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References: This code of conduct is based heavily on that of the INT and the APS. We are also grateful to Roxanne Springer for valuable discussion and guidance.

The hadronic stress-energy tensor on the light front

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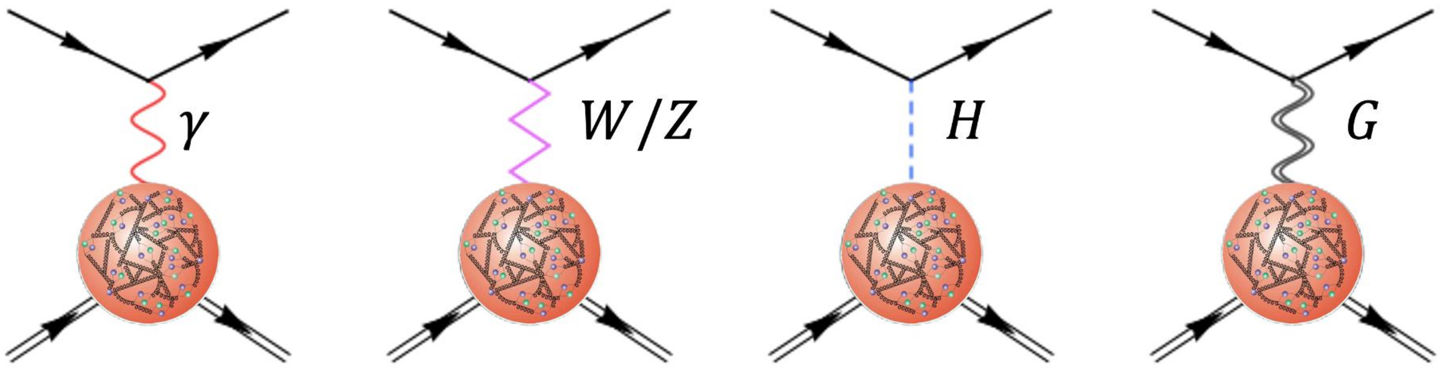
Rising Researchers Seminar Series,
January 21, 2025

Stress-energy tensor



The stress-energy tensor is the conserved current under spacetime translations

[Noether:1918zz, Freese:2021jqs]

$$x^\mu \mapsto x'^\mu = x^\mu + \xi^\mu(x), \quad \partial_\mu \hat{T}^{\mu\nu} = 0$$

Hadron matrix elements and gravitational form factors:

[Kobzarev:1962wt, Pagels:1966zza]

$$\langle p', s'|\hat{T}_i^{\mu\nu}(0)|p, s\rangle = \frac{1}{2M}\bar{u}_{s'}(p')\left[2P^\mu P^\nu A_i(q^2) + iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho J_i(q^2) + \frac{1}{2}(q^\mu q^\nu - g^{\mu\nu}q^2)D_i(q^2) + 2g^{\mu\nu}\bar{c}_i(q^2)\right]u_s(p)$$

where $P = (p + p')/2$, $q = p' - p$.

Gravitational form factor D: the last global unknown

The structure of hadrons can be probed by the other fundamental forces

[Polyakov:2018zvc]

em: $\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\|J^\mu_{\bf em}\|N\rangle$	$\longrightarrow$	Q	$=$	$1.602176487(40)\times 10^{-19}$C
			μ	$=$	$2.792847356(23)\mu_N$
weak: PCAC	$\langle N'\|J^\mu_{\bf weak}\|N\rangle$	$\longrightarrow$	g_A	$=$	$1.2694(28)$
			g_p	$=$	$8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\bf grav} = 0$	$\langle N'\|T^{\mu\nu}_{\bf grav}\|N\rangle$	$\longrightarrow$	m	$=$	$938.272013(23)$ MeV$/c^2$
			J	$=$	$\frac{1}{2}$
			$\boldsymbol{D}$	$=$	**?**

Conservation laws constrain gravitational form factors except *D*

[Cotogno:2019xcl, Lorce:2019sbq]

$$A(0) = 1, \quad J(0) = \frac{1}{2}, \quad \lim_{Q^2\to 0} Q^2 D(Q^2) = 0$$

Mechanical properties of hadrons

$D(q^2)$ is related to the pressure and shear forces inside hadrons

[Polyakov:2018zvc]

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}p(r)$$

$$\Downarrow$$

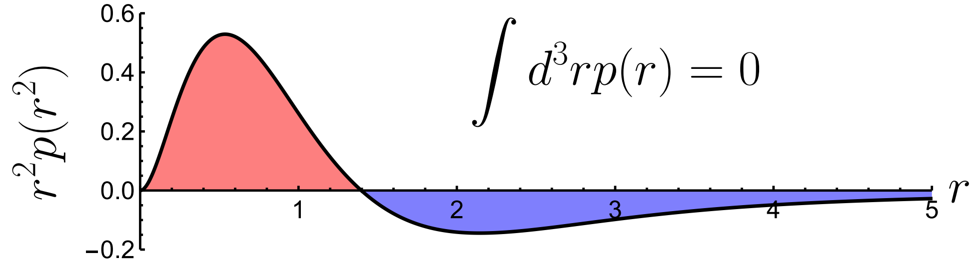
$$p(r) = \frac{1}{6M}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}r^2\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r), \qquad s(r) = -\frac{1}{4M}r\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r)$$

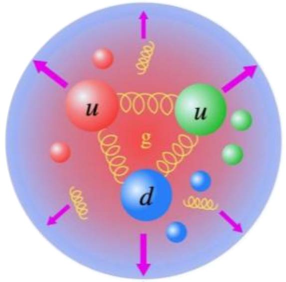
Hadron stability conditions:

[Perevalova:2016dln]

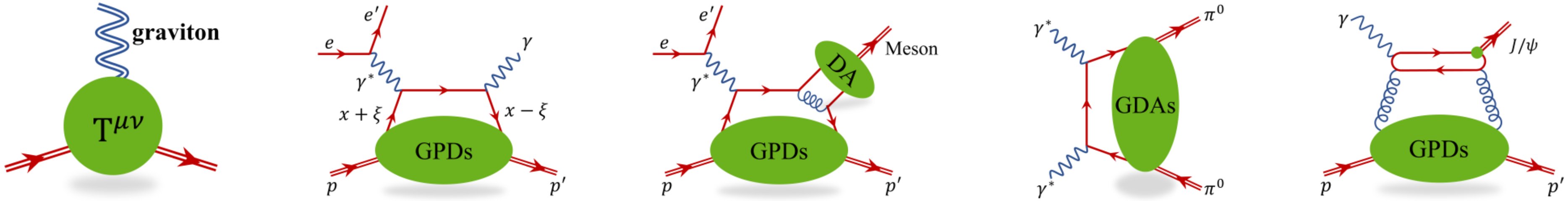
- Force equilibrium (von Laue condition): $\int d^3r p(r) = 0$
- Local stability conjecture: $3D(0) = -2M\int d^3r r^2 \mathrm{d}F_\mathrm{r}/\mathrm{d}S_\mathrm{r} < 0$?

where $\mathrm{d}F_r/\mathrm{d}S_r = 2s(r)/3 + p(r)$





How to access gravitational form factors



[Kumano:2017lhr, Duran:2022xag, Burkert:2023wzr]

