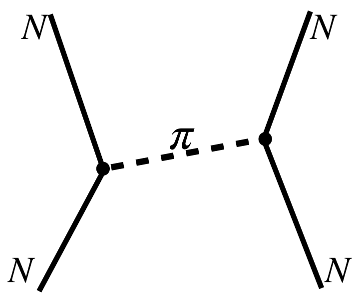
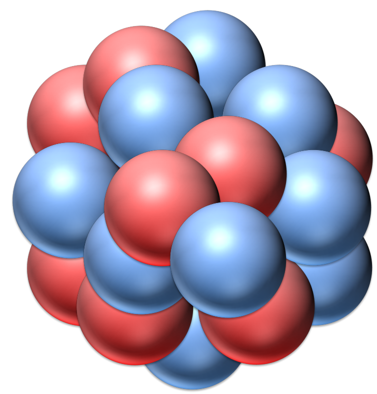
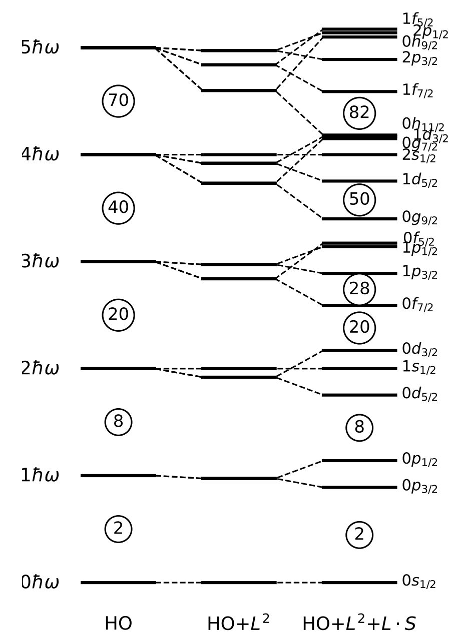
Nuclear Structure
&
Nuclear Forces







National Nuclear Physics Summer School
Seattle, WA, 2026

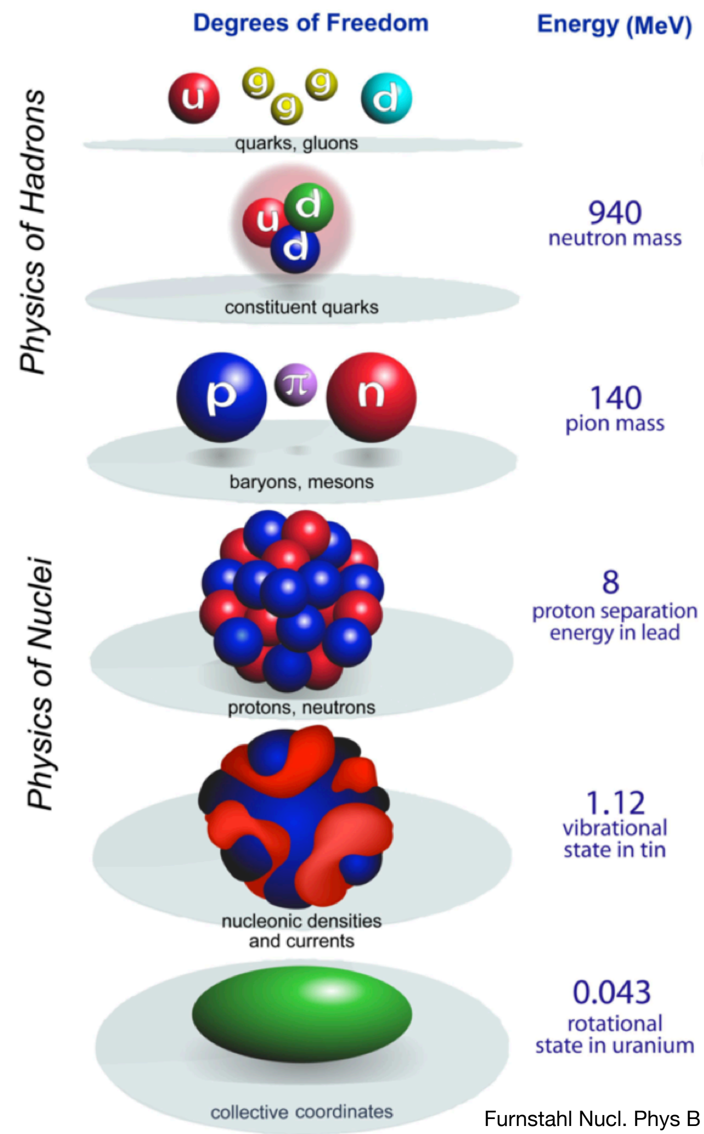
Ragnar Stroberg
University of Notre Dame

The Plan

Lecture 1. Nuclear Structure Phenomena

Lecture 2. Nuclear Forces

Lecture 3. Manifestations of Nuclear Forces in Nuclear Structure



Furnstahl Nucl. Phys B (2012)

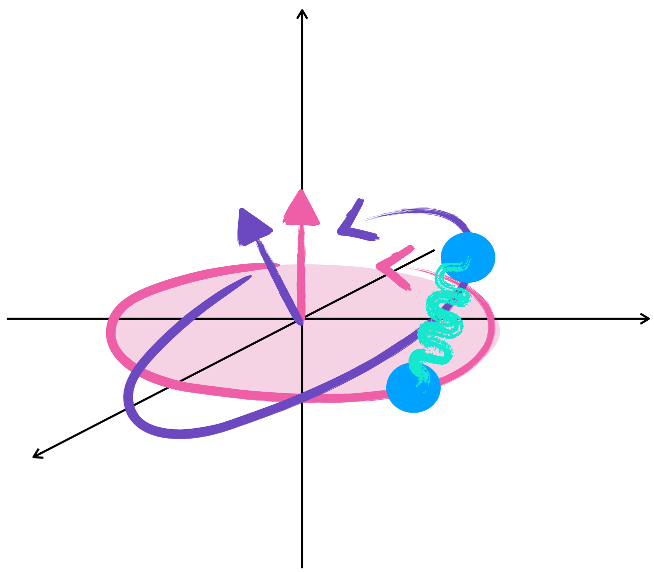
Part 1: Nuclear structure phenomena

1. Saturation

2. Shell structure

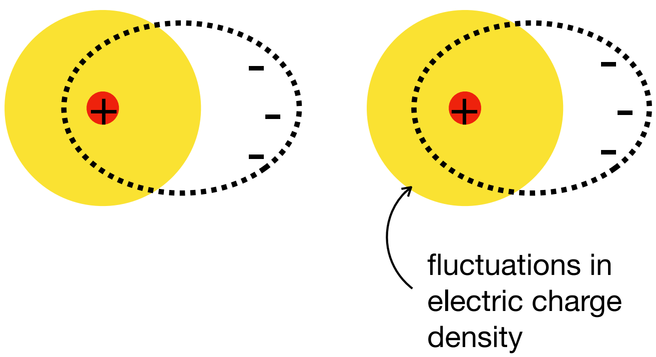
3. Pairing

4. Shapes

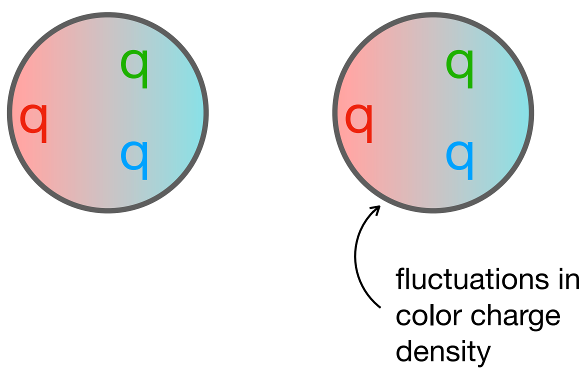


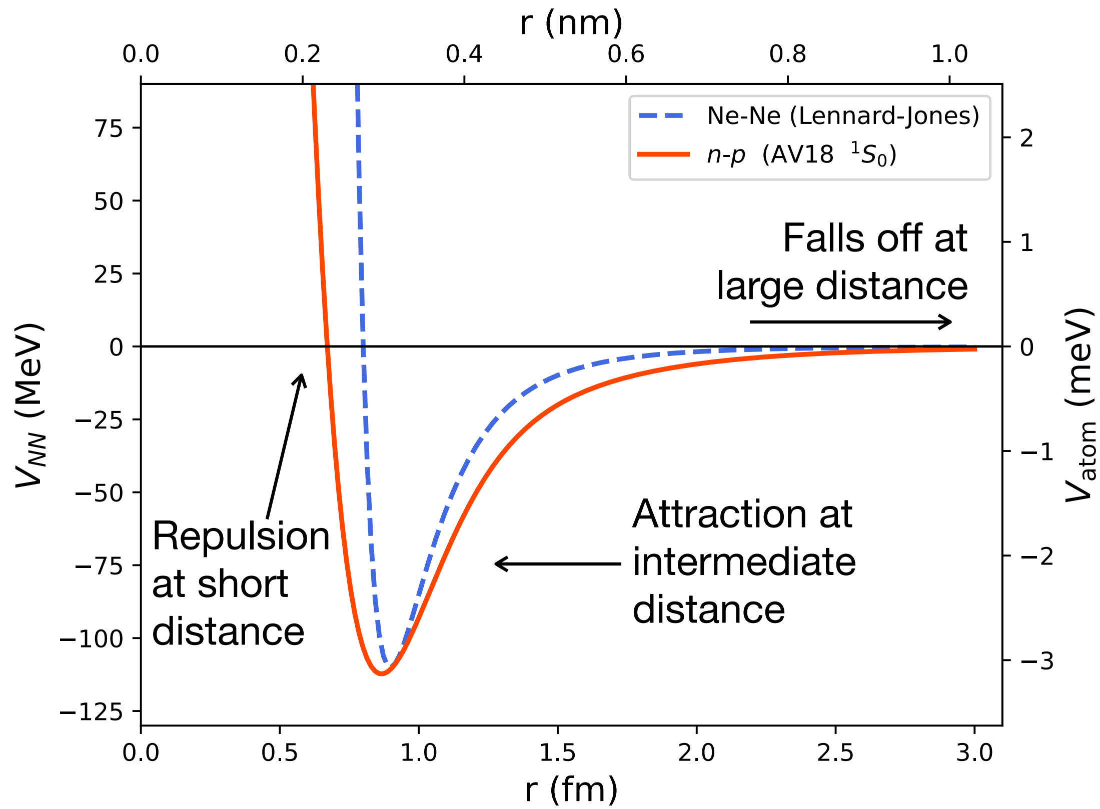
But first: a cartoon of the nuclear force

