

# Interacting mesons: the missing piece of the puzzle to model dense and hot matter

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# Outline

- ➊ Introduction
- ➋ CMF Framework
- ➌ Update 1: Field Redefined CMF
- ➍ Update 2: Interacting Thermal Mesons in CMF
- ➎ Results
- ➏ Summary

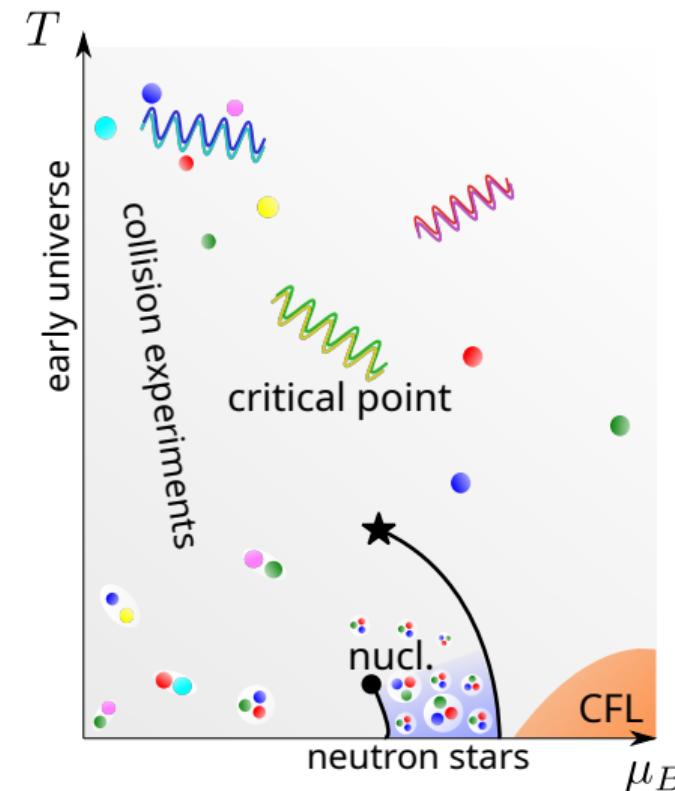
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## Quantum Chromodynamics

QCD exhibits two important features:

- **Asymptotic freedom:** coupling constant becomes small at large energies/temperatures  
→ Quark-Gluon Plasma (QGP).
  - **Confinement:** quarks must be confined in bound states called hadrons at low energies/temperatures.
  - QCD describes the interaction between quarks and gluons.



## QCD and Chiral Symmetry

QCD is approximately invariant under chiral transformations  $SU(3)_L \times SU(3)_R = SU(3)_V \times SU(3)_A$  flavor for the  $(u, d, s)$  quarks.

- Chiral symmetry is an exact symmetry of the QCD Lagrangian when  $m_q = 0$ .
  - Chiral symmetry is slightly explicitly broken by the small finite quark mass.



However, states with different parity have very different mass.  
Chiral symmetry is spontaneously broken

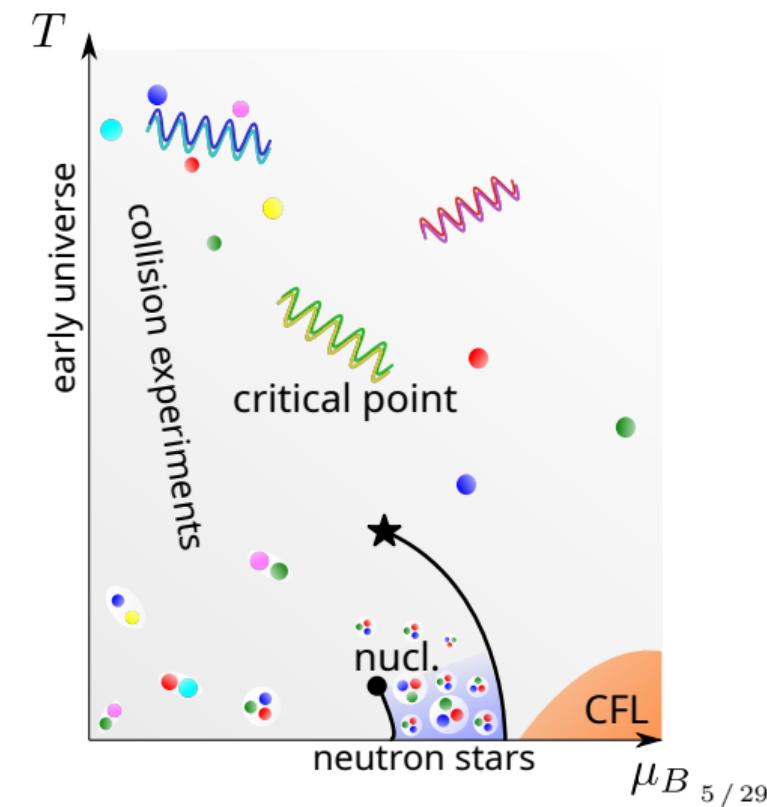
# QCD Phase Diagram

We can probe the QCD phase diagram in relativistic heavy ion collisions and with multi-messenger observations.

Many models predict a first-order phase transition line with a critical point

Lattice QCD is the most reliable theoretical tool at  $\mu_B = 0$ , but at finite  $\mu_B$  it is limited by the sign problem.

Extrapolated Lattice QCD using expansion schemes at  $\mu_B \neq 0$ .



