Rising Researchers Seminar Series

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Participants shall **treat each other professionally and respectfully**. Participants shall refrain from any inappropriate behaviour, including discriminatory actions and comments based on individual characteristics such as age, race, ethnicity, sexual orientation, gender identity, gender expression, marital status, nationality, immigration status, political affiliation, ability status, or educational background. Disruptive or harassing behavior will not be tolerated including but not limited to inappropriate or intimidating language and unwelcome jokes or comments.

Violations of this code of conduct should be reported to any member of the executive committee (see About Us page on the website). The Executive committee may also act of their own accord should they notice violations. Sanctions may range from verbal warning, to ejection from the meeting or, in severe cases, partial or total bans from future meetings.

References: This code of conduct is based heavily on that of the INT and the APS. We are also grateful to Roxanne Springer for valuable discussion and guidance.

QUANTUM SIMULATION OF SU(3) LGT AT LEADING ORDER IN LARGE N

Anthony Ciavarella

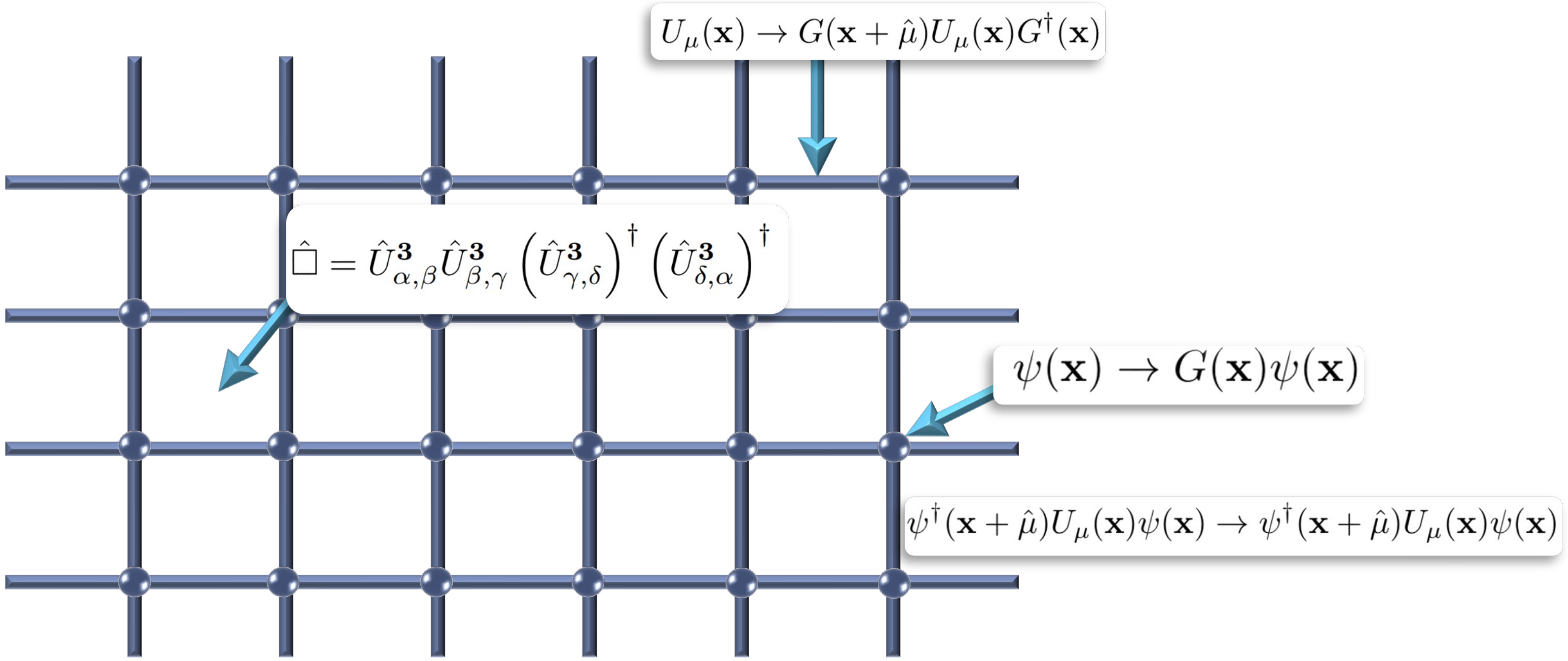
BERKELEY LAB
Bringing Science Solutions to the World

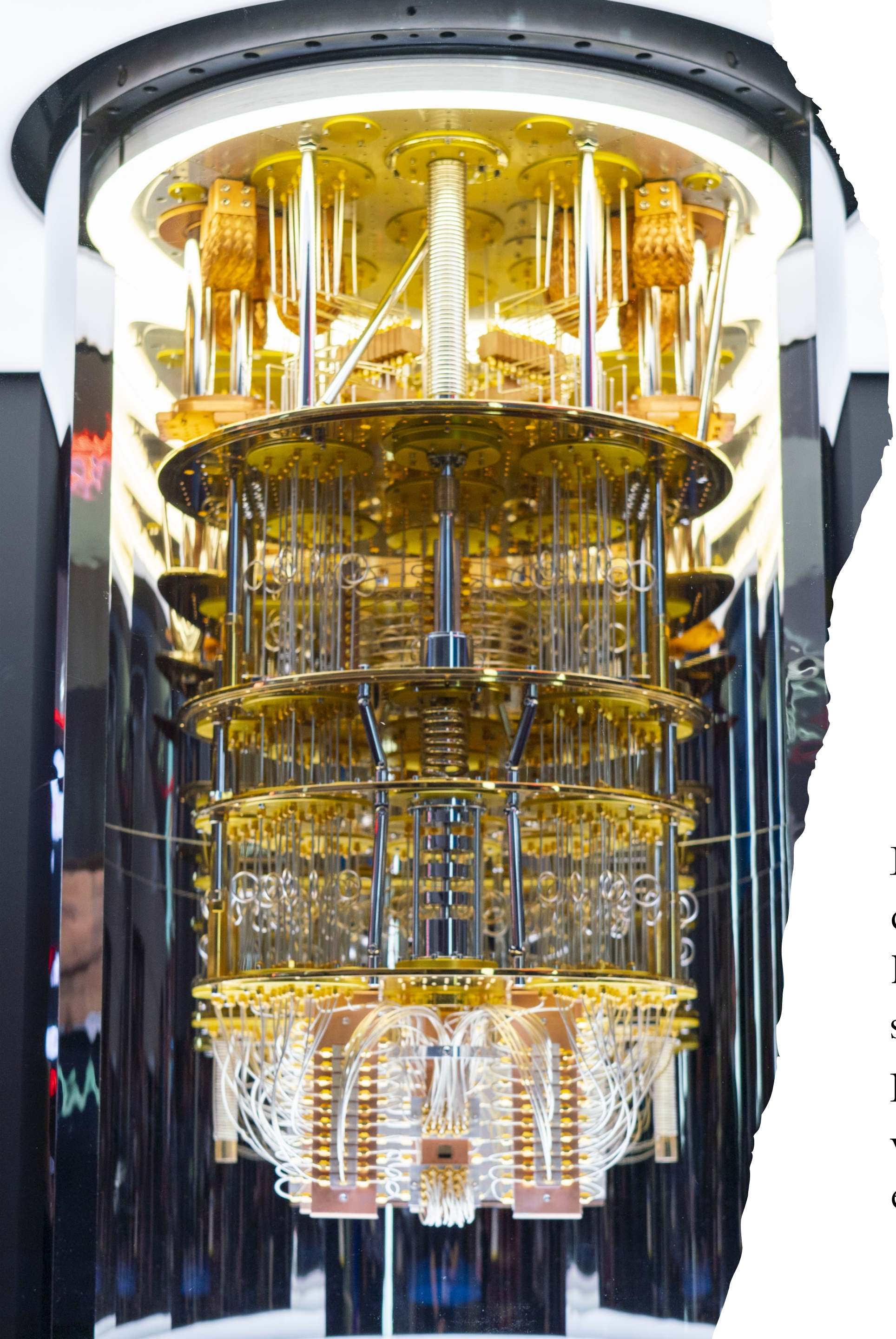
Classical Monte Carlo Simulation

- Hadron masses
- Form factors
- Some Elastic Scattering Amplitudes
- Muon g-2

Sign Problems

- Real time dynamics
- Finite density systems





WHY QUANTUM SIMULATION?