van der Waals forces between neutral atoms

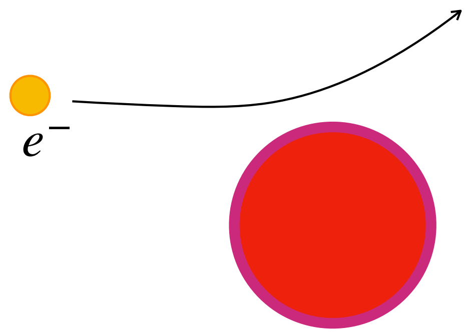


residual strong interaction between nucleons

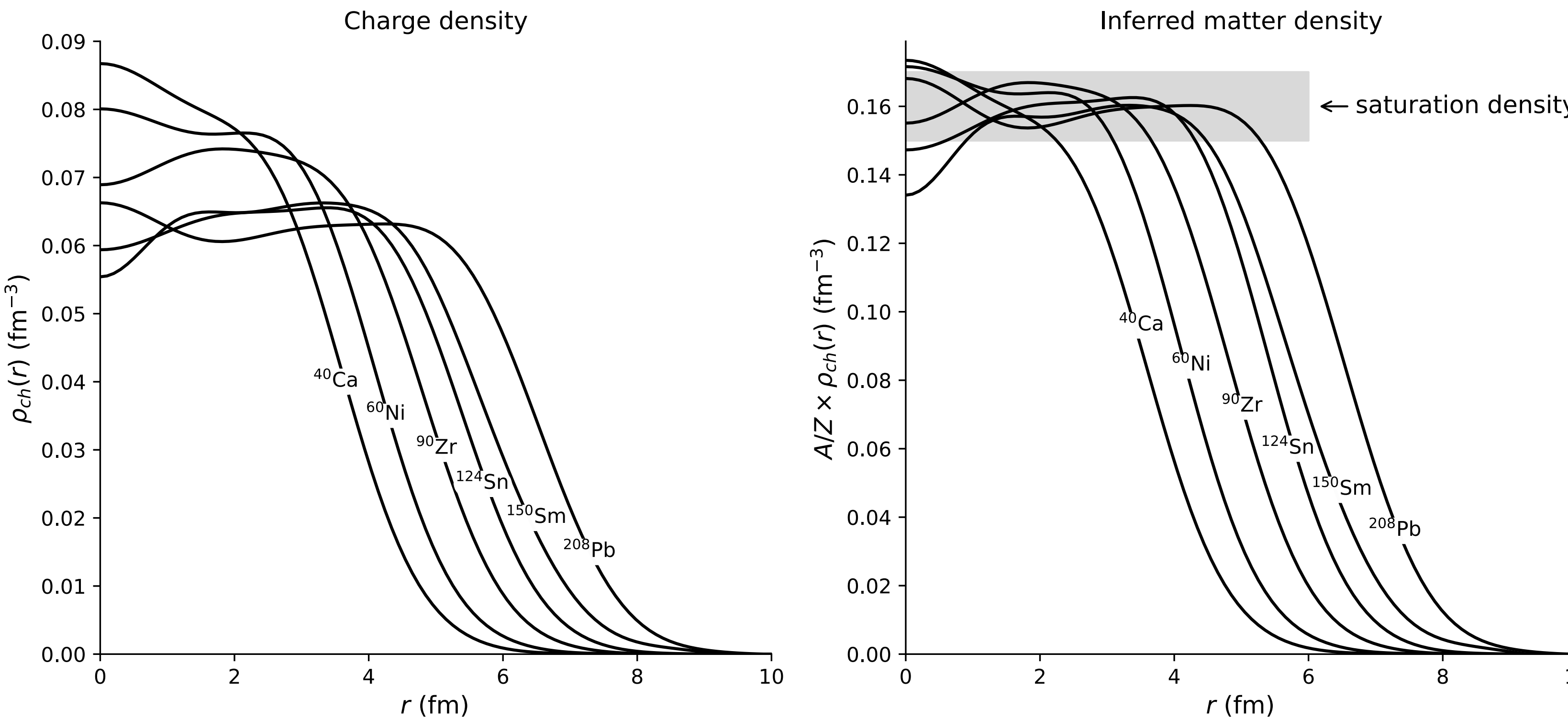




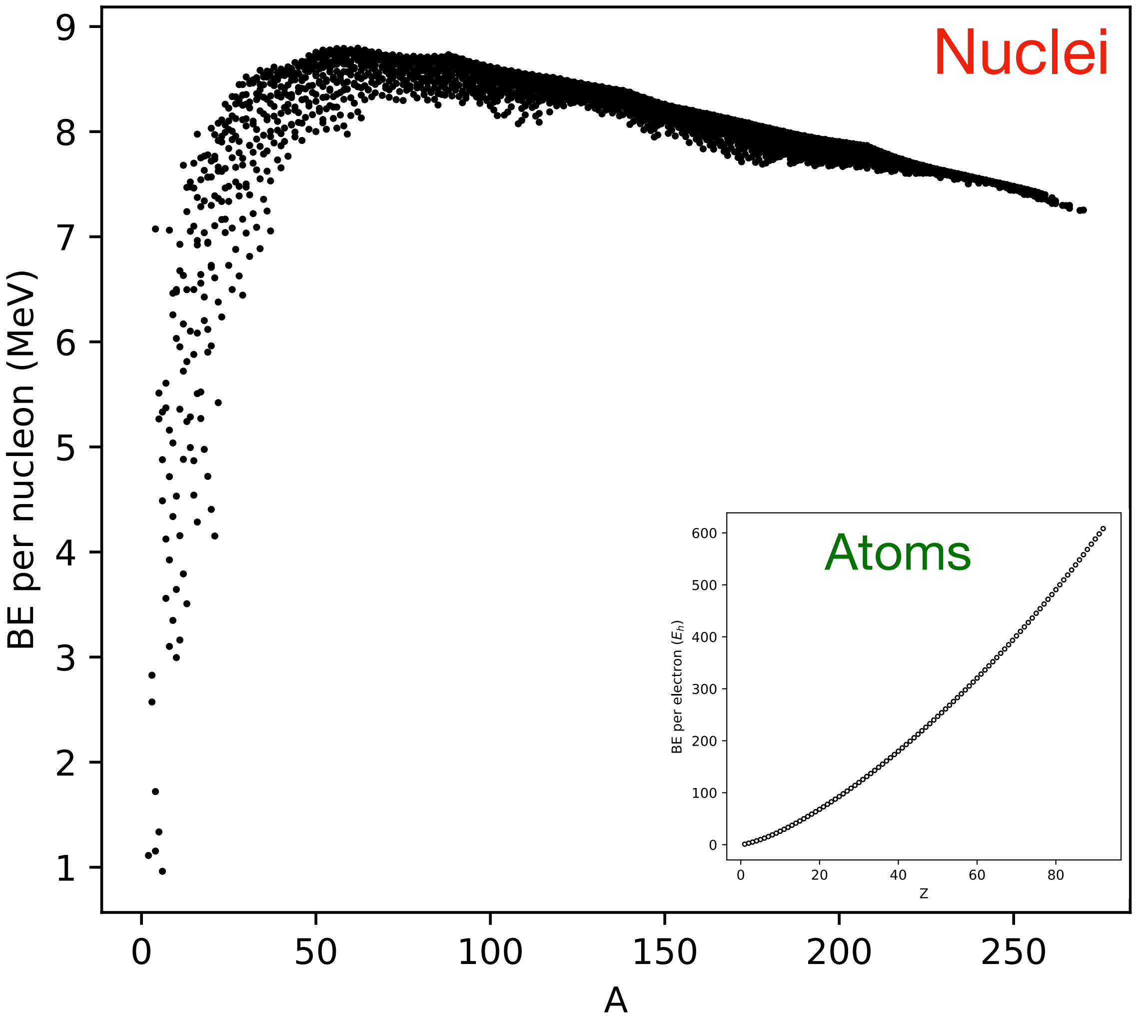
Saturation



$$R = r_0 A^{1/3}$$



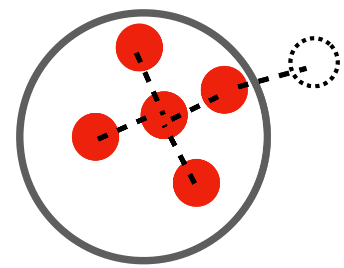
Saturation

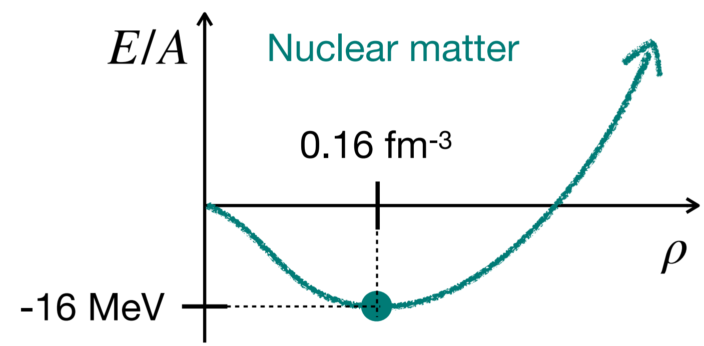


Liquid drop model

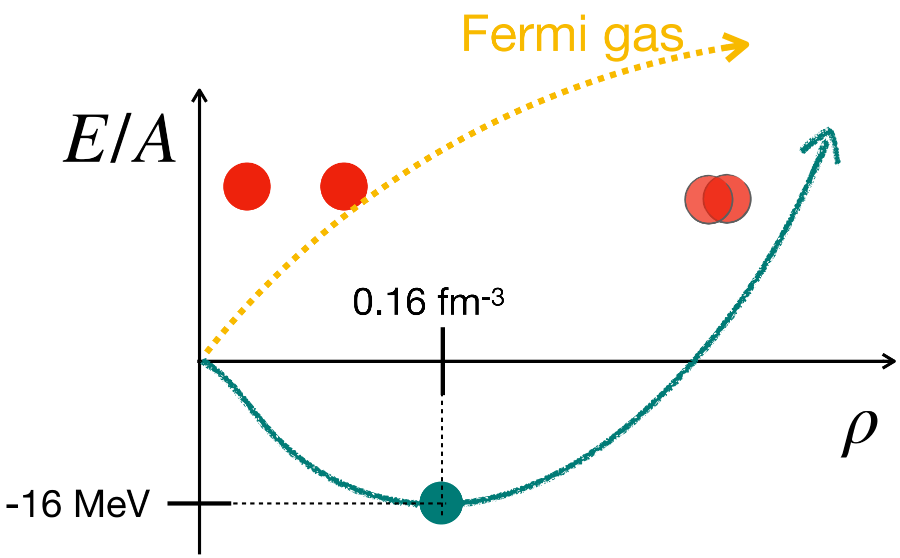
$$BE_{\rm LD} = \alpha_V A - \alpha_S A^{2/3} - \alpha_C \frac{Z^2}{A^{1/3}} - \alpha_A \frac{(N-Z)^2}{A}$$