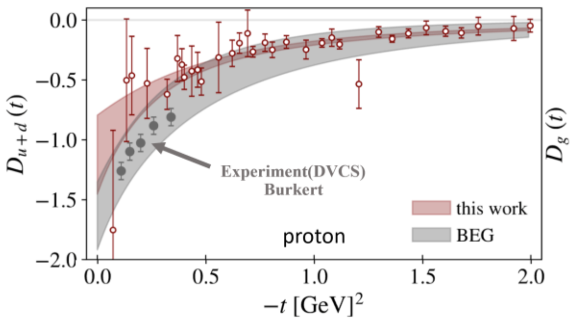
- Deeply virtual Compton scattering
- Deeply virtual meson production
- Two-photon pair production
- J/ψ threshold photoproduction

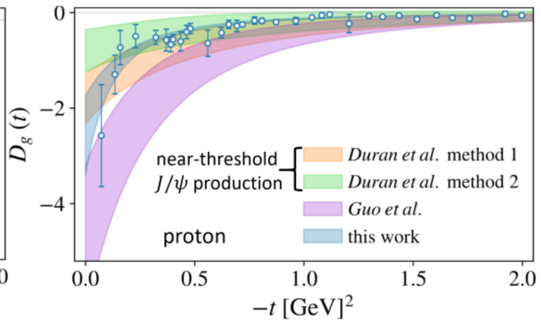
Ji's sum rule:

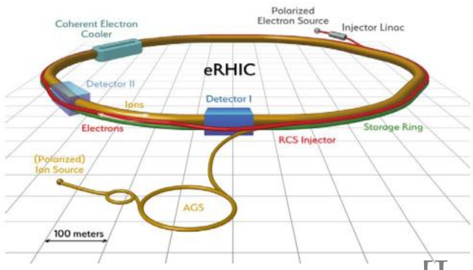
$$\int_{-1}^{1} dx x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t), \quad \int_{-1}^{1} dx x E^{q,g}(x,\xi,t) = B^{q,g}(t) - \xi^2 D^{q,g}(t)$$

[Ji:1996nm]

Here, $H^{q,g}$ and $E^{q,g}$ are generalized parton distributions

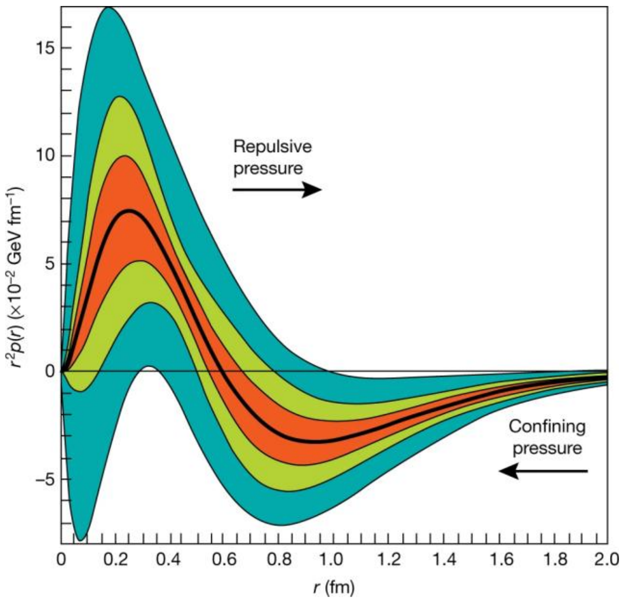


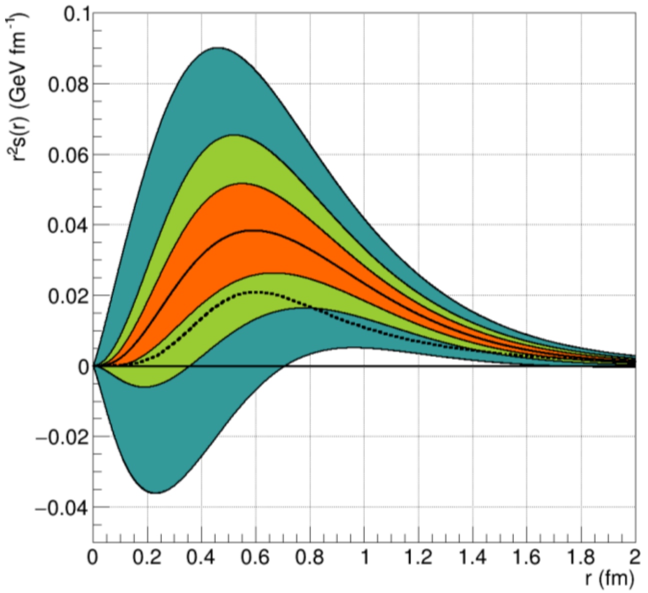




[Lattice'23: Hackett:2023nkr]

The first measurement of the pressure and shear





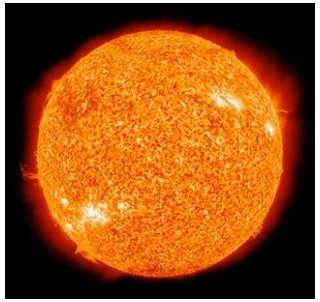
[Burkert:2018bqq]
[Burkert:2021ith]

The proton contains the highest pressure in nature

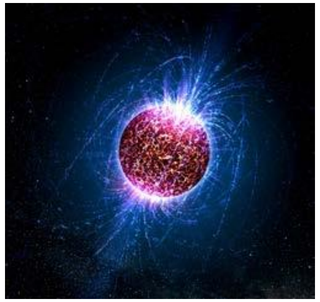
Adapted from Kumano: LC2024

Earth atmosphere
10^5Pa

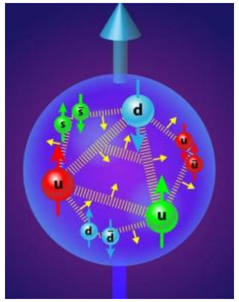
Center of earth
10^{11}Pa



Center of sun
10^{16}Pa



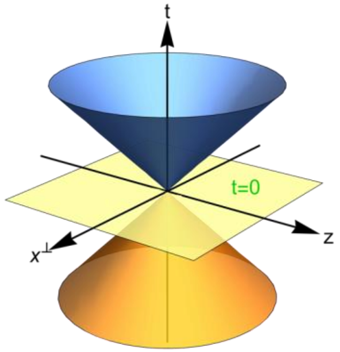
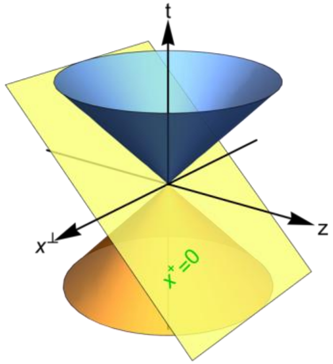
Neutron star
10^{34}Pa



Proton
10^{35}Pa

Light-front quantization

[Dirac:1949cp]

	equal time quantization	light-front quantization
		
■ Time:	$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
■ Hamiltonian:	$H \equiv P^0$	$H \equiv P^- = P^0 - P^3$
■ Dispersion relation:	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$

light-front coordinates

$$x^\pm = x^0 \pm x^3$$

$$\vec{x}^\perp = (x^1, x^2)$$

Light-front quantization is a Hamiltonian method of the quantum field theory

[Brodsky:1997de]

$$(P^+ \hat{P}^- - \vec{P}_\perp^2) \left|\psi_H\right\rangle = M_h^2 \left|\psi_h\right\rangle$$

Light-front wave functions

$$|\psi_h(P,j,\lambda)\rangle = \sum_{n=1}^{\infty} \int [\mathrm{d}x_i \mathrm{d}^2 k_{i\perp}]_n \psi_{h/n}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n) \, |\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

