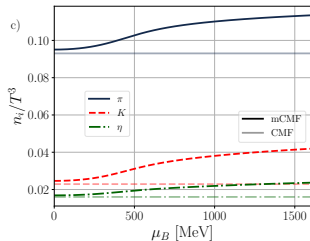
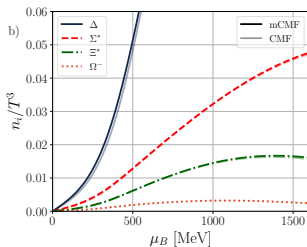
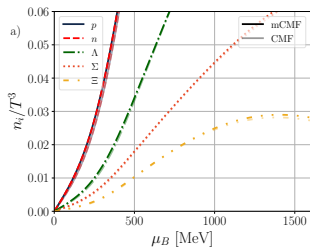


R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)



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 - Their inflection point moves to a smaller μ_B as T increases.
 - The particle population can be obtained from the model.
- Outlook: inclusion of quark degrees of freedom and deconfinement potential

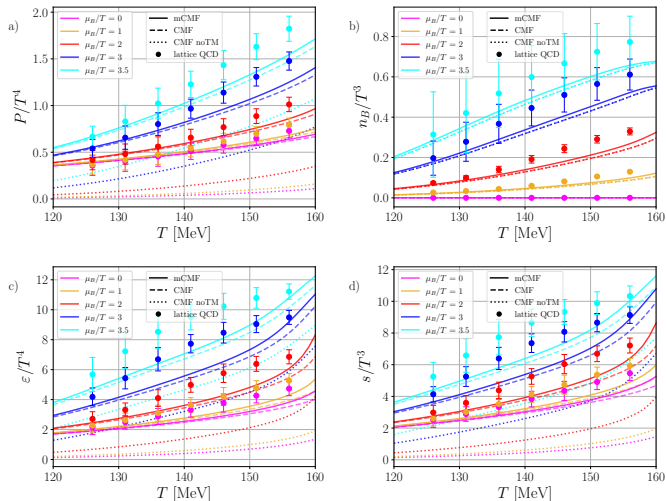
Particle population at $T = 150$ MeV in isospin symmetric matter



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- meson masses reveal the onset of chiral symmetry restoration.
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Outlook: inclusion of quark degrees of freedom and deconfinement potential

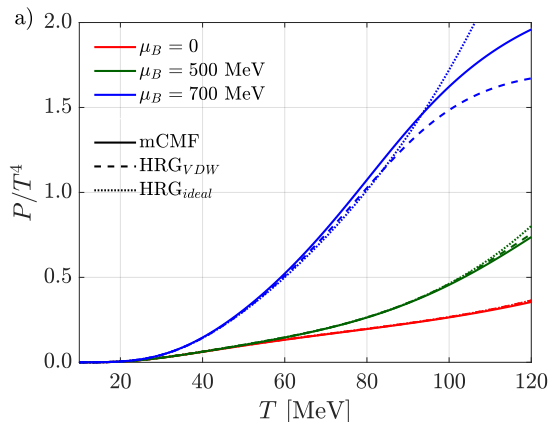
Comparison with lattice QCD



R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

- Matching with state-of-the-art lattice thermodynamics up to $T \sim 160$ MeV. [S. Borsányi, et al. PRL 126 \(2021\)](#)
- **Outlook:** The agreement with lattice data can be improved with the inclusion of quarks and a deconfinement potential.
- Matching with HRG data at low to intermediate temperatures.

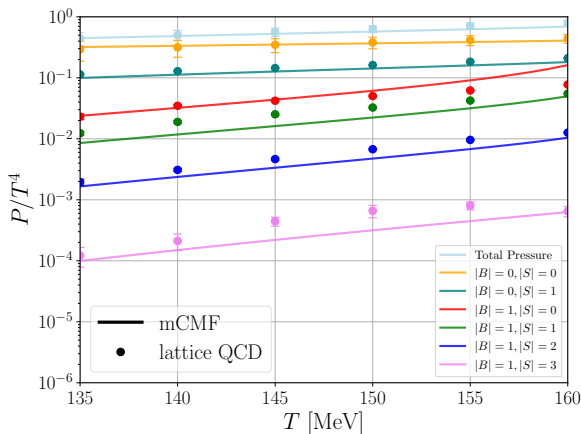
Comparison with HRG thermodynamics



R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

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- **Outlook:** The agreement with lattice data can be improved with the inclusion of quarks and a deconfinement potential.
- Matching with HRG data at low to intermediate temperatures.

Comparison of partial pressure with lattice QCD



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Partial Pressures

Good agreement between the partial pressures of hadrons from different baryonic and strange sectors in CMF and Lattice QCD data at $\mu_B=0$.