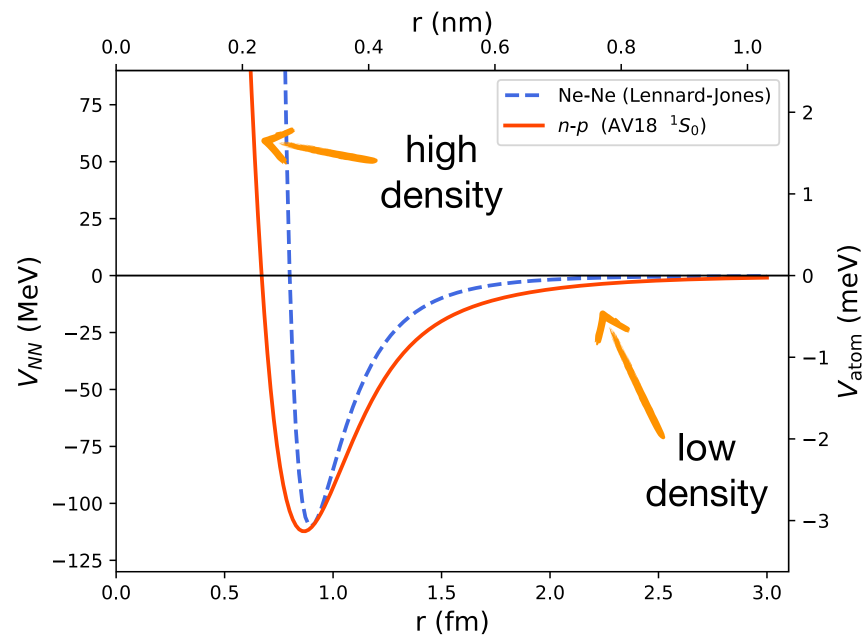
≈ 16 MeV





Saturation





$E_{FG}/A \propto \rho^{2/3}$

$\langle V_{NN}/A \rangle \sim \rho^2$

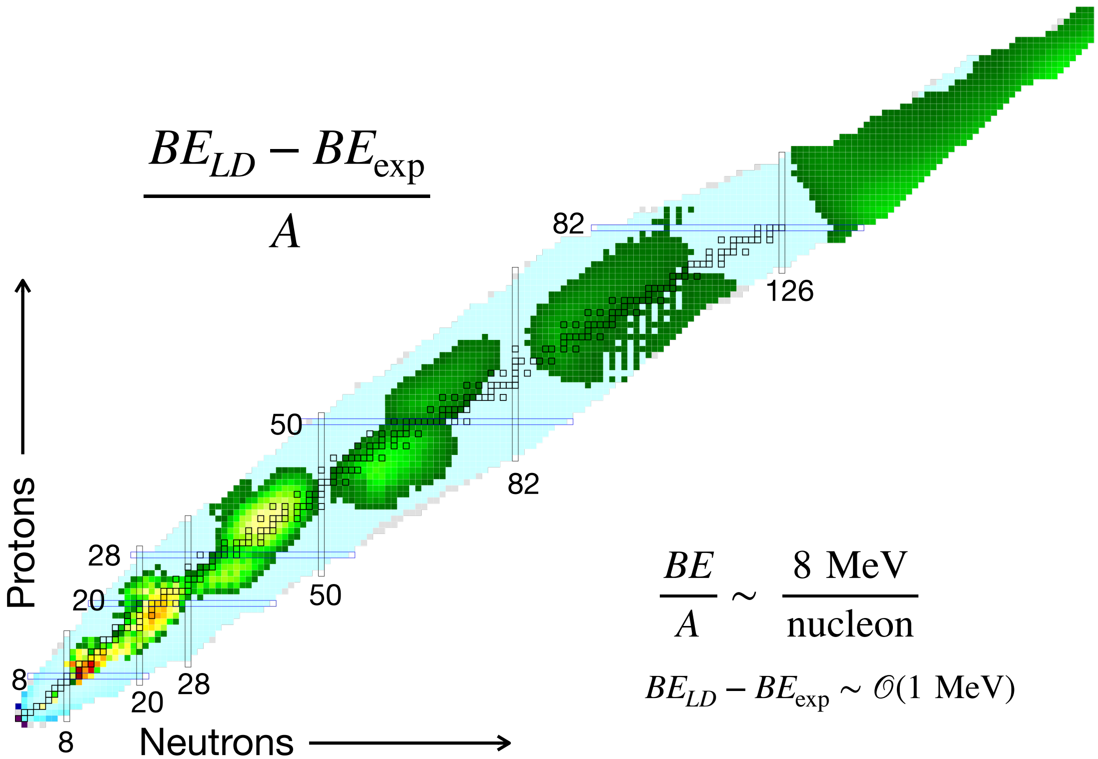
$\langle V_{3N}/A \rangle \sim \rho^3$

Saturation requires either a V_{NN} that changes sign with r, or many-body forces.

Shell structure

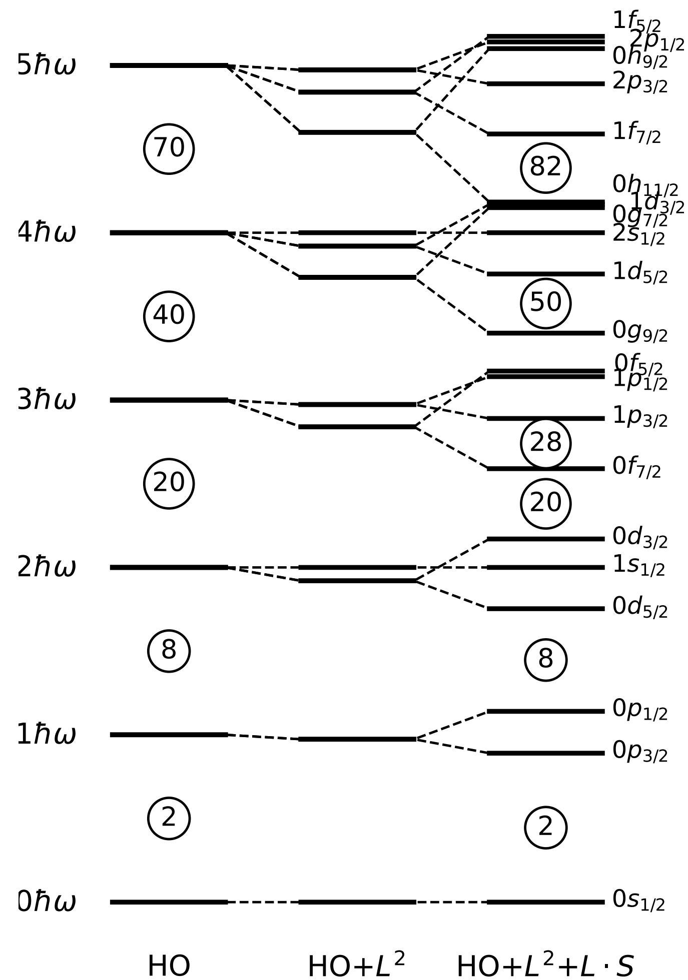
$$BE_{LD} = \alpha_V A - \alpha_S A^{2/3} - \alpha_C \frac{Z^2}{A^{1/3}} + \alpha A \frac{(N-Z)^2}{A} + \Delta_{\text{pair}}$$

$$\frac{BE_{LD} - BE_{\exp}}{A}$$



$$\frac{BE}{A} \sim \frac{8 \text{ MeV}}{\text{nucleon}}$$

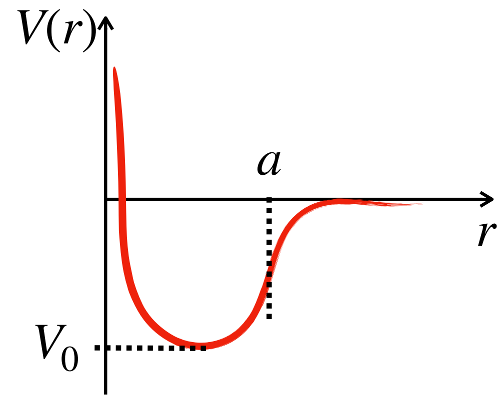
$$BE_{LD} - BE_{\exp} \sim \mathcal{O}(1 \text{ MeV})$$

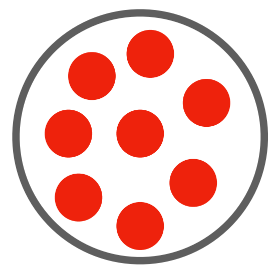


Shell structure

Requirements for shell structure:

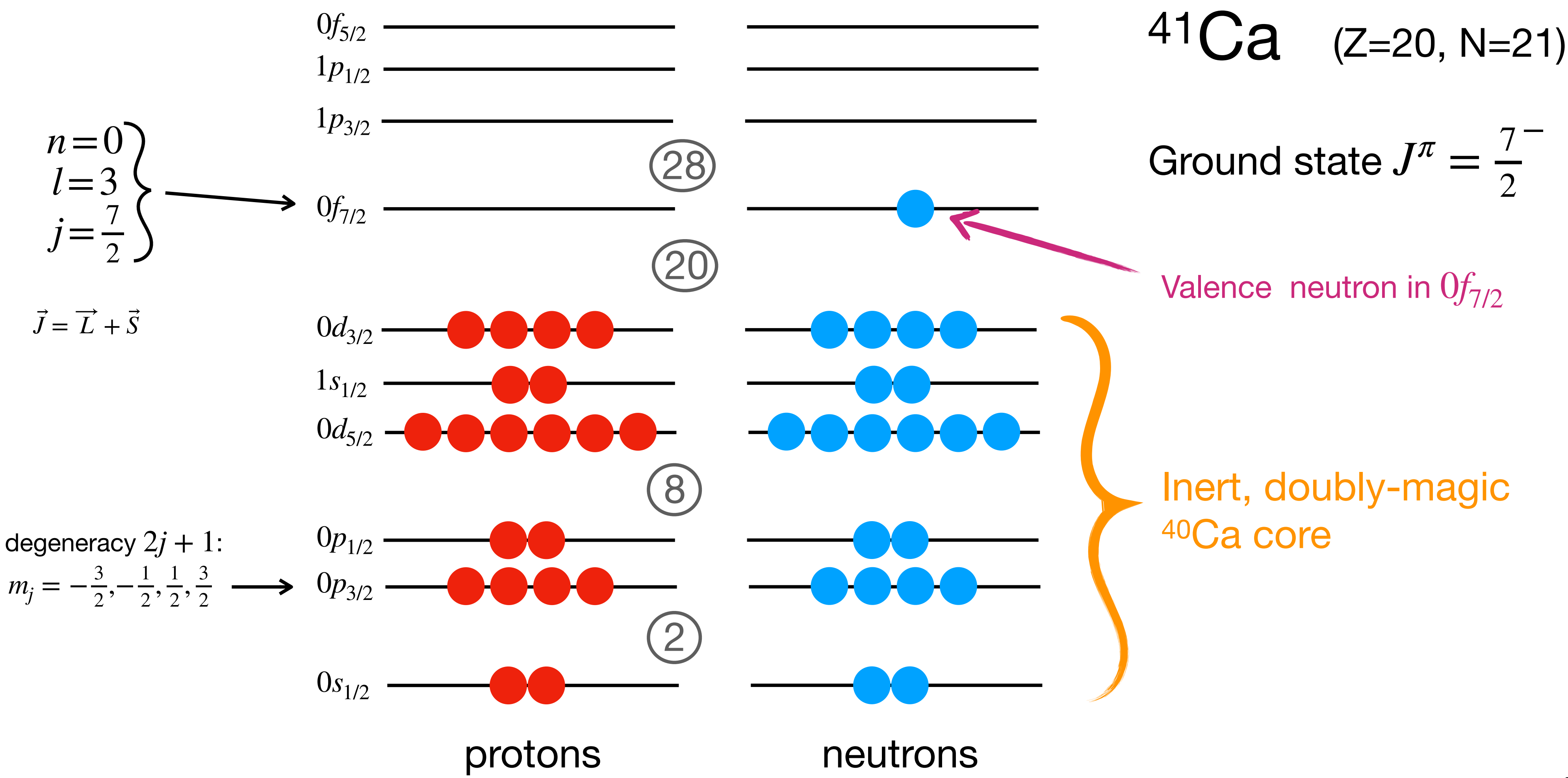
- Attractive interaction
- Symmetry ⇒ degeneracies
- Fermi-Dirac statistics
- Mottelson's "quantality": $\Lambda = \dfrac{\hbar^2}{Ma^2V_0}$