Simulating Physics with Computers (Richard Feynman 1982)

Properties of Efficient Simulation

- Size of the computer scales linearly with the size of space being simulated
- Time the calculation takes is proportional to the amount of time evolution in the simulation
- Computers have only local connectivity between their components

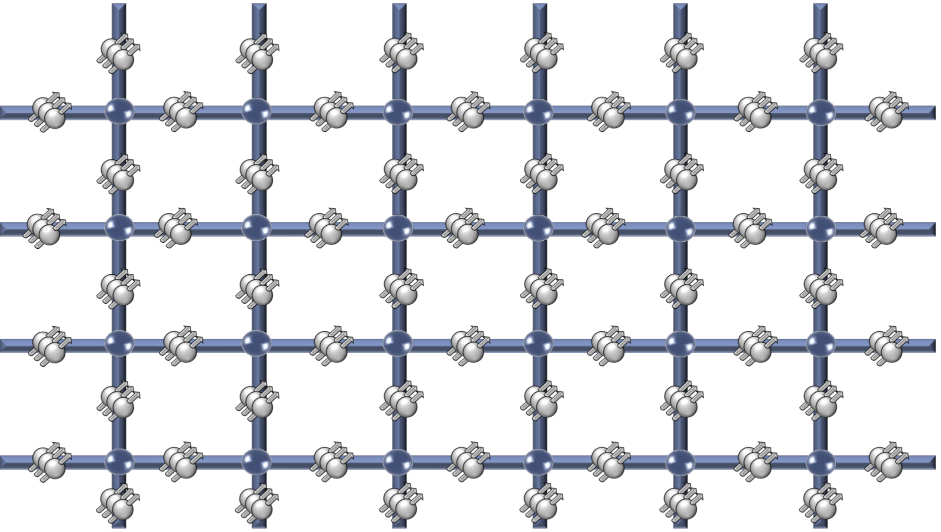
Efficient classical simulation of generic quantum systems forbidden by Bell's Theorem

Instead, we can engineer quantum systems to simulate the theories we wish to study.

DISCLAIMER: Not all quantum states violate Bell inequalities. Some states can be efficiently represented classically.

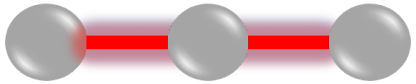
QUANTUM SIMULATION OF GAUGE THEORIES

Pick Lattice Regularization

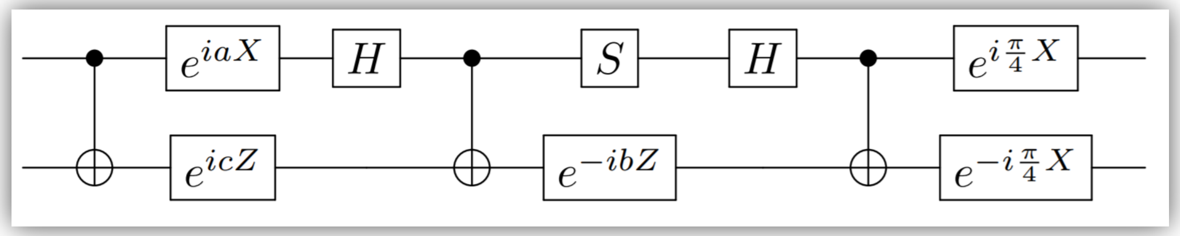


Perform time evolution

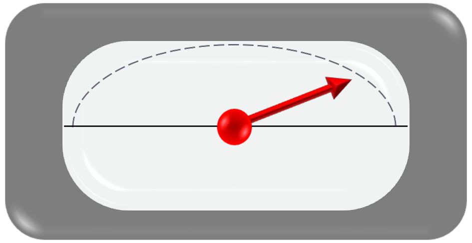
Mitigate Errors



Prepare a physically relevant state



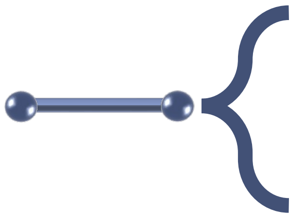
Measure



HAMILTONIAN FORMULATION OF LATTICE GAUGE THEORY

Kogut and Susskind

Phys. Rev. D 11 (1975) 395



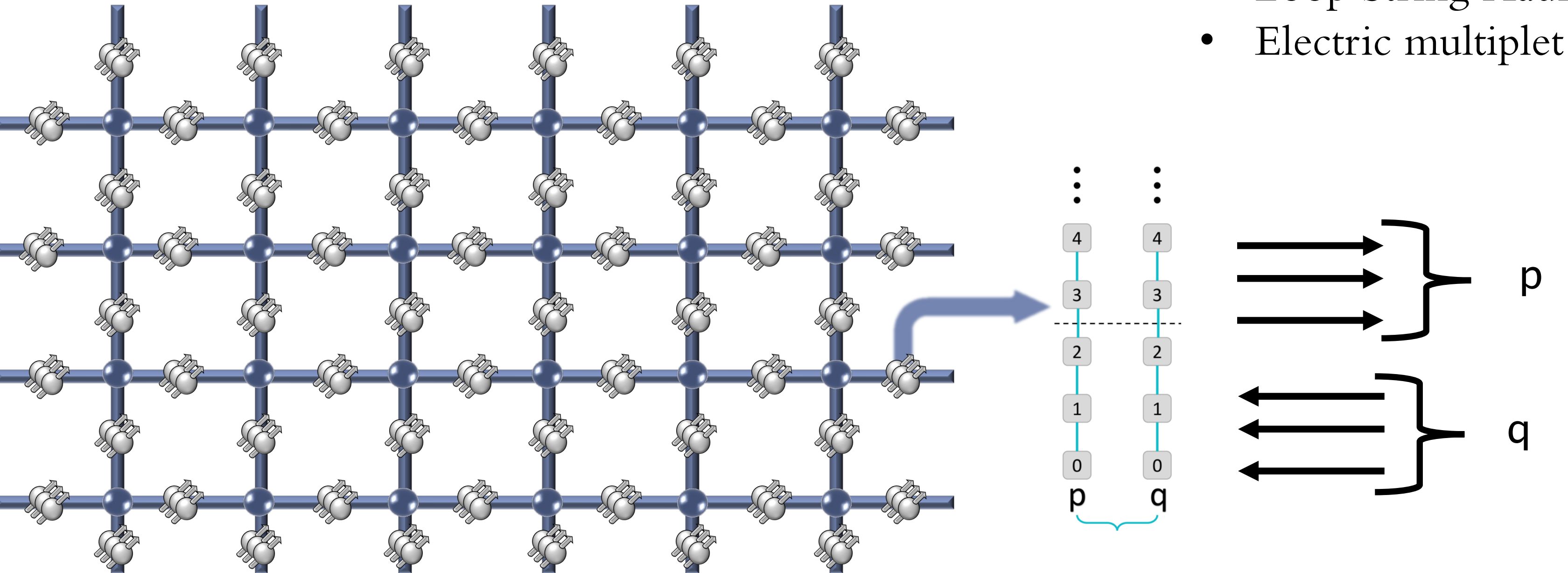
Magnetic Basis $|U\rangle$

SU(2) SU(3)

