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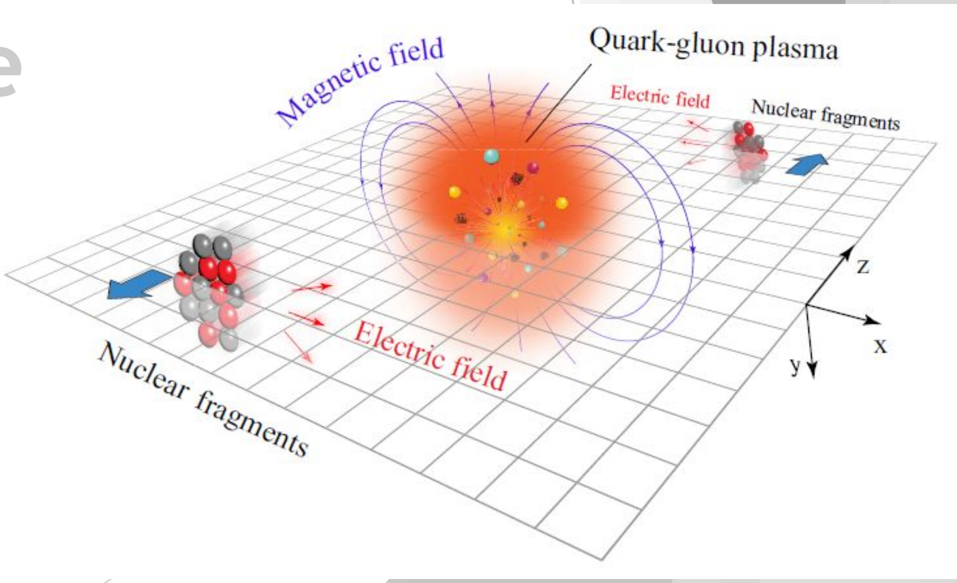
References: This code of conduct is based heavily on that of the INT and the APS. We are also grateful to Roxanne Springer for valuable discussion and guidance.

College of Integrative Sciences and Arts
Arizona State University

February 25, 2025

ELECTRICAL CHARGE TRANSPORT IN STRONGLY MAGNETIZED RELATIVISTIC MATTER

Ritesh Ghosh
Arizona State University



BASED ON:

- **Anisotropic charge transport in strongly magnetized relativistic matter,** Authors: *R Ghosh*, I. A. Shovkovy, Eur. Phys. J. C 84, 1179 (2024)

- **Electrical conductivity of hot relativistic plasma in a strong magnetic field,** Authors: *R Ghosh*, I. A. Shovkovy, Phys.Rev.D 110 (2024) 09, 096009

- **The fermion self-energy and damping rate in a hot magnetized plasma,** Authors: *R Ghosh*, I. A. Shovkovy, Phys.Rev.D 109 (2024) 9, 096018

Outline:

- Background and motivation
- Fermion in magnetic field
- damping rate
  - from self-energy
  - from the poles of the propagator
- Electric conductivity in extreme condition
  - QED conductivity
  - QCD conductivity

MATTER IN EXTREME CONDITIONS

- **High temperature** $\sim 10^{12}$ K

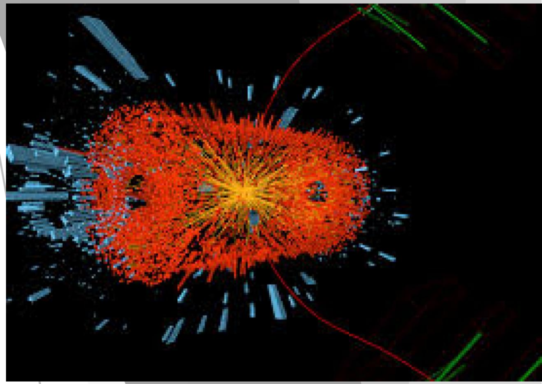
  Quark gluon plasma in heavy ion collisions, Early universe

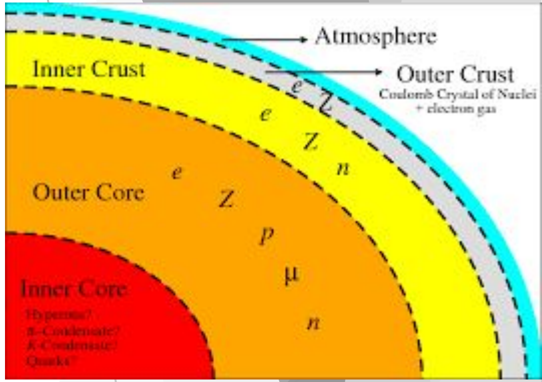
- **High densities** [redacted]

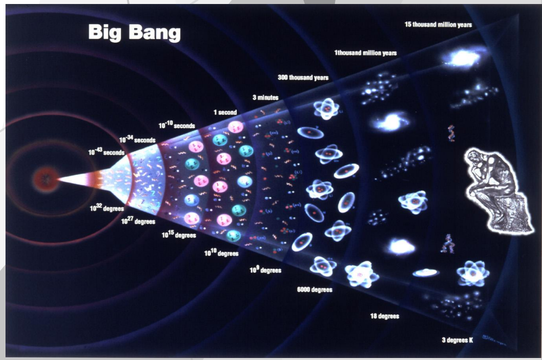
  Compact stars core, experimental lab (FAIR…)

- **High magnetic field** $\sim 10^{14} - 10^{18}$ G

  Inside compact star, non-central heavy-ion collisions







Magnetic field:

- Compact stars
  - equation of state, mass-radius relation, gravitational collapse/merger, neutrino emission from star

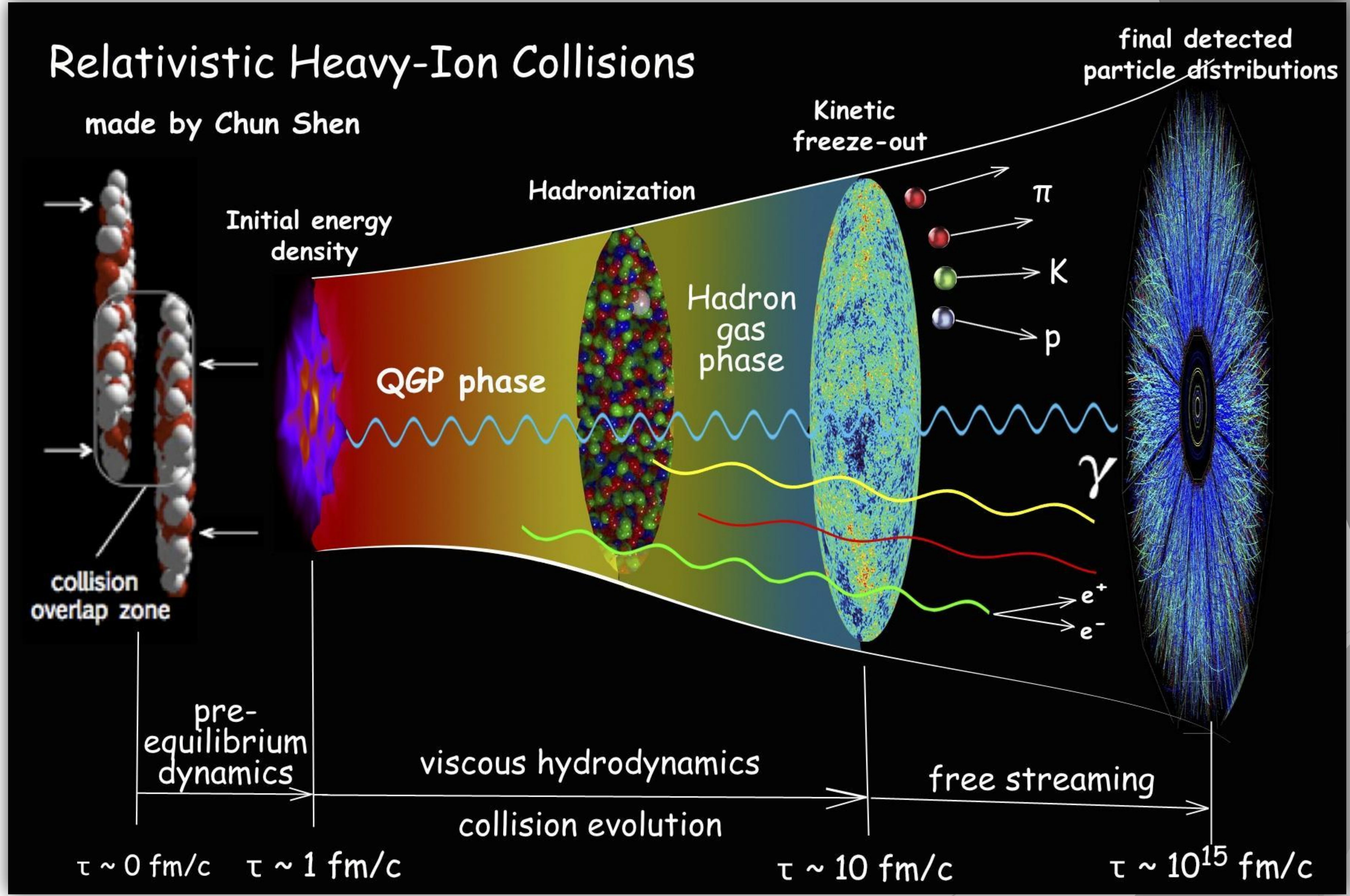
    [Duncan et al. *Astrophys.J.Lett*. 392 (1992) L9]
    [Ferrar et al. arXiv: 1009.3521]
    [Anderson et al. *Phys.Rev.Lett*. 100 (2008) 191101]
    [Ghosh & Shovkovy, arXiv: 2501.03318]

- In the early universe
  - Magnetic fields from cosmological phase transitions

    [Vachaspati et al. *Phys.Lett.B* 265 (1991) 258-261]
    [Enqvist, Olesen, *Phys.Lett.B* 319 (1993) 178-185]

- Non-central heavy-ion collisions
  - chiral magnetic effect, anisotropies, elliptic flow

    [Kharzeev et al., arXiv:0711.0950]
    [Fukushima et al. arXiv:1209.5064]