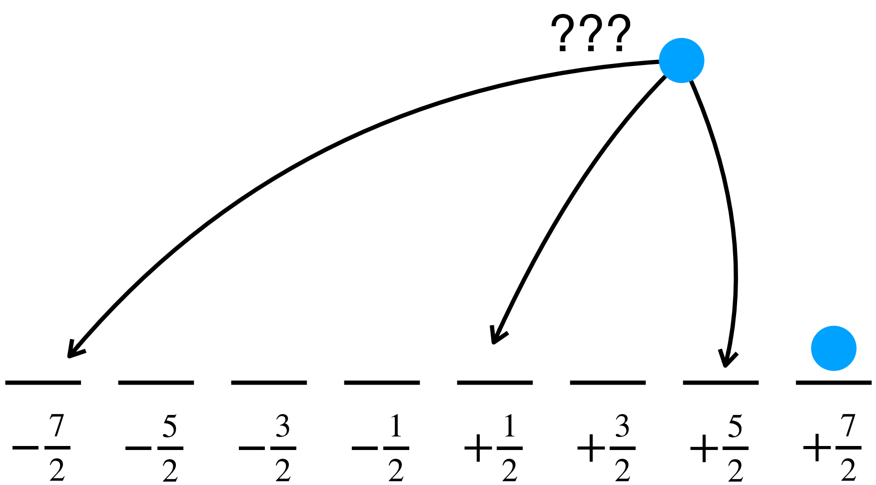


For observed nuclear shell gaps, spin-orbit splitting is essential

The Shell Model



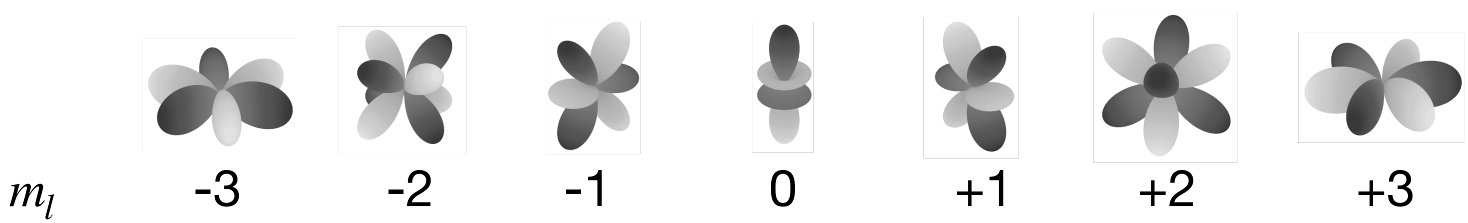
Multiple valence particles



Which m_j states get filled first depends on the *residual interaction*.

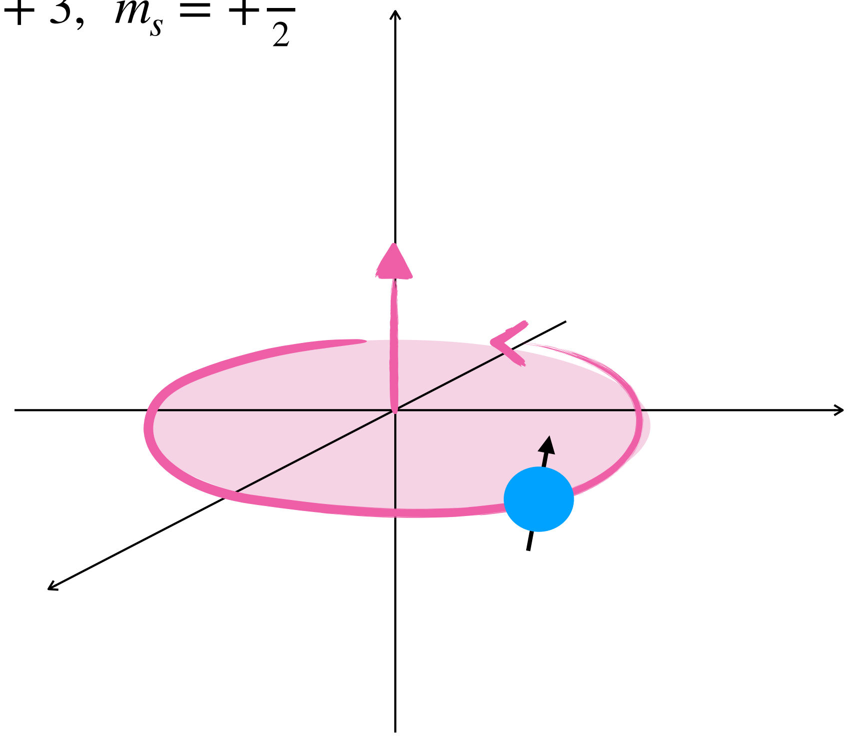
Let's consider an interaction that is short-range and attractive.

$l = 3$ spherical harmonics:



$0f_{7/2}$

$$m_j = +\frac{7}{2} \quad \Rightarrow \quad m_l = +3, \;\; m_s = +\frac{1}{2}$$



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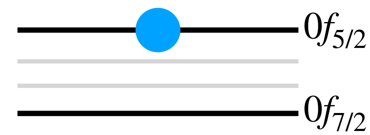
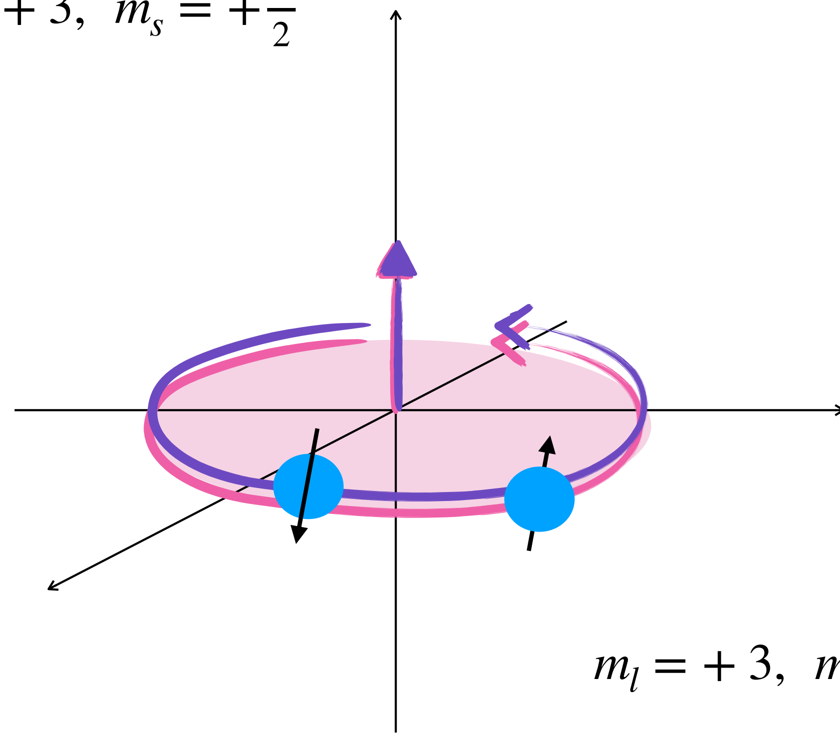




$m_l = +3, \; m_s = -\frac{1}{2} \quad \Rightarrow \quad j \sim \frac{5}{2}$

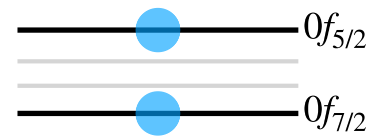
$M_L = +6 \Rightarrow L = 6$

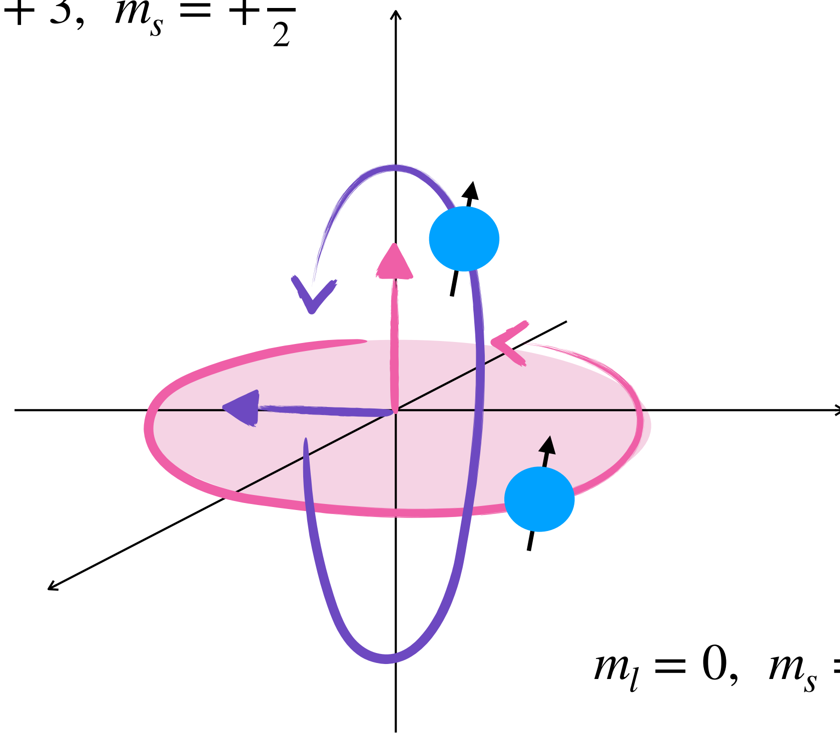
centrifugal barrier

Large overlap?