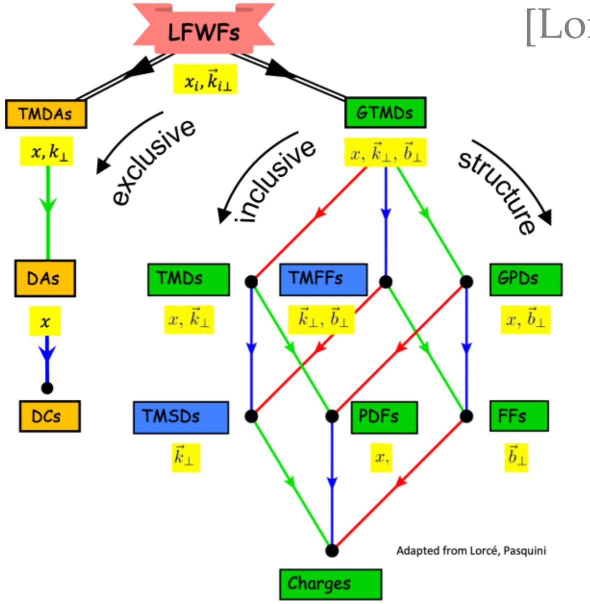
Light-front wave functions (LFWFs) are boost invariant and only depend on relative variables

$$x_i \equiv p_i^+/P^+, \quad \vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i \vec{P}_\perp \Longrightarrow \sum_i x_i = 0, \sum_i \vec{k}_{i\perp} = 0$$

Light-front wave functions provide intrinsic information of the structure of hadrons

- Overlap of LFWFs: Structure functions (e.g. PDFs), form factors
- Integrating out LFWFs: light-cone distributions (e.g. DAs)

[Lorce:2011kd]



Light-front wave function representation

Diagonal representation for charge form factor and GFF $A(q^2)$

[Drell:1969km, West:1970av, Brodsky:1980zm]

$$F_1(q_\perp^2) = \sum_j \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\})\psi_n(\{x_i, \boldsymbol{r}_{i\perp}\}) e_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp}$$

[Brodsky:2000ii]

$$A(q_\perp^2) = \sum_j \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\})\psi_n(\{x_i, \boldsymbol{r}_{i\perp}\}) x_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp}$$

Number densities:

$$\rho_{\mathrm{ch}}(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} F_1(q_\perp^2) = \Big\langle \sum_j e_j \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{j\perp}) \Big\rangle$$

$$\mathcal{A}(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} A(q_\perp^2) = \Big\langle \sum_j x_j \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{j\perp}) \Big\rangle$$

where the quantum average is defined as

$$\langle \hat{O} \rangle = \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \hat{O} \psi_n(\{x_i, \boldsymbol{r}_{i\perp}\})$$

Light-front wave function representation for $D(q^2)$

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| Reviews

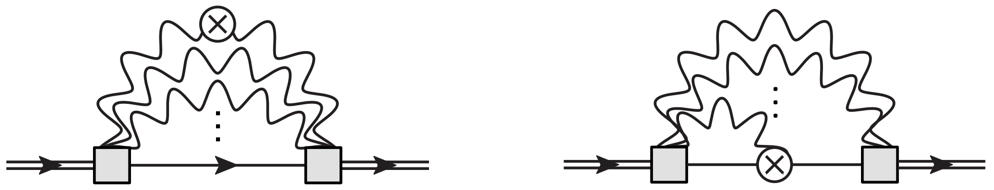
Forces inside hadrons: Pressure, surface tension, mechanical radius, and all that

Maxim V. Polyakov and Peter Schweitzer

https://doi.org/10.1142/S0217751X18300259 | **Cited by:** 241 **(Source: Crossref)**

$\hat{T}_{++}$ of the EMT. Being related to the stress tensor $\hat{T}_{ij}$ the form factor $D(t)$ naturally "mixes" good and bad light-front components and is described in terms of transitions between different Fock state components in overlap representation. As a quantity intrinsically nondiagonal in a Fock space, it is difficult to study the D-term in approaches based on light-front wave functions. This is due to the rela-

[Polyakov:2018zvc]



diagonal   non-diagonal

- $D(q^2)$ contains the overlap between different Fock state components
- A complete description requires the inclusion of all Fock components
- Renormalization plays a vital role in the cancelation of non-diagonal terms

Scalar Yukawa model

$$\mathcal{L} = \partial_\mu N^\dagger \partial^\mu N - m^2 N^\dagger N + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi - \frac{1}{2}\mu^2\pi^2 + g_0 N^\dagger N \pi + \delta m^2 N^\dagger N$$

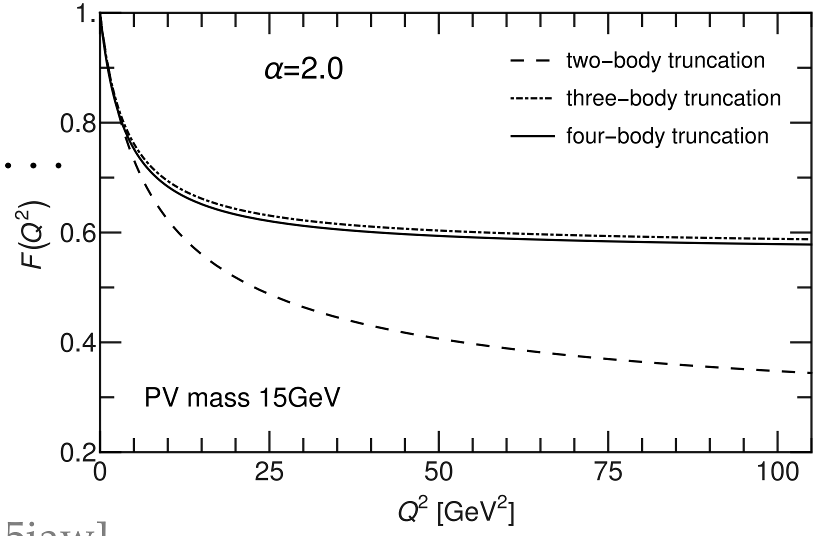
$$\Downarrow$$

$$\hat{T}^{\mu\nu} = \partial^{\{\mu} N^\dagger \partial^{\nu\}} N - g^{\mu\nu}\left[\partial_\sigma N^\dagger \partial^\sigma N - (m^2 - \delta m^2) N^\dagger N\right] - g^{\mu\nu} g_0 N^\dagger N \pi + \partial^\mu \pi \partial^\nu \pi - \frac{1}{2} g^{\mu\nu}\left(\partial^\rho \pi \partial_\rho \pi - \mu_0^2 \pi^2\right)$$

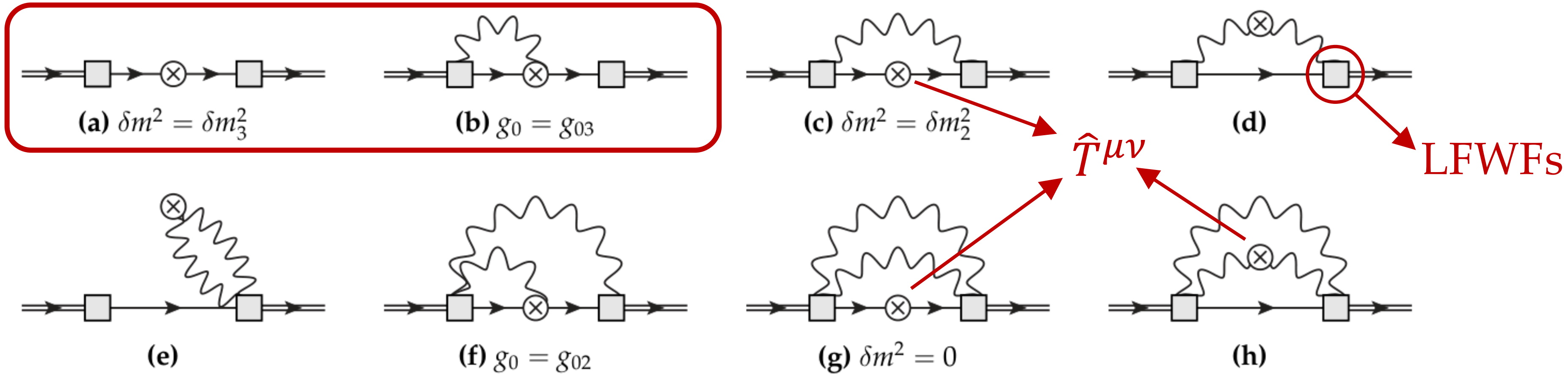
where $m = 0.94\text{GeV}$, $\mu = 0.14\text{GeV}$. g_0 and δm^2 are bare parameters.