ASU

Chun Shen

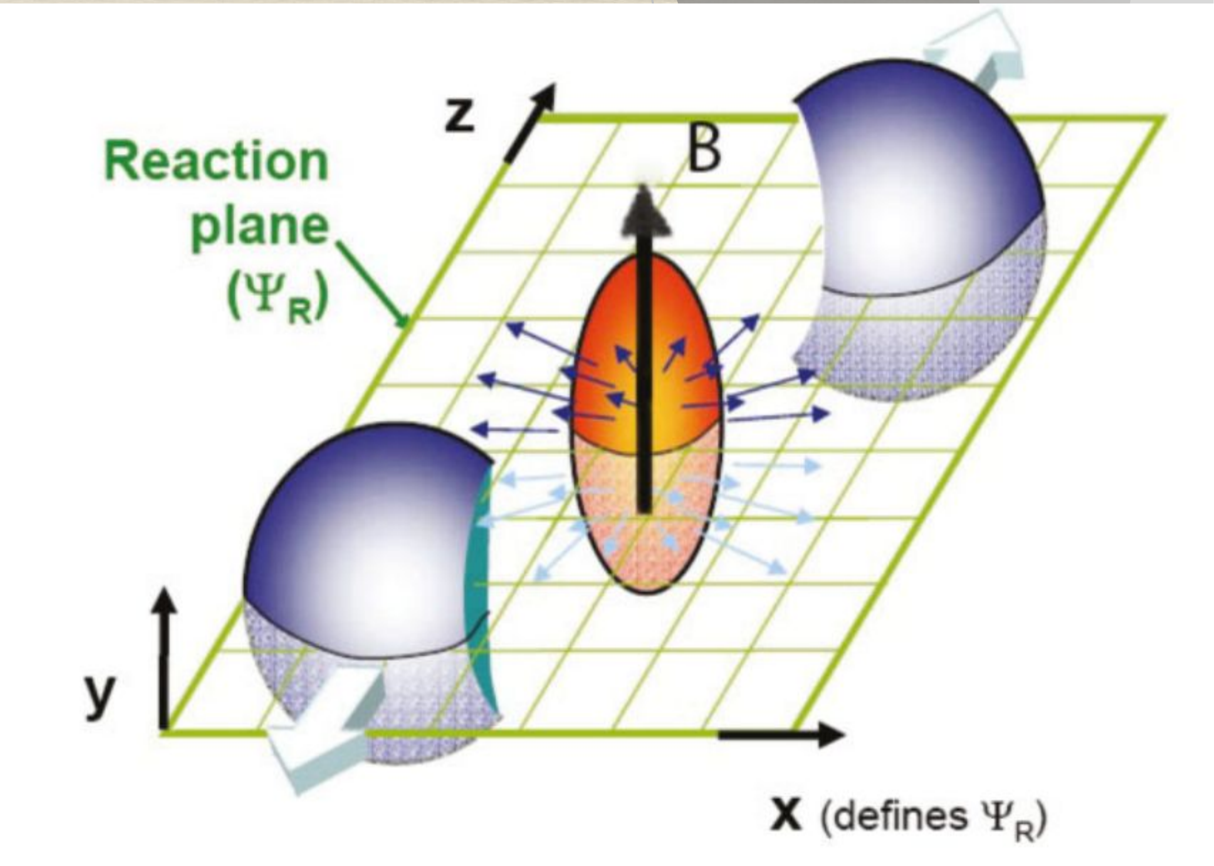
Magnetic field in HICs

Off-center heavy ion collision

Magnetic field strength $|eB| \sim m_\pi^2$

$m_\pi^2 \sim 10^{18} G$

Short lived 10^{-24} s



[Ferrer, Incera, arXiv: 1603.08226]

Earth magnetic field~ 10^{-1} G

refrigerator magnets ~ 100 G

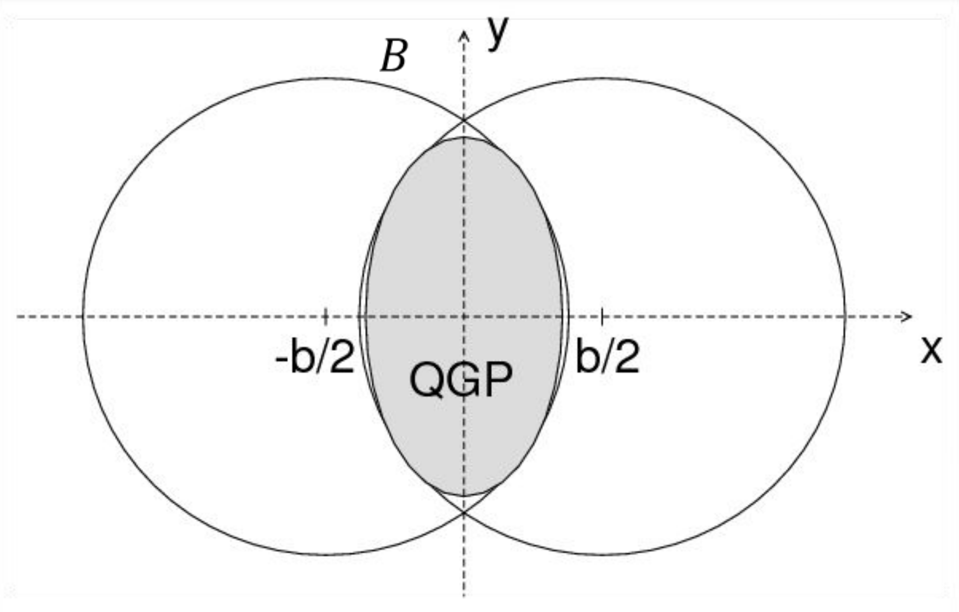
Neutron star~ $10^{14} - 10^{17}$ G

[Skokov et. al, 0907.1396 [nucl-th]]
[Zhong et. al, 1408.5694 [hep-ph]]

LIENARD-WIECHERT POTENTIAL:

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$



Non-relativistic limit,

$$v_n \ll 1$$

Coulomb's law:

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n}{R_n^3},$$

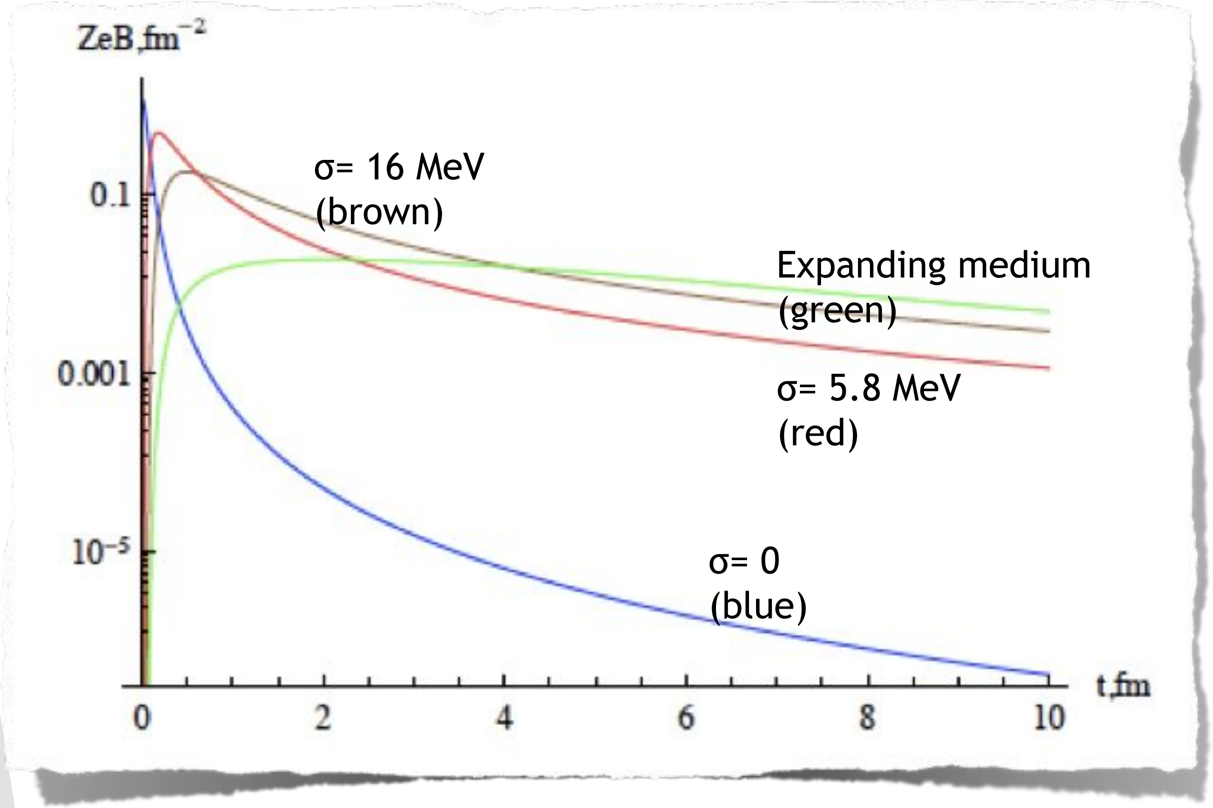
Biot-Savart law:

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{R_n^3}.$$

[Rafelski & Müller, PRL, 36, 517 (1976)]
[Kharzeev et al., arXiv:0711.0950]
[Skokov et al., arXiv:0907.1396]
[Voronyuk et al., arXiv:1103.4239]

Conductivity?

[Tuchin, Adv.High Energy Phys. 2013 (2013) 490495]



High electric conductivity can prolong the lifetime of magnetic fields

$1 fm/c = 3.3 \times 10^{-24} sec$ $\quad$ $\mathrm{fm}^{-2} \approx 2m_\pi^2$

CONDUCTIVITY @ B≠0

Phenomenological models: (Unreliable !!)

[Mamo, JHEP 08, 083 (2013)]
[Fukushima & Okutsu, Phys. Rev. D 105, 054016 (2022)]
[Kurian & Chandra, Phys. Rev. D 96, 114026 (2017)]
[Das, Mishra, Mohapatra, Phys. Rev. D 101, 034027 (2020)]
[Satapathy, Ghosh, Ghosh, Phys. Rev. D 104, 056030 (2021)]
[Bandyopadhyay et al. EPJC 83, 489 (2023)]

limitations of kinetic theory @B≠0!