$0f_{7/2}$

$m_j = +\frac{7}{2} \quad \Rightarrow \quad m_l = +3, \; m_s = +\frac{1}{2}$



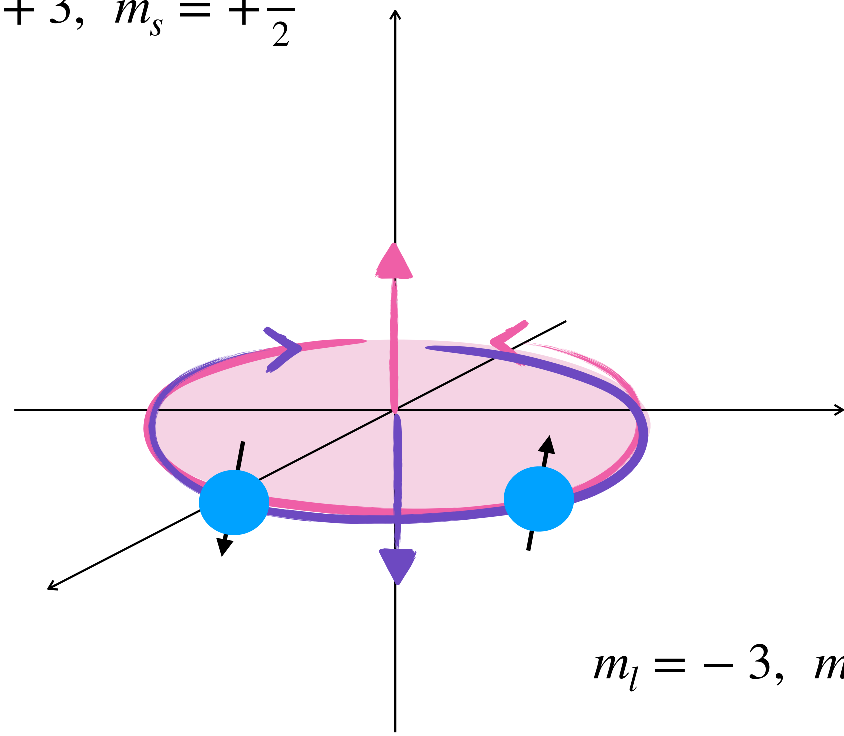


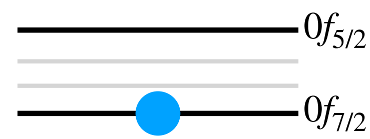
$m_l = 0, \; m_s = \frac{1}{2} \quad \Rightarrow \quad j \sim \frac{5}{2}, \frac{7}{2}$

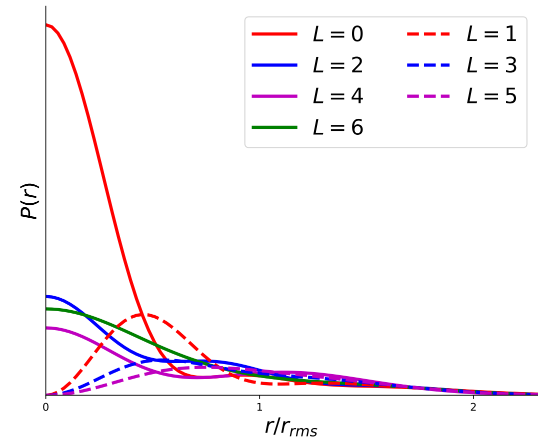
Small overlap

$0f_{7/2}$

$$m_j = +\frac{7}{2} \quad \Rightarrow \quad m_l = +3, \;\; m_s = +\frac{1}{2}$$



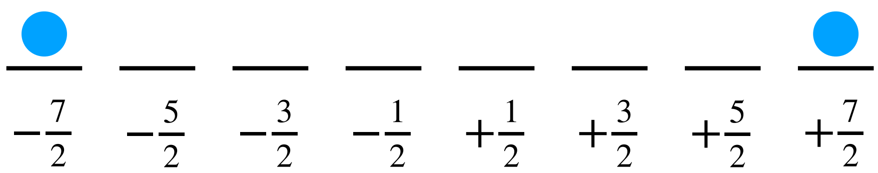




$$m_l = -3, \;\; m_s = -\frac{1}{2} \quad \Rightarrow \quad j = \frac{7}{2}$$

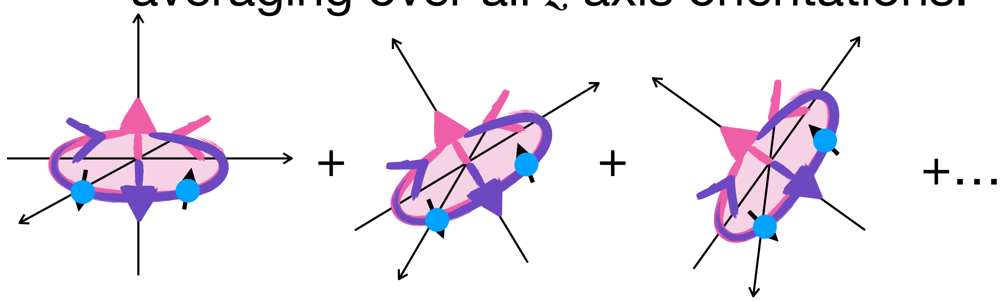
Large overlap

Multiple valence particles

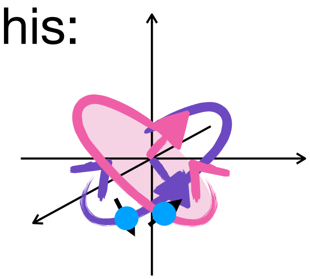


For a short-ranged, attractive interaction, in the presence of spin-orbit splitting, we should add the second particle to $m_j' = -m_j$.

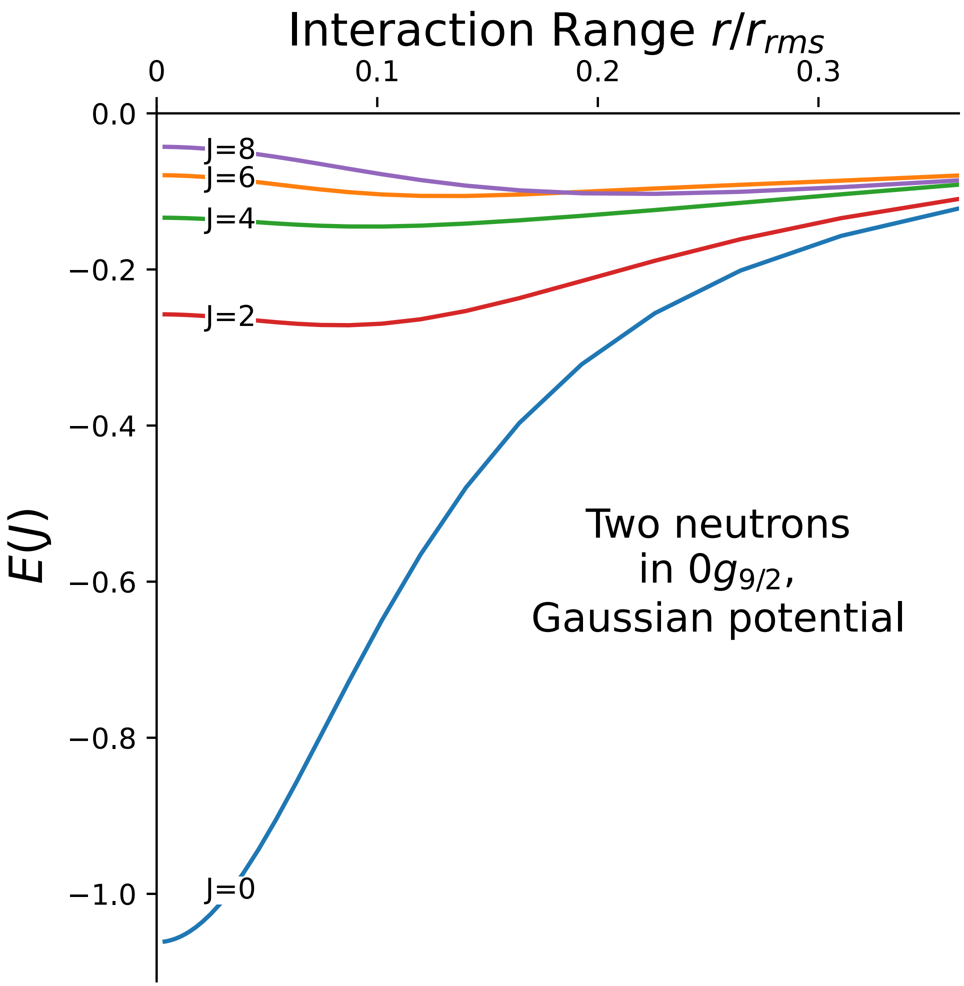
But this doesn't have good total J.
Coupling to $J = 0$ is equivalent to averaging over all z axis orientations.



Other $J \neq 0$ include configurations like this:



Upshot: 2 identical particles will prefer to couple to $J = 0$.



As the range of the interaction gets shorter, the $J = 0$ configuration drops in energy.

A simple pairing model

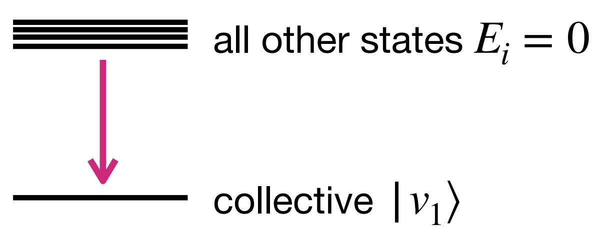
$$\langle m, -m | V | m', -m' \rangle = -G$$

Ω states $m > 0$

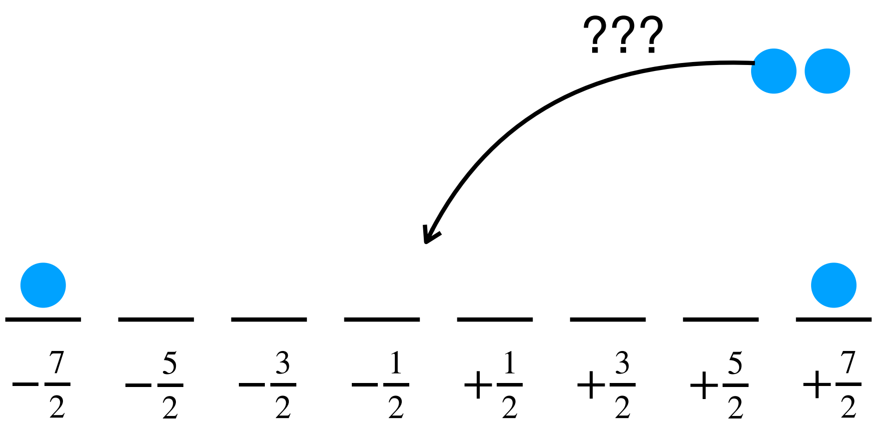
$$H = -G \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & 1 \\ 1 & 1 & \dots & 1 \end{pmatrix}$$

Ground state: $|v_1\rangle = \frac{1}{\sqrt{\Omega}} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$

$$\boxed{E_1 = -G\Omega}$$



Multiple valence particles



Now we add 2 more neutrons.
Where do they go?