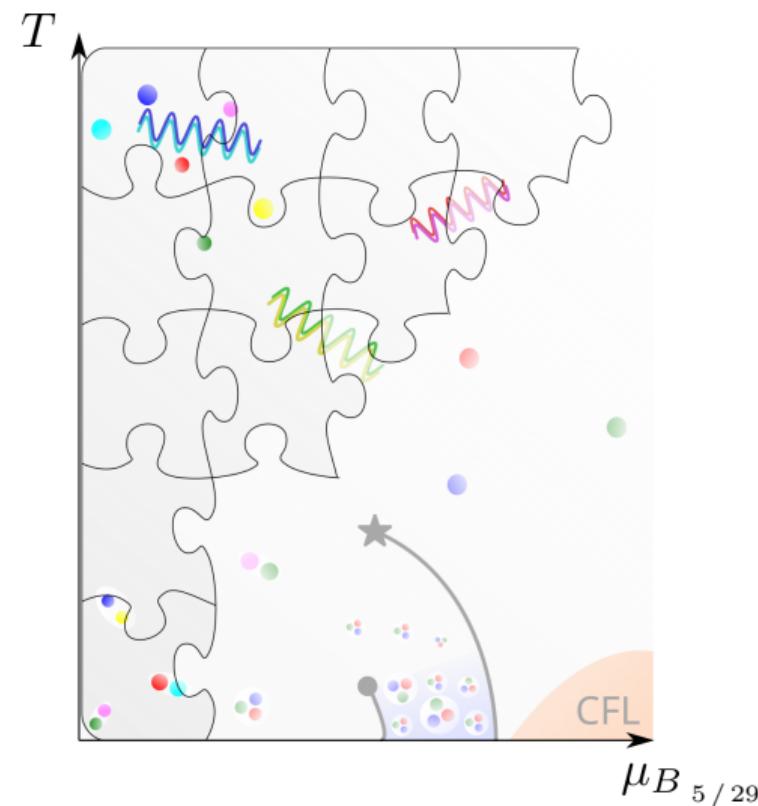
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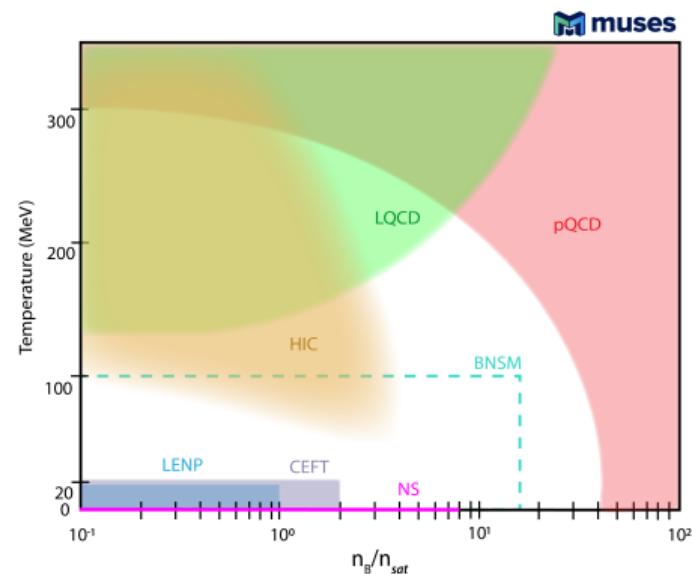
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# QCD Phase Diagram as per Modern Theories and Experiments

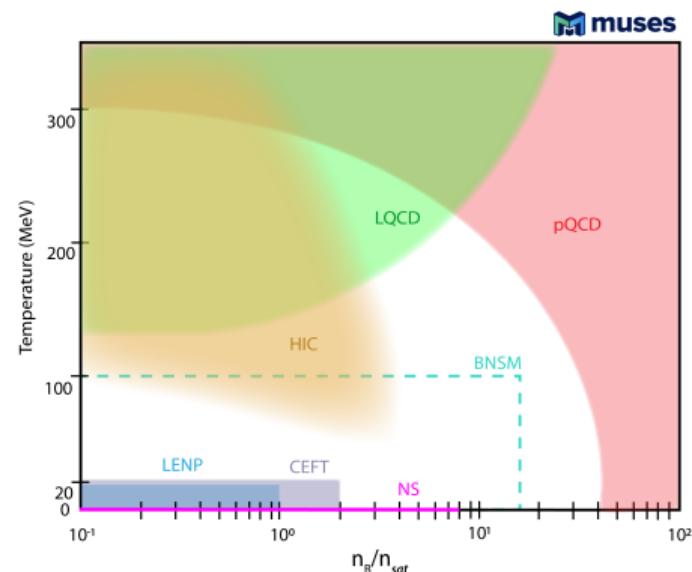
- Theoretical insights include:
  - Lattice QCD results
  - Perturbative QCD results
  - Chiral effective field theory results
- Experimental insights include:
  - Heavy-ion collision results
  - Low-energy nuclear physics results
  - Observations from neutron stars and their mergers
- White Region: Effective models.
- Living Review (to be updated regularly).



R.K., et al., Living Rev. Rel. 27, 3 (2024)

# Role of Interacting Mesons in the QCD Equation of State

- Interacting mesons are important at finite temperature because of their large thermal population.
- Medium-dependent meson properties are crucial for a realistic QCD equation of state.
- Meson condense at finite isospin fraction, therefore important for neutron star physics.
- Key question: how can interacting mesons be modeled consistently at finite  $\mu_B$ , and  $\mu_Q$ ?



R.K., et al., *Living Rev. Rel.* 27, 3 (2024)

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# Effective model for Nuclear Matter

We need a simplified theoretical framework that describes QCD in the desired energy range

Interpret data  $\iff$  make predictions

## Requirements:

- QCD Chiral symmetry.
- Broken scale invariance.
- Nuclear matter degrees of freedom and interactions.
- Constrained by first-principles results and/or experiments/observations.

# Chiral Mean-field (CMF) Model

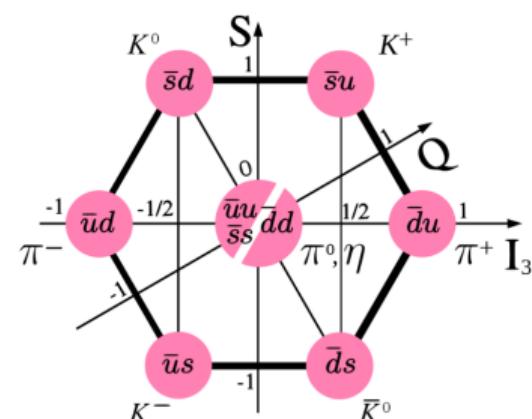
- Quarks and hadrons interactions are mediated via the exchange of scalar ( $\sigma$ ,  $\zeta$  and  $\delta$ ) and vector ( $\omega$ ,  $\phi$  and  $\rho$ ) mesons.
- A comprehensive equation of state encompassing the baryon octet, decuplet, mesons, and quarks.
- Pseudoscalar and Vector mesons are added as a non-interacting gas.
- CMF is a non-linear extension of the sigma model and is fitted to agree with low- and high-energy physics data.
- CMF uses a Polyakov loop-inspired deconfinement potential to describe the deconfinement phase transition.
- Uses the mean field approximation, neglecting thermal fluctuations.

