Part 3: Theory Meets Data:

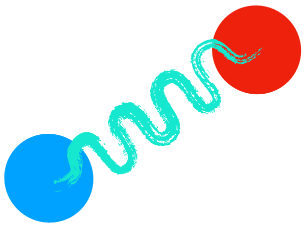
Manifestations of Nuclear Forces in Nuclear Structure

1. Nuclear matter

2. Shell structure

3. β decays

4. Sensitivity studies

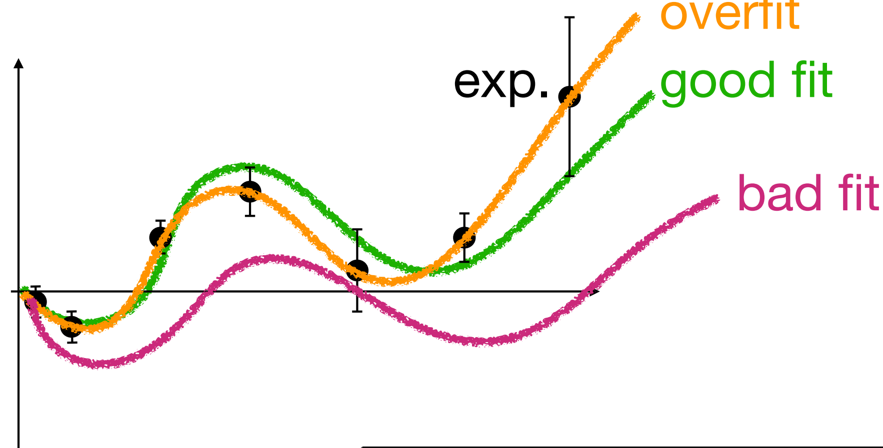


Fitting LECs

The “old way”:

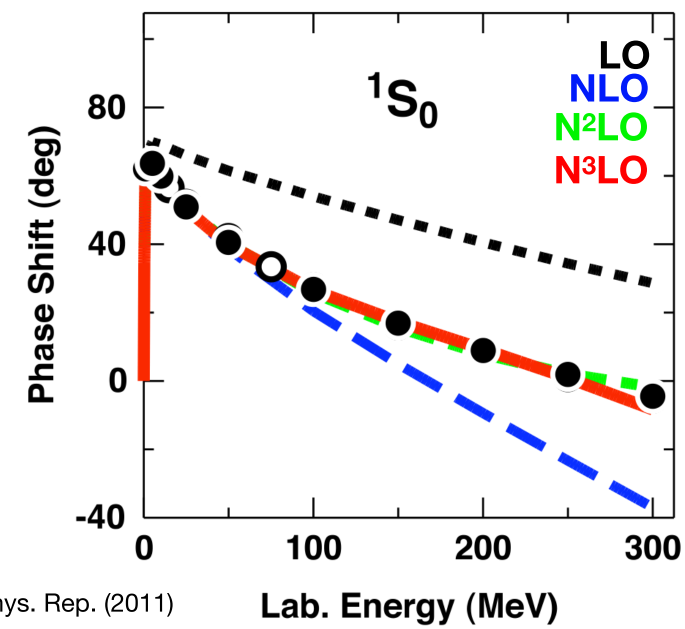
$$\chi^2 = \sum_{i=1}^{\nu} \frac{(x_i^{\rm th} - x_i^{\rm exp})^2}{\sigma_i^2}$$

A good fit has $\chi^2/\nu \approx 1$.



This assumes errors $x_i^{\rm th} - x_i^{\rm exp}$ are uncorrelated random numbers drawn from a Gaussian of variance σ_i^2.

Difficulty: usually, one takes σ_i^2 only from experiment (e.g. statistical uncertainty). **But the theory has error too!** Moreover, often theory errors are correlated.



Entem&Machleidt Phys. Rep. (2011)

Fitting LECs

The “old” philosophy (for “realistic” forces):

2N terms must be fit to 2N data,

3N terms must be fit to 3N data.

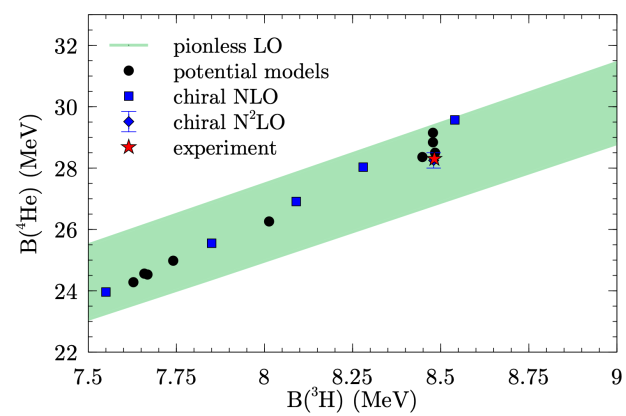
Fitting to heavier nuclei is “dirty phenomenology”.

A more modern point of view:

You can fit to whatever you like, so long as you can properly assess the full error with correlations.

$A = 3,4$ data are highly correlated, enhancing the possibility of an overfit.

Fitting to $A \gg 3$ is fine, so long as the disagreement with $A \sim 3$ is of the size of higher order corrections.



Fierz Rearrangement and Regulators

At leading order, there are 4 independent NN operator structures:

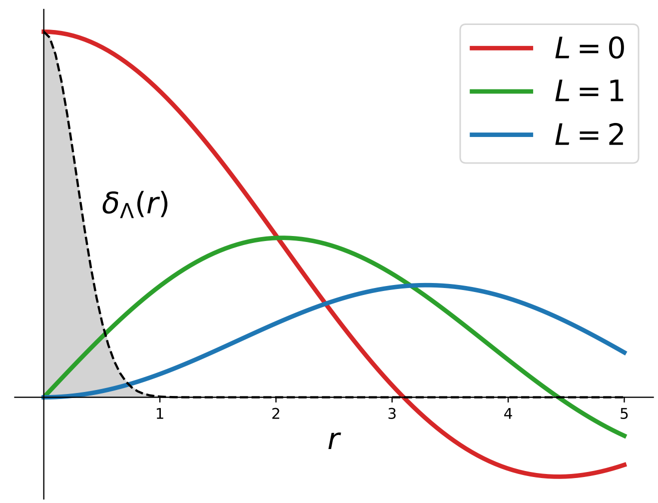
$$1, \quad \sigma_1 \cdot \sigma_2, \quad \tau_1 \cdot \tau_2, \quad (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)$$

But valid NN wave functions must be antisymmetrized under exchange ⇒

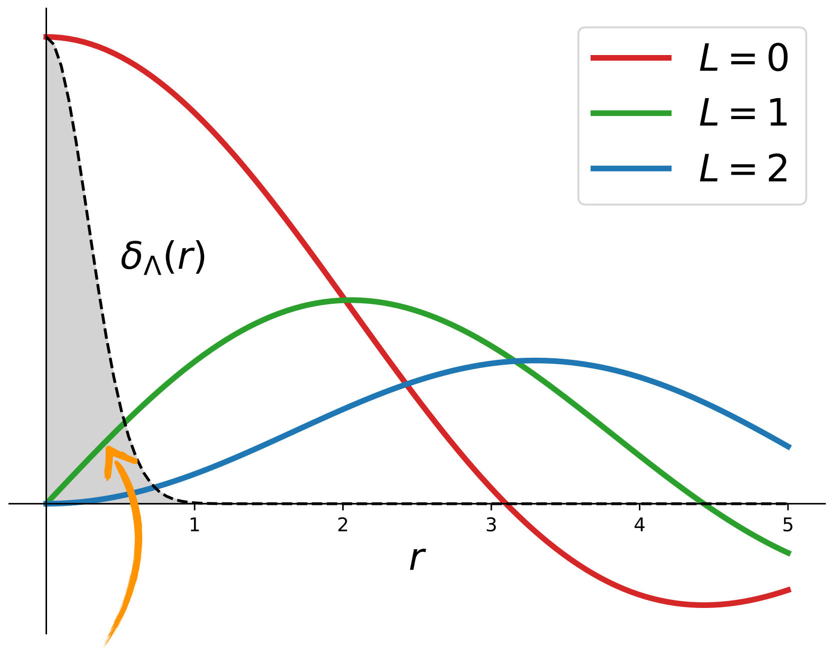
$$(-1)^{L+S+T} = -1$$

Contact term $\delta(\vec{r})$ only acts in $L = 0$.
So $(S, T) = (0,1)$ or $(1,0)$

If we know S, we know T, so $\sigma_1 \cdot \sigma_2$ and $\tau_1 \cdot \tau_2$ are redundant. Really, we only 2 operators, e.g. $1, \ \sigma_1 \cdot \sigma_2$



Fierz Rearrangement and Regulators

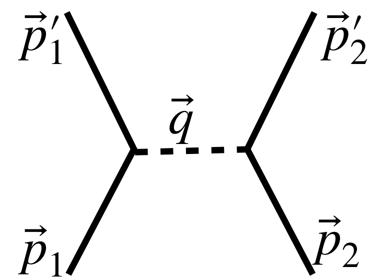


"Regular artifact"

$\langle kL \,|\, \delta_\Lambda \,|\, kL \rangle \sim (k/\Lambda)^{2L}$

Local regulator: $f_{\text{loc}}(q) = e^{-(q/\Lambda)^2}$

Non-local regulator: $f_{\text{nl}}(p, p') = e^{-(p'/\Lambda)^2} e^{-(p/\Lambda)^2}$



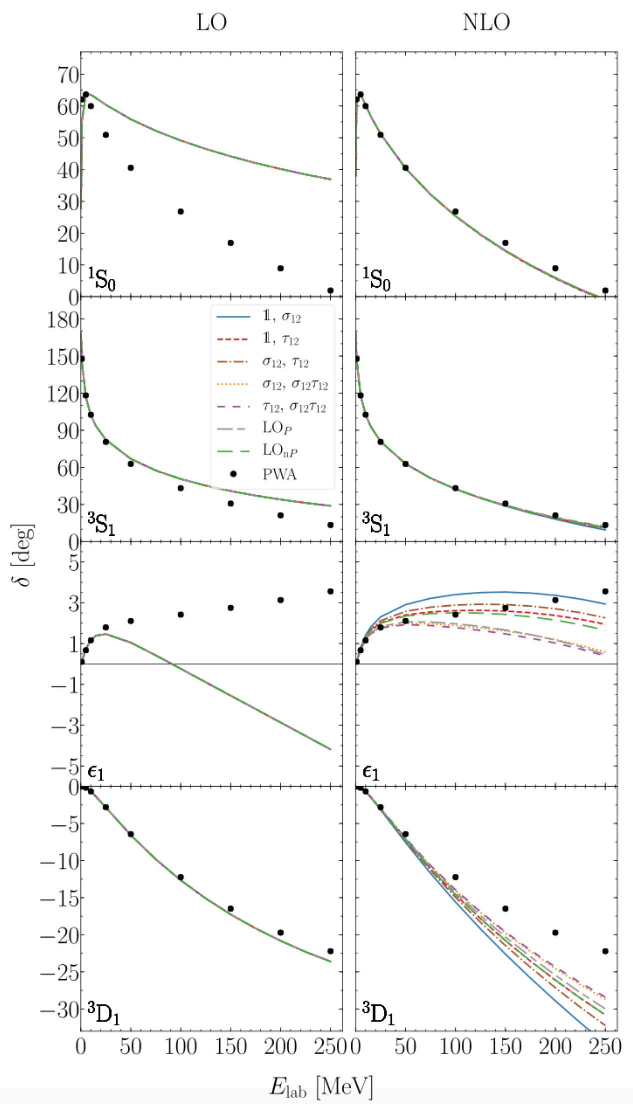
$$\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$$

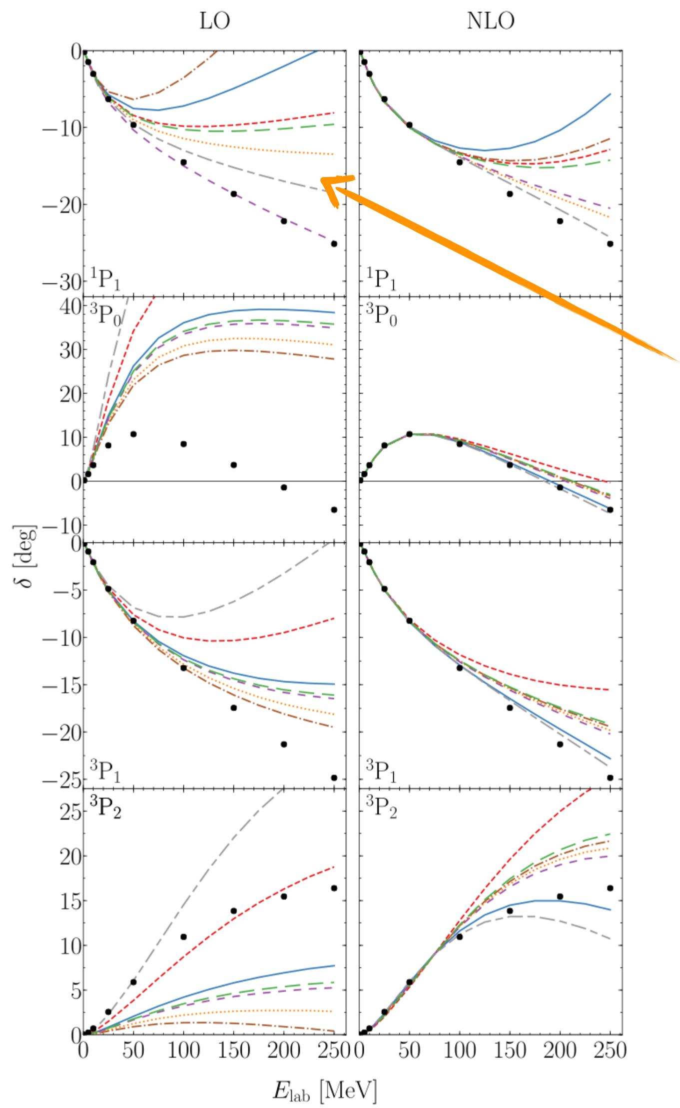
$$\vec{p}' = \frac{1}{2}(\vec{p}_1' - \vec{p}_2')$$

$$\vec{q} = \vec{p}' - \vec{p}$$

Swap $\vec{p}_1 \leftrightarrow \vec{p}_2$ should leave regulator unchanged.

