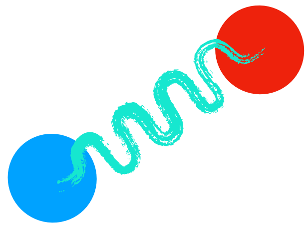
# Part 2: Nuclear Forces

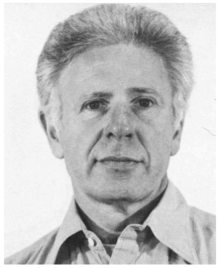
**1. Meson exchange picture**

**2. 3N forces**

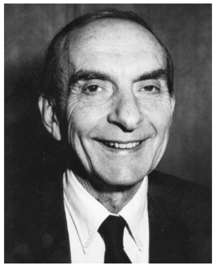
**3. Chiral Effective Field Theory**



“Never have so many contributed so little to so few.”
-Marvin Goldberger, in reference to the nuclear interaction

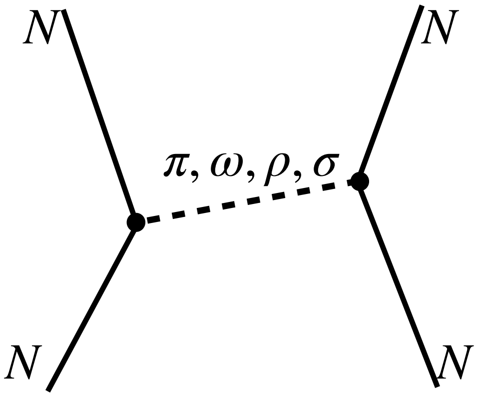


“Eugene [Wigner] asked Gerry [Brown], as he went into the building, what he planned to work on. ‘I plan to work out the nucleon–nucleon interaction in nuclei.’ Eugene said that it would take someone cleverer than him, to which Gerry replied that they probably disagreed what it meant to ‘work out’.”



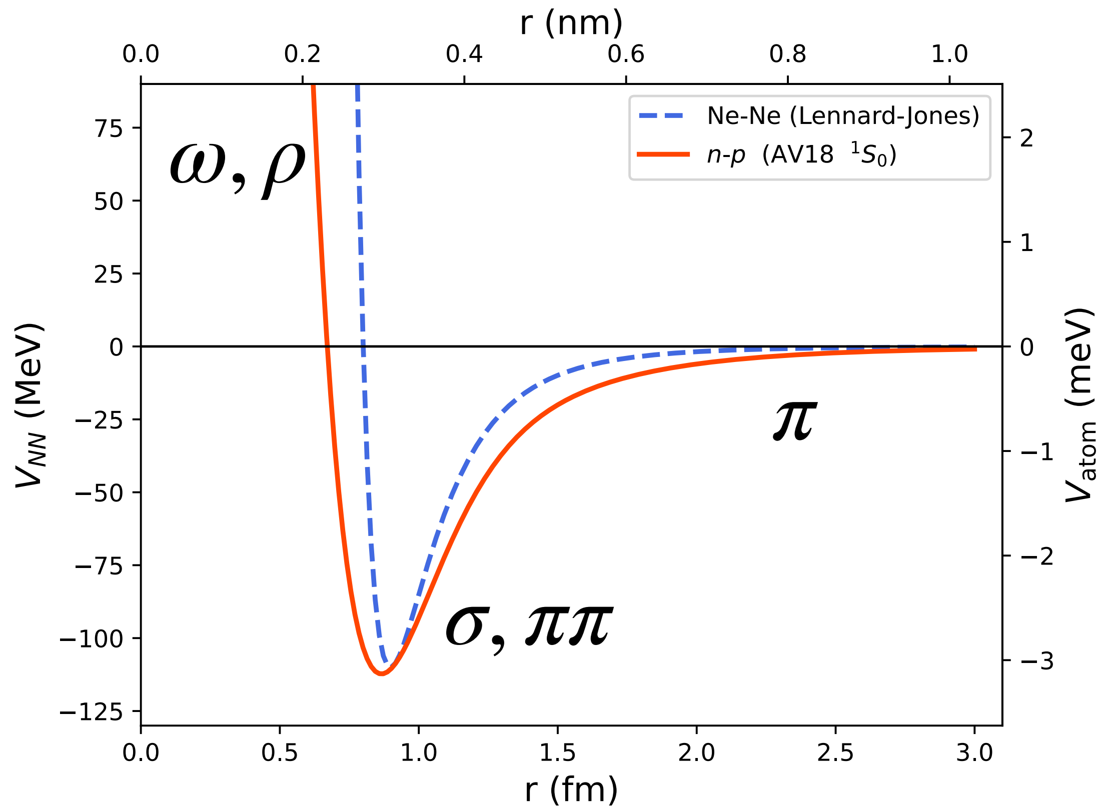
quotes from Kuo and Holt Nuc. Phys. A (2014)

# Meson exchange



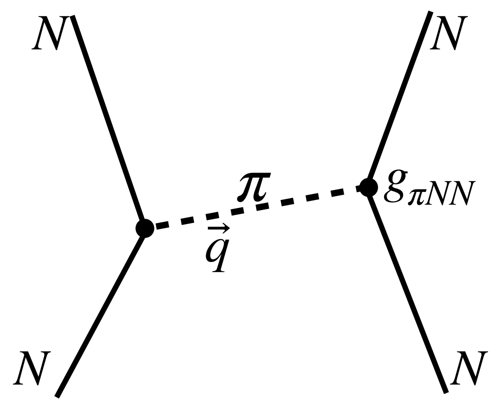

$$V(r) \sim g^2 \frac{e^{-mr}}{r}$$

Coupling strength $g$,
meson mass $m$

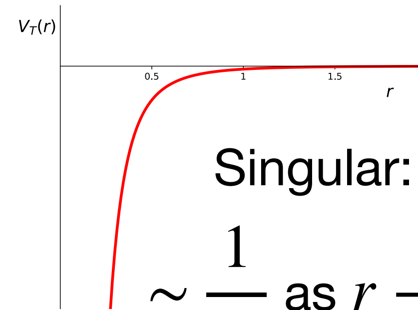



$m_\pi \approx 140$ MeV $\rightarrow 1.4$ fm
$m_\rho \approx 770$ MeV $\rightarrow 0.26$ fm
$m_\omega \approx 783$ MeV $\rightarrow 0.25$ fm
$m_\sigma \sim 500$ MeV $\rightarrow 0.4$ fm?

# One pion exchange potential




$$V_{1\pi}(q) = -\left(\frac{g_{\pi NN}}{2M_N}\right)^2 (\tau_1 \cdot \tau_2) \frac{(q \cdot \sigma_1)(q \cdot \sigma_2)}{q^2 + m_\pi^2}$$



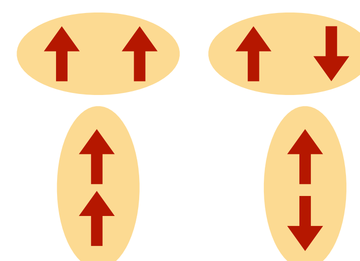

Singular:

$\sim \dfrac{1}{r^3}$ as $r \to 0$

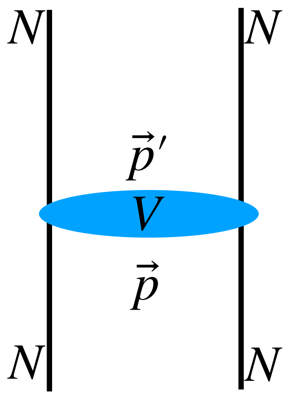
$$V_{1\pi}(r) = \left(\frac{g_{\pi NN}}{2M_N}\right)^2 \frac{m_\pi^2}{12\pi} (\tau_1 \cdot \tau_2) \left[ S_{12}(\hat{r}) \left( 1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) + \sigma_1 \cdot \sigma_2 \right] \frac{e^{-m_\pi r}}{r}$$