Attempts within a gauge theory (LLL approximation or "longitudinal" kinetic theory):

[Hattori & Satow, PRD 94, 114032 (2016)]
[Hattori, Satow, Yee, Phys. Rev. D 95, 076008 (2017)]
[Fukushima & Hidaka, Phys. Rev. Lett. 120, 162301 (2018)]
[Fukushima & Hidaka, JHEP 04, 162 (2020)]

Lattice calculations:

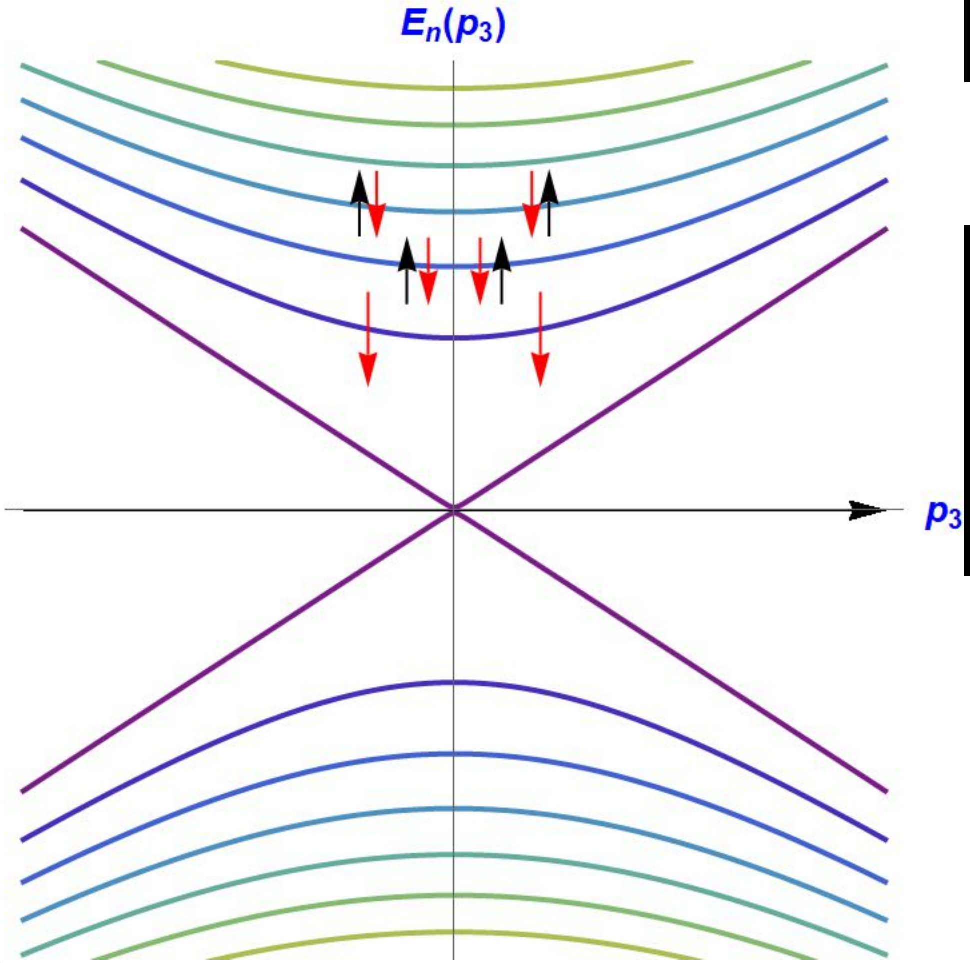
[Buividovich et al. Phys. Rev. Lett. 105, 132001 (2010)]
[Astrakhantsev et al. Phys. Rev. D 102, 054516 (2020)]
[Almirante et al. arXiv:2406.18504]

Landau Levels

Relativistic:

$$E_{kin} \geq E_{rest}$$

$$v \sim c$$



Dirac Equation

[redacted]

Energy spectrum:

[redacted]

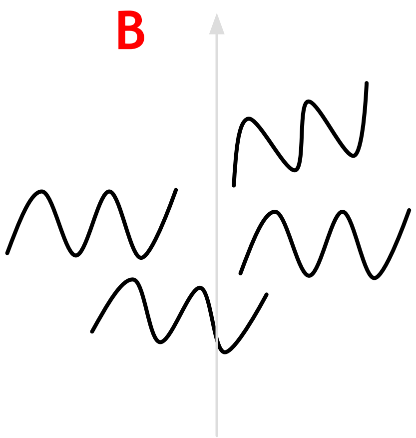
Spin quantum number: s=± 1/2

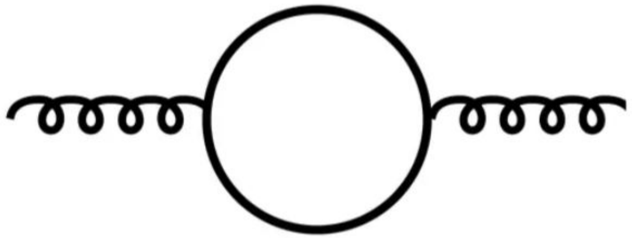
Orbital: k=0,1,2,…

[Miranski & Shovkovy, *Phys.Rept*. 576 (2015) 1-209]

Gluons or photons in strong field

No ‘electric charge’
-does not feel B at zeroth order





Effective mass through coupling with fermions

$$M^2 \propto \alpha |eB|$$

[Miranski & Shovkovy, Phys. Rev. D 66, 045006 (2002)]

FERMION PROPAGATOR @B≠0

Field theoretical approach

The fermion propagator in coordinate space:

[redacted]

$u \equiv (t, x, y, z)$

translation invariant part

Schwinger phase:

[redacted]

$p_{\perp}^{\mu} = (p_x, p_y)$

$p_{\parallel}^{\mu} = (p_0, p_z)$

Fourier transform of the translation invariant part (free propagator)

spin projectors:

$$\mathcal{P}_{\pm} = \left(1 \pm s_{\perp} i\gamma^1\gamma^2\right)/2$$

$$s_{\perp} = \text{sign}(qB)$$

$$\ell = 1/\sqrt{|qB|}$$

[redacted]

[redacted]

generalized Laguerre polynomials

Fermion propagator in the spectral form:

[redacted]

ELECTRICAL CONDUCTIVITY

$$j^i = \sigma^{ij} E^j$$

$$B = 0$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

$$\vec{J} = \sigma_0 \vec{E}$$

$$\vec{B} = B\hat{z}$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_\perp & -\sigma_H & 0 \\ \sigma_H & \sigma_\perp & 0 \\ 0 & 0 & \sigma_\parallel \end{bmatrix}$$

ELECTRICAL CONDUCTIVITY

[Ghosh, Shovkovy, Phys.Rev.D 110 (2024) 09, 096009]
& [Ghosh, Shovkovy, Eur. Phys.J.C 84, 1179 (2024)]

Kubo formula:

$$\sigma_{ij} = \lim_{\Omega\to 0} \frac{\mathrm{Im}\left[\Pi_{ij}(\Omega + i0; \mathbf{0})\right]}{\Omega} = -\frac{\alpha}{8\pi T} \sum_{f=1}^{N_f} q_f^2 \int \frac{dk_0 d^3\mathbf{k}}{\cosh^2 \frac{k_0-\mu_f}{2T}} \mathrm{tr}\left[\gamma^i A_{\mathbf{k}}^f(k_0)\gamma^j A_{\mathbf{k}}^f(k_0)\right].$$

Fermion spectral function

Spectral function ∝ fermion damping rate (Γ_n)

[redacted]

[redacted]

$\Gamma_n \to 0$

$\sigma_{\parallel} \to \infty$

$\sigma_{\perp} \to 0$

When interactions included:

$\sigma_{\parallel}$ Finite because of scattering

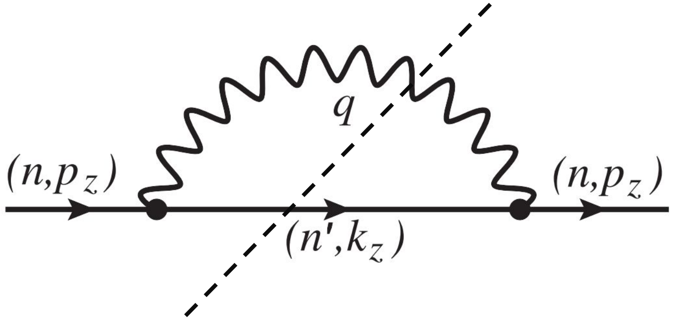
$\sigma_{\perp} \neq 0$ Because of Landau level hopping

1. FERMION DAMPING RATE FROM SELF-ENERGY:

[Ghosh, Shovkovy, Phys. Rev. D 109, 096018 (2024)]

$B \neq 0$

[redacted]

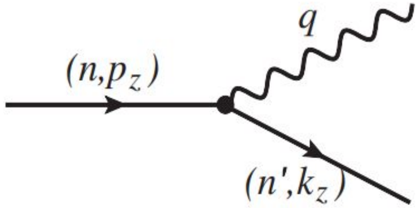


$= \mathrm{Im}\Sigma_n(p_z)$

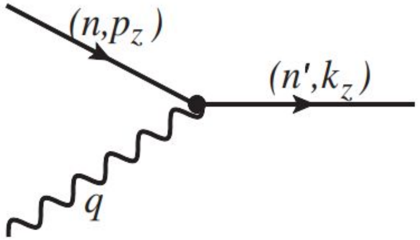
Γ_n Order of $\alpha|eB|/T$

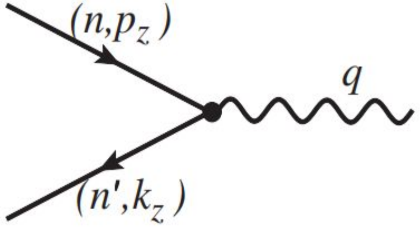
- Underlying processes: $eB \gg \alpha T^2$

