# Rising Researchers Seminar Series

The Rising Researchers Seminar Series is committed to creating an **environment which is welcoming and safe** for all speakers, attendees, organizers, and other affiliated persons, hereinafter ``participants". It is the responsibility of all participants to foster such an environment.

Participants shall **treat each other professionally and respectfully**. Participants shall refrain from any inappropriate behaviour, including discriminatory actions and comments based on individual characteristics such as age, race, ethnicity, sexual orientation, gender identity, gender expression, marital status, nationality, immigration status, political affiliation, ability status, or educational background. Disruptive or harassing behavior will not be tolerated including but not limited to inappropriate or intimidating language and unwelcome jokes or comments.

**Violations of this code of conduct should be reported** to any member of the executive committee (see About Us page on the website). The Executive committee may also act of their own accord should they notice violations. Sanctions may range from verbal warning, to ejection from the meeting or, in severe cases, partial or total bans from future meetings.

References: This code of conduct is based heavily on that of the INT and the APS. We are also grateful to Roxanne Springer for valuable discussion and guidance.

Massachusetts Institute of Technology

LNS
Laboratory for Nuclear Science

# Using the atomic nucleus as a probe for BSM physics.

## INT Rising Researchers Seminar

Antoine Belley

11 Feb. 2025

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NSERC CRSNG

# Shortcomings of the Standard Model

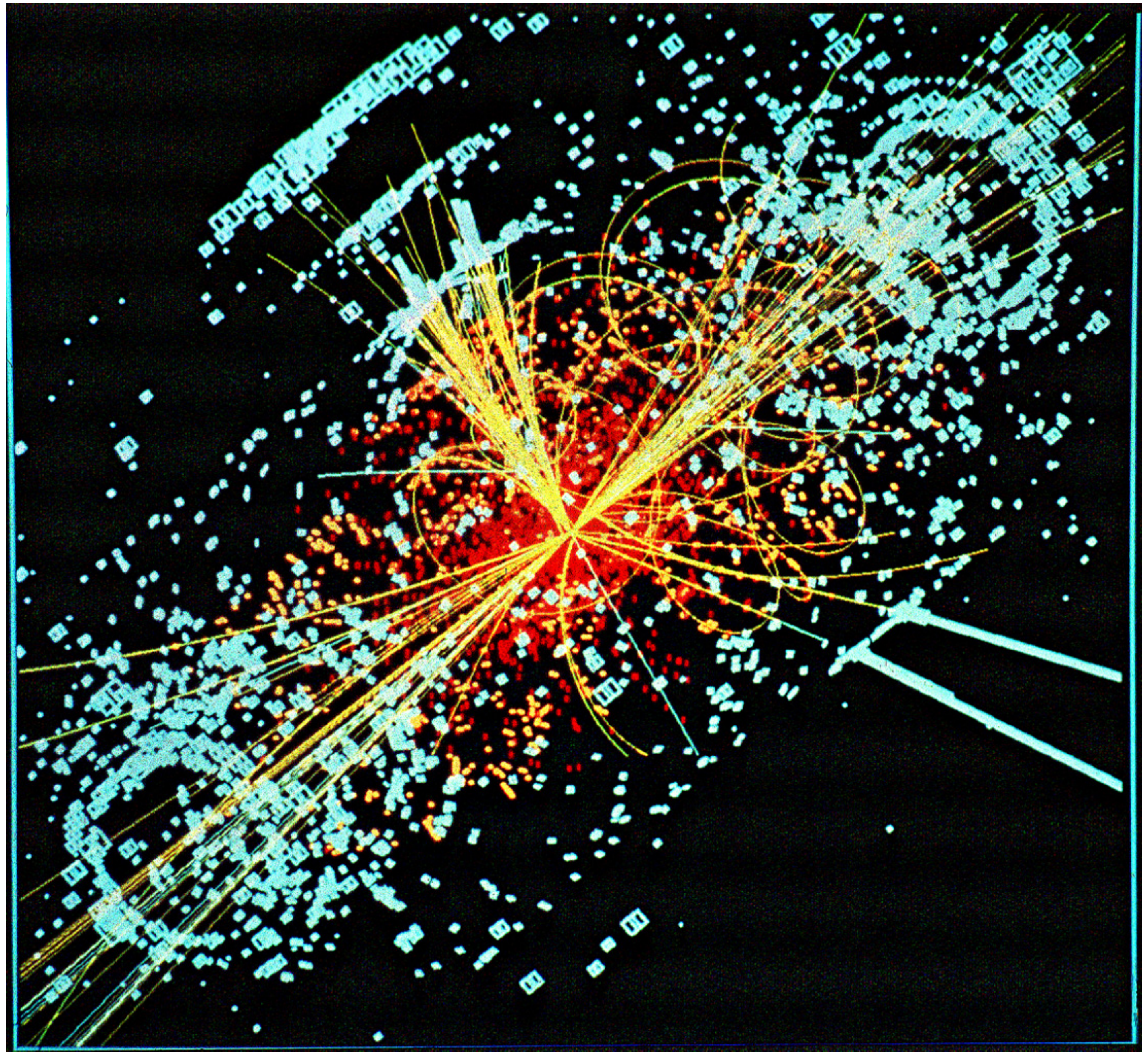
While being the most successful scientific model ever, the Standard Model has some major shortcomings:

- Fails to explain gravity
- Fails to explain most of the mass and energy of the universe (only predicts ~ 5% of it)
- Fails to explain neutrino mass and oscillation.
- Fails to explain the matter-antimatter asymmetry.

How should we search beyond the Standard Model to explain these shortcomings?

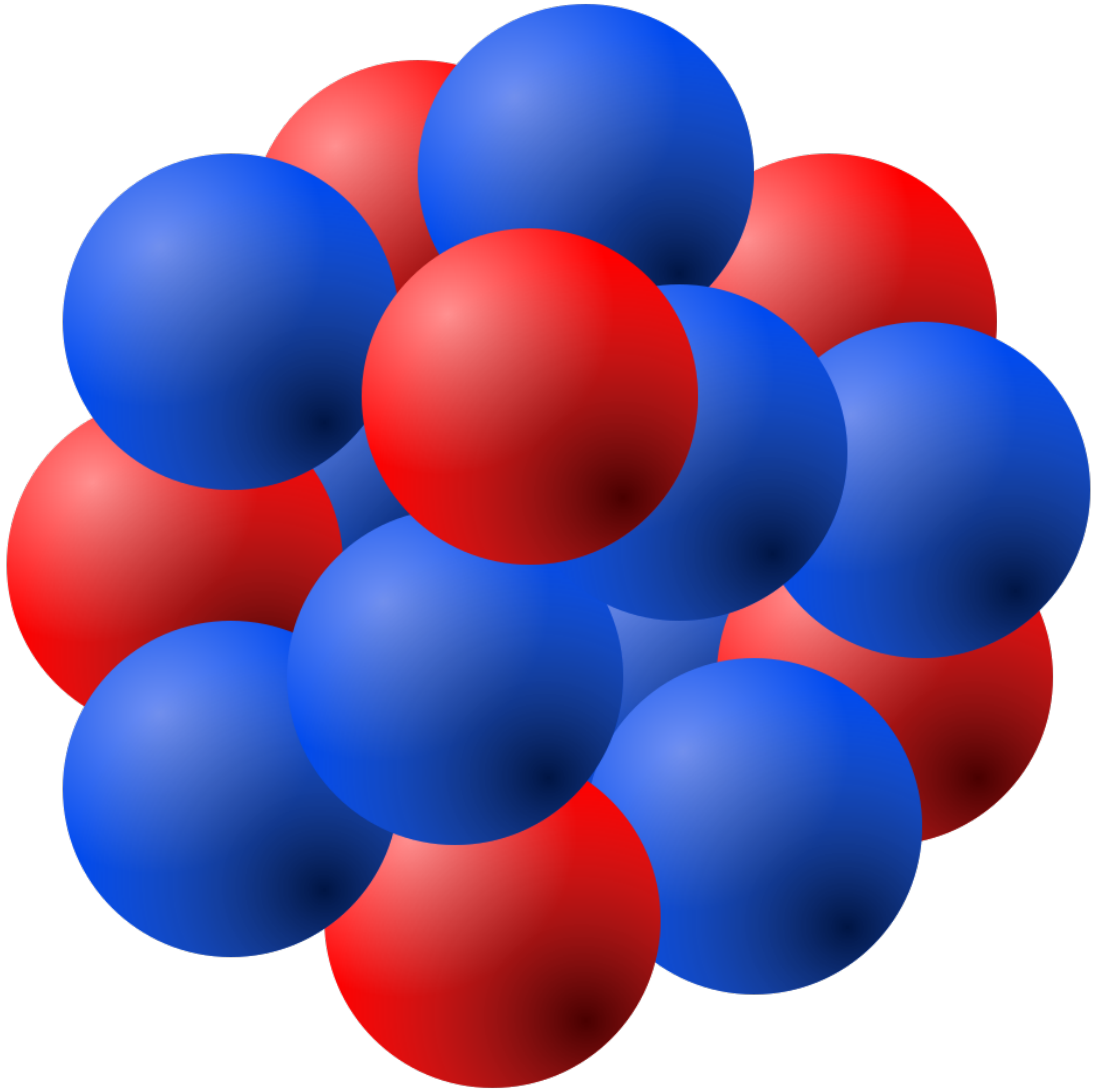
# Going Beyond

## 1. High-energy route



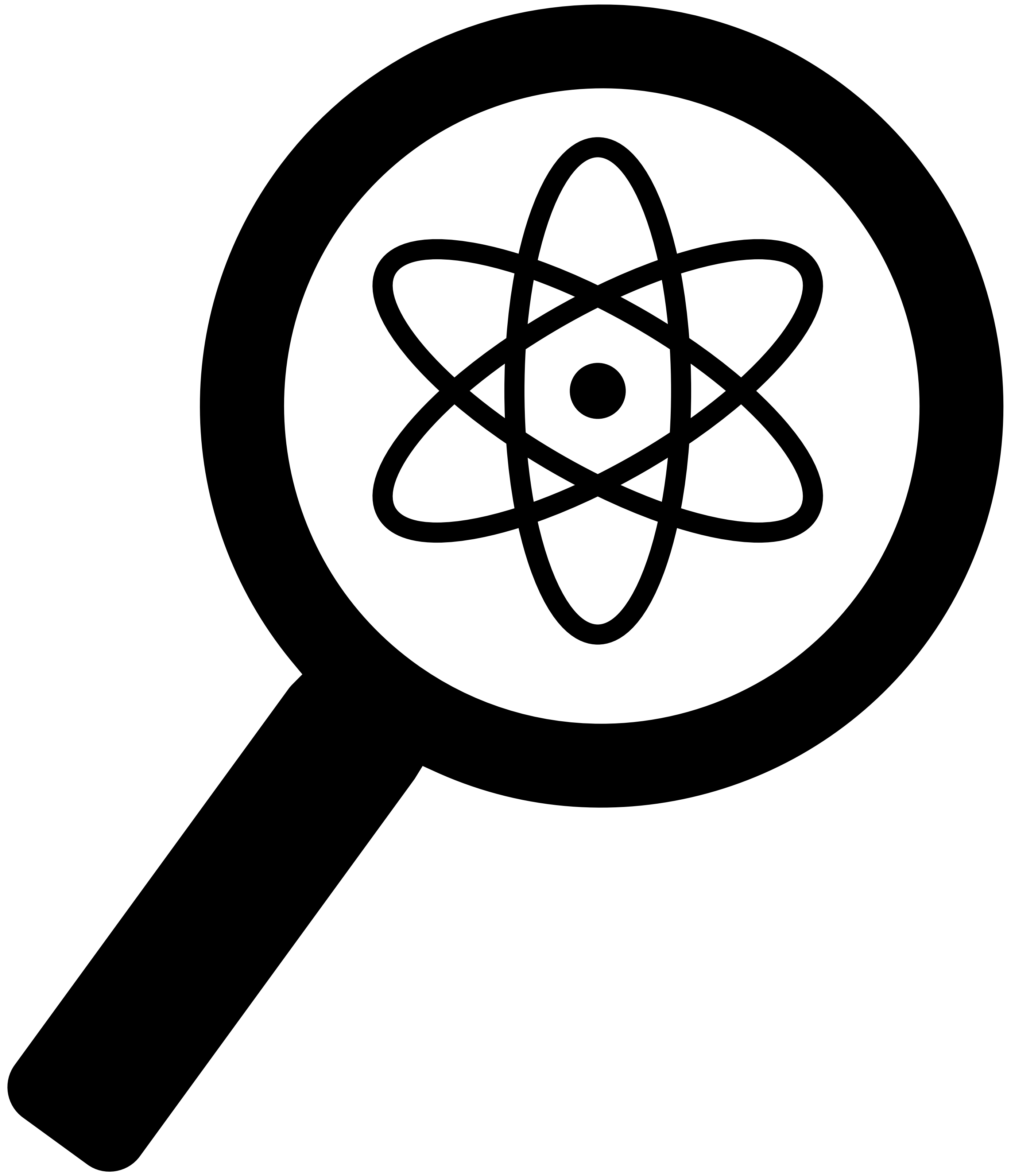
https://cds.cern.ch/record/628469

## 2. High-precision low-energy route



By Marekich - Own work (vector version of PNG image), CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=21701588

# Atomic Nucleus as a Probe



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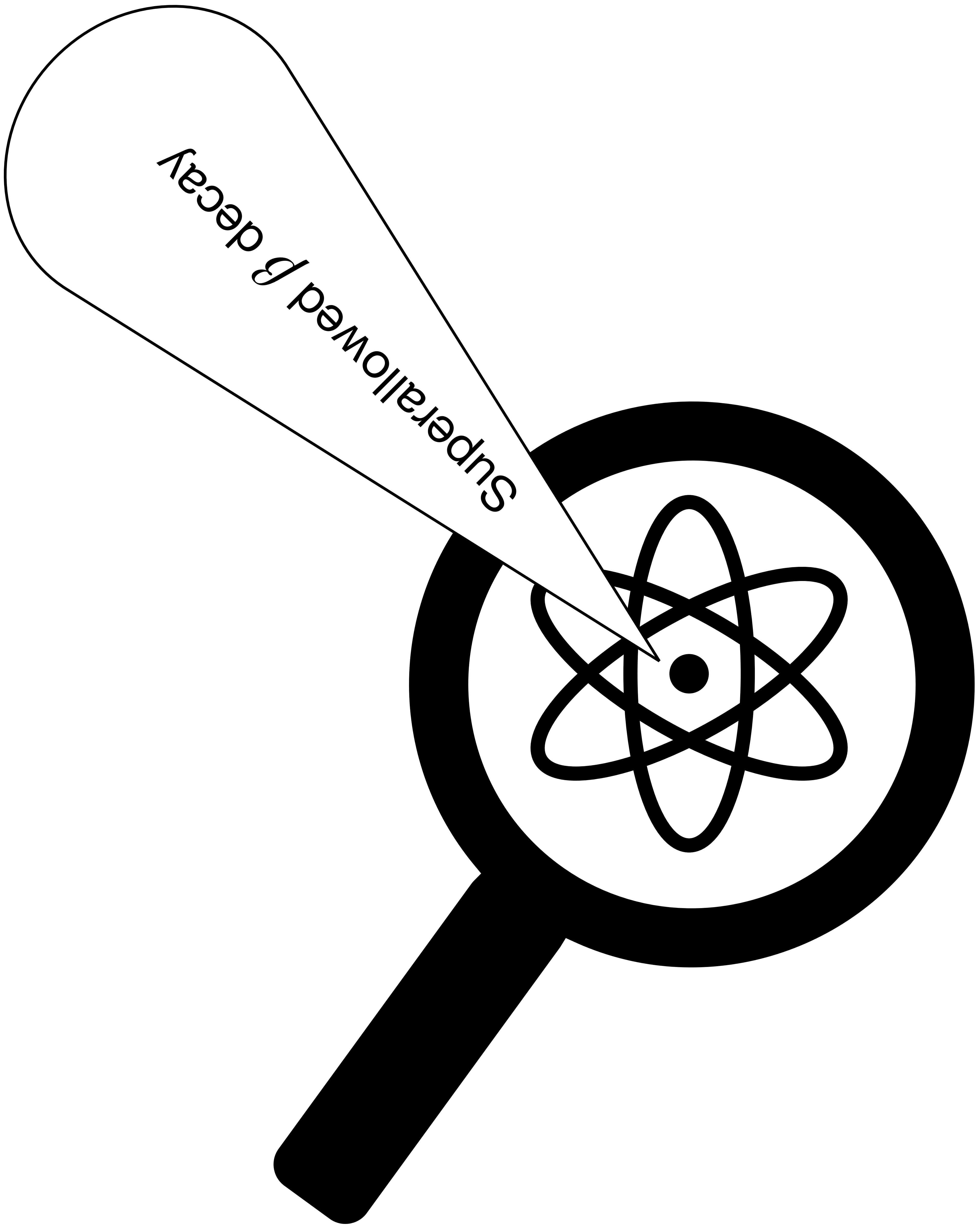

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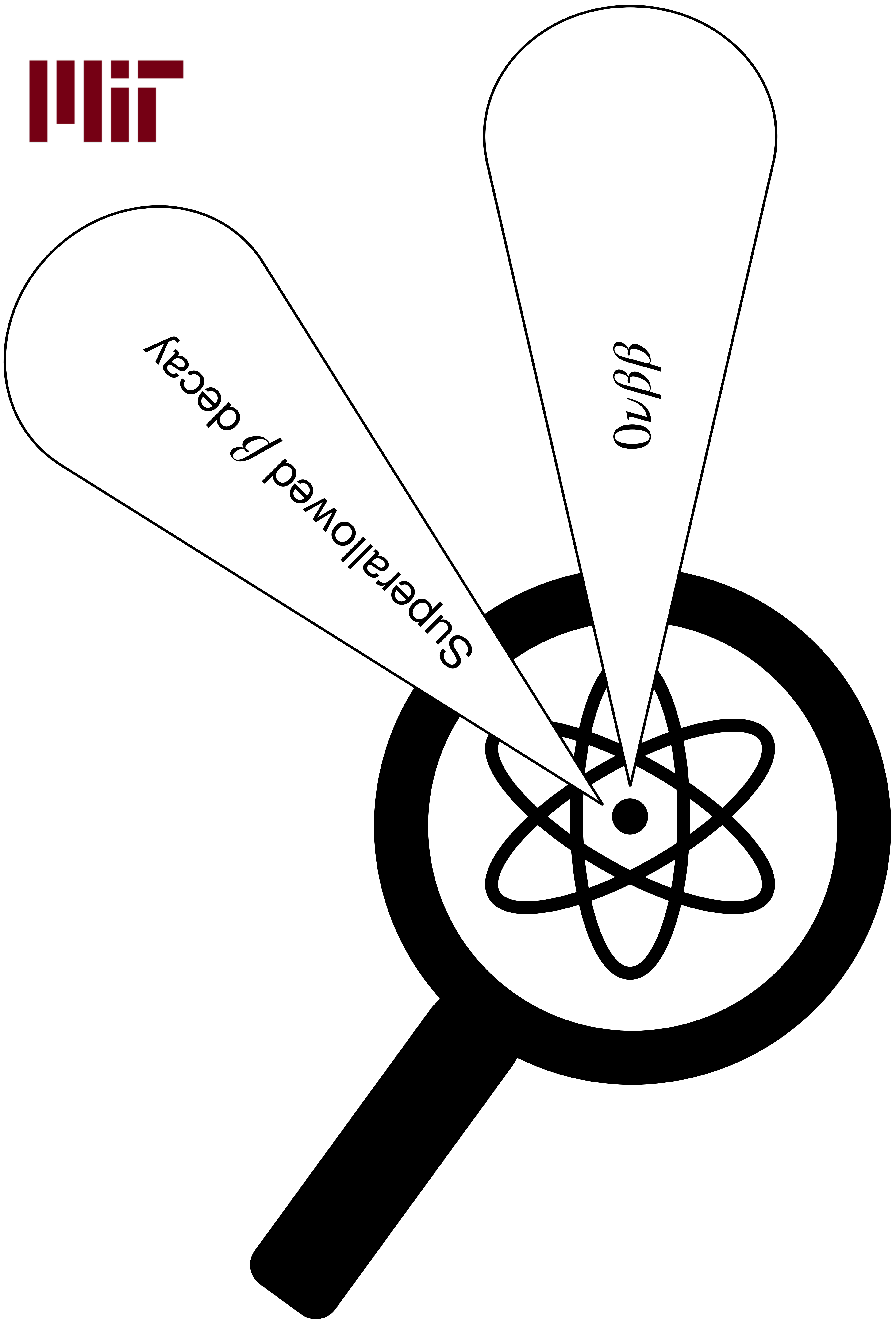

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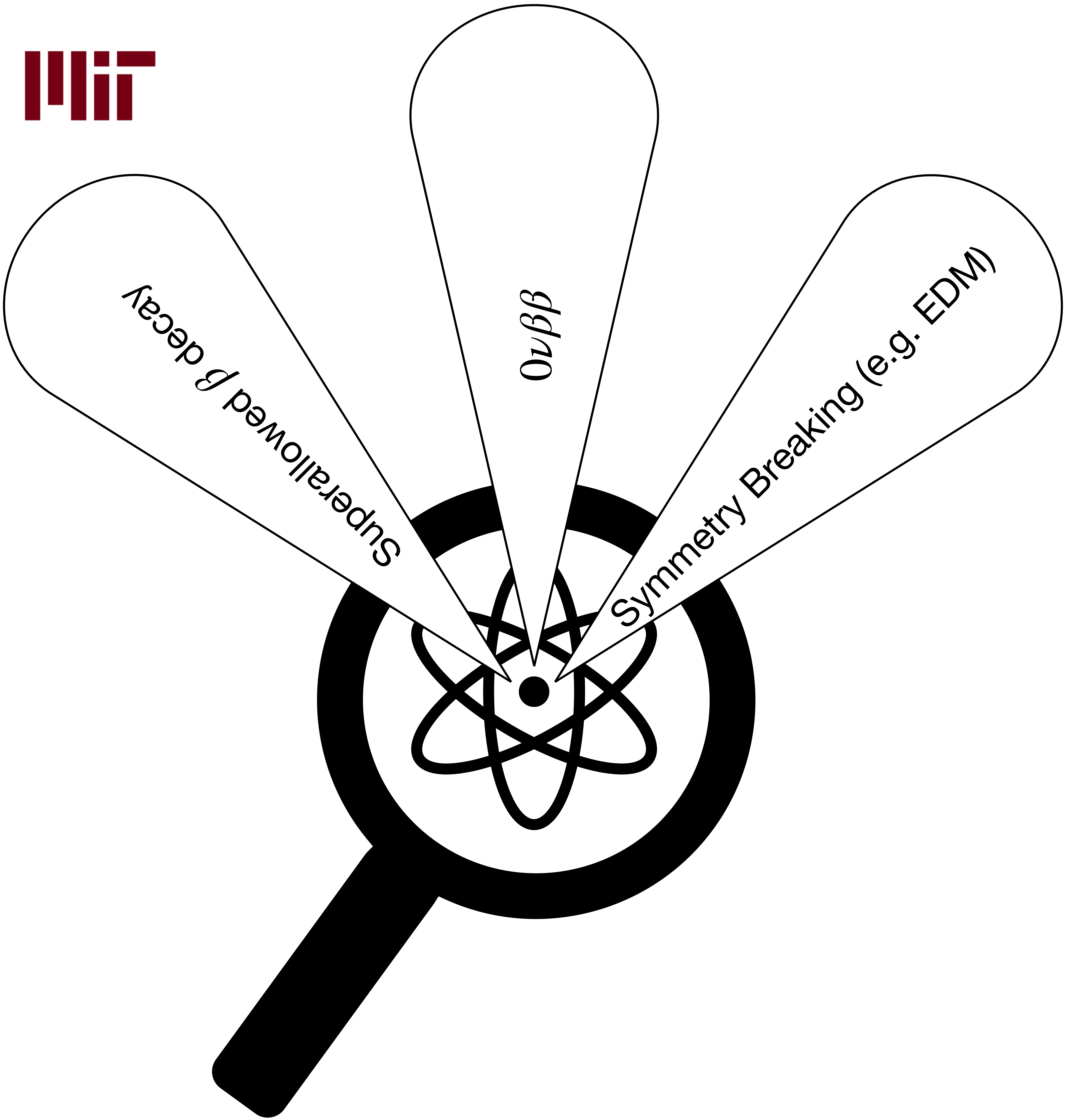

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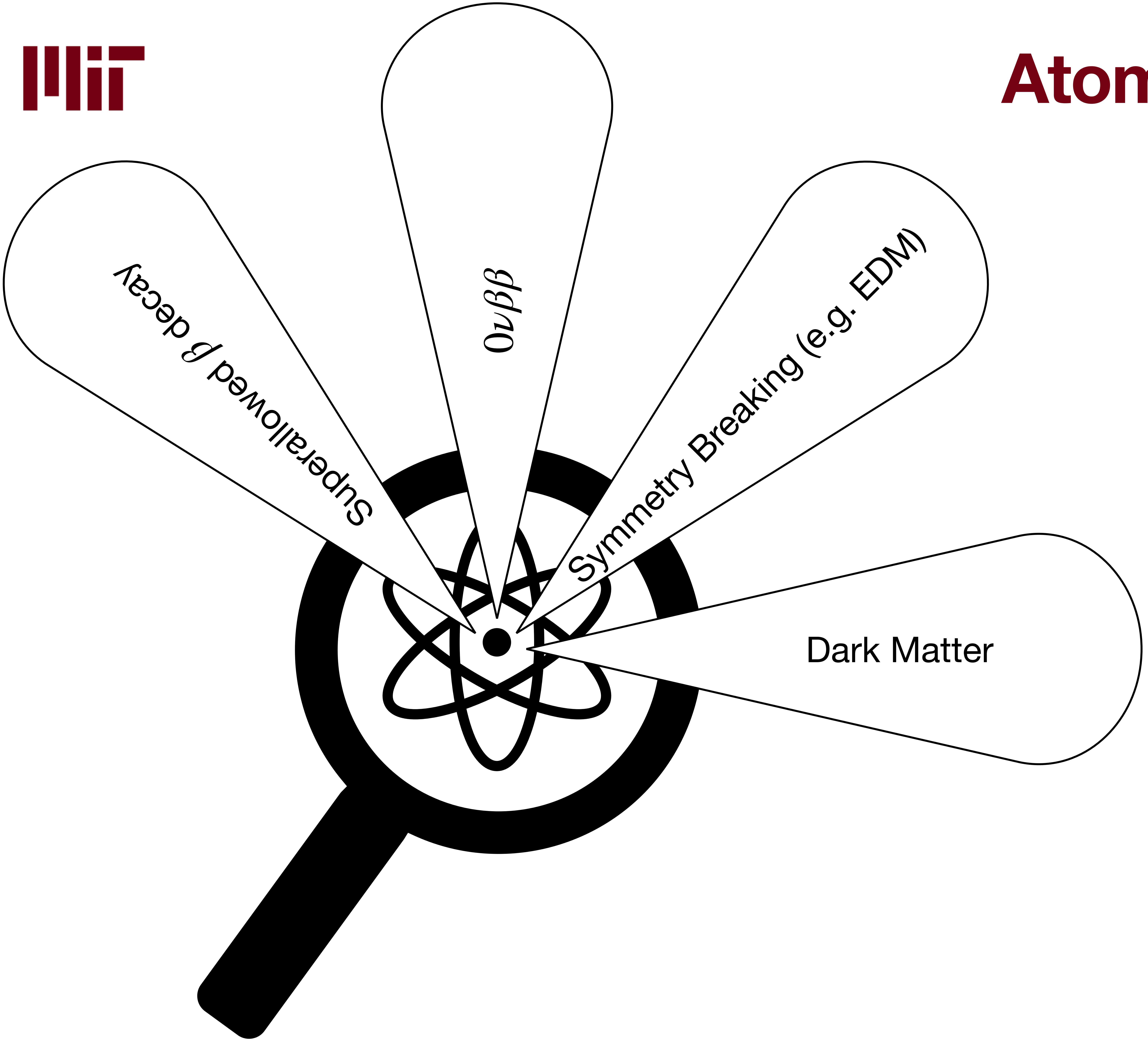

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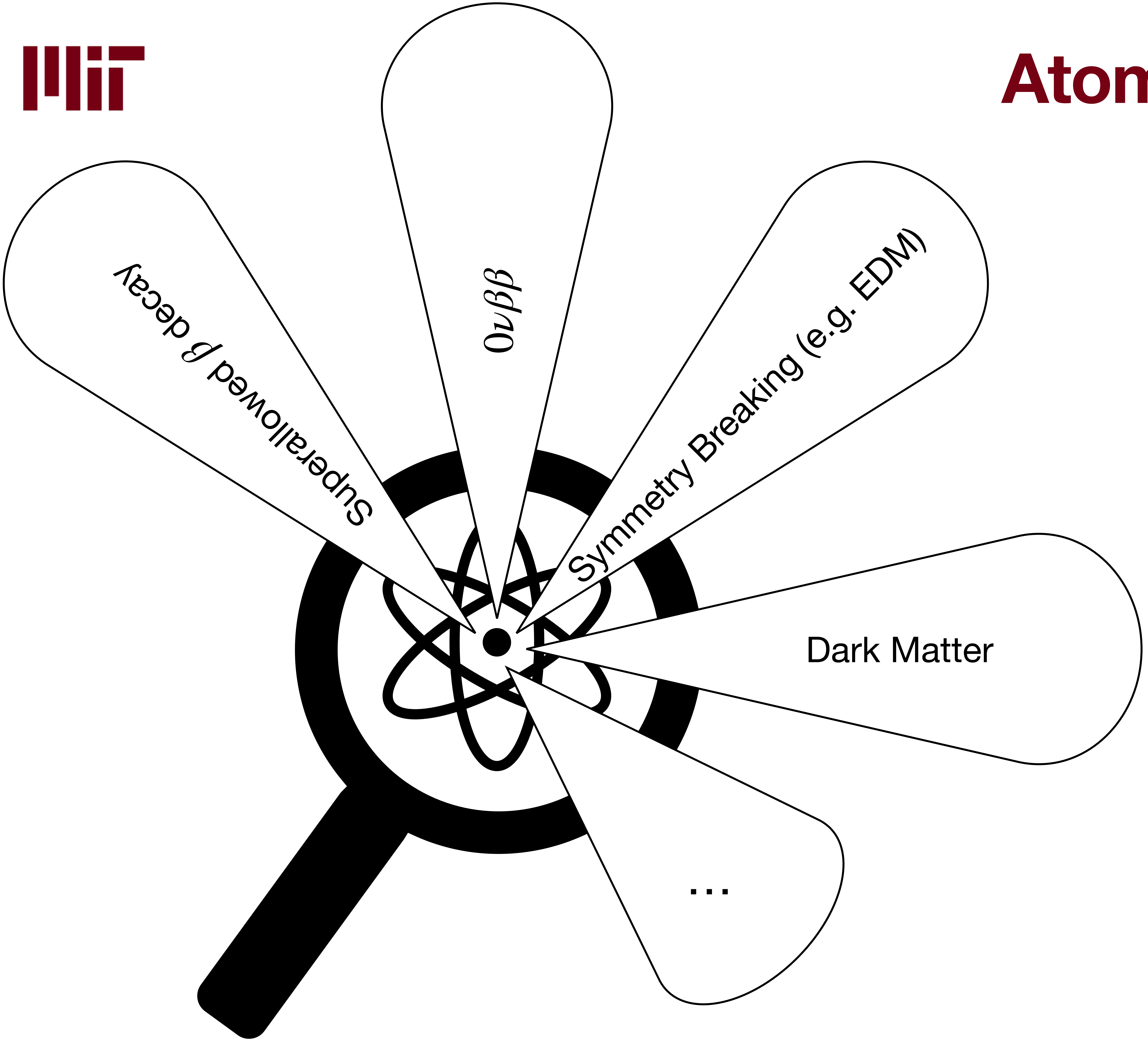

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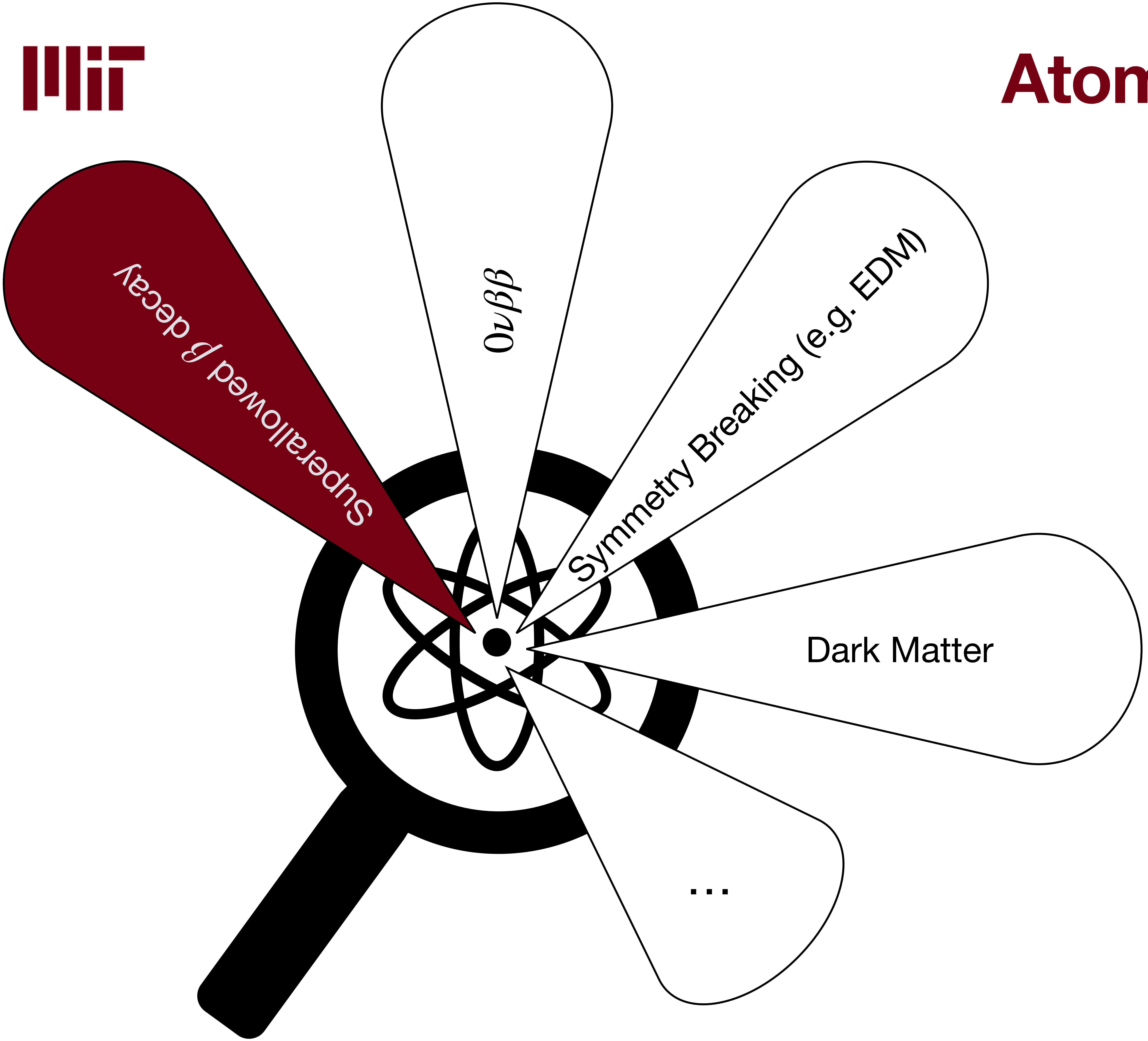


1. Looking at deviation from Standard Model prediction.

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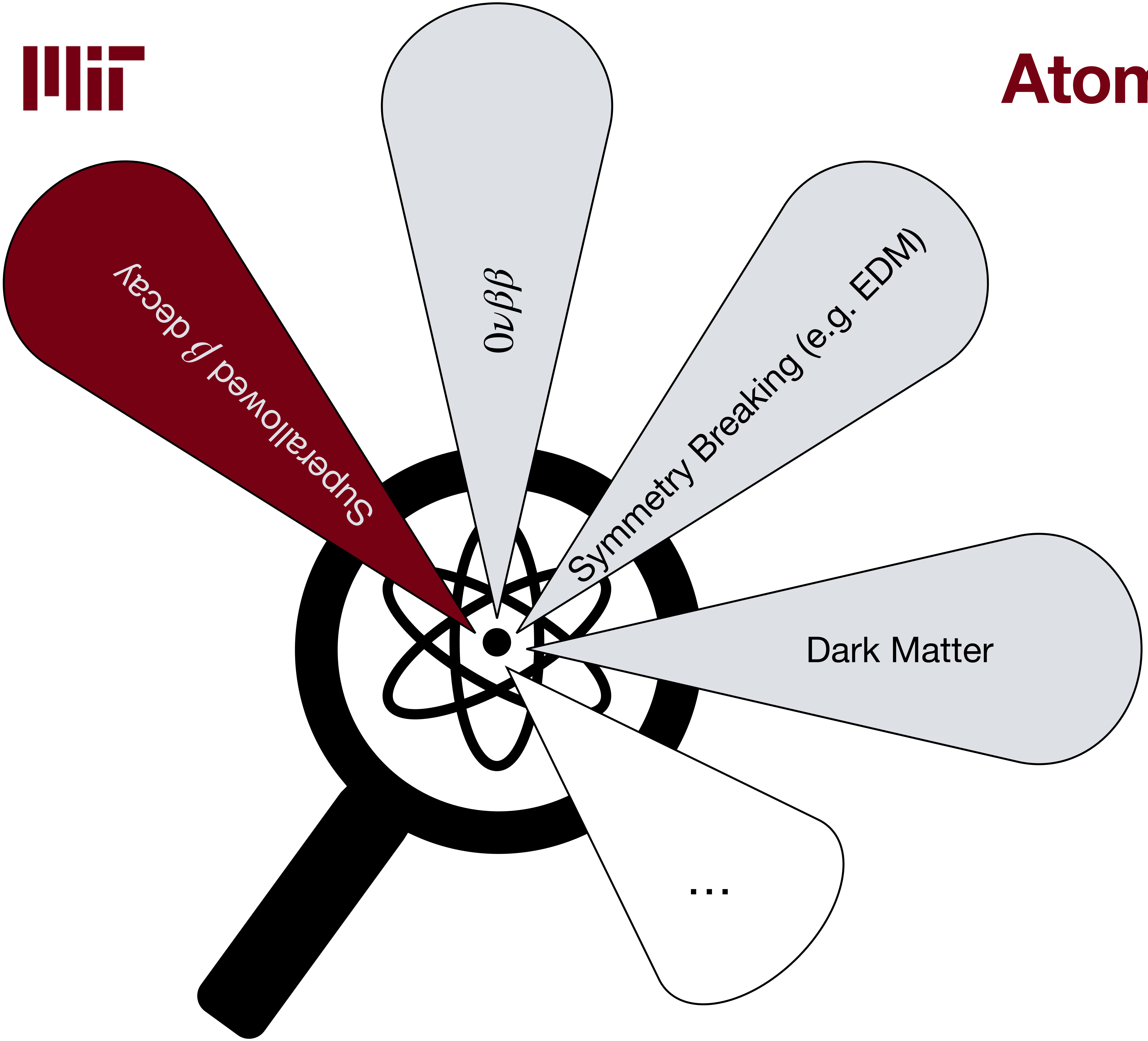



1. Looking at deviation from Standard Model prediction.

2. Search for phenomena not predicted by the Standard Model.

# The Need for Nuclear Theory

In all cases, nuclear theory inputs are required in order to interpret experimental results:

- **Superallowed $\beta$ decay:** Corrections to Standard Model $\delta_{NS}$ and $\delta_C$.
- $\mathbf{0\nu\beta\beta}$**:** Nuclear matrix elements $M^{0\nu}$.
- **Electric Dipole Moment:** The nuclear Schiff Moment.
- **Dark Matter Scattering:** WIMP Scattering structure factor $S_A$.
- …

# The Need for Nuclear Theory

In all cases, nuclear theory inputs are required in order to interpret experimental results

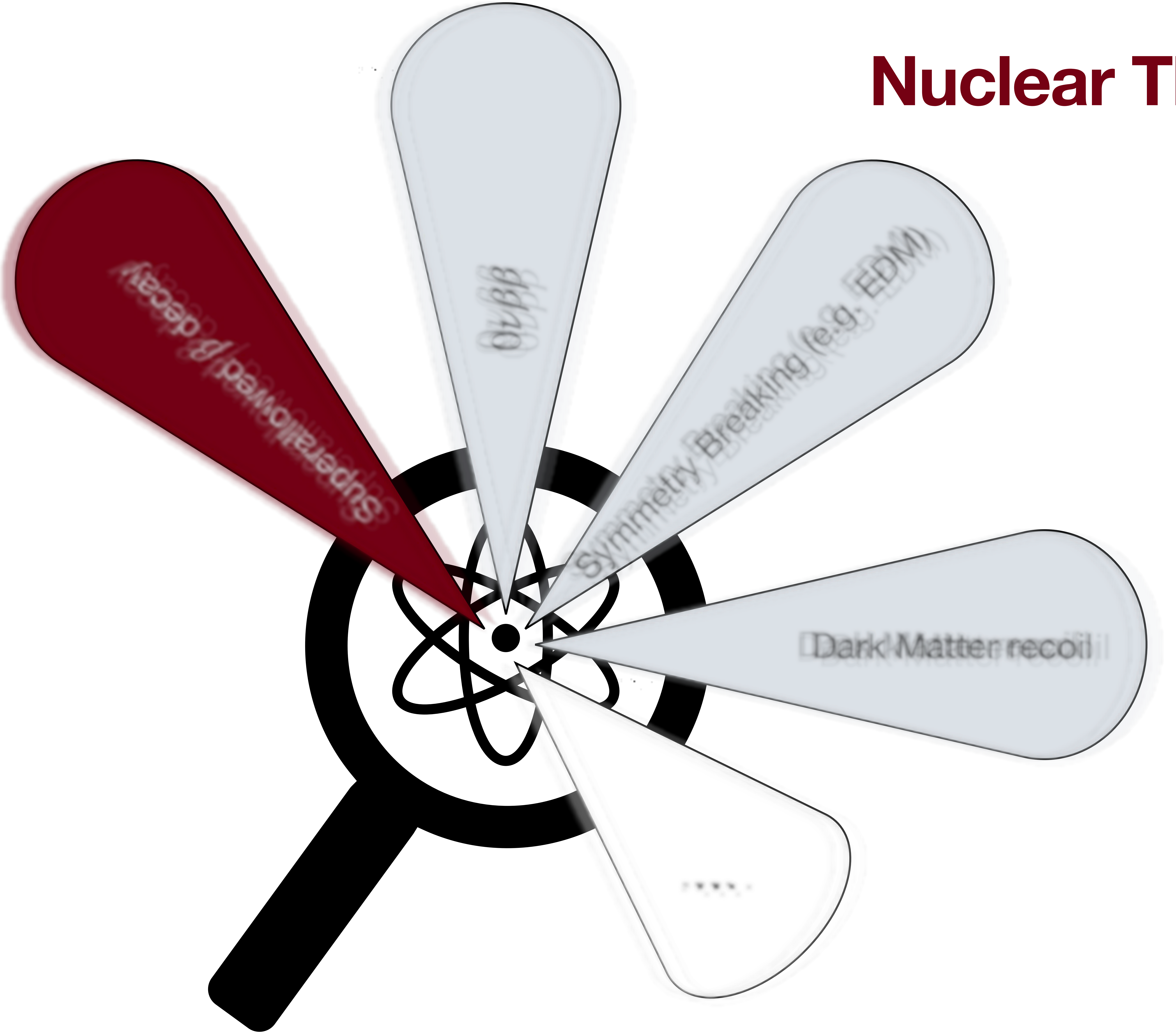
- Sup
- $0\nu\beta$
- Elec
- Dark
- …

Most calculations so far rely on phenomenological nuclear models with uncontrollable approximations, i.e. uncertainty quantification is really difficult .