*V. A. Dexheimer and S. Schramm, Phys. Rev. C 81, 045201 (2010)*

# The mean field approximation (MFA)

- Mesons are not dynamical degrees of freedom.
- We assume a dense, spatially isotropic, and rotationally invariant system.
- Negative parity states ( $\pi, K, \eta$ ) are neglected.
- Only mesons with  $I_z^3 = 0$  survive.

$$\begin{aligned}\sigma &\rightarrow \langle \sigma \rangle \equiv \sigma_0, \\ V^\mu &\rightarrow \langle V^\mu \rangle \equiv \langle V_0, 0 \rangle, \\ \langle \pi_i \rangle &= 0\end{aligned}$$



# Chiral Mean-Field Model

The chiral mean-field Lagrangian is written as

$$\mathcal{L}_{\text{CMF}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{\text{esb}} - U_{\Phi}.$$

where  $\mathcal{L}_{\text{kin}}$  stands for kinetic,  $\mathcal{L}_{\text{int}}$  for meson-baryon interactions,  $\mathcal{L}_{\text{scal}}$  for scalar self interactions,  $\mathcal{L}_{\text{vec}}$  for vector self interactions (C1-C4),  $\mathcal{L}_{\text{esb}}$  for explicit symmetry breaking, and  $U_{\Phi}$  is a Polyakov loop inspired potential may be written as

$$U_{\Phi} = \left( a_0 T^4 + a_1 \mu_B^4 + a_2 T^2 \mu_B^2 \right) \Phi^2 + a_3 T_0^4 \ln \left( 1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4 \right).$$

*R.K., et al., Phys. Rev. D 109, 074008 (2024)*

# Motivation

## Including Interacting Thermal Mesons in CMF

- Old CMF
  - Degenerate vector mesons masses
  - Non-interacting pseudoscalar and vector mesons
- For consistent treatment of mesons, we need to break the mass degeneracy of vector mesons.
- To match thermodynamics from extrapolated Lattice QCD data at finite  $T$  and  $\mu_B$ , we need an EoS with interacting mesons.

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# Field Redefined Chiral Mean-Field Model

The field redefined chiral mean-field Lagrangian is written as

$$\mathcal{L}_{\text{CMF}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{\text{esb}} - U_{\Phi}.$$

where  $\mathcal{L}_{\text{kin}}$  stands for kinetic,  $\mathcal{L}_{\text{int}}$  for meson-baryon interactions,  $\mathcal{L}_{\text{scal}}$  for scalar self interactions,  $\mathcal{L}_{\text{vec}}$  for vector self interactions (RC1-RC4),  $\mathcal{L}_{\text{esb}}$  for explicit symmetry breaking, and  $U_{\Phi}$  is a Polyakov loop inspired potential may be written as

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*R.K., et al., Phys. Rev. D 109, 074008 (2024)*

# Breaking the vector mass degeneracy in CMF

The mass of the vector meson nonet is degenerate in the previous CMF. To break the vector nonet mass degeneracy, a chiral invariant term is added in  $\mathcal{L}_{\text{vec}}$

$$\tilde{\mathcal{L}}_{\text{vec}}^{\text{CI}} = \frac{1}{4}\mu \text{Tr} \left[ \tilde{V}_{\mu\nu} \tilde{V}^{\mu\nu} \langle X \rangle^2 \right].$$

which gives

$$m_{K^*}^2 = Z_{K^*} m_V^2, \quad m_{\omega/\rho}^2 = Z_{\omega/\rho} m_V^2, \quad m_\phi^2 = Z_\phi m_V^2, \\ \tilde{\xi} = Z_\xi^{1/2} \xi, \quad \xi = \rho, \omega, K^*, \phi$$

Meson	$\omega$	$\rho$	$K^*$	$\phi$
Old Mass (MeV)	687.33	687.33	687.33	687.33
New Mass (MeV)	770.87	770.87	865.89	1007.76

# Lagrangian terms and corresponding constraints

Lagrangian Term	Used to constrain
$\mathcal{L}_{\text{int}} + \mathcal{L}_{\text{vec}}^{\text{SI}}$	Nuclear saturation properties (BE/A, $E_{\text{sym}}$ , $L$ , $K$ )
$\mathcal{L}_{\text{vec}}^{\text{m}} + \mathcal{L}_{\text{vec}}^{\text{CI}}$	Vacuum mass of vector mesons
$\mathcal{L}_{\text{esb}}$	Hyperon-nucleon potential
	Deconfinement transition temperature (pure glue Lattice QCD)
$U_{\Phi}$	Pseudocritical transition temperature (Lattice QCD)
	Crossover region (Lattice QCD)