$n > n'$



$n' > n$





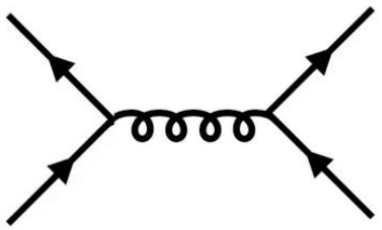
Subleading $\alpha^2 T$

kinematically forbidden In the absence of a magnetic field

Leading order processes:

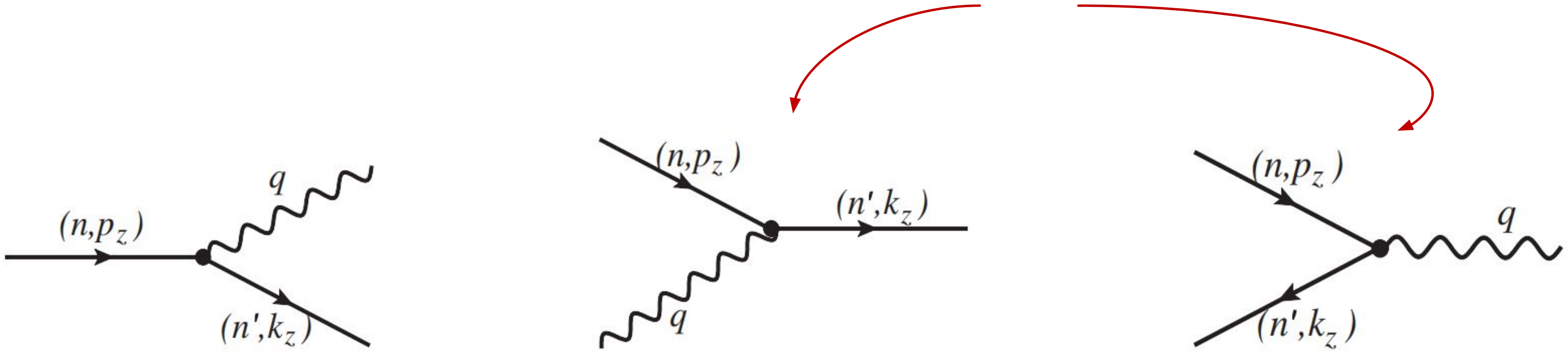
$B = 0$: $2 \to 2$ are leading processes.

$1 \leftrightarrow 2$ Kinematically forbidden



$B \neq 0$: $1 \leftrightarrow 2$ are leading processes.

$n = 0$



$n > n'$ $n' > n$

Analytic expression:

[redacted]

$\propto$ Matrix amplitude squared

- positive definite quantity

[redacted]

$$\xi^{\pm} = \frac{1}{2}\left[\sqrt{2n' + (\bar{m}_0/qB)^2} \pm \sqrt{2n + (\bar{m}_0/qB)^2}\right]^2$$

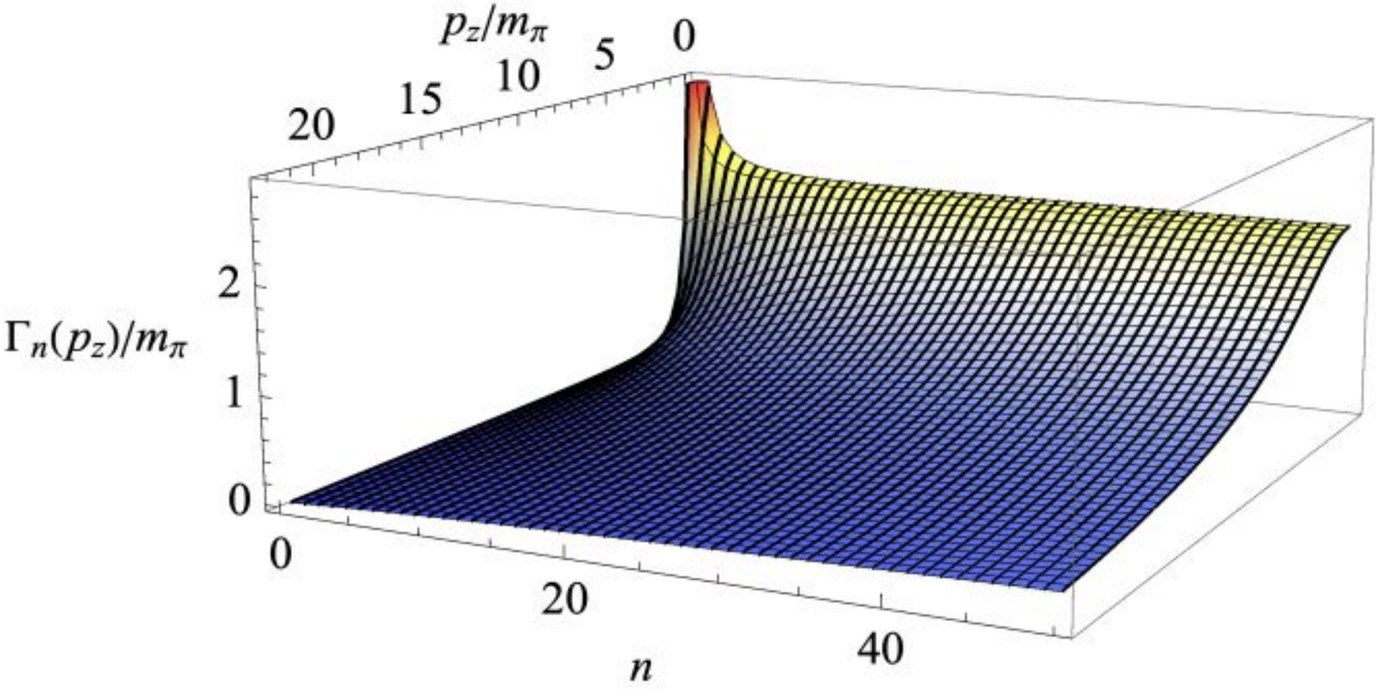
Energy conservation:

$$p_0 = s_1 E_{n',k_z} + s_2 E_q$$

$$\begin{aligned}
&\psi_n \to \psi_{n'} + \gamma \quad (s_1 > 0,\ s_2 > 0): && 0 < \xi < \xi^-, \\
&\psi_n + \gamma \to \psi_{n'} \quad (s_1 > 0,\ s_2 < 0): && 0 < \xi < \xi^-, \\
&\psi_n + \bar{\psi}_{n'} \to \gamma \quad (s_1 < 0,\ s_2 > 0): && \xi^+ < \xi < \infty.
\end{aligned}$$

Damping rate:

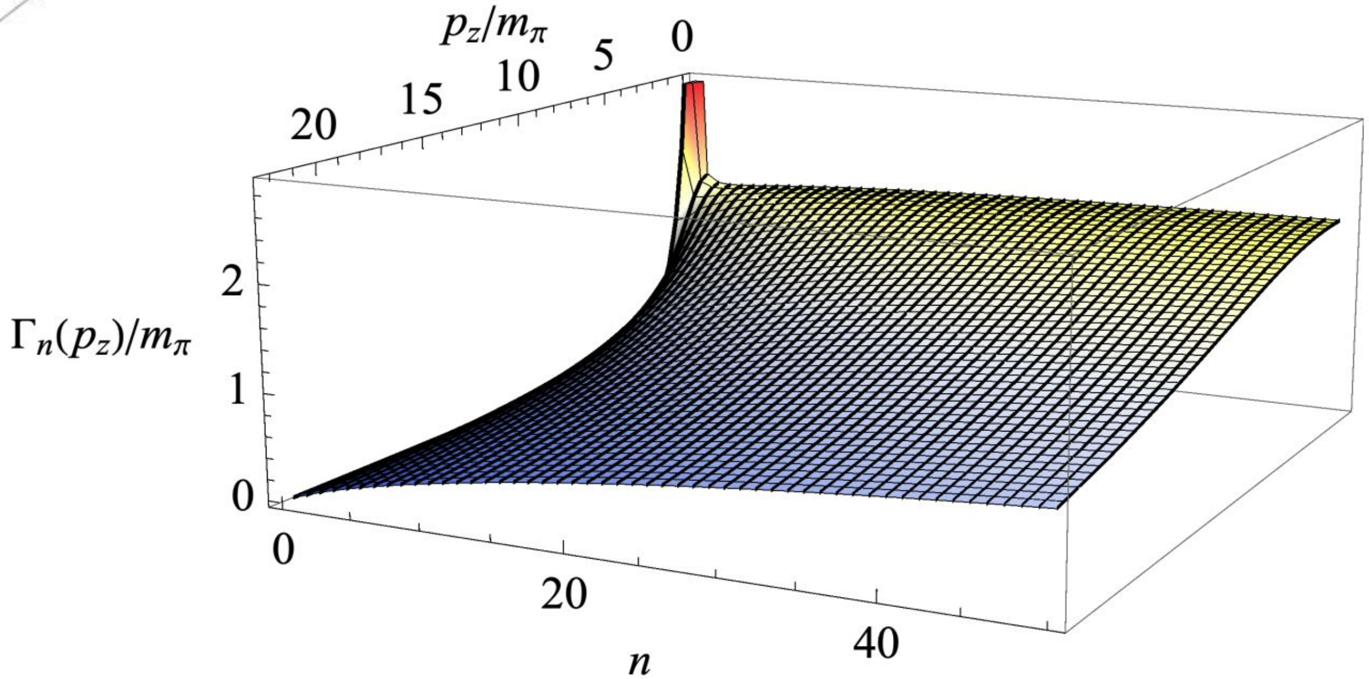
$$n'_{\max} = 2n_{\max} = 100$$



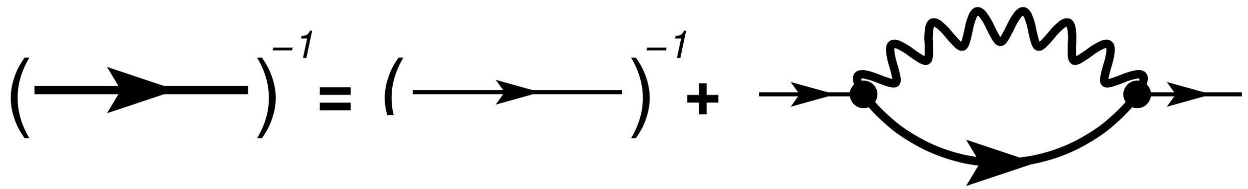
T=400 MeV

$$|qB| = (200\,\text{MeV})^2$$

$$|qB| = (75\,\text{MeV})^2$$



2. Damping rates from the poles of the propagator:



Quark self-energy

$$\bar{G}^{-1} = \bar{S}^{-1} + i\Sigma$$

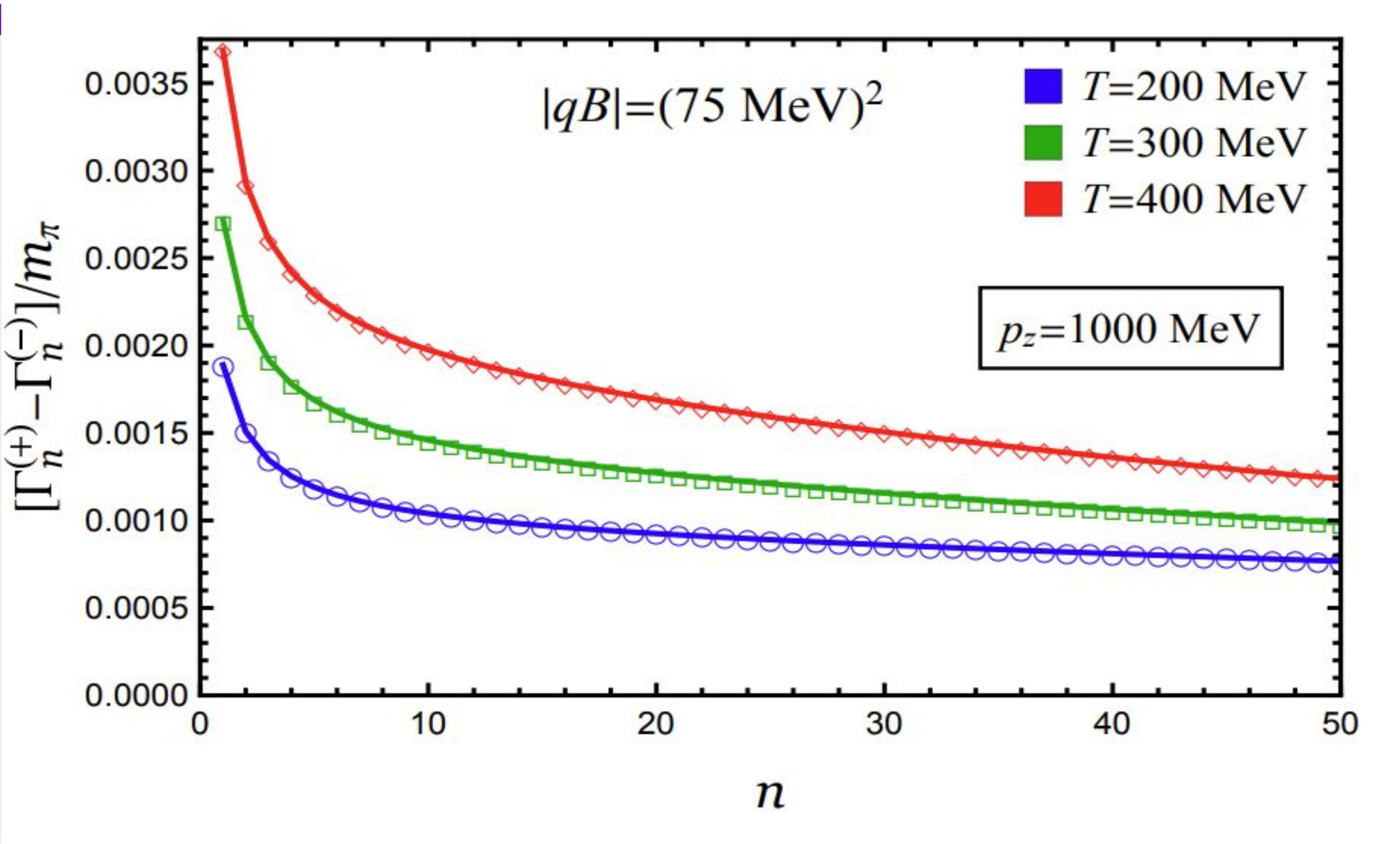
$$\begin{aligned}\bar{\Sigma}(p_\parallel, \boldsymbol{p}_\perp) &= -2e^{-p_\perp^2\ell^2}\sum_{n=0}^{\infty}(-1)^n\left[\delta v_{\parallel,n}(p_\parallel\cdot\gamma_\parallel) + i\gamma^1\gamma^2(p_\parallel\cdot\gamma_\parallel)\tilde{v}_n - \delta m_n - i\gamma^1\gamma^2\tilde{m}_n\right]\left[\mathcal{P}_+ L_n(2p_\perp^2\ell^2) - \mathcal{P}_- L_{n-1}(2p_\perp^2\ell^2)\right] \\ &- 4e^{-p_\perp^2\ell^2}\sum_{n=0}^{\infty}(-1)^n\delta v_{\perp,n}(\boldsymbol{\gamma}_\perp\cdot\boldsymbol{p}_\perp)L^1_{n-1}(2p_\perp^2\ell^2)\end{aligned}$$

splitting of the parallel velocities and masses of the two spin states

Poles:

$$\det[\bar{G}^{-1}] = 0$$

Two spin-split Landau-level states : different da[...]



$$\Gamma_n^{(\pm)}$$

$$|\Gamma_n^{(+)} - \Gamma_n^{(-)}| \ll \Gamma_n^{(\mathrm{ave})}$$

$$\Gamma_n^{(\mathrm{ave})} \equiv (\Gamma_n^{(+)} + \Gamma_n^{(-)})/2$$

CONDUCTIVITY OF QED PLASMA

[Ghosh, Shovkovy, Phys.Rev.D 110 (2024) 09, 096009]

$$\sigma_{\parallel}/T = \tilde{\sigma}_{\parallel}(|eB|/T^2)$$

$$\sigma_{\perp}/T = \tilde{\sigma}_{\perp}(|eB|/T^2)$$

$$T >> m_e$$

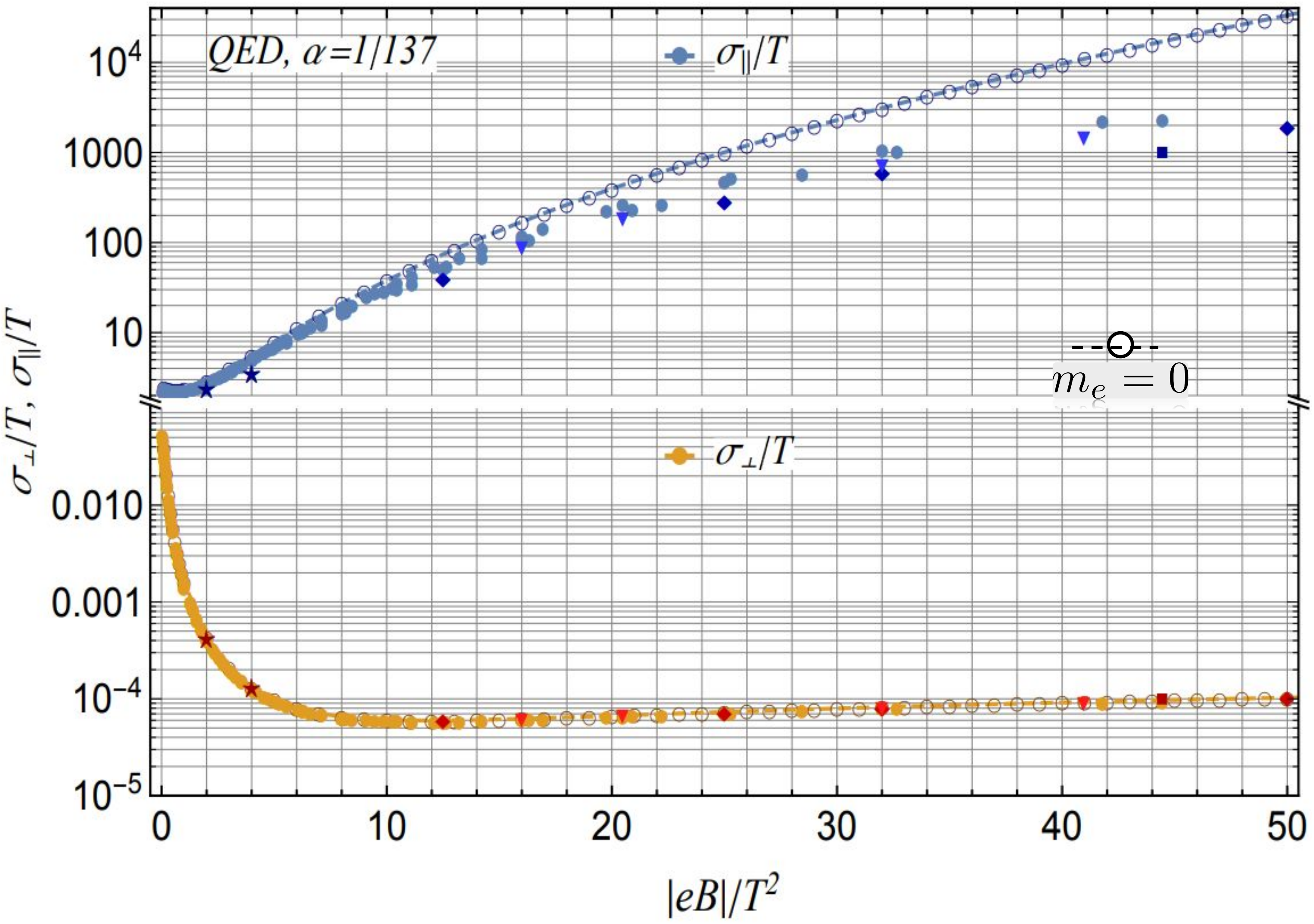
$$\sqrt{|eB|} >> m_e$$

$$\sigma_{\parallel} \propto 1/\Gamma_n(p_z)$$

- tends to decrease with temperature (like metals)

$$\sigma_{\perp} \propto \Gamma_n(p_z)$$

- tends to increase with temperature (like semiconductors)



$$15m_e \leq T \leq 80m_e \quad \text{and} \quad (15m_e)^2 \leq |eB| \leq (200m_e)^2$$

TRANSPORT MECHANISM

- Longitudinal conductivity:

[redacted]

- Conductivity is hindered by transitions or scattering events.