Electric Basis $|j\ m_L\rangle \otimes |j\ m_R\rangle$ $|p, q, T_L, T_L^z, Y_L, T_R, T_R^z, Y_R\rangle$

$$U^R_{s_L\ s_R}|j\ m_L\rangle\otimes|j\ m_R\rangle = \sum_{j' m'_L m'_R}\sqrt{\frac{\dim(j)}{\dim(j')}}\,C^{j' m'_L}_{j\ m_L;R\ s_L}\,C^{j' m'_R}_{j\ m_R;R\ s_R}\,|j'\ m'_L\rangle\otimes|j'\ m'_R\rangle$$

$$\hat{H} = \frac{g^2}{2a^{d-2}}\sum_{b,\text{links}}|\hat{\mathbf{E}}^{(b)}|^2 + \frac{1}{2a^{4-d}g^2}\sum_{\text{plaquettes}}\left[\,6 - \hat{\Box}(\mathbf{x}) - \hat{\Box}^\dagger(\mathbf{x})\,\right]$$



Magnetic Formulations

- Discrete Subgroups – PRD 102, 114513 (2020)
- Discrete Jacobi Transform - arXiv:2311.11928
- Axis-Angle Coordinates – arXiv:2307.11829

Electric Formulations

- Spin Operators – PRA 73 (2006) 022328
- Loop String Hadron - PRD 101 (2020) 11, 114502
- Electric multiplet – PRD 103 (9), 094501

3+1D LATTICE QCD HAMILTONIAN

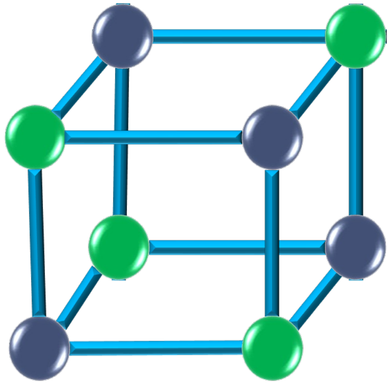
$$\hat{H} = \hat{H}_K + \hat{H}_E + \hat{H}_B$$

$$\hat{H}_K = \sum_{\vec{r},\hat{\mu},a,b} \eta_{\vec{r},\hat{\mu}} \frac{1}{2} \hat{\psi}^{\dagger}_{\vec{r},a} \hat{U}^{a,b}_{\vec{r},\vec{r}+\hat{\mu}} \hat{\psi}_{\vec{r}+\hat{\mu},b} + \text{h.c.}$$

$$\hat{H}_E = \frac{g^2}{2} \sum_{l \in \text{links},c} \hat{E}^c_l \hat{E}^c_l$$

$$\hat{H}_B = -\frac{1}{2g^2} \sum_{p \in \text{plaquettes}} \Box_p \quad ,$$

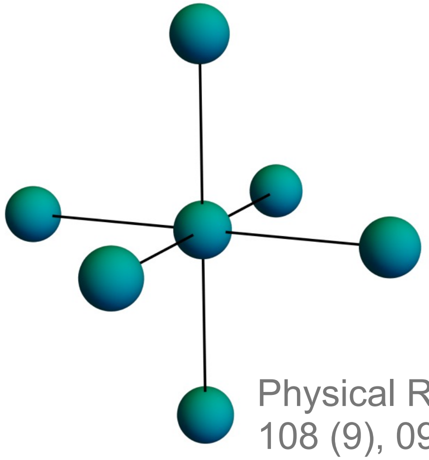
Staggered fermions in 3D $\Rightarrow$ 2 Flavors in the continuum limit



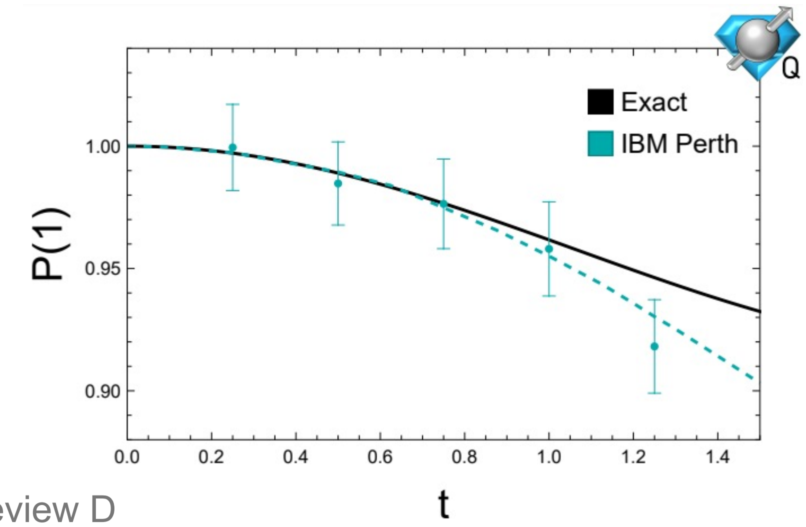
Symmetries

- Translation by two sites
- Translation by one site = Chiral symmetry (Broken)
- Diagonal Translations = Isospin symmetry
- SU(4) Wigner Spin-Flavor Symmetry at Strong Coupling

PRD 15, 1111 (1977)



Physical Review D 108 (9), 094513



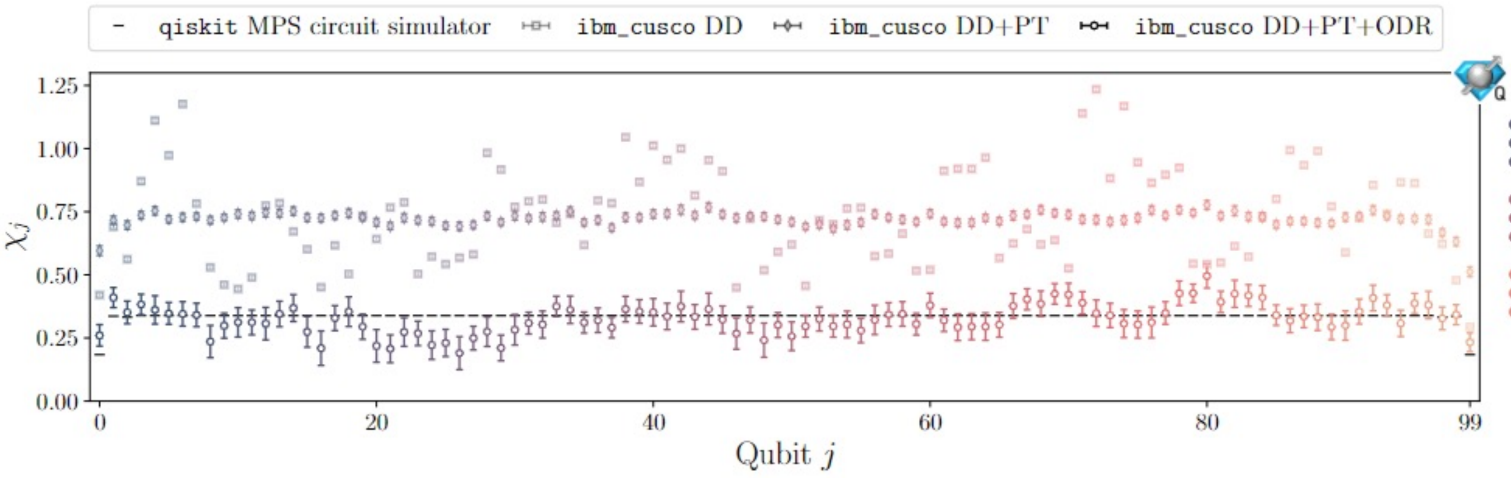
Resources need to probe continuum physics

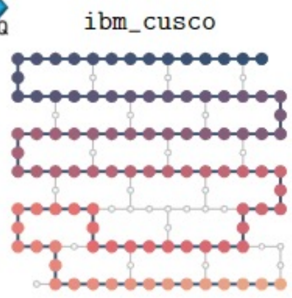
- Glueballs need lattice sizes of 24x24x24 PRD, 73:014516 (2006)
- Continuum physics reached by g~0.8 PRD 15, 1111 (1977) (Rough estimate from strong coupling expansions)
- Requires p,q truncation at 3 $O(10^5)$ qubits PRD 102 (2020) 9, 094515
- Number of time steps = lattice length $O(10^{15})$ gates PRD 103 (9), 094501 (2021)