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Summary

- Extended the CMF model by including pseudoscalar and vector meson interactions.
- We achieved an unprecedented level of agreement with extrapolated lattice QCD and HRG thermodynamics that could be improved by adding quark degrees of freedom.
- Our goal is to obtain a realistic equation of state for dense matter at finite temperature, which will be useful in neutron stars and neutron star merger simulations.

Ongoing Work and Outlook:

- Obtained the EoS at finite isospin fraction with meson condensation.
- Extension to neutron star mergers simulations.
- Combine quarks and a deconfinement potential.

Acknowledgements



Backup Slides...

Discussion Points on the CMF Model

- **Finite μ_B Fitting:** Current work focuses on zero baryon chemical potential; extension to finite μ_B is planned for future studies to capture full QCD phase structure.
- **Coincidence of Transition Temperatures:** In the CMF model, the chiral and deconfinement transitions coincide for finite quarks due to the strong coupling between the Polyakov loop and effective baryon/quark masses.
- **Finite Size Scaling in QCD:** Employed to probe critical behavior by analyzing susceptibilities. No clear signatures of criticality have been observed so far in the explored parameter range.

Future work will address finite μ_B and explore potential critical points with improved resolution.

Estimation of error in the model

- A better agreement with the results from LIGO and VIRGO concerning the radius of the neutron star by adding vector-isovector interactions ($\omega\rho$) and using a different crust.
- Low-energy nuclear physics observations are not well-constrained, therefore more wider range of constraints should be studied.

RK, VD et al., Effects of hyperon potentials and symmetry energy in quark deconfinement, Physics Letters B 849, 138475 (2024).

- Use statistical methods, such as Bayesian analysis, to constrain model parameters.
- Optimizing CMF code by transforming it into C++ from Fortran at finite temperature.

Multiplets

- Baryon Octet

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda_0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

- Scalar Matrix: Mean-Fields

$$X = \begin{pmatrix} \frac{\delta^0 + \sigma}{\sqrt{2}} & 0 & 0 \\ 0 & -\frac{\delta^0 + \sigma}{\sqrt{2}} & 0 \\ 0 & 0 & \zeta \end{pmatrix}$$

- A_p matrix

$$A_p = \frac{1}{\sqrt{2}} \begin{pmatrix} m_\pi^2 f_\pi & 0 & 0 \\ 0 & m_\pi^2 f_\pi & 0 \\ 0 & 0 & 2m_K^2 f_K - m_\pi^2 f_\pi \end{pmatrix}.$$

Multiplets

- Pseudoscalar Nonet

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} \left(\pi^0 + \frac{\eta^8}{\sqrt{1+2w^2}} \right) & \pi^+ & 2 \frac{K^+}{w+1} \\ \pi^- & \frac{1}{\sqrt{2}} \left(-\pi^0 + \frac{\eta^8}{\sqrt{1+2w^2}} \right) & 2 \frac{K^0}{w+1} \\ 2 \frac{K^-}{w+1} & 2 \frac{\bar{K}^0}{w+1} & -\sqrt{\frac{2}{1+2w^2}} \eta^8 \end{pmatrix}$$

where $w = \sqrt{2}\zeta_0/\sigma_0$.

- Vector Meson Nonet : Mean Fields (ω, ρ and ϕ)

$$V = \begin{pmatrix} \frac{\rho^0 + \omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0 + \omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

- Pseudoscalar Singlet

$$Y = \sqrt{\frac{1}{3}} \eta_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The CMF Lagrangian

$$\mathcal{L}_{\text{CMF}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{\text{esb}}$$

$$\mathcal{L}_{\text{kin}} = i \text{Tr} \left(\bar{B} \gamma_\mu D^\mu B \right) = i \sum_{i \in B} \left(\bar{\psi}_i \gamma_\mu \partial^\mu \psi_i \right)$$

$$\begin{aligned} \mathcal{L}_{\text{scal}} = & -\frac{1}{2} k_0 \chi_0^2 \left(\sigma^2 + \zeta^2 + \delta^2 \right) + k_1 \left(\sigma^2 + \zeta^2 + \delta^2 \right)^2 + k_2 \left[\frac{\sigma^4 + \delta^4}{2} + \zeta^4 + 3(\sigma\delta)^2 \right] \\ & + k_3 \chi_0 \left(\sigma^2 - \delta^2 \right) \zeta + k_{3N} \chi_0 \left(\frac{\sigma^3}{\sqrt{2}} + \frac{3}{\sqrt{2}} \sigma \delta^2 + \zeta^3 \right) - k_4 \chi_0^4 + \frac{\epsilon}{3} \chi_0^4 \ln \left[\frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \zeta_0} \right] \end{aligned}$$

$$\mathcal{L}_{\text{vec}} = \frac{1}{2} \left(m_\omega^2 \omega^2 + m_\phi^2 \phi^2 + m_\rho^2 \rho^2 \right) + \mathcal{L}_{\text{vec}}^{\text{SI}}$$

$$\mathcal{L}_{\text{esb}}^u = - \left[m_\pi^2 f_\pi \sigma + \left(\sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right]$$

$$\mathcal{L}_{\text{int}} = - \sum_{i \in B} \bar{\psi}_i \left[\gamma_0 (g_{i\omega} \omega + g_{i\rho} \rho + g_{i\phi} \phi) + g_{i\sigma} \sigma + g_{i\zeta} \zeta + g_{i\delta} \delta \right] \psi_i$$