The Cooper pairs effectively act like bosons. Put them in the same state!

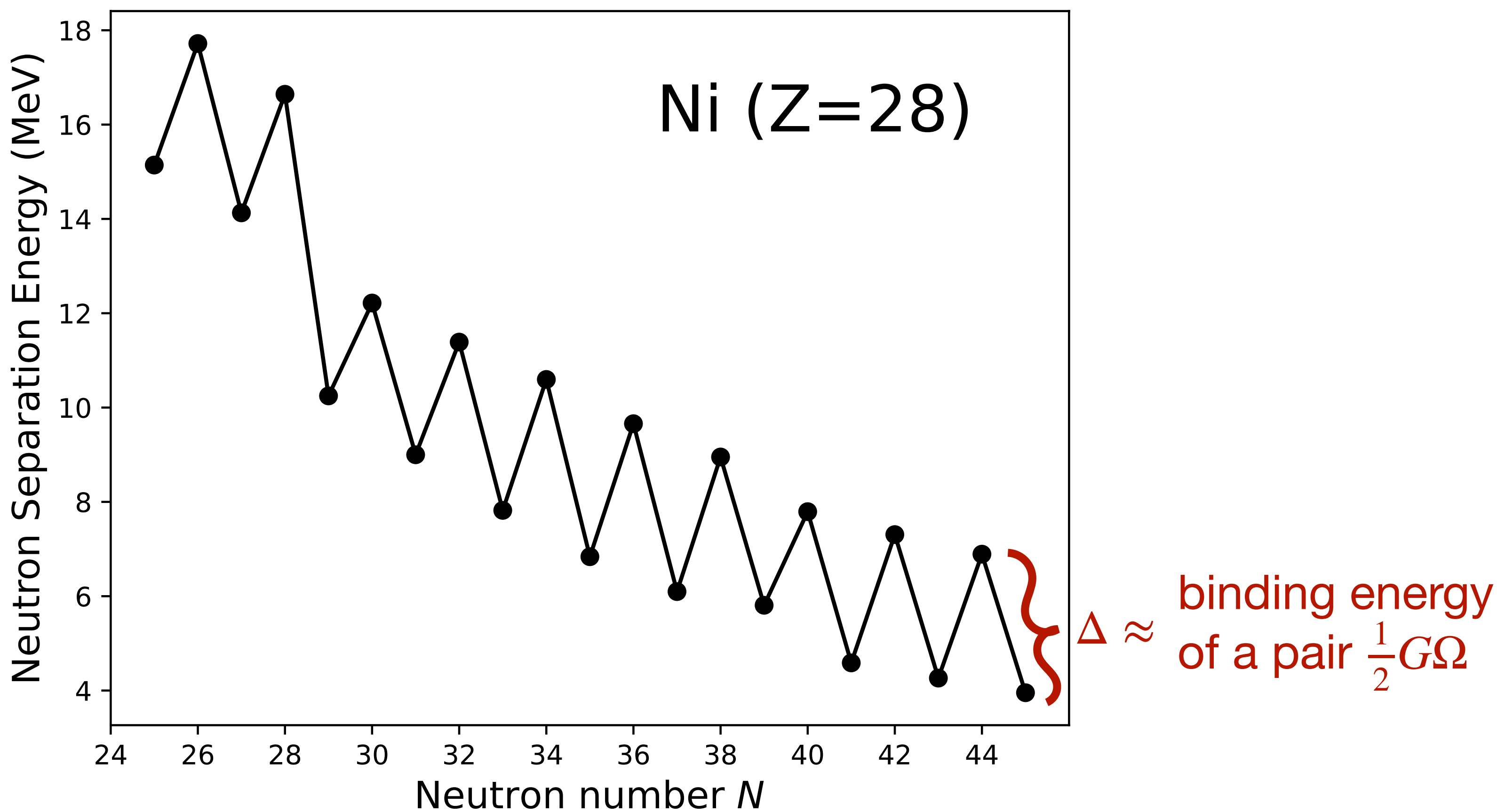
$$P^\dagger = [a^\dagger a^\dagger]^{J=0} = \sum_m \frac{(-1)^{j-m}}{\sqrt{2j+1}} a_m^\dagger a_{-m}^\dagger$$

$$|\Psi^{A=4}\rangle \approx P^\dagger P^\dagger |0\rangle$$

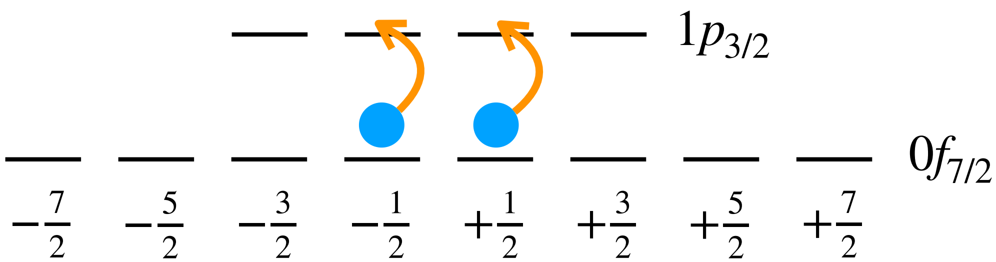
But they're still really fermions! The chance that nucleons from different Cooper pairs try to occupy the same state is $\propto 1/\Omega$, where $\Omega = \frac{1}{2}(2j+1)$.

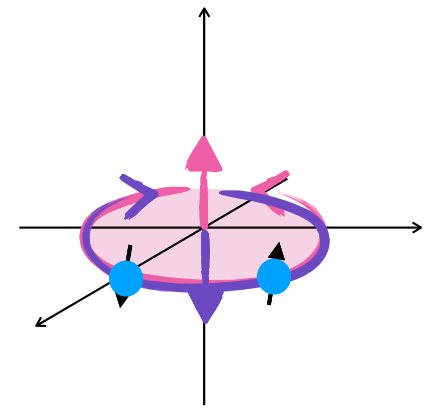
To the extent that the interaction is dominantly in $J=0$, the Cooper pairs do not interact with each other.

Odd-even staggering



Multiple j shells





With a short-range interaction, the nucleons “forget” which l orbit they were in, and can scatter into different shells.

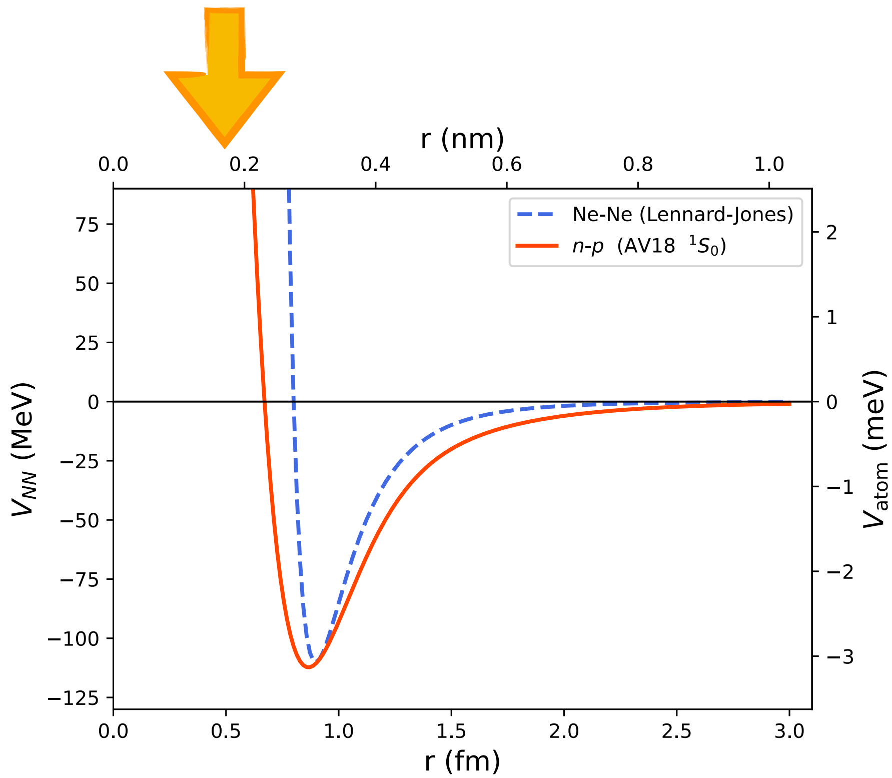
$$P^\dagger = \sum_i C_i \, [a_i^\dagger a_{\bar{i}}^\dagger]^{J=0}$$

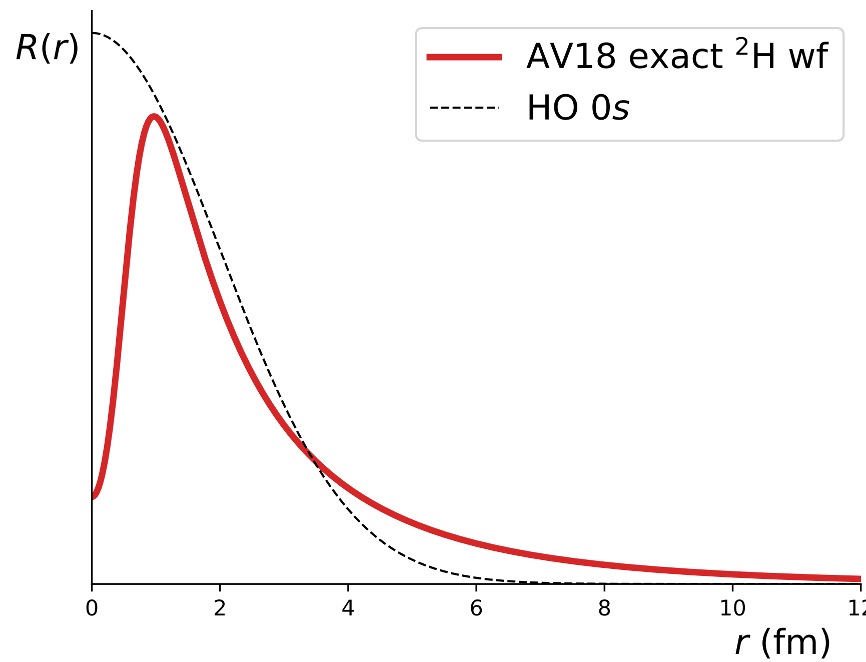
Now the Cooper pair is a superposition of $J = 0$ configurations built from different j shells.

And now Ω counts the total number of states across all the j shells.