1D SIMULATIONS AT SCALE

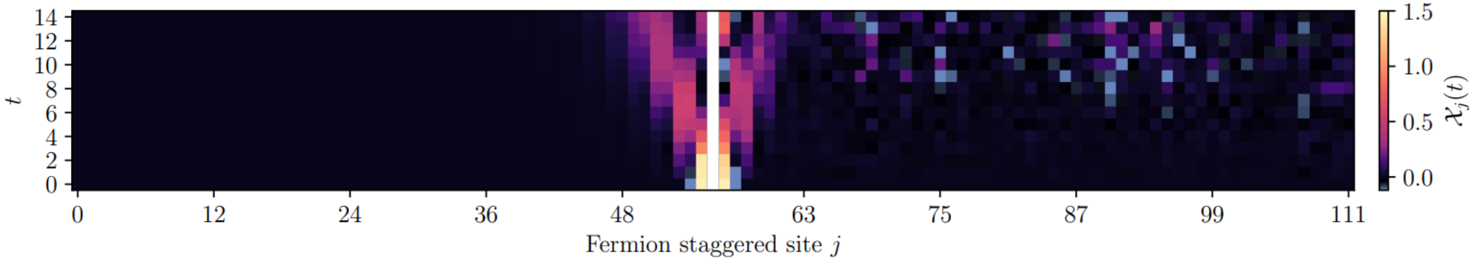
- Recent hardware advances have enabled simulations of large 1D lattices

Vacuum State Preparation
PRX Quantum 5, 020315





Hadron State Preparation
Phys. Rev. D 109, 114510



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QUANTUM SCIENCE CENTER

Scalable Circuits for Preparing Ground States on Digital Quantum Computers: The Schwinger Model Vacuum on 100 Qubits

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arXiv: 2308.04481 [quant-ph]

Quantum Simulations of Hadron Dynamics in the Schwinger Model using 112 Qubits

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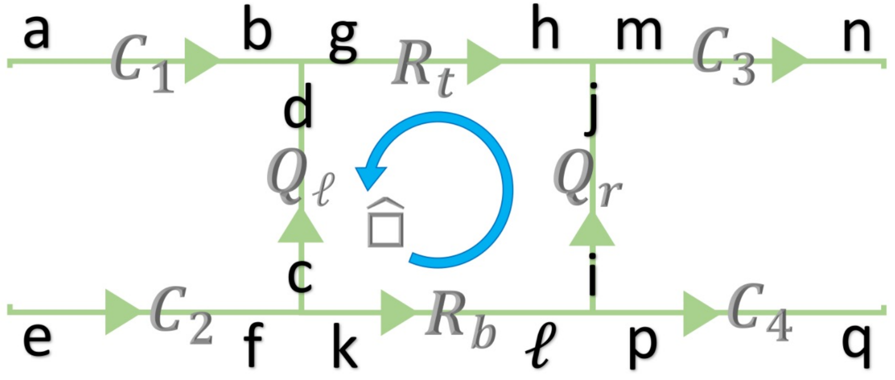
GAUGE FIELD OPERATORS IN MULTIPLE DIMENSIONS

Electric Matrix Element

$$\sum_b |\hat{\mathbf{E}}^{(b)}|^2 \, |p,q\rangle = \frac{p^2+q^2+pq+3p+3q}{3} \, |p,q\rangle$$

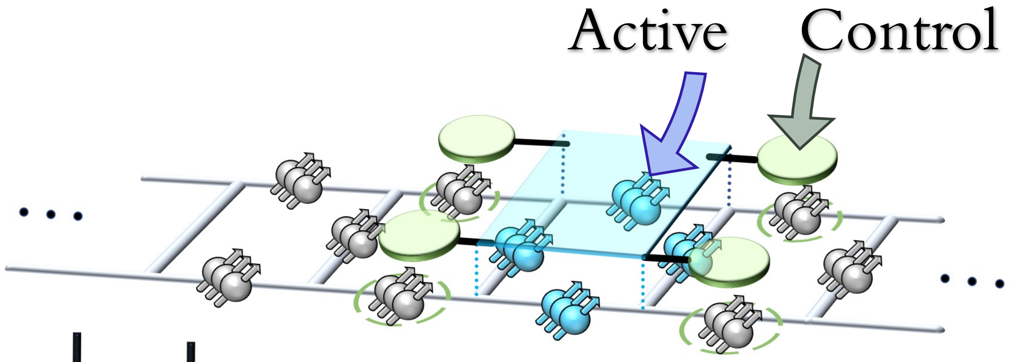


Magnetic Matrix Element

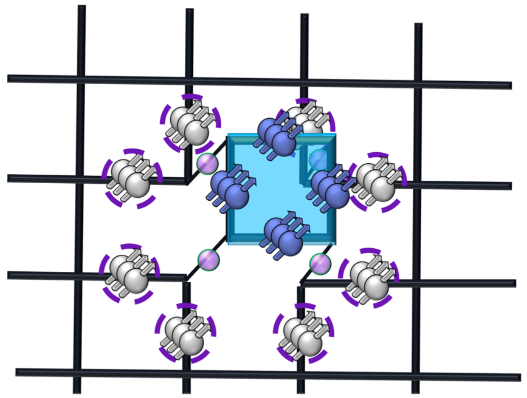
$$\left\langle \begin{pmatrix} \mathbf{C}_1, \mathbf{R}_t', \mathbf{C}_3 \\ \mathbf{Q}_\ell', \mathbf{Q}_r' \\ \mathbf{C}_2, \mathbf{R}_b', \mathbf{C}_4 \end{pmatrix} \middle| \hat{\square} \middle| \begin{pmatrix} \mathbf{C}_1, \mathbf{R}_t, \mathbf{C}_3 \\ \mathbf{Q}_\ell, \mathbf{Q}_r \\ \mathbf{C}_2, \mathbf{R}_b, \mathbf{C}_4 \end{pmatrix} \right\rangle =$$

$$\sqrt{\frac{\dim(\mathbf{R}_t)\dim(\mathbf{R}_b)}{\dim(\mathbf{R}_t')\dim(\mathbf{R}_b')\dim(\mathbf{Q}_\ell)\dim(\mathbf{Q}_r)\dim(\mathbf{Q}_\ell')^3\dim(\mathbf{Q}_r')^3}}$$

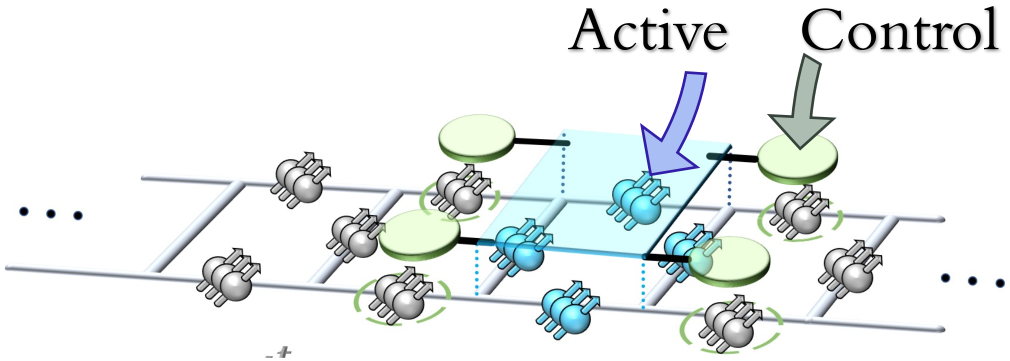
$$\begin{Bmatrix} \overline{\mathbf{R}}_t & \mathbf{C}_1 & \overline{\mathbf{Q}}_\ell \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \overline{\mathbf{R}}_t' & \mathbf{C}_1 & \overline{\mathbf{Q}}_\ell' \end{Bmatrix} \begin{Bmatrix} \mathbf{R}_t & \overline{\mathbf{C}}_3 & \overline{\mathbf{Q}}_r \\ \overline{\mathbf{3}} & \mathbf{1} & \overline{\mathbf{3}} \\ \mathbf{R}_t' & \overline{\mathbf{C}}_3 & \overline{\mathbf{Q}}_r' \end{Bmatrix} \begin{Bmatrix} \overline{\mathbf{R}}_b & \mathbf{C}_2 & \mathbf{Q}_\ell \\ \overline{\mathbf{3}} & \mathbf{1} & \overline{\mathbf{3}} \\ \overline{\mathbf{R}}_b' & \mathbf{C}_2 & \mathbf{Q}_\ell' \end{Bmatrix} \begin{Bmatrix} \mathbf{R}_b & \overline{\mathbf{C}}_4 & \mathbf{Q}_r \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{R}_b' & \overline{\mathbf{C}}_4 & \mathbf{Q}_r' \end{Bmatrix}$$