## Results: Field Redefined CMF Model

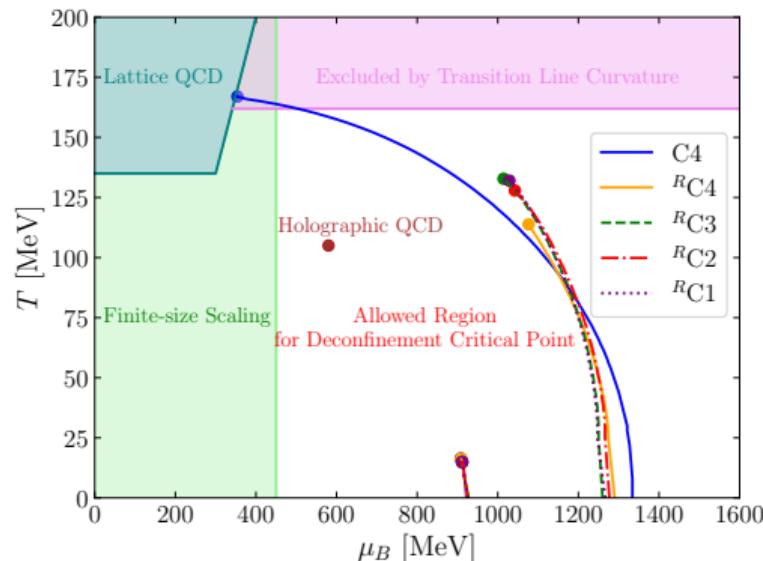


Figure: QCD Phase Diagram

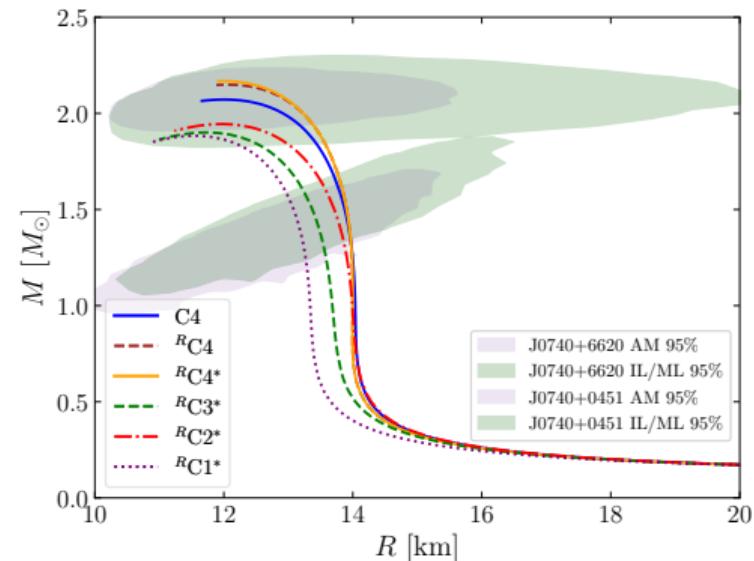


Figure: Mass-Radius Curve

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# Interacting Thermal Mesons in CMF (mCMF)

## Motivation

- In the field redefined CMF, mesons were also added as a non-interacting gas.
- In-medium masses of baryons and quarks in the CMF

$$M_B^* = g_{B\sigma}\sigma + g_{B\delta}\tau_3\delta + g_{B\zeta}\zeta + M_{0_B} + g_{B\Phi}\Phi^2.$$

$$M_q^* = g_{q\sigma}\sigma + g_{q\delta}\tau_3\delta + g_{q\zeta}\zeta + M_{0_q} + g_{q\Phi}(1 - \Phi).$$

- In-medium masses of mesons

$$M_M^* = M_M^*(\sigma, \zeta, \delta, \omega, \rho, \phi) + g_{M\Phi}f(\Phi).$$

- Comparison of EoS with interacting mesons from CMF with extrapolated lattice QCD, as well as the HRG model.

# Interacting Thermal Mesons in Hadronic CMF

## Starting with the hadronic CMF

- Add in-medium baryon masses at finite temperature and density.

$$M_B^* = g_{B\sigma}\sigma + g_{B\delta}\tau_3\delta + g_{B\zeta}\zeta + M_{0_B}.$$

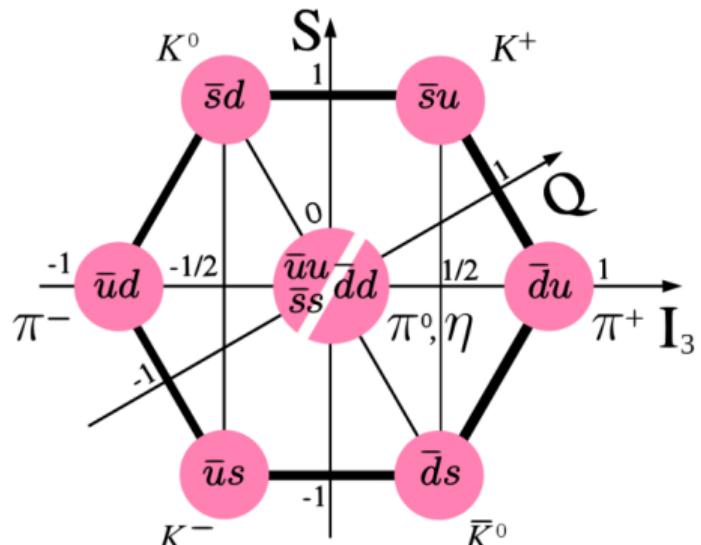
- Add in-medium meson masses at finite temperature and density

$$M_M^* = M_M^*(\sigma, \zeta, \delta, \omega, \rho, \phi).$$

- Comparison of EoS with interacting mesons from CMF with extrapolated lattice QCD, as well as the HRG model up to 160 MeV in temperature.

# Including pseudoscalar and vector mesons as dynamical degrees of freedom

- Computed in-medium mass of pseudoscalar mesons from an explicit symmetry-breaking term in the CMF lagrangian.
- Before applying the MFA.
- Computed in-medium mass of vector mesons from the vector meson self-interaction term.



# In-medium mass of thermal mesons in CMF

The explicit chiral symmetry-breaking Lagrangian term in the CMF model is written as

$$\mathcal{L}_{\text{esb}}^u = \left( -\frac{1}{2}m_{\eta^0}^2 \text{Tr}Y^2 - \frac{1}{2}\text{Tr} \left[ A_p \left( u(X + iY)u + u^\dagger(X - iY)u^\dagger \right) \right] \right)$$

The second derivative of the  $\mathcal{L}_{\text{esb}}^u$  at its minimum with respect to the respective mesons  $\varphi_i$ , gives the in-medium mass

$$m_{\varphi_{ij}}^{*2} = - \lim_{\varphi \rightarrow \langle \varphi \rangle} \frac{\partial^2}{\partial \varphi_i \partial \varphi_j} \mathcal{L}_{\text{esb}}^u$$

with  $\varphi_i = \pi, \eta, \eta', K$ , and for the vacuum expectation for the mesons we consider  $\langle \varphi \rangle = 0$ . Simillarly, the in-medium mass can be obtained for vector mesons using  $\mathcal{L}_{\text{vec}}$ .