- N: mock nucleon, π: mock pion [Gross:2001ha]
- Quenched approximation: to avoid vacuum instability
- Fock sector expansion: $|p\rangle = |N\rangle + |N\pi\rangle + |N\pi\pi\rangle + |N\pi\pi\pi\rangle \cdots$
- Solved up to $|N\pi\pi\pi\rangle$ sector at non-perturbative couplings
- Fock sector dependent renormalization [Karmanov:2008br]
- Fock sector expansion converged up to $|N\pi\pi\rangle$ sector [Li:2015iaw]

$\alpha = g^2/(16\pi m^2), \quad Q^2 = -q^2$



Stress-energy tensor renormalization



- Light-front wave functions (LFWFs) & sector dependent counterterms from [Li, Karmanov & Vary:2015iaw,2016yzu]
- Light-front graphical rules extended to non-perturbative regime using LFWFs [Carbonell:1998rj]
- All divergences cancel out with sector dependent counterterms, e.g. $(a) + (b)$:

$$t_a^{\alpha\beta} = Z[2P^\alpha P^\beta + (\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} - \frac{1}{2}q^\alpha q^\beta]$$

$$t_b^{\alpha\beta} = -\sqrt{Z} g^{\alpha\beta} \int \frac{dx}{2x(1-x)} \int \frac{d^2 k_\perp}{(2\pi)^3} g_{03}\psi_2(x, k_\perp) = g^{\alpha\beta} Z \delta m_3^2$$

$$t^{\alpha\beta} = \langle p' | \hat{T}^{\alpha\beta}(0) | p \rangle$$

Covariant decomposition on the light-front

[Carbonell:1998rj, Karmanov:2002qu]

The hadronic matrix element for spin-0 particles:

$$\langle p'|\,\hat{T}_i^{\alpha\beta}(0)\,|p\rangle = 2P^\alpha P^\beta A_i(q^2) + \frac{1}{2}(q^\alpha q^\beta - q^2 g^{\alpha\beta})D_i(q^2) + 2M^2 g^{\alpha\beta}\bar{c}_i(q^2) + \frac{M^4\omega^\alpha\omega^\beta}{(\omega\cdot P)^2}S_{1i}(q^2) + (V^\alpha V^\beta + q^\alpha q^\beta)S_{2i}(q^2)$$

where $P = (p+p')/2$, $q = p' - p$, $V^\alpha = \epsilon^{\alpha\beta\rho\sigma}P_\beta q_\rho \omega_\sigma/(\omega\cdot P)$. $\omega^\mu = (\omega^+, \omega^-, \boldsymbol{\omega}_\perp) = (0,2,0)$ is a null vector indicating the light-front direction.

- $S_{1,2}(q^2)$ are two spurious gravitational form factors (GFFs) which usually contain uncanceled divergences
- The spurious GFFs appear due to the violation of the full Lorentz symmetry

Components to extract gravitational form factors

In Drell-Yan-Breit frame ($q^+ = 0$, $\vec{P}_\perp = 0$):

$$
\begin{aligned}
t_i^{++} &= 2(P^+)^2 A_i(q_\perp^2), \\
t_i^{+-} &= 2(M^2 + \frac{1}{4}q_\perp^2)A_i(q_\perp^2) + q_\perp^2 D_i(q_\perp^2) + 4M^2\bar{c}_i(q_\perp^2), \\
t_i^{12} &= \frac{1}{2}q_\perp^1 q_\perp^2 D_i(q_\perp^2), \\
t_i^{11} + t_i^{22} &= -\frac{1}{2}q_\perp^2 D_i(q_\perp^2) - 4M^2\bar{c}_i(q_\perp^2) + 2q_\perp^2 S_{2i}(q_\perp^2), \\
t_i^{--} &= 2\Big(\frac{M^2 + \frac{1}{4}q_\perp^2}{P^+}\Big)^2 A_i(q_\perp^2) + \frac{4M^4}{(P^+)^2}S_{1i}(q_\perp^2)
\end{aligned}
$$

- $\hat{T}_i^{++}$, $\hat{T}_i^{12}$ and $\hat{T}_i^{+-}$ are three “good currents” which are free of spurious form factors
- Gravitational form factors derived from these currents are consistent with the covariant field theory in the perturbative limit

[Cao:2024rul]

Light-front wave function representation

[Cao:2023ohj, Cao:2024fto]

$$t^{12} = \frac{1}{2}\Big\langle \sum_j e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{j\perp}} \frac{i\overleftrightarrow{\nabla}_{j1}\overleftrightarrow{\nabla}_{j2} - q_1 q_2}{x_j} \Big\rangle$$

$$\int d^3x T^{+\mu}(x) = P^\mu$$

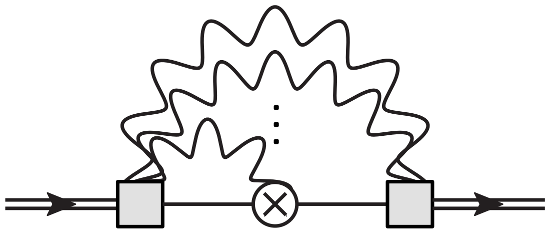
$$t^{+-} = 2\Big\langle \underbrace{\sum_j e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{j\perp}} \frac{-\frac{1}{4}\overleftrightarrow{\nabla}^2_{j\perp} + m_j^2 - \frac{1}{4}q_\perp^2}{x_j}}_{\text{kinetic part}} + \underbrace{V e^{i\boldsymbol{r}_{N\perp}\cdot\boldsymbol{q}_\perp}}_{\text{potential part}} \Big\rangle$$

$$P^- = \frac{P_\perp^2 + M^2}{P^+}$$

where $V = M^2 - \sum_j \frac{-\nabla^2_{j\perp} + m_j^2}{x_j}$ in the scalar Yukawa model. The quantum average is defined as

$$\langle \hat{O} \rangle = \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \hat{O} \psi_n(\{x_i, \boldsymbol{r}_{i\perp}\})$$

- Modify V in phenomenological models
- $e^{i\boldsymbol{r}_{N\perp}\cdot\boldsymbol{q}_\perp} \xrightarrow{\text{F.T.}} \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{N\perp})$ indicates the location of interaction



Energy and momentum densities

2D transverse densities on the light-front:

[Xu:2024hfx, Freese:2021czn]

$$\mathcal{T}^{\alpha\beta}(\vec{r}_\perp; P) = \frac{1}{2P^+} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \langle P + \frac{1}{2} q | \hat{T}^{\alpha\beta}(0) | P - \frac{1}{2} q \rangle$$