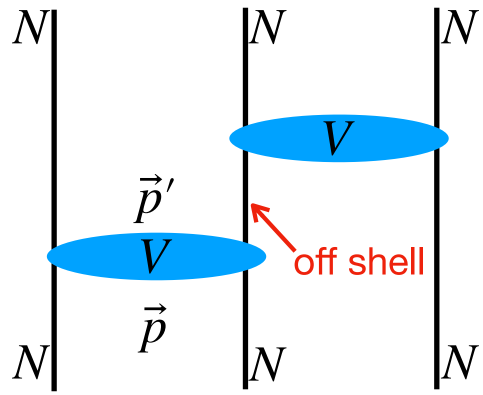
Tensor operator

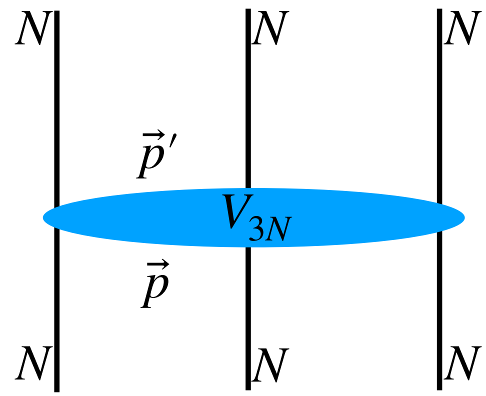
$$S_{12}(\hat{r}) \equiv 3(\hat{r} \cdot \sigma_1)(\hat{r} \cdot \sigma_2) - \sigma_1 \cdot \sigma_2$$
$$\propto Y_2(\hat{r}) \cdot [\sigma_1 \otimes \sigma_2]_2$$



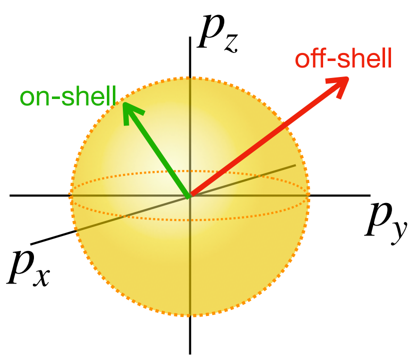
# Off-shell NN potentials



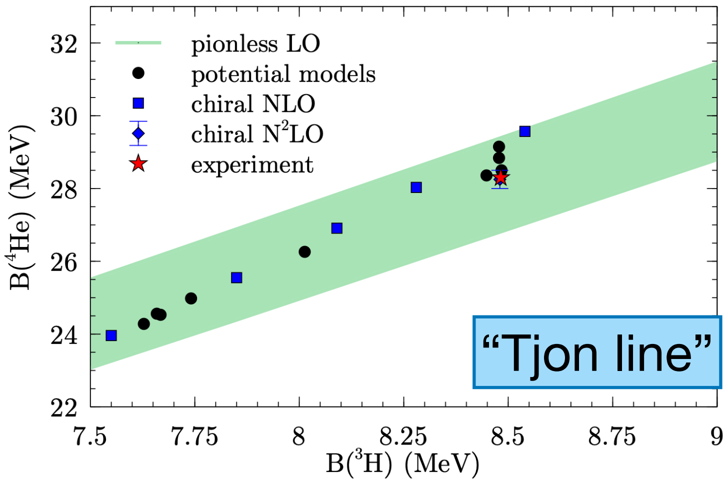

"On-the-energy-shell": $\dfrac{p^2}{2M} = \dfrac{p'^2}{2M} = E$

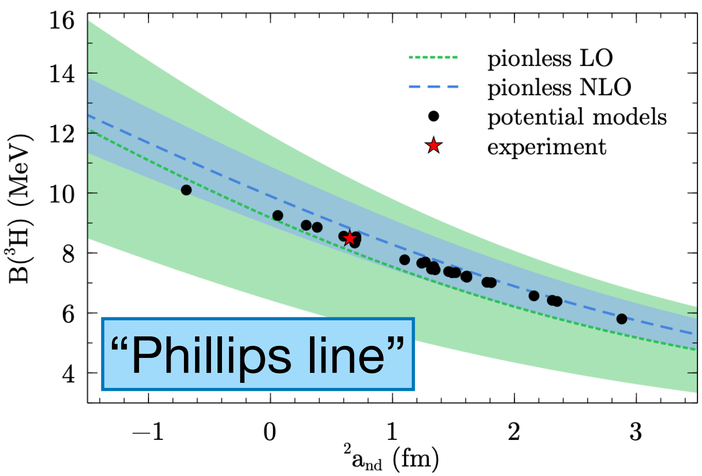



The off-shell $T$ matrix does not affect NN systems, but it *does* affect $A > 2$. Moreover, any two on-shell equivalent potentials are related by a 3N potential.

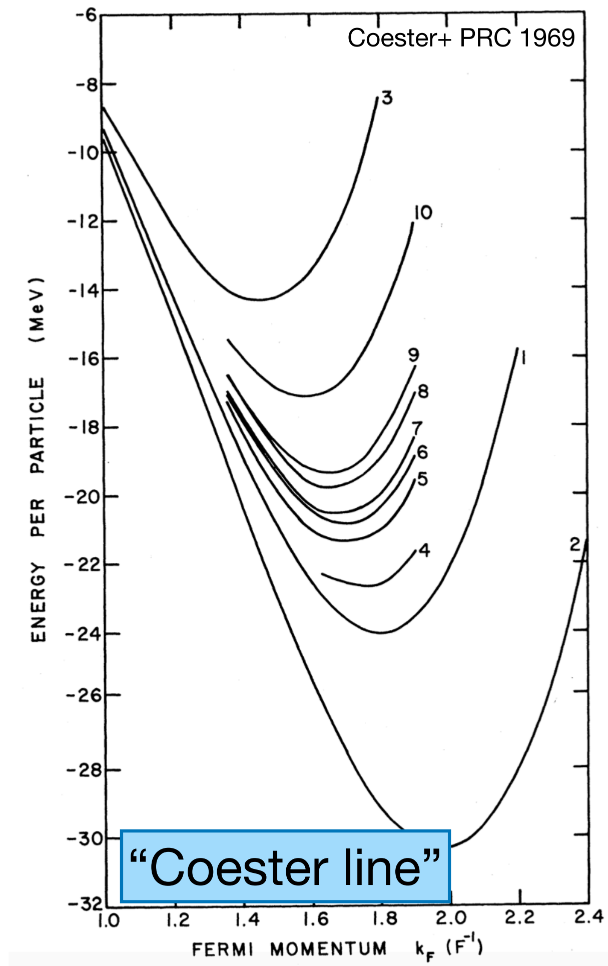
see Polyzou & Glöckle Few Body Syst. (1990)