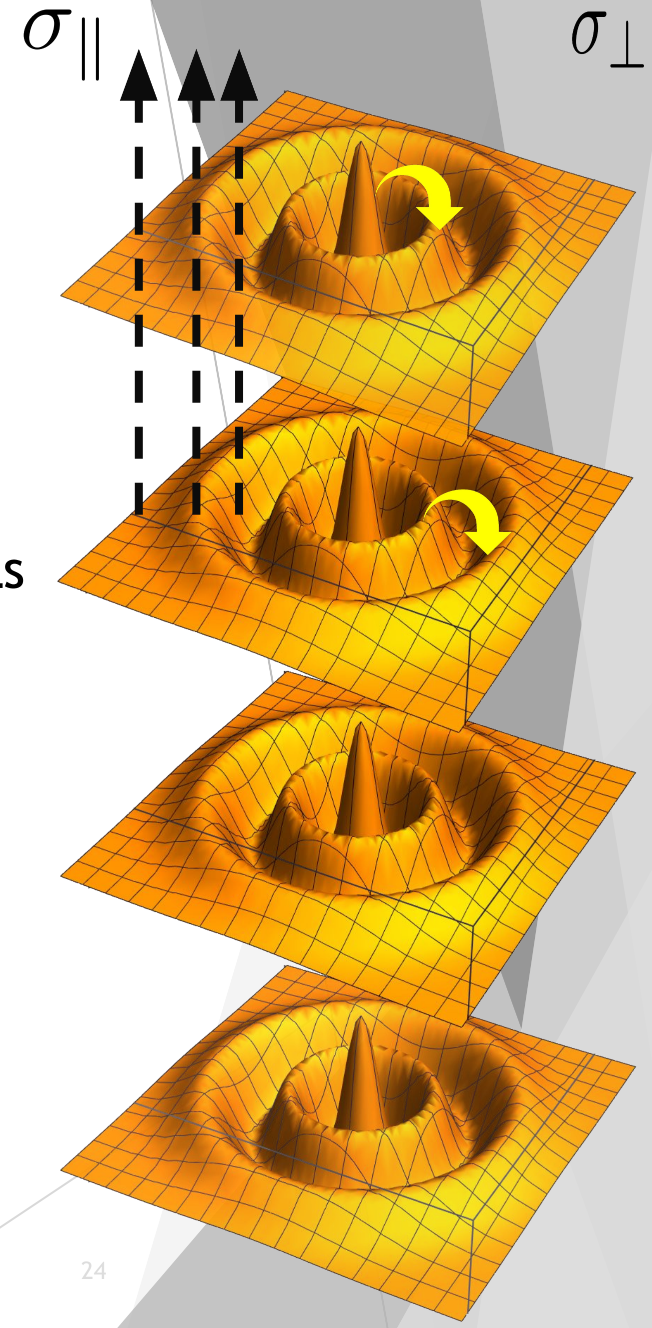
- damping rates are determined by scattering and transitions to other LLs
  ---Individual LLs contribute like independent species

- Transverse conductivity:

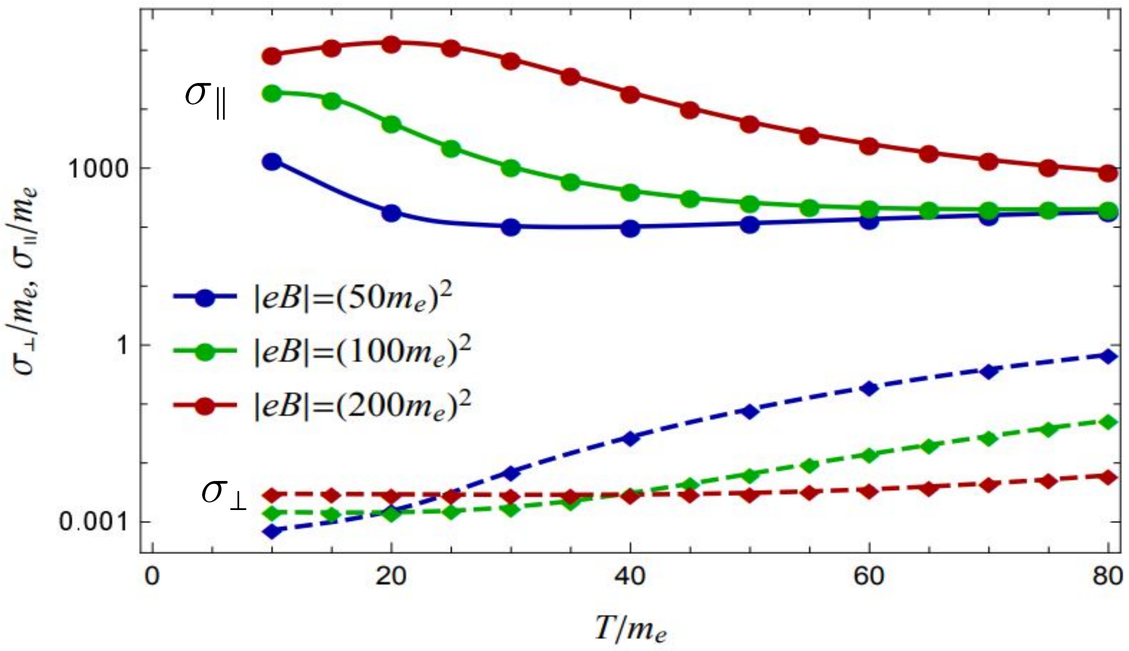
[redacted]

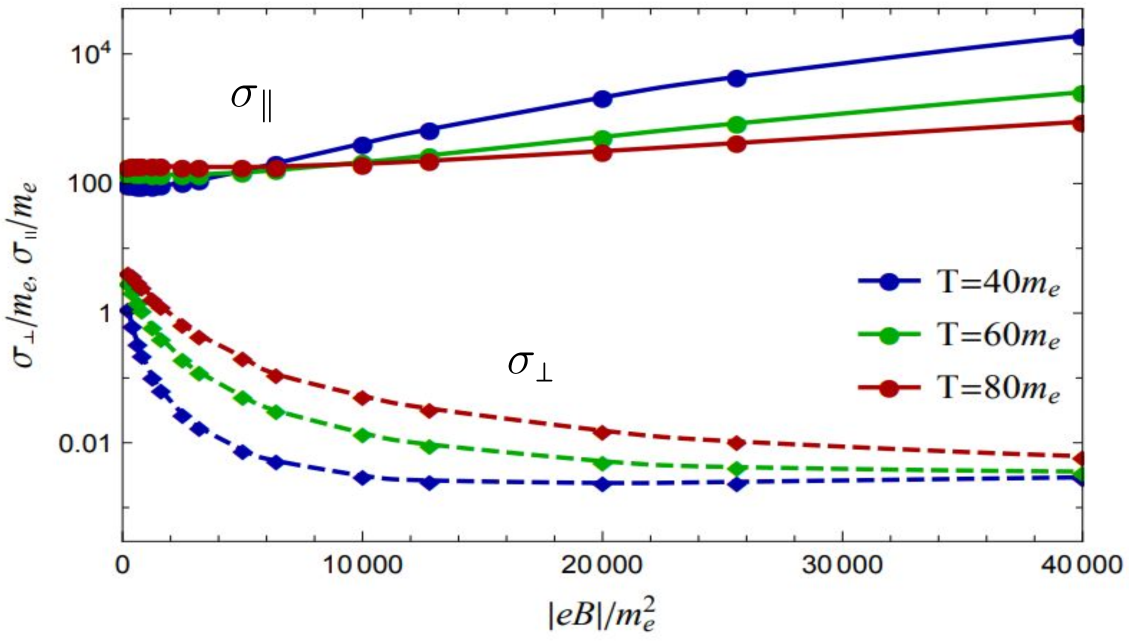
– Conductivity is driven by transitions (hopping) between LLs

– At least, transitions between 0th and 1st LLs are required



T and B-dependence





T- dependence:

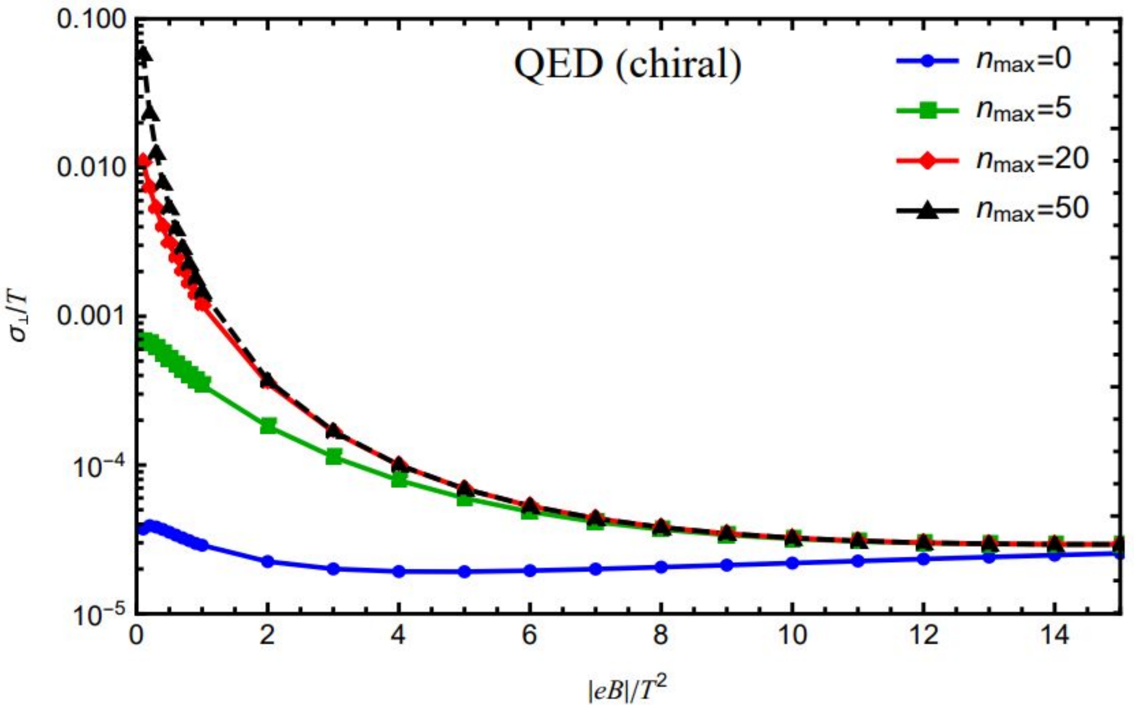
$\sigma_\parallel \propto 1/\Gamma_n(p_z)$
- tends to decrease with temperature (like metals)