$\vec{p} \to -\vec{p}, \;\; \vec{q} \to \vec{p}' + \vec{p}$. $\quad f_{\text{nl}}(p, p')$ unchanged, but $f_{\text{loc}}(q)$ changes.

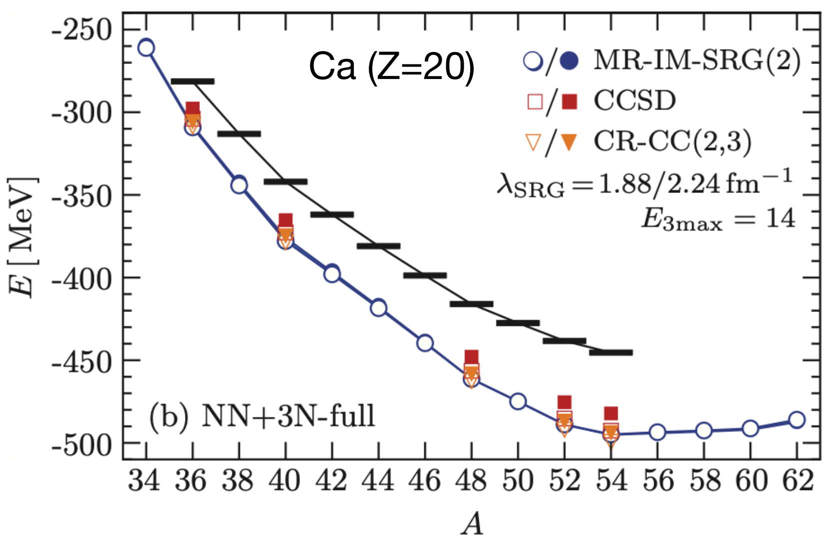


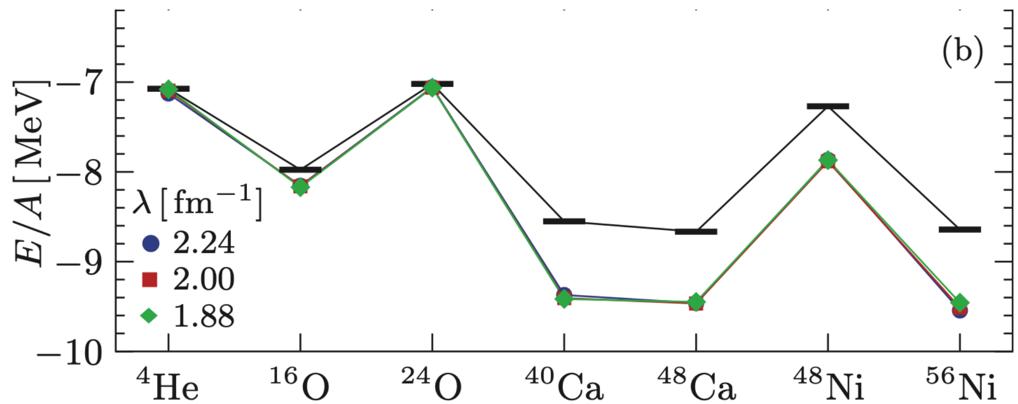


At leading order, P-waves should be unaffected by the contact term, but a local regulator leads to a significant effect.

Huth+ PRC (20170

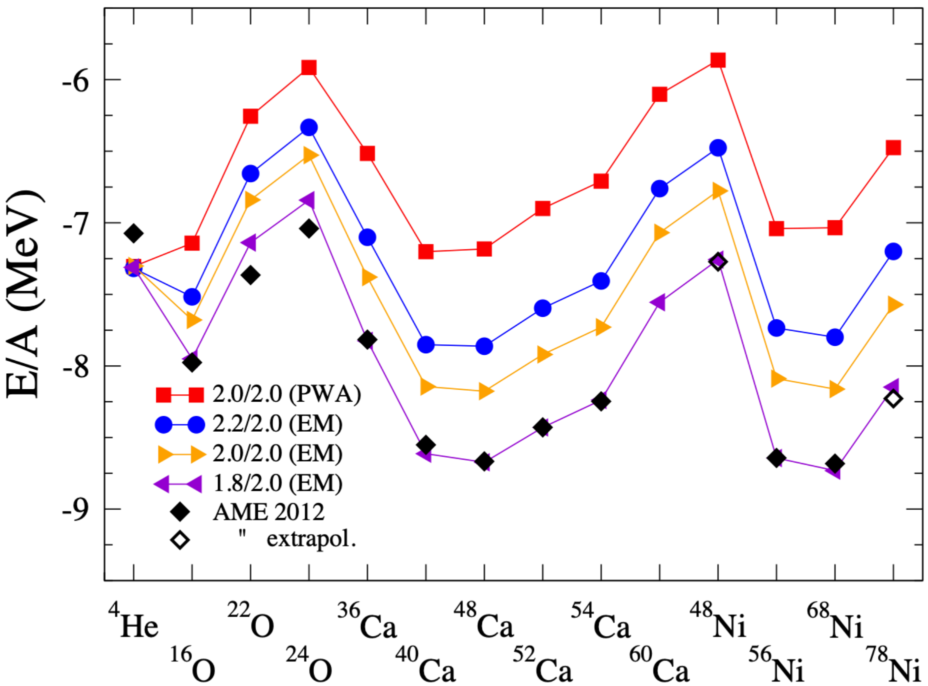
Ab inito calculations of ground state energies with chiral potentials

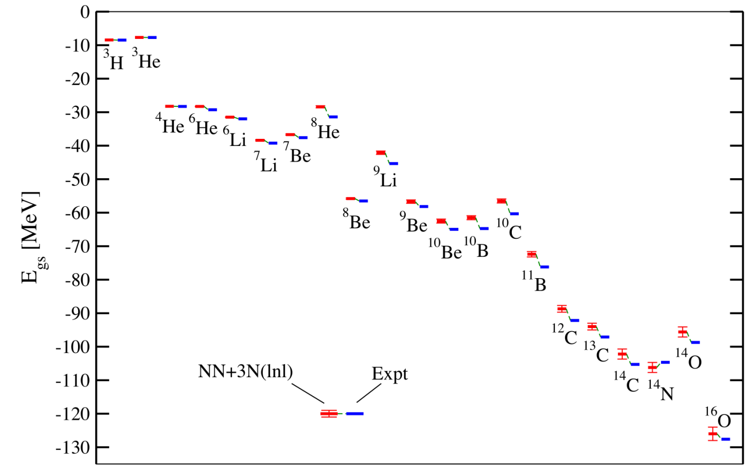


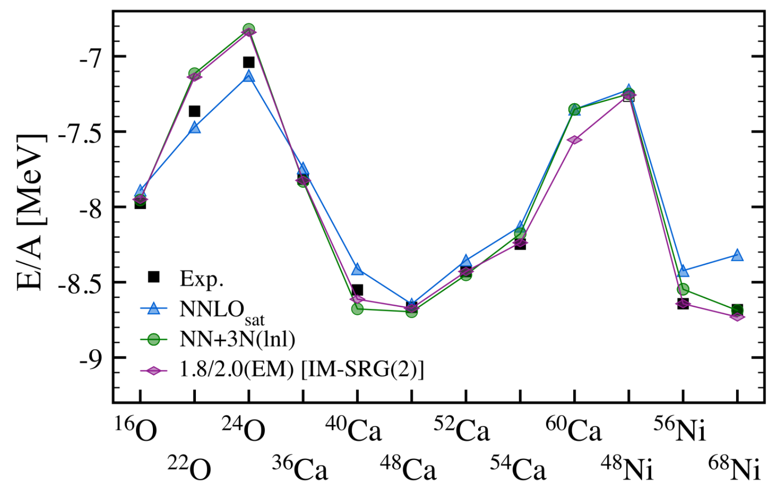


Hergert+ PRC (2013)
Hergert+ PRC (2014)

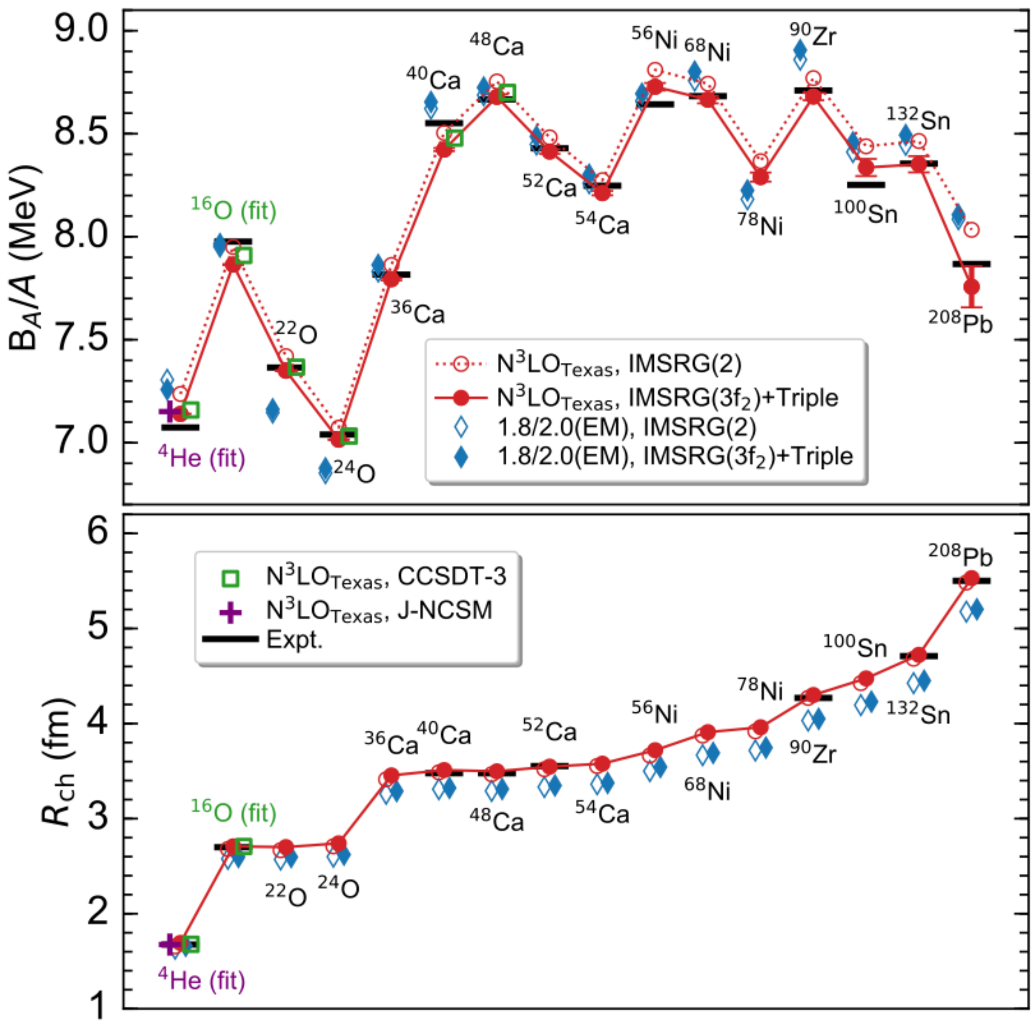
Secret sauce? Nonlocal regulator for 3N force