$\Rightarrow$

In most cases, the largest source of uncertainty on experimental results/limits is from nuclear theory inputs.

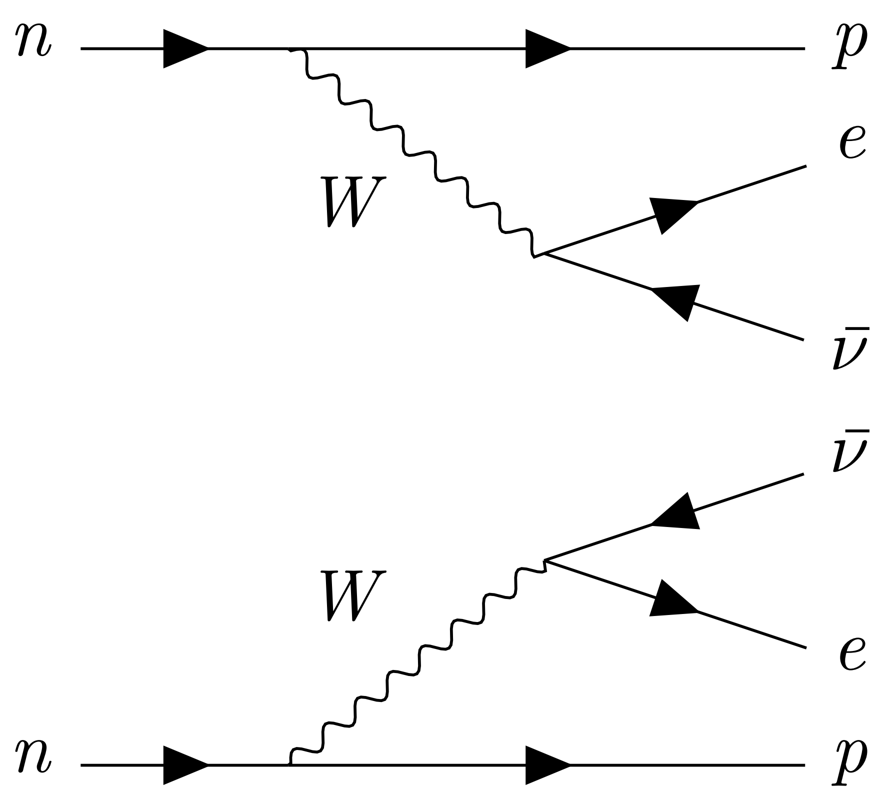
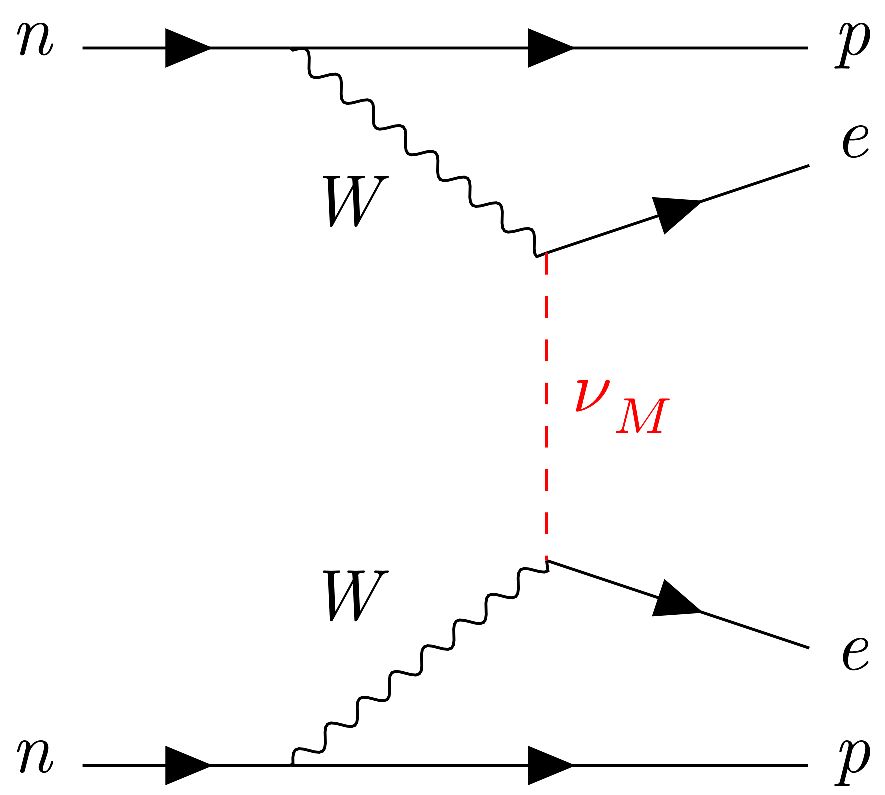
# Nuclear Theory Blur



## Goal of the talk

Show how, by using ab initio methods that rely on systematically improvable expansions rather than nuclear models, a coherent picture can be achieved for BSM observables using $0\nu\beta\beta$ as an example.

# $2\nu\beta\beta$ vs $0\nu\beta\beta$

| Decay | $2\nu\beta\beta$ | $0\nu\beta\beta$ |
|---|---|---|
| Diagram |  |  |
| Half-life Formula | $[T^{2\nu}_{1/2}]^{-1} = g_A^4 G^{2\nu} \lvert M^{2\nu} \rvert^2$ | $[T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu} \lvert M^{0\nu} \rvert^2 \left( \frac{\langle m_{\beta\beta} \rangle}{m_e} \right)^2$ |
| NME Formula | $M^{2\nu} \approx M^{2\nu}_{GT}$ | $M^{0\nu} = M^{0\nu}_{GT} - (\frac{g_V}{g_A})^2 M^{0\nu}_F + M^{0\nu}_T - 2g_{\nu\nu} M^{0\nu}_{CT}$ |
| LNV | No | Yes! |
| Observed | Yes | No |

*NME : Nuclear matrix elements

**LNV : Lepton number violation

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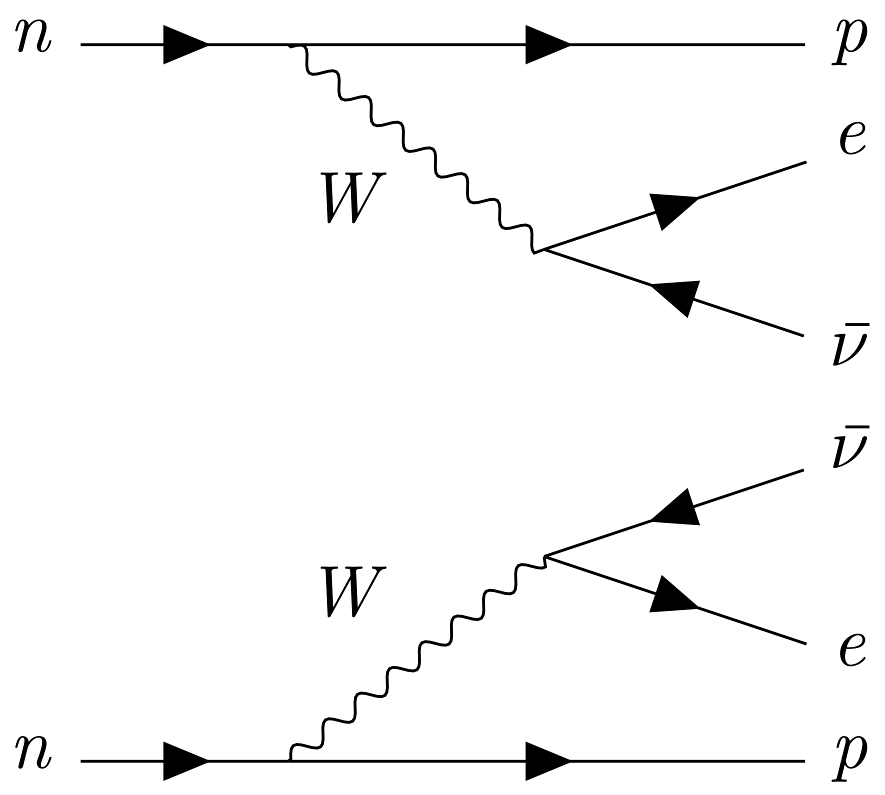
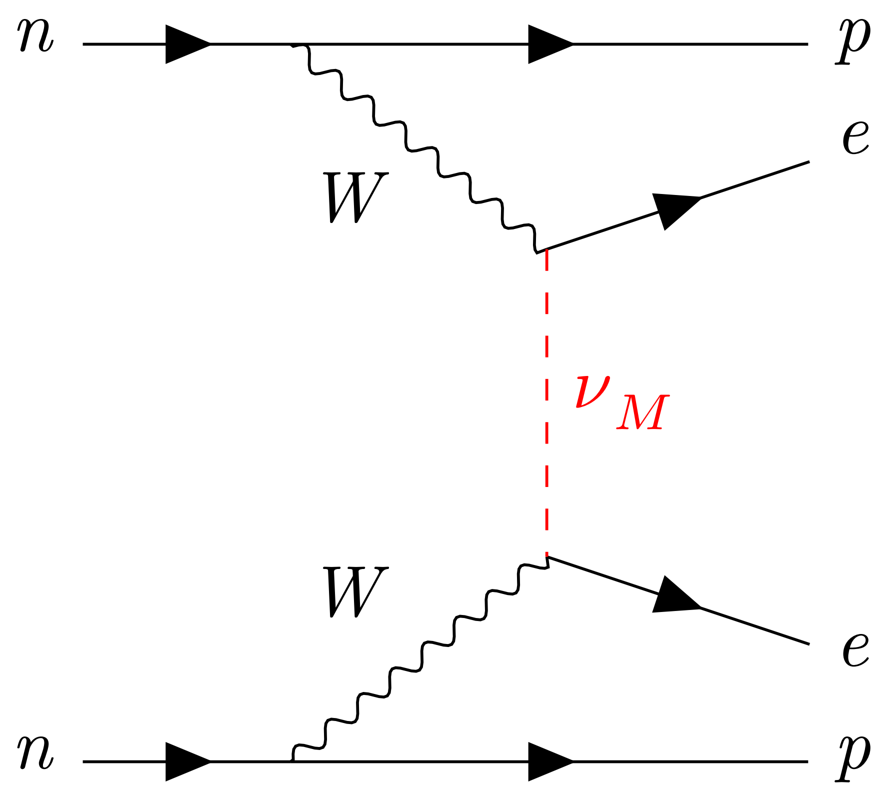
| Diagram |   |   |
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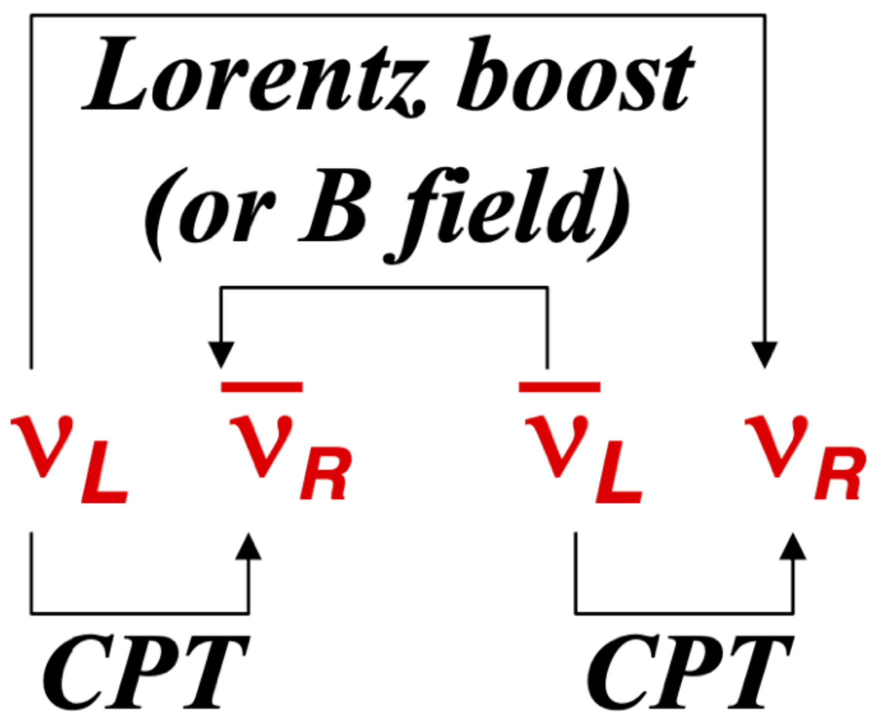
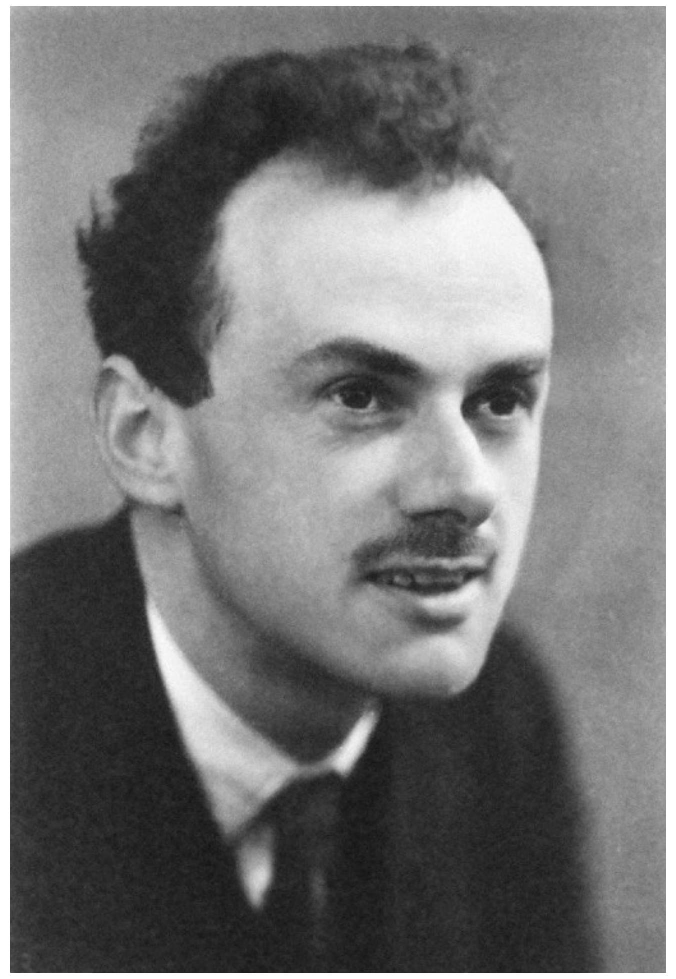
# The Nature of the Neutrino Puzzle

## The classic picture: The Dirac neutrino



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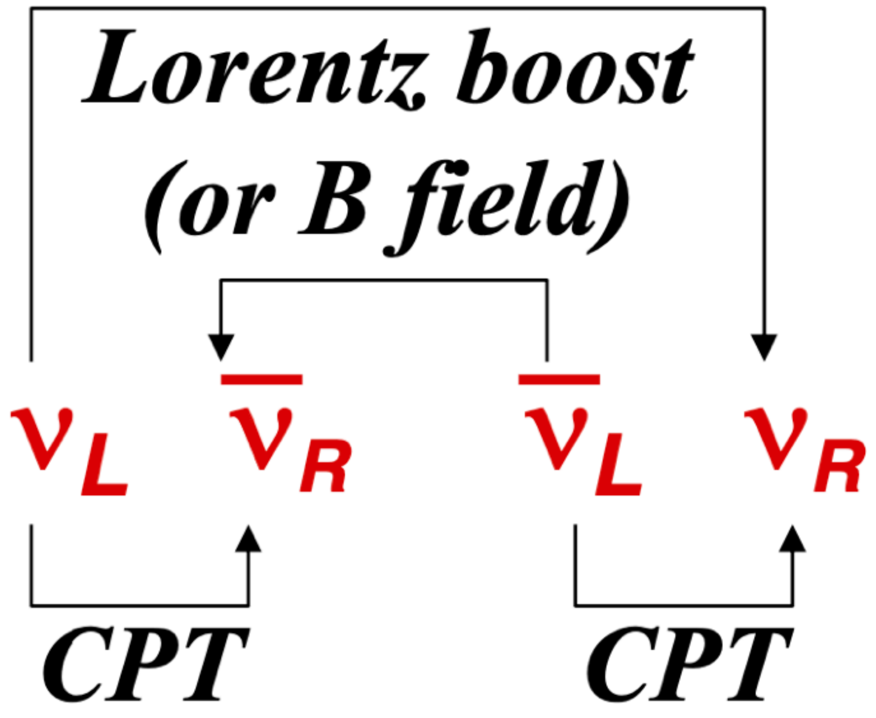
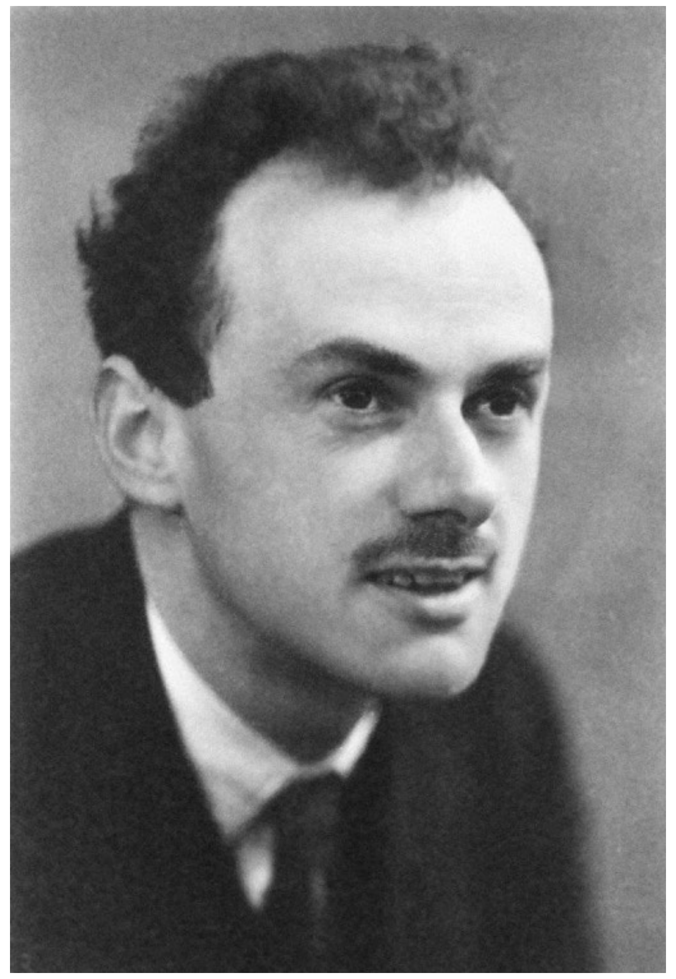
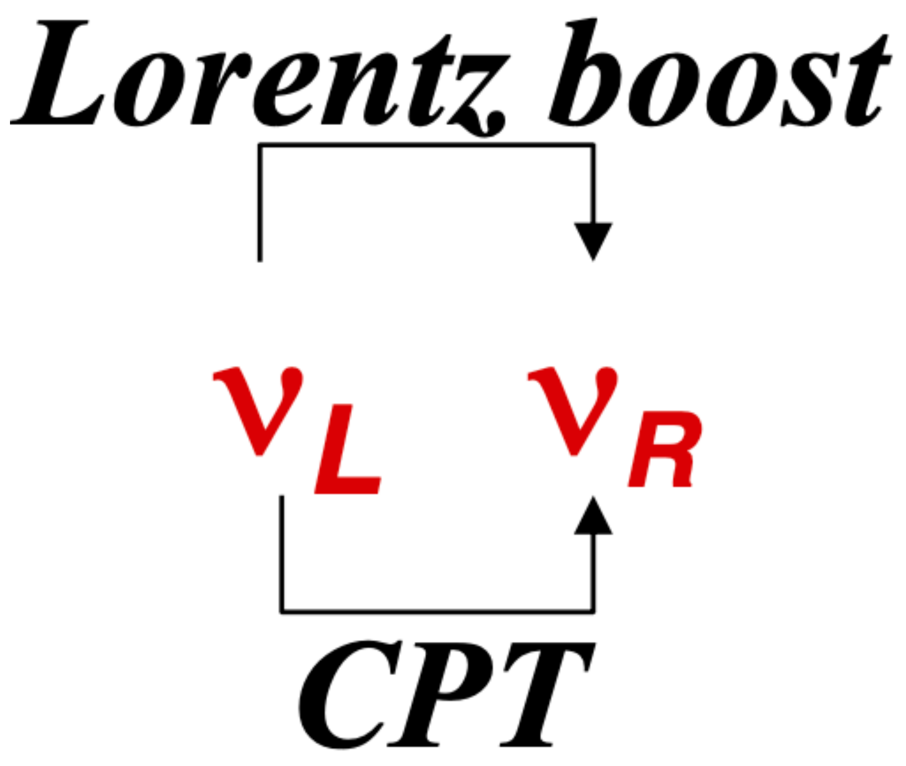


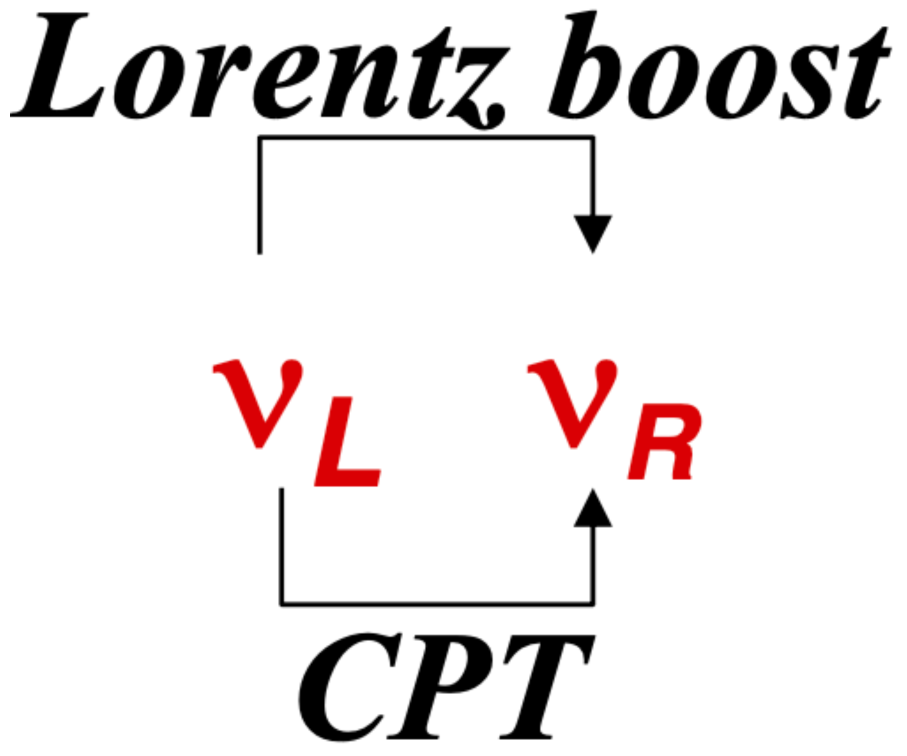



## The Majorana neutrino




# Why Majorana Neutrinos?

- The neutrinos are the only neutral fermions: only candidates.
- Explains why we only observe left-handed neutrinos and right-handed anti-neutrinos.
- Gives natural explanation to the small neutrino masses via the seesaw mechanism.



# The Black Box Theorem (Schechter & Valle 1982)

Assuming only that:

- Quarks and electrons are massive.
- The standard left-handed weak interaction exists.

Loop corrections for any mechanism require a neutrino Majorana mass term.

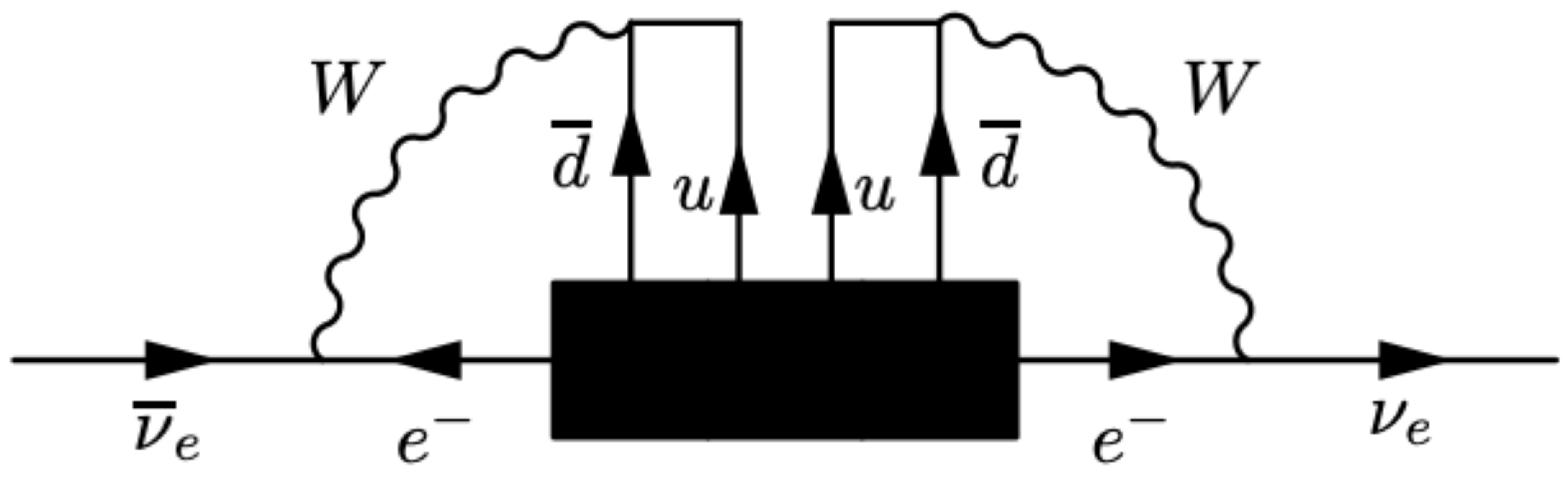



Figure from JHEP 1106:091,2011

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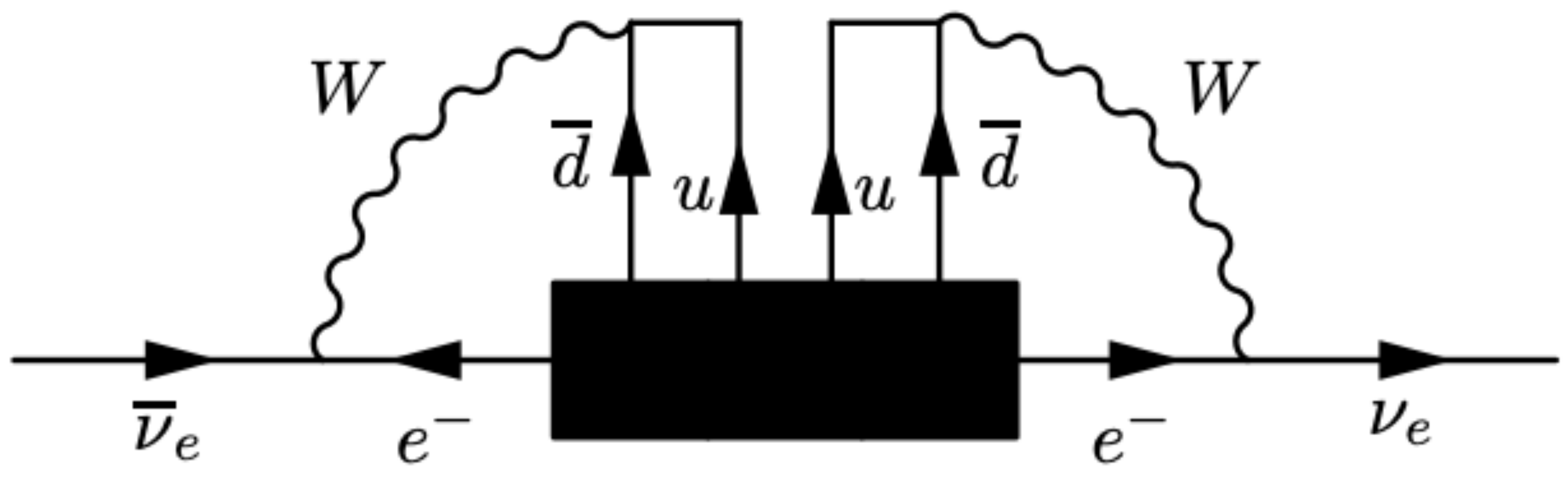


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Observation of $0\nu\beta\beta \Rightarrow$ Neutrinos are Majorana

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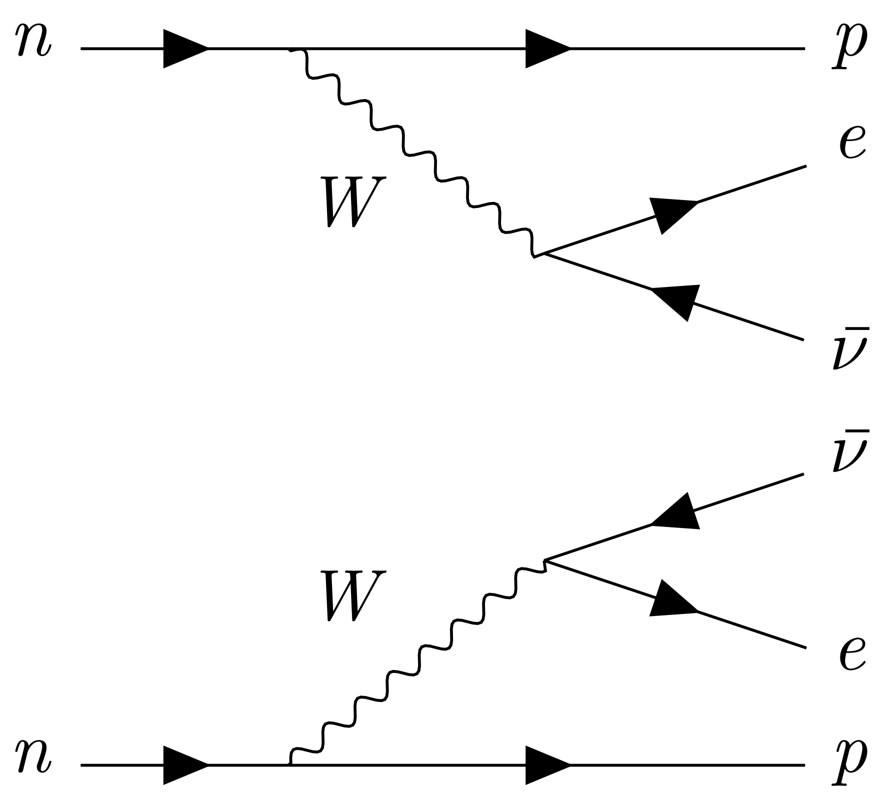
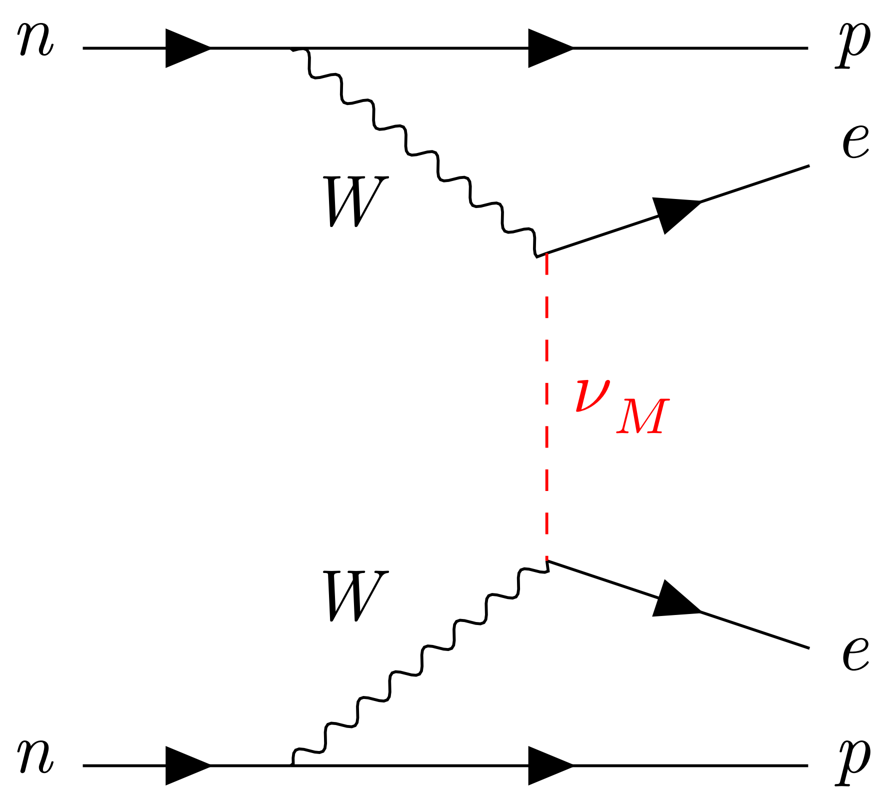
| Diagram |   |   |
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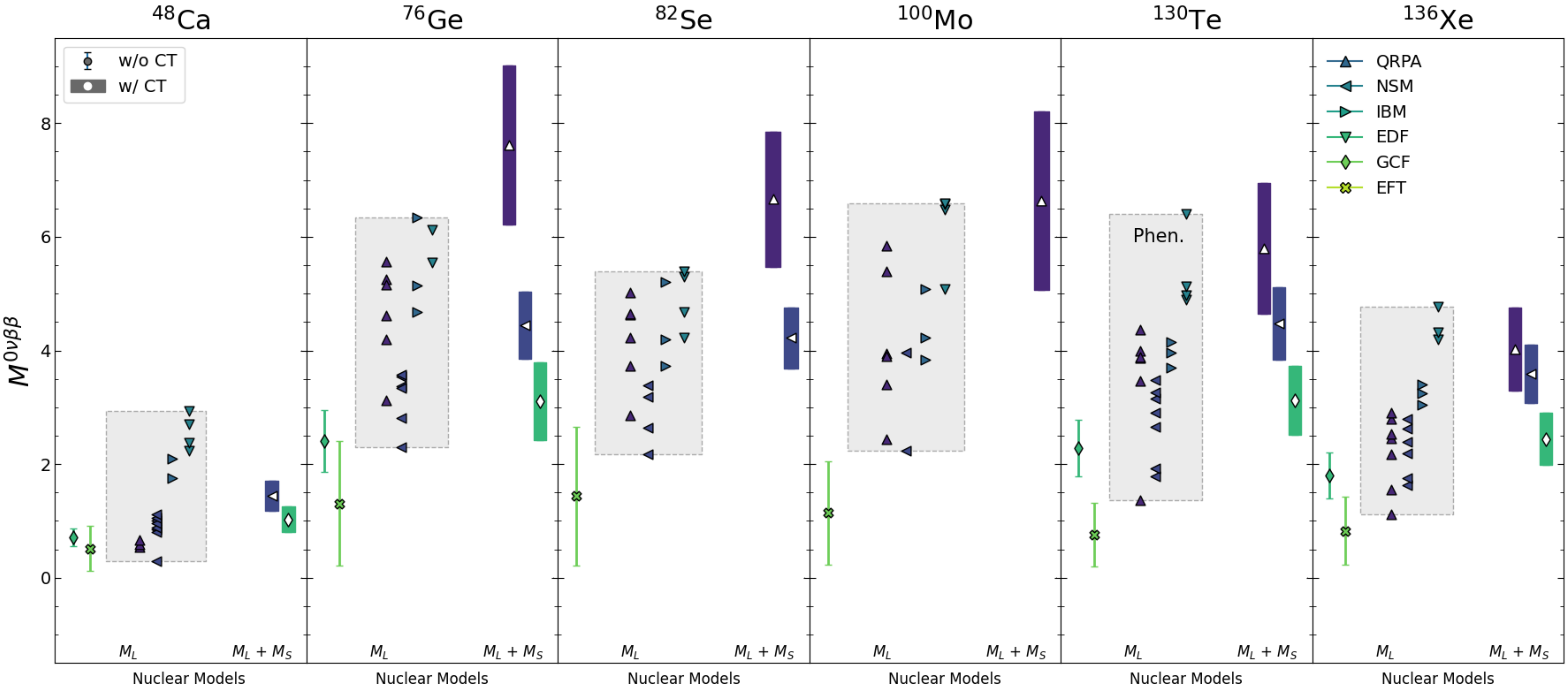
# Status of $0\nu\beta\beta$-decay Matrix Elements




Values from Engel and Menéndez, Rep. Prog. Phys. **80** 046301 (2017); Yao, Sci. Bull. 10.1016 (2020); Brase et al, Phys. Rev. C **106**, 034309 (2021)