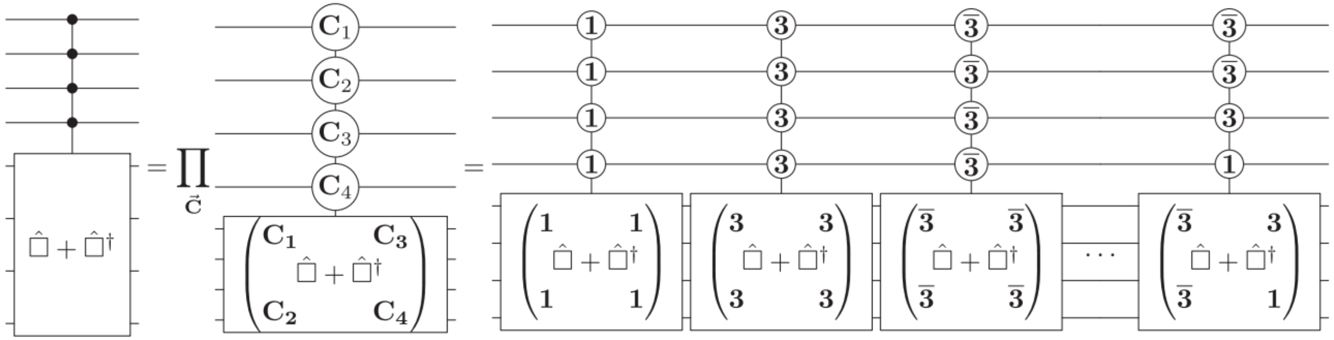


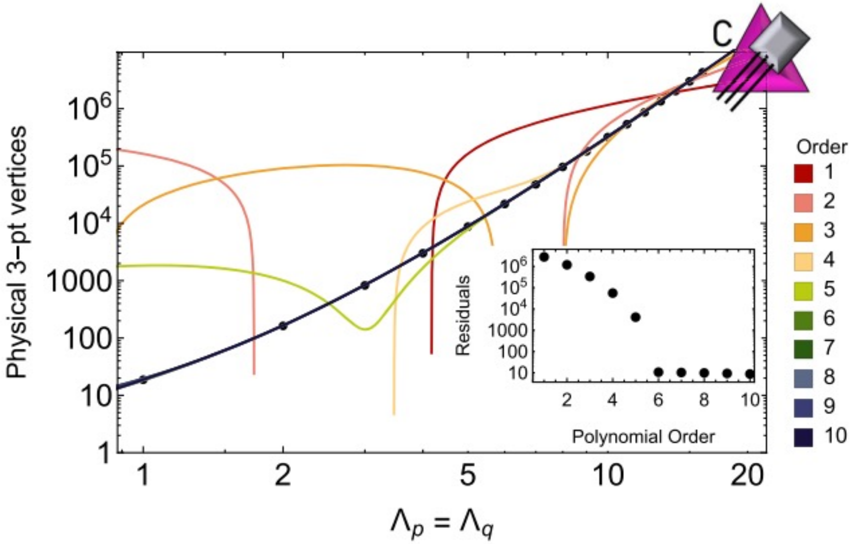
$$\begin{Bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{D} & \mathbf{B} & \mathbf{E} \end{Bmatrix} = \sum \langle \mathbf{D}, y', \mathbf{B}, x | \mathbf{E}, q' \rangle_{\Gamma_1} \langle \mathbf{A}, y, \mathbf{B}, x | \mathbf{C}, q \rangle_{\Gamma_2} \langle \mathbf{A}, y, \mathbf{3}, c | \mathbf{D}, y' \rangle_{\Gamma_3} \langle \mathbf{C}, q, \mathbf{3}, c | \mathbf{E}, q' \rangle_{\Gamma_4}$$



CHALLENGES OF GOING TO SCALE IN MULTIPLE DIMENSIONS





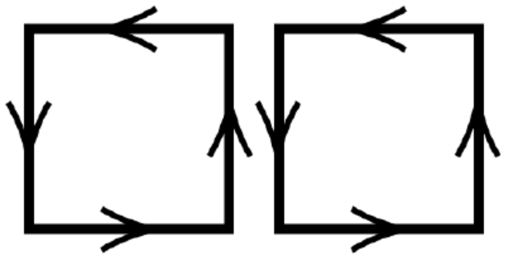


- Gate count for time evolution scales as Λ^{16}

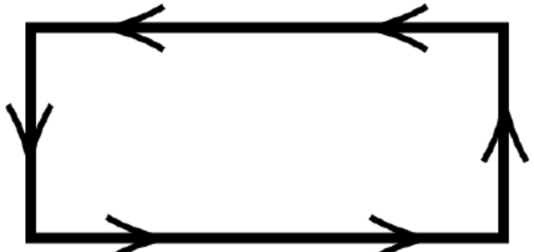
LARGE N EXPANSION

SU(3) → SU(N), Expand in 1/N

- Qualitatively reproduces many aspects of QCD
- Provides a starting point for describing interactions between baryons
- Used in event generators that simulate collider physics



= O(1)



= O(1/N)

Phys. Rev. Lett. **133**, 111901

- Expand operators in powers of 1/N
- Truncate both in powers of 1/N and electric energy
- The only electric basis states that are relevant are those that can be created from the electric vacuum by applying plaquette operators
- The large N scaling of a state is determined by the maximum overlap of the state with

$$\left|\{P_p, \bar{P}_p\}\right\rangle \equiv \prod_p \hat{\Box}_p^{P_p} \hat{\Box}_p^{\dagger \bar{P}_p} |0\rangle$$