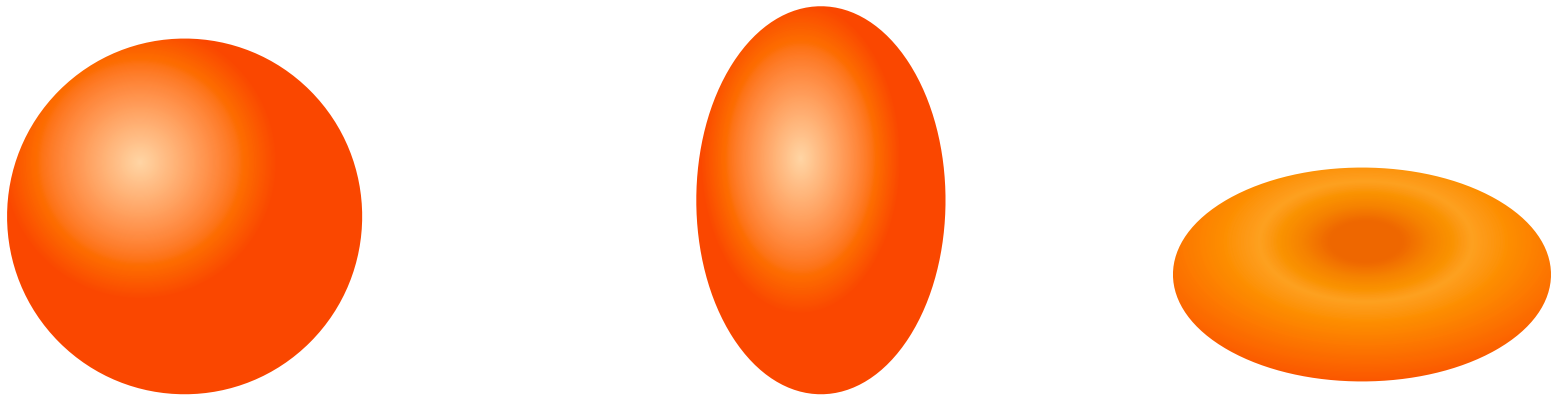
The different Cooper pairs still do not interact with each other.

But what about that repulsive core?



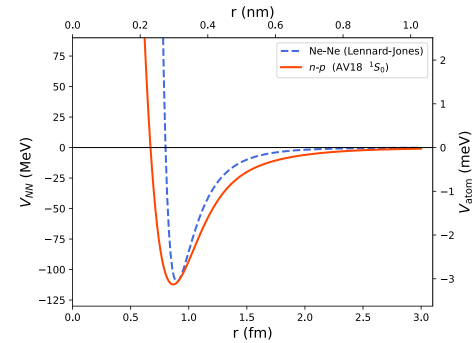


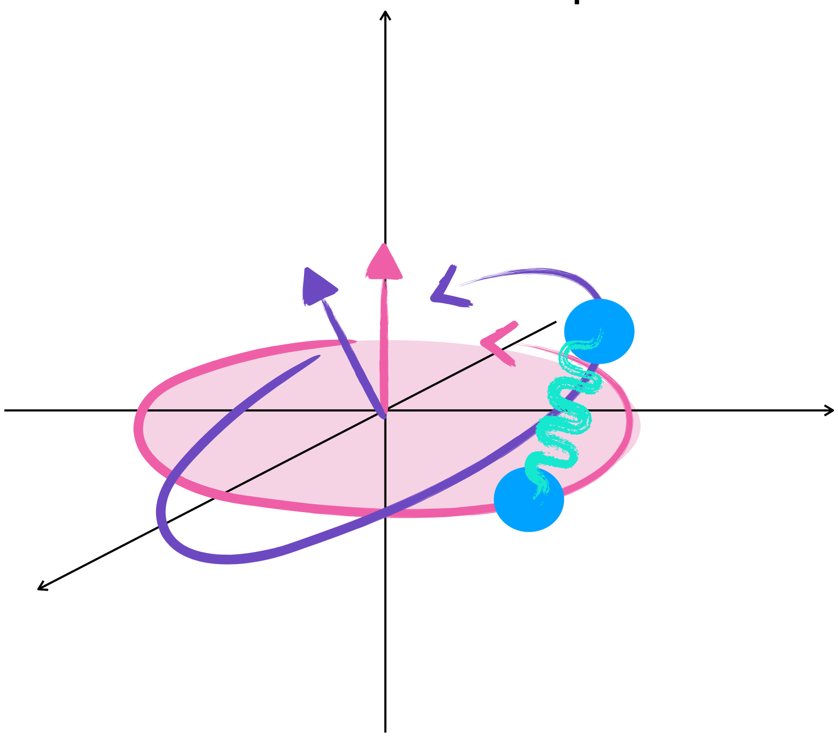
Shapes



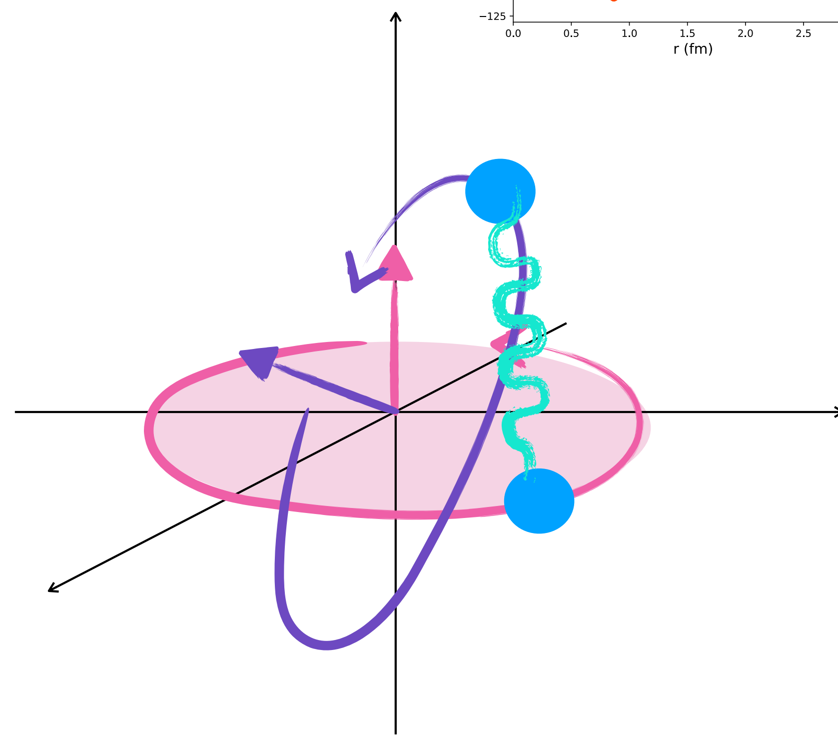
If the interaction is intermediate-range, there will be preference for orbits near the x-y plane

If the interaction is *constant* (~infinite range), there is no preference.