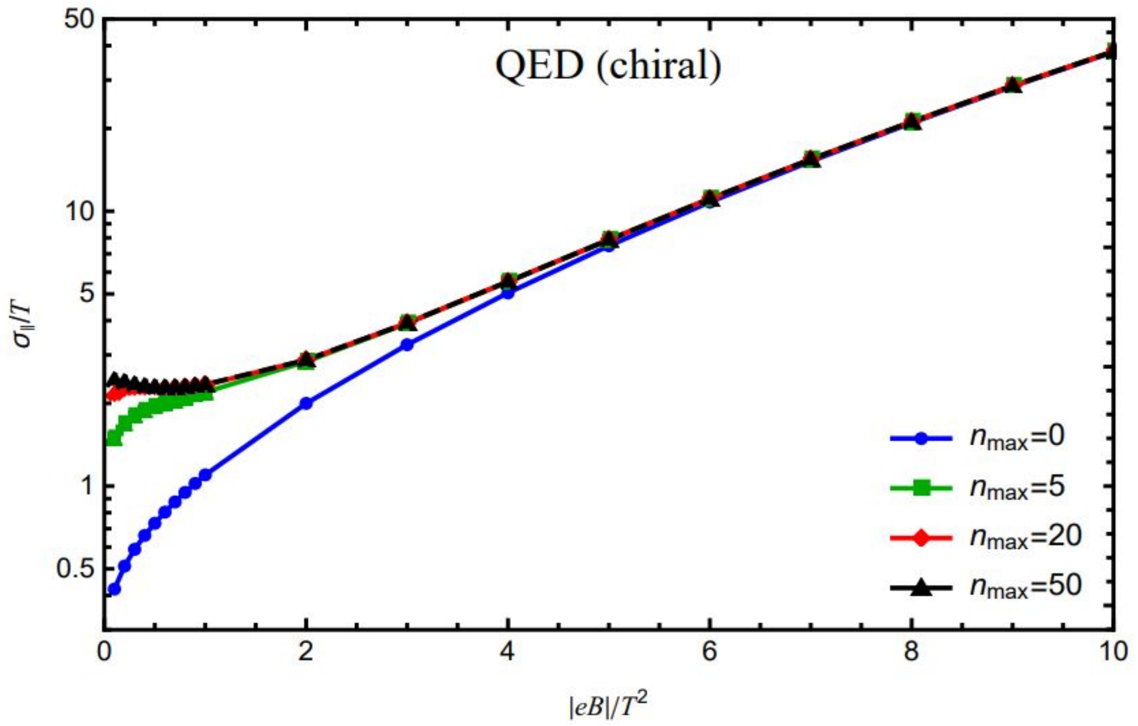
$\sigma_\perp \propto \Gamma_n(p_z)$
- tends to increase with temperature (like semiconductors)

B- dependence:

$\sigma_\parallel$ increases and $\sigma_\perp$ decreases with B

Landau-Level Sum Convergence





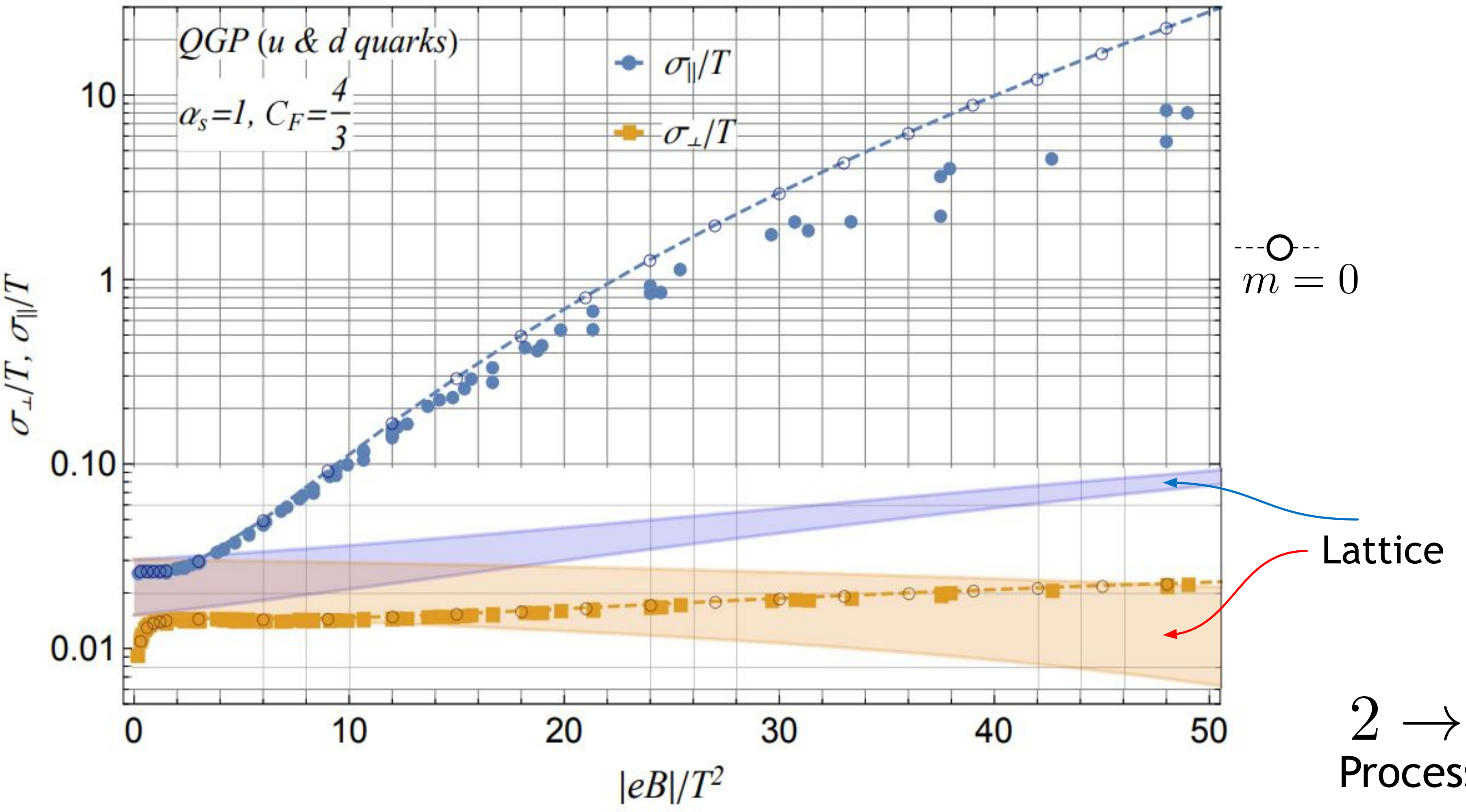
- A significant number of Landau levels must be included across a broad range of parameters.

$\sigma_\parallel$: Requires $n_{\max} \gtrsim 10T^2/|eB|$

$\sigma_\perp$: Requires $n_{\max} \gtrsim 30T^2/|eB|$

CONDUCTIVITY OF QCD PLASMA

[Ghosh, Shovkovy, Eur. Phys.J.C 84, 1179 (2024)]



$$\frac{\Delta\sigma_{\parallel}}{TC_{\rm em}} = \frac{3\,|eB|}{4\pi^2T^2}C_\tau(|eB|)$$

$C_\tau(4\ \mathrm{GeV}^2) \approx 0.134$

$C_\tau(9\ \mathrm{GeV}^2) \approx 0.142$

[Almirante et al. arXiv:2406.18504]

$2 \to 2$ Processes?