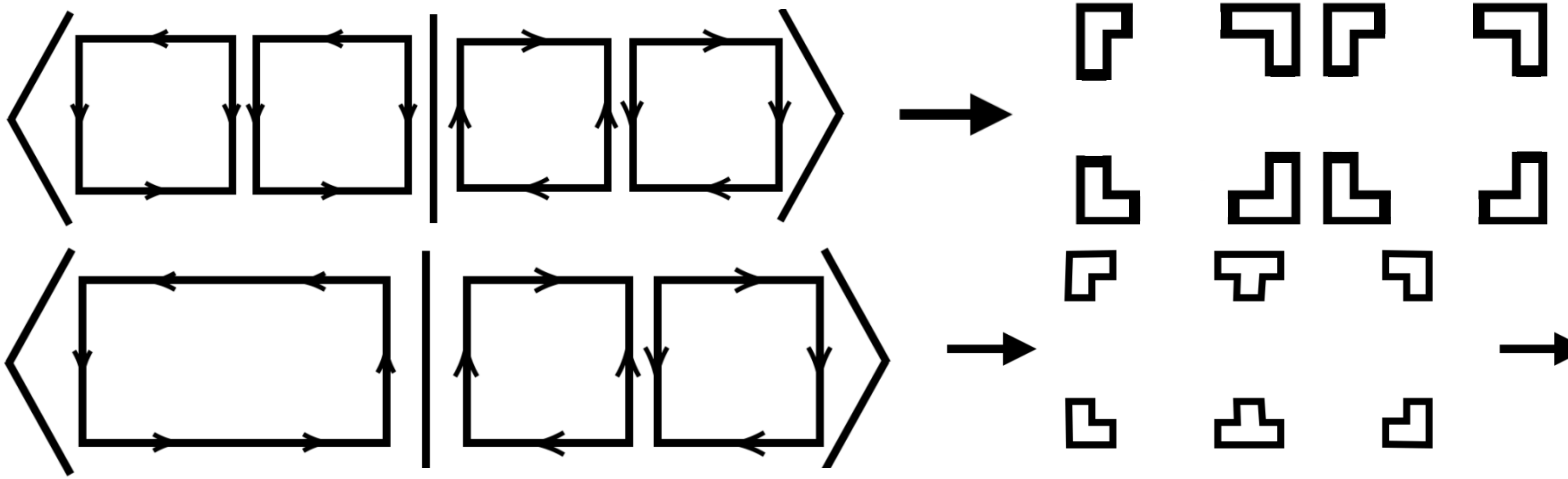
LARGE N SCALING OF STATES

- The Large N scaling is determined by $\left\langle \{L_i\} \middle| \{P_p, \bar{P}_p\} \right\rangle$ $\quad |\{P_p, \bar{P}_p\}\rangle \equiv \prod_p \hat{\Box}_p^{P_p} \hat{\Box}_p^{\dagger \bar{P}_p} |0\rangle$
- This can be evaluated in the magnetic basis, $\mathbf{1} = \prod_{\text{links } l} \int dU_l \, |U_l\rangle \langle U|_l$

  using the identity

$$\int dU \prod_{n=1}^{q} U_{i_n j_n} U^*_{i'_n j'_n} =$$

$$\frac{1}{N_c^q} \sum_{\text{permutations k}} \prod_{n=1}^{q} \delta_{i_n i'_{k_n}} \delta_{j_n j'_{k_n}} + \mathcal{O}\left(\frac{1}{N_c^{q+1}}\right)$$



$$\rightarrow \frac{1}{N_c^8} \times N_c^8 = 1$$

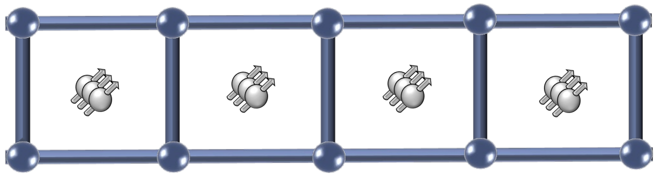
$$\rightarrow \frac{1}{N_c^7} \times N_c^6 = \frac{1}{N_c}$$

Simple Scaling Rule $\quad \left\langle \{L_i, a_\ell\} \middle| \{P_p, \bar{P}_p\} \right\rangle \propto \prod_i N_c^{1-m_i}$

$m_i =$# Plaquettes enclosed by loop i

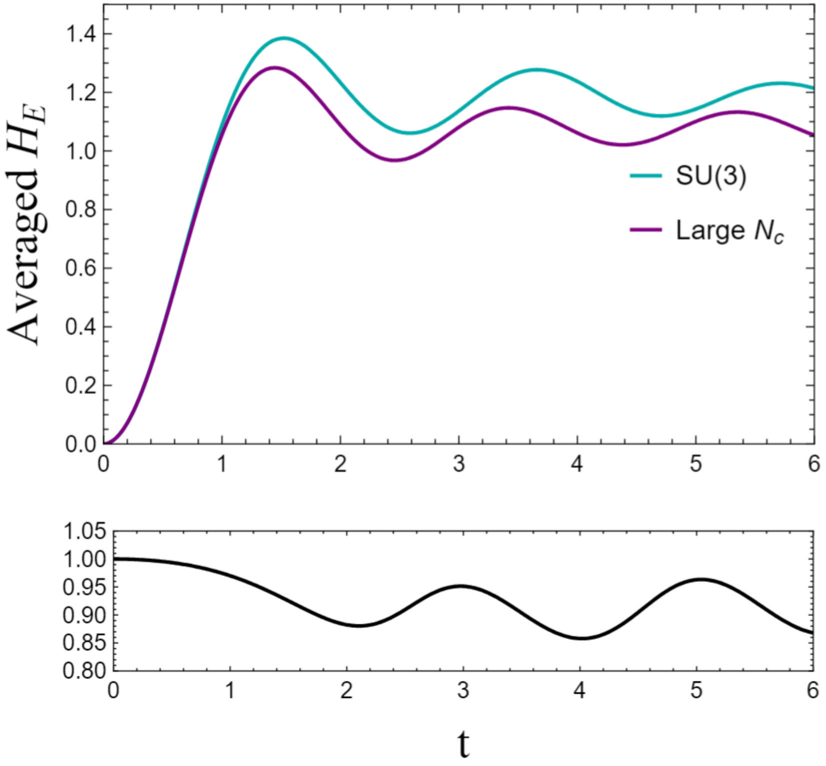
LARGE N TRUNCATION

- The Hamiltonian can be truncated in 1/N as well as in irreps
- This reduces both the qubit count and computational cost
- At the harshest truncation, only one qubit is required per plaquette
- Resources can be compared for a small lattice at this truncation (4x1)



	Naïve Encoding	Multiplet Basis	Large N_c
Qubit Count	60	24	4
Gauss's law enforced	No	Partially	Partially

$$\hat{H} = \sum_p \left(\frac{8}{3}g^2 - \frac{1}{2g^2}\right)\hat{P}_{1,p} - \frac{1}{g^2\sqrt{2}}\hat{P}_{0,p+\hat{x}}\hat{P}_{0,p-\hat{x}}\hat{P}_{0,p+\hat{y}}\hat{P}_{0,p-\hat{y}}\hat{X}_p$$