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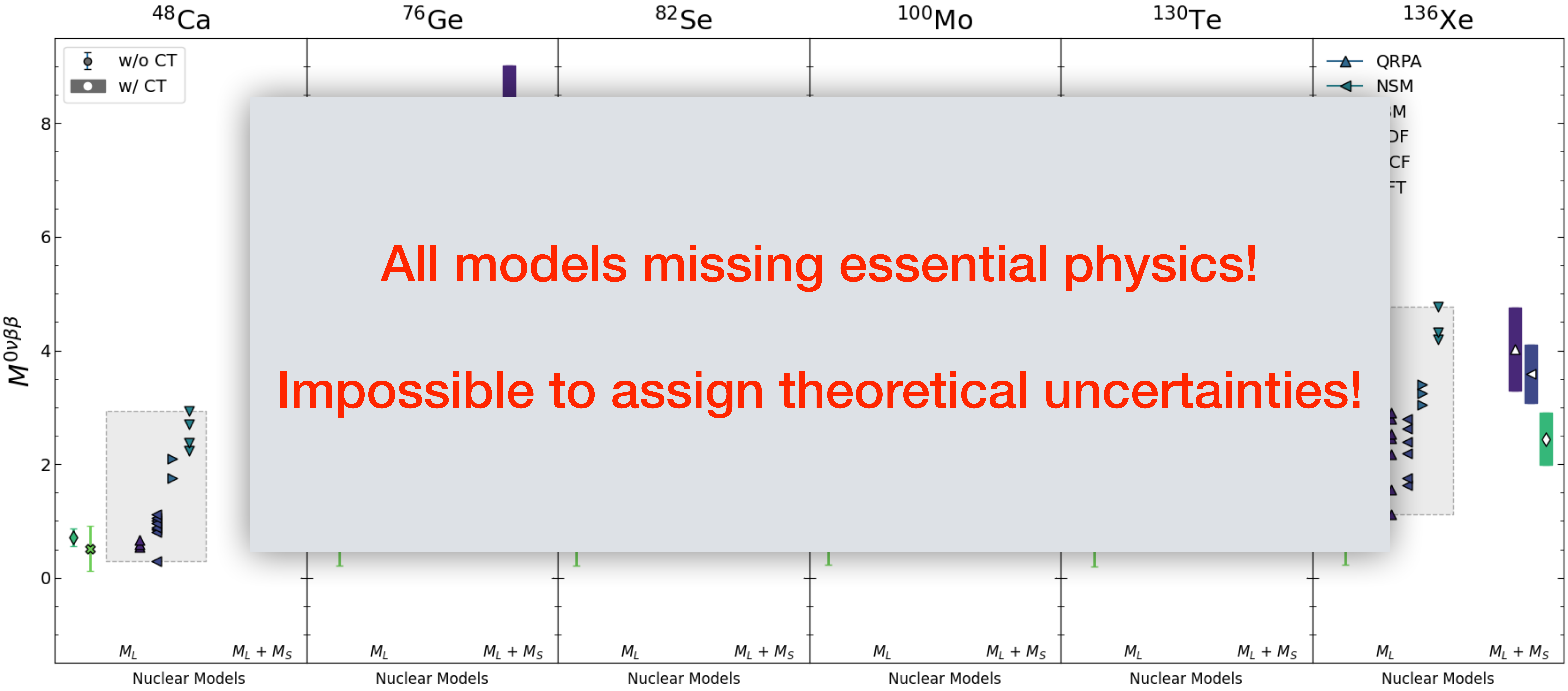


**All models missing essential physics!**

**Impossible to assign theoretical uncertainties!**

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# List of Challenges

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  - Deriving an expression for the nuclear potential
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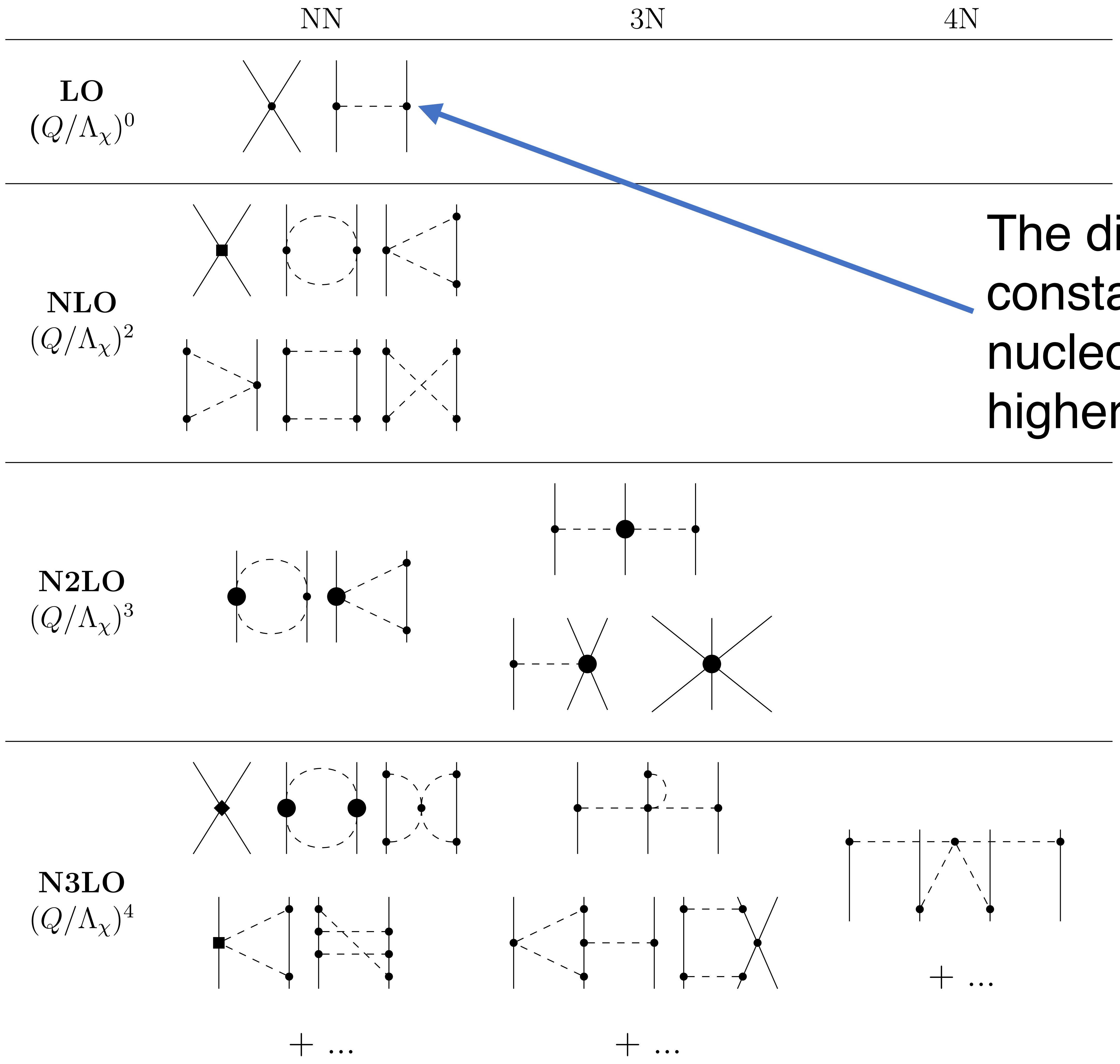
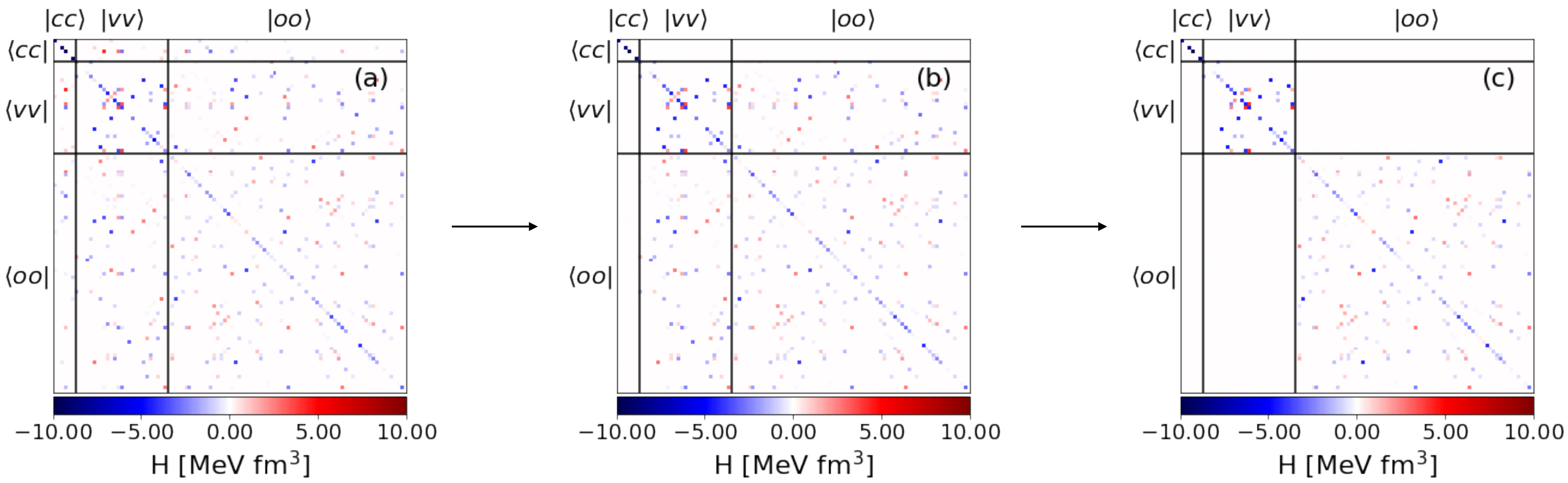
# Expansion order by order of the nuclear forces

Reproduces symmetries of low-energy QCD using nucleons as fields and mesons as force carriers.




The different low energy coupling constants (LECs) are fitted to few-nucleon data to absorb the effect of higher order terms

Valence-Space In Medium Similarity Renormalization Group




Bare Hamiltonian

$$\hat{H}(0)$$

Core is decoupled

$$\hat{H}(s) = e^{\Omega_c(s)}\hat{H}(0)e^{-\Omega_c(s)}$$

$$\hat{H}_c = e^{\Omega_c(\infty)}\hat{H}(0)e^{-\Omega_c(\infty)}$$

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# The VS-IMSRG

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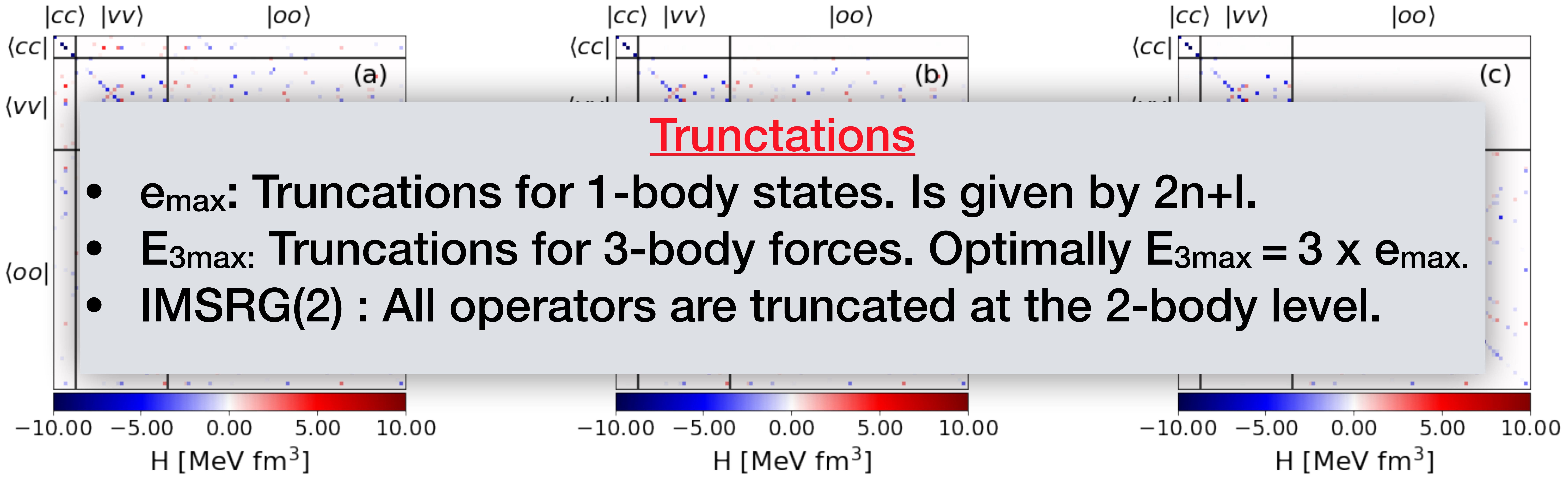


## Truncations

- $e_{max}$: Truncations for 1-body states. Is given by 2n+l.
- $E_{3max}$: Truncations for 3-body forces. Optimally $E_{3max} = 3 \times e_{max}$.
- IMSRG(2) : All operators are truncated at the 2-body level.

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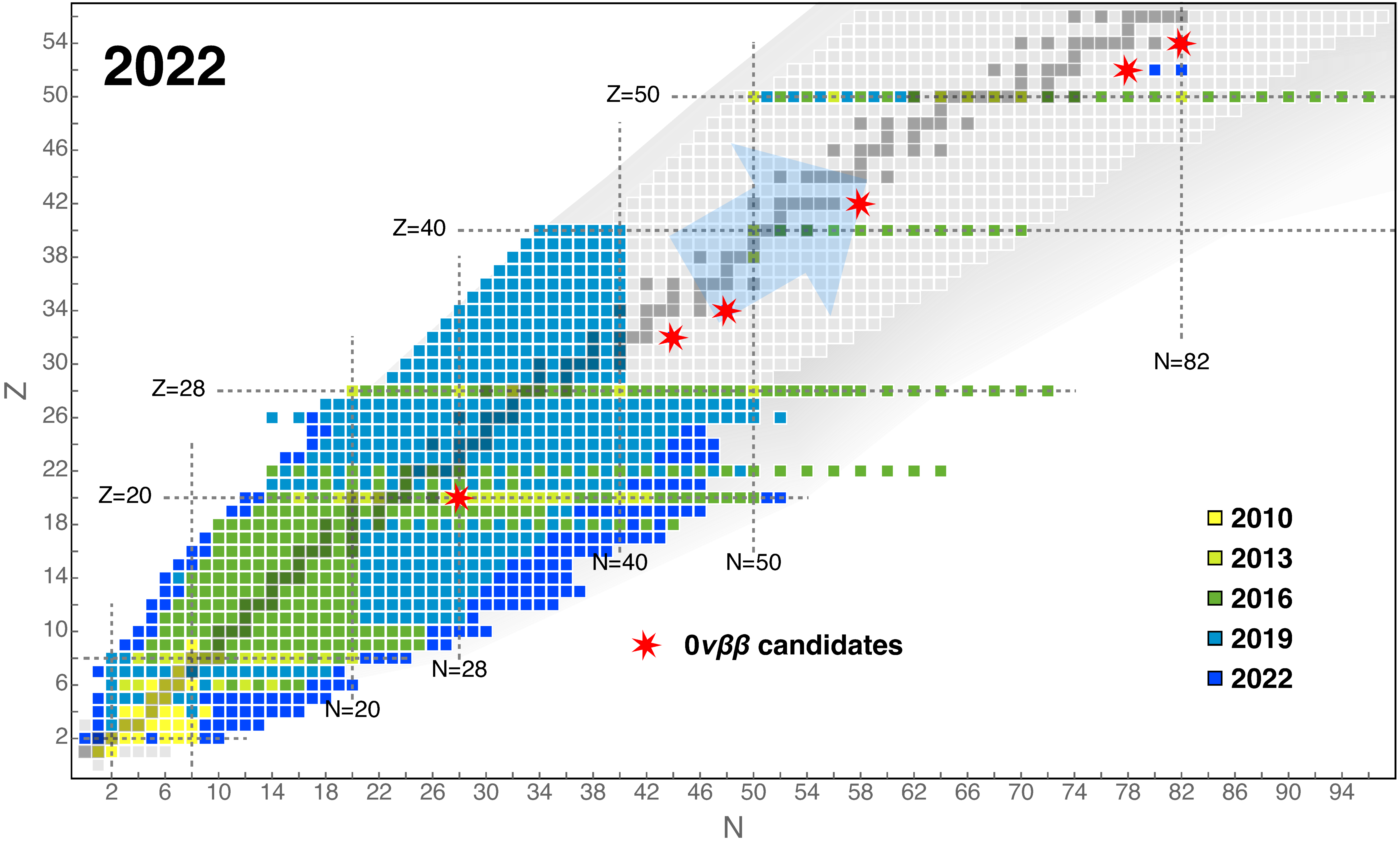
# The Ab Initio Revolution



# EFT Corrections to the Operator

A more complete approach based on EFT allows to find corrections to these operators:

$$[T_{1/2}^{0\nu}]^{-1} = g_A^4 G^{0\nu} \, | M_{LR}^{0\nu} + M_{SR}^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{\text{loops}}^{0\nu} |^2 \left( \frac{m_{\beta\beta}}{m_e} \right)^2$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)

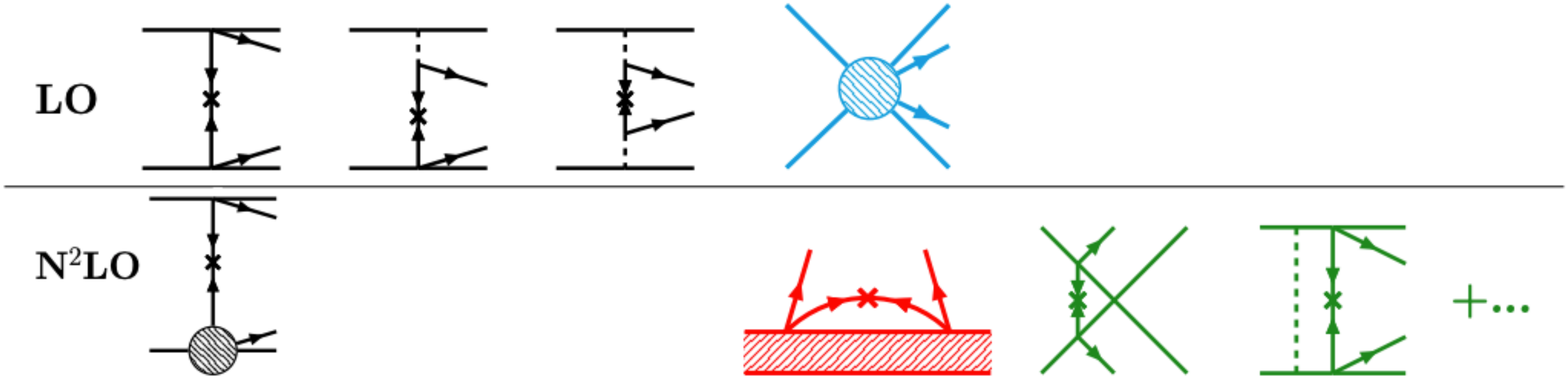



Figure courtesy of L. Jokiniemi

# The Short-Range Contact Operator

A short-range contact operator previously thought to be at higher order is promoted to first order for renormalization:

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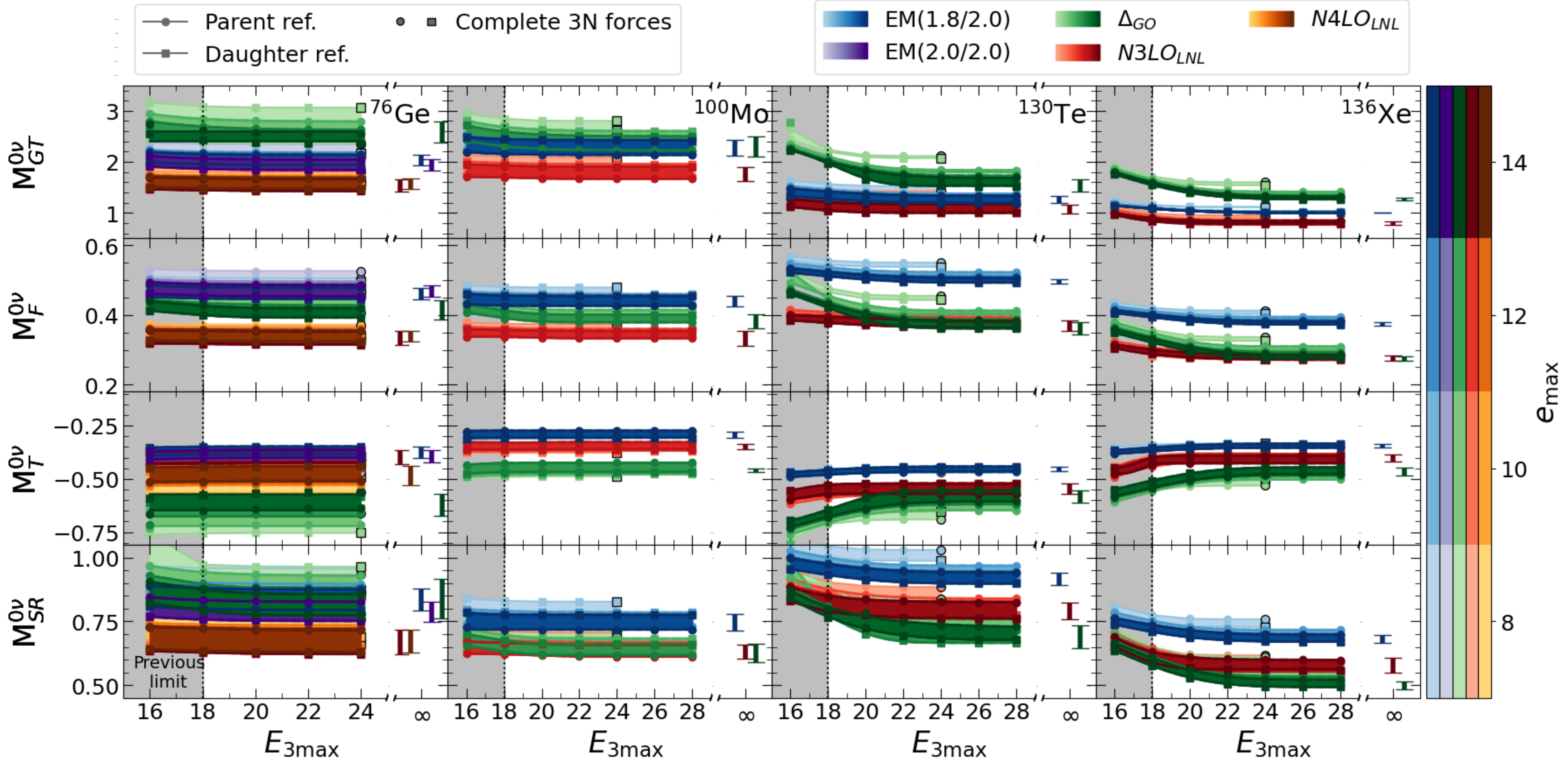
Contact operator regularized with non-local regulator matching the nuclear interaction used:

$$M_{CT}^{0\nu} = \langle 0_f^+ | \frac{R_{Nucl}}{8\pi^3}\left(\frac{m_N g_A^2}{4f_\pi^2}\right)^2 \exp(-(\frac{p}{\Lambda_{int}})^{2n_{int}})\exp(-(\frac{p'}{\Lambda_{int}})^{2n_{int}}) | 0_i^+ \rangle$$

# Results in Heavy Nuclei




**Belley**, et al., arXiv:2307.15156 (2023)

# List of Challenges

- Obtaining a result:

$$NME = \langle \psi_f | O | \psi_i \rangle$$

  - Deriving an expression for the nuclear potential ($\chi$-EFT)
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  - Deriving operators consistently with the nuclear interactions (EFTs)

- Obtaining a **reliable** result:
  - Uncertainty Quantification

# Uncertainty quantification

# Propagating the LECs Error

Recall that the nuclear potential depends  on a set of LECs $\alpha$:

$$M^{0\nu\beta\beta}(\alpha) = \langle \psi_f(\alpha) \,|\, O \,|\, \psi_i(\alpha) \rangle$$

that are fitted to NN and few-nucleon data, i.e. each LEC has an uncertainty $\delta\alpha$ associated with it.

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Bayesian Statistics!

# Bayesian Approach

Value of the nuclear matrix elements (what we are interested in)

Different values obtained with different interactions/ methods

Any other relevant information we have beforehand

We read $prob(A \mid B)$ as probability of A given B

$$\underbrace{prob(y \mid y_k, I)} \propto \underbrace{prob(y_k \mid y, I)} \times \underbrace{prob(y \mid I)}$$

**Posterior distribution**
Probability distribution for the final value given the data and our previous knowledge (what we want to obtain).

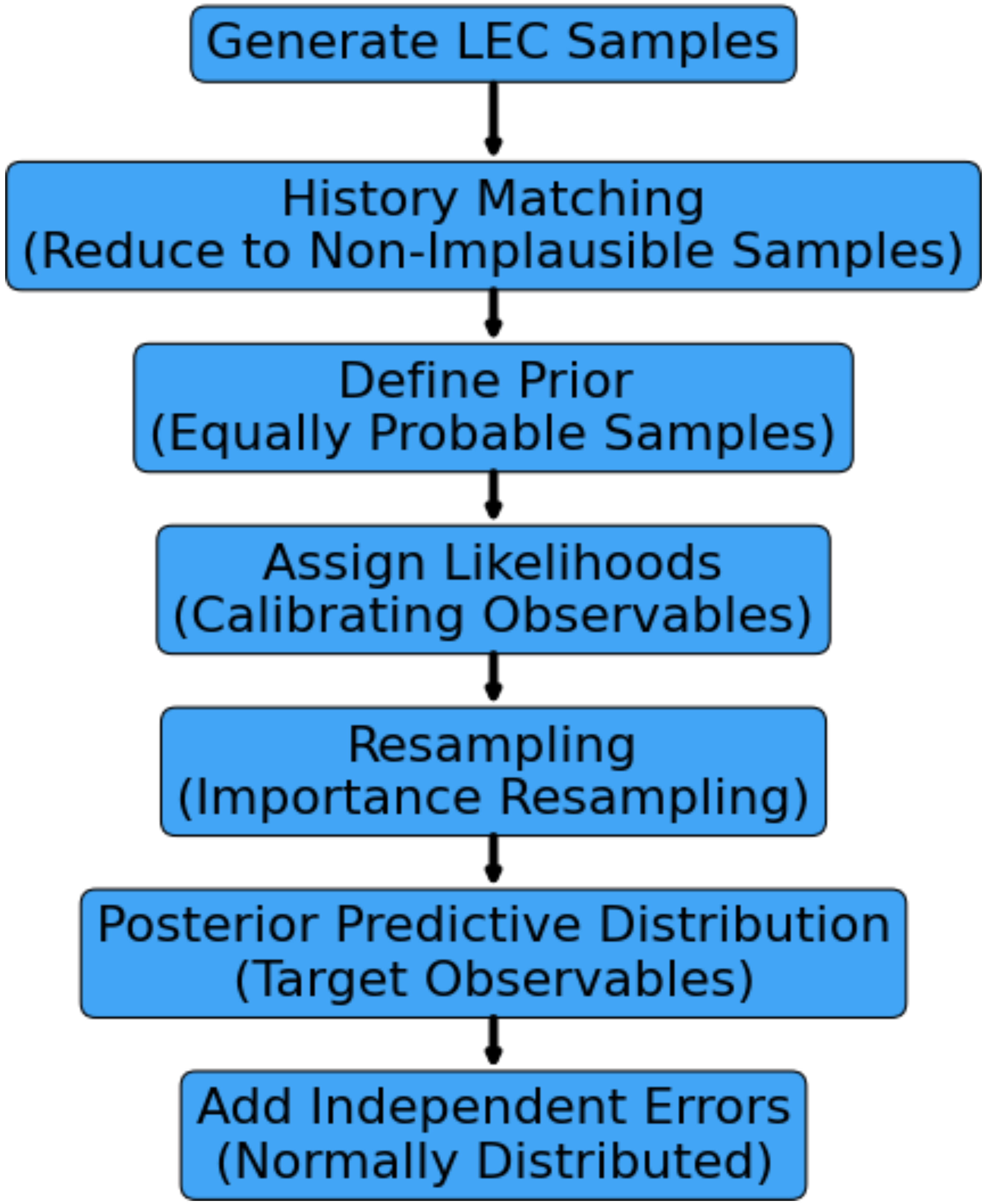
For finite samples, we use sampling/importance resampling to obtain the final PDF.

**Likelihood**
Probability that this sample gives a result that is representative of experimental values.

Chosen to be a multivariate normal centred at the experimental value for few observables we have data on (calibrating observables).

**Prior**
Assume a uniform prior for low energy constants of natural size. Then use history matching to remove implausible samples from the set. Assume each of the remaining samples to be as likely as the others.

# Procedure for UQ in the Bayesian Approach



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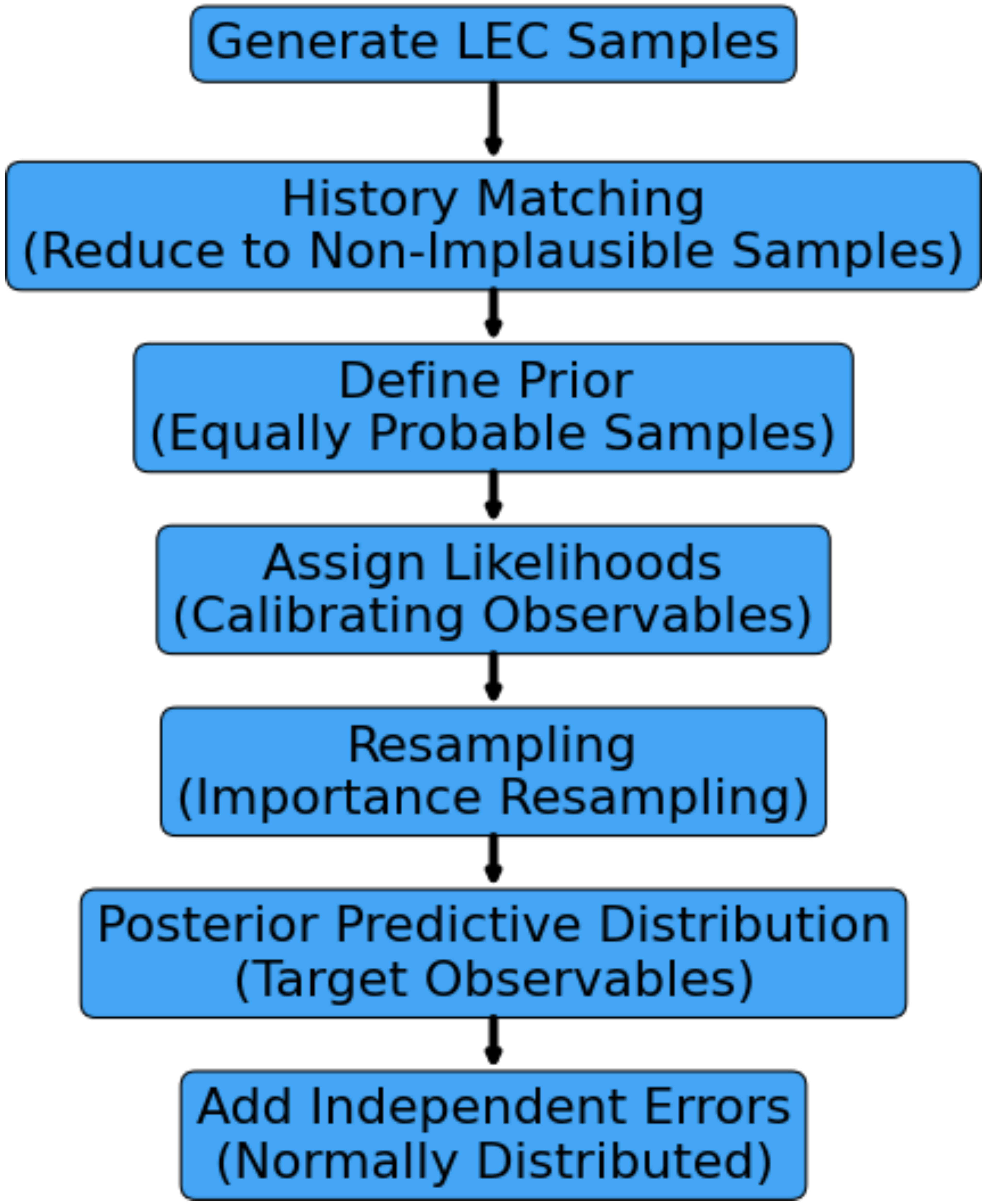


## The catch

Need to be able to compute the observables for all the non-implausible samples.

Due to the large cost of many-body methods, this becomes quickly infeasible as the number of samples grows.

# Emulators for Many-Body Methods

There are two ways to build an emulator for nuclear physics:

1. **<u>Physics driven</u>**
   - Incorporates some knowledge about the physics into the model.
   - Requires little data to be trained.
   - Is limited to the purpose it was constructed for.

E.g. Eigenvector continuation emulator for the Coupled Cluster method, Parametric matrix models.

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# Using Gaussian Process as an Emulator

- Idea behind Gaussian Process regressions is to assume that the distribution of the observable we want to fit is Gaussian:

$$f(\mathbf{x}) = \mathcal{N}(\mu, K(\mathbf{x}, \mathbf{x}))$$

where $\mu$ is a mean function and $K(\mathbf{x}, \mathbf{x})$ is the covariance matrix between the inputs.

- Want to infer the distribution of potentially unobserved Y* points from the observed points Y. This can be done via a property of Gaussian distribution called Conditioning, i.e.:

$$P_{Y*|Y} \sim \mathcal{N}\left(\mu_Y^* + \Sigma_{X*X}\Sigma_{XX}^{-1}(Y - \mu_Y), \Sigma_{X*X*} - \Sigma_{X*X}\Sigma_{XX}^{-1}\Sigma_{XX*}\right).$$

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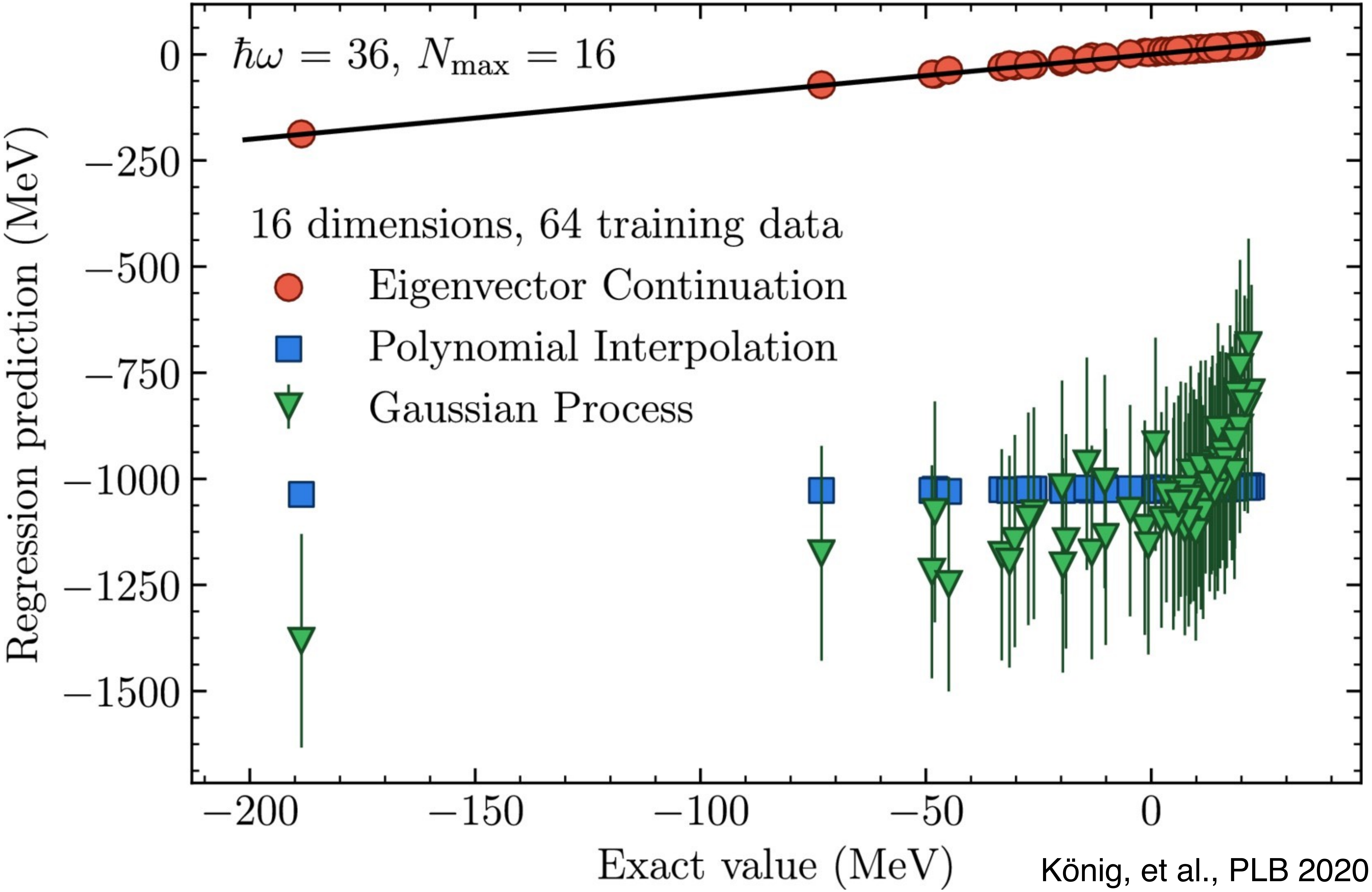
$$P_{Y*|Y} \sim \mathcal{N}\left(0 + \Sigma_{X*X}\Sigma_{XX}^{-1}(Y - 0), \Sigma_{X*X*} - \Sigma_{X*X}\Sigma_{XX}^{-1}\Sigma_{XX*}\right).$$

Normalizing inputs

Only need to optimize hyperparameters of $K(\mathbf{x}, \mathbf{x})$!

# Using Gaussian Process as an Emulator




König, et al., PLB 2020.135814

# Using Gaussian Process as an Emulator

- **Multi-Tasks Gaussian Process**: Leverages multiple correlated outputs from shared inputs by defining a combined kernel. This approach increases data points without additional costly computations.

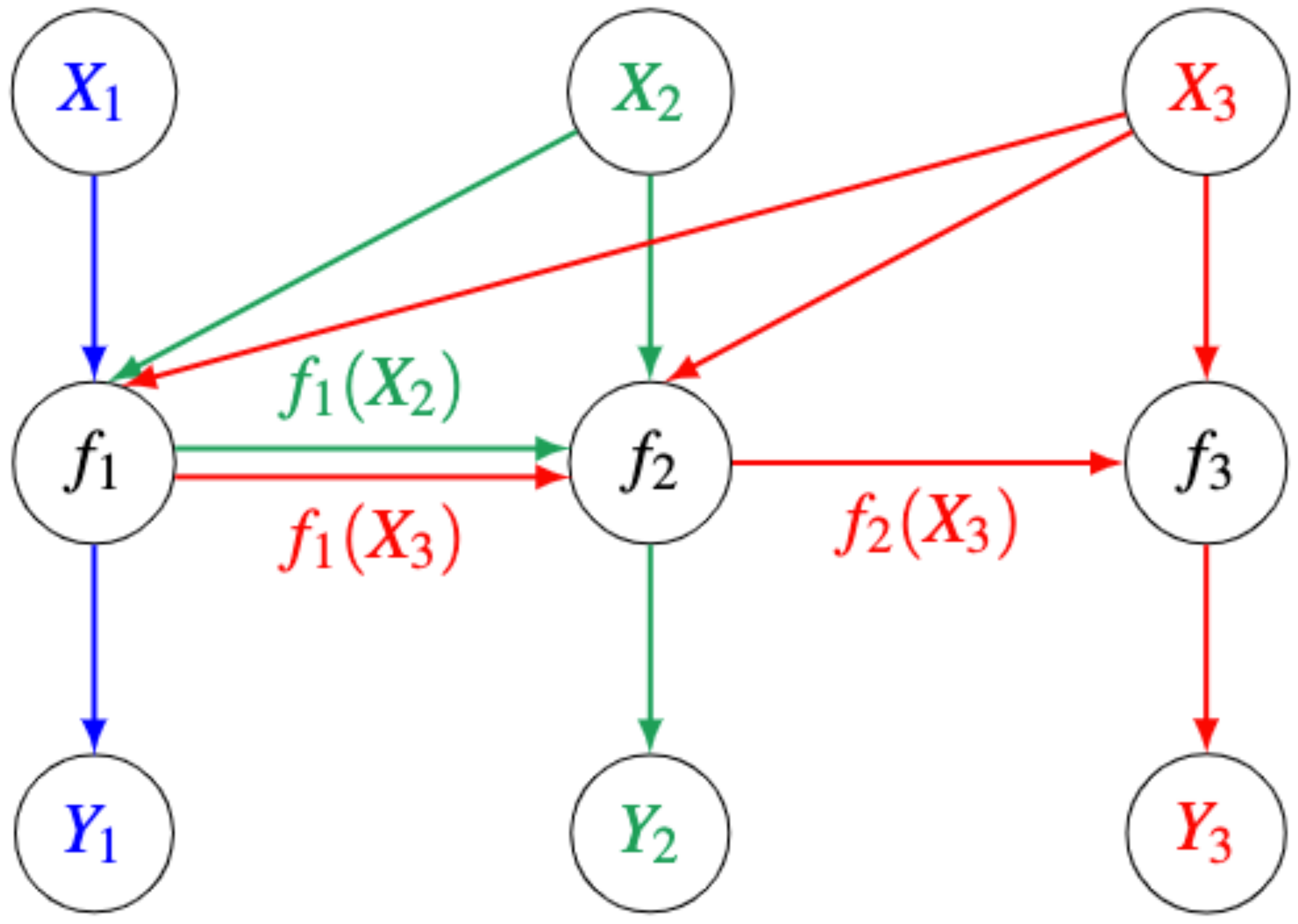
- **Multi-Fidelity Gaussian Process**: Uses limited high-fidelity data and abundant low-fidelity data and model differences with a Gaussian Process. This enables predicting high-fidelity outcomes from low-fidelity inputs, assuming a linear scaling between the fidelity levels.