# In-medium mass of thermal mesons in CMF

The explicit chiral symmetry-breaking Lagrangian term in the CMF model is written as

$$\mathcal{L}_{\text{esb}}^u = \left( -\frac{1}{2}m_{\eta^0}^2 \text{Tr}Y^2 - \frac{1}{2}\text{Tr} \left[ A_p \left( u(X + iY)u + u^\dagger(X - iY)u^\dagger \right) \right] \right)$$

The feedback in CMF equation of motion

$$\frac{\partial \left( \Omega^H/V \right)}{\partial \vartheta} = \frac{\partial(\Omega^{\text{orig}}/V)}{\partial \vartheta} + \sum_{i \in M} n_{s_i}^M \frac{\partial m_i^*}{\partial \vartheta}.$$

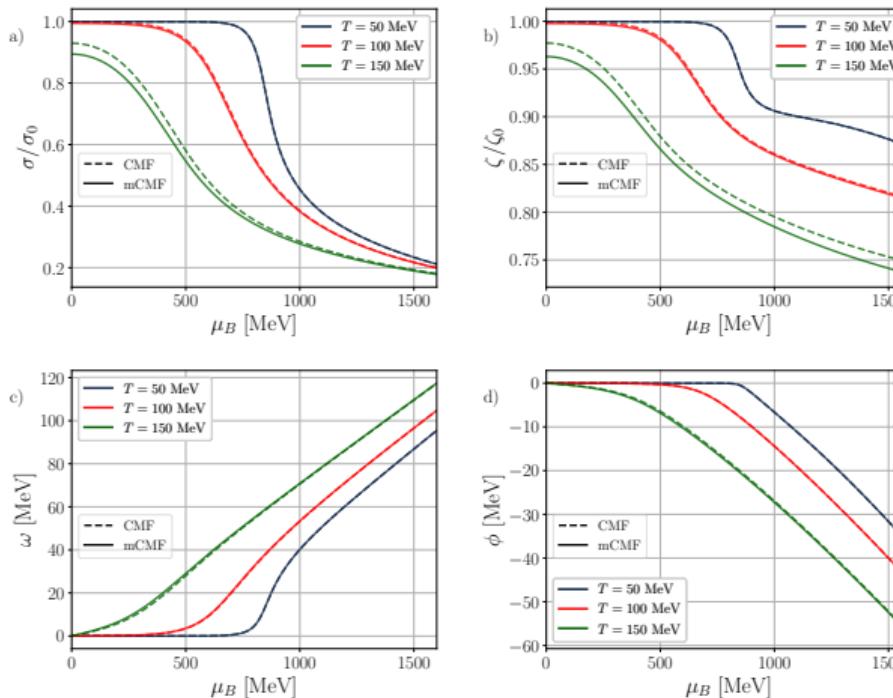
where  $\vartheta = \sigma, \zeta, \delta, \omega_0, \rho_0, \phi_0$ . This generates a backreaction in the equations of motion, which in turn contributes directly to the thermodynamic quantities, including the pressure, energy density, and entropy density.

*R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)*

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# Mean fields in isospin symmetric matter



R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

- pion masses depend directly on the  $\sigma$  meson (proxy for the chiral condensate).

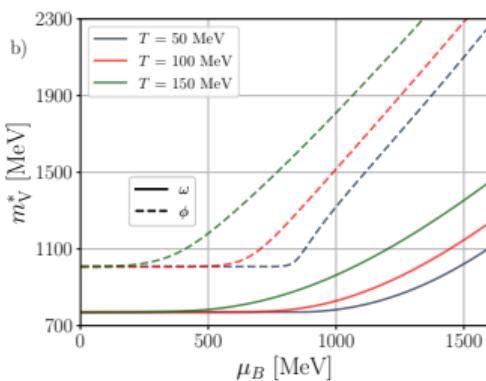
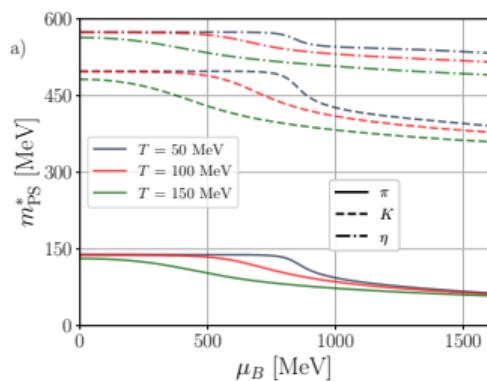
- meson masses reveal the onset of chiral symmetry restoration.

- Their inflection point moves to a smaller  $\mu_B$  as  $T$  increases.

- The particle population can be obtained from the model.

Outlook: inclusion of quark degrees of freedom and deconfinement potential

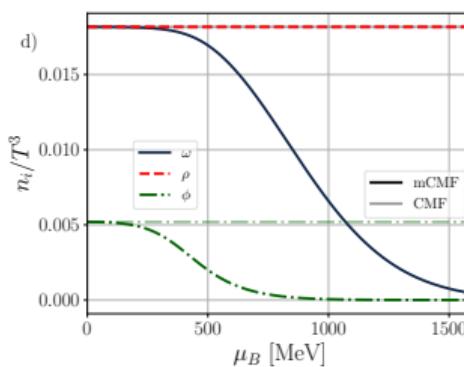
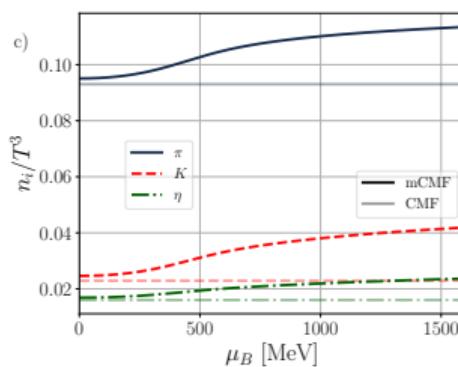
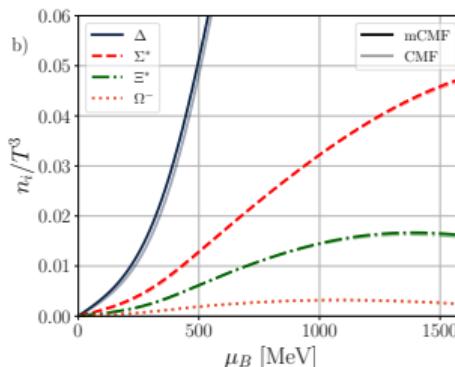
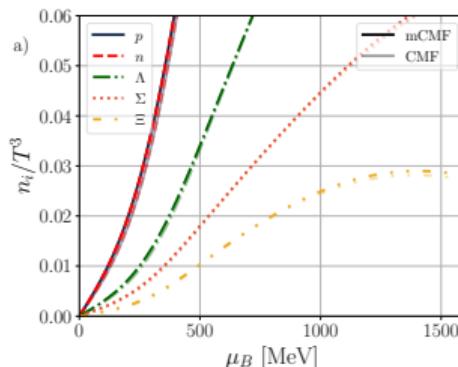
# In-medium meson mass in isospin symmetric matter