REAL TIME EVOLUTION ON IBM'S QUANTUM COMPUTERS

Interaction Picture Trotterization

$$\hat{H}_{B,I}(t) = e^{i\hat{H}_E t}\hat{H}_B e^{-i\hat{H}_E t}$$

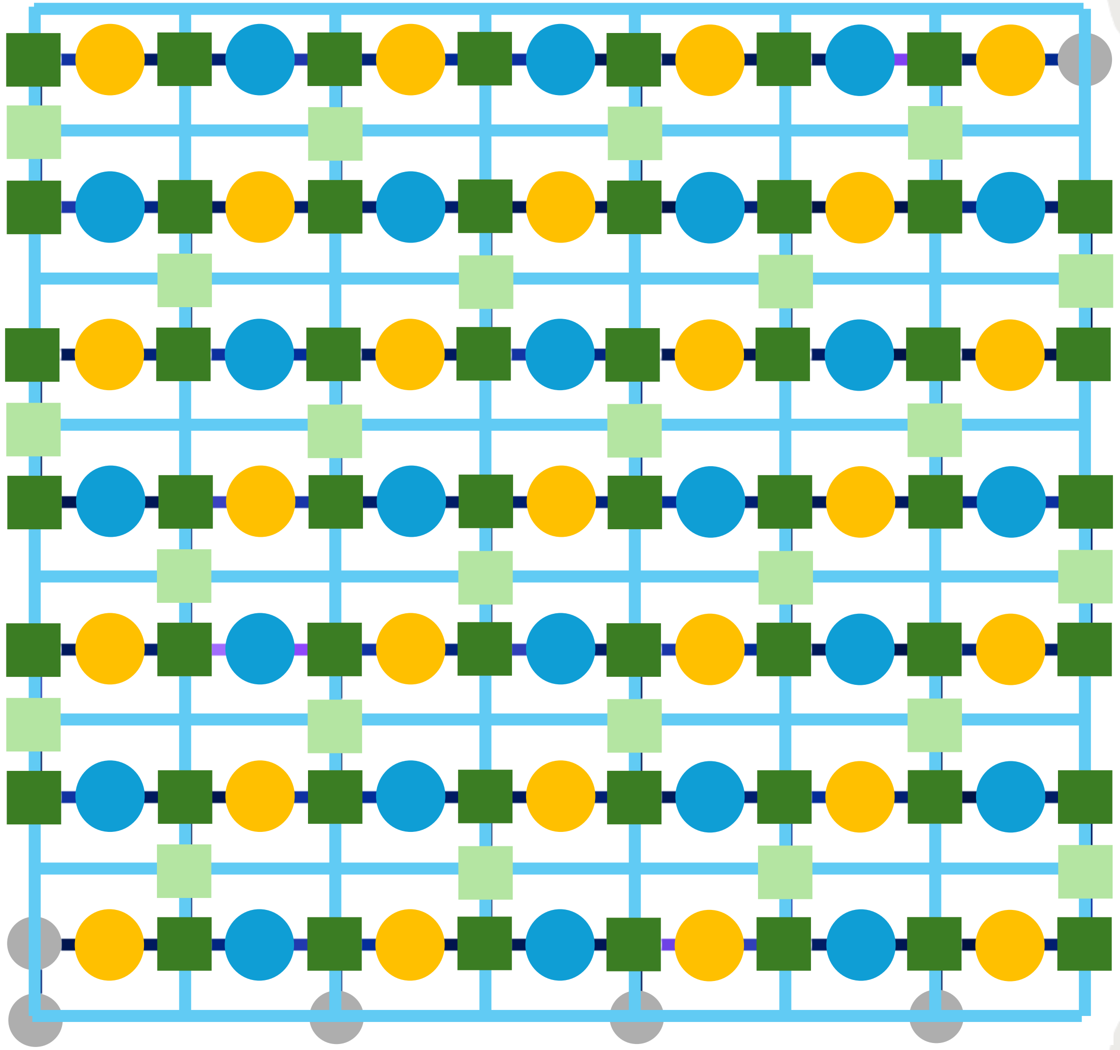
$$e^{-i\hat{H}t} = e^{-i\hat{H}_E t}\,\mathcal{T}e^{-i\int_0^t ds\hat{H}_{B,I}(s)}$$

$$e^{-i\int_0^{\Delta t} ds\hat{H}_{B,I}(s)} = e^{-i\int_0^{\Delta t} ds\hat{H}_{B,E}(s)}e^{-i\int_0^{\Delta t} ds\hat{H}_{B,O}(s)} + \mathcal{O}\left(\frac{\Delta t^2}{g^4}\right)$$

$$e^{-i\int_0^{\Delta t}\mathrm{ds}\,\hat{H}_{B,E}(s)}e^{-i\int_0^{\Delta t}\mathrm{ds}\,\hat{H}_{B,O}(s)} =$$

$$\left[e^{i\phi\sum_p \hat{Z}_p}\right]\left[e^{i\theta\sum_{p\in E}\hat{X}_p\prod_{q\in\partial p}\hat{P}_{0,q}}\right]\left[e^{i\theta\sum_{p\in O}\hat{X}_p\prod_{q\in\partial p}\hat{P}_{0,q}}\right]\left[e^{-i\phi\sum_p \hat{Z}_p}\right]$$

- Yellow and blue qubits are used to represent the state of the system
- Square qubits are used to enable communication between those used to represent the system
- One Trotter step = CNOT depth 45

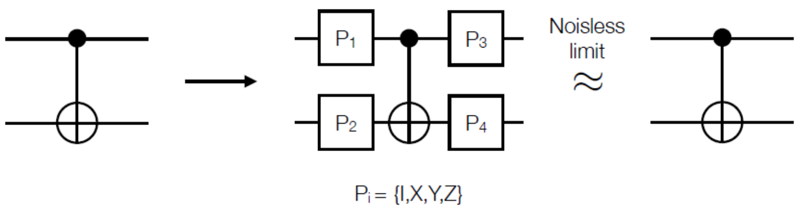


ERROR MITIGATION

- Quantum simulations have errors coming from inherent hardware errors
- Pauli twirling converts coherent errors into a Pauli error channel
- Decoherent Pauli noise renormalizes Pauli operators

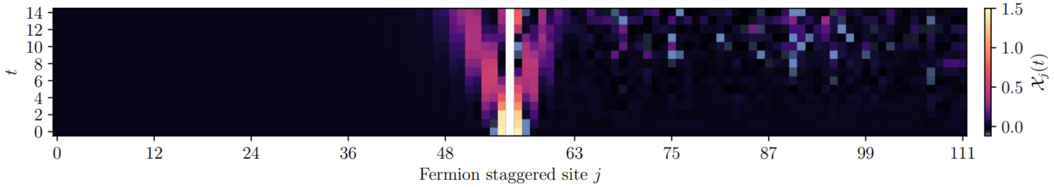
$$\langle\psi|\hat{P}|\psi\rangle \rightarrow \eta_P \langle\psi|\hat{P}|\psi\rangle$$

Pauli Twirling (or randomized compiling)



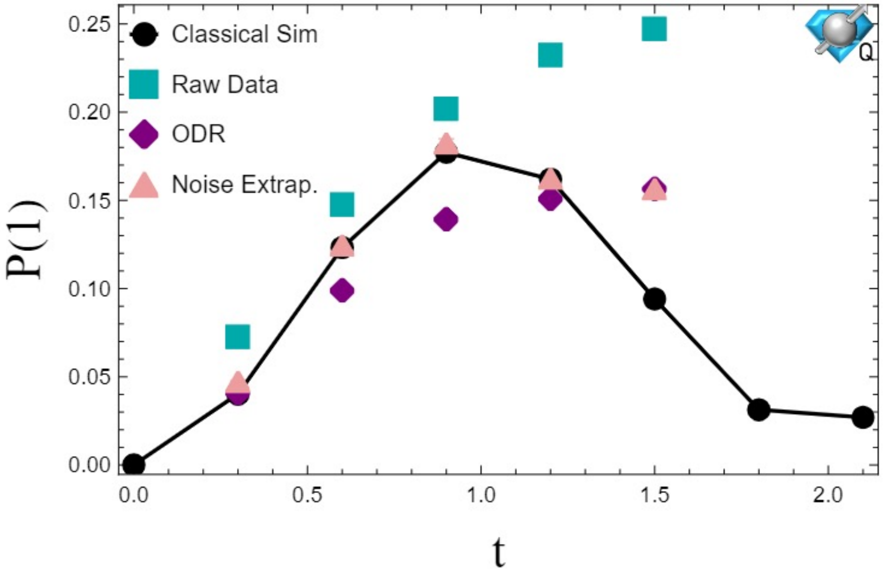
- This can be mitigated by running a circuit with a known answer to determine η_P
- Other sources of hardware error can be mitigated by artificially introducing noise by applying more CNOT gates and extrapolating to zero noise.