# The MM-DGP Algorithm

- **Deep Gaussian Processes [1]**: Stack multiple GPs in a neural network-like architecture for improved hierarchical learning.

- **Multi-Fidelity Modelling**: Model low-to-high fidelity differences by passing outputs from one fidelity as inputs to the next.

- **MM-DGP Extension**: Adapted to handle multiple outputs across fidelity levels, creating the Multi-output Multi-fidelity Deep Gaussian Process (MM-DGP).

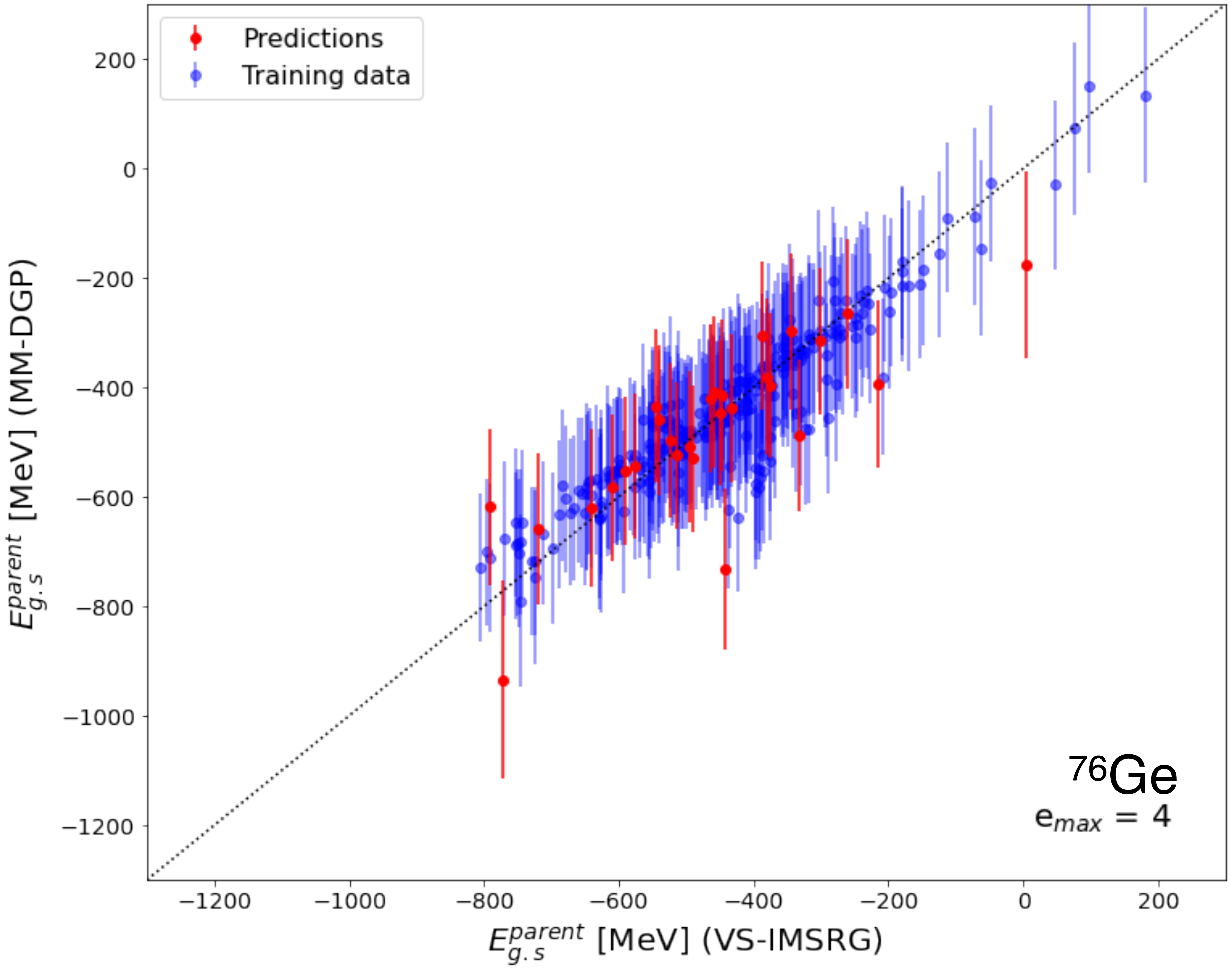



[1] Kurt Cutajar, Mark Pullin, Andreas Damianou, Neil Lawrence, Javier González arXiv:1903.07320 (2021).

# The MM-DGP Algorithm: Energies

Using Δ-full chiral EFT interactions at N2LO:

250 training points



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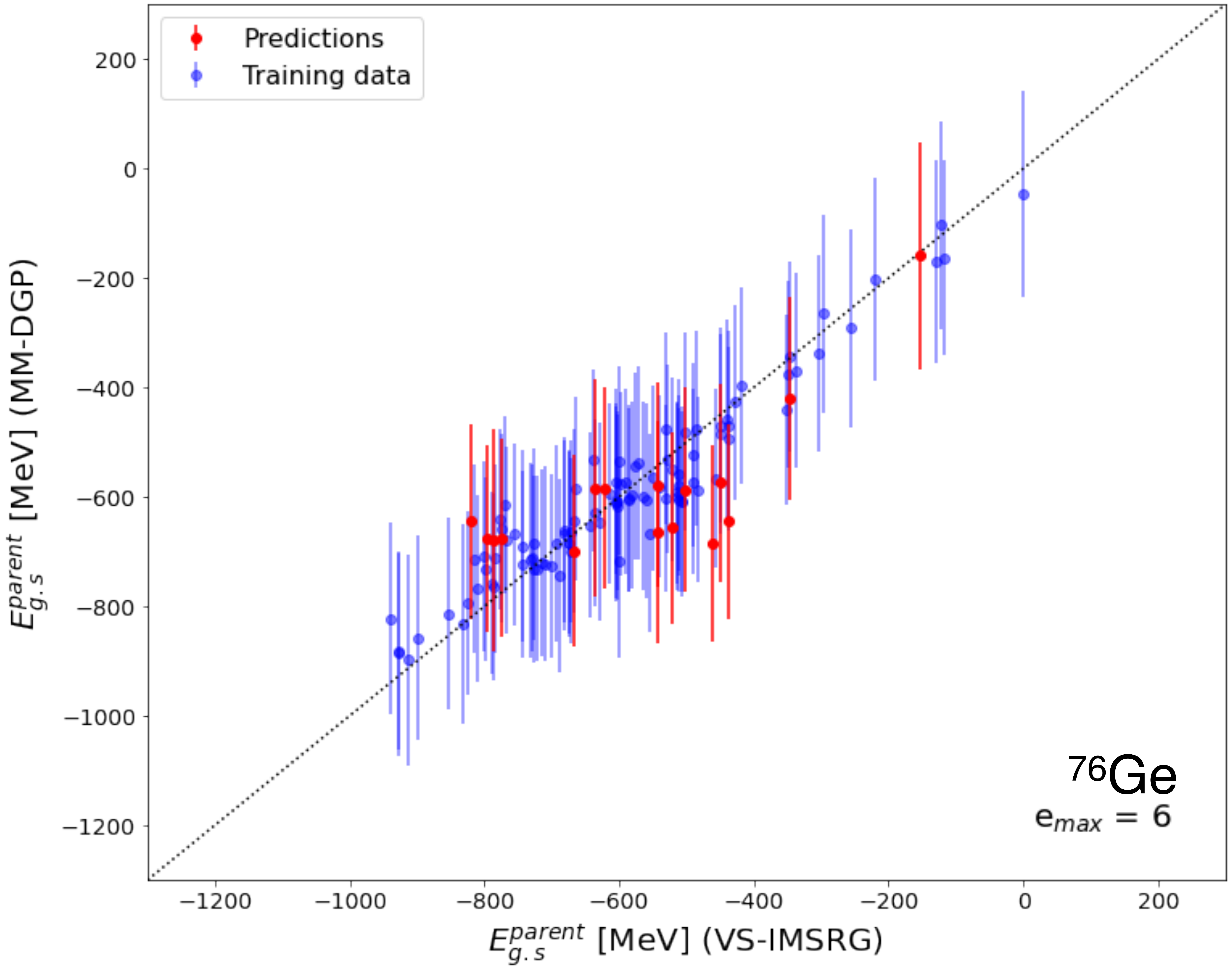
Using Δ-full chiral EFT interactions at N2LO:

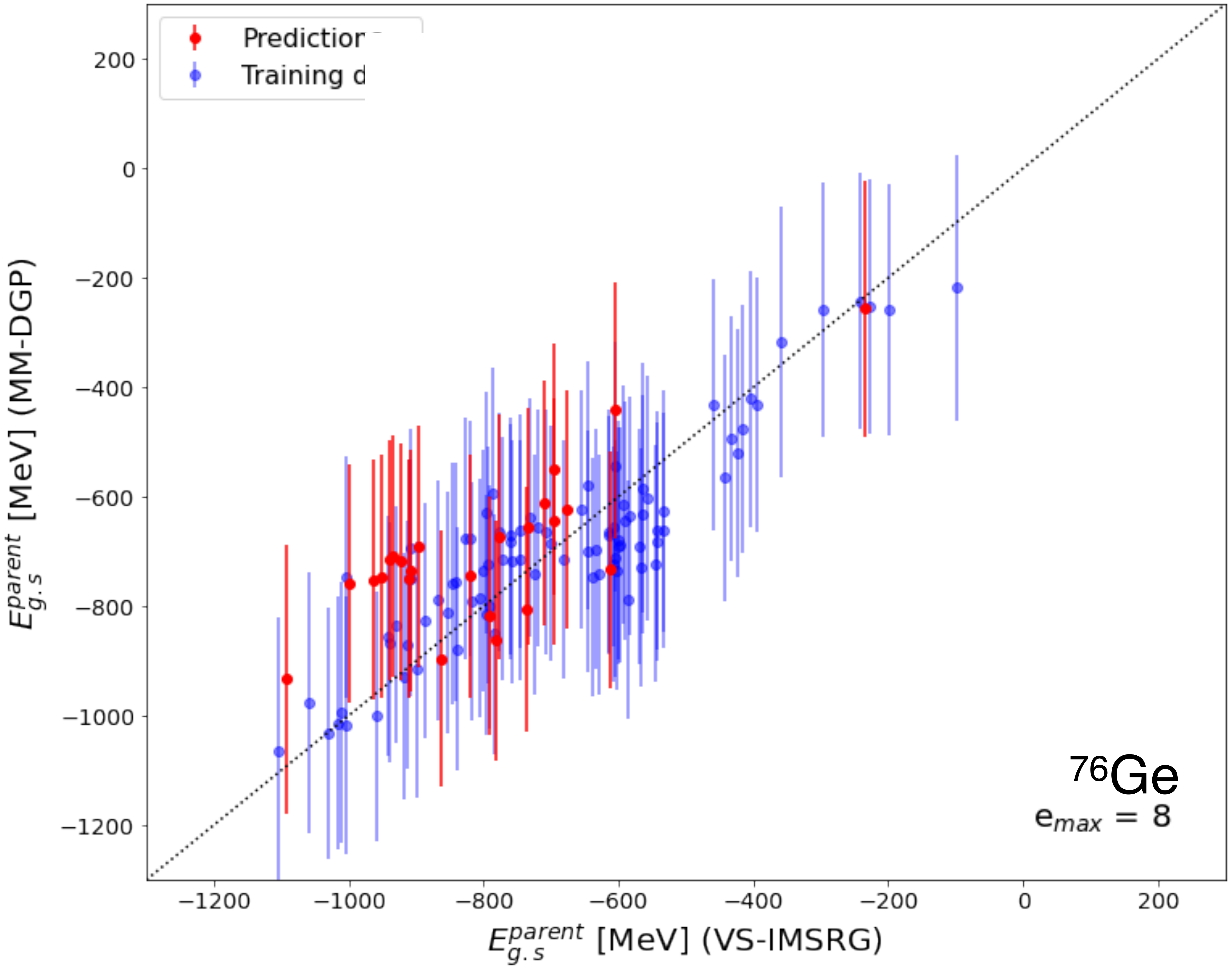
100 training points



# The MM-DGP Algorithm: Energies

Using $\Delta$-full chiral EFT interactions at N2LO:

90 training points



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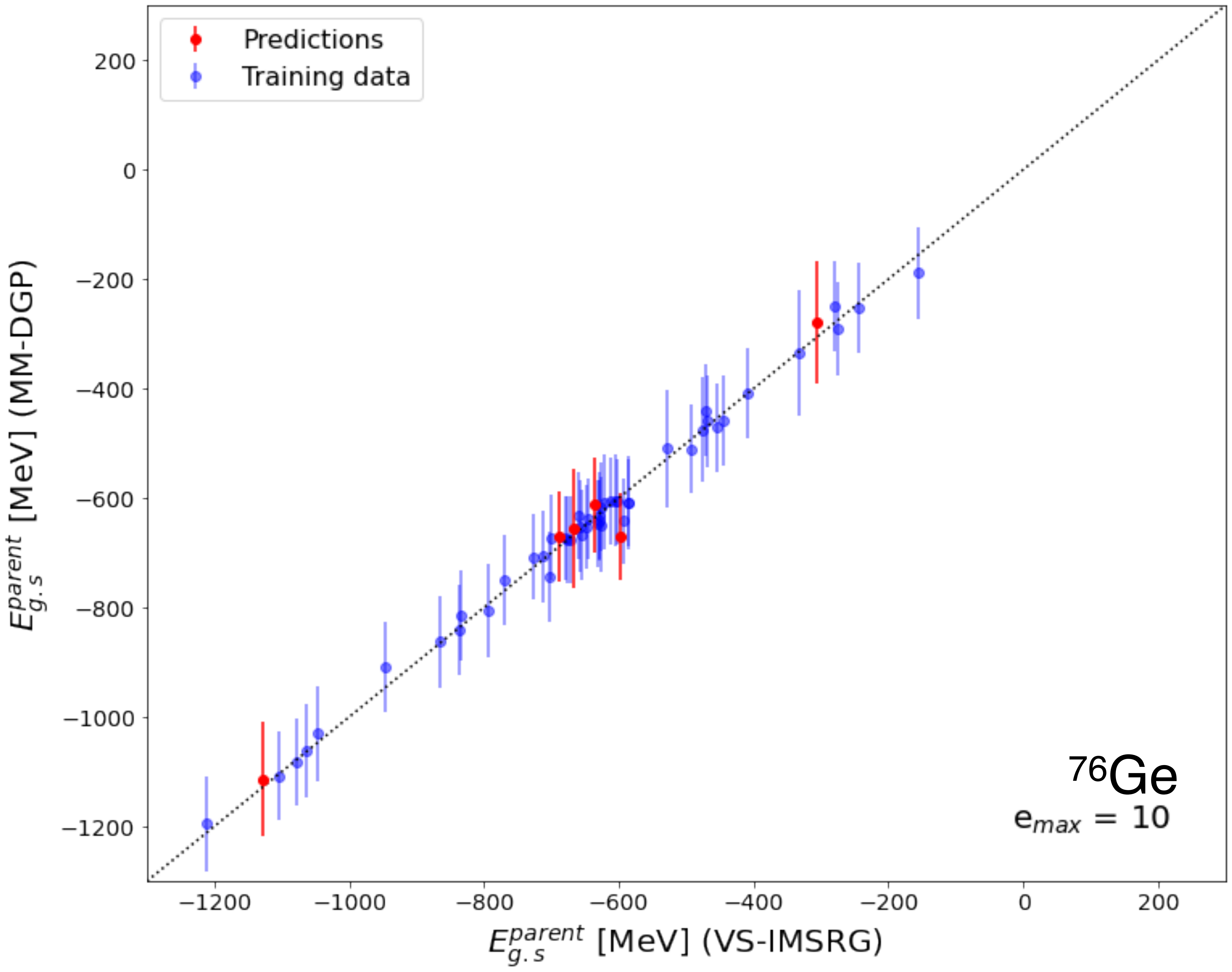

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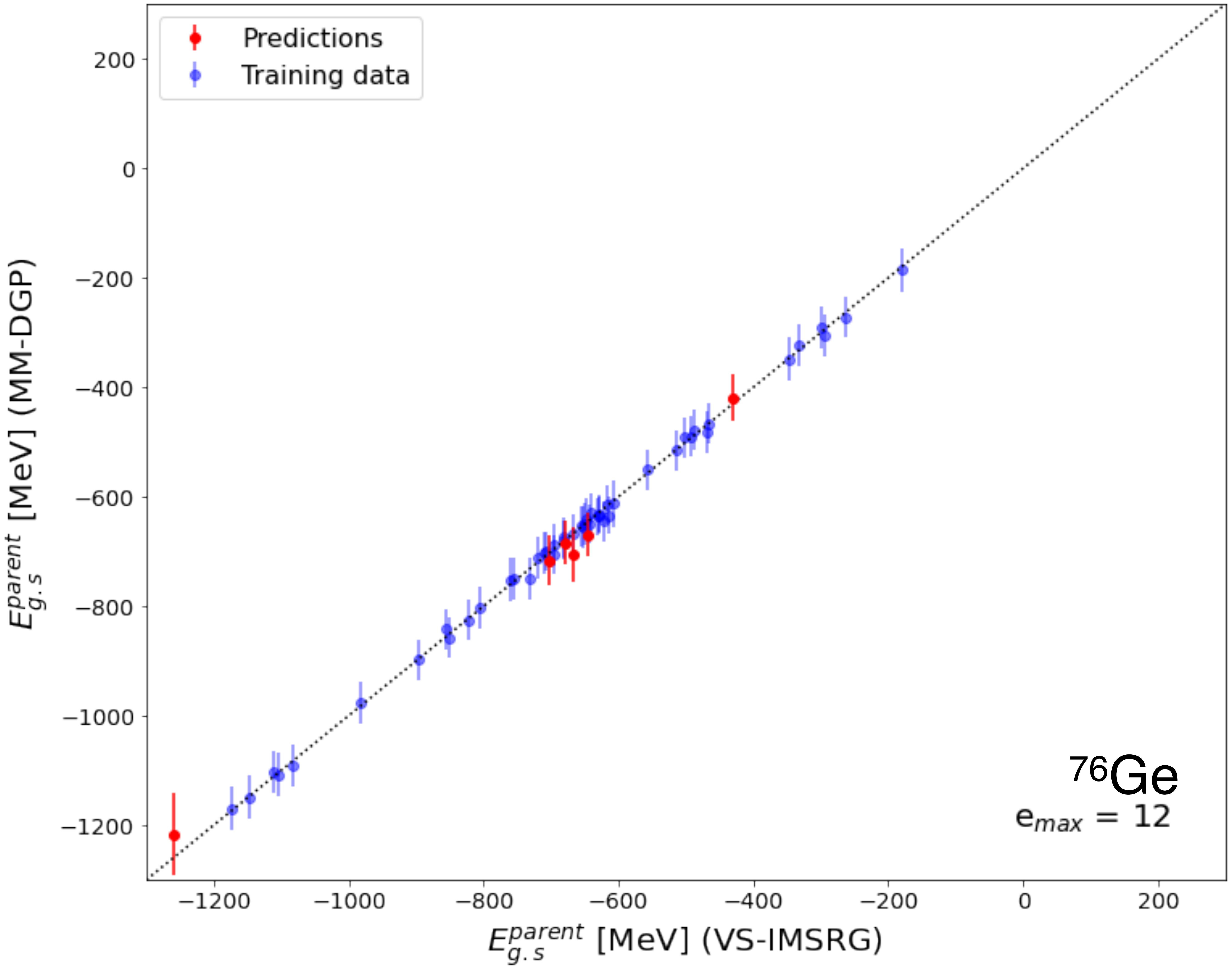


Root Mean Square Error = 11 MeV

# The MM-DGP Algorithm: $0\nu\beta\beta$ NMEs

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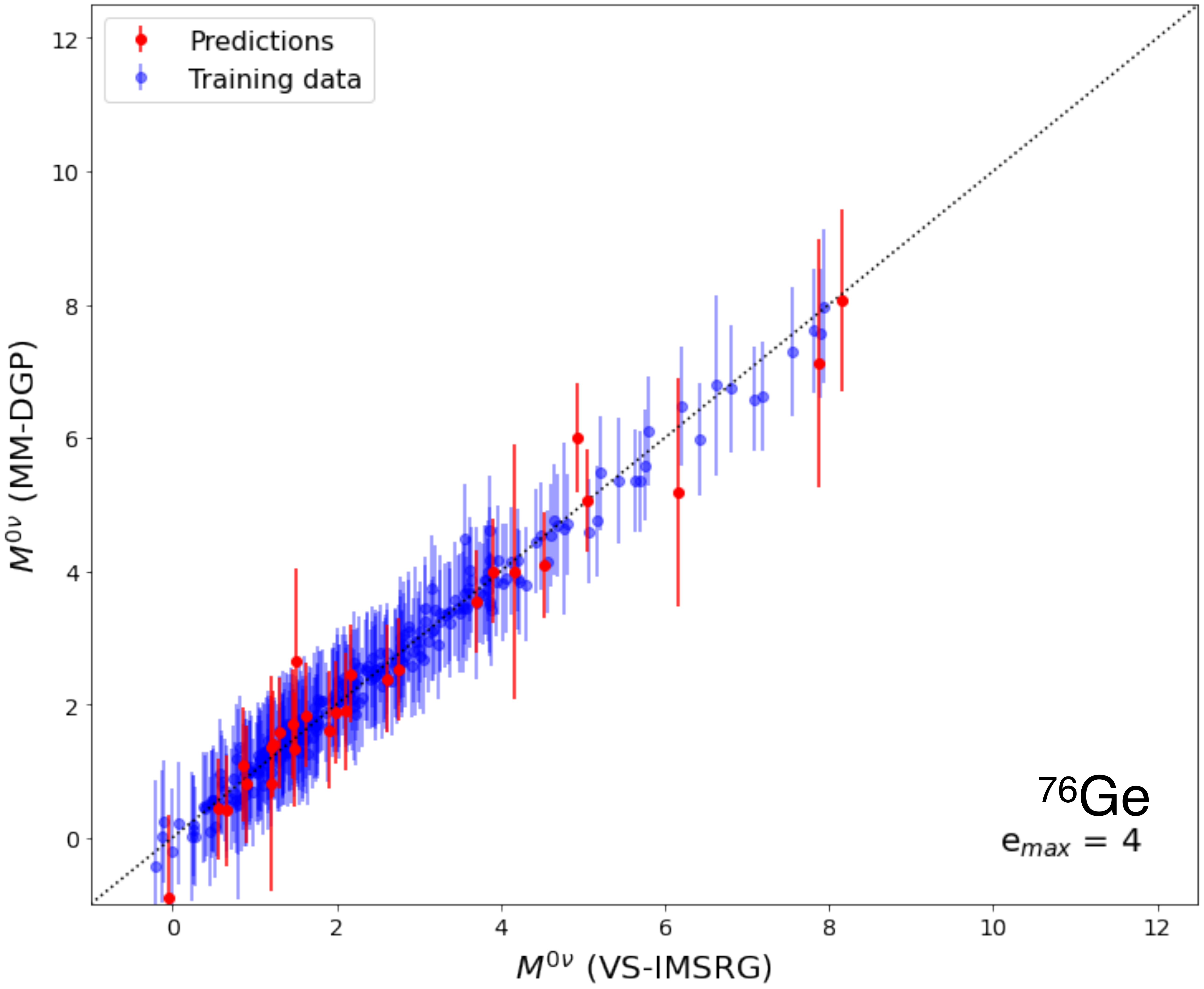

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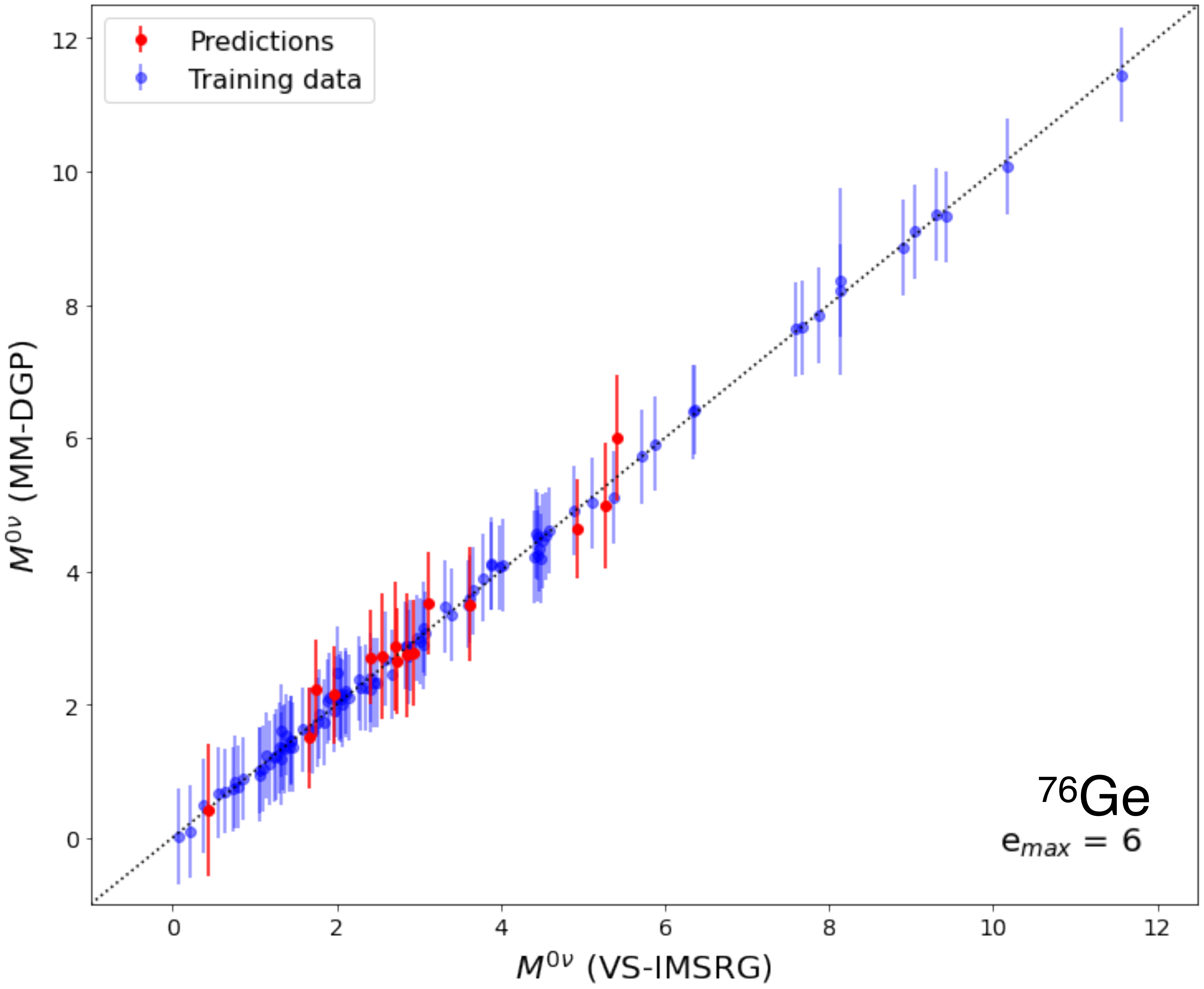

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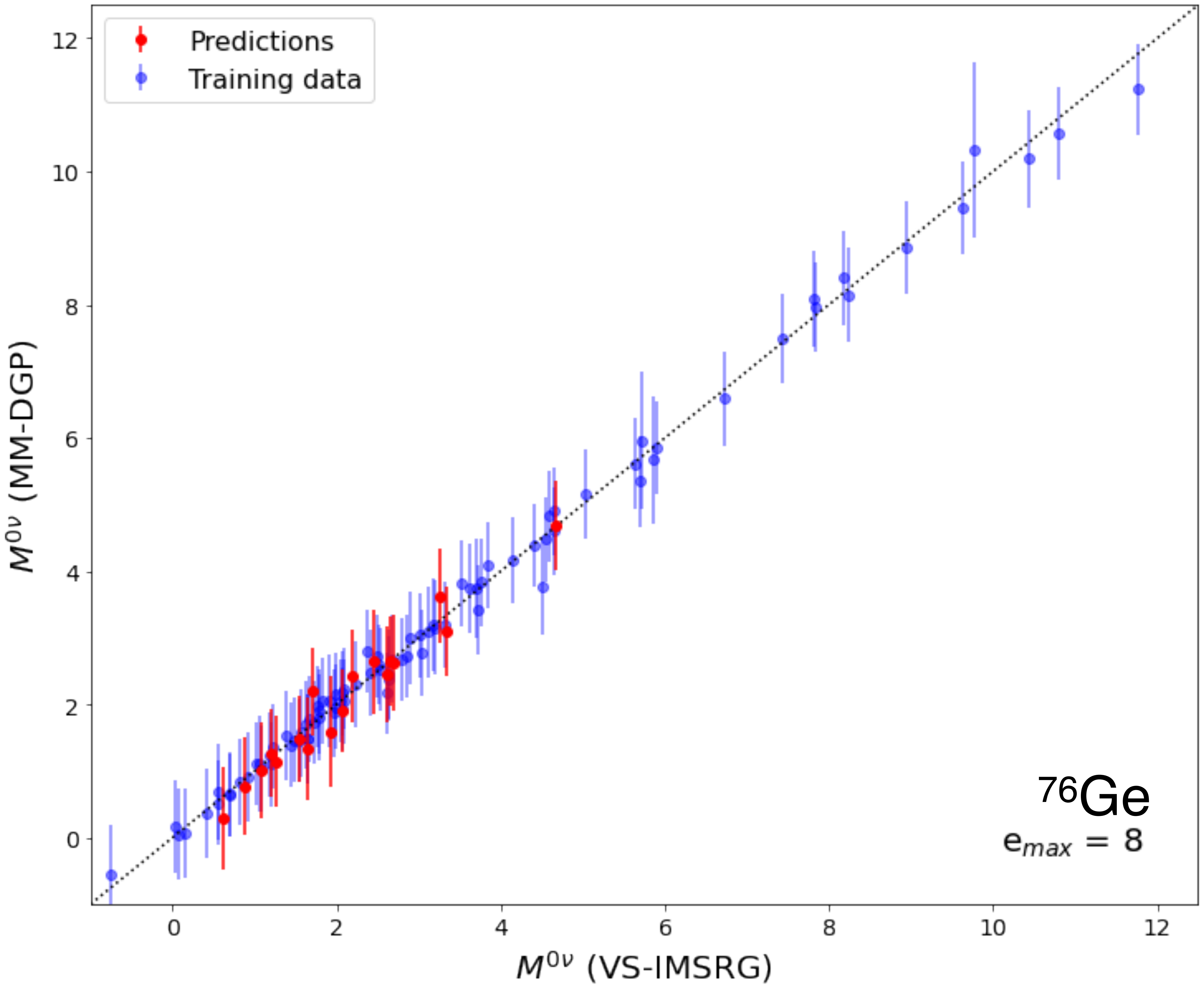

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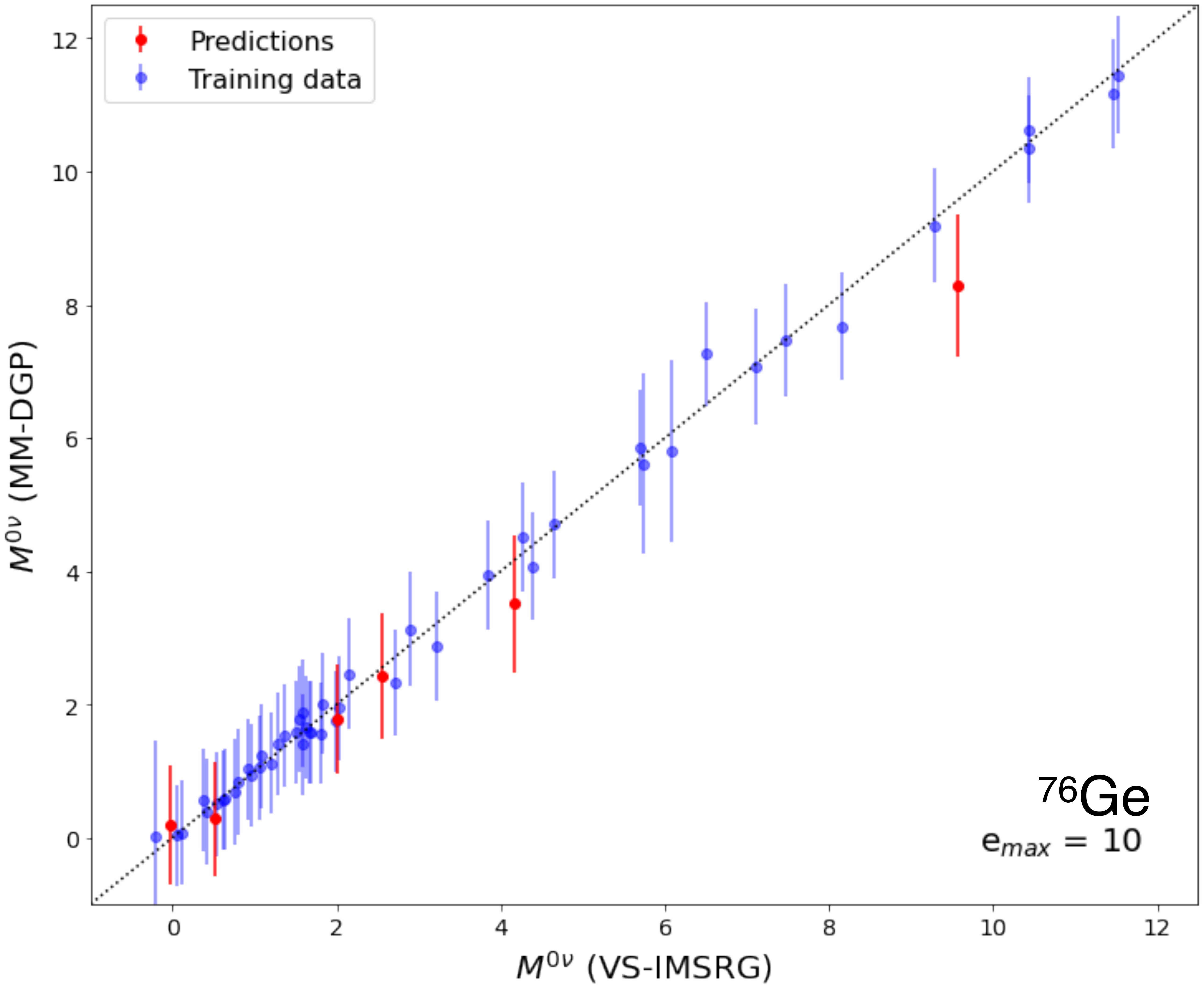

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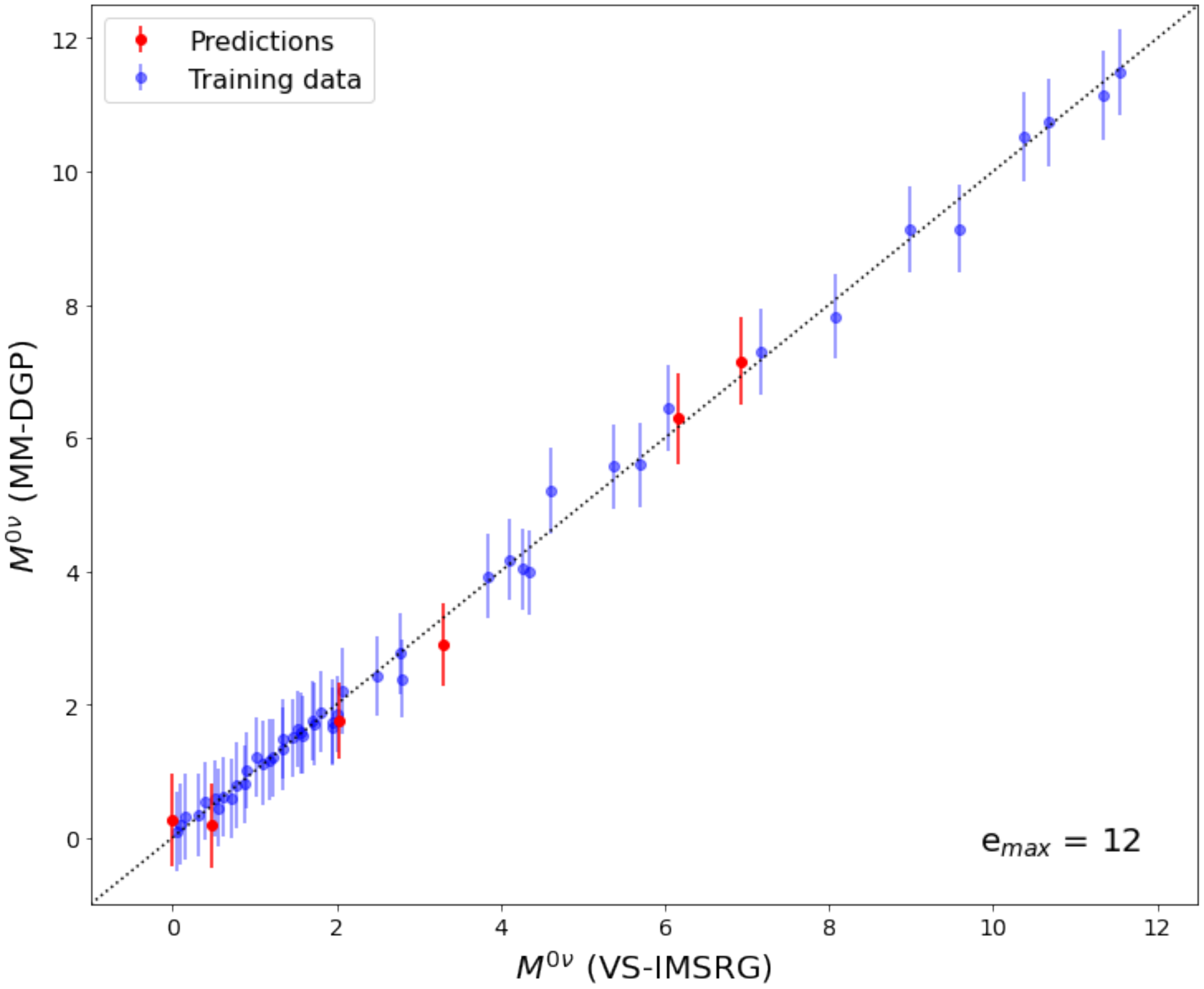
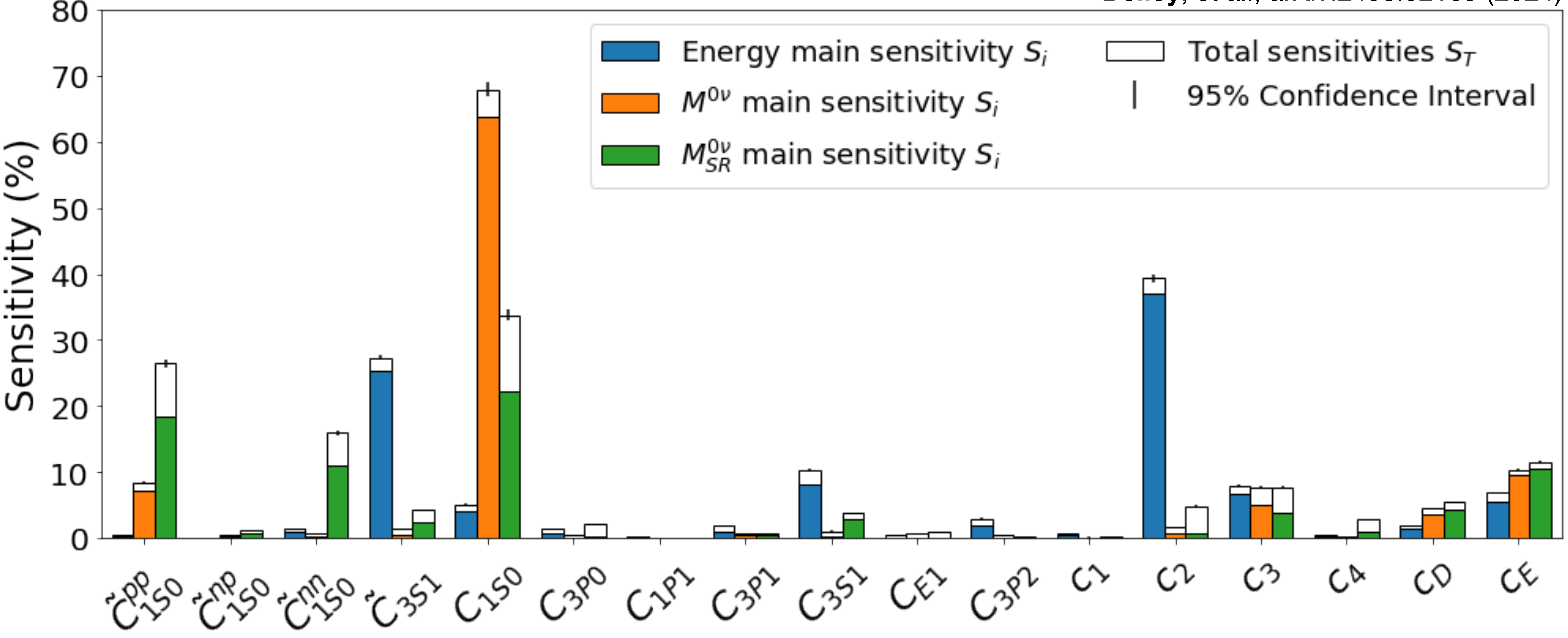



Root Mean Square Error = 0.13

# The MM-DGP Algorithm: GSA

**Belley**, et al., arXiv:2408.02169 (2024)



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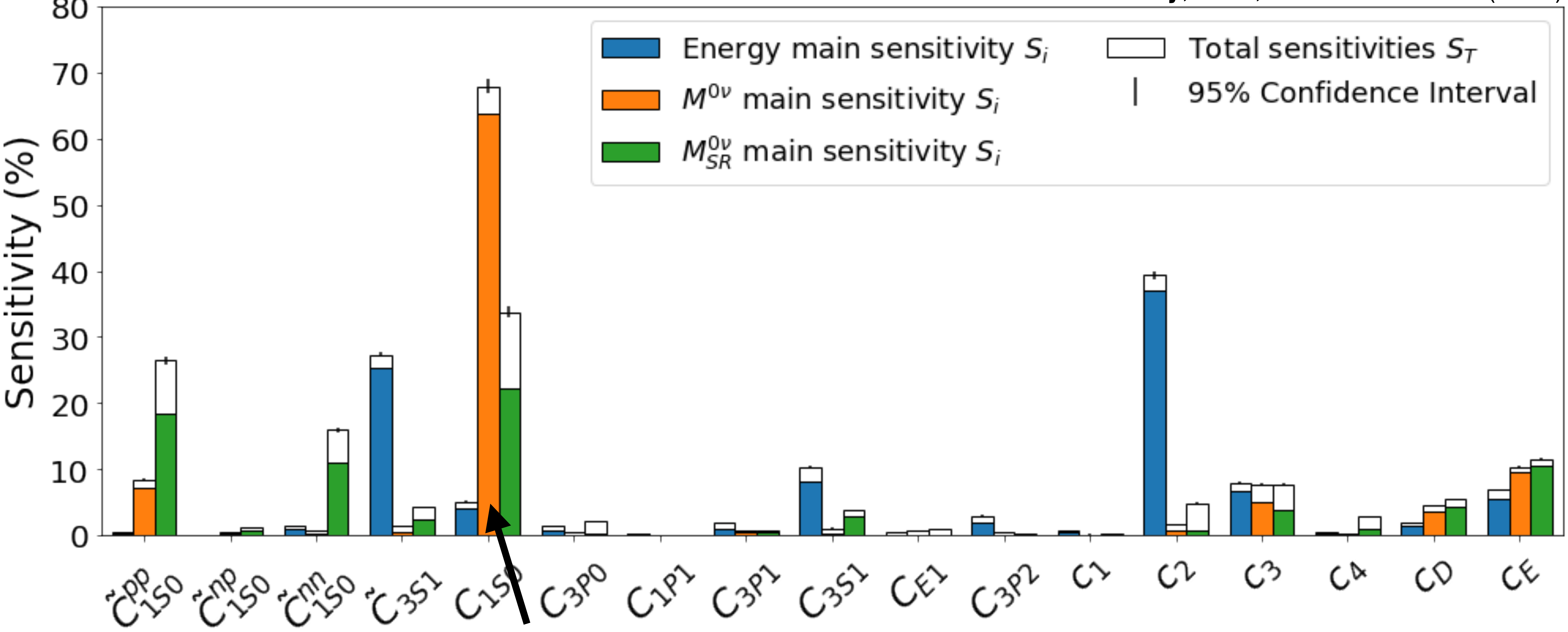


The total matrix element mostly depends on one LEC!