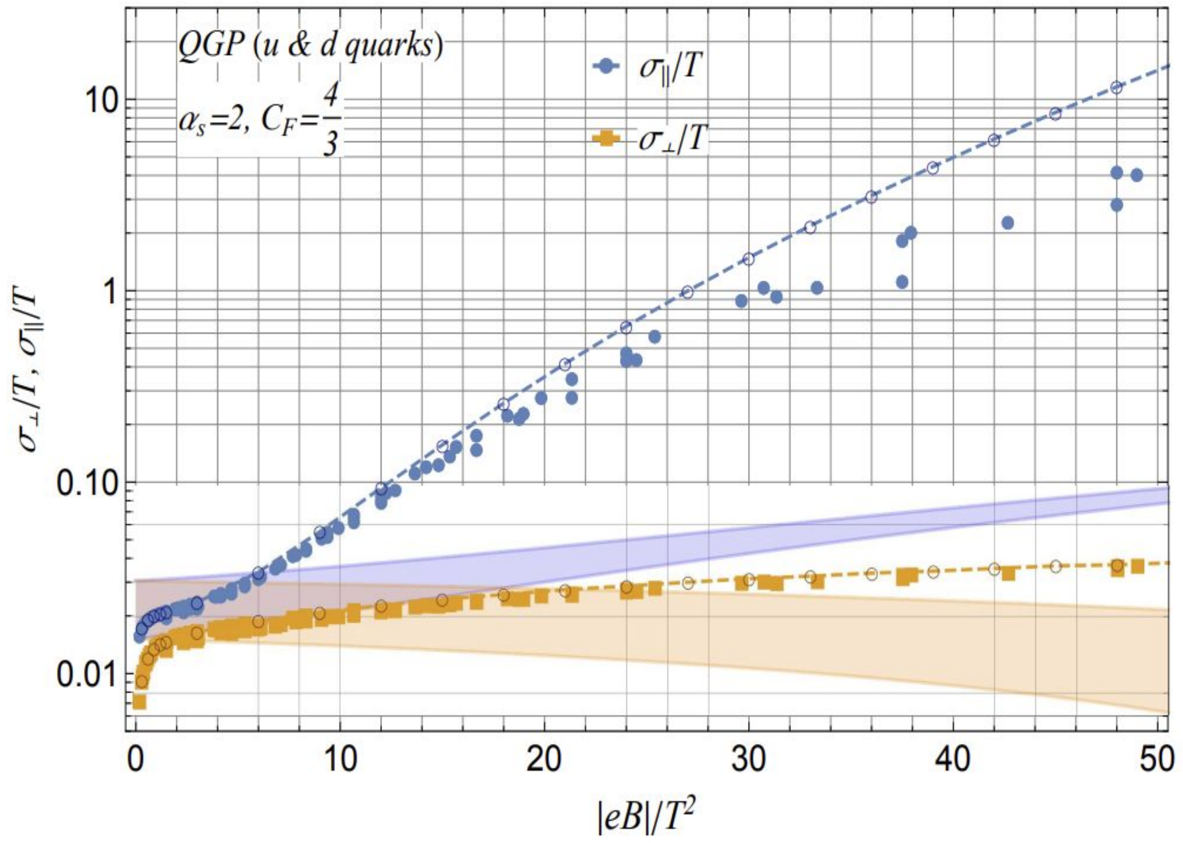
α_s?

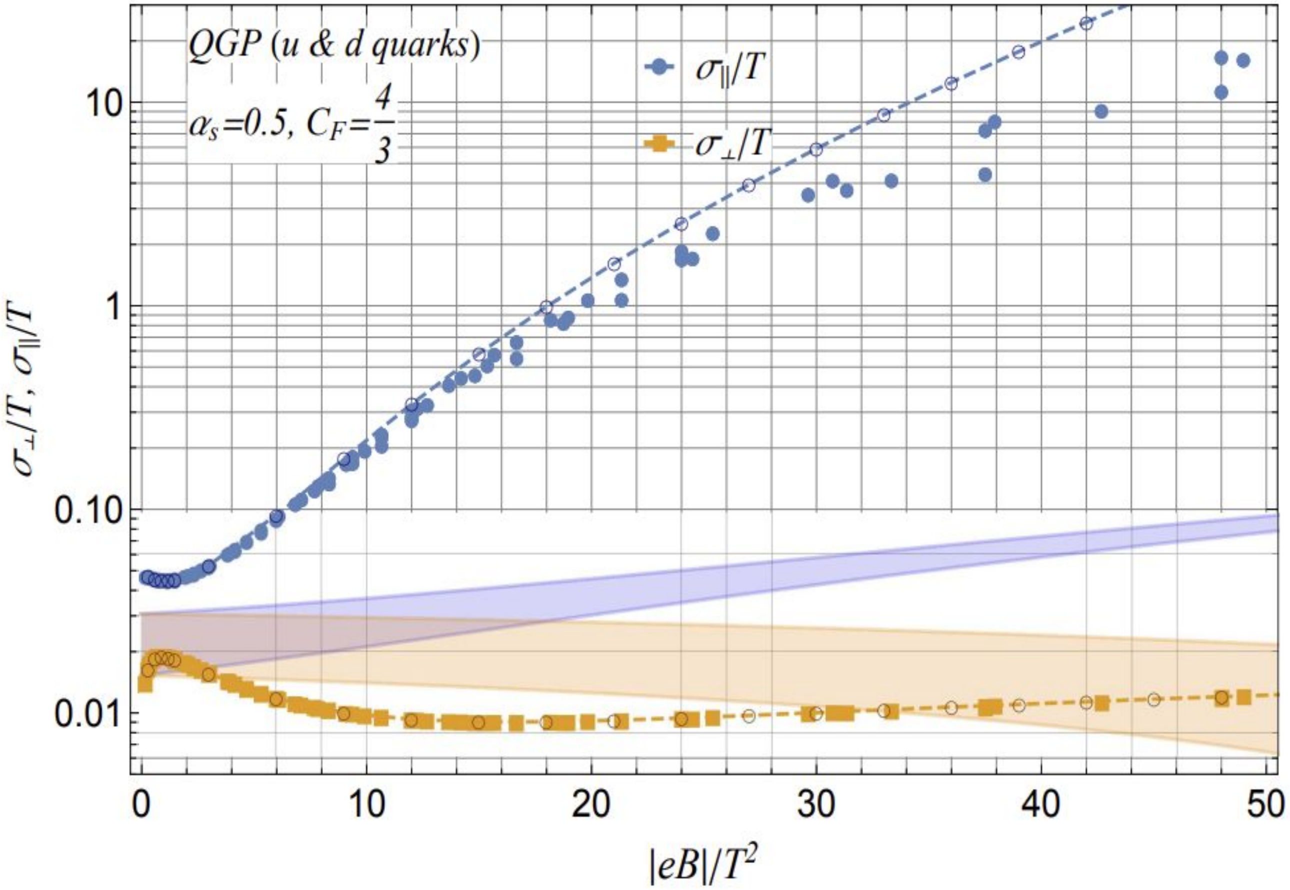
$B=0$ (Lattice)

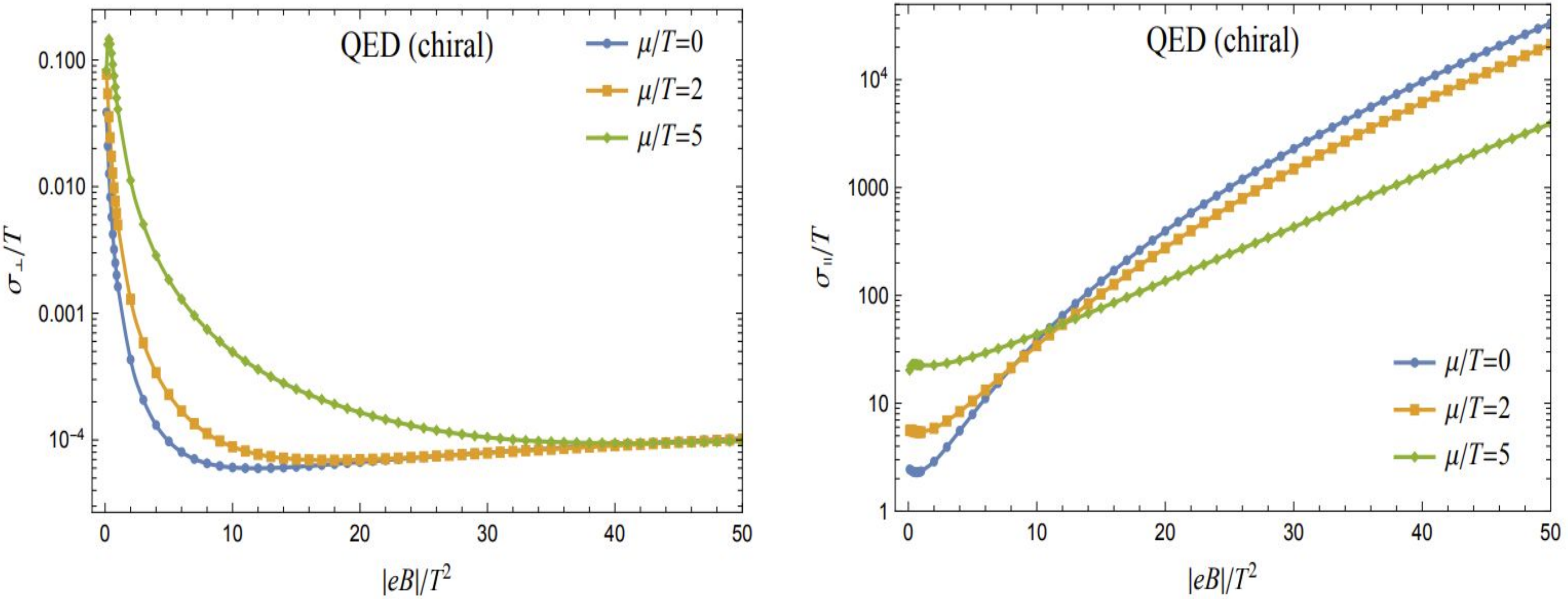
$\sigma \approx 1.1\,\mathrm{MeV}@T = 200\,\mathrm{MeV}$ to $5.6\,\mathrm{MeV}@T = 350\,\mathrm{MeV}$

[Aarts et al. JHEP 1502, 186 (2015)]

Conductivities in two-flavor QGP for two different choices of strong coupling







$$\sigma^{ij} = -\frac{\alpha}{8\pi T} \sum_{f=1}^{N_f} q_f^2 \int \frac{dk_0 d^3\boldsymbol{k}}{\cosh^2 \frac{k_0 - \mu_f}{2T}} \mathrm{tr}\left[\gamma^i A_{\boldsymbol{k}}^f(k_0) \gamma^j A_{\boldsymbol{k}}^f(k_0)\right].$$

SUMMARY: CONDUCTIVITY @ $B \neq 0$

- Conductivity is calculated for a plasma in a strong magnetic field from first principles within a gauge theory (QED/QCD).

- The transverse conductivity is suppressed, while the longitudinal conductivity is enhanced by a strong magnetic field. ---Anisotropic nature …Different mechanisms

- Transverse conduction critically relies on quantum transitions between Landau levels, effectively lifting charge trapping in localized Landau orbits.

- The damping rates are determined by $1 \leftrightarrow 2$ processes.

- The results are relevant for understanding a wide range of extreme astrophysical environments, such as those found in neutron stars, supernovae, and heavy-ion collisions.

<u>Outlook:</u>

- How other sub-leading processes contribute to the conductivity?
- Behavior at finite chemical potential?


Thank you