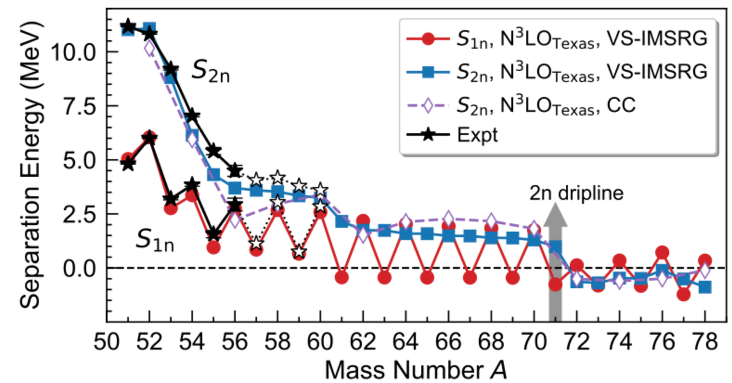






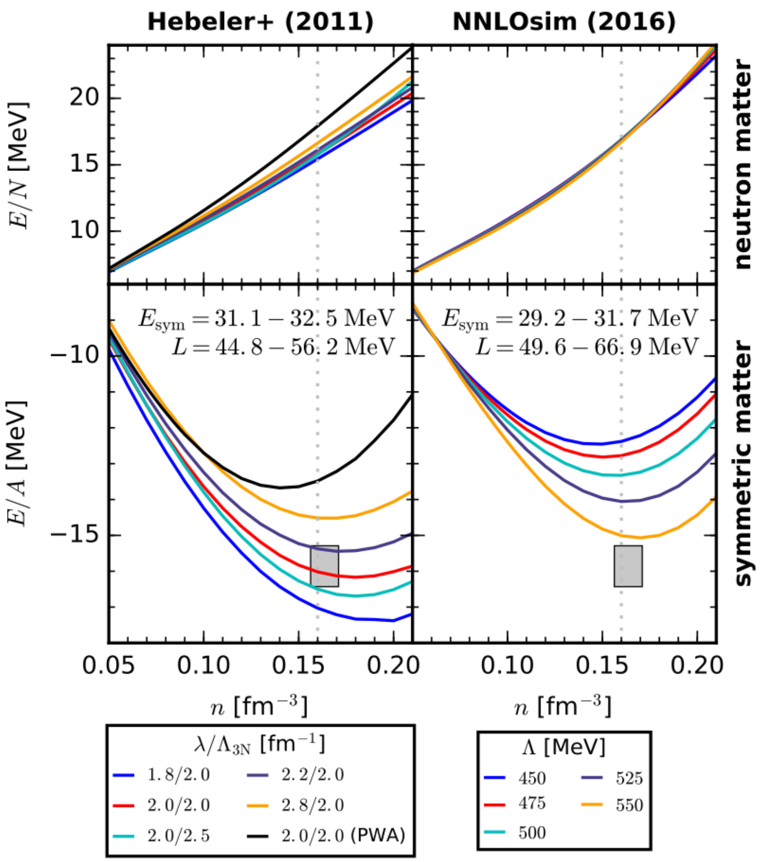
Simonis, SRS+ PRC (2017)
Ekström+ PRC (2015)
Somà+ PRC (2020)

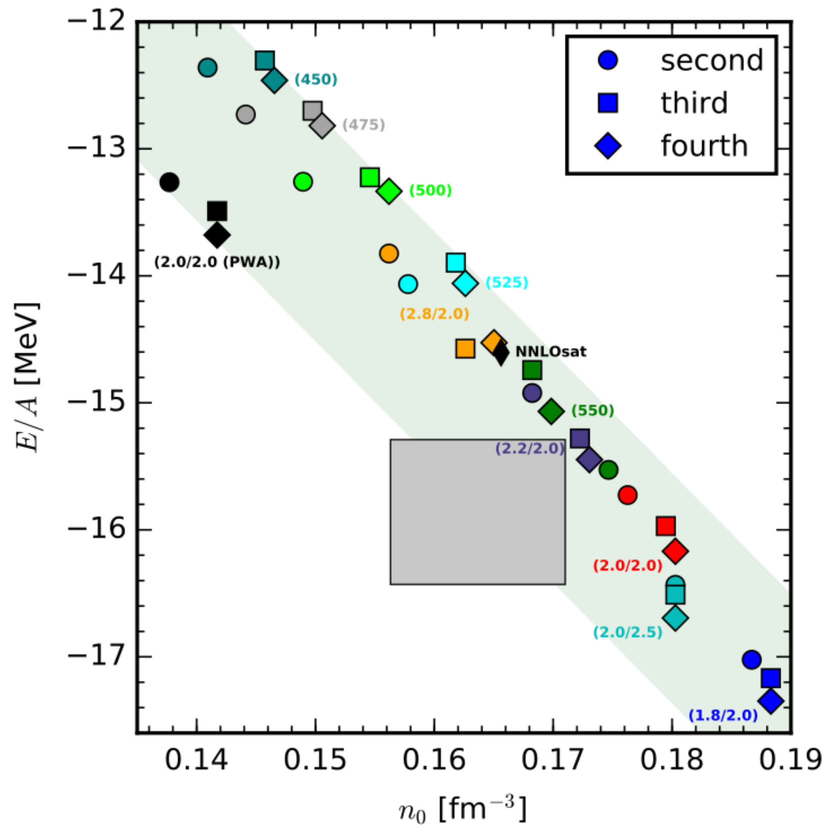




Hu+ arXiv:2512.11723 (2025)

Nuclear Matter

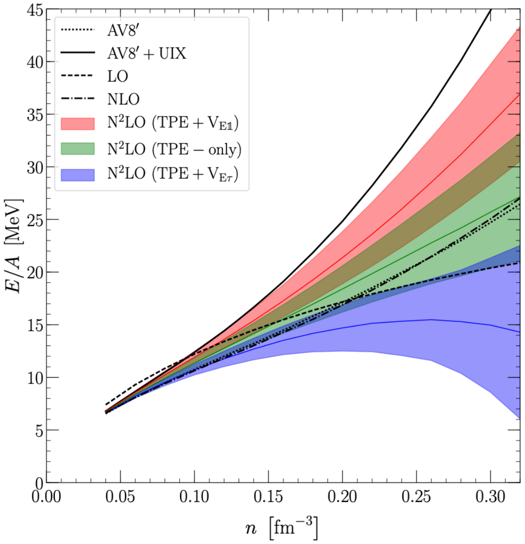




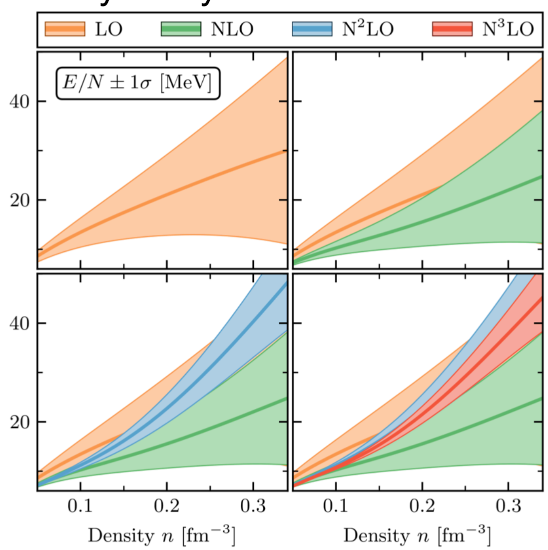
Drischler, Hebeler & Schwenk, PRL (2019)

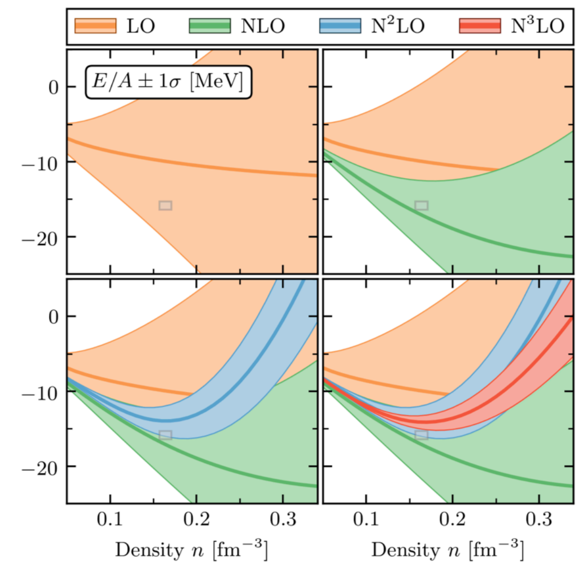
Nuclear matter with uncertainty quantification

Quantum Monte Carlo



Many-body Perturbation Theory

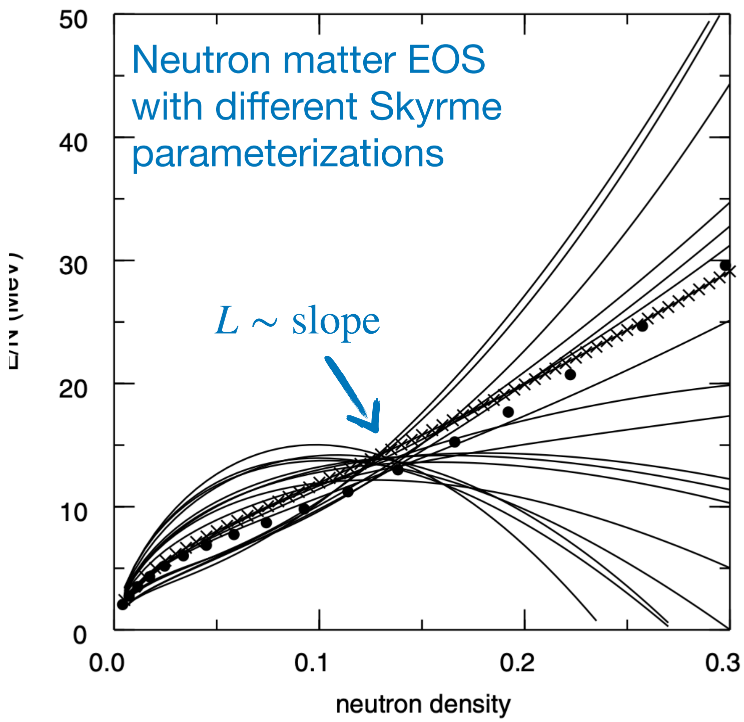


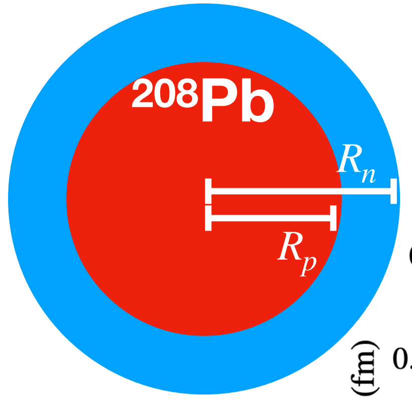


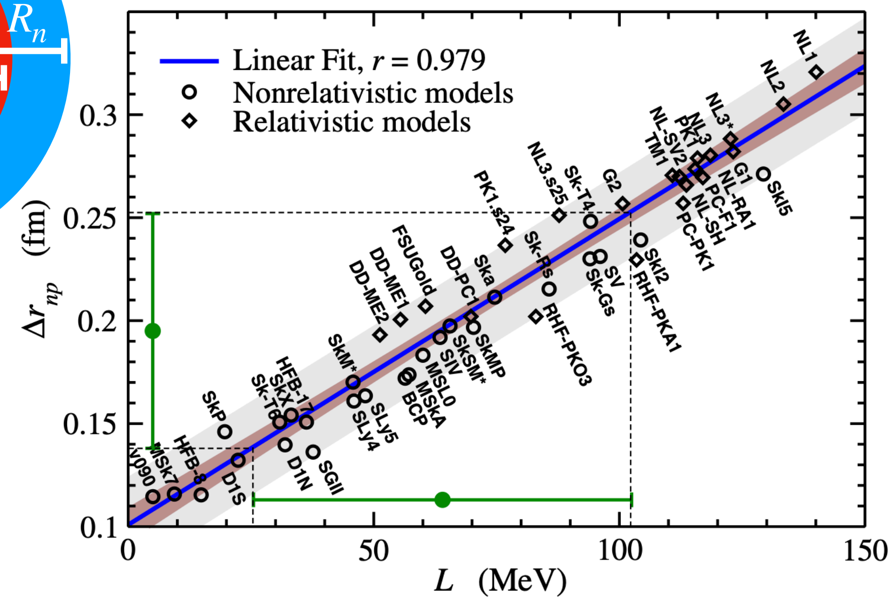
Tews+ ApJ (2018)

Drischler+ PRC (2020)

Neutron skins and the neutron matter equation of state



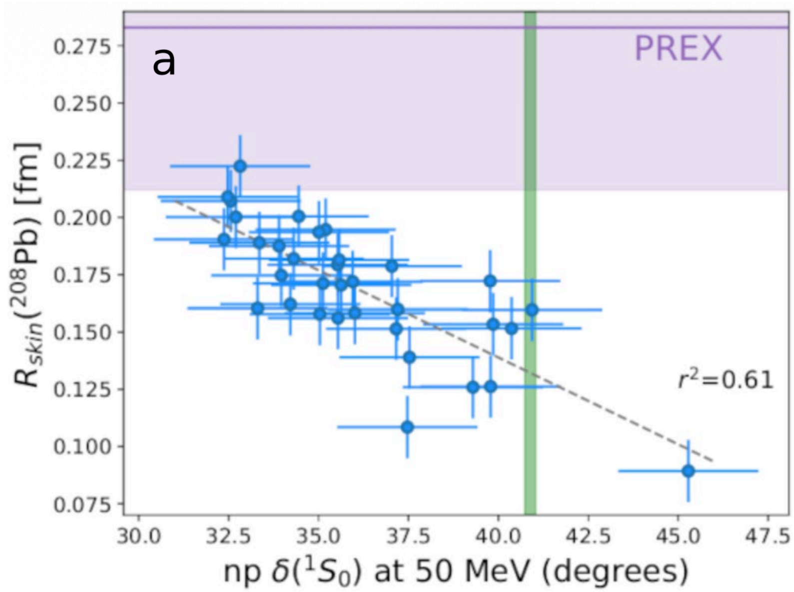


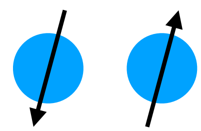


Relativistic and non-relativistic mean field models accurately reproduce E/A and r_{ch}, but are essentially unconstrained for $\Delta r_{np}, L$.