Operator Decoherence Renormalization
Phys. Rev. Lett. 127, 270502
arXiv:2210.11606
PRX Quantum 5, 020315
Phys. Rev. D 109, 114510



112 qubits, CNOT depth 370 (13,858 CNOTs)

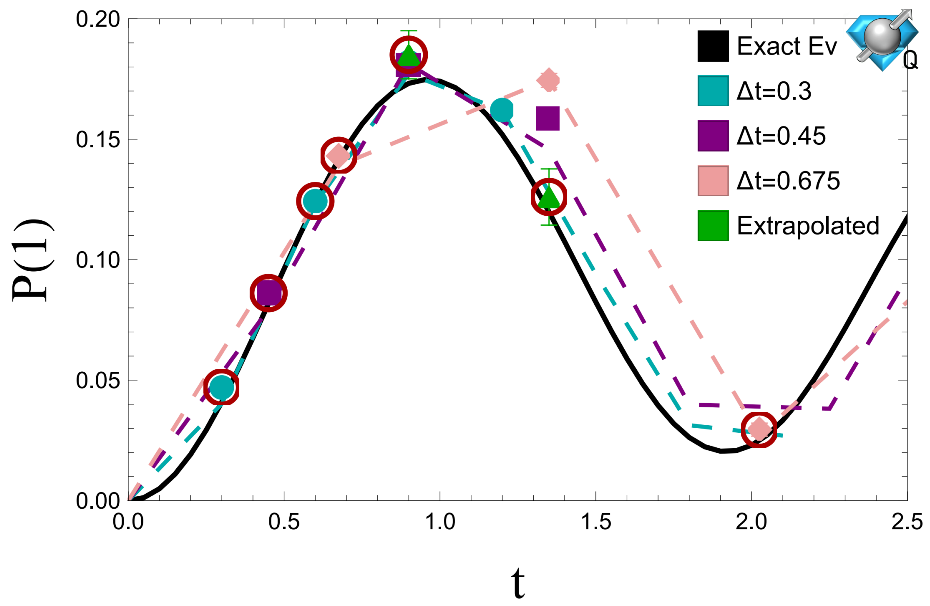
5x5



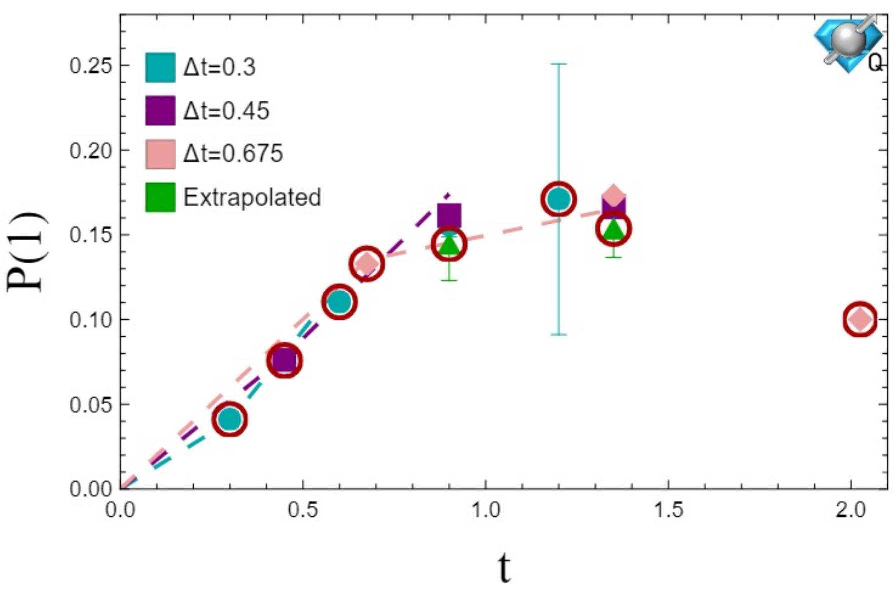
ALGORITHMIC ERRORS

- Errors also come from the Trotterization of the time evolution operator.
- This can be mitigated by performing the evolution with multiple step sizes that sample the same points in time and extrapolating.
- Noise in circuits scales with circuit depth not system size so small simulations can be used to validate the results of larger ones.
- CuQuantum was used to perform a classical simulation for a 8x8 lattice.

5x5



8x8

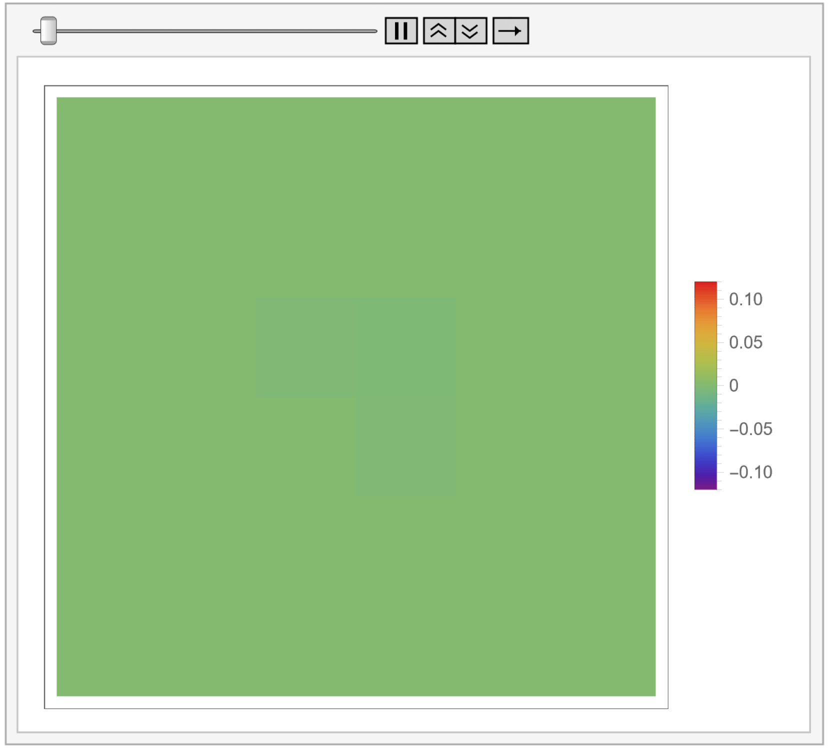


129 Qubits
CZ Depth 113
1500 CZ Gates
8733 1 Qubit Gates

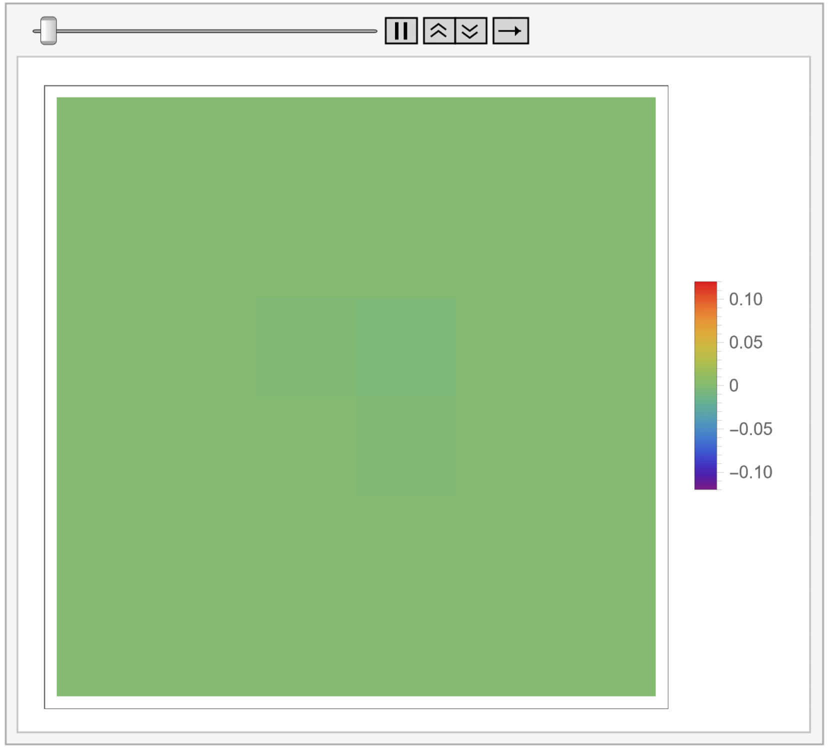
RYDBERG ATOMS

- The LGT Hamiltonian at this truncation is the same as that describing a Rydberg atom array
- This can enable analog simulations in the near term

SU(3) Yang Mills



Neutral Atom Array



SUMMARY & FUTURE GOALS

- The large N expansion can be used to reduce the resources needed for simulation.
- This also allows for straightforward implementation on neutral atom platforms.
- The truncated Hamiltonian is similar to the PXP model indicating there may be connections to condensed matter work on scarring and confinement in spin models.
- Future work will look at including quarks and 1/N corrections.