Formalism: CMF thermodynamic potential

$$\begin{aligned}\frac{\Omega^H}{V} &= \frac{\Omega}{V} + \frac{\Omega_{\text{th}}^M}{V}, \\ &= U_M + \frac{\Omega_{\text{th}}^B}{V} + \frac{\Omega_{\text{th}}^M}{V},\end{aligned}\tag{1}$$

$$U_M = \mathcal{L}_{\text{vec}} - \mathcal{L}_{\text{scal}} - \mathcal{L}_{\text{esb}} + \mathcal{L}_{\text{vac}},\tag{2}$$

$$\frac{\Omega_{\text{th}}^B}{V} = -T \sum_{i \in \text{baryons}} \frac{\gamma_i}{2\pi^2} \int dk k^2 \left(\ln \left[1 + e^{-\frac{1}{T}(E_i^*(k) - \mu_i^*)} \right] + \ln \left[1 + e^{-\frac{1}{T}(E_i^*(k) + \mu_i^*)} \right] \right),\tag{3}$$

$$\frac{\Omega_{\text{th}}^M}{V} = T \sum_{i \in \text{mesons}} \frac{\gamma_i}{2\pi^2} \int dk k^2 \ln \left[1 - e^{-\frac{1}{T}(E_i^*(k) - \mu_i^*)} \right].\tag{4}$$

Parameters used to fit the constraints

Parameter	Term	Used to constrain
$g_1^V, g_8^V, \alpha_V, g_4$	$\mathcal{L}_{\text{int}} + \mathcal{L}_{\text{vec}}^{\text{SI}}$	$g_{N\phi} = 0, g_1^V = \sqrt{6}g_8^V, n_{\text{sat}} \approx 0.15 \text{ fm}^{-3}, B^{\text{sat}}/A \approx -15.70 \text{ MeV},$ $E_{\text{sym}}^{\text{sat}} \approx 28.9 \text{ MeV}, 66 \leq L^{\text{sat}}(\text{MeV}) \leq 87, 275 \leq K(\text{MeV}) \leq 305$
m_V, μ	$\mathcal{L}_{\text{vec}}^{\text{m}} + \mathcal{L}_{\text{vec}}^{\text{CI}}$	$m_\omega = 770.87 \text{ MeV}, m_\rho = 770.87 \text{ MeV}, m_\phi = 1007.76 \text{ MeV}$
m_3	\mathcal{L}_{esb}	$U_\Lambda \approx -28 \text{ MeV}$
a_0	U_Φ	$T_c^d \approx 270 \text{ MeV}$
a_1		$n_{B,c}^d \approx 3.5 n_{\text{sat}}$
a_2		$T_c^{\text{HQ}} > 135 \text{ MeV}, \mu_{B,c} > 400 \text{ MeV}$
a_3		$\Phi \in 0, 1$
$T_0(\text{gauge})$		$T_c^d, \Phi \in 0, 1$
$T_0(\text{quarks})$		$T_c^p \approx 159 \text{ MeV}, \Phi \in 0, 1$
$g_{q\Phi}, g_{B\Phi}$		T_c^p

Dense QCD Phase Diagram: Theory vs Constraints

- Theory allows independent chemical potentials μ_B and μ_q
- Rich phase structure predicted at high density and low temperature
- Experiments limited to high temperature and low to moderate μ_B
- No direct experimental access to large μ_B region
- Vorticity effects seen in heavy ion collisions
- Vorticity is macroscopic, not true microphysics input
- Color superconductivity expected only in cold, fully evolved neutron stars
- Not relevant for proto neutron stars or neutron star mergers
- Neutron stars provide main constraints on dense QCD matter

Vorticity

- Vorticity measures local rotation of a fluid or medium
- Defined as the curl of the velocity field, $\boldsymbol{\omega} = \nabla \times \mathbf{v}$
- It is a macroscopic property of the system
- Describes rotation of matter, not particle spin
- Generated by large angular momentum in non central heavy ion collisions
- Very large vorticity expected in the quark gluon plasma
- Leads to spin polarization of emitted hadrons
- Not a fundamental QCD interaction or microphysics input