R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

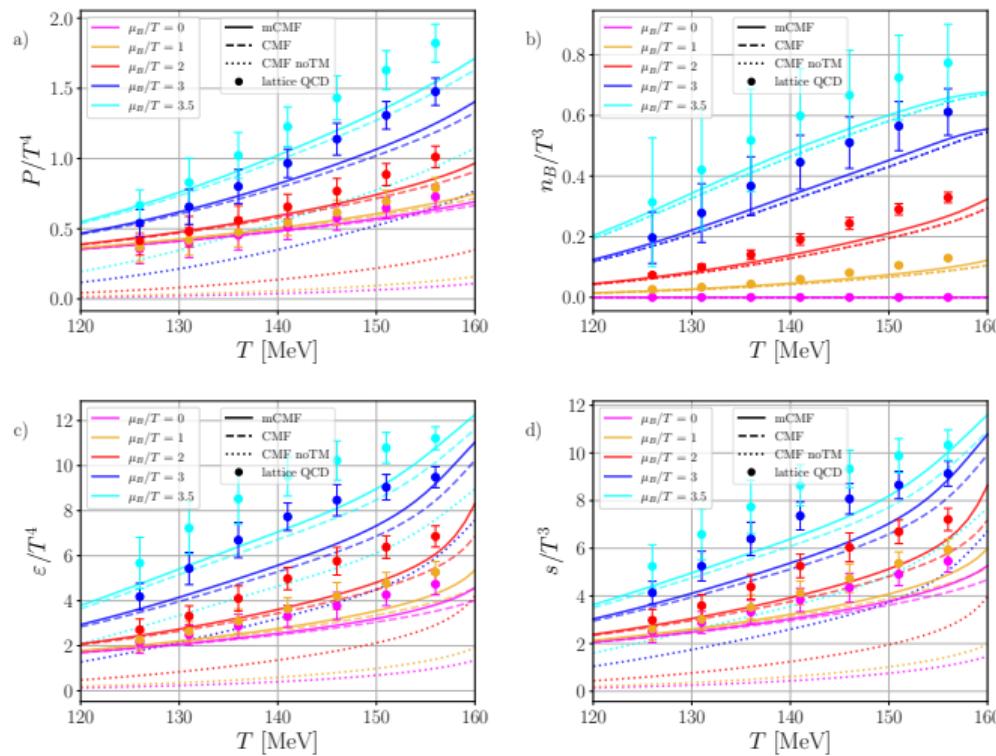
- pion masses depend directly on the  $\sigma$  meson (proxy for the chiral condensate).
  - meson masses reveal the onset of chiral symmetry restoration.
  - Their inflection point moves to a smaller  $\mu_B$  as  $T$  increases.
  - The particle population can be obtained from the model.
- Outlook: inclusion of quark degrees of freedom and deconfinement potential

# Particle population at $T = 150$ MeV in isospin symmetric matter



- pion masses depend directly on the  $\sigma$  meson (proxy for the chiral condensate).
  - meson masses reveal the onset of chiral symmetry restoration.
  - Their inflection point moves to a smaller  $\mu_B$  as  $T$  increases.
  - The particle population can be obtained from the model.
- Outlook: inclusion of quark degrees of freedom and deconfinement potential**

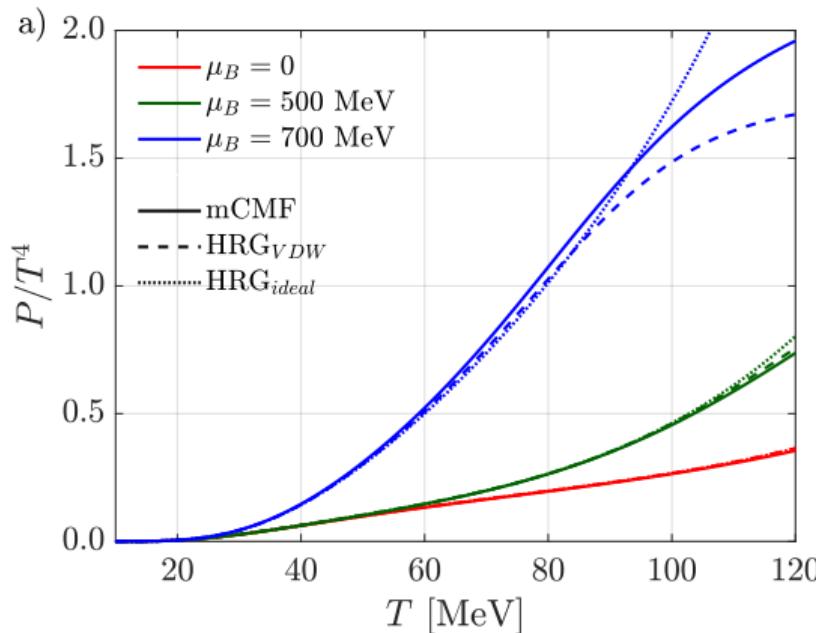
# Comparison with lattice QCD



R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

- Matching with state-of-the-art lattice thermodynamics up to  $T \sim 160$  MeV. [S. Borsányi, et al. PRL 126 \(2021\)](#)
- **Outlook:** The agreement with lattice data can be improved with the inclusion of quarks and a deconfinement potential.
- Matching with HRG data at low to intermediate temperatures.