Momentum ($\mu = +, 1, 2$) and energy ($\mu = -$) densities:

$$\mathcal{P}^\mu(r_\perp) \equiv \mathcal{T}^{+\mu}(r_\perp; P) = P^\mu \mathcal{A}(r_\perp),$$

$$\mathcal{P}^-(r_\perp) \equiv \mathcal{T}^{+-}(r_\perp; P) = \frac{P_\perp^2 \mathcal{A}(r_\perp) + \mathcal{M}^2(r_\perp)}{P^+}$$

$$\int d^3x T^{+\mu}(x) = P^\mu$$

Where (for spin-0 particles):

$$\mathcal{A}(r_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} A(q_\perp^2),$$

$$\mathcal{M}^2(r_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \left[(M^2 + \frac{1}{4} q_\perp^2) A(q_\perp^2) + \frac{1}{2} q_\perp^2 D(q_\perp^2)\right]$$

$$P^- = \frac{P_\perp^2 + M^2}{P^+}$$

- $\mathcal{A}(r_\perp)$ can be interpreted as the momentum density
- $\mathcal{M}^2(r_\perp)$ can be interpreted as the invariant mass squared density

Hadron as a relativistic medium

[Li:2024vgv]

- The quantum expectation value of the stress-energy tensor:

$$\langle\Psi|\hat{T}^{\alpha\beta}(x)|\Psi\rangle = \langle\mathcal{E}\mathcal{U}^\alpha\mathcal{U}^\beta - \mathcal{P}\Delta^{\alpha\beta} + \Pi^{\alpha\beta} - g^{\alpha\beta}\Lambda\rangle_\Psi$$

where $\mathcal{U}^\alpha$ is hadronic 4-velocity ($\mathcal{U}^\alpha\mathcal{U}_\alpha = 1$), $\Delta^{\alpha\beta} = g^{\alpha\beta} - \mathcal{U}^\alpha\mathcal{U}^\beta$

- Physical densities:

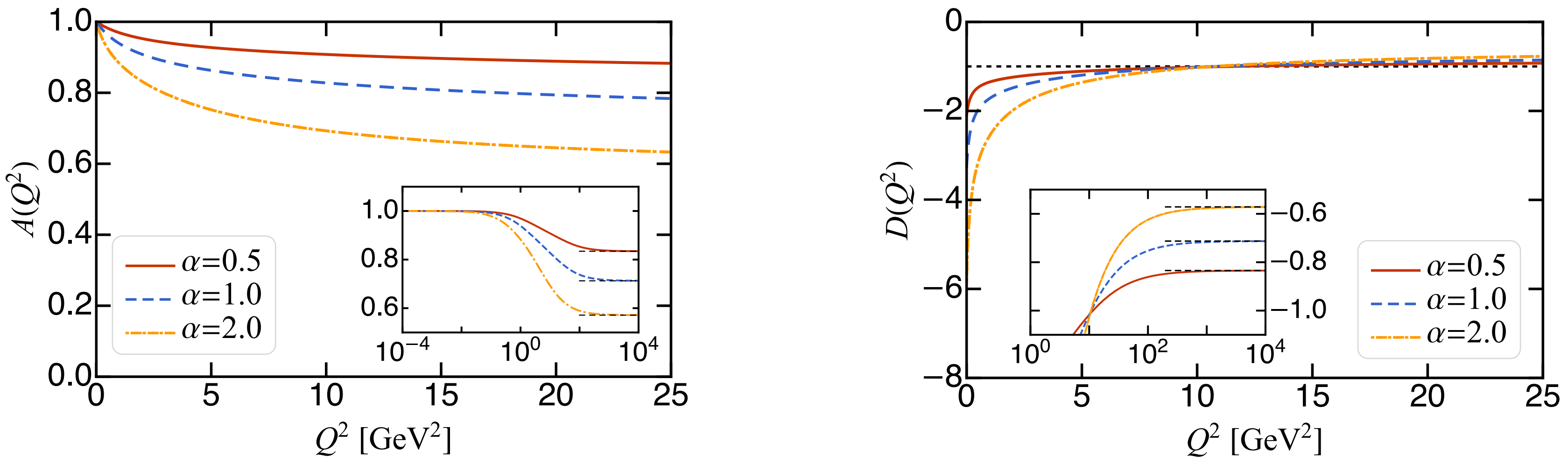
$$\text{Energy density: } \mathcal{E}(x) = M\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\left\{A(q^2) - \frac{q^2}{4M^2}[A(q^2)+D(q^2)]\right\}$$

$$\text{Pressure: } \mathcal{P}(x) = \frac{1}{6M}\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot x}q^2D(q^2)$$

$$\text{Shear tensor: } \Pi^{\alpha\beta}(x) = \frac{1}{4M}\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot x}(q^\alpha q^\beta - \frac{q^2}{3}\Delta^{\alpha\beta})D(q^2)$$

$$\text{Cosmological constant: } \Lambda = -M\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot x}\bar{c}(q^2)$$

Strongly-coupled scalar nucleon



[Cao:2023ohj]

$$\alpha = \frac{g^2}{16\pi m^2}$$

- For small coupling, $D(Q^2)$ is close to -1, the free scalar particle's result
- In the forward limit ($Q^2 = 0$),

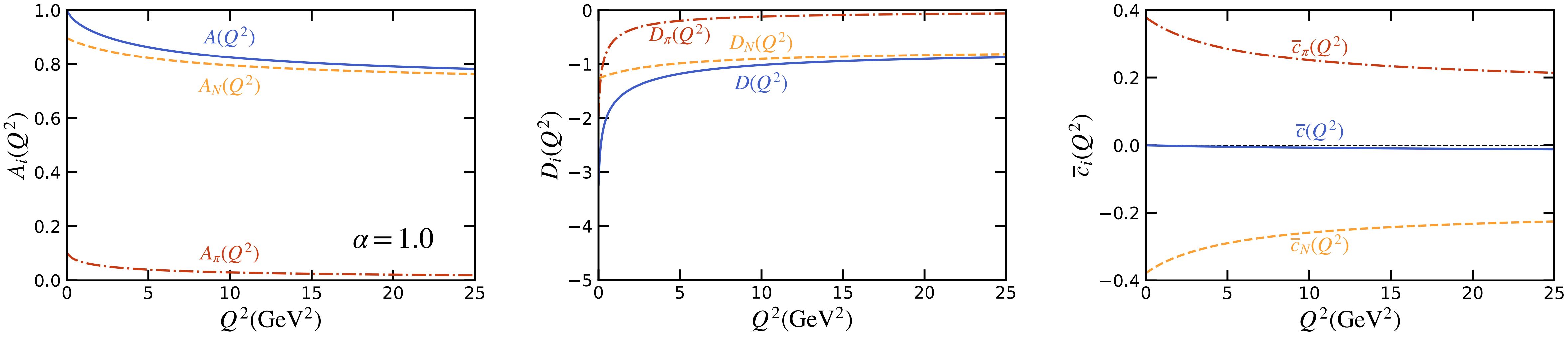
$$\lim_{Q^2=0} A(Q^2) = 1, \qquad \lim_{Q^2\to 0} Q^2 D(Q^2) = 0$$