Brown PRL (2000)
Roca-Maza+ PRL (2011)

Neutron skins and the neutron matter equation of state

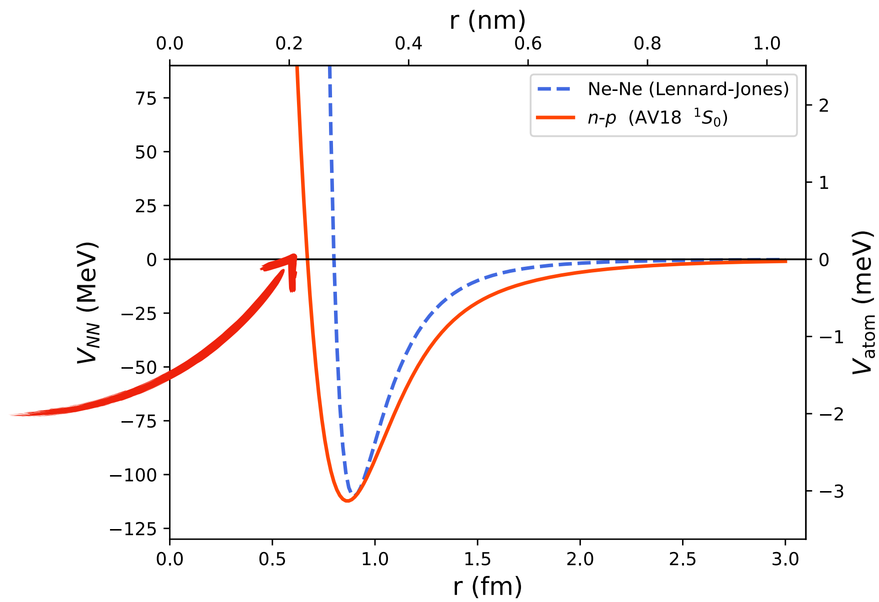




$T_{\rm lab} = 50$ MeV

$\Rightarrow k_{\rm rel} \sim 1.5\ {\rm fm}^{-1}$

$r \sim k^{-1} \sim 0.65$ fm



$$V_{\rm skyrme} \sim t_0\delta(\vec{r}) + \frac{1}{6}t_3\delta(\vec{r})\rho + \ldots$$

Nuclei have $\rho \sim {\rm const}.$ so t_0, t_3 have degeneracy, density dependence is unconstrained. NN scattering breaks the degeneracy.

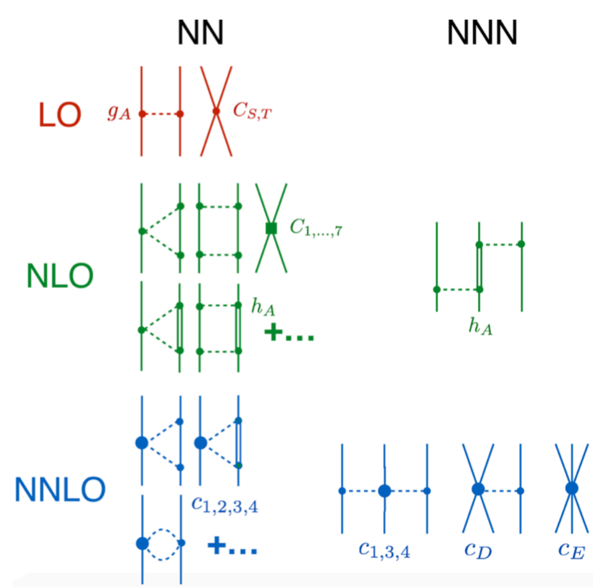
Hu+ Nat. Phys. (2022)

Why do 3N forces seem to be so important when they come at N2LO? Shouldn't they be a small correction? Does this mean 4N forces will be important too?

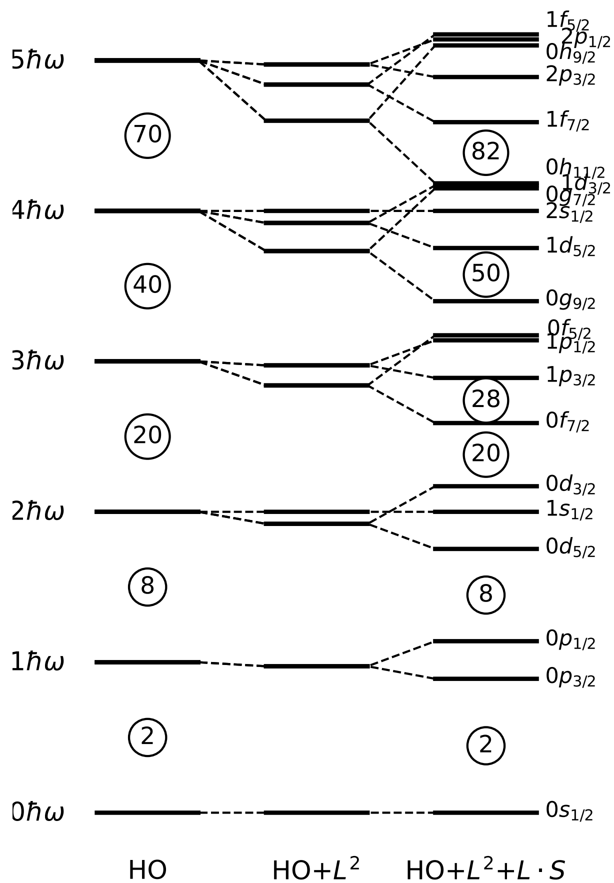
- Actually, 3N forces arise at NLO if we include the Δ -nucleon mass splitting as a small scale.
- Nuclei form bound systems, with $E/A \sim 8 \text{ MeV}$

$$KE/A \approx \frac{3}{5}\epsilon_F \approx 22 \text{ MeV}$$

$$V/A \approx 30 \text{ MeV}$$

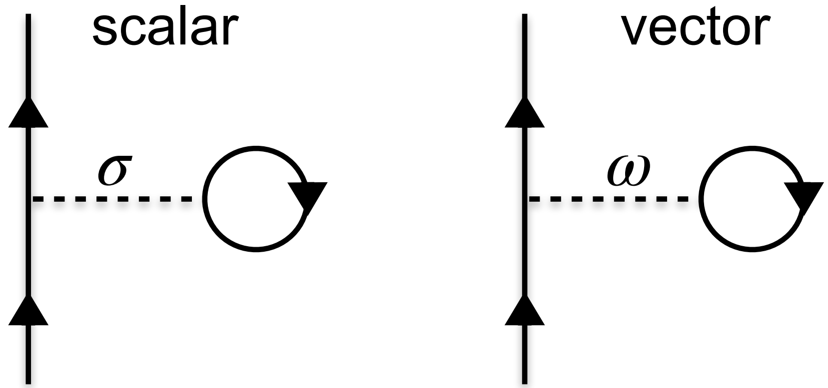


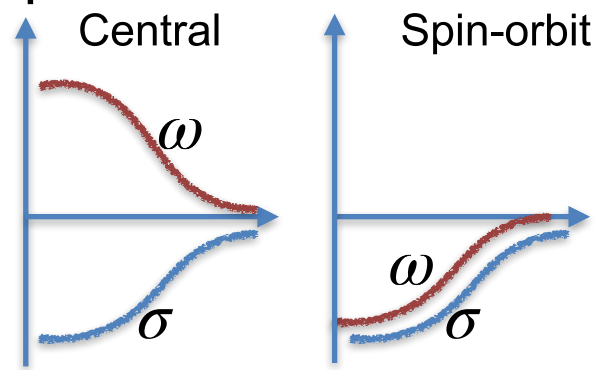
Origin of the spin-orbit mean field potential



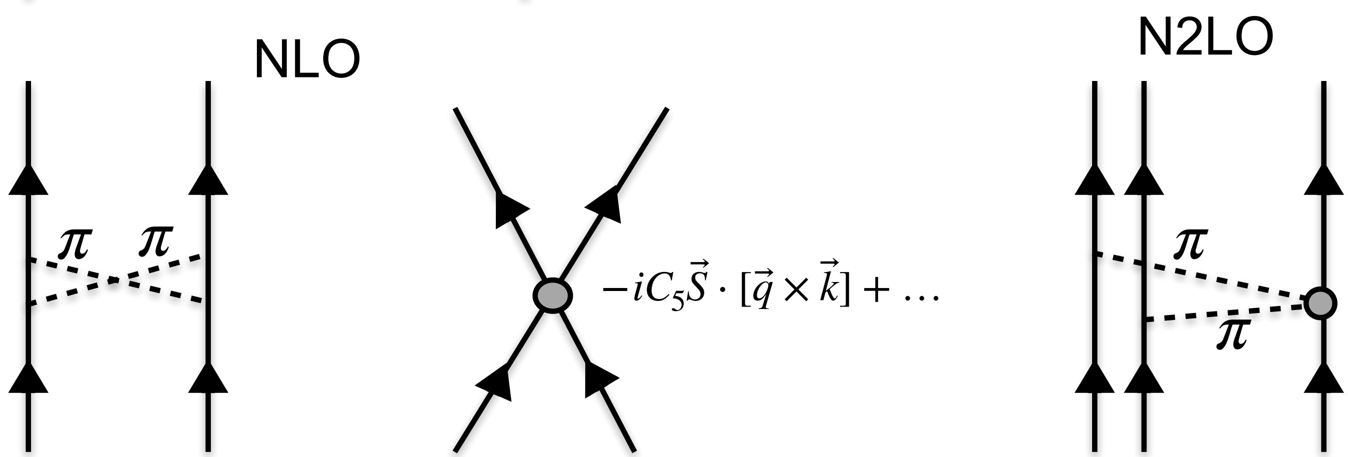
Origin of the spin-orbit mean field potential

RMF:

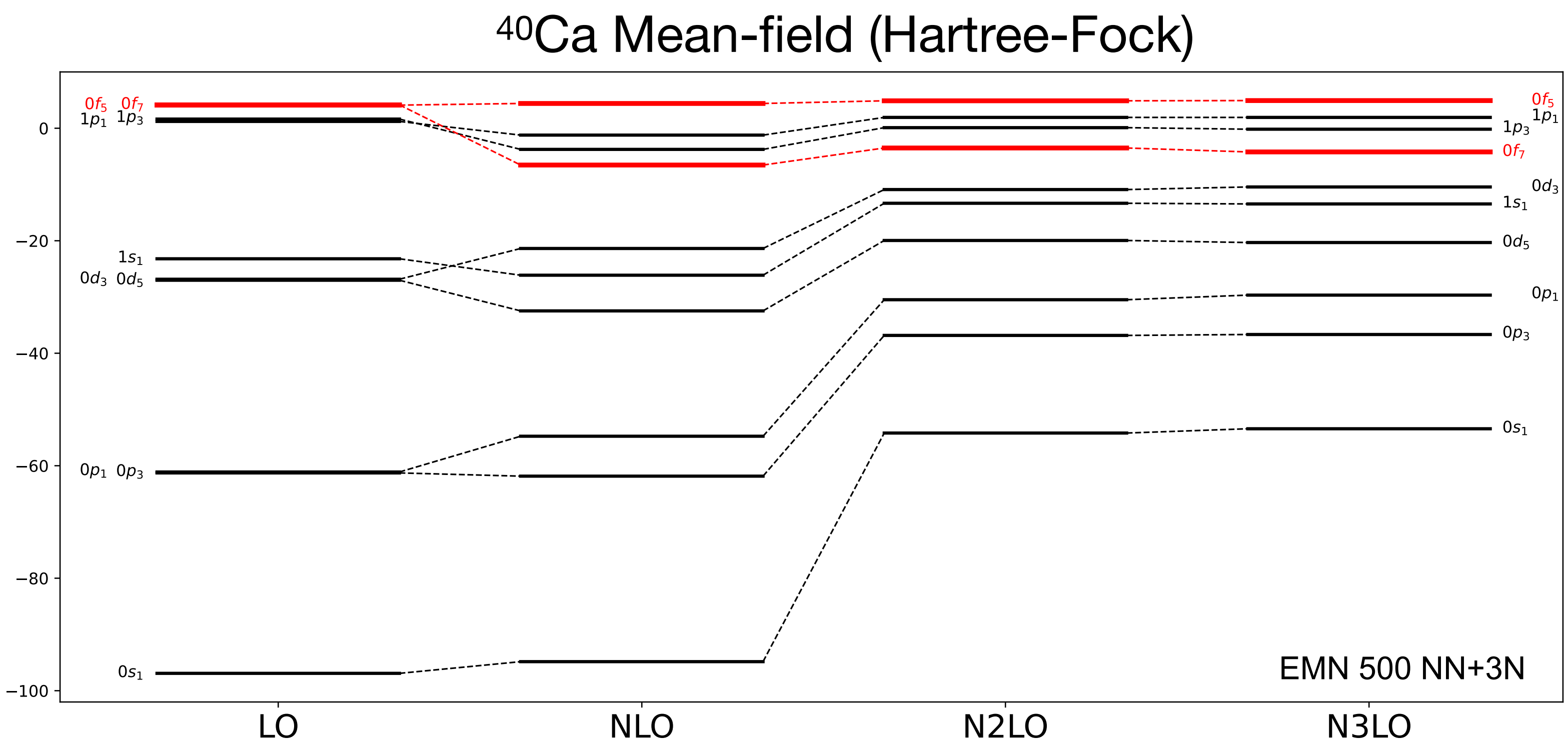




EFT:

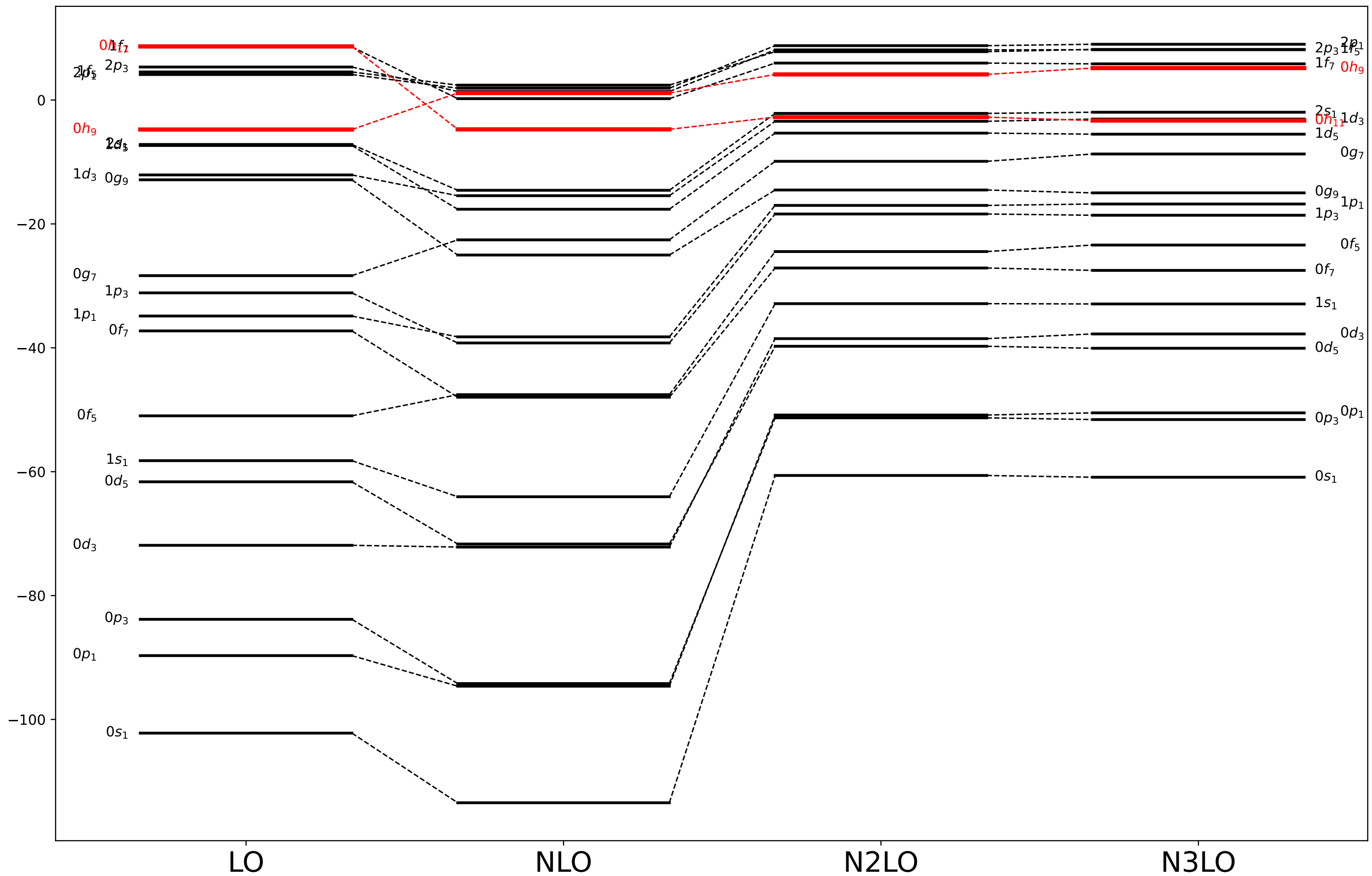


Origin of the spin-orbit mean field potential

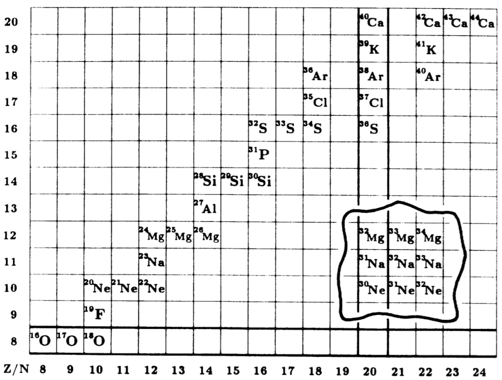


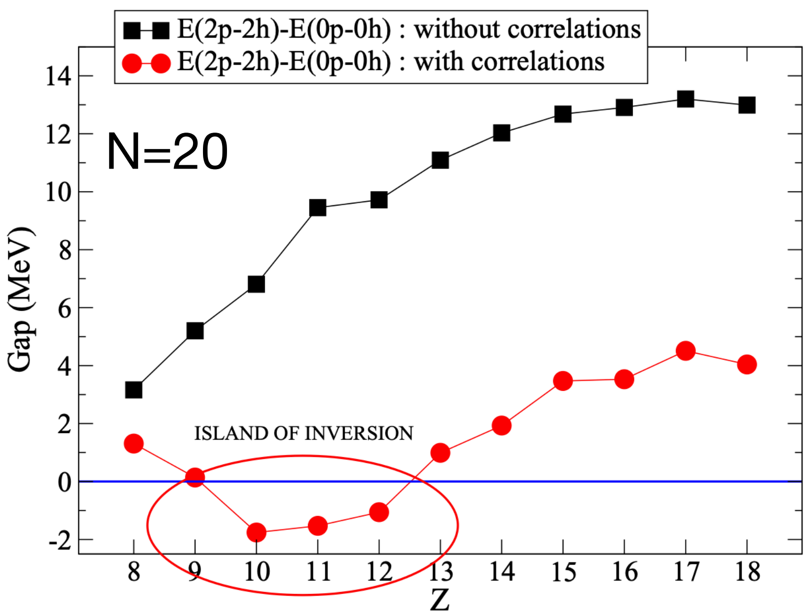
Entem, Machleidt, Nosyk PRC (2017)

^{132}Sn Mean-field (Hartree-Fock)

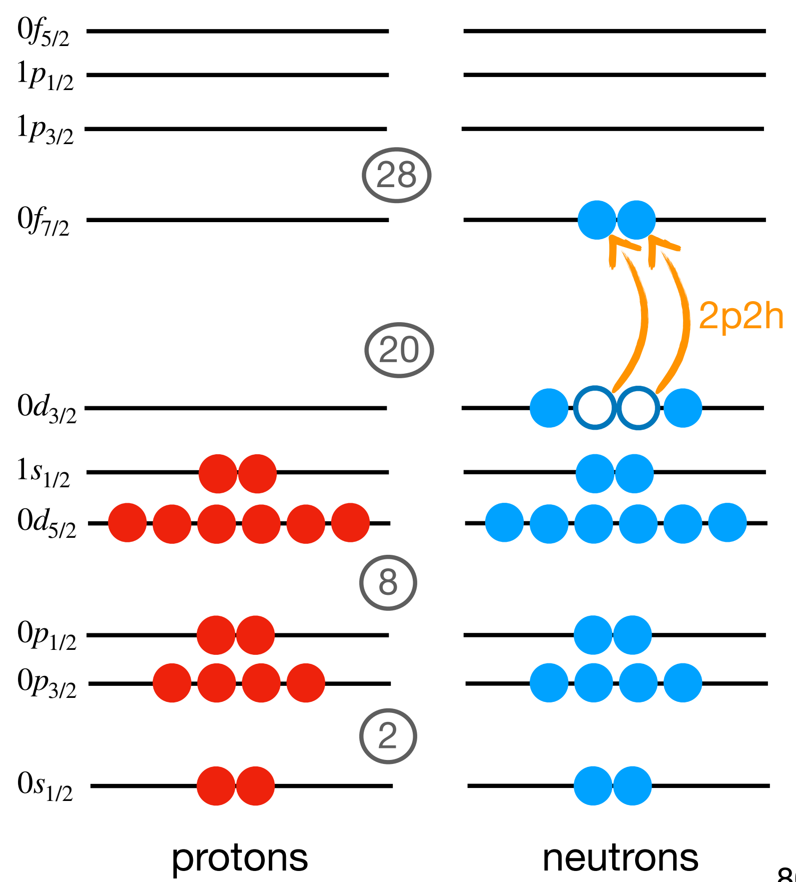


Shell evolution with isospin

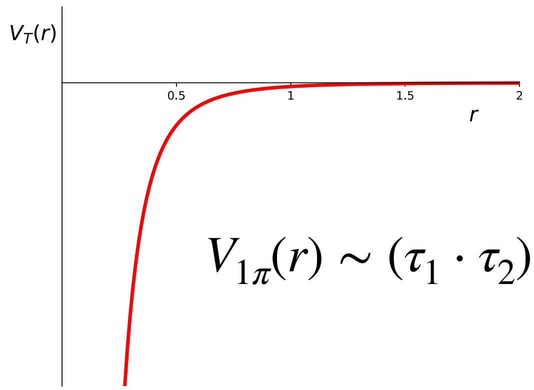




Warburton, Becker, Brown PRC (1990)
Caurier, Nowacki, Poves PRC (2014)



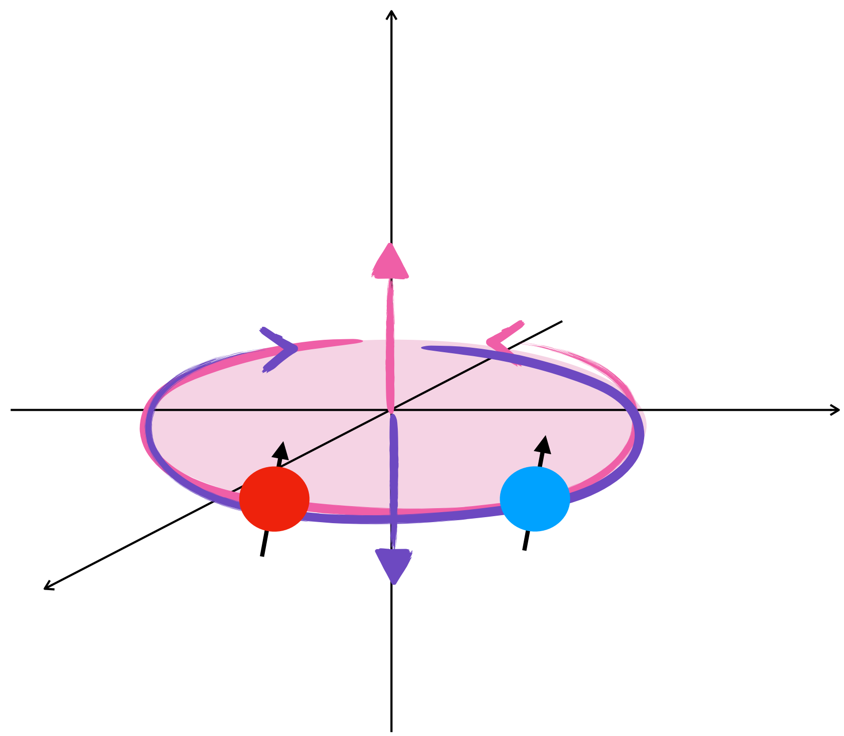
Tensor force



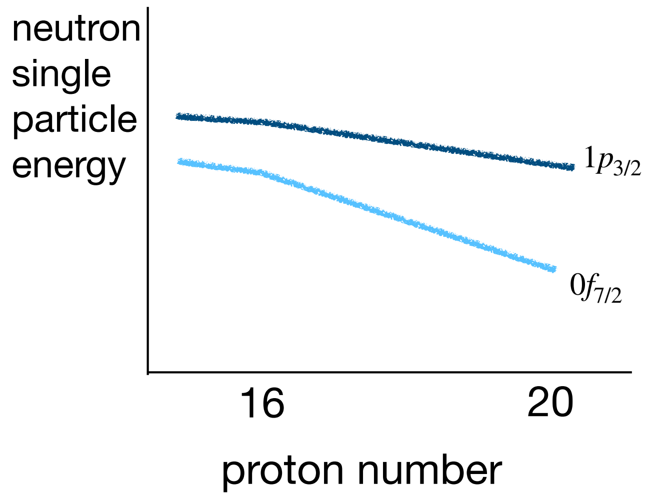
$$V_{1\pi}(r) \sim (\tau_1 \cdot \tau_2)\ S_{12}(\hat{r})\frac{e^{-mr}}{r^3} + \ldots$$