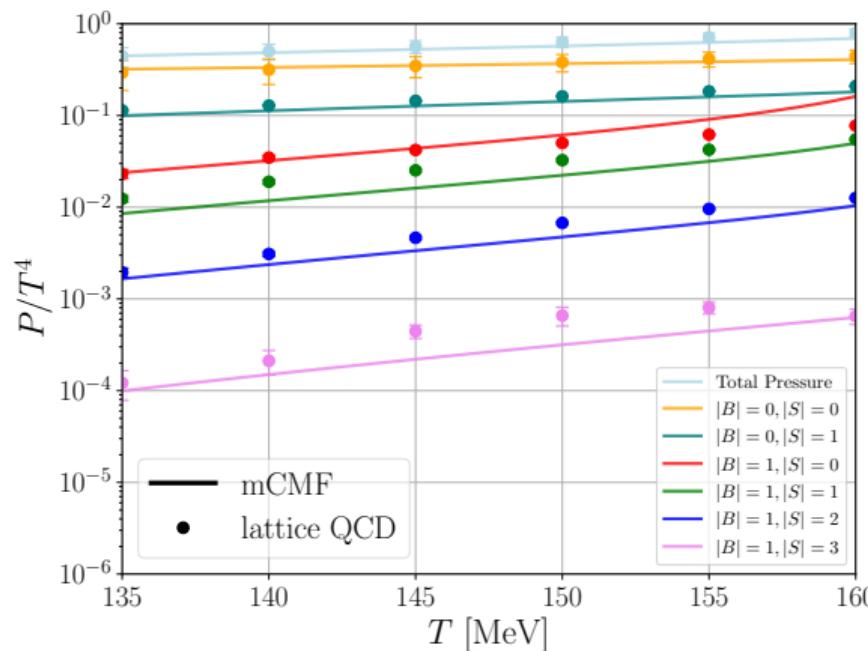
# Comparison with HRG thermodynamics



R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

- Matching with state-of-the-art lattice thermodynamics up to  $T \sim 160$  MeV. [S. Borsányi, et al.](#)  
[PRL 126 \(2021\)](#)
- **Outlook:** The agreement with lattice data can be improved with the inclusion of quarks and a deconfinement potential.
- Matching with HRG data at low to intermediate temperatures.

# Comparison of partial pressure with lattice QCD



R.K., V.D. et al. Phys. Rev. D 111, 074029 (2025)

## Partial Pressures

Good agreement between the partial pressures of hadrons from different baryonic and strange sectors in CMF and Lattice QCD data at  $\mu_B=0$ .

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# Summary

- Extended the CMF model by including pseudoscalar and vector meson interactions.
- We achieved an unprecedented level of agreement with extrapolated lattice QCD and HRG thermodynamics that could be improved by adding quark degrees of freedom.
- Our goal is to obtain a realistic equation of state for dense matter at finite temperature, which will be useful in neutron stars and neutron star merger simulations.

## Ongoing Work and Outlook:

- Obtained the EoS at finite isospin fraction with meson condensation.
- Extension to neutron star mergers simulations.
- Combine quarks and a deconfinement potential.

# Acknowledgements



*Backup Slides...*

# Discussion Points on the CMF Model

- **Finite  $\mu_B$  Fitting:** Current work focuses on zero baryon chemical potential; extension to finite  $\mu_B$  is planned for future studies to capture full QCD phase structure.
- **Coincidence of Transition Temperatures:** In the CMF model, the chiral and deconfinement transitions coincide for finite quarks due to the strong coupling between the Polyakov loop and effective baryon/quark masses.
- **Finite Size Scaling in QCD:** Employed to probe critical behavior by analyzing susceptibilities. No clear signatures of criticality have been observed so far in the explored parameter range.

*Future work will address finite  $\mu_B$  and explore potential critical points with improved resolution.*

# Estimation of error in the model

- A better agreement with the results from LIGO and VIRGO concerning the radius of the neutron star by adding vector-isovector interactions ( $\omega\rho$ ) and using a different crust.
- Low-energy nuclear physics observations are not well-constrained, therefore more wider range of constraints should be studied.

*RK, VD et al., Effects of hyperon potentials and symmetry energy in quark deconfinement, Physics Letters B 849, 138475 (2024).*

- Use statistical methods, such as Bayesian analysis, to constrain model parameters.
- Optimizing CMF code by transforming it into C++ from Fortran at finite temperature.

# Multiplets

- Baryon Octet

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

- Scalar Matrix: Mean-Fields

$$X = \begin{pmatrix} \frac{\delta^0 + \sigma}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{-\delta^0 + \sigma}{\sqrt{2}} & 0 \\ 0 & 0 & \zeta \end{pmatrix}$$

- $A_p$  matrix

$$A_p = \frac{1}{\sqrt{2}} \begin{pmatrix} m_\pi^2 f_\pi & 0 & 0 \\ 0 & m_\pi^2 f_\pi & 0 \\ 0 & 0 & 2m_K^2 f_K - m_\pi^2 f_\pi \end{pmatrix}.$$

# Multiplets

- Pseudoscalar Nonet

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} \left( \pi^0 + \frac{\eta^8}{\sqrt{1+2w^2}} \right) & \pi^+ & 2 \frac{K^+}{w+1} \\ \pi^- & \frac{1}{\sqrt{2}} \left( -\pi^0 + \frac{\eta^8}{\sqrt{1+2w^2}} \right) & 2 \frac{K^0}{w+1} \\ 2 \frac{K^-}{w+1} & 2 \frac{\bar{K}^0}{w+1} & -\sqrt{\frac{2}{1+2w^2}} \eta^8 \end{pmatrix}$$

where  $w = \sqrt{2}\zeta_0/\sigma_0$ .