- For large Q^2,

$$\lim_{Q^2\to\infty} A(Q^2) = Z, \qquad \lim_{Q^2\to\infty} D(Q^2) = -Z$$

revealing a pointlike core, consistent with the physical picture of the model

Dissecting the strongly-coupled scalar nucleon

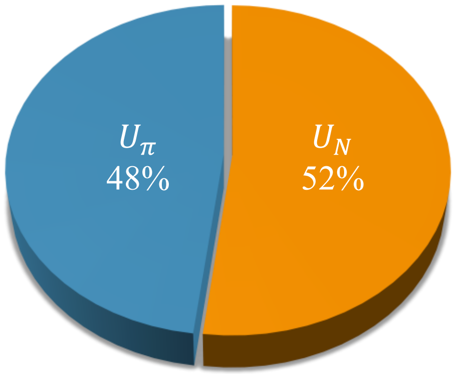


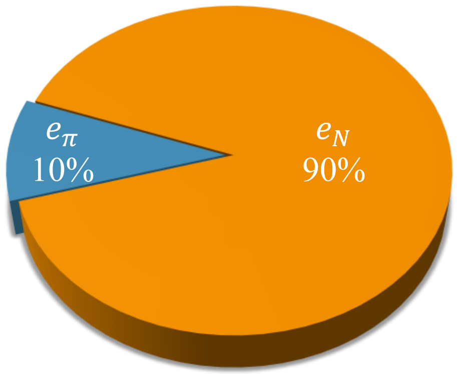
- A nonvanishing but small $\bar{c}(q^2)$ because of Fock space truncation $\sum_i \bar{c}_i(q^2) \neq 0$ [Cao:2024fto]
- Mass decomposition: [Lorce:2017xzd]

$$e_i = \int d^2 r_\perp \mathcal{E}(r_\perp) = A_i(0)$$

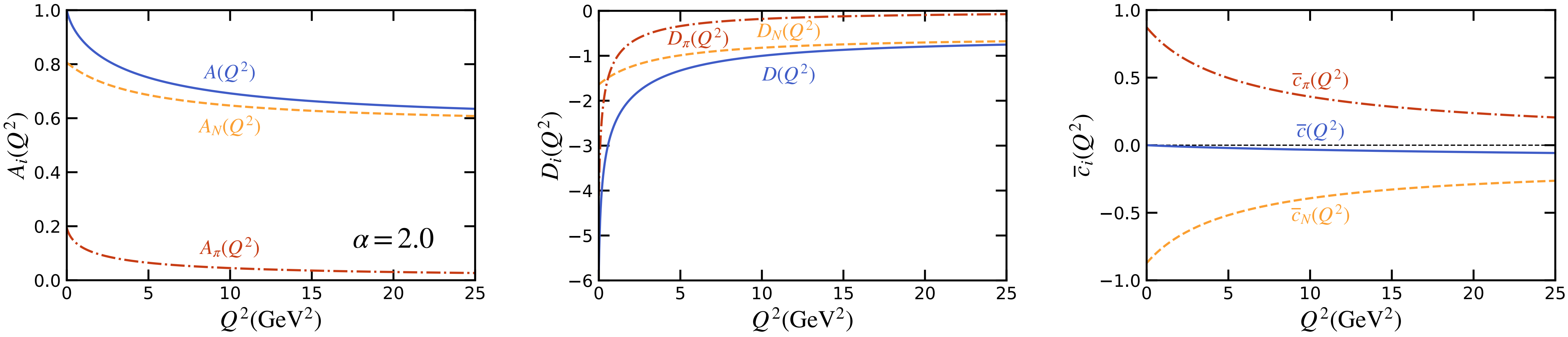
$$\lambda_i = \int d^2 r_\perp \Lambda_i(r_\perp) = \bar{c}_i(0)$$

$$U_i = e_i + \lambda_i$$





Dissecting the strongly-coupled scalar nucleon



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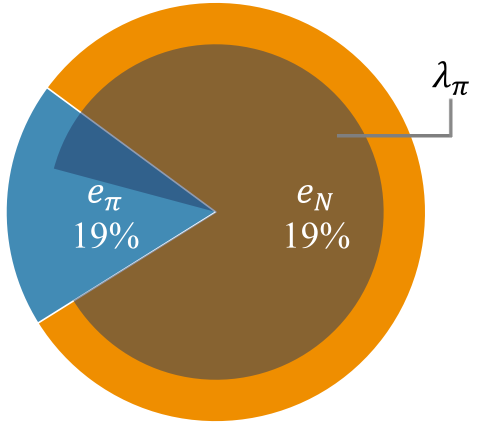


Charmonium: hydrogen atom of QCD

Effective Hamiltonian in the $q\bar{q}$ Fock sector [Li:2015zda, Li:2017mlw, Li:2021ejv]

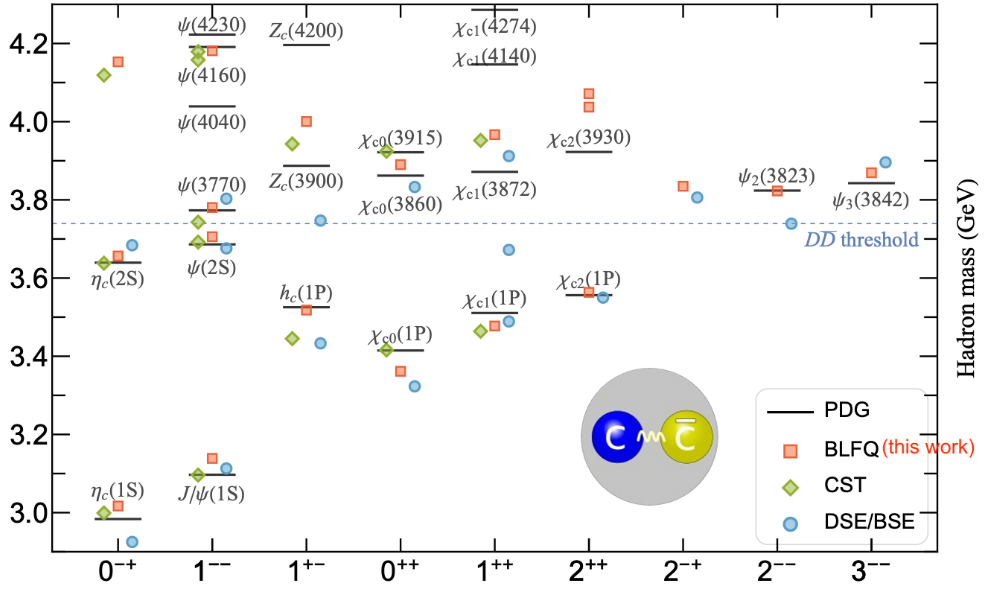
kinetic energy — confinement

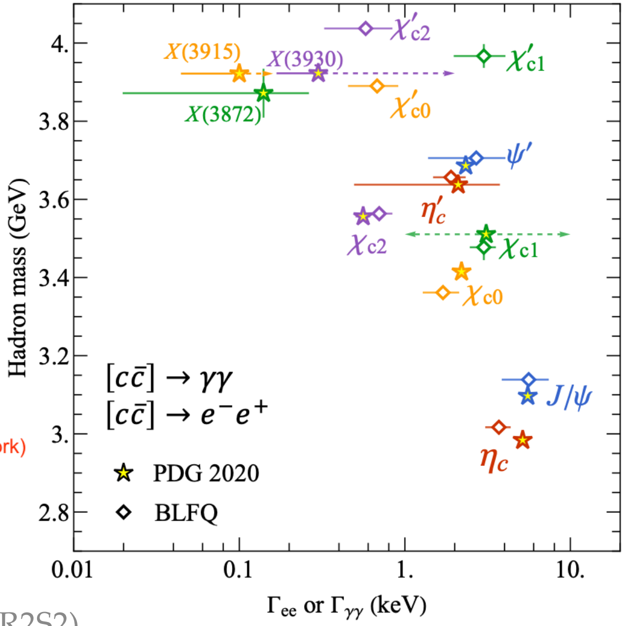
$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x\big(x(1-x)\partial_x\big)$$