# NN potentials in $A > 2$ systems

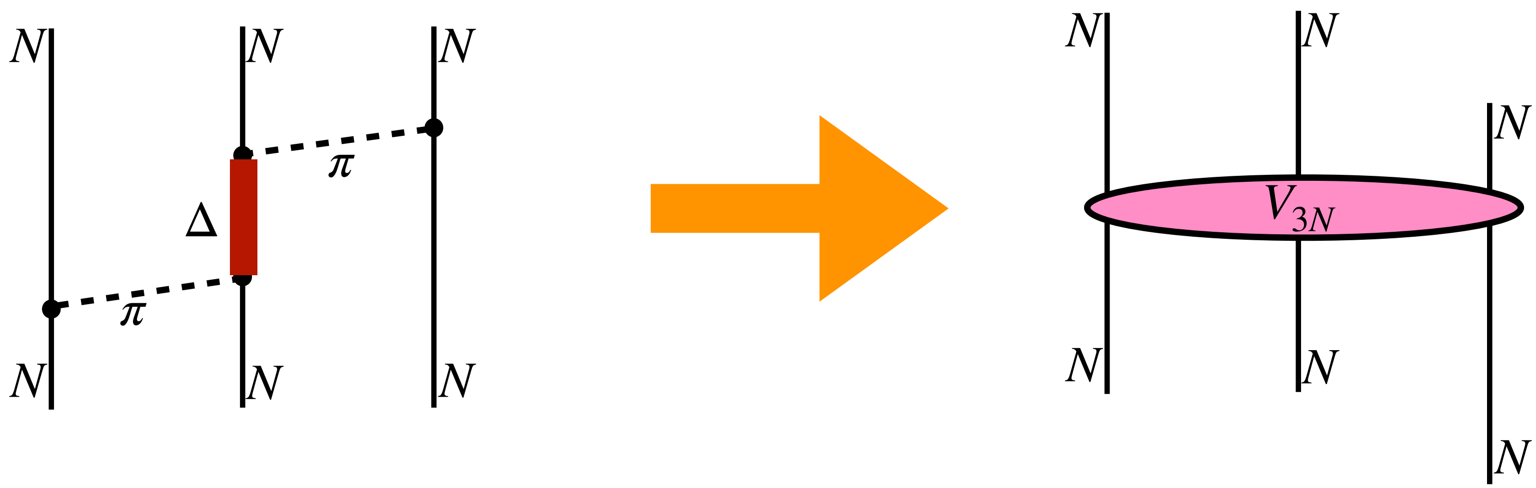


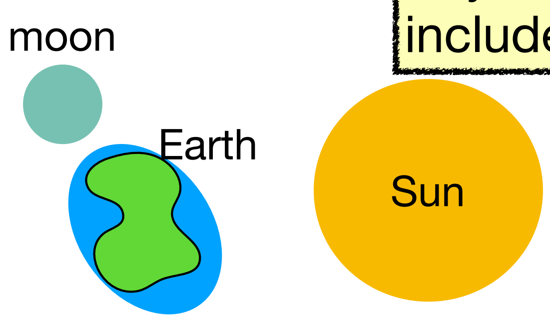

Hammer, König, van Kolck RMP 2020



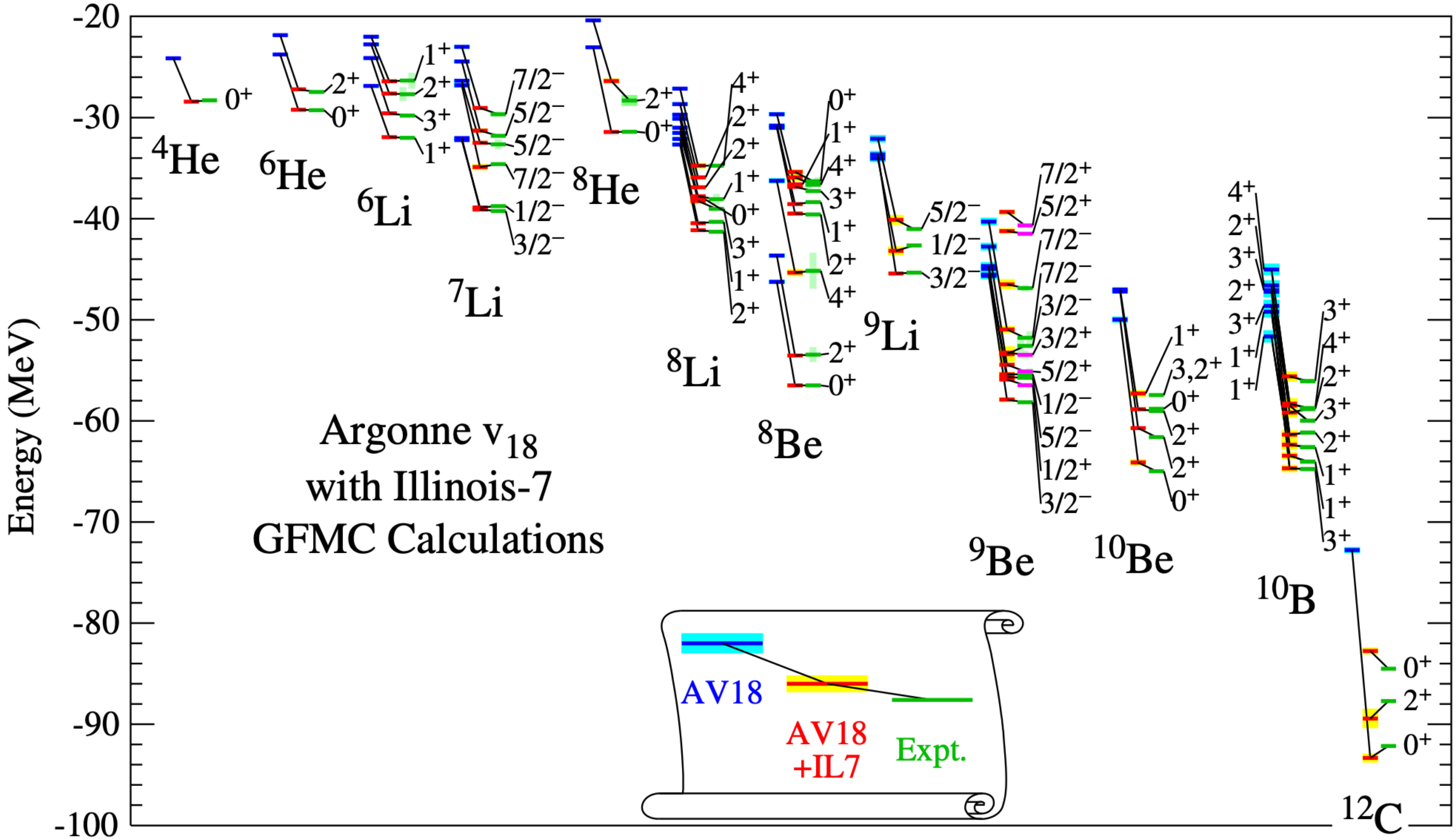
# 3N Forces




Any degrees of freedom that are not explicitly included will generally lead to many-body forces.