- Vector Meson Nonet : Mean Fields ( $\omega, \rho$  and  $\phi$ )

$$V = \begin{pmatrix} \frac{\rho^0 + \omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0 + \omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

- Pseudoscalar Singlet

$$Y = \sqrt{\frac{1}{3}} \eta_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# The CMF Lagrangian

$$\mathcal{L}_{\text{CMF}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{\text{esb}}$$

$$\mathcal{L}_{\text{kin}} = i \text{Tr} \left( \bar{B} \gamma_\mu D^\mu B \right) = i \sum_{i \in B} \left( \bar{\psi}_i \gamma_\mu \partial^\mu \psi_i \right)$$

$$\mathcal{L}_{\text{scal}} = -\frac{1}{2} k_0 \chi_0^2 \left( \sigma^2 + \zeta^2 + \delta^2 \right) + k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right)^2 + k_2 \left[ \frac{\sigma^4 + \delta^4}{2} + \zeta^4 + 3(\sigma\delta)^2 \right]$$

$$+ k_3 \chi_0 \left( \sigma^2 - \delta^2 \right) \zeta + k_{3N} \chi_0 \left( \frac{\sigma^3}{\sqrt{2}} + \frac{3}{\sqrt{2}} \sigma \delta^2 + \zeta^3 \right) - k_4 \chi_0^4 + \frac{\epsilon}{3} \chi_0^4 \ln \left[ \frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \zeta_0} \right]$$

$$\mathcal{L}_{\text{vec}} = \frac{1}{2} \left( m_\omega^2 \omega^2 + m_\phi^2 \phi^2 + m_\rho^2 \rho^2 \right) + \mathcal{L}_{\text{vec}}^{\text{SI}}$$

$$\mathcal{L}_{\text{esb}}^u = - \left[ m_\pi^2 f_\pi \sigma + \left( \sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right]$$

$$\mathcal{L}_{\text{int}} = - \sum_{i \in B} \bar{\psi}_i \left[ \gamma_0 (g_{i\omega} \omega + g_{i\rho} \rho + g_{i\phi} \phi) + g_{i\sigma} \sigma + g_{i\zeta} \zeta + g_{i\delta} \delta \right] \psi_i$$

# Formalism: CMF thermodynamic potential

$$\begin{aligned} \frac{\Omega^H}{V} &= \frac{\Omega}{V} + \frac{\Omega_{\text{th}}^M}{V}, \\ &= U_M + \frac{\Omega_{\text{th}}^B}{V} + \frac{\Omega_{\text{th}}^M}{V}, \end{aligned} \quad (1)$$

$$U_M = \mathcal{L}_{\text{vec}} - \mathcal{L}_{\text{scal}} - \mathcal{L}_{\text{esb}} + \mathcal{L}_{\text{vac}}, \quad (2)$$

$$\frac{\Omega_{\text{th}}^B}{V} = -T \sum_{i \in \text{baryons}} \frac{\gamma_i}{2\pi^2} \int dk k^2 \left( \ln \left[ 1 + e^{-\frac{1}{T}(E_i^*(k) - \mu_i^*)} \right] + \ln \left[ 1 + e^{-\frac{1}{T}(E_i^*(k) + \mu_i^*)} \right] \right), \quad (3)$$

$$\frac{\Omega_{\text{th}}^M}{V} = T \sum_{i \in \text{mesons}} \frac{\gamma_i}{2\pi^2} \int dk k^2 \ln \left[ 1 - e^{-\frac{1}{T}(E_i^*(k) - \mu_i^*)} \right]. \quad (4)$$

# Parameters used to fit the constraints

Parameter	Term	Used to constrain
$g_1^V, g_8^V, \alpha_V, g_4$	$\mathcal{L}_{\text{int}} + \mathcal{L}_{\text{vec}}^{\text{SI}}$	$g_{N\phi} = 0, g_1^V = \sqrt{6}g_8^V, n_{\text{sat}} \approx 0.15 \text{ fm}^{-3}, B^{\text{sat}}/A \approx -15.70 \text{ MeV}, E_{\text{sym}}^{\text{sat}} \approx 28.9 \text{ MeV}, 66 \leq L^{\text{sat}}(\text{MeV}) \leq 87, 275 \leq K(\text{MeV}) \leq 305$
$m_V, \mu$	$\mathcal{L}_{\text{vec}}^{\text{m}} + \mathcal{L}_{\text{vec}}^{\text{CI}}$	$m_\omega = 770.87 \text{ MeV}, m_\rho = 770.87 \text{ MeV}, m_\phi = 1007.76 \text{ MeV}$
$m_3$	$\mathcal{L}_{\text{esb}}$	$U_\Lambda \approx -28 \text{ MeV}$
$a_0$		$T_c^d \approx 270 \text{ MeV}$
$a_1$		$n_{B,c}^d \approx 3.5 n_{\text{sat}}$
$a_2$		$T_c^{\text{HQ}} > 135 \text{ MeV}, \mu_{B,c} > 400 \text{ MeV}$
$a_3$	$U_\Phi$	$\Phi \in 0, 1$
$T_0(\text{gauge})$		$T_c^d, \Phi \in 0, 1$
$T_0(\text{quarks})$		$T_c^p \approx 159 \text{ MeV}, \Phi \in 0, 1$
$g_{q\Phi}, g_{B\Phi}$		$T_c^p$

# Dense QCD Phase Diagram: Theory vs Constraints

- Theory allows independent chemical potentials  $\mu_B$  and  $\mu_q$
- Rich phase structure predicted at high density and low temperature
- Experiments limited to high temperature and low to moderate  $\mu_B$
- No direct experimental access to large  $\mu_B$  region
- Vorticity effects seen in heavy ion collisions
- Vorticity is macroscopic, not true microphysics input
- Color superconductivity expected only in cold, fully evolved neutron stars
- Not relevant for proto neutron stars or neutron star mergers
- Neutron stars provide main constraints on dense QCD matter

# Vorticity

- Vorticity measures local rotation of a fluid or medium
- Defined as the curl of the velocity field,  $\omega = \nabla \times \mathbf{v}$
- It is a macroscopic property of the system
- Describes rotation of matter, not particle spin
- Generated by large angular momentum in non central heavy ion collisions
- Very large vorticity expected in the quark gluon plasma
- Leads to spin polarization of emitted hadrons
- Not a fundamental QCD interaction or microphysics input