$$- \frac{C_F 4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}')$$

one gluon exchange

Two parameters (m_q, κ) are fixed by fitting the charmonium mass spectrum



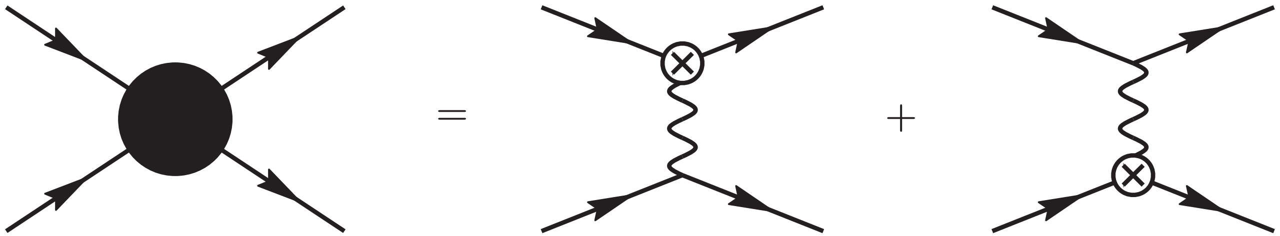


Impulse ansatz

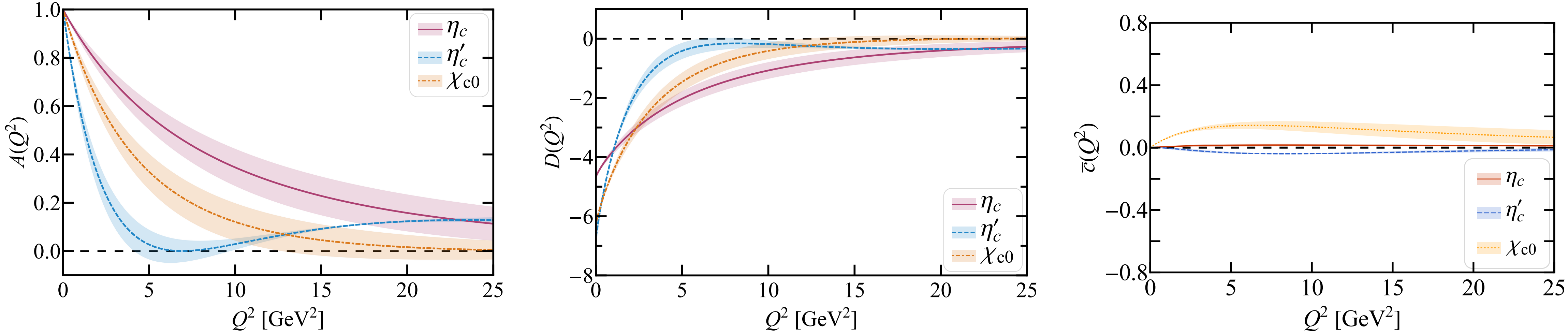
- From the effective Hamiltonian, we can't give the exact stress-energy operator directly
- We adopt impulse ansatz for the interaction term in T^{+-}

$$t^{+-}_{\text{int}} = \frac{1}{2}\sum_{s,\bar{s}}\int \frac{\mathrm{d}x}{4\pi x(1-x)}\int \mathrm{d}^2 r_\perp \psi^*_{s\bar{s}}(x,\vec{r}_\perp)\left[e^{i\vec{q}_\perp\cdot\vec{r}_{1\perp}} + e^{i\vec{q}_\perp\cdot\vec{r}_{2\perp}}\right]v(x,\vec{r}_\perp,-i\nabla_\perp)\psi_{s\bar{s}}(x,\vec{r}_\perp)$$

where $v(x,\vec{r}_\perp,-i\nabla_\perp) = M^2 - \frac{-\nabla^2_\perp+m^2_q}{x} - \frac{-\nabla^2_\perp+m^2_{\bar{q}}}{1-x}$



Charmonium gravitational form factors



[Xu:2024hfx, Hu:2024edc]

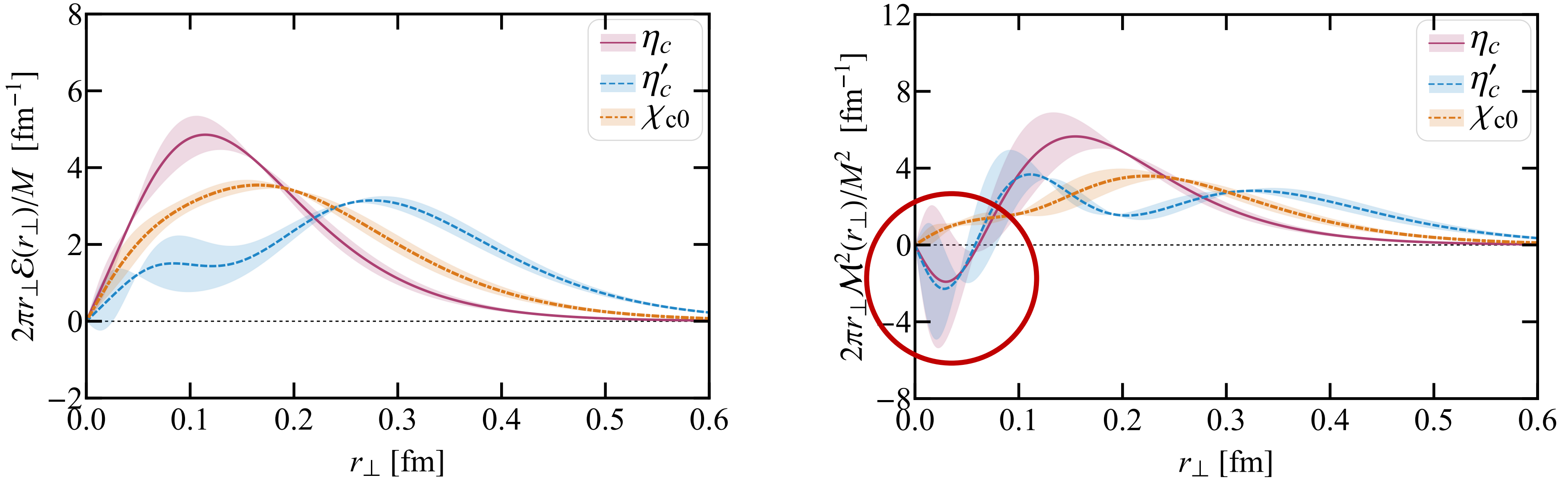
- GFF $\bar{c}(q^2)$ can measure the violation of Lorentz symmetries
- The S-wave charmonium η_c retains more Lorentz symmetries than the P-wave χ_{c0}

Energy density and invariant mass squared density

$$\mathcal{E}(r_\perp) = M \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \left\{ \left(1 + \frac{q_\perp^2}{4M^2}\right) A\left(q_\perp^2\right) + \frac{q_\perp^2}{4M^2} D\left(q_\perp^2\right) \right\},$$

$$\mathcal{M}^2(r_\perp) = M^2 \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \left\{ \left(1 + \frac{q_\perp^2}{4M^2}\right) A\left(q_\perp^2\right) + \frac{q_\perp^2}{2M^2} D\left(q_\perp^2\right) \right\}$$

- Energy density is positive
- Invariant mass squared density becomes negative at small $r_\perp$: tachyonic core?