# The MM-DGP Algorithm: GSA

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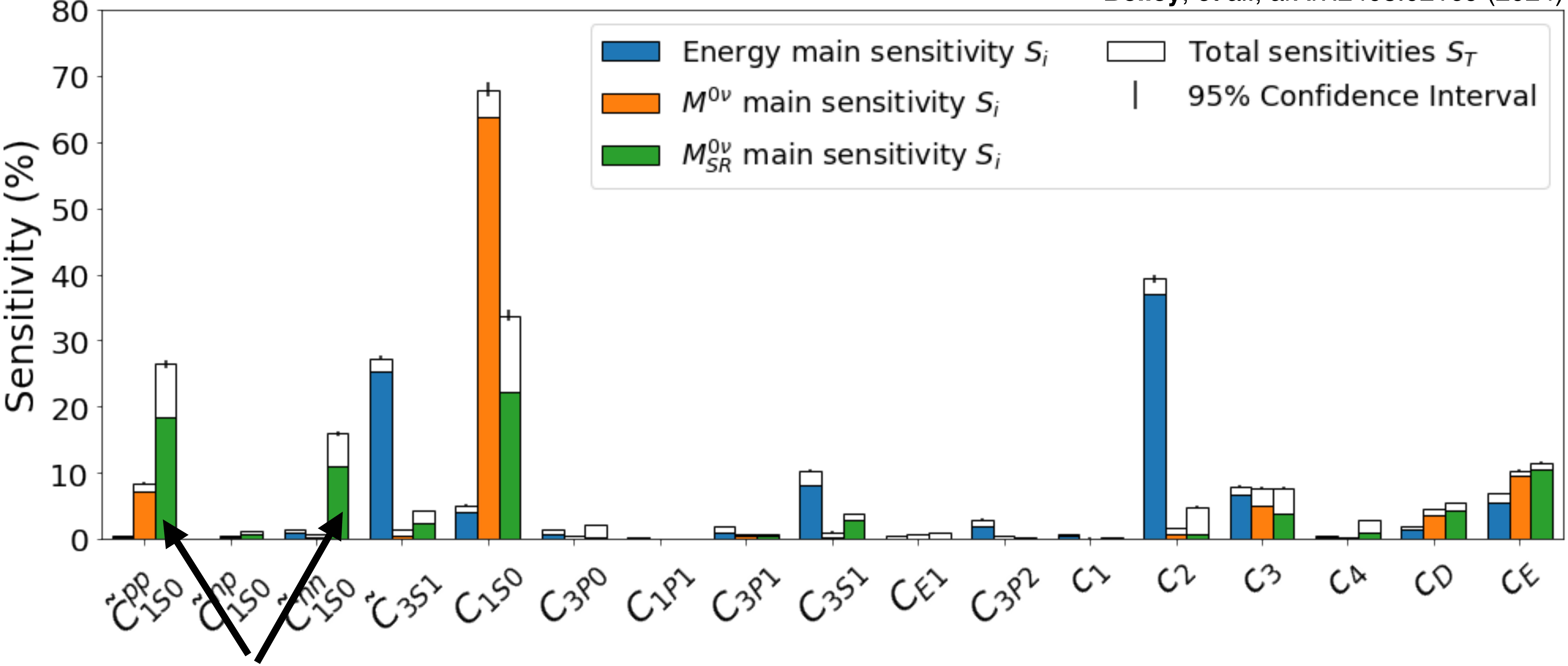


The short-range matrix element however sees other contributions from LECs associated to the short-range nuclear interaction.

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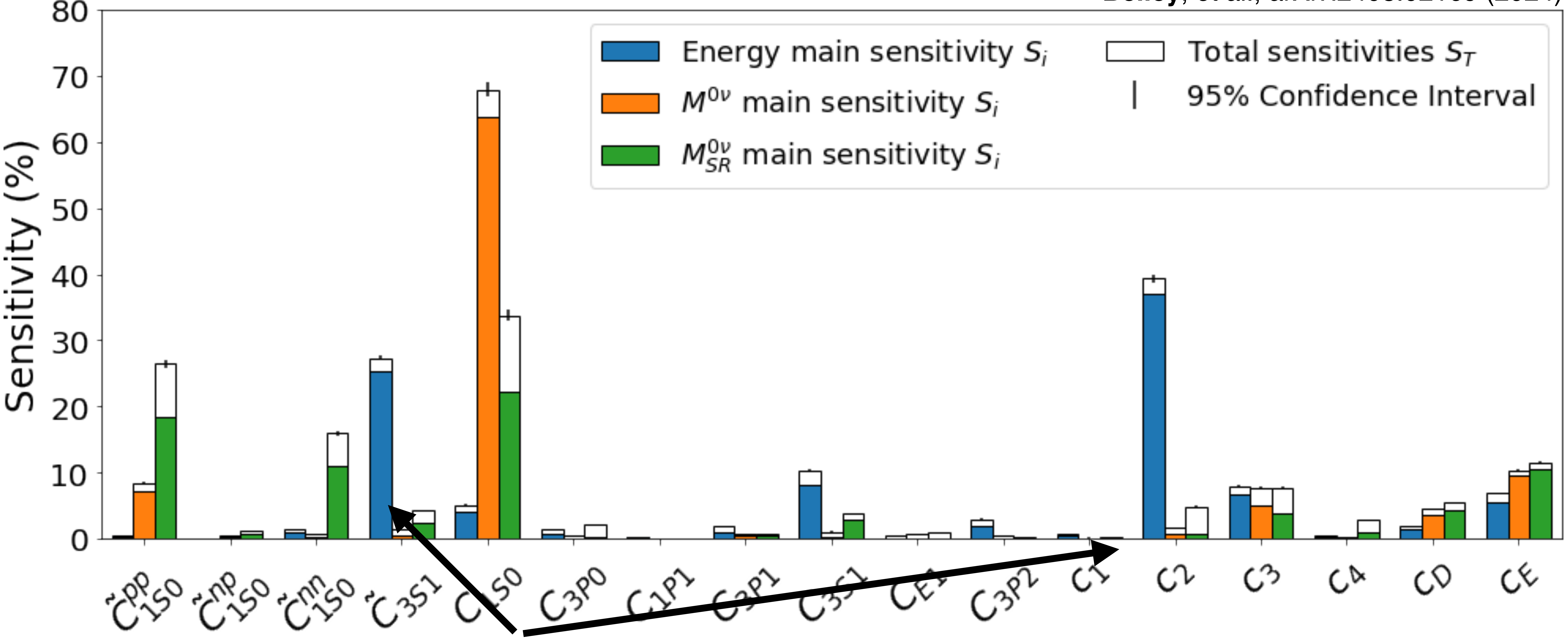
**Belley**, et al., arXiv:2408.02169 (2024)

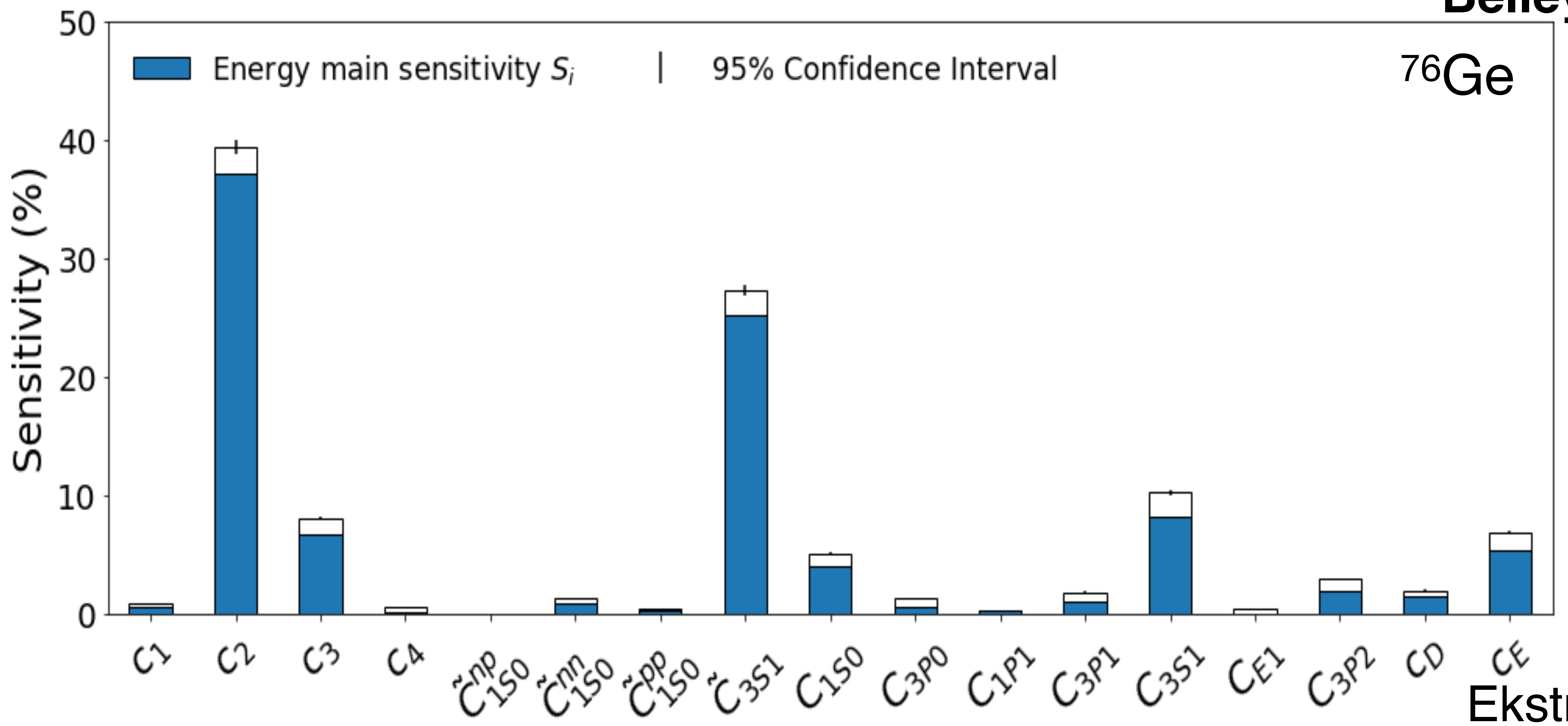



Results for energies are consistent with results of physics-based emulators of the coupled cluster method.

# The MM-DGP Algorithm: GSA

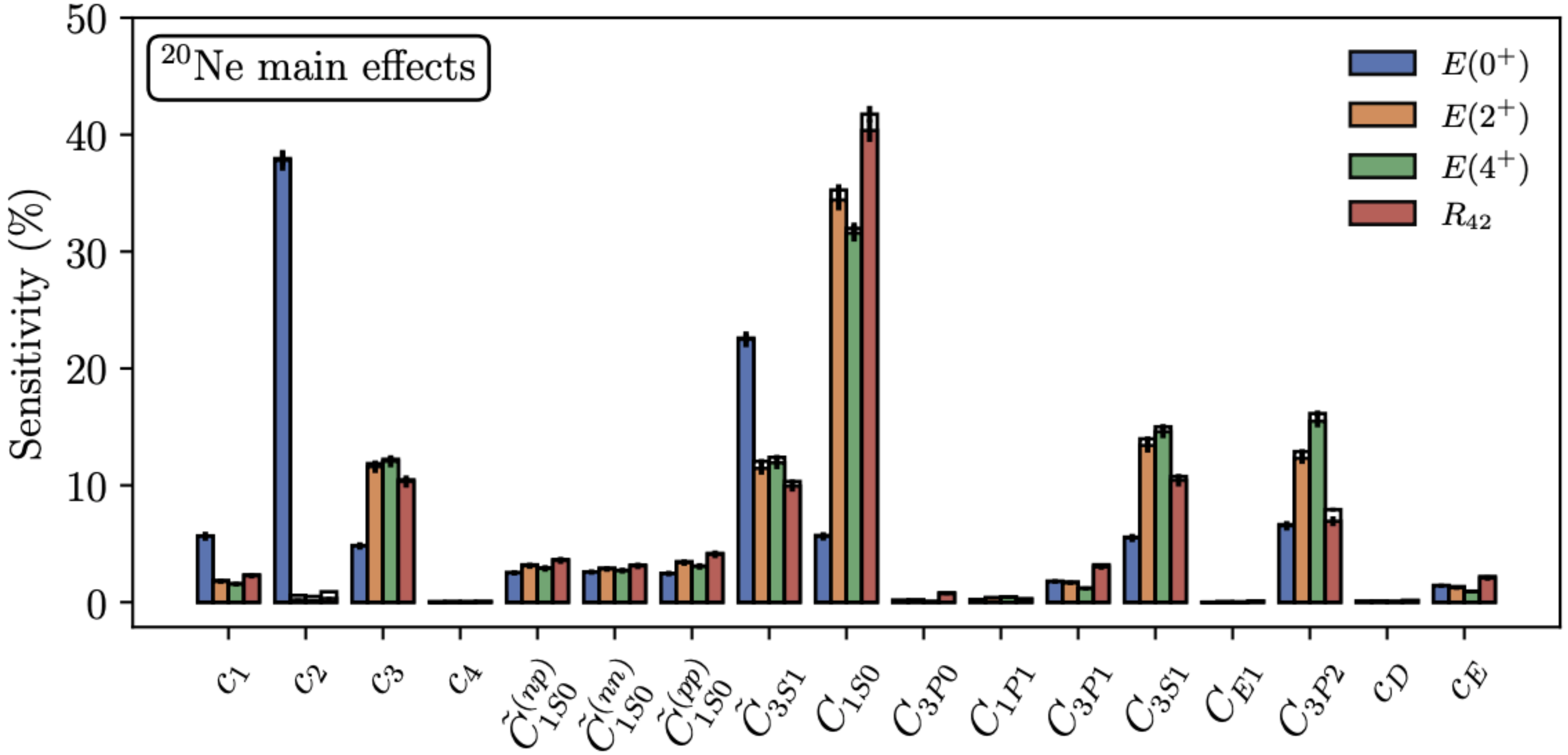
**Belley**, et al., arXiv:2408.02169 (2024)

MM-DGP

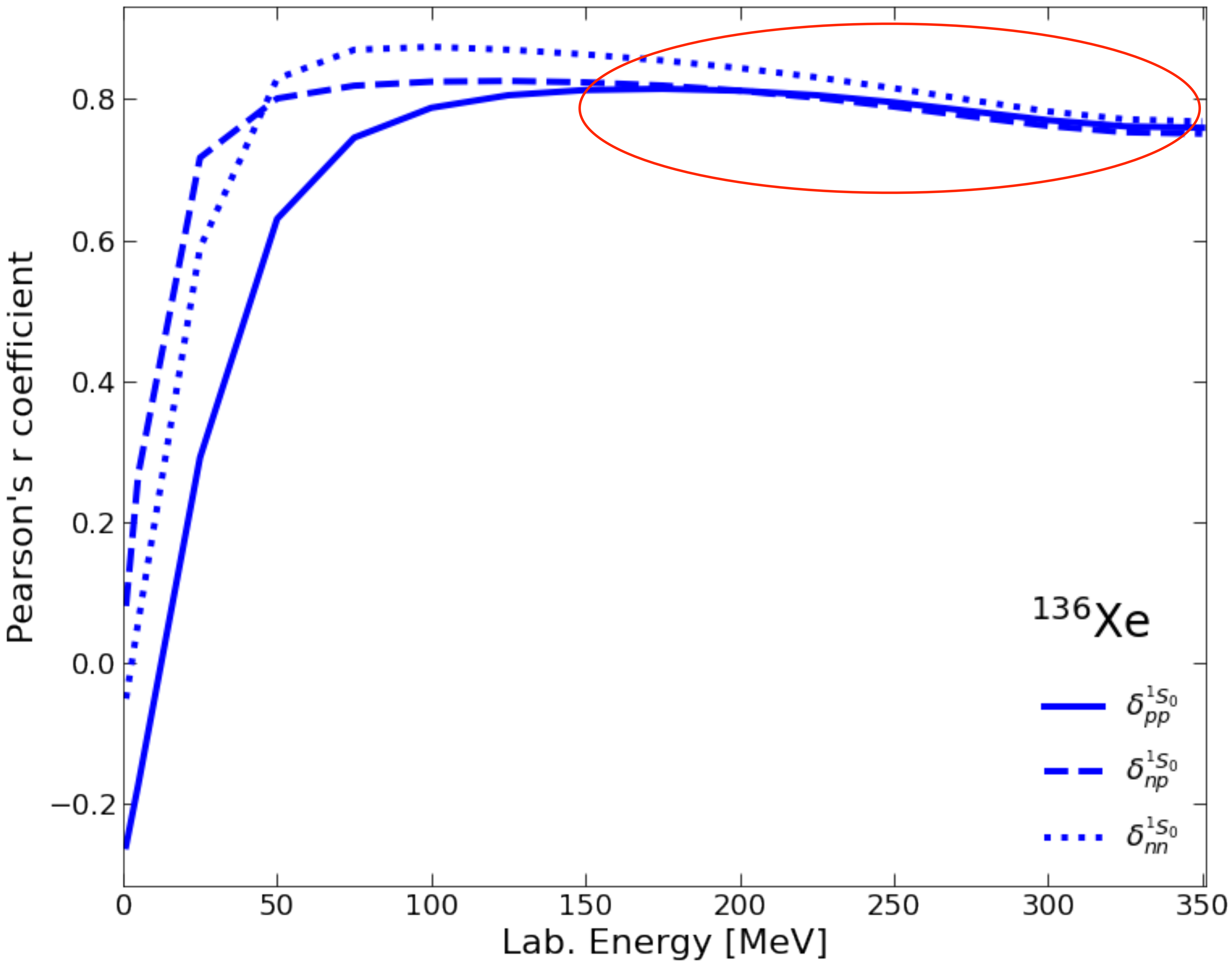



Ekström, et al., arXiv:2305.06955 (2023)

Eigenvector continuation



# Correlation with Phase Shift




Strong correlation for energies > 50 MeV

⇒

The size of matrix elements is mostly constrained by the interaction between the two nucleons that undergo the decay, given they are close enough from each other.

**Belley**, et al., arXiv:2408.02169 (2024)

# Posterior Distribution of the NMEs

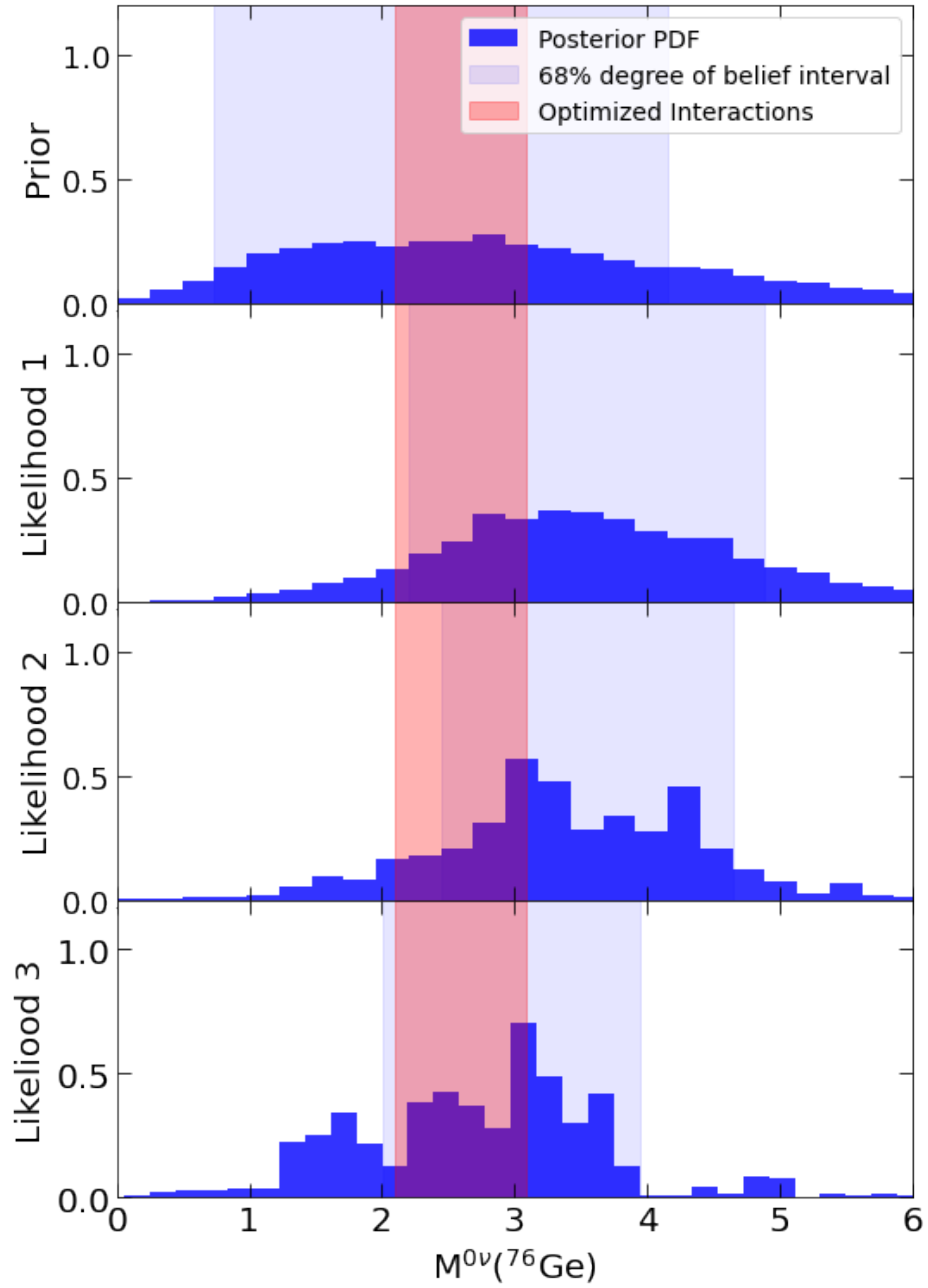
- Use 8188 “non-implausible” samples obtain by Jiang, W. G. et al. (Phys. Rev. C **109**, 064314) .
- Many-body problem is “solved” with the MM-DGP.
- Consider all sources of uncertainties by taking:

$$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

where the $\epsilon$’s are the errors coming from different sources and are assumed to be normally distributed and independent.

- Interactions are weighted by the $^1S_0$ neutron-proton phase shifts at 50 MeV and observables for mass A=2-4,16.

# Choosing a Likelood




Likelihood 1: Only contains $^1S_0$ neutron-proton phase shifts at 50 MeV.

Likelihood 2: Contains $^1S_0$ neutron-proton phase shifts at 50 MeV and observables for A=2-4.

Likelihood 3: Contains $^1S_0$ neutron-proton phase shifts at 50 MeV and observables for A=2-4,16.

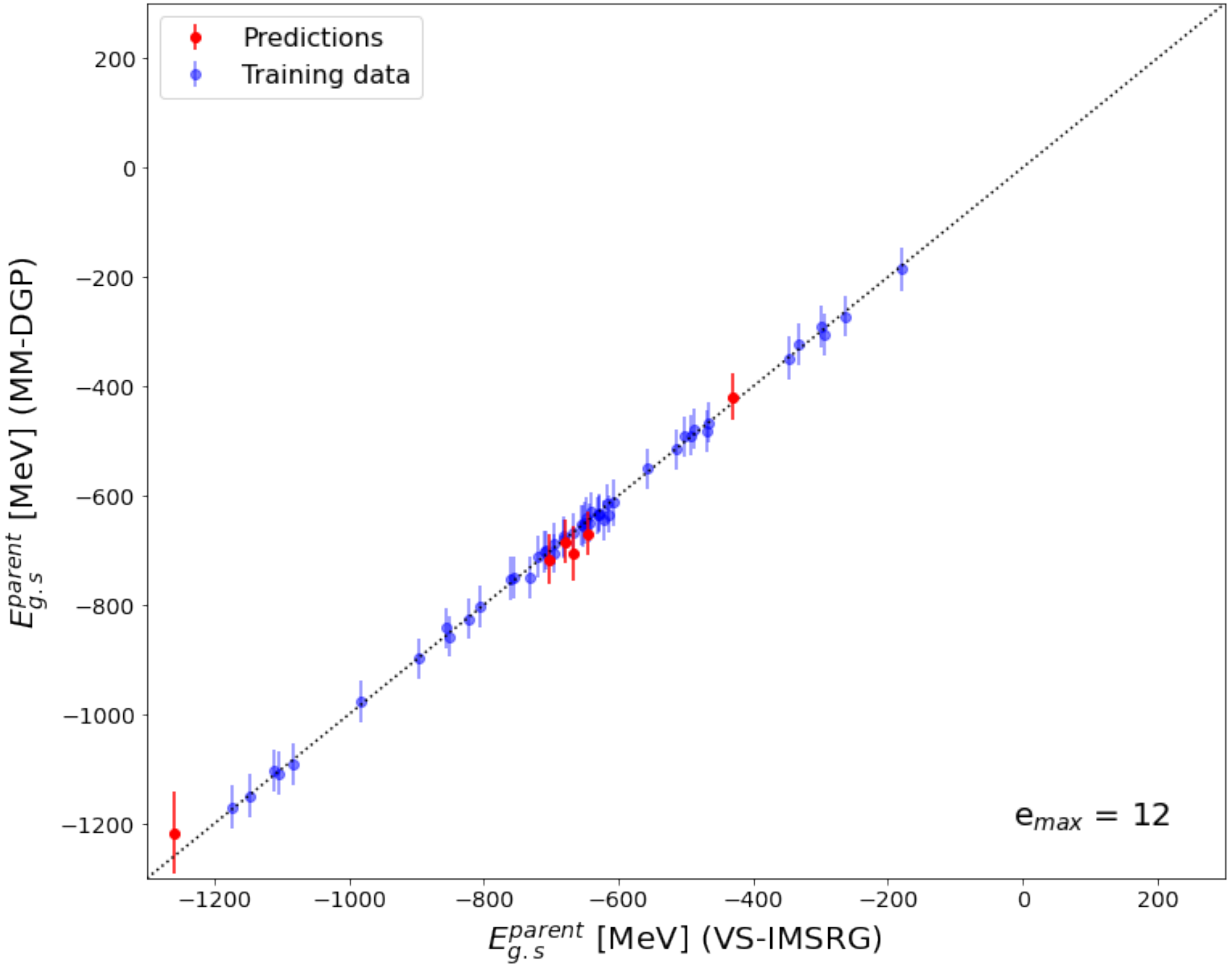
A2-4: $E(^2H)$, $r_p(^2H)$, $Q(^2H)$, $E(^3H)$, $E(^4He)$, $r_p(^4He)$

A16: $E(^{16}O)$, $r_p(^{16}O)$

# Emulator error

$$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

This error is given directly by the Gaussian Process and depends on the LECs (i.e. each predicted point has its own error).



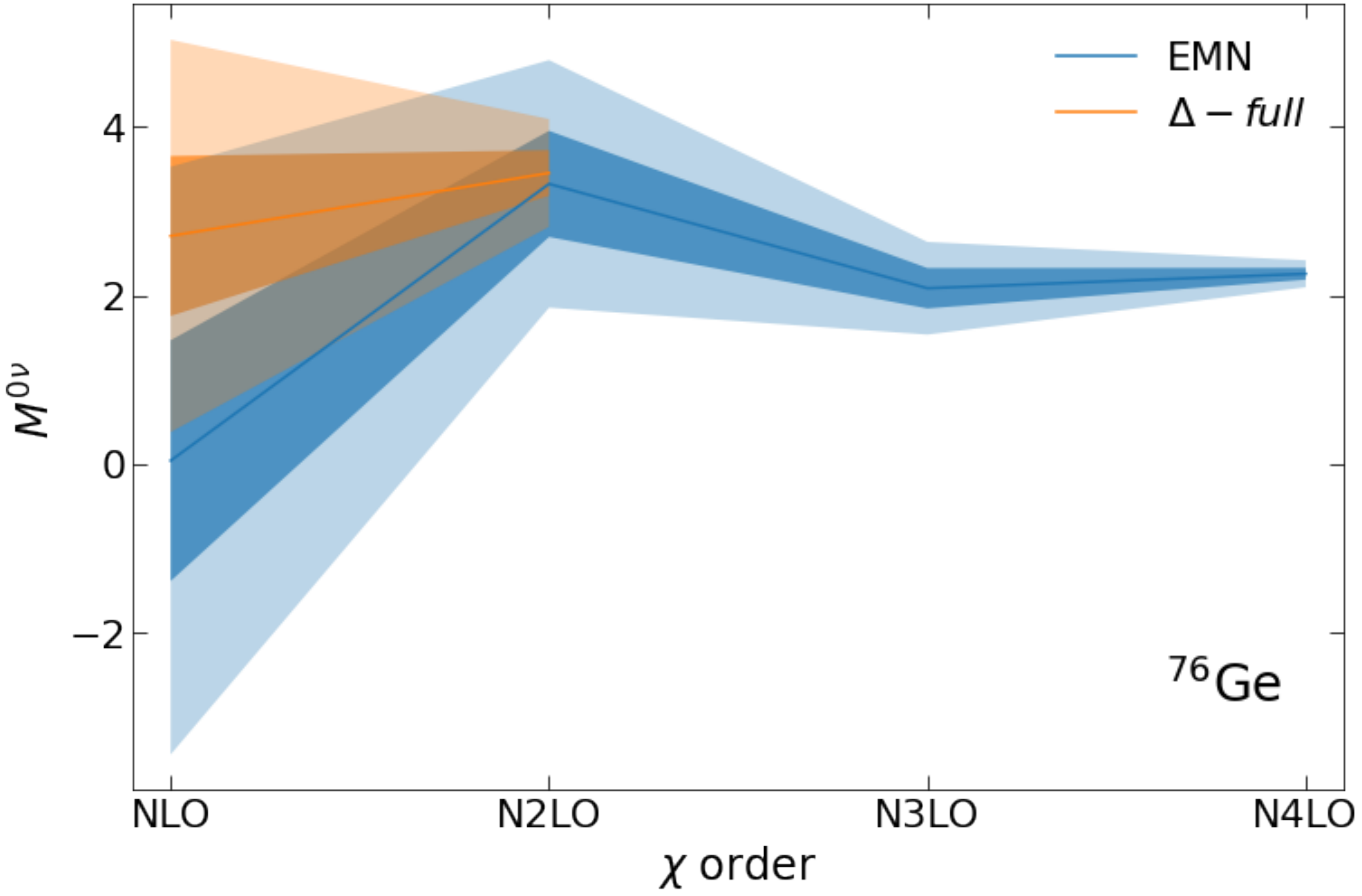
# EFT Truncation error

$$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

Error due to the truncation of the nuclear interactions (the samples are truncated at N2LO, including delta excitations).

Use EMN interaction at NLO, N2LO, N3LO and N4LO, without delta excitations, to verify convergence of chiral expansion.

Using the $\Delta$-full interaction of this work, only NLO and N2LO orders are available. Using expansion from BUQEYE collaboration, we get $\epsilon_{EFT} = 0.3$.



# EFT Truncation error

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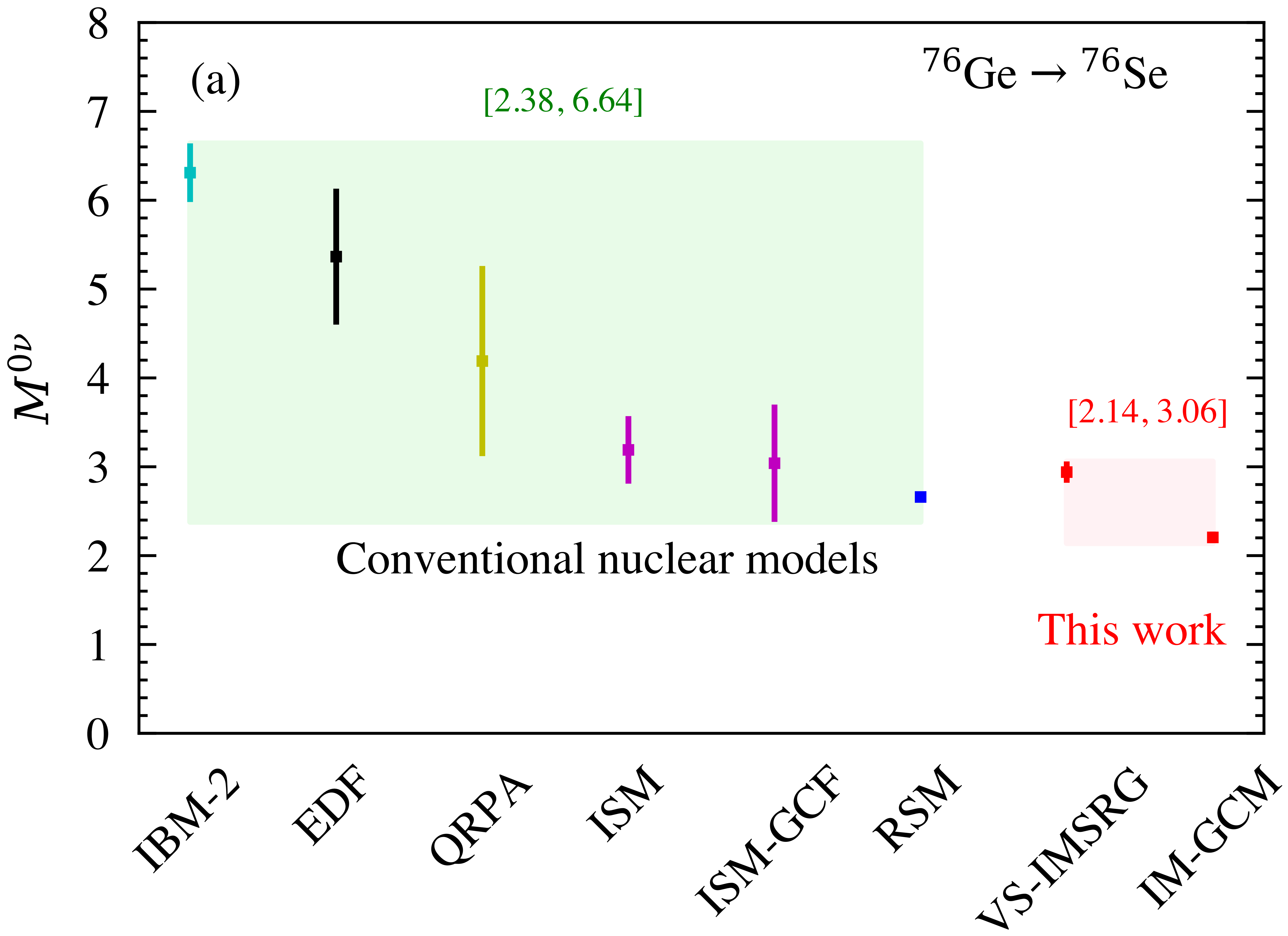
Error due to the truncation of the many-body method. This is studied by comparing the results of the IM-GCM and VS-IMSRG using the magic interaction.

This error is surprisingly large as we find $\epsilon_{many-body}$ = 0.88.