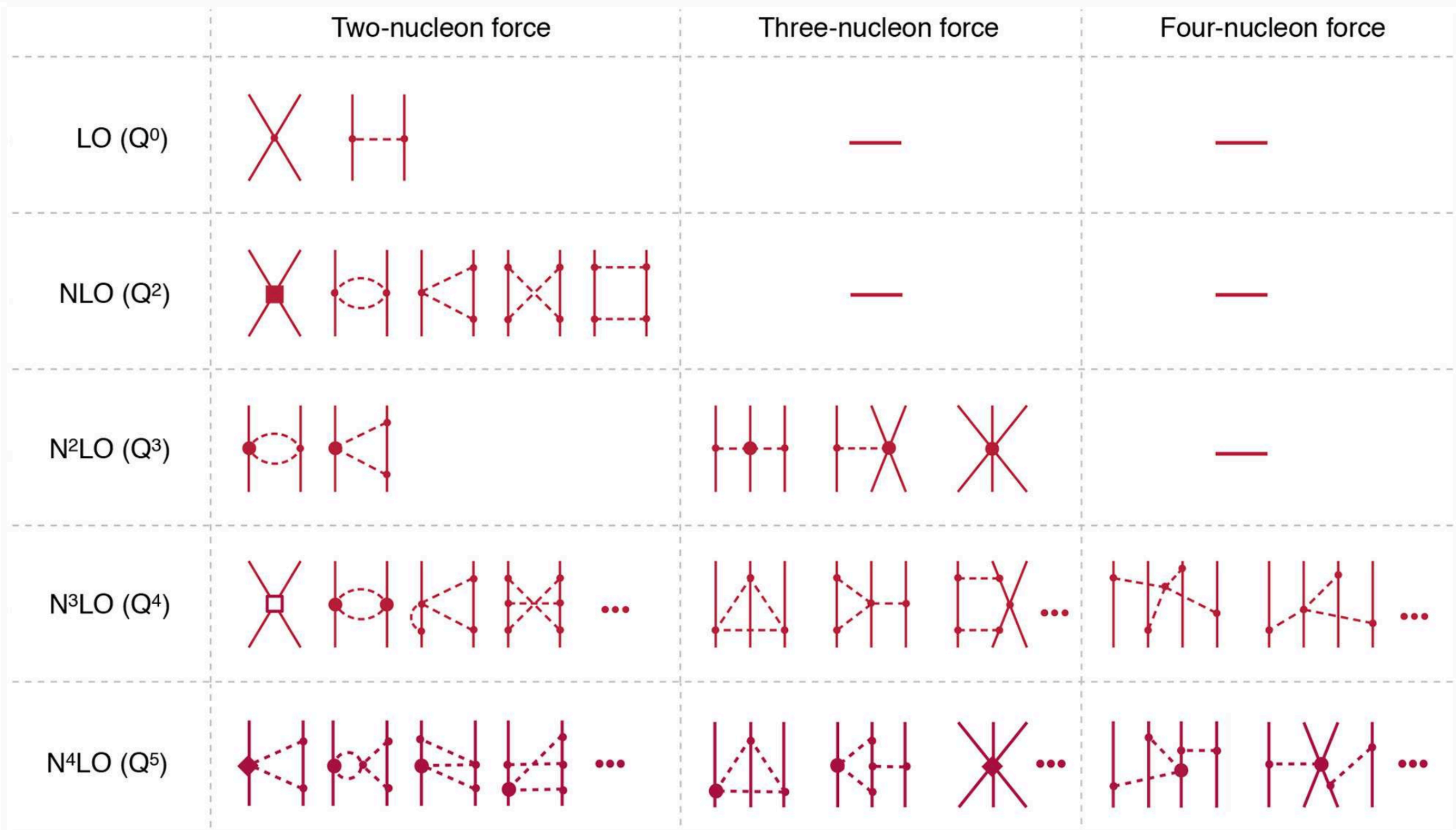


# 3N Forces and Spectroscopy of Light Nuclei




Carlson+ RMP 2015
Wiringa and Pieper, PRL 2002

# Chiral EFT




Epelbaum, Krebs, Reinert, Front. Phys. 2020

# Effective Field Theory

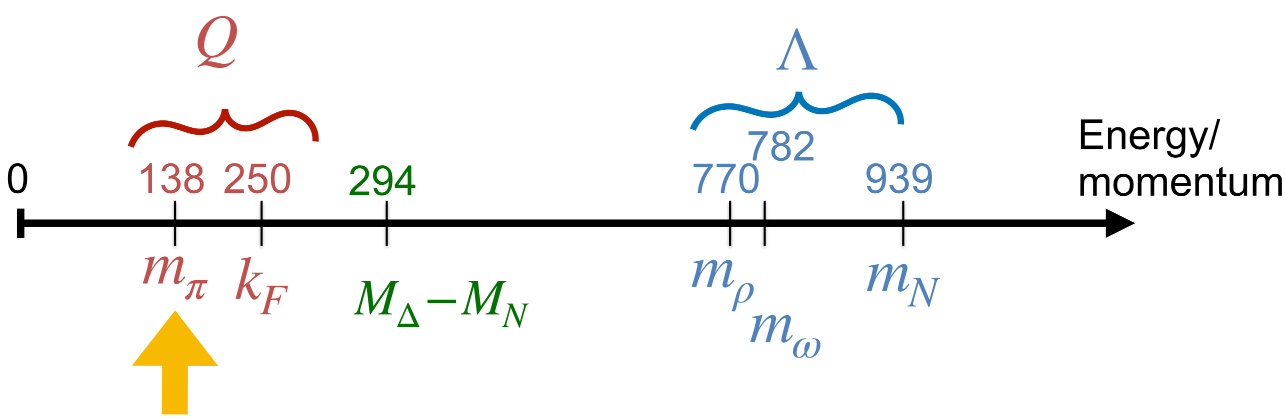
This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. The "theorem" says that although individual quantum field theories have of course a good deal of content, quantum field theory itself has no content beyond analyticity, unitarity, cluster decomposition, and symmetry. This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible $S$-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. As I said, this has not been proved, but any counterexamples would be of great interest, and I do not know of any.

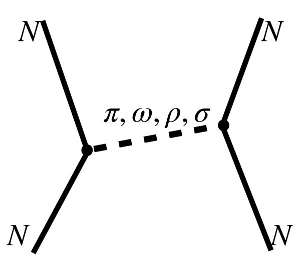
Ok, great. But if we're going to do perturbation theory, we need a small parameter. And what symmetries should we use?

Weinberg, "Phenomenological Lagrangians" Physica 96A 327 (1979)

https://utphysicshistory.net/StevenWeinberg.html

We're interested in strongly interacting particles at low energies, and the QCD interaction strength $\alpha_S$ isn't perturbative. But the relevant momenta $Q$ are small compared with typical QCD scales $\Lambda$, so we can attempt an expansion in powers of $Q/\Lambda$.

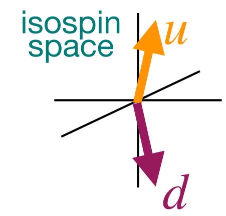



Taylor expand heavy meson propagator: $$\frac{1}{q^2+m^2} = \frac{1}{m^2}\left(1 - \frac{q^2}{m^2} + \frac{q^4}{m^4} + \ldots\right)$$

# Chiral Symmetry

$$\mathscr{L} = \bar{q}(i\gamma^\mu D_\mu - M)q \;-\; \tfrac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} \qquad q = \begin{pmatrix} u \\ d \end{pmatrix}$$ (2 flavor case)




If $M \to 0$, then $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$ *decouple*.

$$\mathscr{L}^0 = \bar{q}_L(i\gamma^\mu D_\mu)q_L + \bar{q}_R(i\gamma^\mu D_\mu)q_R \;-\; \tfrac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}$$

$\mathscr{L}^0$ is unchanged under independent isospin rotations of $q_R, q_L$ (chiral symmetry).

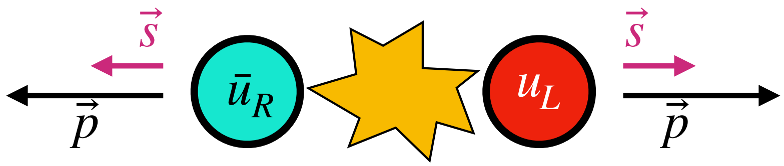
Equivalently, we may use:

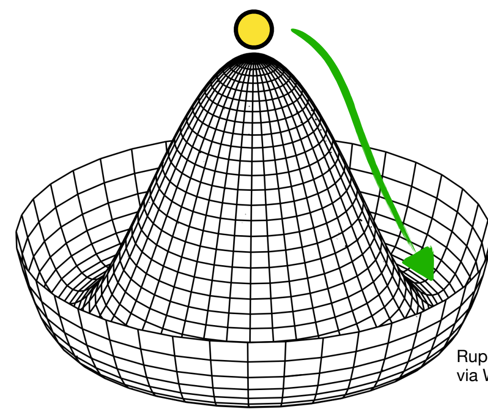
$$q \to \exp\left[-i\theta_V \cdot \frac{\tau}{2}\right]\begin{pmatrix} u \\ d \end{pmatrix}$$ "vector" (left and right rotate together)

$$q \to \exp\left[-i\gamma_5\theta_A \cdot \frac{\tau}{2}\right]\begin{pmatrix} u \\ d \end{pmatrix}$$ "axial" (left and right rotate opposite)

# Chiral Symmetry

The axial symmetry $q \rightarrow \exp\left[-i\gamma_5\theta_A \cdot \frac{\tau}{2}\right]\begin{pmatrix} u \\ d \end{pmatrix}$ is **spontaneously broken** in the QCD vacuum.

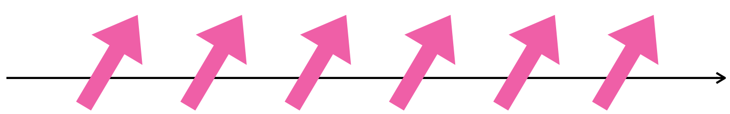


RupertMillard, Public domain, via Wikimedia Commons

The vacuum can lower its energy by allowing $q\bar{q}$ pairs to pop into existence in a coherent way. The specific phase of the various flavors and chiralities is arbitrary, but it should be the same everywhere.

## Analogy: 1D chain of spins

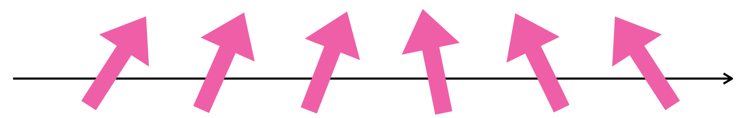


1D chain of atoms where the g.s. has no magnetic moment, but excited state has a magnetic moment. Nearest-neighbor interaction between spins wants them to line up. If the interaction is strong enough, the ground state of the system will have all atoms in the excited state, with moments all pointing in the same arbitrary direction.

# Consequences of spontaneously broken chiral symmetry

Rotating all the spins doesn't change the energy

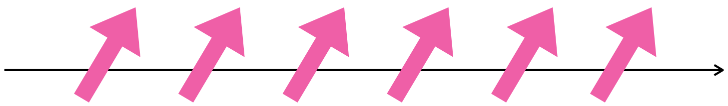
Applying an $x$-dependent rotation *does* change the energy, $\propto \dfrac{\partial \vec{s}}{\partial x} \leftrightarrow k$

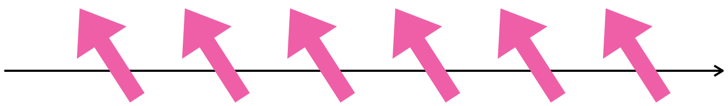


$E \sim k$, for infinitely long wavelength $k \to 0$, $E \to 0$. Looks like a massless particle. This is a **Goldstone mode** → **Goldstone boson**.