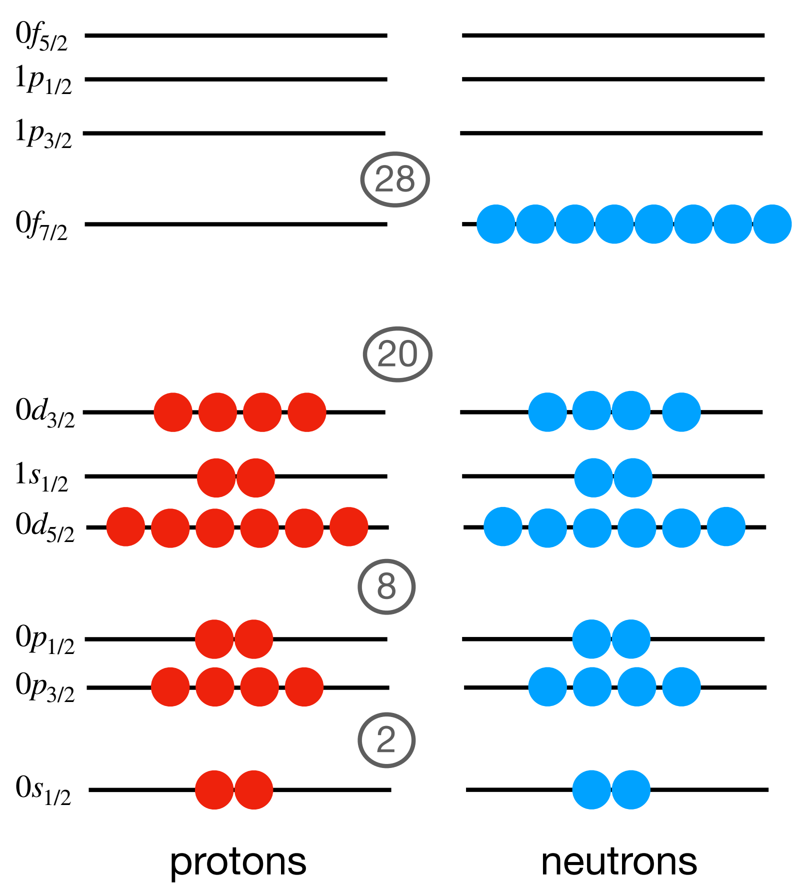
$S_{12}(\hat{r}) \sim [\sigma_1 \otimes \sigma_2]_2 \otimes Y_2$ is only active in spin $S = 1$ channel

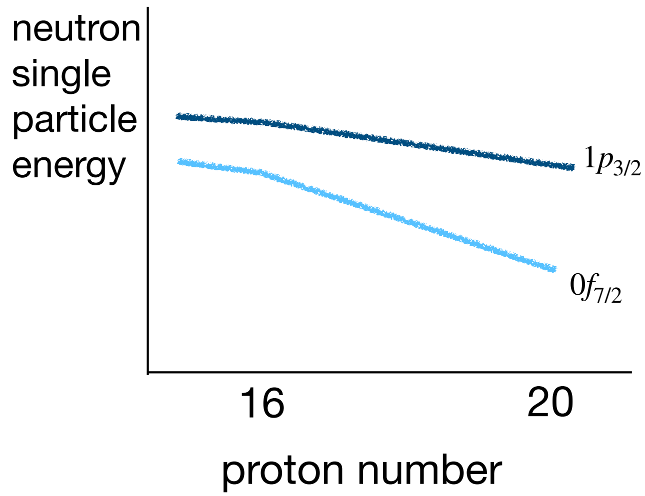
pn: $T = 0 \;\Rightarrow\; \tau_1 \cdot \tau_2 = -3$



The tensor force generates a strong attractive pn interaction with spins aligned, orbits anti-aligned.


neutron single particle energy
$1p_{3/2}$
$0f_{7/2}$
16
20
proton number


$0f_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
28
$0f_{7/2}$
20
$0d_{3/2}$
$1s_{1/2}$
$0d_{5/2}$
8
$0p_{1/2}$
$0p_{3/2}$
2
$0s_{1/2}$
protons
neutrons


neutron
single
particle
energy
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20
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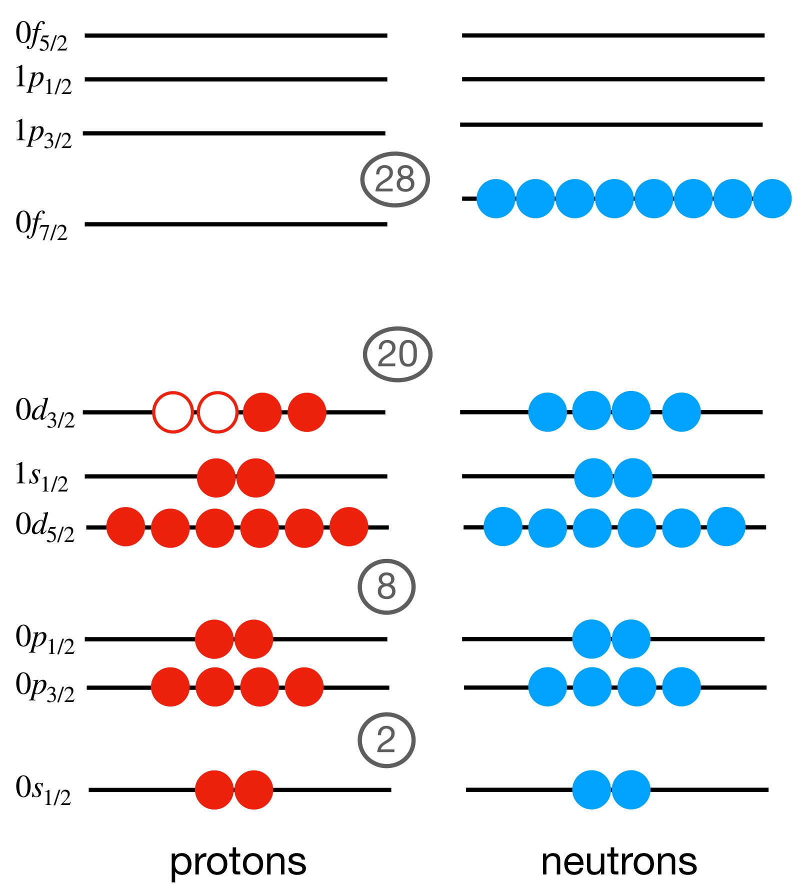
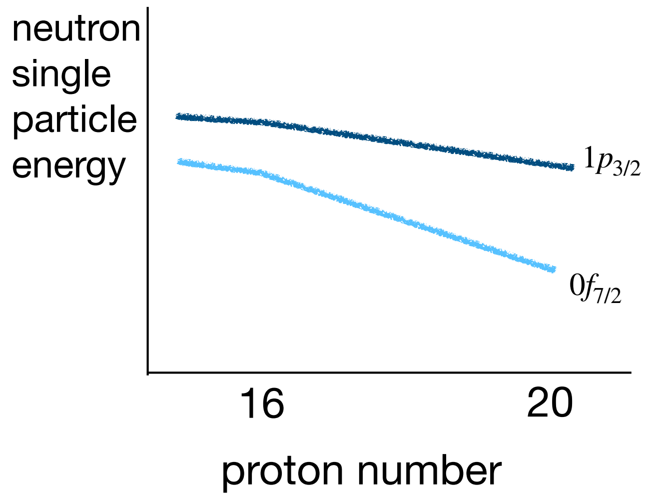
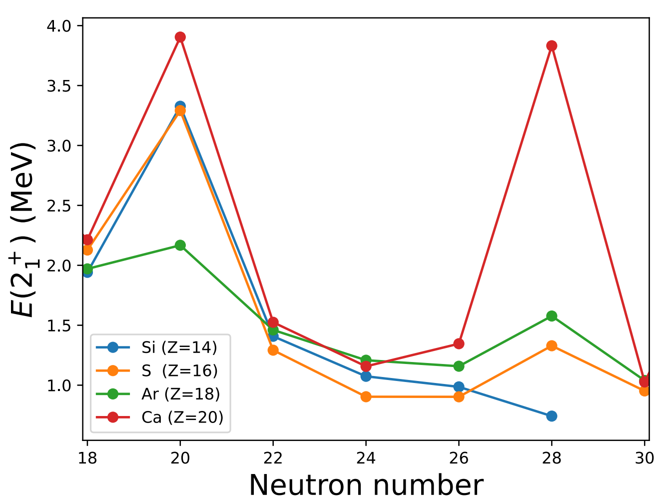
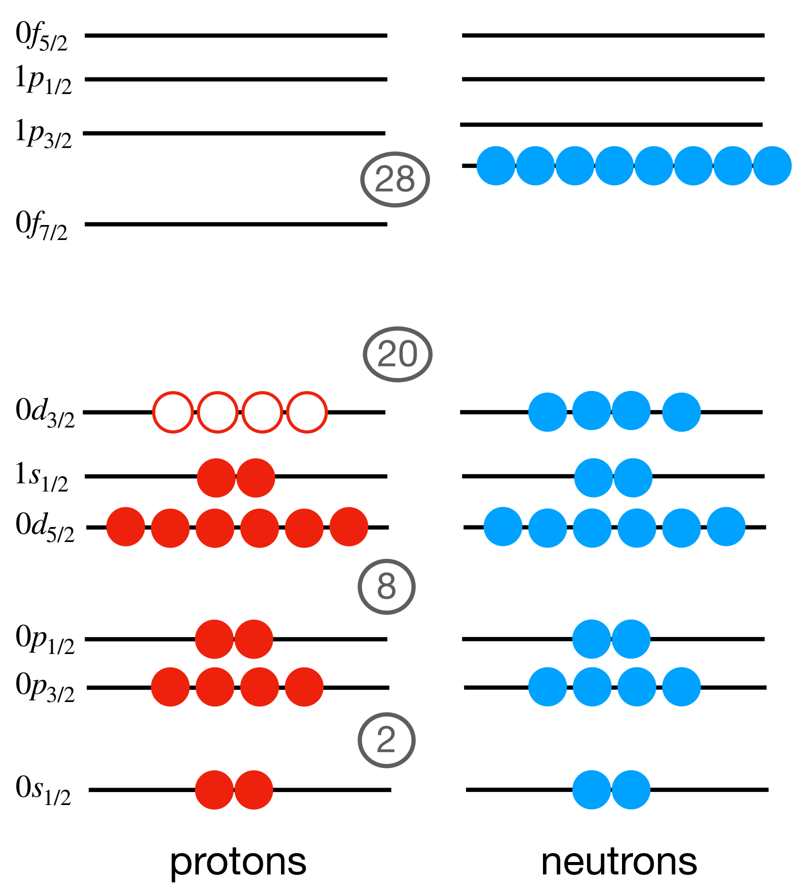
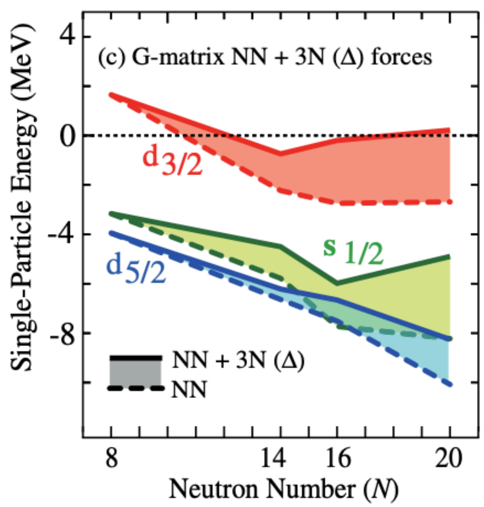

$0f_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
28
$0f_{7/2}$
20
$0d_{3/2}$
$1s_{1/2}$
$0d_{5/2}$
8
$0p_{1/2}$
$0p_{3/2}$
2
$0s_{1/2}$
protons
neutrons