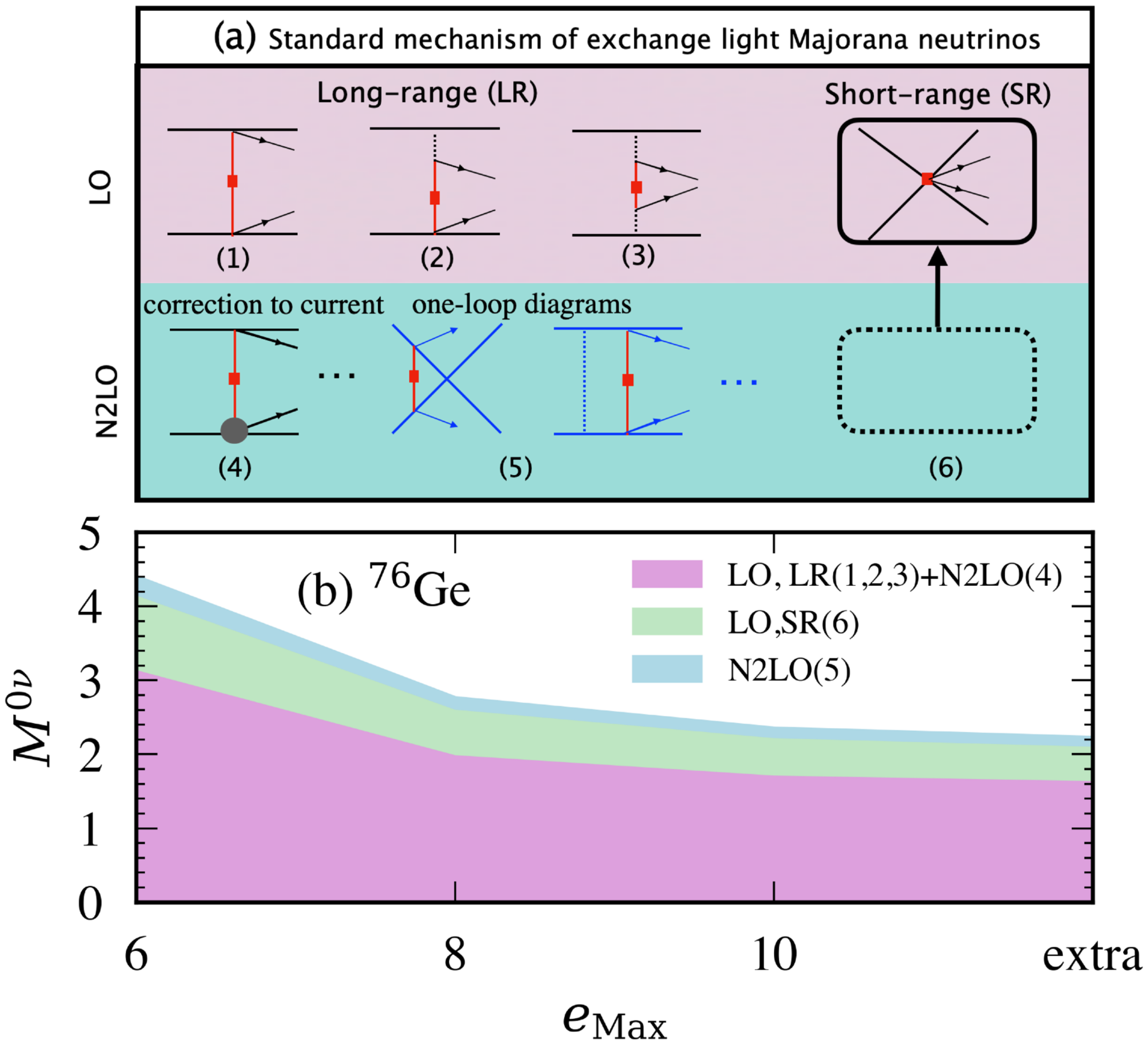
# Operator error

$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$

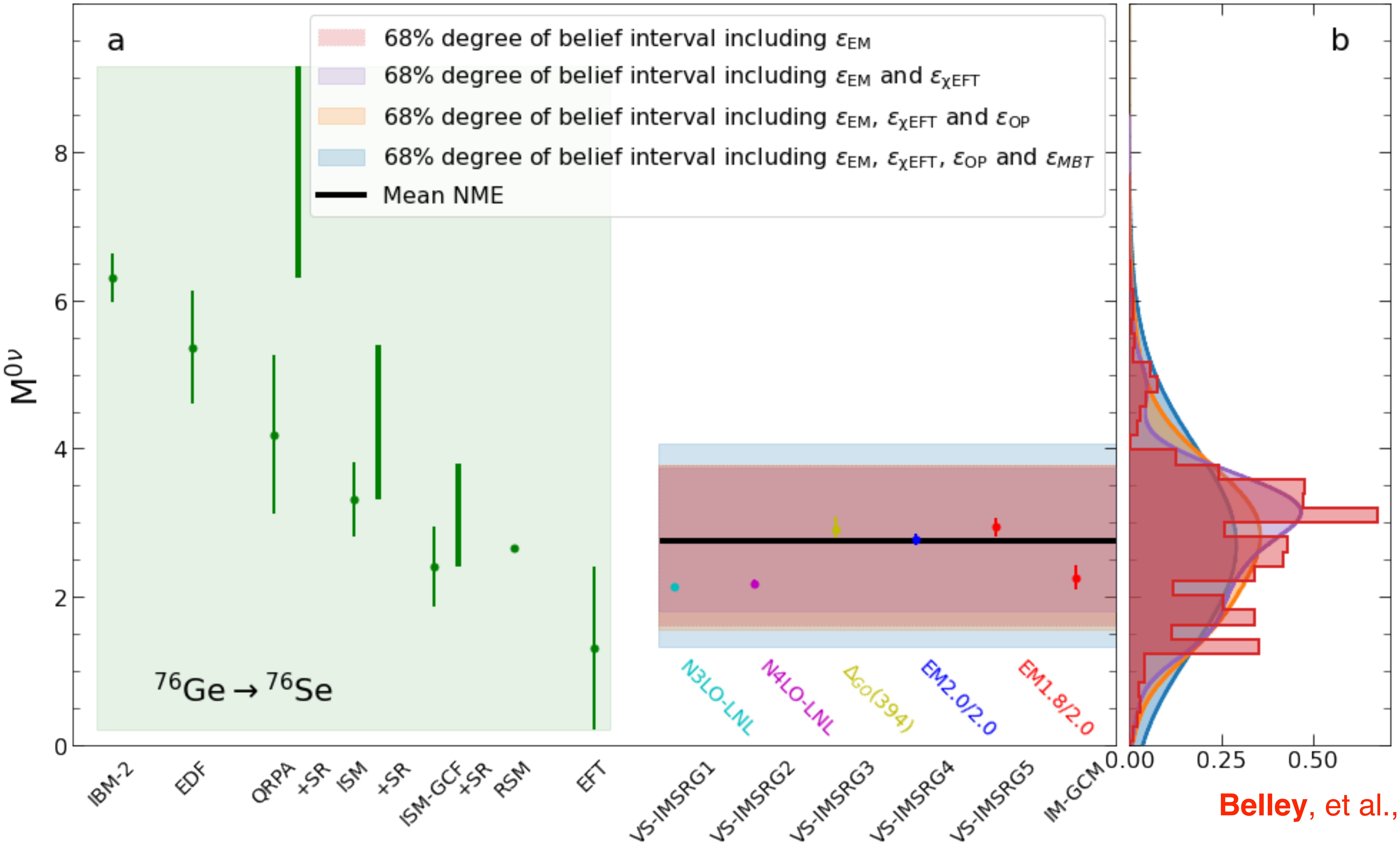
Error due to the truncation of the operator in chiral expansion + closure energy correction + value of the contact LEC.

Adding N2LO operators has very small contribution (< 0.2). Biggest contribution comes from determination of contact term.

Total error amounts to $\epsilon_{operator}$ = 0.47.



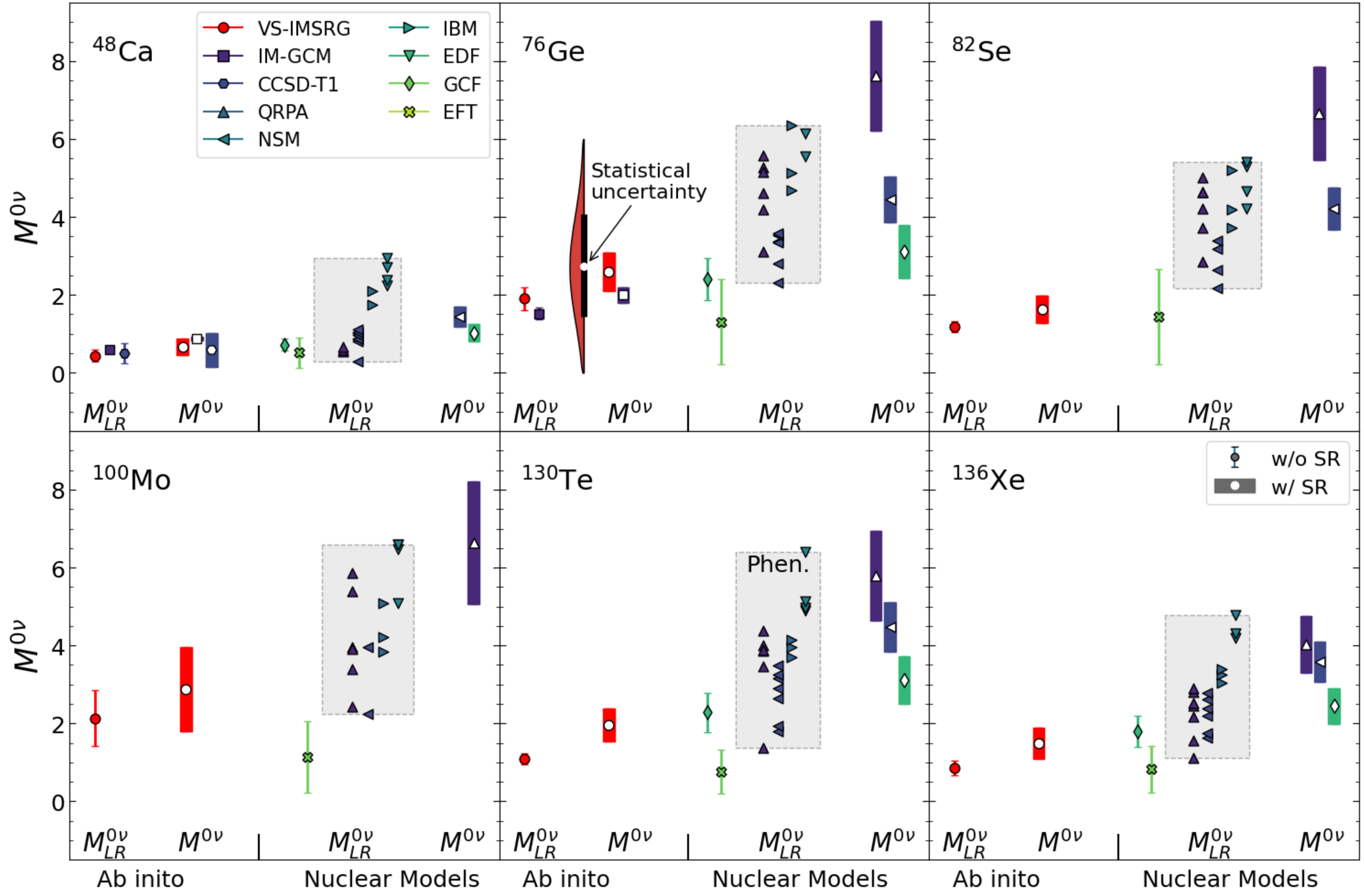
# Combining all sources of uncertainty




$$M^{0\nu\beta\beta} = 2.60^{+1.28}_{-1.36}$$

**Belley**, et al., Phys. Rev. Lett. 132, 182502

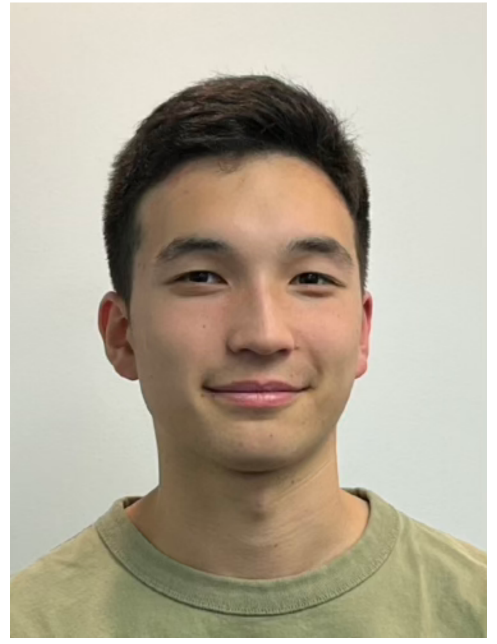
# The Current Picture



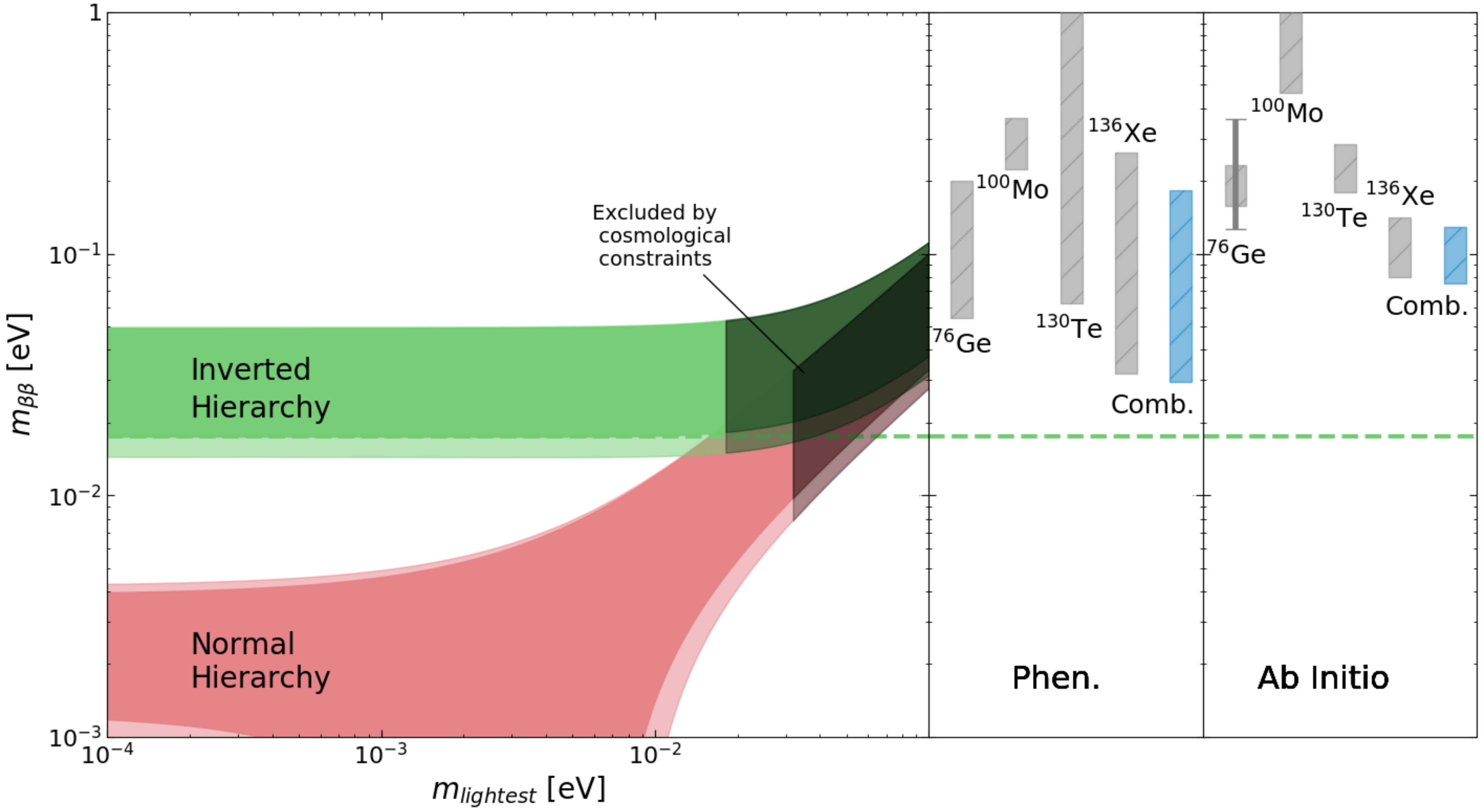

Values from:

- Agostini et al., Rev. Mod. Phys. 95, 025002 (2023);
- Yao, Sci. Bull. 10.1016 (2020);
- **Belley** et al., Phys. Rev. Lett. 126, 042502 (2020);
- Brase et al., Phys. Rev. C 106, 034309 (2021);
- Weiss et al., Phys. Rev. C 106, 064401 (2022);
- **Belley** et al., arXiv:2307.15156 (2023);
- **Belley** et al., Phys. Rev. Lett. 132, 182502 (2024).

# Combining Limits of Different Isotopes



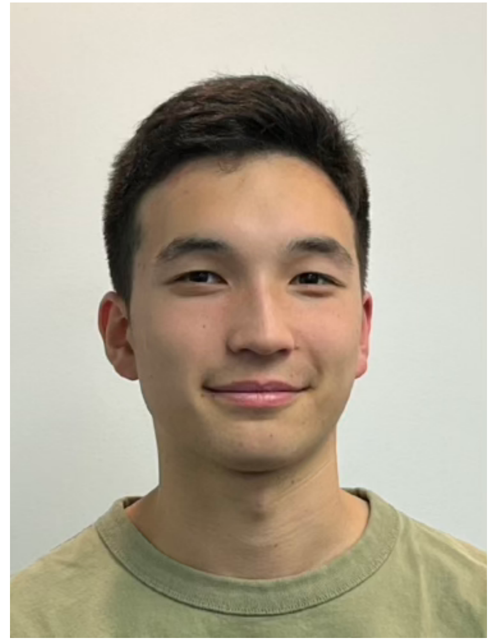
Taiki Shickele




Current

Experimental limits: **GERDA ($^{76}$Ge)** Phys. Rev. Lett. 125, 252502, **CUPID-Mo ($^{100}$100)** Eur. Phys. J. C 82 11, 1033, **CUORE($^{130}$Te)** arXiv:2404.04453, **EXO($^{136}$Xe)** Phys. Rev. Lett. 123, 161802 and **Kamland Zen ($^{136}$Xe)** arXiv:2406.11438

# Combining Limits of Different Isotopes



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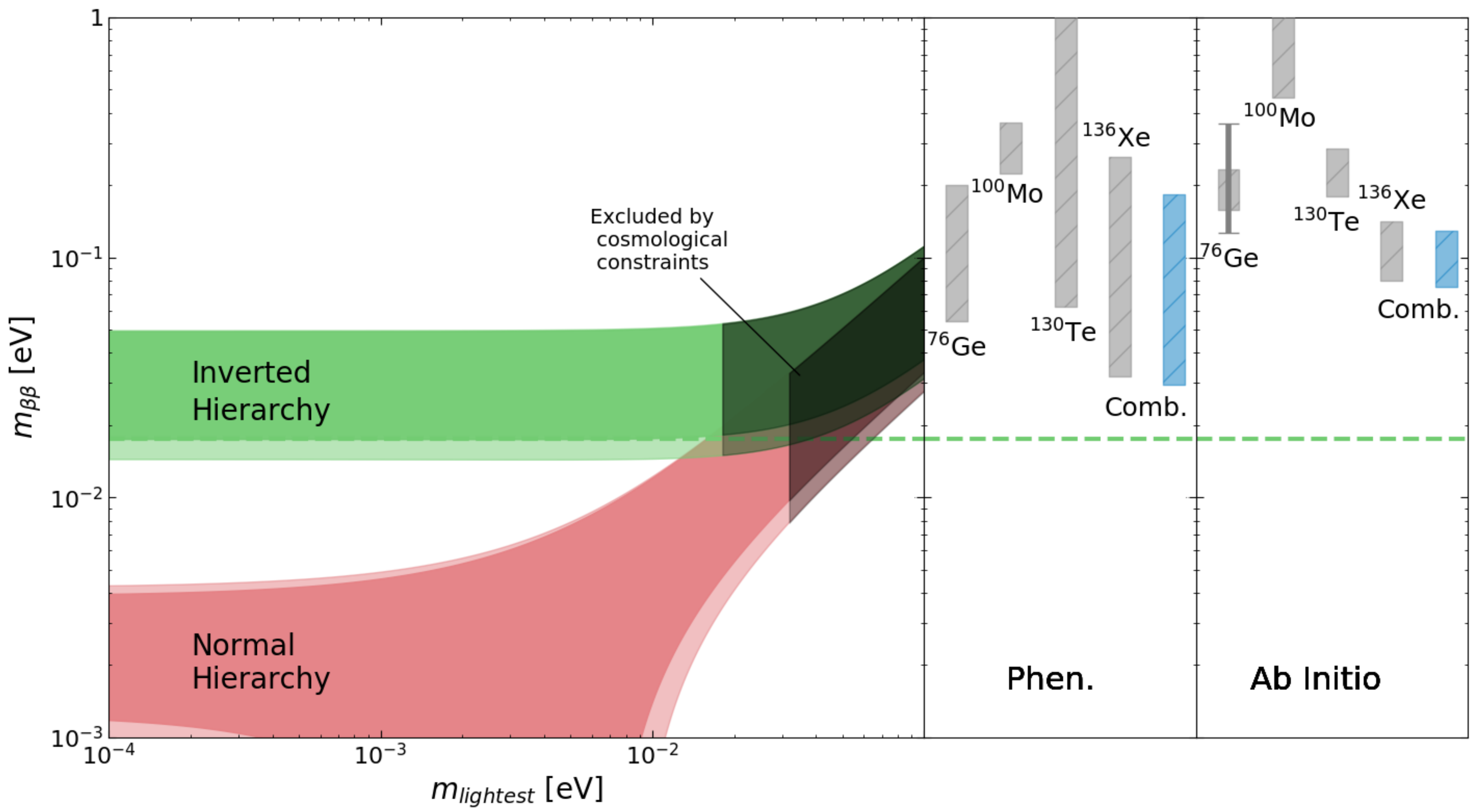


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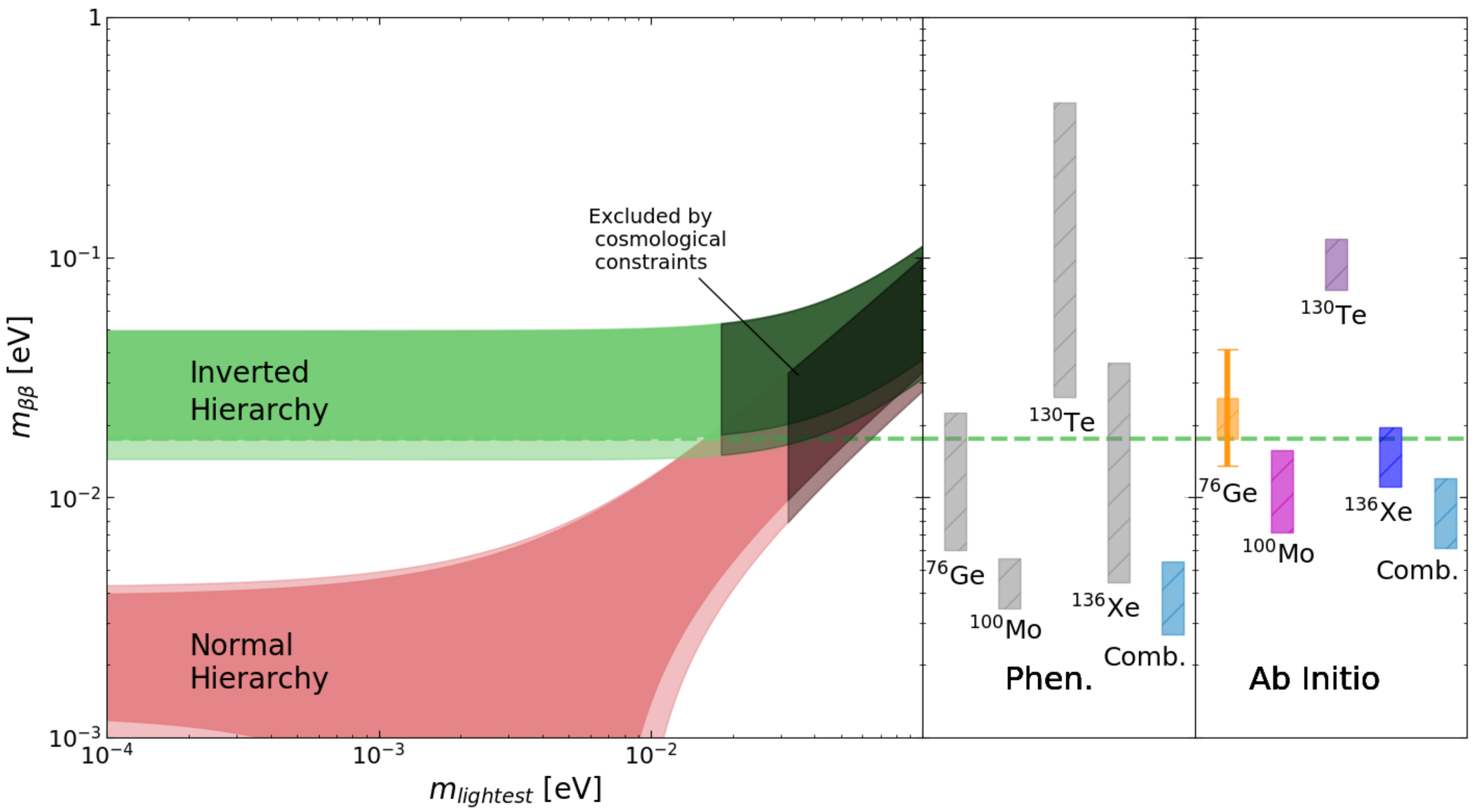


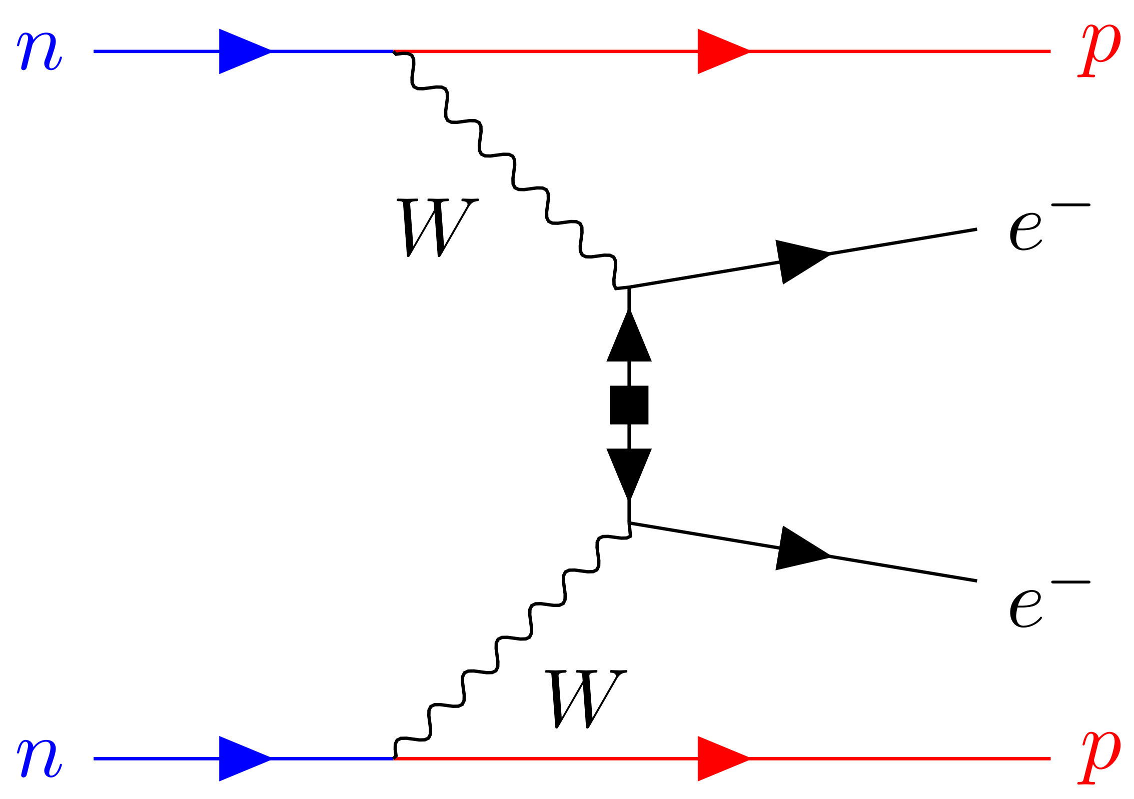

Future

Experimental limits: **GERDA ($^{76}$Ge)** Phys. Rev. Lett. 125, 252502, **CUPID-Mo ($^{100}$100)** Eur. Phys. J. C 82 11, 1033, **CUORE($^{130}$Te)** arXiv:2404.04453, **EXO($^{136}$Xe)** Phys. Rev. Lett. 123, 161802 and **Kamland Zen ($^{136}$Xe)** arXiv:2406.11438

Expected limits: **LEGEND ($^{76}$Ge)** arXiv:2107.11462, **CUPID ($^{100}$100)** arXiv:1907.09376, **AMoRE ($^{100}$100)** arXiv:2406.09698, **SNO+($^{130}$Te)** arXiv:2104.11687, **NEXT ($^{136}$Xe)** JHEP09(2023)190 and **nEXO ($^{136}$Xe)** J. Phys .G 49 1, 015104.

# Going Past the Standard Mechanism

$$[T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu} M^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2$$

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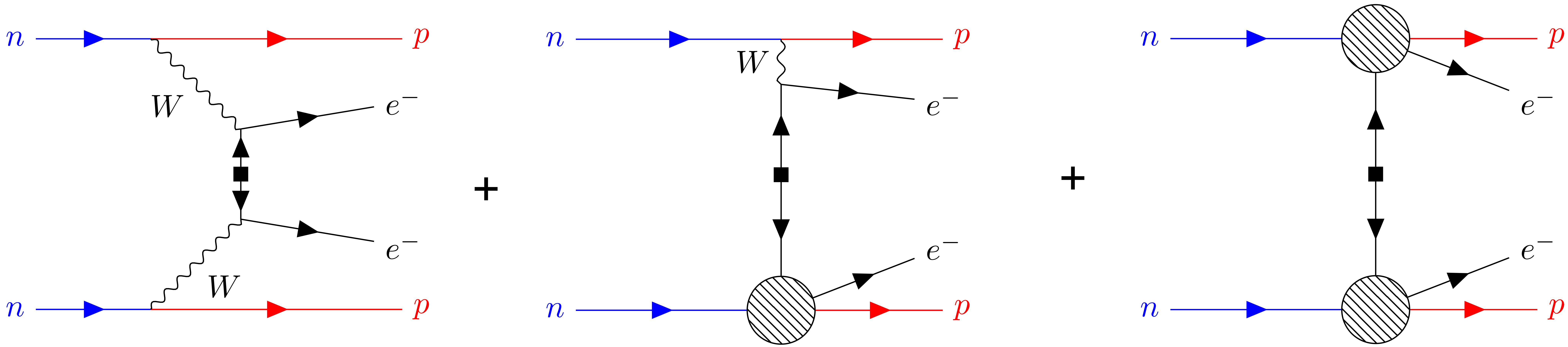
$$[T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu} M^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 \longrightarrow [T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu}|M^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 + \sum_i G_i |M_i|^2 \eta_i^2$$



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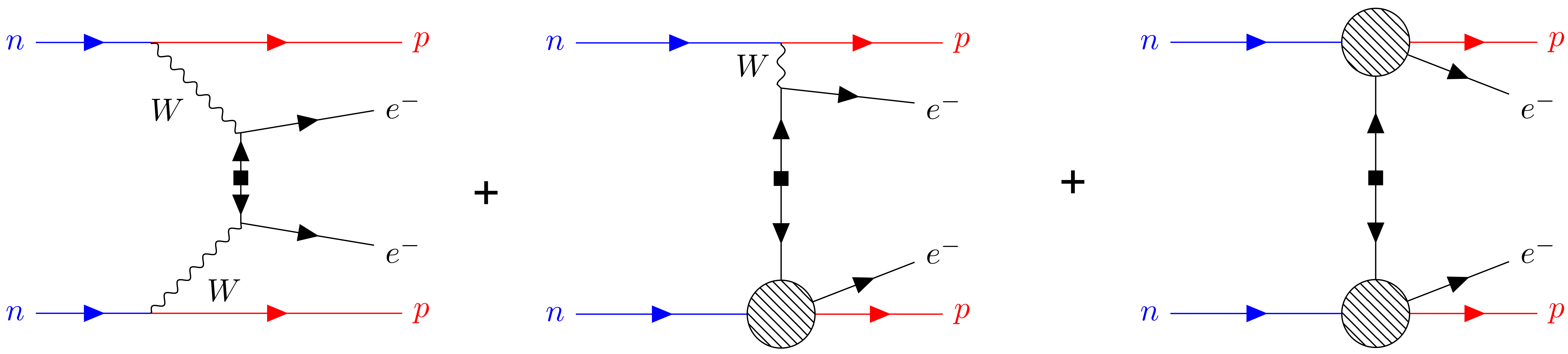


Simplest extension is to add heavy sterile neutrinos $\Rightarrow [T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu} \left| M^{0\nu}\left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right) + M^{0N}\left(\frac{m_p}{m_N}\right)\right|^2$

Mass of heavy neutrino

# Heavy Sterile Neutrino NMEs

Alex Todd

$$M^{0N} = M^{0N}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0N}_F + M^{0N}_T$$

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→ All operators are short-range contact operators.

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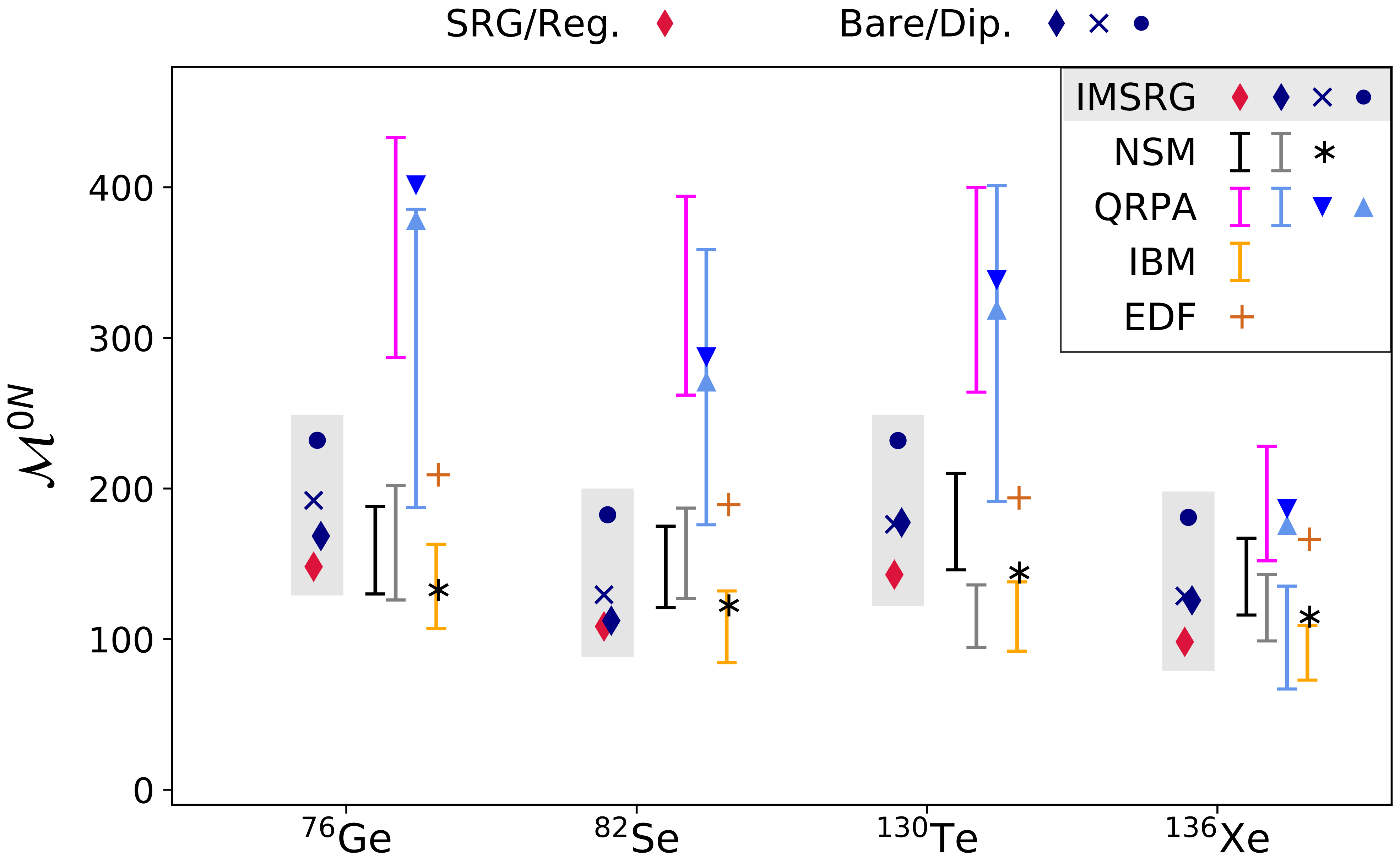

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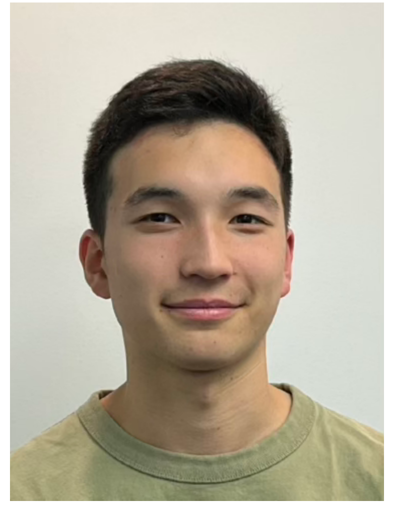

Taiki Shickele  Alex Todd

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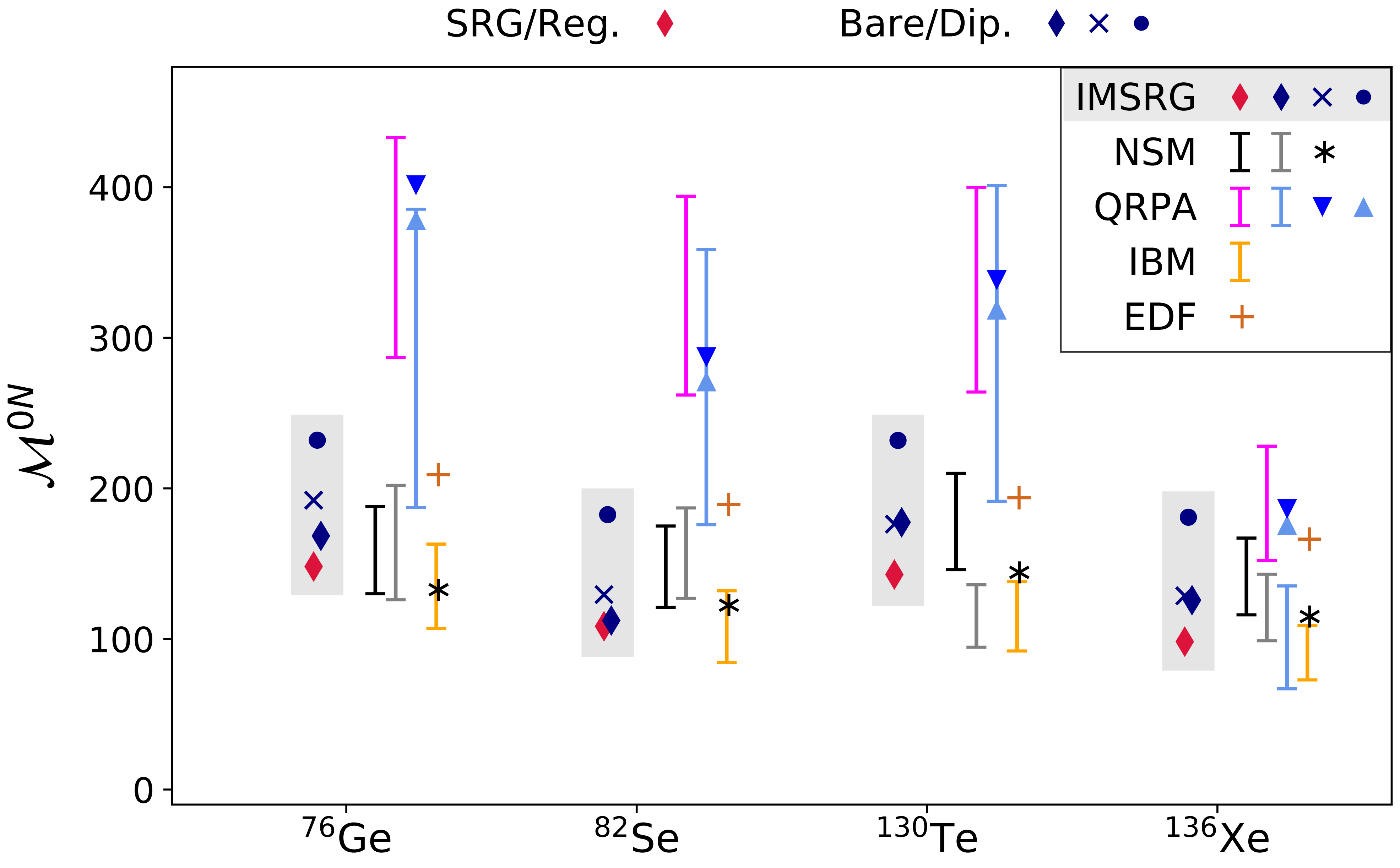
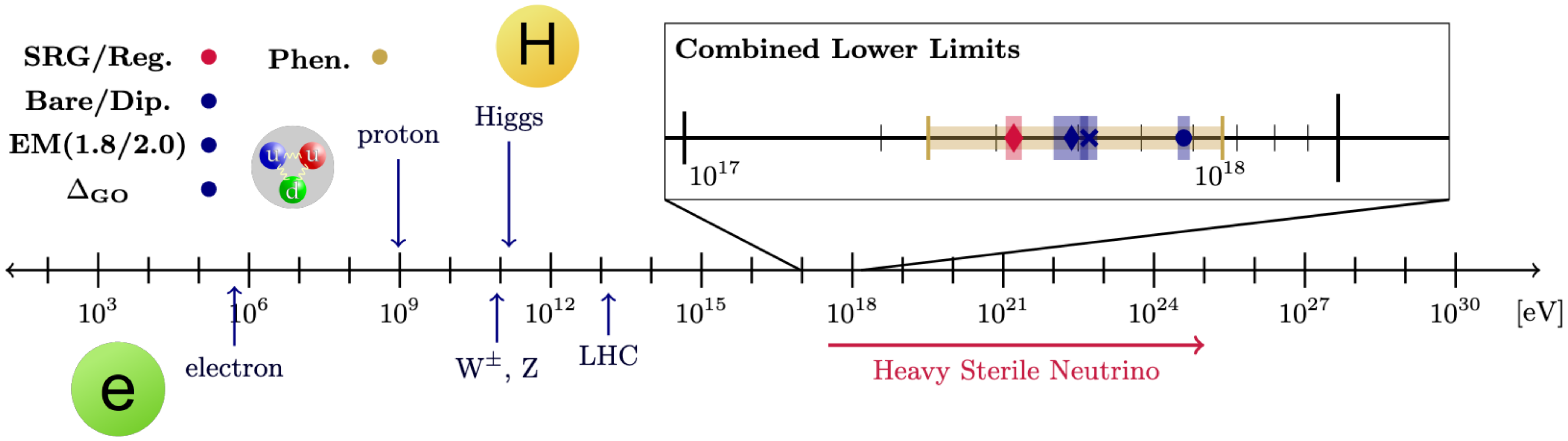
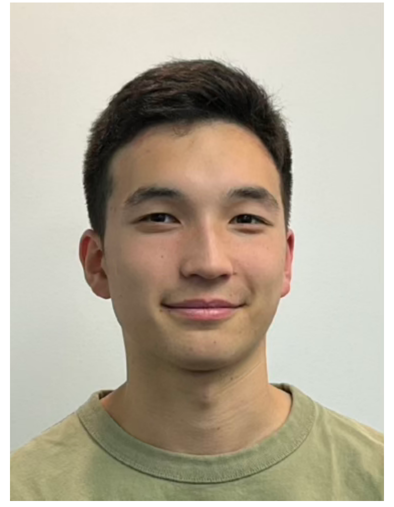