In QCD, the Goldstone bosons of spontaneously broken axial symmetry are the pions $\pi^0, \pi^\pm$, which should then be massless.

# Consequences of spontaneously broken chiral symmetry





In the limit $k \to 0$ the creation of a Goldstone mode corresponds to a transformation to an equivalent vacuum. The only way their presence can be detected is when they change with $x$.

$\Rightarrow$ Goldstone bosons have no non-derivative couplings.

"Soft" pions are weakly interacting, so we can do perturbation theory!

# Naive Dimensional Analysis

Lagrangian has mass dimension $[\mathscr{L}] = M^4$

Building blocks:
- Nucleon field: $[N] = M^{3/2}$
- Pion field: $[\pi] = M$
- Derivative: $[\partial] = M$

Dimensonful parameters:
- pion mass $m_\pi \approx 140$ MeV
- pion decay const. $f_\pi \approx 92$ MeV
- $M_N \sim m_\rho \sim 4\pi f_\pi \equiv \Lambda_\chi \sim 700$ MeV

$$\mathscr{L} \sim c_{lmn} f_\pi^2 \Lambda_\chi^2 \left(\frac{\bar{N}N}{f_\pi^2 \Lambda_\chi}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{\partial, m_\pi}{\Lambda_\chi}\right)^n$$

Low Energy Constant, $\mathscr{O}(1)$

"Naturalness"

$$\mathscr{L}_{\text{free}} = \bar{N}(i\partial - M)N + \frac{1}{2}\partial\pi\partial\pi + \frac{1}{2}m_\pi^2\pi^2$$

Manohar & Georgi, Nuc. Phys. B 234,189 (1984),
Friar, Few Bod. Syst. 22, 161 (1997)

# Naive Dimensional Analysis

$$\mathscr{L} \sim c_{lmn} f_\pi^2 \Lambda_\chi^2 \left(\frac{\bar{N}N}{f_\pi^2 \Lambda_\chi}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{\partial, m_\pi}{\Lambda_\chi}\right)^n$$

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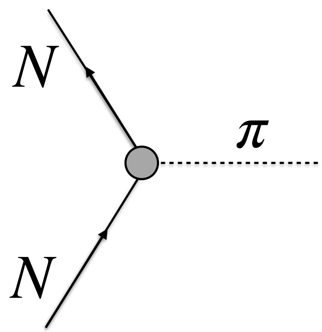
Example: $\pi NN$ vertex, $l = m = n = 1.$




$$\mathscr{L} \sim c_{\pi NN} f_\pi^2 \Lambda_\chi^2 \left(\frac{\bar{N}N}{f_\pi^2 \Lambda_\chi}\right) \left(\frac{\pi}{f_\pi}\right) \left(\frac{\partial}{\Lambda_\chi}\right)$$

$$\mathscr{L}_{\pi NN} = \frac{c_{\pi NN}}{f_\pi} \left(\bar{N}\partial N\right) \pi$$

$$\mathscr{L}_{\pi NN} = \frac{g_A}{2f_\pi} \bar{N}\sigma \cdot \partial \tau^a N \pi^a$$ (Goldberger-Treiman)

$$c_{\pi NN} = \frac{g_A}{2} = \frac{1.27}{2} = \mathcal{O}(1)$$ ✓

# Naive Dimensional Analysis

$$\mathscr{L} \sim c_{lmn} f_\pi^2 \Lambda_\chi^2 \left( \frac{\bar{N}N}{f_\pi^2 \Lambda_\chi} \right)^l \left( \frac{\pi}{f_\pi} \right)^m \left( \frac{\partial, m_\pi}{\Lambda_\chi} \right)^n$$

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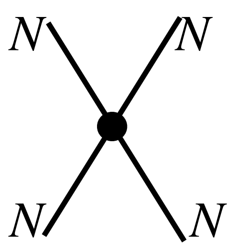
Example: contact, $l=2, m=n=0$




$$\mathscr{L} \sim c_{NN} f_\pi^2 \Lambda_\chi^2 \left( \frac{\bar{N}N}{f_\pi^2 \Lambda_\chi} \right)^2$$

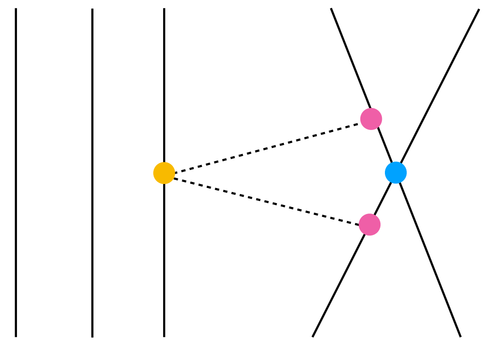
$$\mathscr{L}_{NN} = \frac{c_{NN}}{f_\pi^2} \left( \bar{N}N \right)^2$$

From Entem & Machleidt, $C_S \approx -0.01 \times 10^4 \text{ GeV}^{-2} \Rightarrow c_{NN} \approx -0.8$ ✓

# Power counting for the amplitude

We want to estimate the order $\left(\frac{Q}{\Lambda_\chi}\right)^\nu$ of an arbitrary amplitude. There will be:

- $Q^4$ for each loop $\sim \int d^4p$
- $Q^{-2}$ for each pion propagator $\frac{1}{p^2 - m^2}$
- $Q^{-1}$ for each nucleon propagator $\frac{1}{\gamma^\mu p_\mu - M}$
- $Q^1$ for each derivative at a vertex
- $Q^{-4}$ for momentum conserving $\delta$ functions



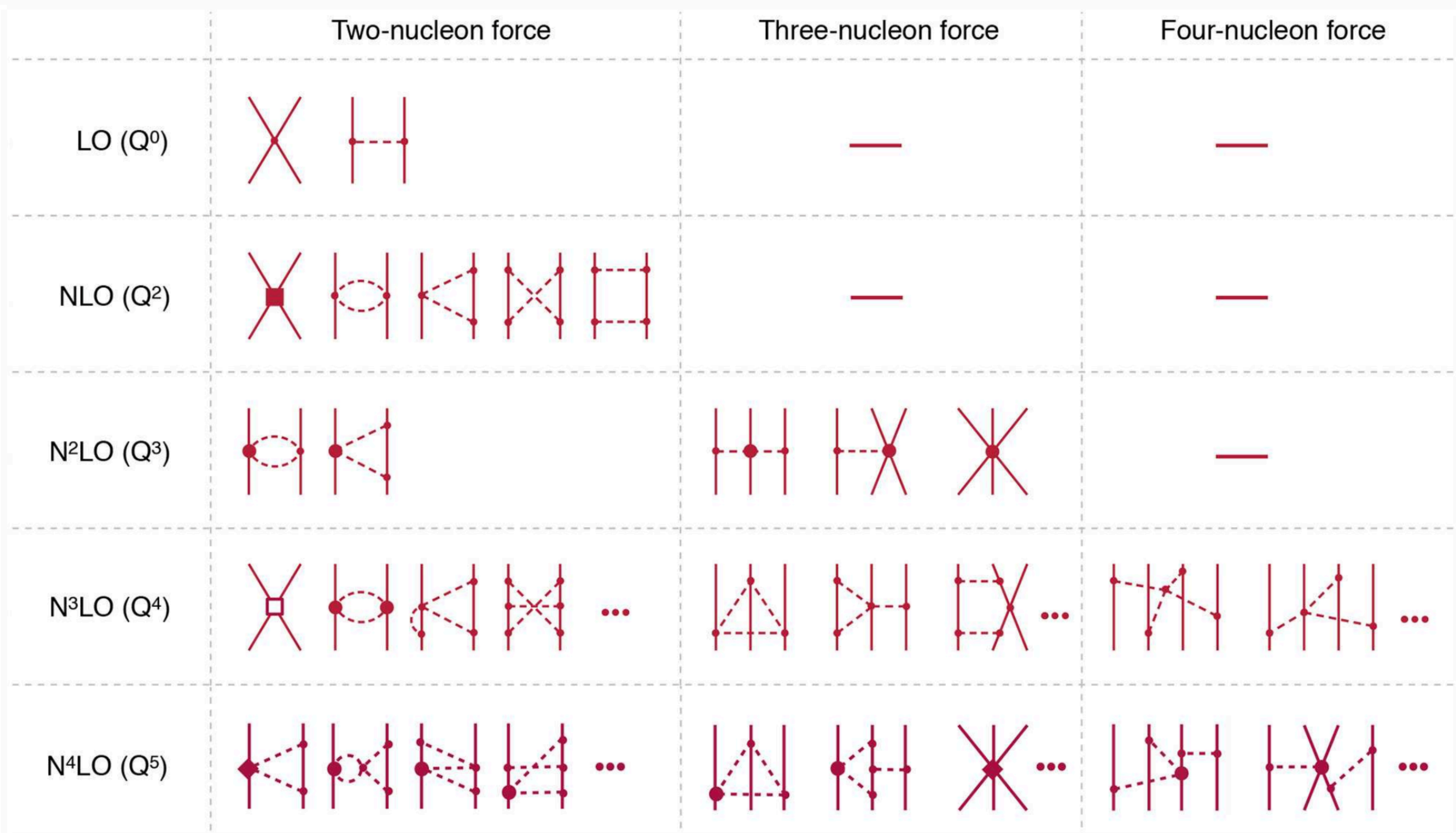
$$\nu = 1 + 2(A - C + L) + \sum_i \Delta_i$$