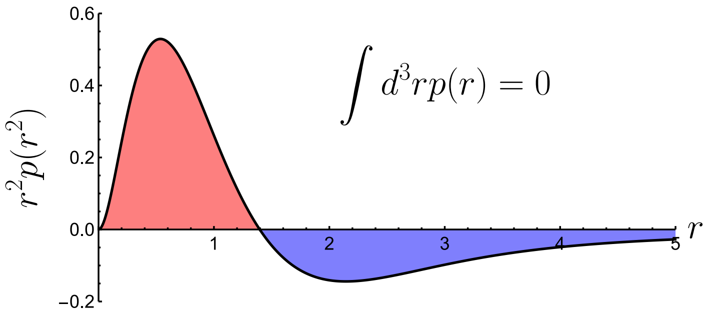


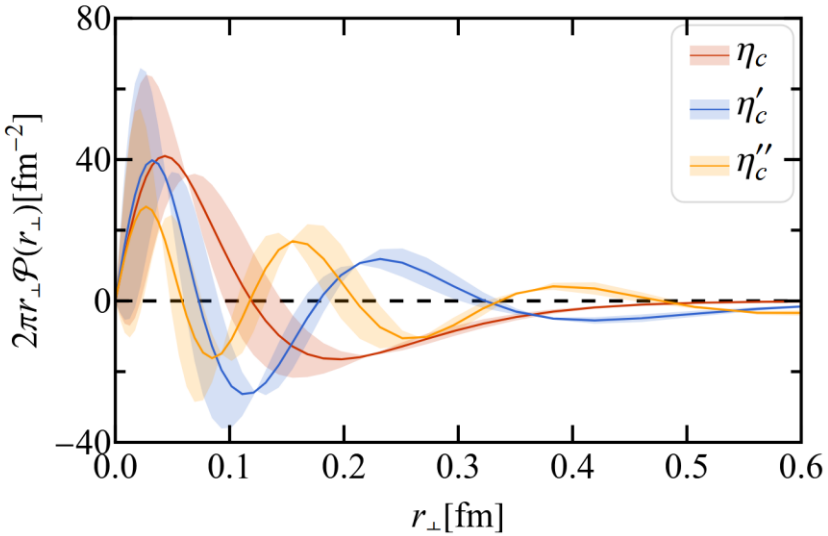
Pressure inside charmonium

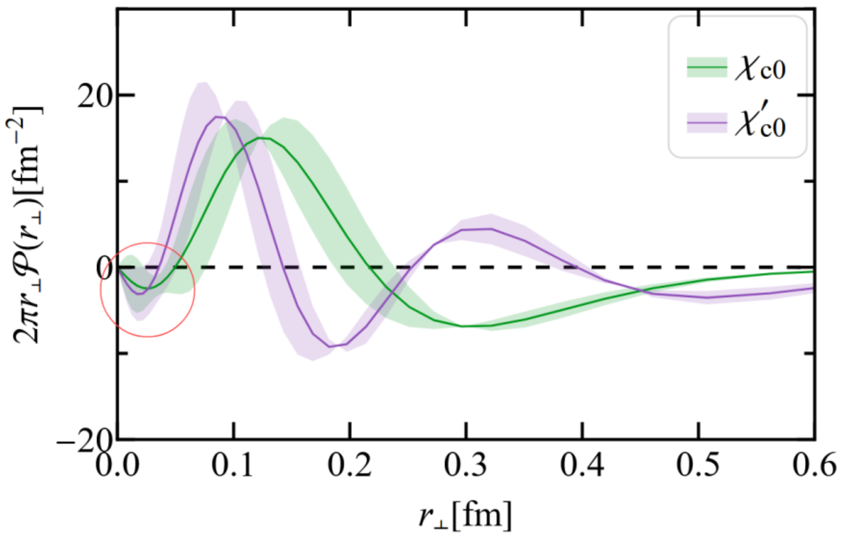
Pressure $\mathcal{P}(r_\perp)$

$$\mathcal{P}(r_\perp) = -\frac{1}{6M}\int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} q_\perp^2 D(q_\perp^2)$$

- η_c has a repulsive core while χ_{c0} has an attractive core
- Particles with higher radial excitation have more complicated mechanical structures







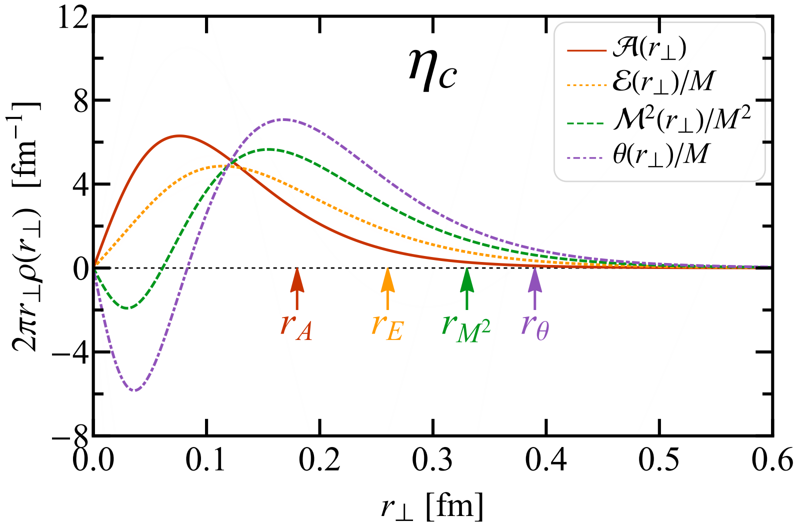
Physical densities

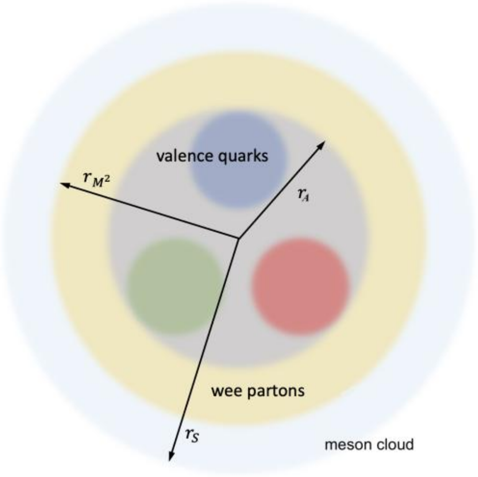
- Momentum density $\mathcal{A}(r_\perp)$, energy density $\mathcal{E}(r_\perp)$, invariant mass squared density $\mathcal{M}^2(r_\perp)$ and the trace scalar density $\theta(r_\perp) = \mathcal{T}^\alpha_\alpha(r_\perp) = \mathcal{E}(r_\perp) - 3\mathcal{P}(r_\perp)$
- The negative D suggests a chain of inequalities about different radii

$$r_A < r_E < r_{M^2} < r_\theta$$

$$r_A^2 = -6A'(0), \quad r_E^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+D), \quad r_{M^2}^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+2D), \quad r_\theta^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+3D)$$

$$\lambda_C = \frac{1}{M}$$





Summary

- We obtain a non-perturbative light-front wave function representation to evaluate the gravitational form factors
- We apply the light-front wave function representation to two systems: the strongly-coupled scalar theory and the charmonium
- The extracted densities provide novel insights into the hadron structures

Based on:

Cao, Li, Vary, PRD 108, 056026 (2023)
Xu, Cao, Hu, Li, Zhao, Vary, PRD 109, 114024 (2024)
Cao, Xu, Li, Chen, Zhao, Karmanov, Vary, JHEP (2024) 95
Cao, Li, Vary, PRD 110, 076025 (2024)
Hu, Cao, Xu, Li, Zhao, Vary, arXiv:2408.09689

Thank you!