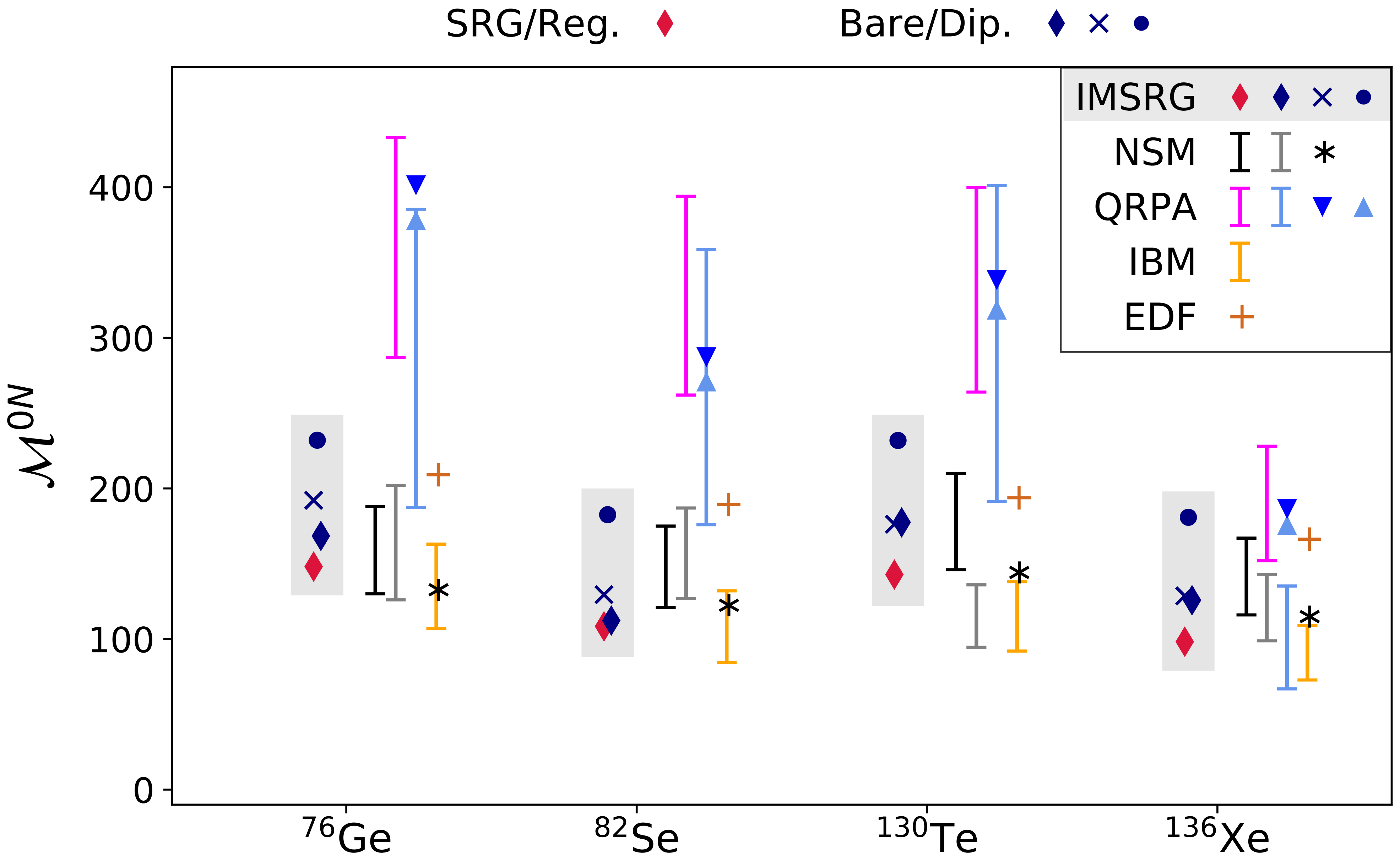
# Heavy Sterile Neutrino NMEs



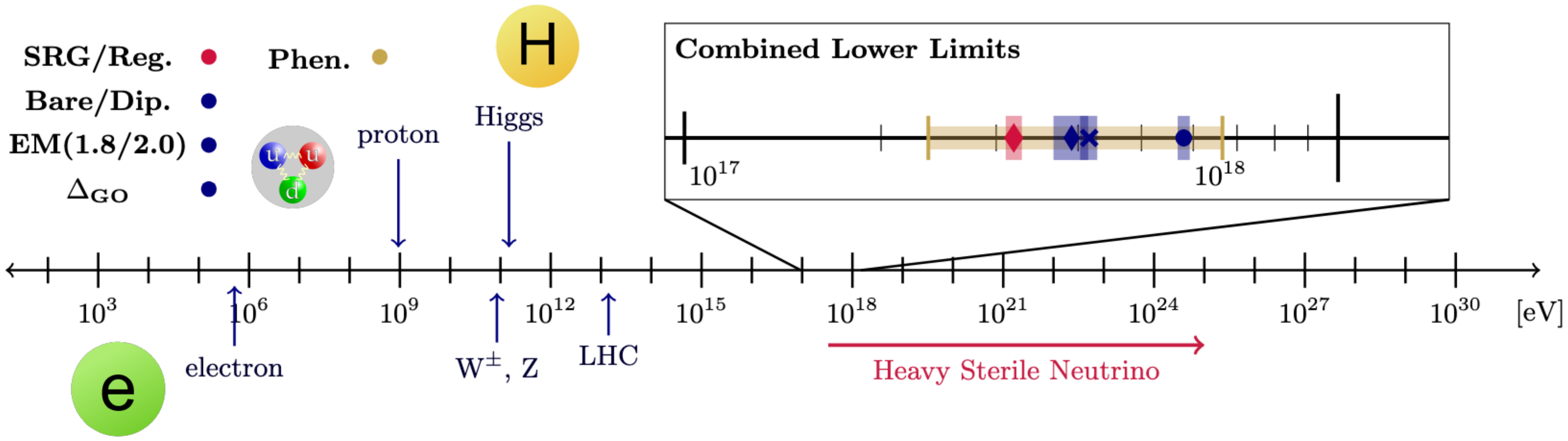
Taiki Shickele    Alex Todd

$$M^{0N} = M^{0N}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0N}_F + M^{0N}_T$$

→ All operators are short-range contact operators.




Can probe energy scales many orders of magnitude higher than particle accelerators!



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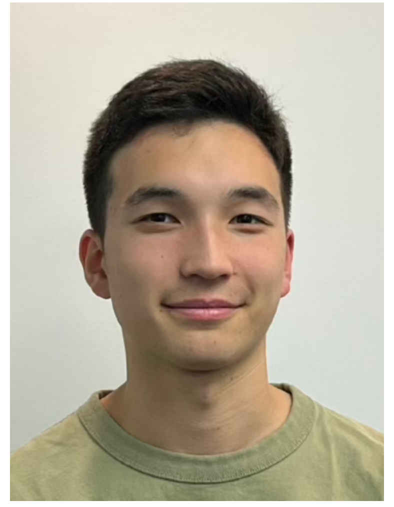

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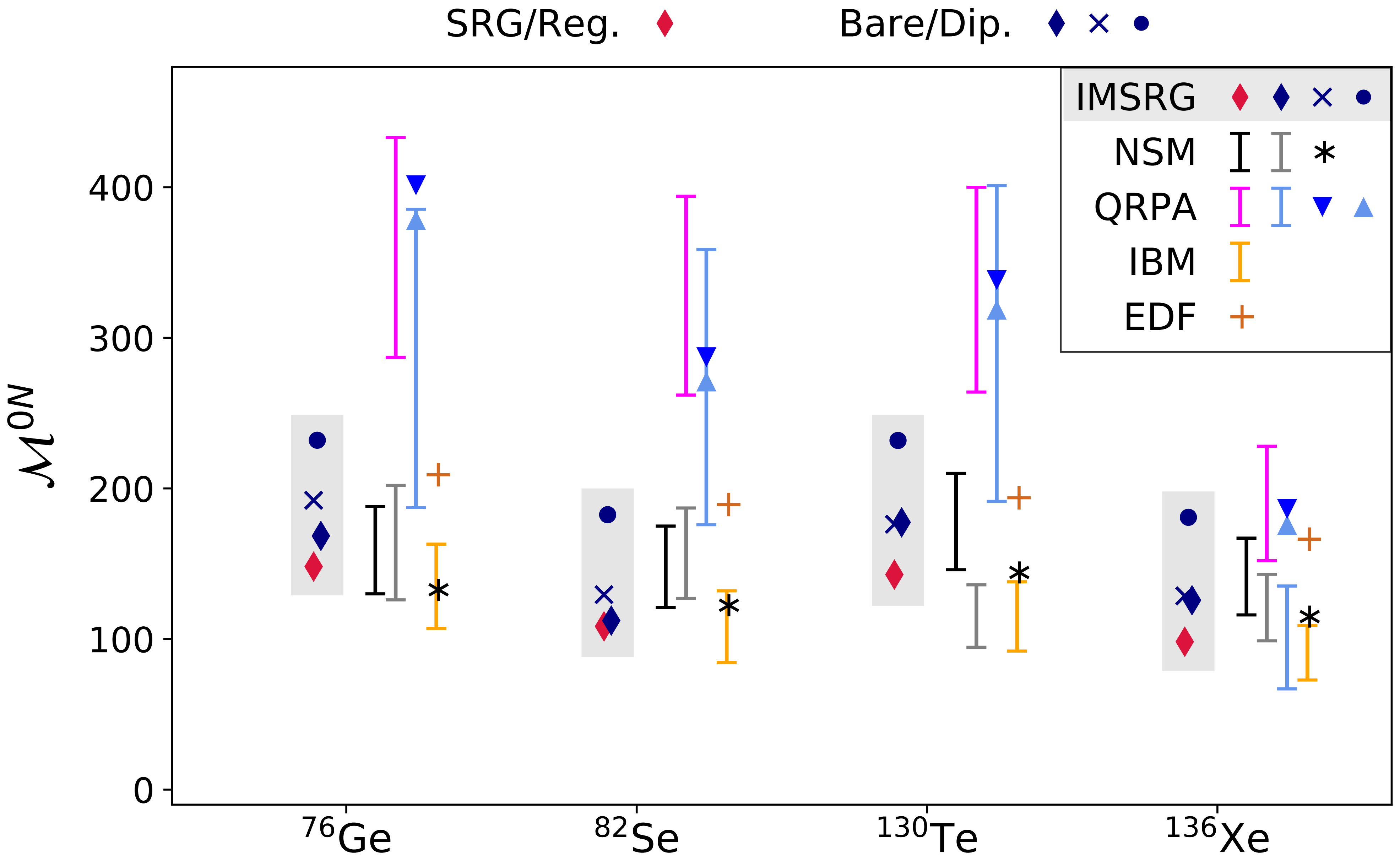


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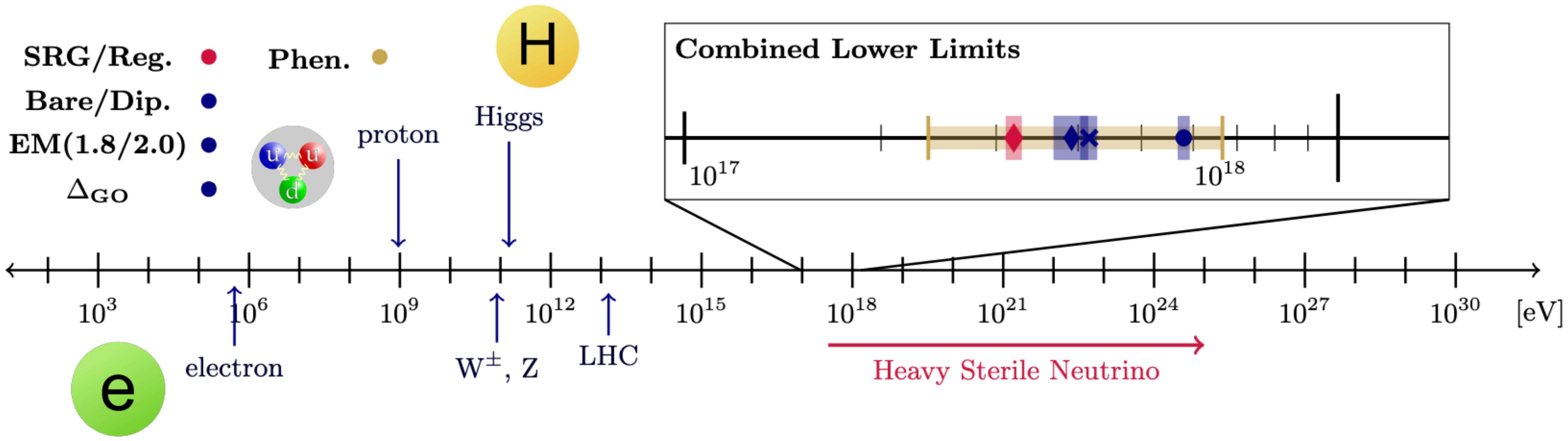



From V. Cirigliano, et al., JHEP12(2018)097, only 15 different nuclear matrix elements can contribute to mechanisms involved in $0\nu\beta\beta$. Observation in many isotopes is required to identify (or at least constrain) the mechanisms at play.

# Global emulation

# Need For Global Emulator

- **Current emulators are still pretty limited and costly to train.**

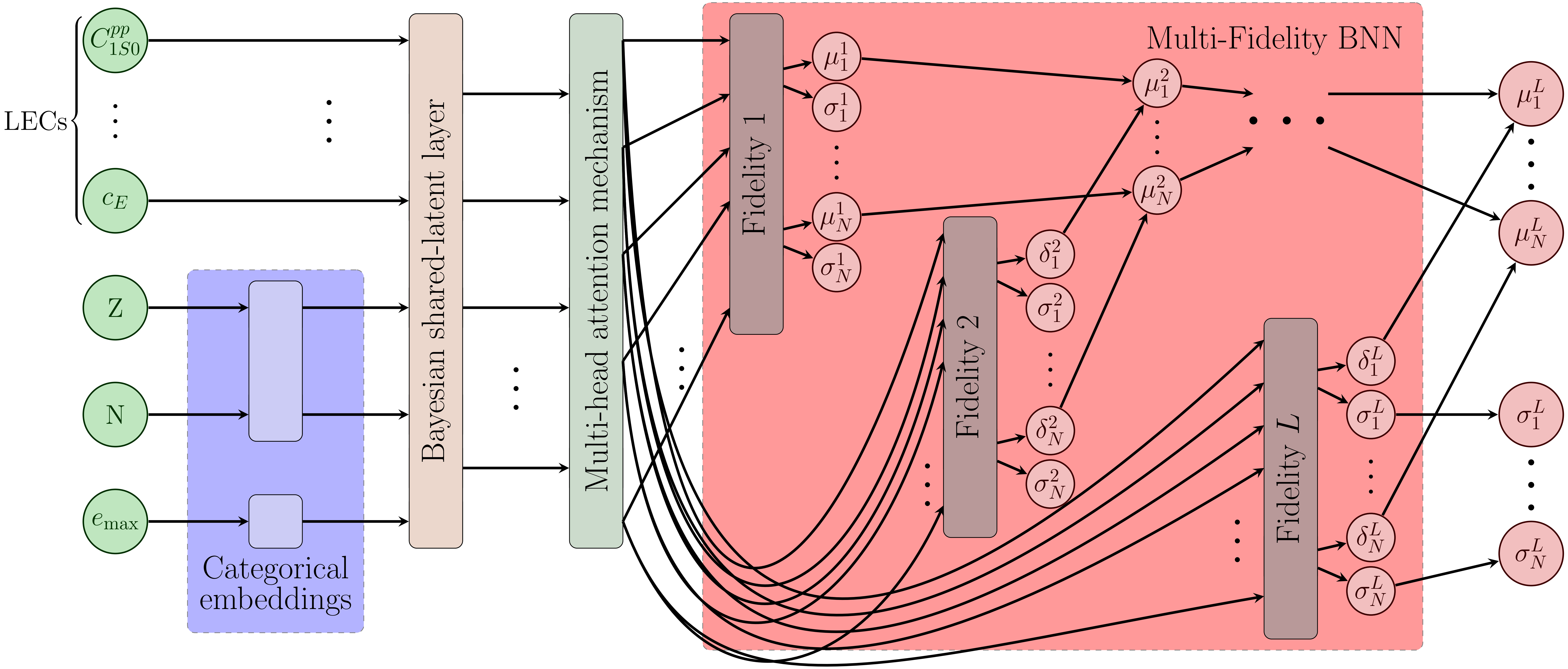
- **They do not/cannot make use of the fact that results from different interactions correlate strongly in different nuclei $\Rightarrow$ need to retrain the emulator for each nucleus.**

- **Need a global emulator that can leverage this advantage.**

Jose Miguel Muñoz Arias

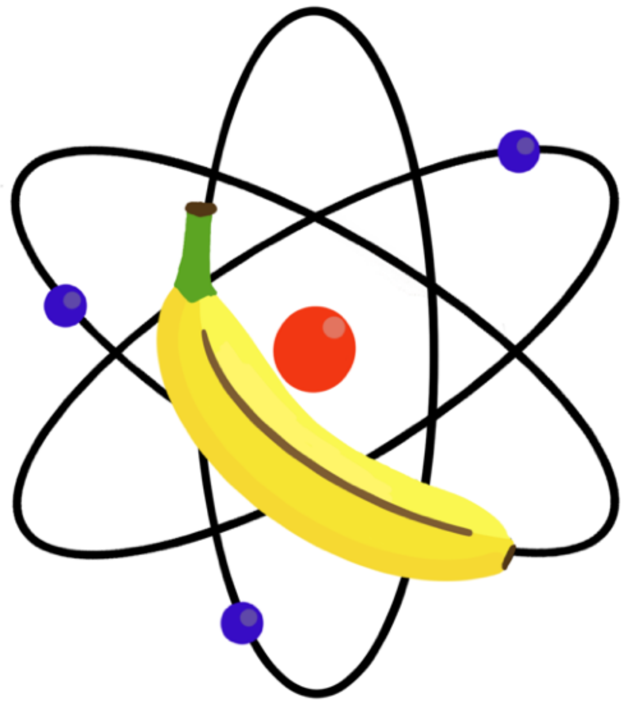
BANNANE

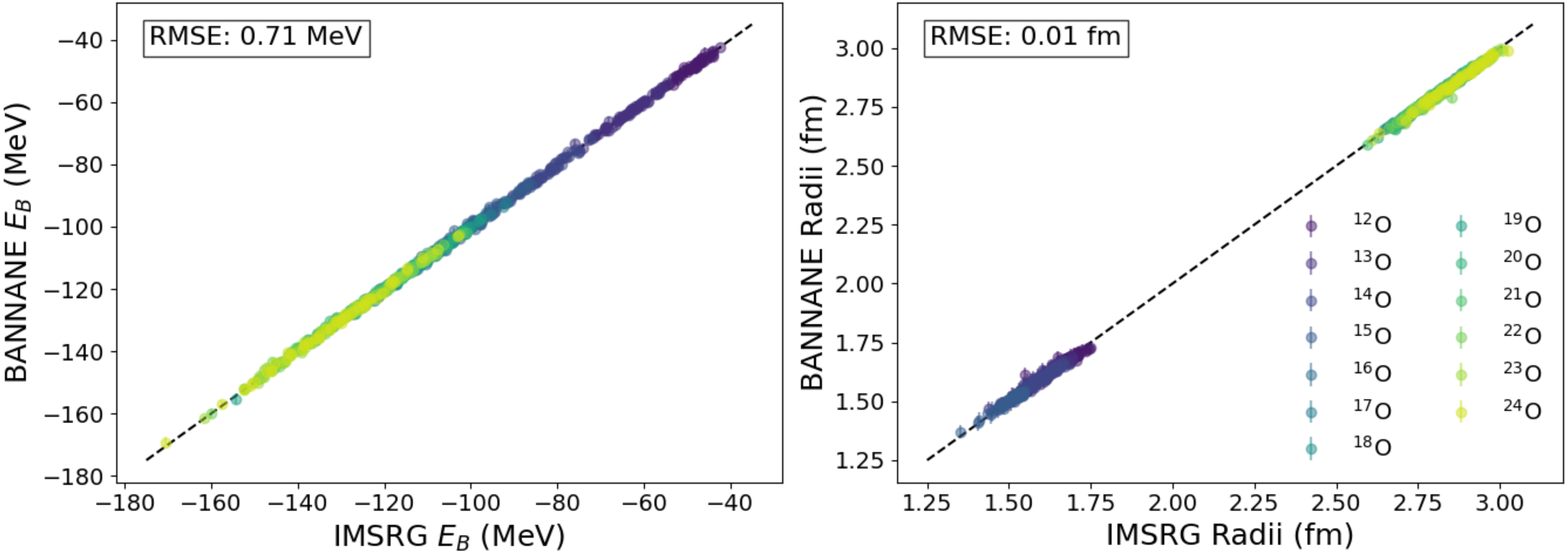
# BAyesian Neural Network: an Atomic and Nuclear Emulator



# Emulating Multiple Isotopes

Jose Miguel Muñoz Arias






BANNANE achieves state-of-the-art emulation, with smaller errors than both MM-DGP and EC while emulating over a full isotopic chain.

# Emulating Multiple Isotopes

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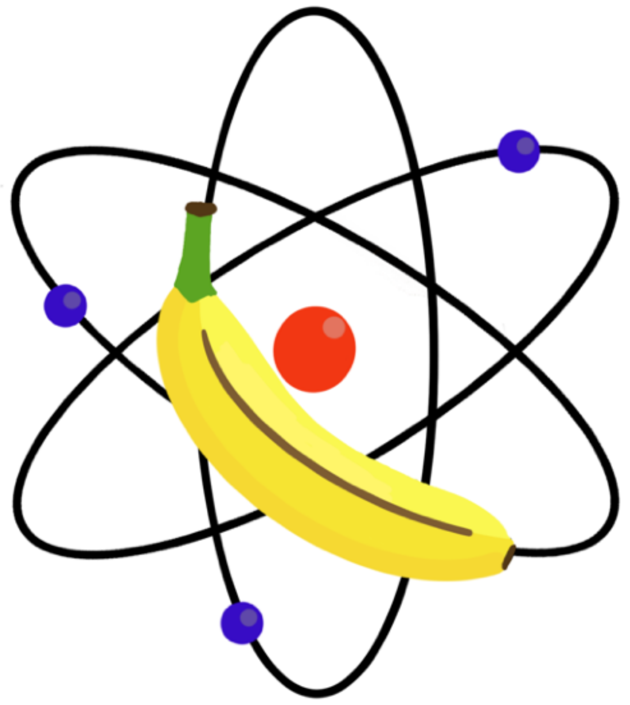
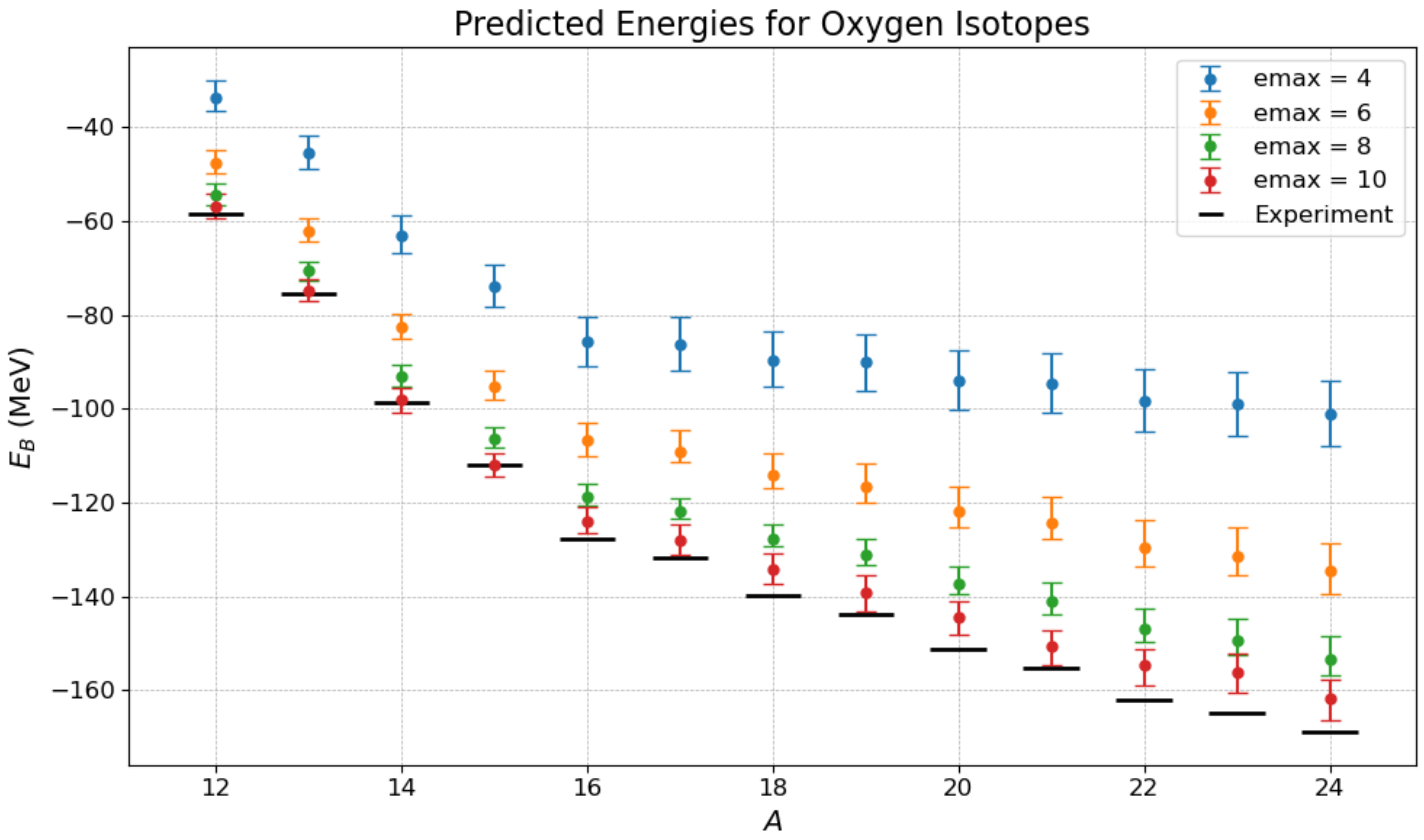
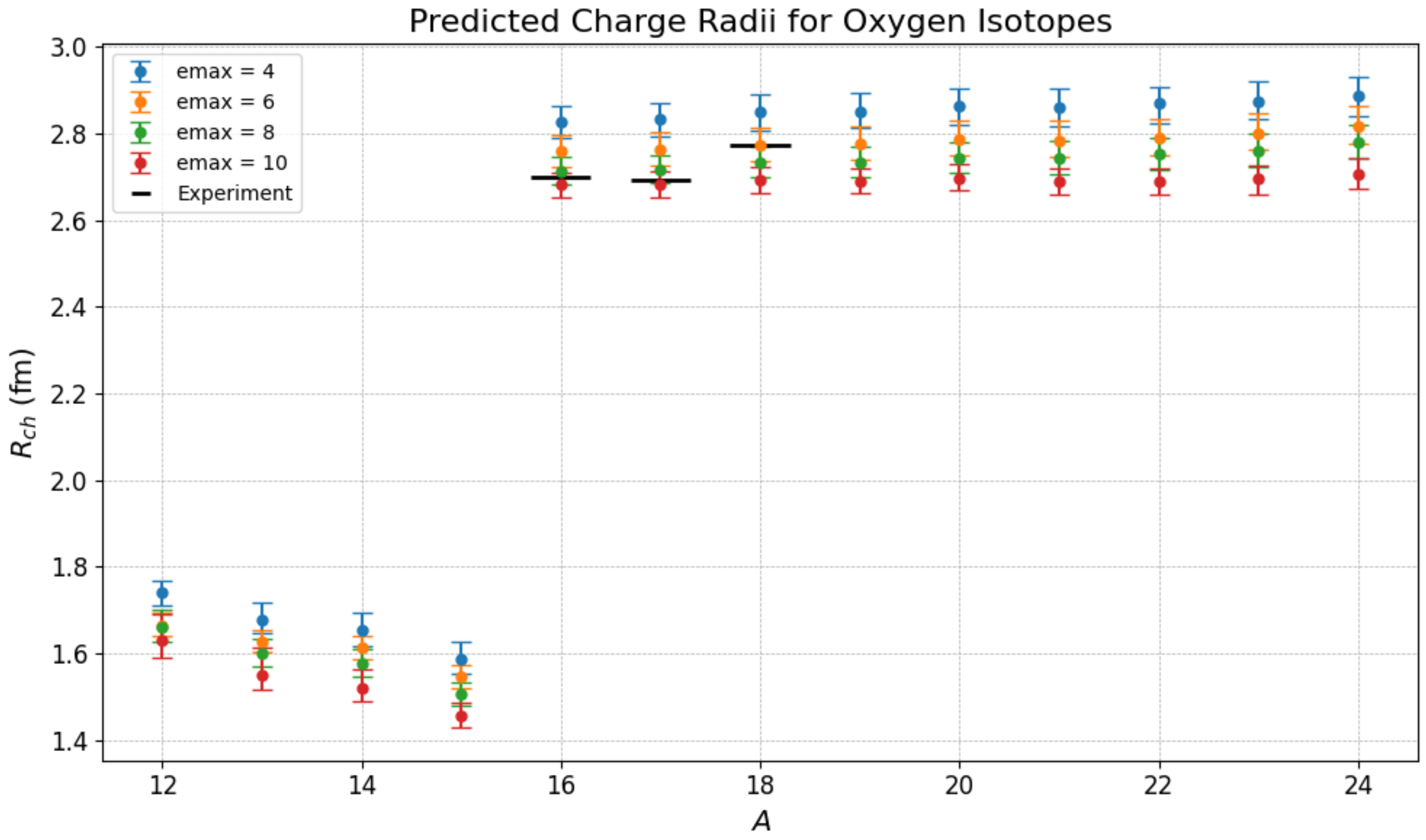


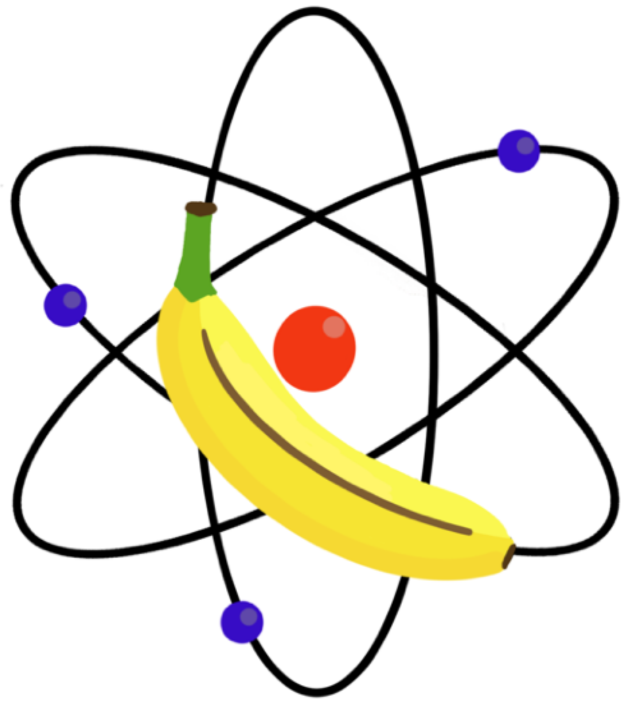


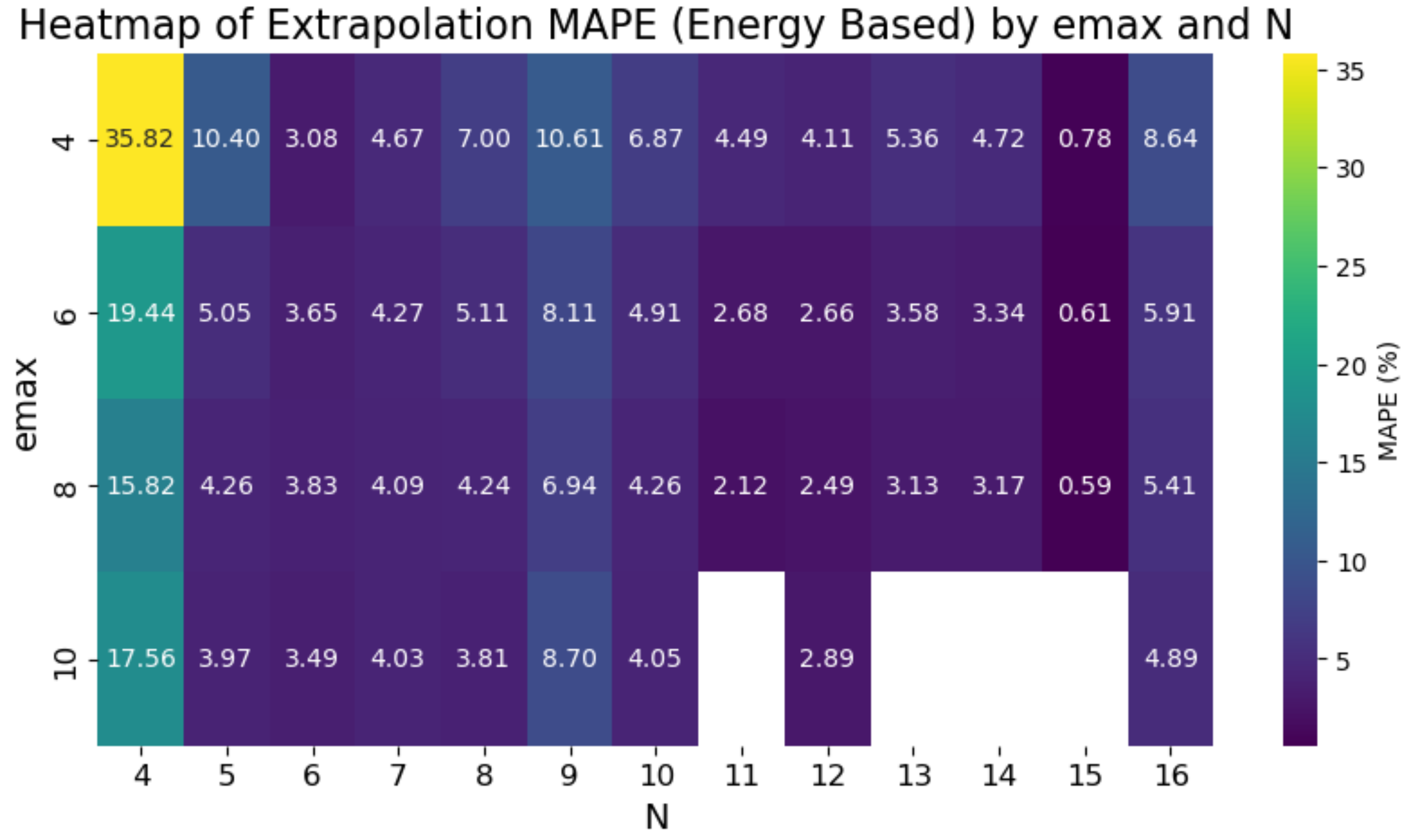



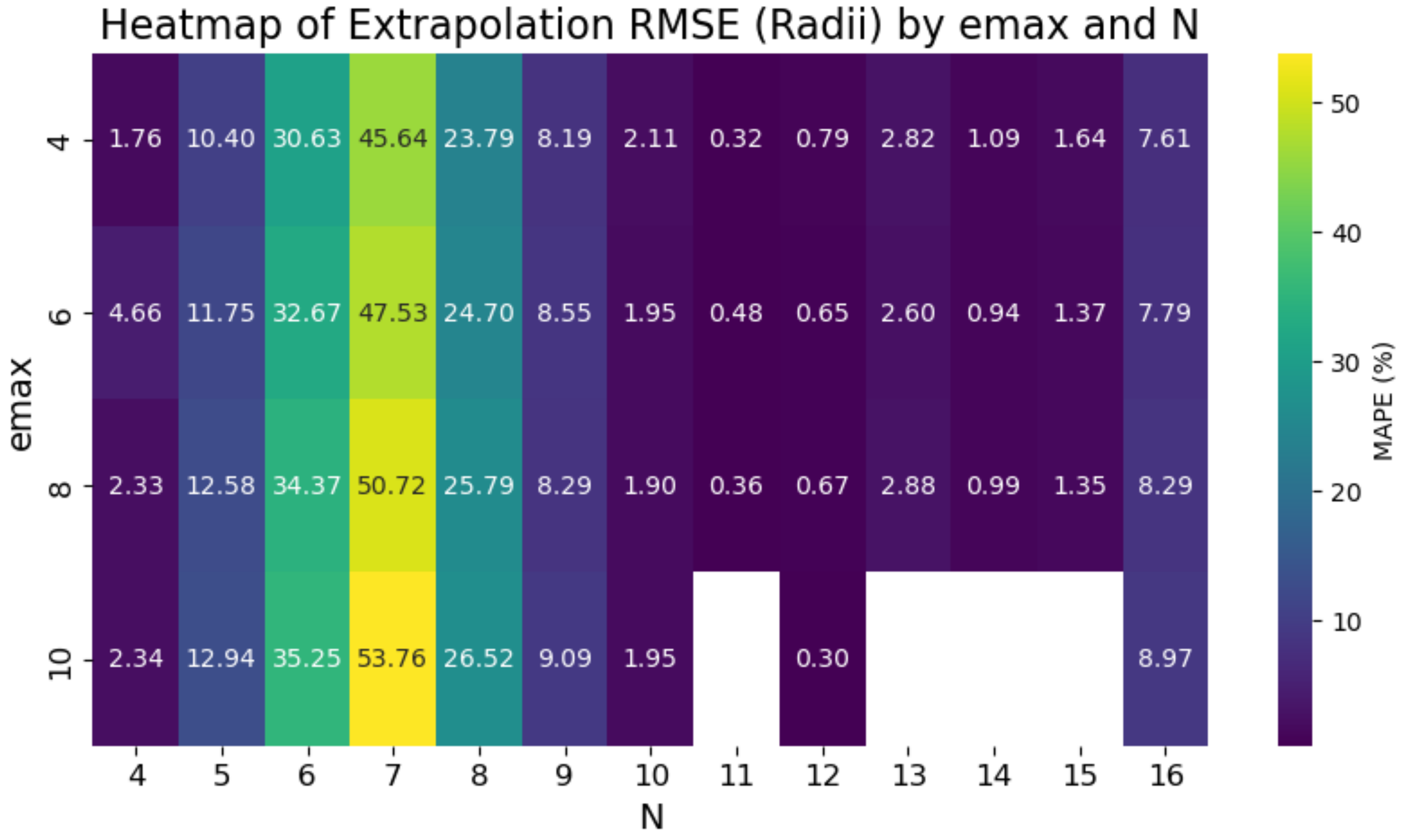

Combining this with previous UQ technique, we can predict observables with associated uncertainties over the full isotopic chains in a few minutes.