nucleons

clusters

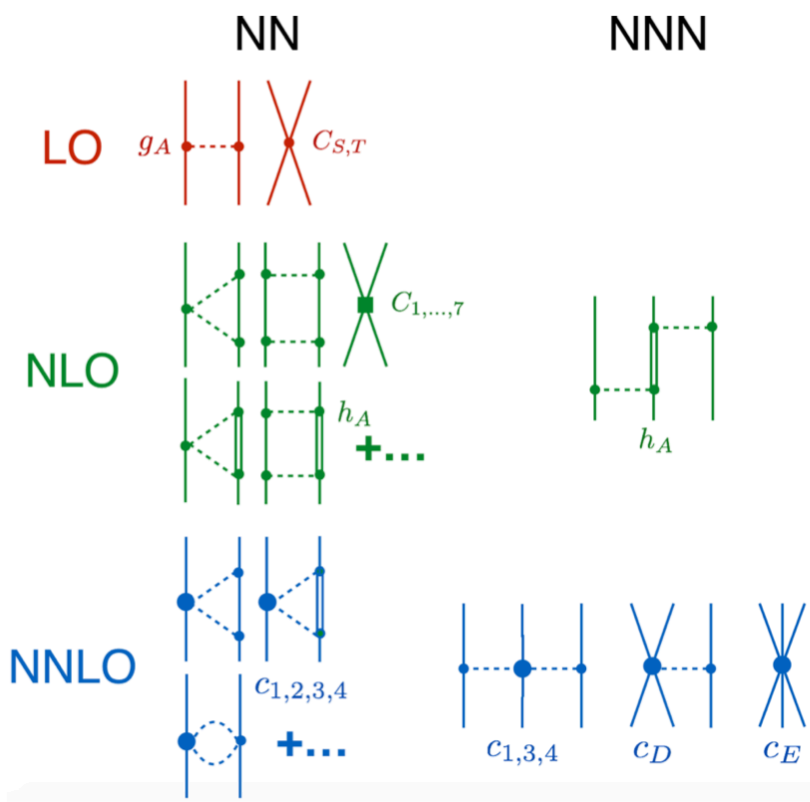
loops

vertices



Two-nucleon force
Three-nucleon force
Four-nucleon force
LO (Q⁰)
NLO (Q²)
N²LO (Q³)
N³LO (Q⁴)
N⁴LO (Q⁵)

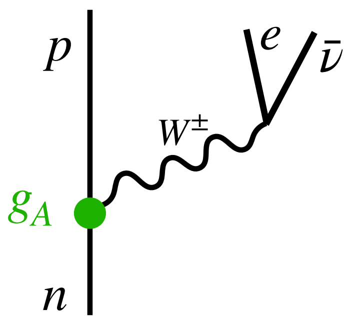
# Explicitly including Δ isobars

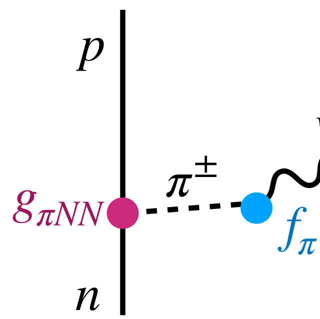


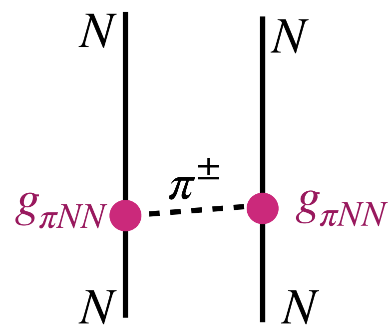



Ekström+ PRC (2018)

# Weak interactions

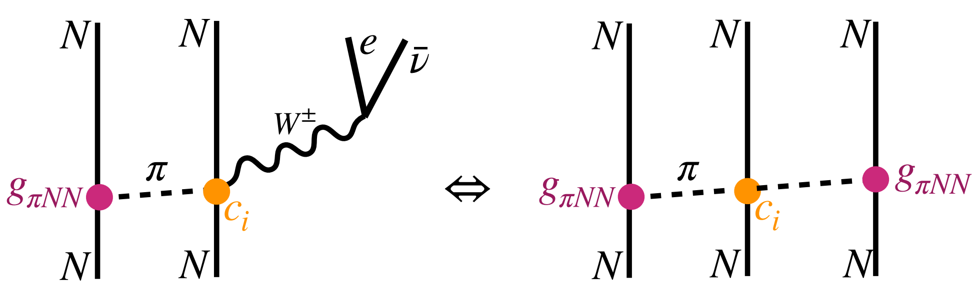


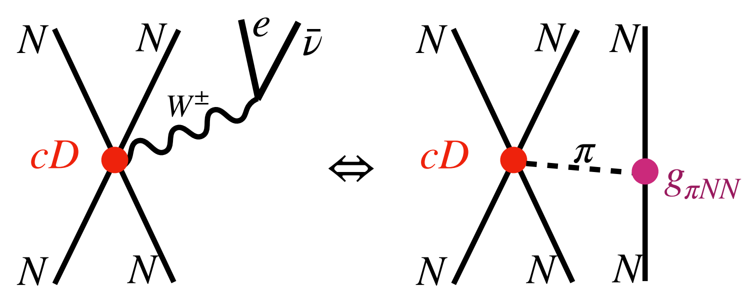




Goldberger-Treiman

$$\frac{g_{\pi NN}}{M_N} = \frac{g_A}{f_\pi}$$



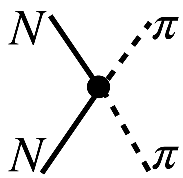



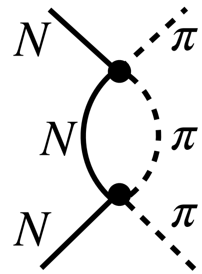

Gardestig + Phillips, PRL (2006)

# Iteration of LO vertices

Born series

$T = V + VGV + VGVGV + \ldots$



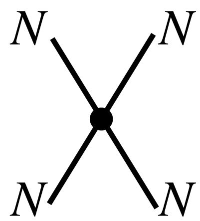

$\sim \bar{N}N(\pi\partial_0\pi)$

$$\sim \int d^3q \bar{N}N(\pi\partial_0\pi)\frac{1}{q}\bar{N}N(\pi\partial_0\pi)$$

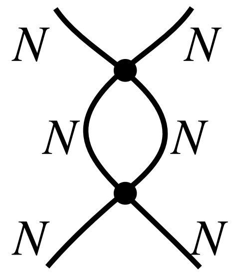
Near $q \to 0$, denominator gets small, but coupling to pion vanishes.