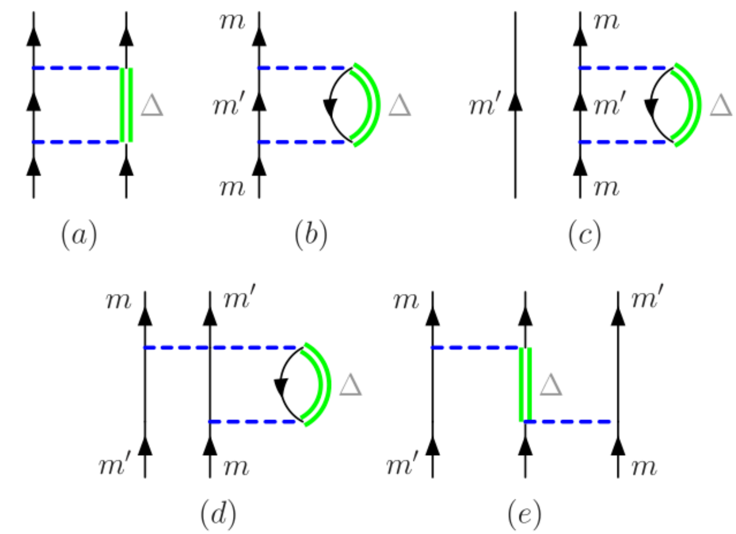

neutron single particle energy
$1p_{3/2}$
$0f_{7/2}$
16
20
proton number

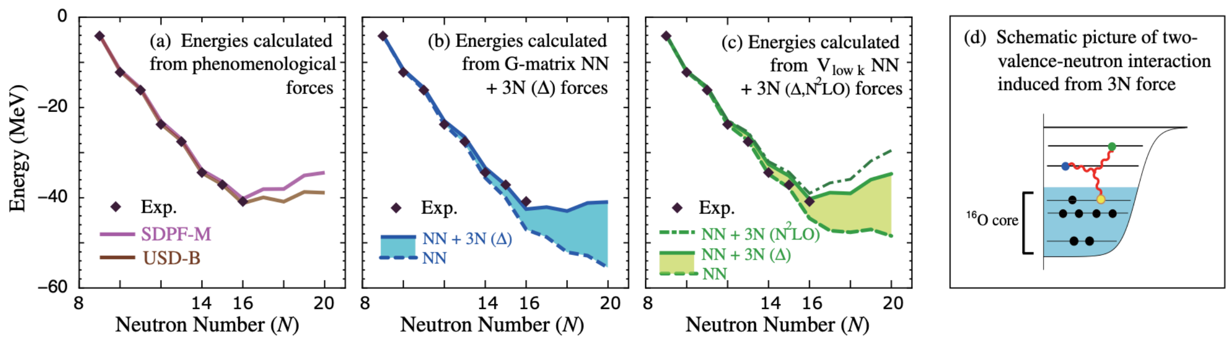

$E(2_1^+)$ (MeV)
Si (Z=14)
S (Z=16)
Ar (Z=18)
Ca (Z=20)
Neutron number


$0f_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
28
$0f_{7/2}$
20
$0d_{3/2}$
$1s_{1/2}$
$0d_{5/2}$
8
$0p_{1/2}$
$0p_{3/2}$
2
$0s_{1/2}$
protons
neutrons

3N forces and shell evolution

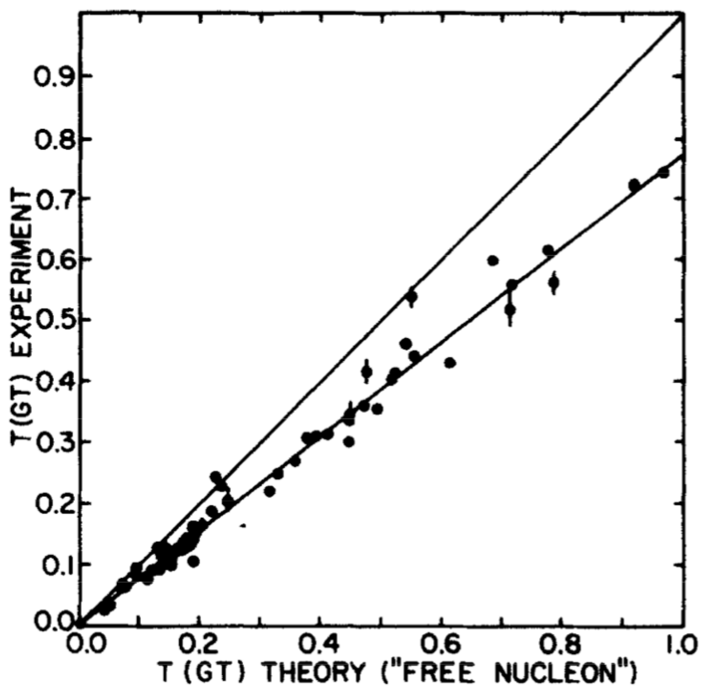


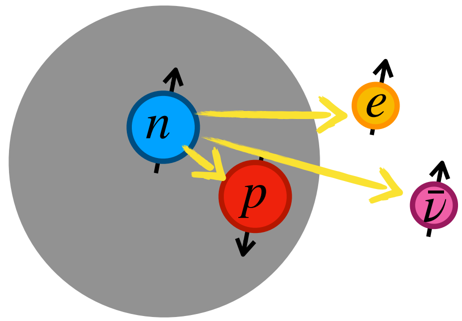




Otsuka+ PRL (2010)

Quenching in Gamow-Teller β decays



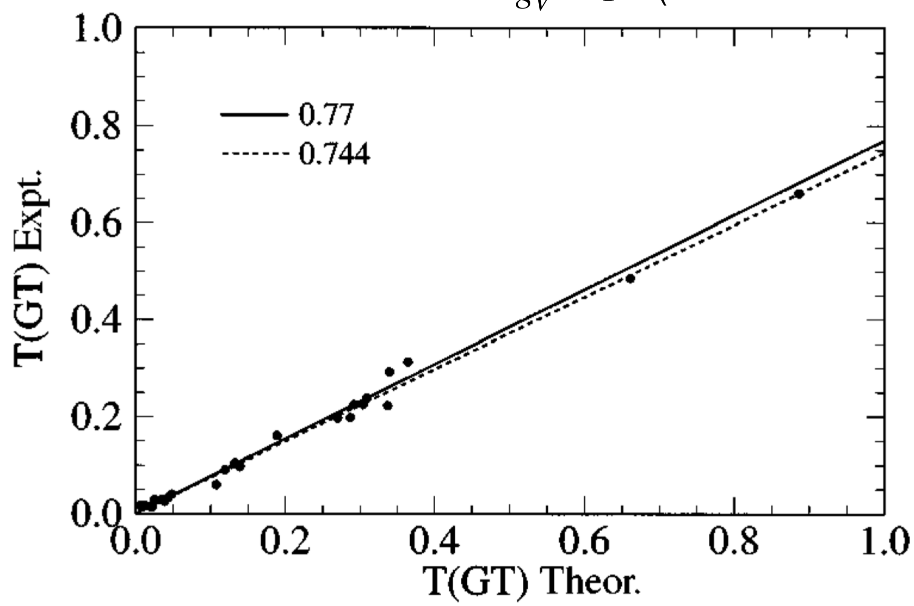


Hadronic axial current

$$\mathcal{J}^A \rightarrow \langle \psi_f | \sum_i^A g_A \sigma_i \tau_i | \psi_i \rangle$$

$g_A = 1.27$

$g_V = 1$ (conserved vector current)



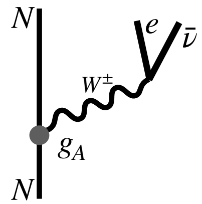
Brown & Wildenthal At Data Nucl Data Tab (1985)

Martinez-Pinedo+ PRC (1996)

Quenching in β decays

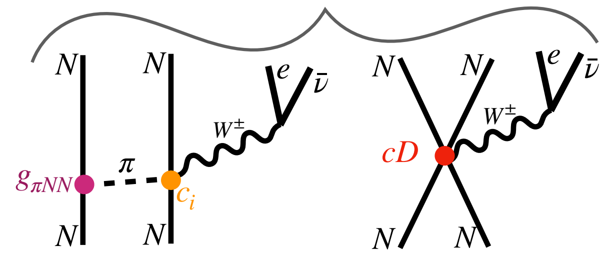
Leading term

1bc

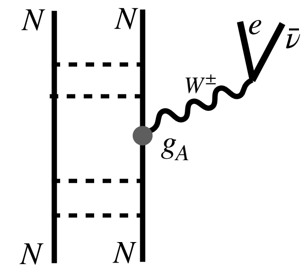


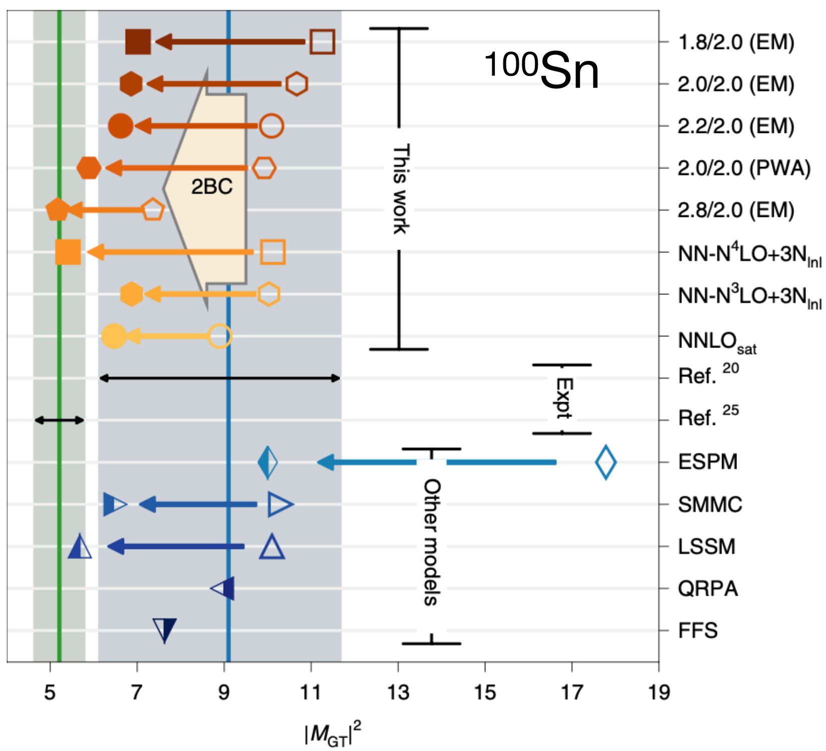
What's missing:

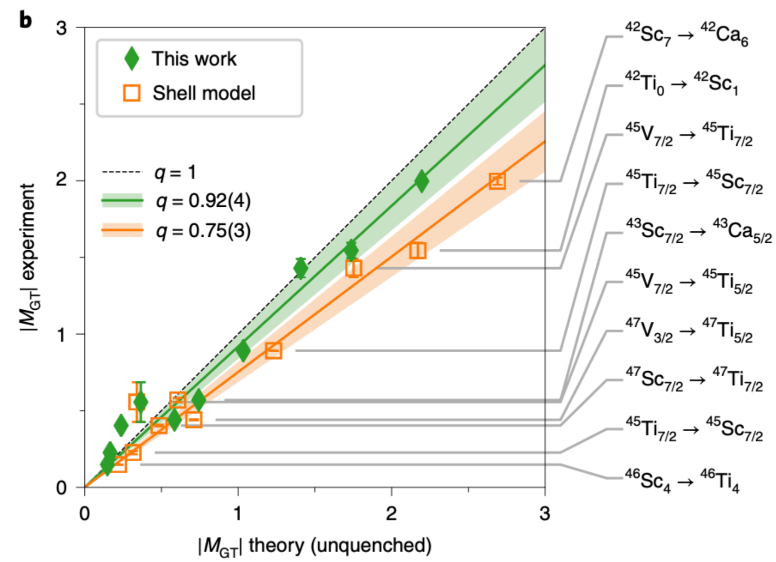
2bc

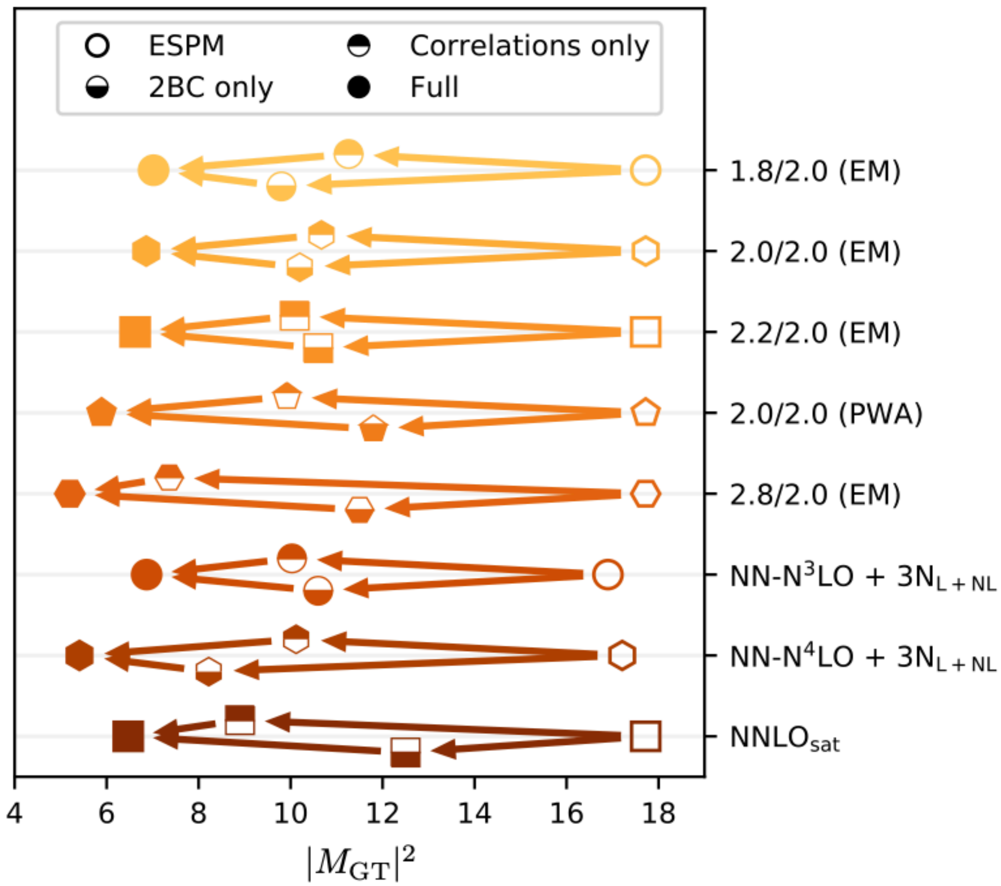


correlations







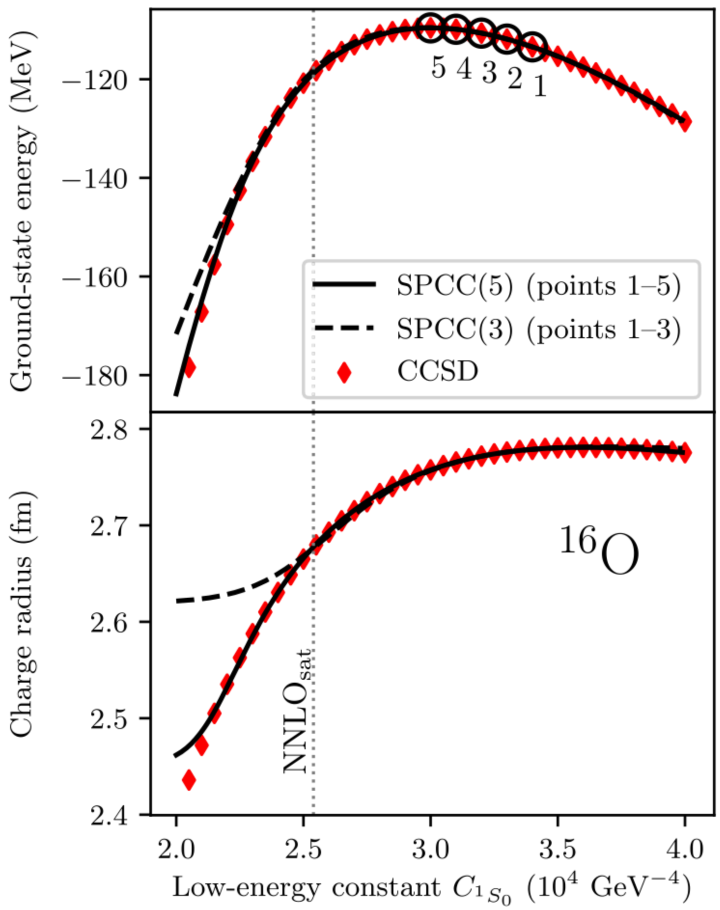


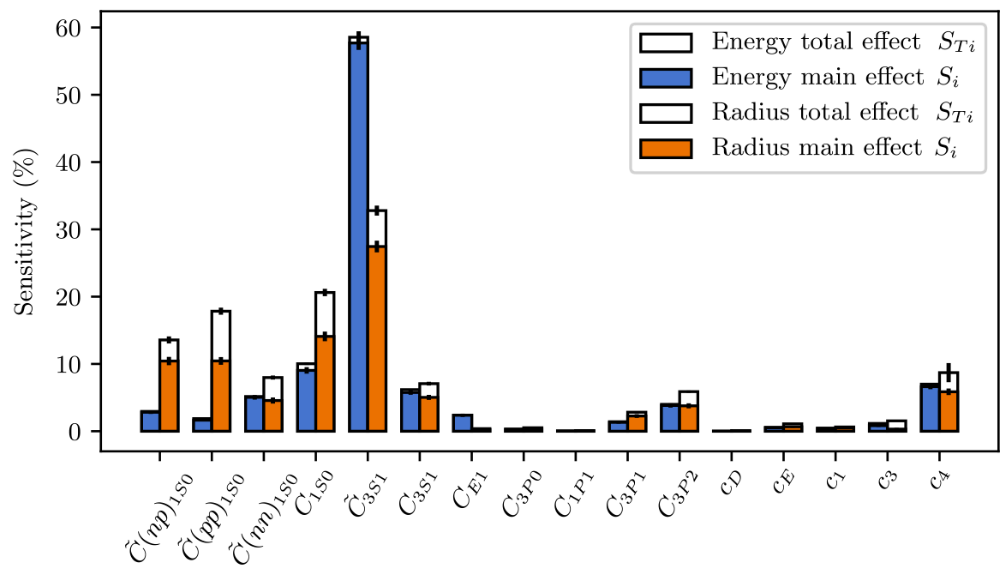
Correlations and 2b currents interact in a non-trivial way. Their effects don't simply add together.

Gysbers+ Nat. Phys. (2019)

Sensitivity studies

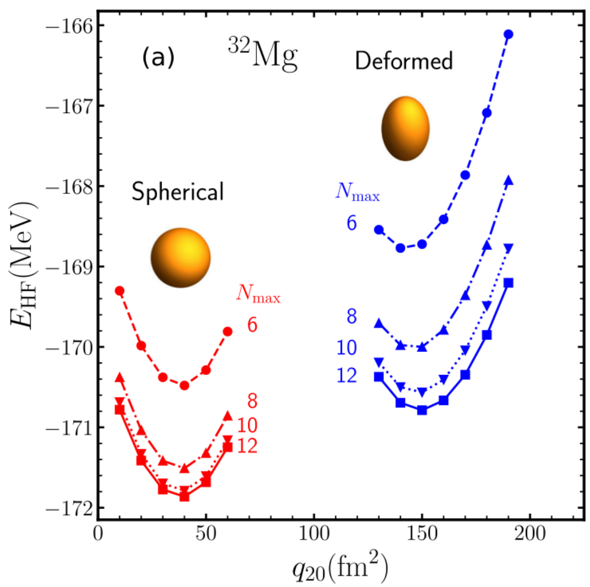
Emulators



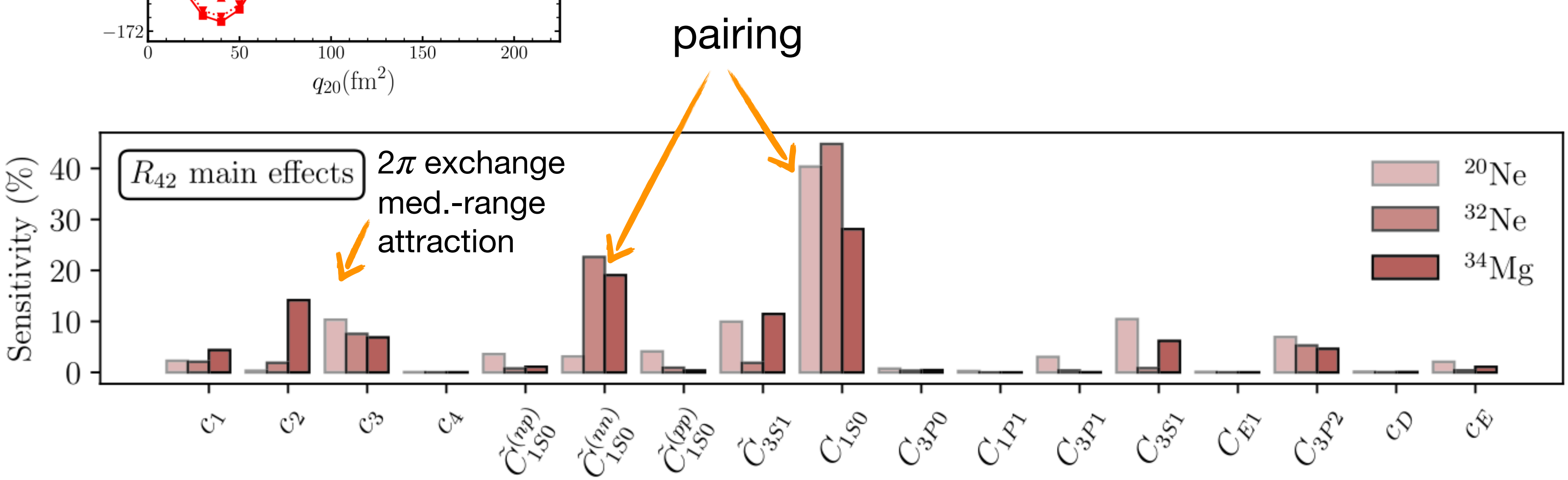


Ekstrom+Hagen PRL (2019)

Sensitivity studies



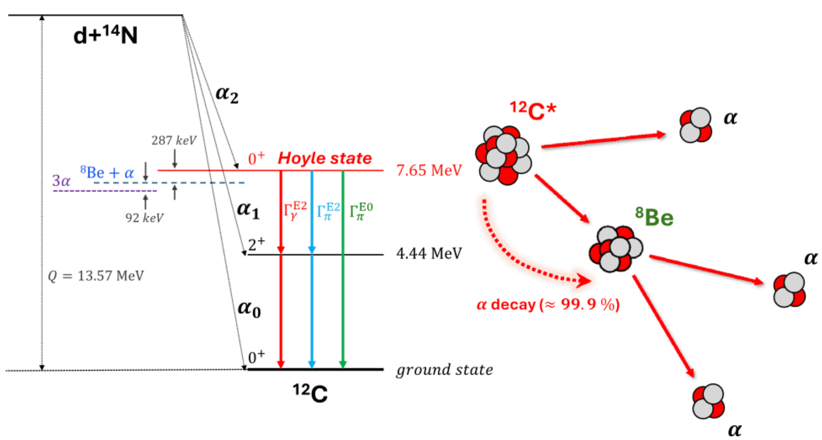
What aspect of the nuclear force drives deformation?

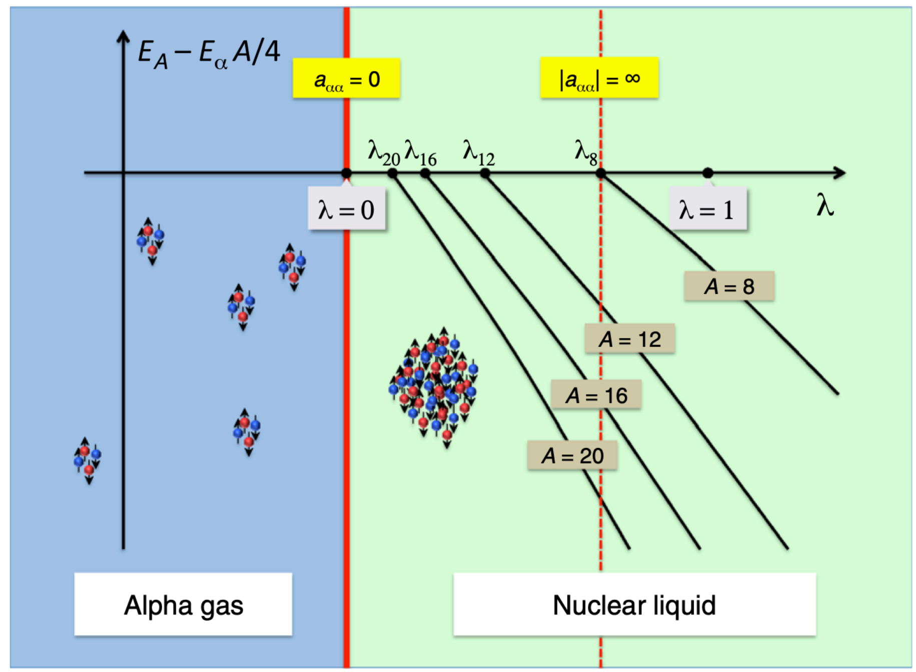


Sun+ PRX (2025)

How fine-tuned is our universe?

A	$BE(A)$	$A/4 \times BE(\alpha)$
4	28.296	28.296
8	56.499	56.591
12	92.162	84.887
16	127.619	113.183





Dell'Aquila+ Nat. Sci. Rep (2024)

Elhatisari+ PRL (2016)

Summary

- Nuclei are interesting and complicated many-body quantum systems. Many features can be traced to generic properties, like a short-range attractive force.

- Competition between various collective modes depends more sensitively on the details of the interaction and the system.

- Chiral effective field theory provides a beautiful and systematic way to construct a nuclear interaction, while maintaining a connection to QCD. But some conceptual issues remain to be ironed out.

- The availability of ab initio methods allows us to directly test the impact of nuclear forces in nuclei.