# Predicting Unseen Data

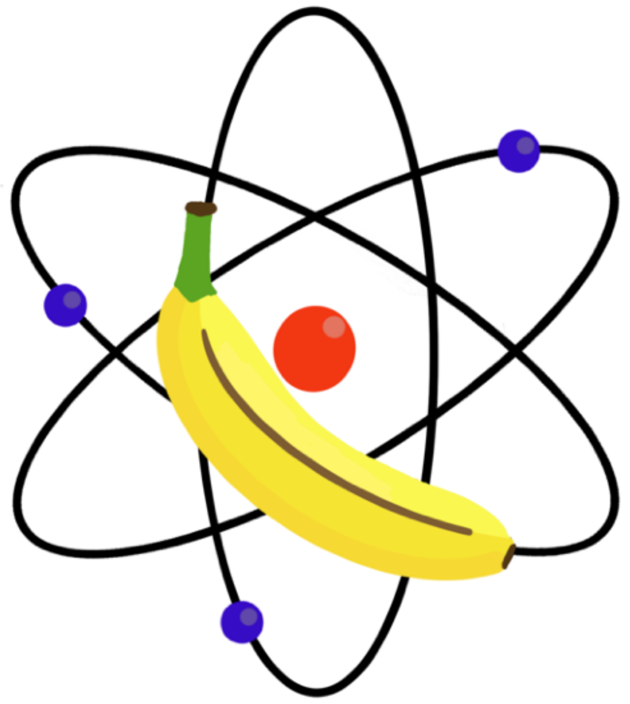
Jose Miguel Muñoz Arias





Jose Miguel Muñoz Arias



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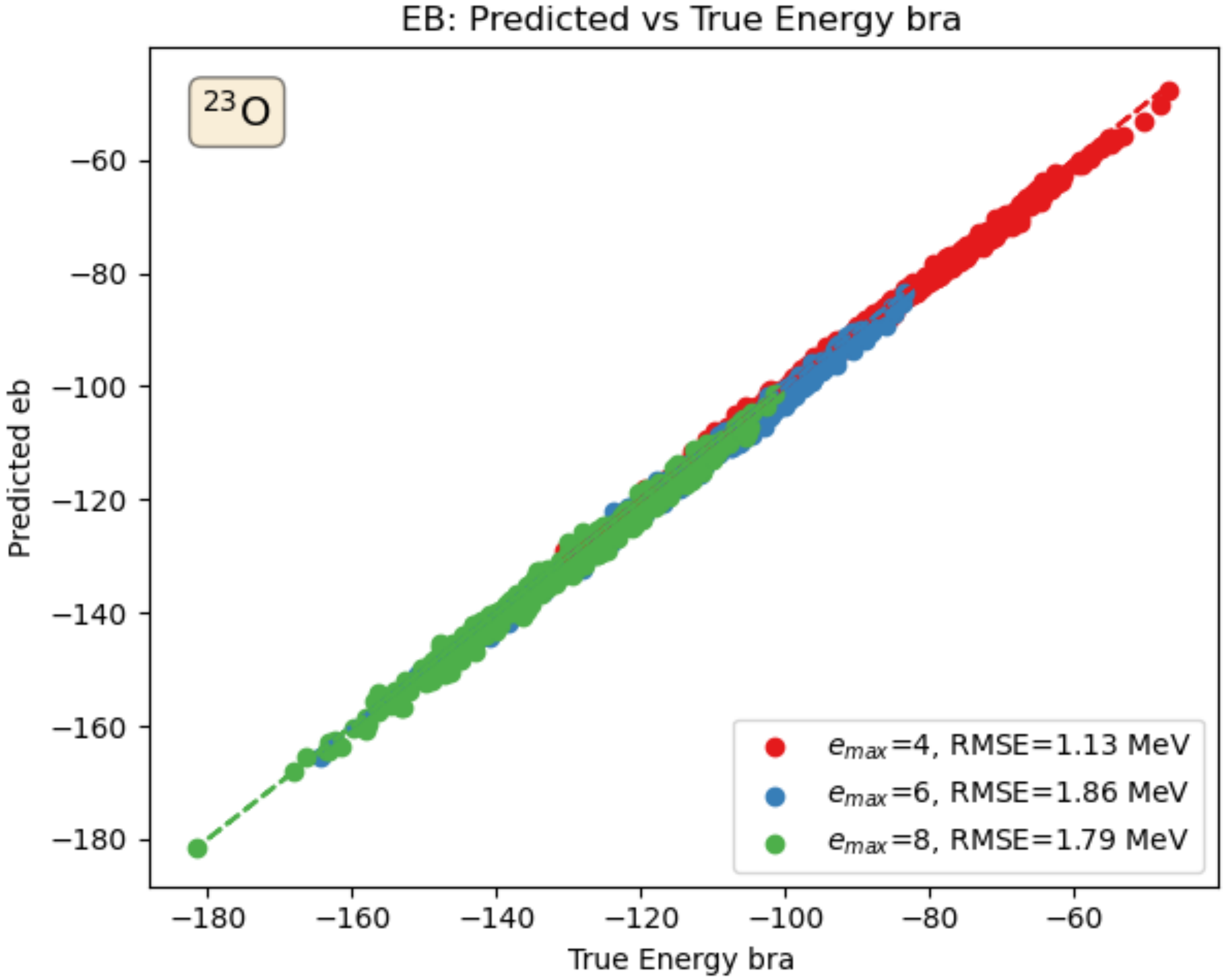
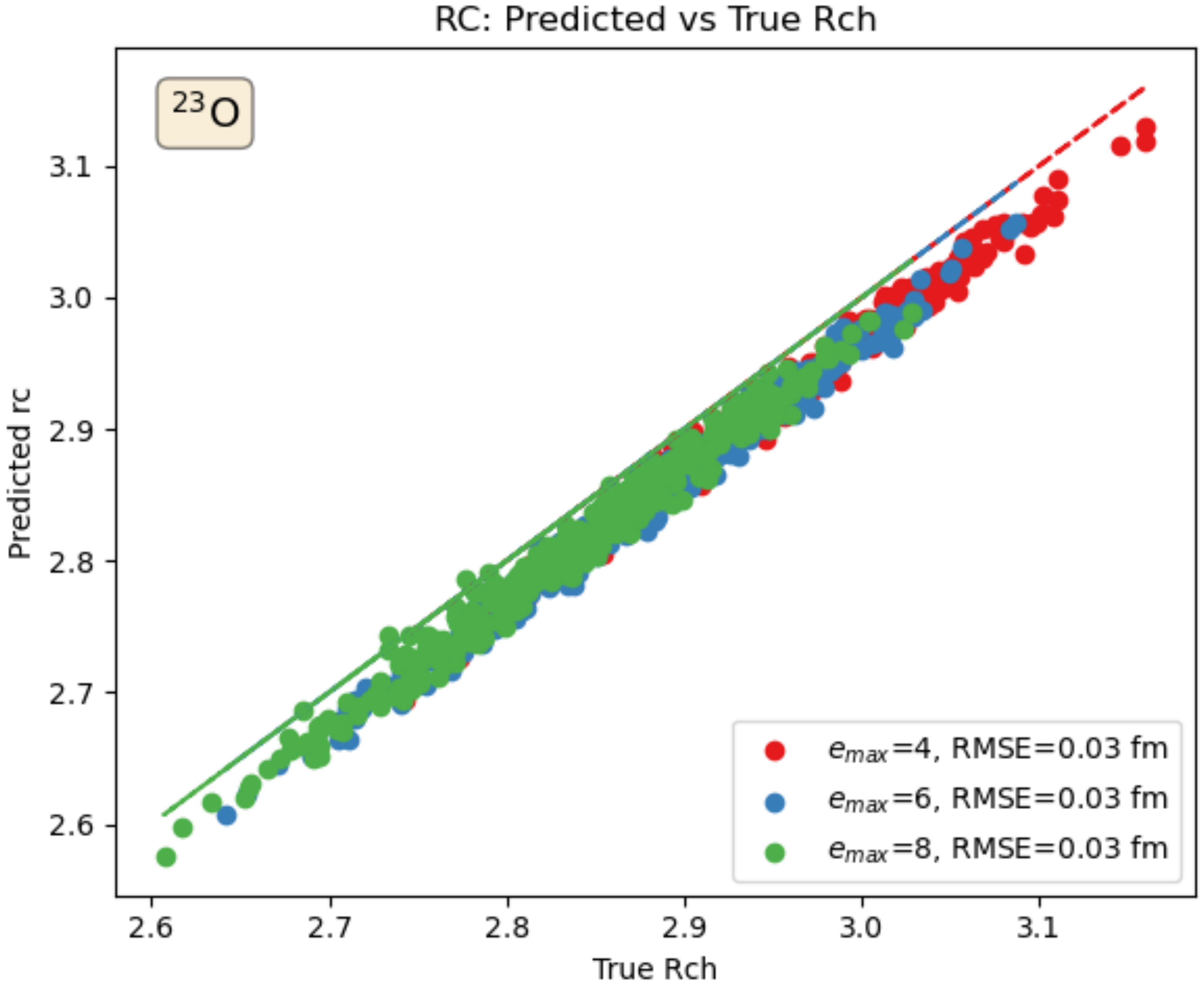
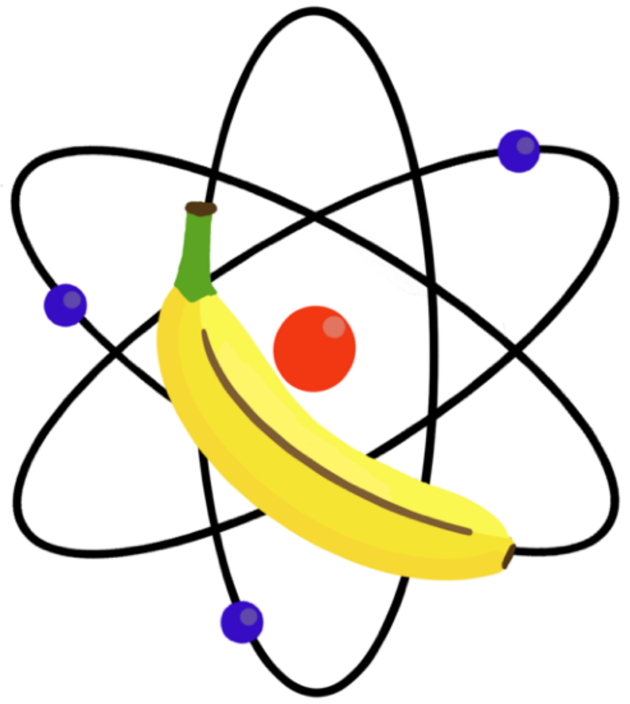


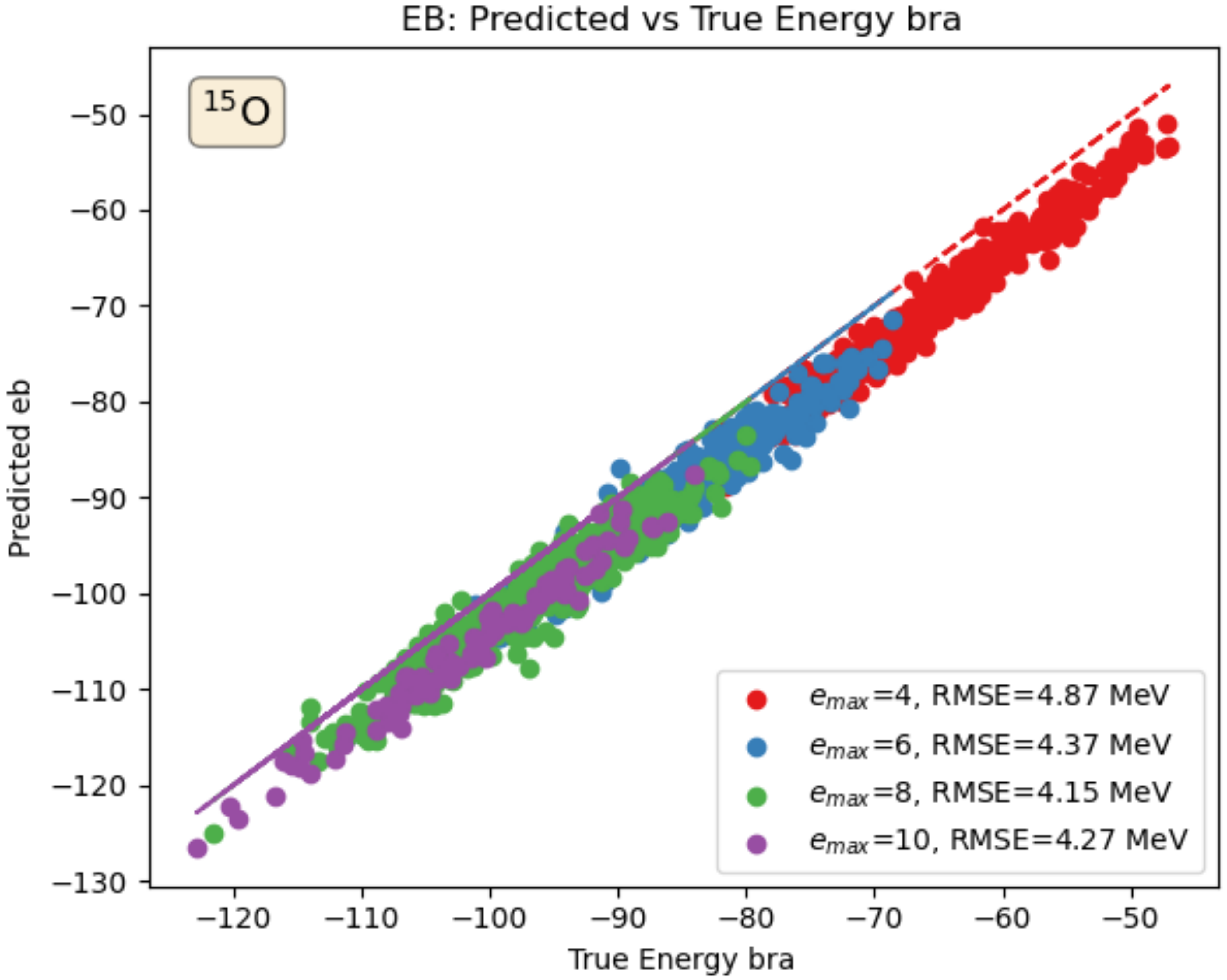



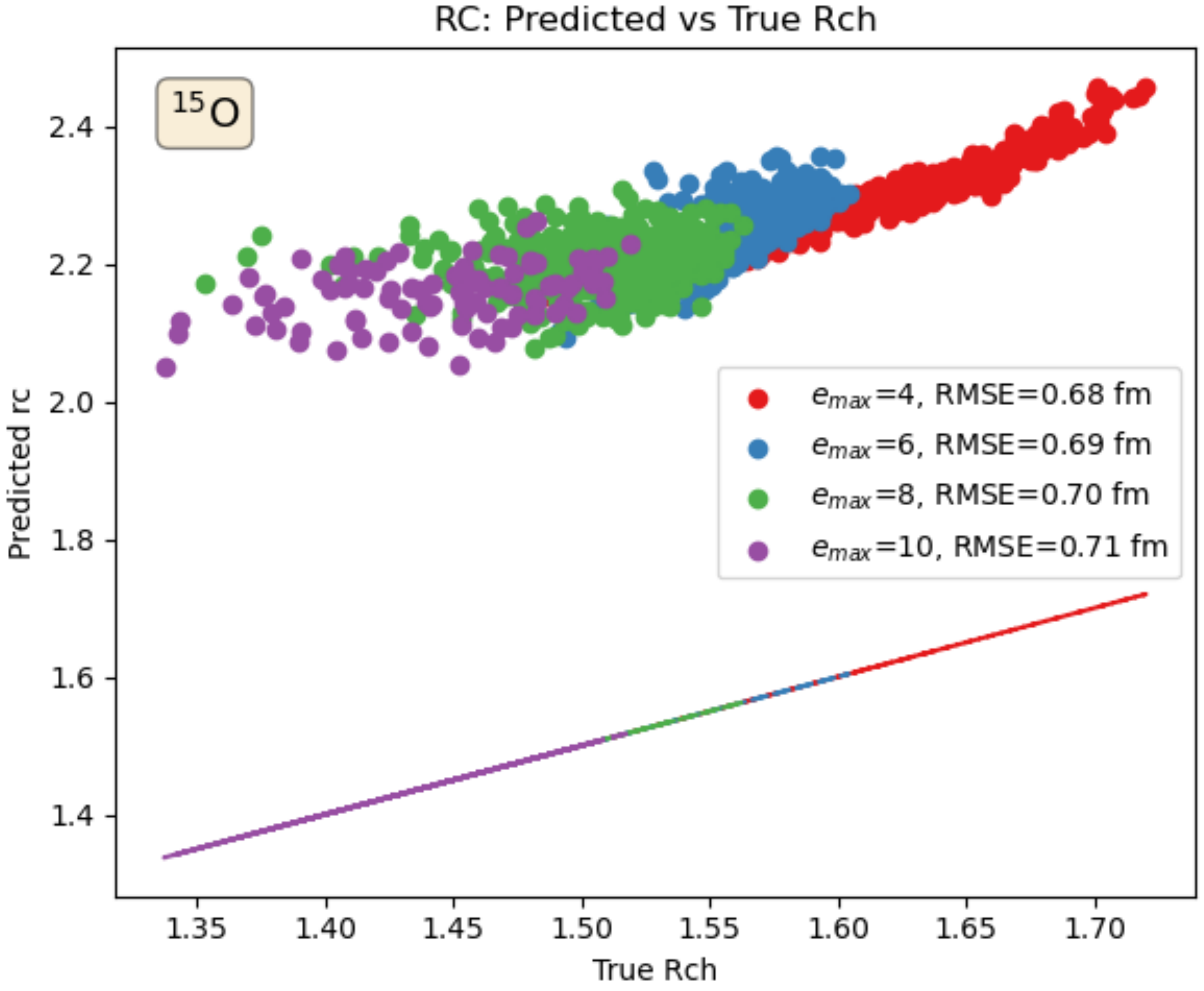

Removing $^{23}O$ from the training data and prediction observables still show amazing agreement!

# Predicting Unseen Data

Jose Miguel Muñoz Arias






Removing $^{15}$O from the training data, the model struggle to find the nuclear shell…

# Predicting From Low Fidelity

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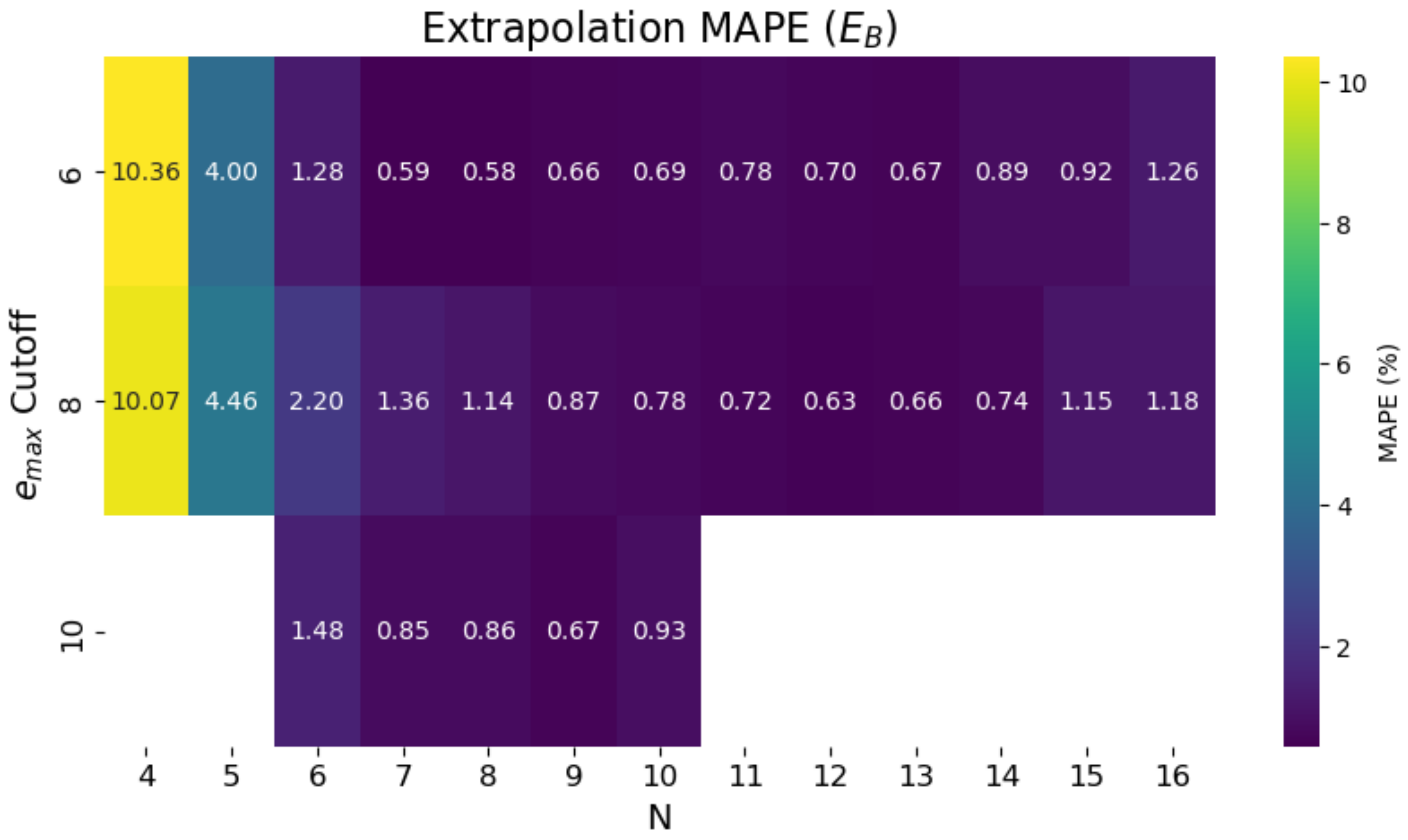


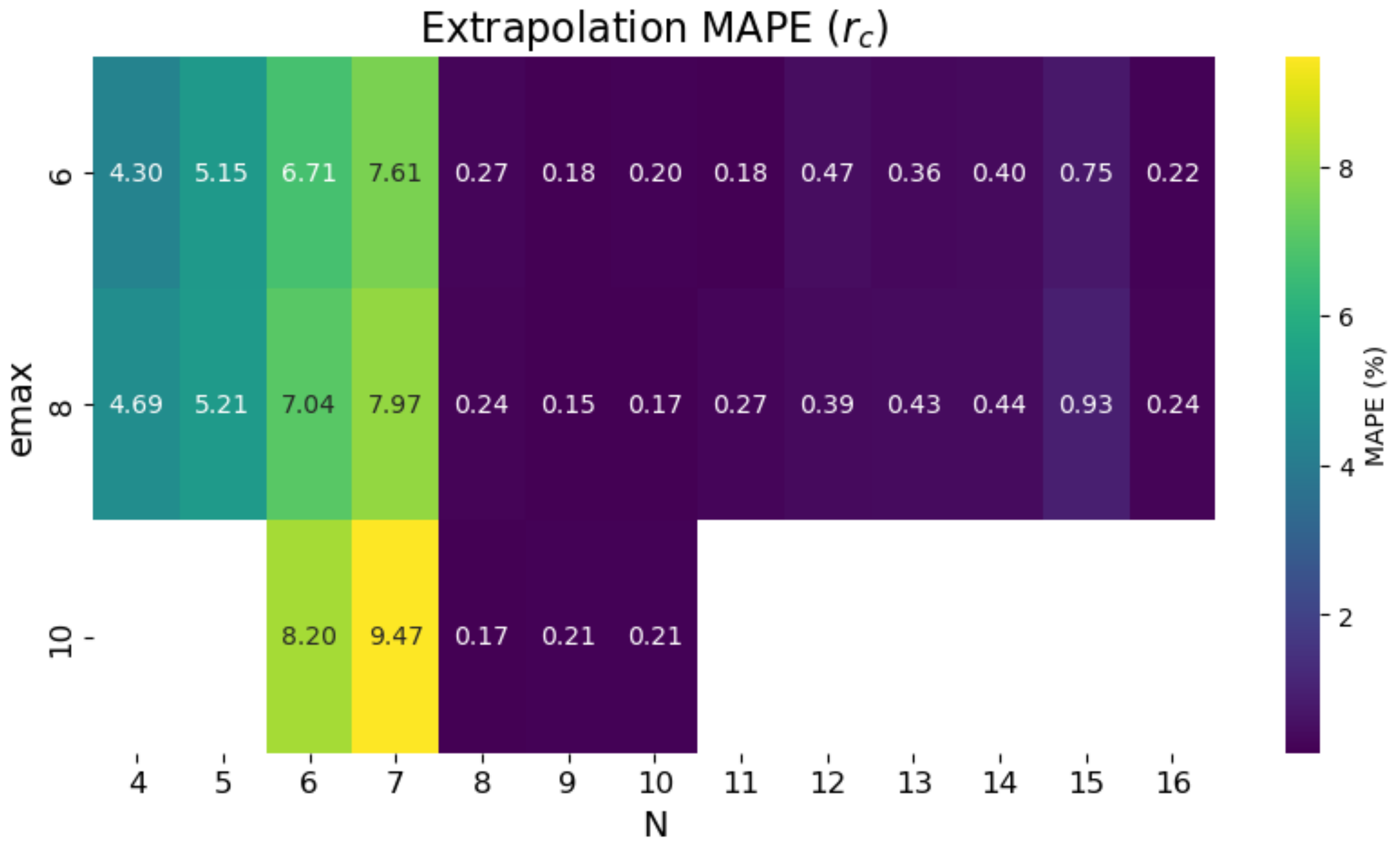


Including the lowest  fidelity greatly improve the predictions of the model!

# Emulating Multiple Isotopes

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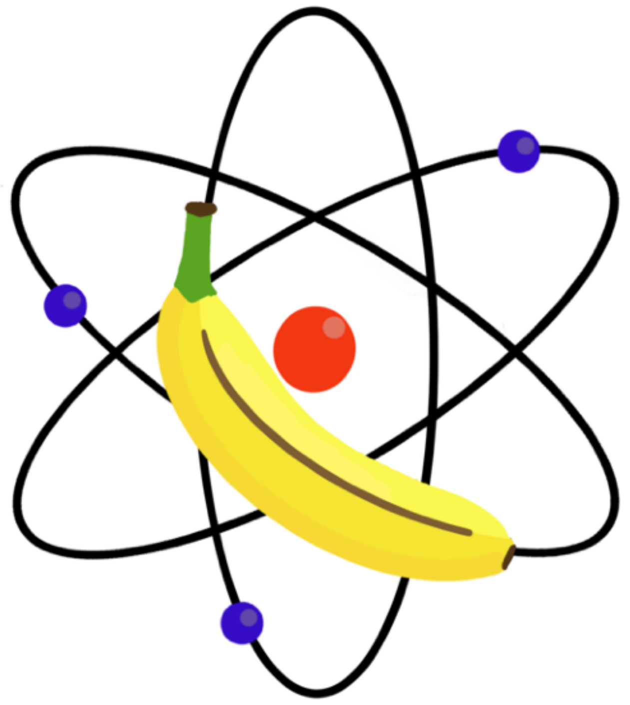

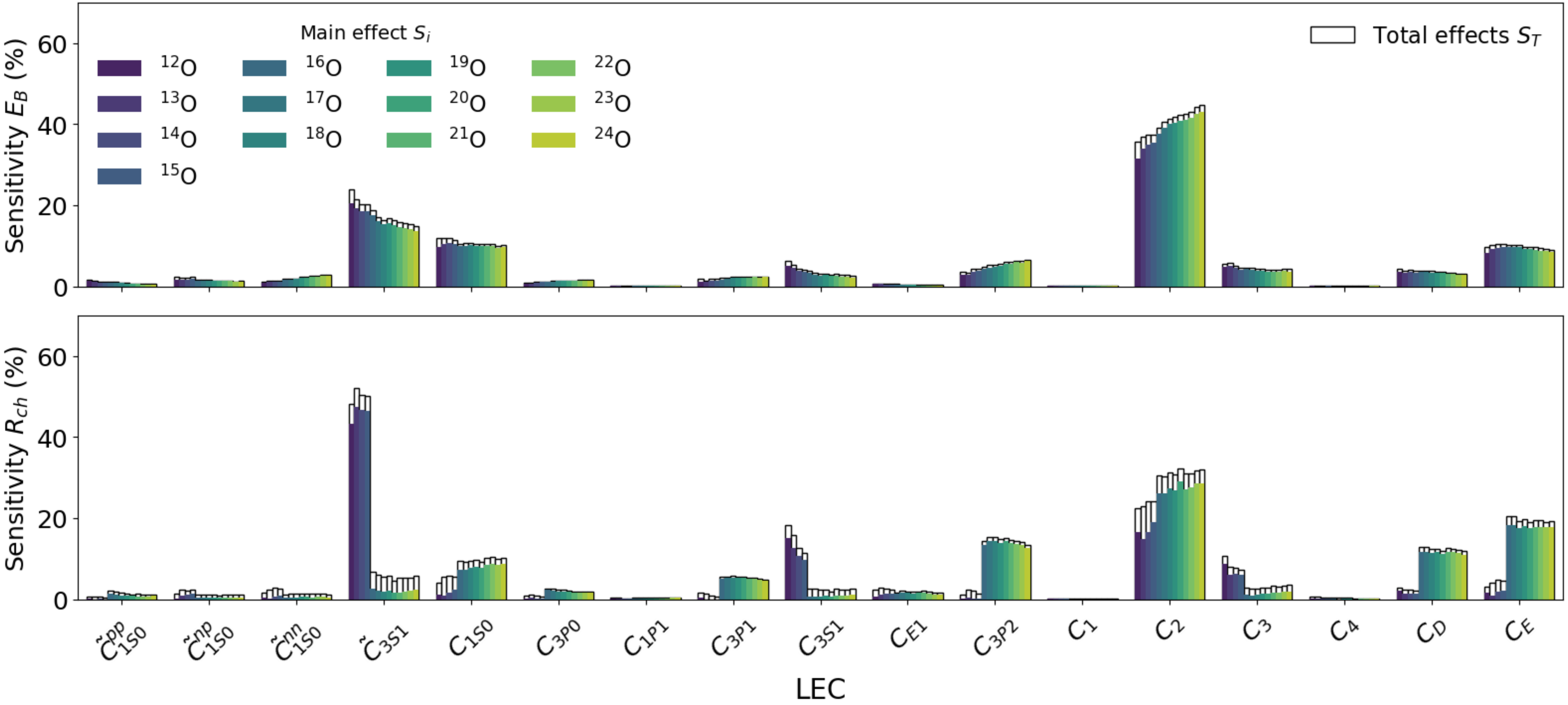


Global sensitivity analysis is consistent with other emulators!

# Visualizing the Embeddings

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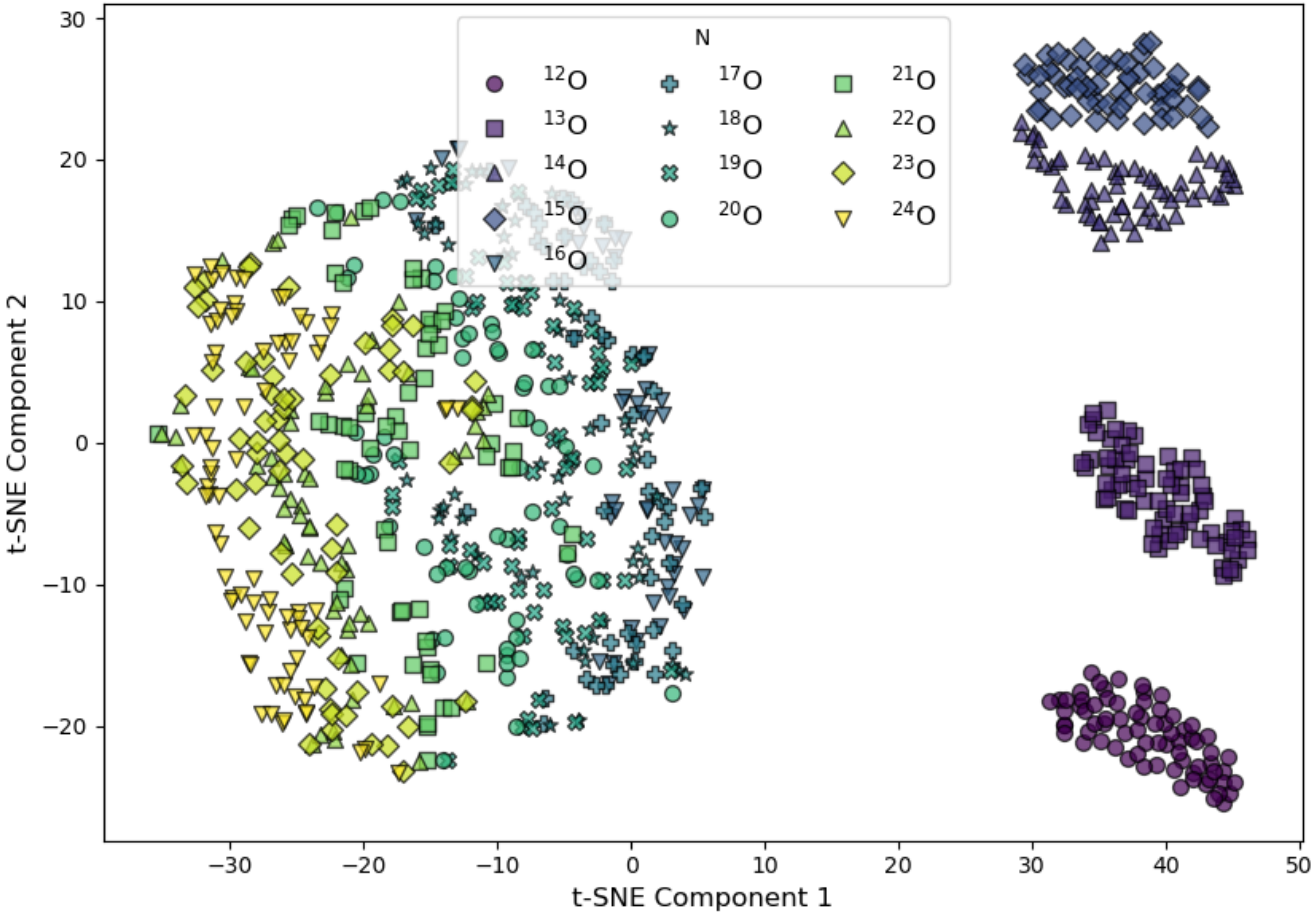
- Projection of the embeddings from the attention mechanism.



# Visualizing the Embeddings

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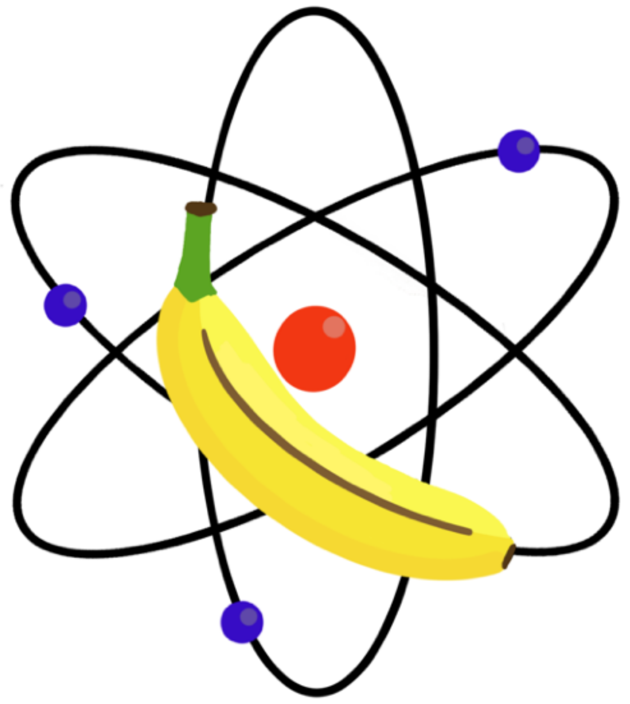
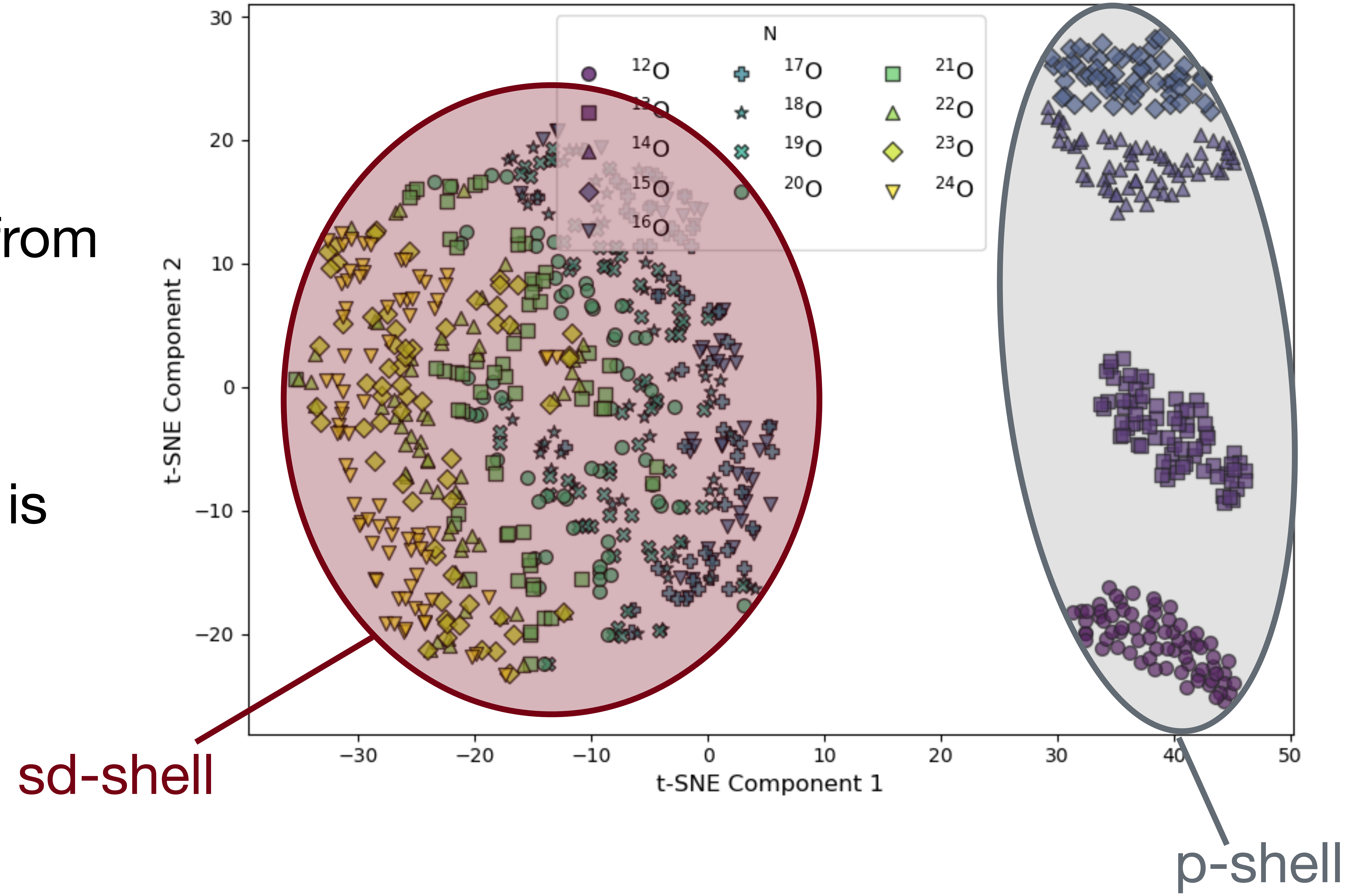


- Projection of the embeddings from the attention mechanism.

- Can clearly see that the model is learning nuclear shells!



# Summary …

- Emulators are required to obtain uncertainty quantification of nuclear theory observables required for searches of new physics.
- Emulator further allows the use of other statistical tools like global sensitivity analysis.
- Many-body uncertainty is the main source of uncertainty in current calculations.

# … and Outlook

- Improving the emulator with other machine learning models.
- Reducing  the many-body error using methods that probe the IMSRG(3).
- Doing a similar analysis for other nuclear processes.
- Computing other observables for BSM searches with uncertainties.

# Thank you!

# Questions?

abelley@mit.edu