This is sub-leading. 👍

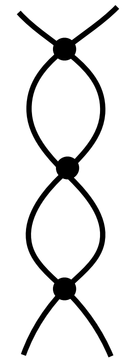
# Iteration of the NN interaction




$$\sim \frac{1}{f_\pi^2}$$




$$\sim \int d^3q \frac{1}{f_\pi^2} \frac{1}{q^2/M} \frac{1}{f_\pi^2}$$

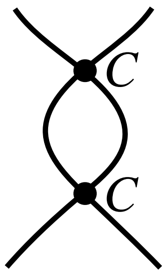


…

Enhancement as $q \to 0$, no derivative couplings to suppress it. 😬

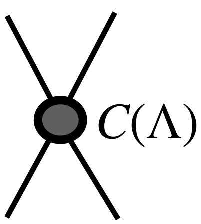
Nuclei form bound states which are *inherently* non-perturbative. Use $\chi_{\mathrm{PT}}$ to generate the **potential** $V$ order by order, and iterate $V$ to all orders by solving the Schrödinger or Lippman-Schwinger equation.

# Renormalization

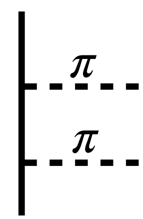



$$\sim \int_0^\infty d^3q\, C \frac{1}{q^2/M} C \qquad \to \infty$$

replace: $$\sim \int_0^\Lambda d^3q\, C \frac{1}{q^2/M} C$$ $\to$ finite, but depends on $\Lambda$



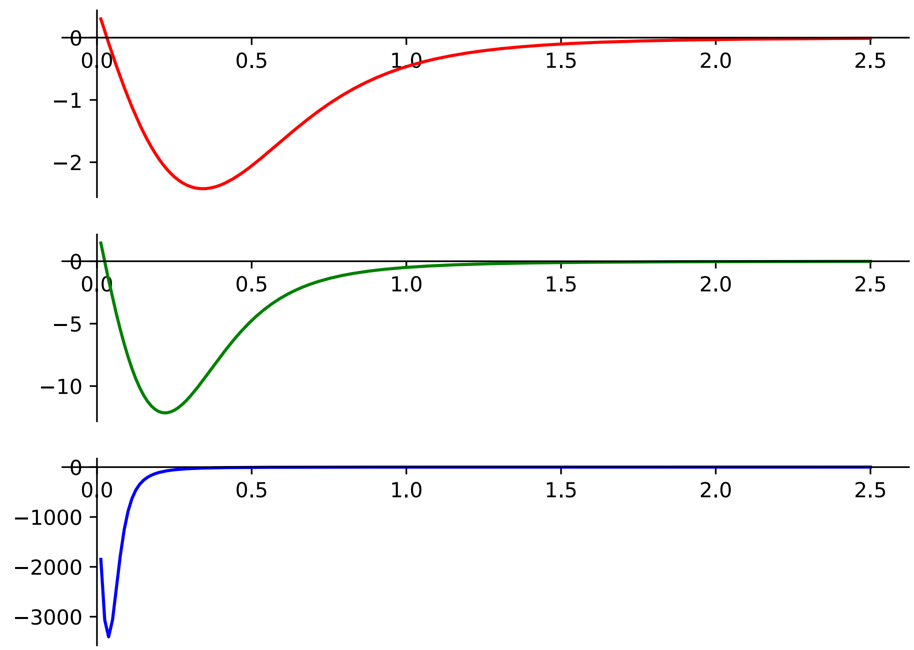

"Counter term": encodes stuff happening at momenta above $\Lambda$. Looks just like our leading order contact! Coefficient $C(\Lambda)$ changes with $\Lambda$ so that physical prediction is independent of $\Lambda$. Then we can take $\Lambda \to \infty$ and declare victory.




This also diverges as $q \to \infty$

$$V_{1\pi}(r) = \left(\frac{g_{\pi NN}}{2M_N}\right)^2 \frac{m_\pi^2}{12\pi}(\tau_1 \cdot \tau_2)\left[S_{12}(\hat{r})\left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2}\right) + \sigma_1 \cdot \sigma_2\right]\frac{e^{-m_\pi r}}{r}$$

$1/r^3$ potential is too singular. As $\Lambda \to \infty$ ($r$ cutoff range $\to 0$) we see more of the deep potential, and the wave function gets sucked in. Once that happens, momenta get $p \gg m_\pi$ and the whole thing breaks down.

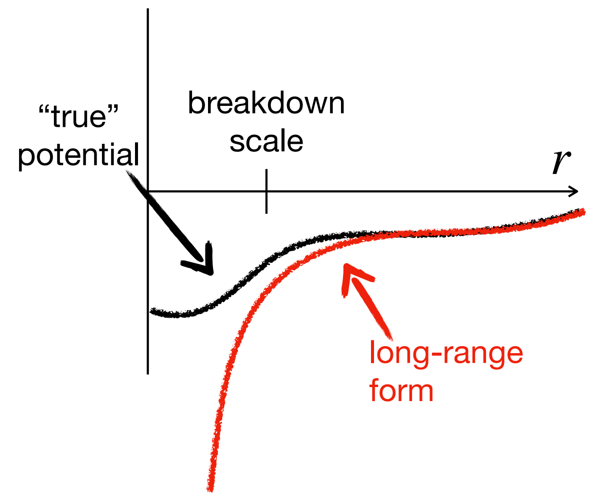

Doc! It hurts when I take my cutoff to infinity!
Then don't do that.

# An argument why you shouldn't take $\Lambda \rightarrow \infty$

- The EFT is valid for $Q \lesssim \Lambda_\chi$. Beyond that, the form of the interaction is almost certainly wrong.

- Taking $\Lambda > \Lambda_\chi$ cannot improve the result, and may introduce errors/instabilities.

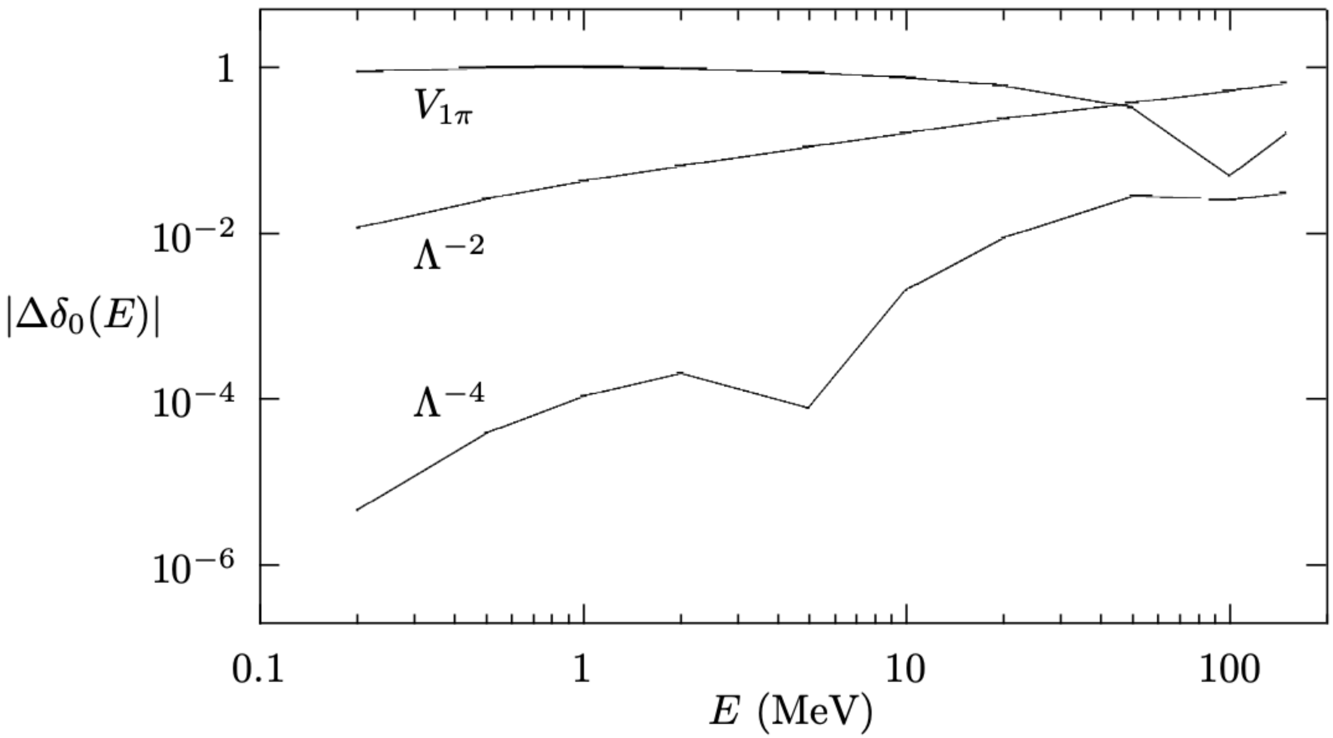
- As long as the regulator artifacts are within the EFT truncation error, there is no reason to do better.




$$e^{-(p/\Lambda)^2} = 1 - \left(\frac{p}{\Lambda}\right)^2 + \frac{1}{2}\left(\frac{p}{\Lambda}\right)^4 + \ldots$$

Entem & Machleidt Phys. Rep. (2011)
Lepage nucl-th/9706029 (1997)
Epelbaum & Gegalia EPJA (2009)

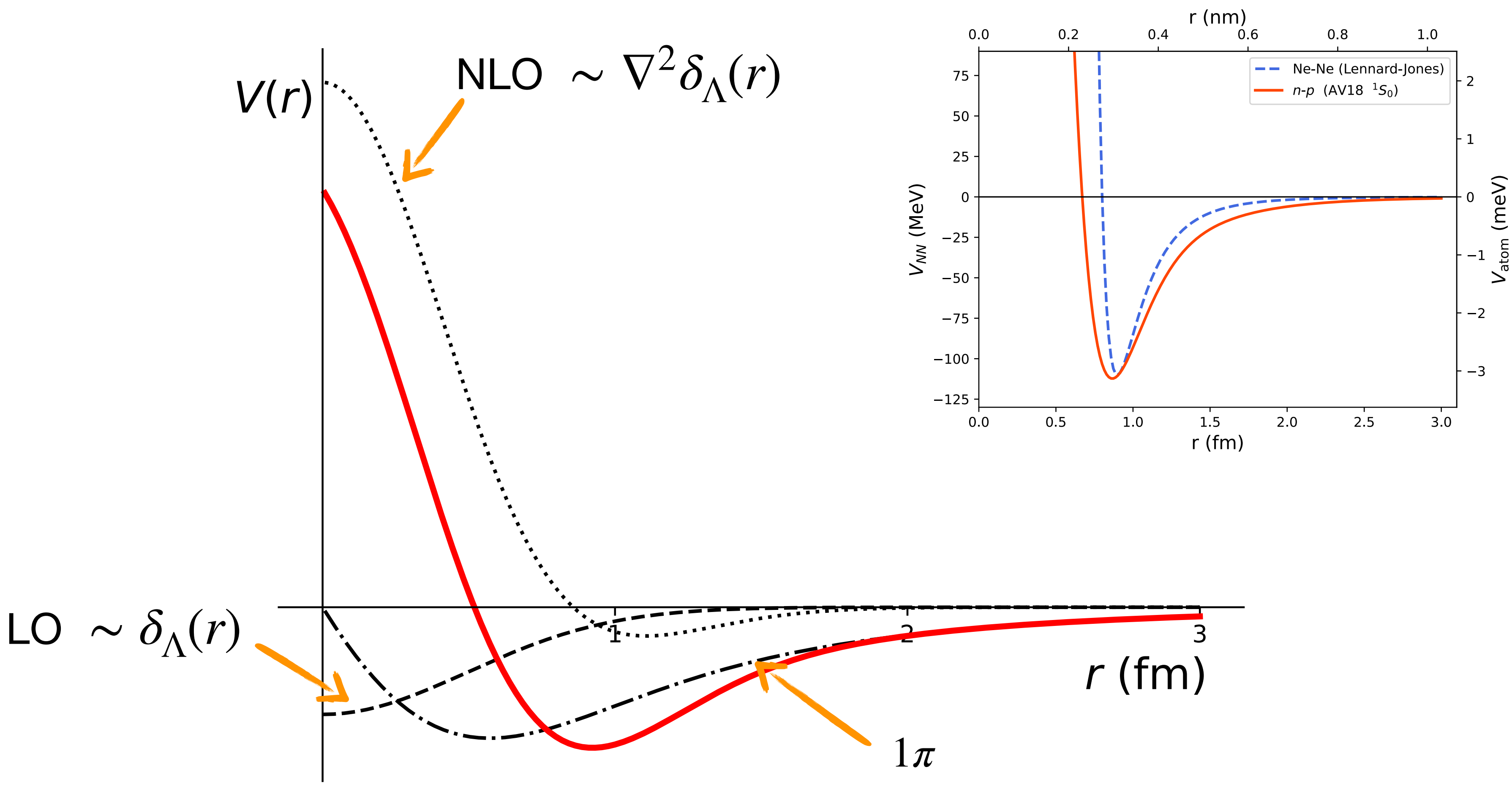
# Lepage Plot




When going to higher orders in the EFT, the most improvement should come at low momentum/energy. At $E \sim \Lambda_\chi$, there should be no improvement.

Lepage nucl-th/9706029 (1997)

# Connecting back to our simple picture of the potential



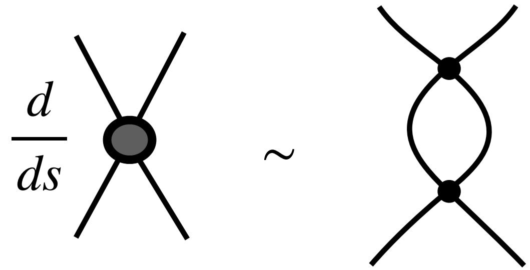
# Similarity renormalization group

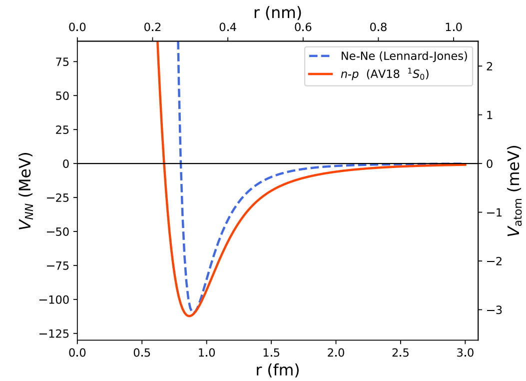
$$H(s) = U(s) H U^{\dagger}(s)$$

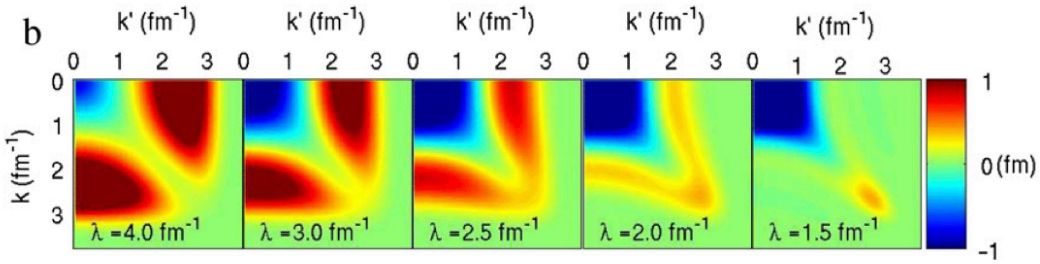
$$\frac{dU}{ds} = \eta(s) U(s)$$

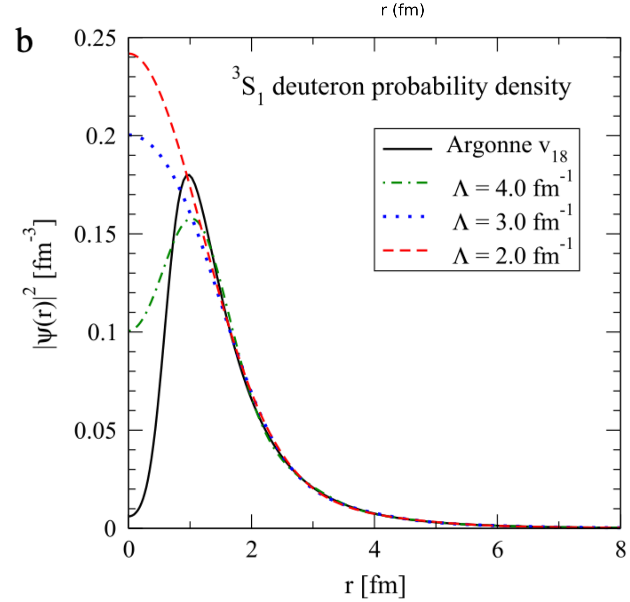
$$\frac{dH}{ds} = [\eta, H]$$

$$\eta = [T, V]$$




Bogner+ PPNP (2010)