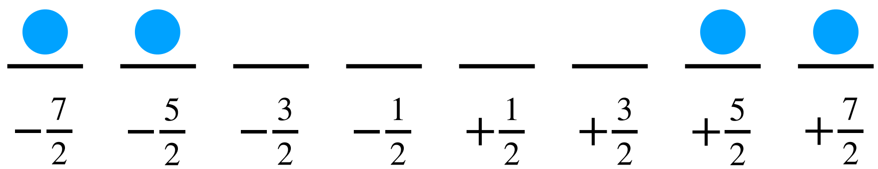


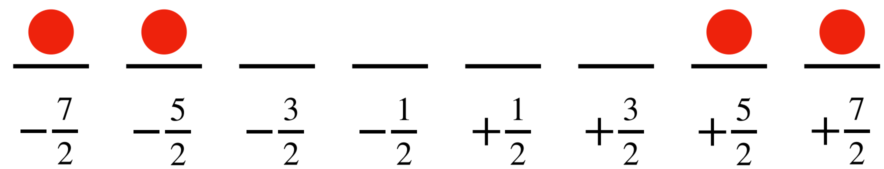
More attraction

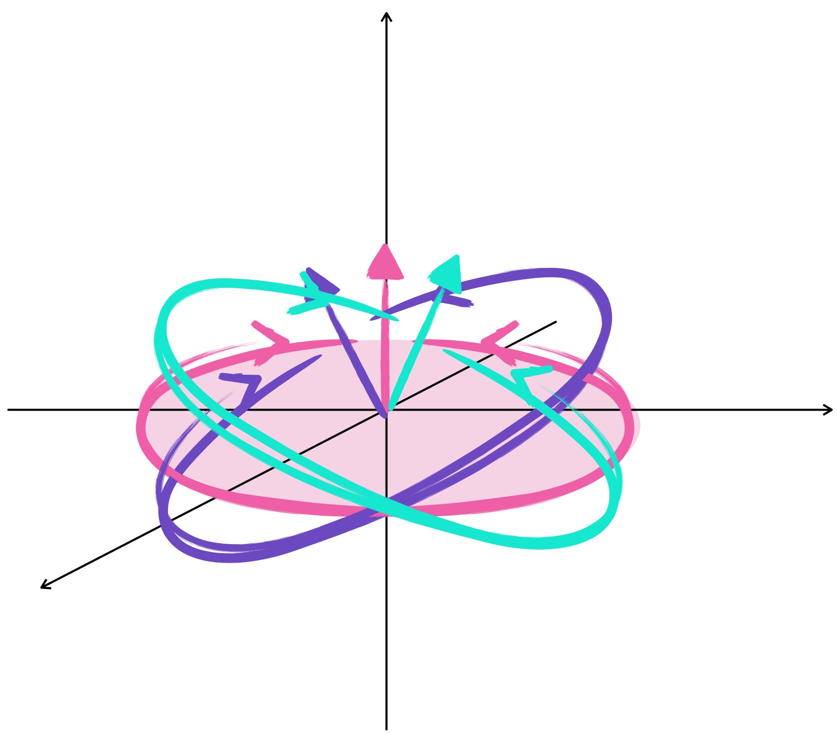


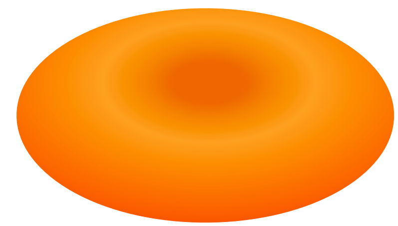
Less attraction

Filling a single j shell





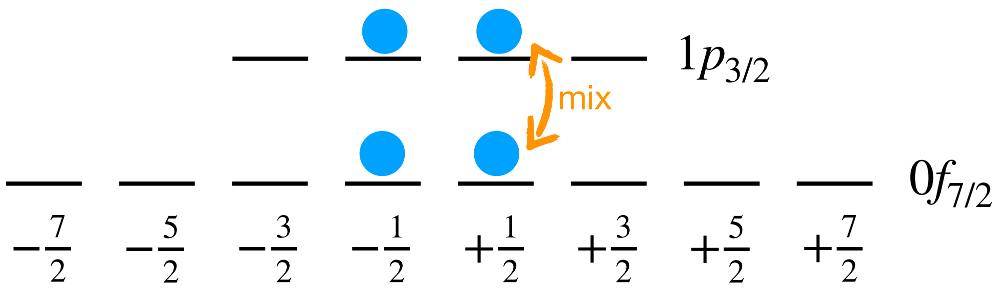


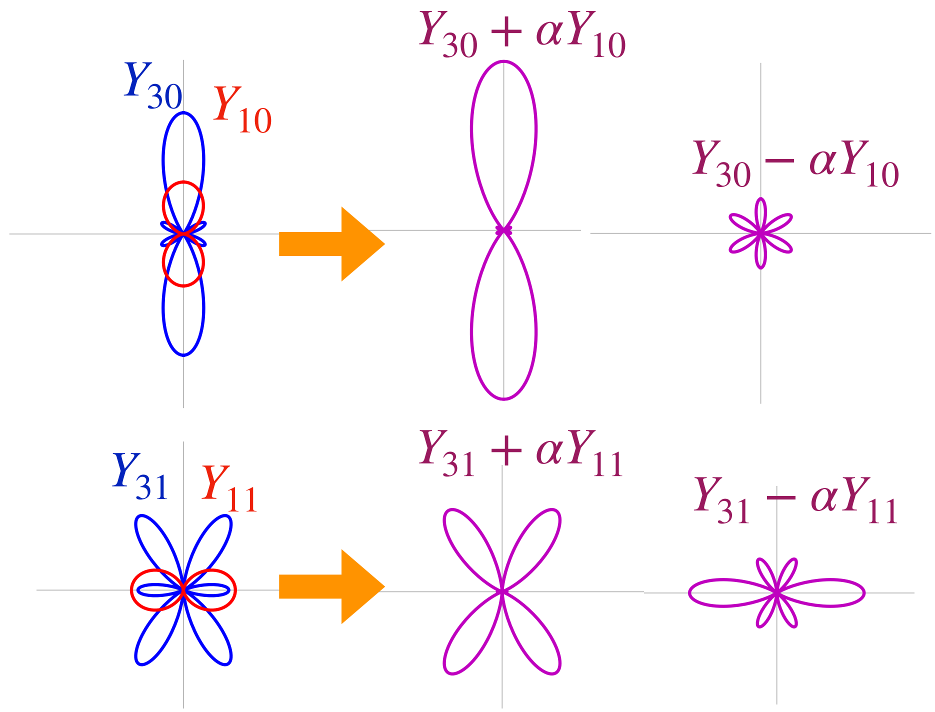


Intrinsic oblate shape

⇒ Prolate
in lab frame

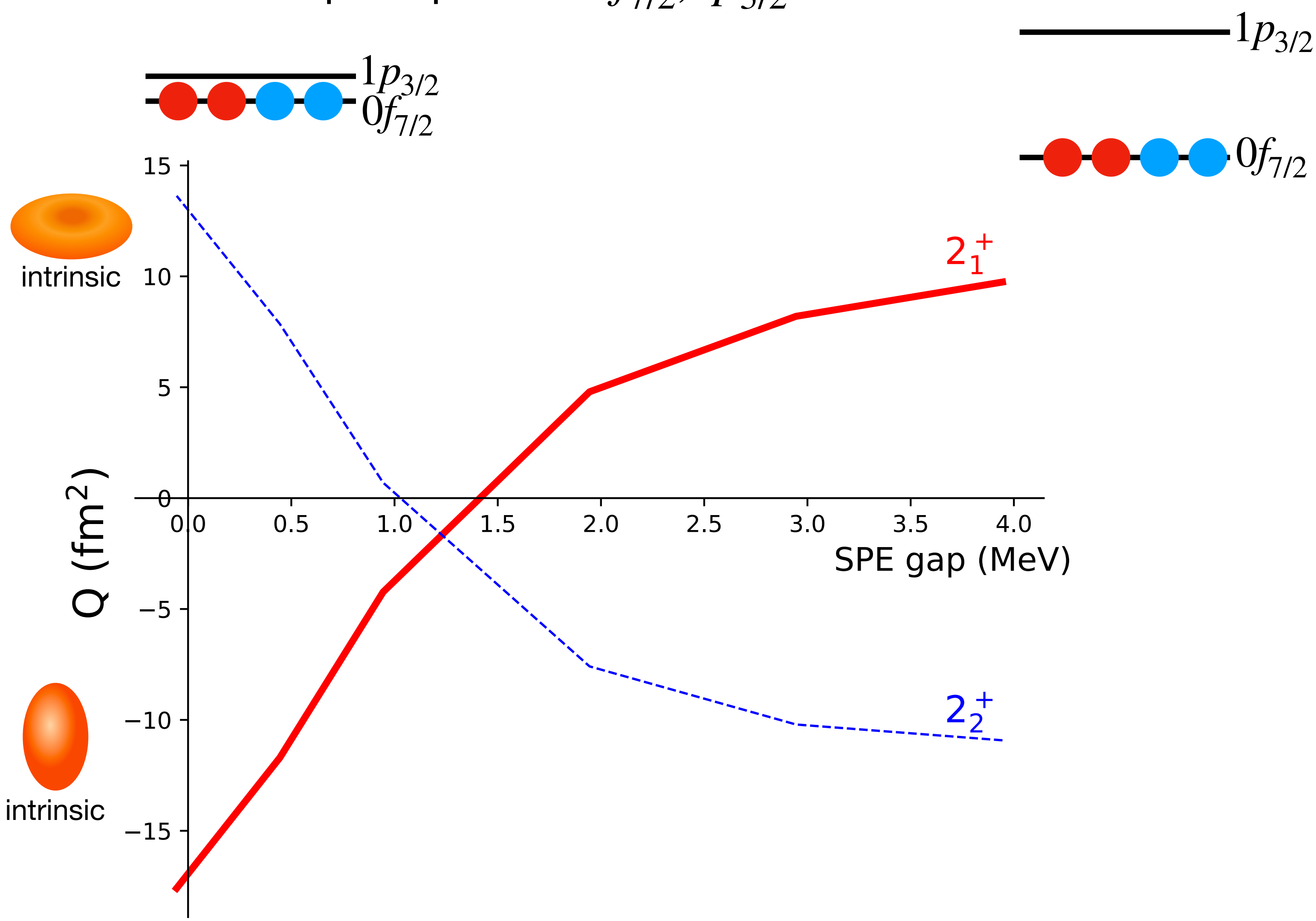
Multiple j shells



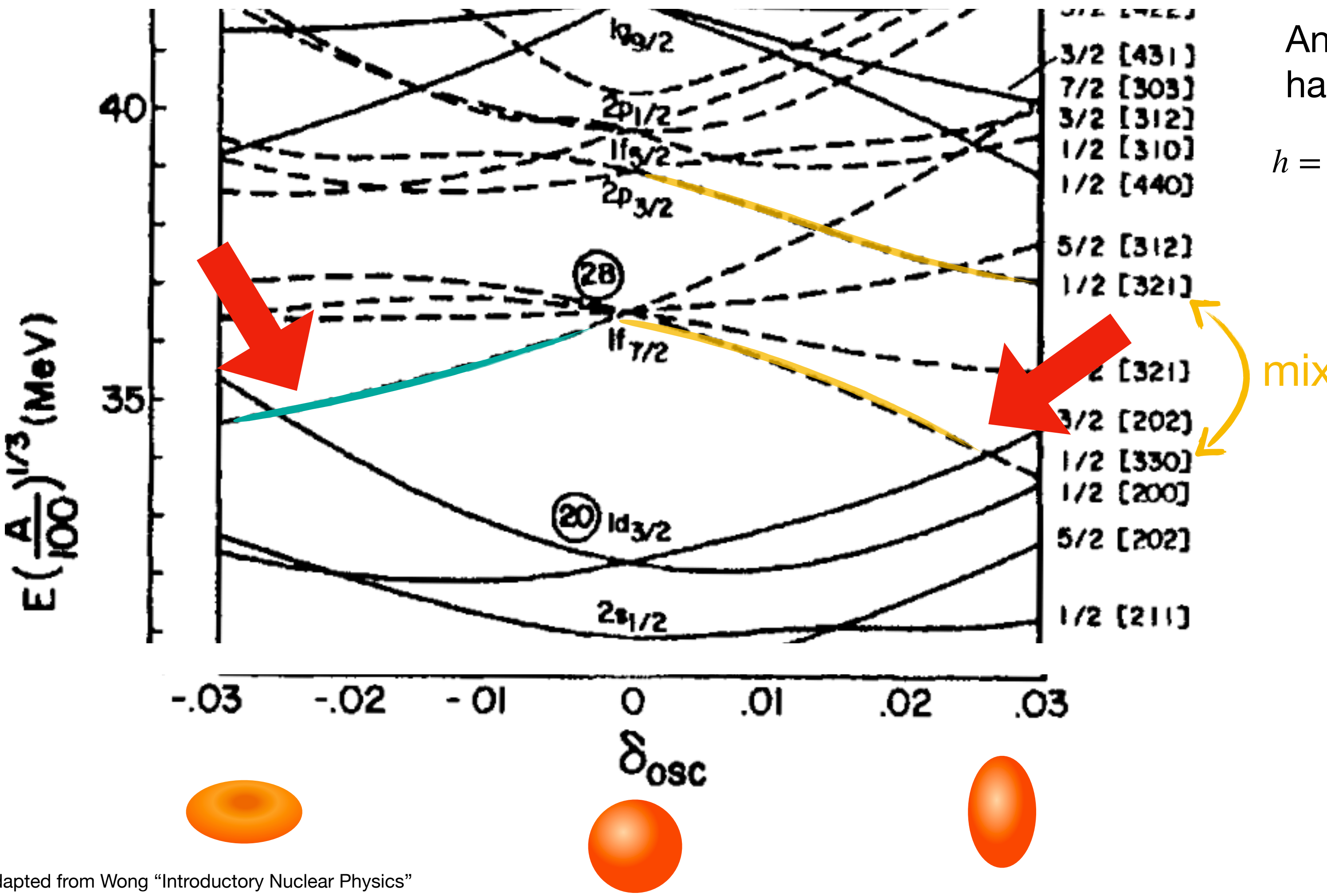


By **hybridizing** orbits, we can increase or decrease the concentration along the z axis.

Example: 2p+2n in $0f_{7/2}$,$1p_{3/2}$ shells



Nilsson model



Anisotropic 3D harmonic oscillator

$$h = \frac{1}{2}m\omega_\perp^2(x^2+y^2)+\frac{1}{2}m\omega_z^2 z^2$$
$$+c_L L^2 + c_{LS} L \cdot S$$

Figure adapted from Wong “Introductory Nuclear Physics”

The competition between shapes

Pairing:

- Short-range interaction
- Single pair has some intrinsic deformation
- Different Cooper pairs are not correlated ⇒ **spherical**.
- $E \sim -G\Omega\, N_{\text{pairs}}$ ⇒ prefer high degeneracy of single particle levels.

Deformation:

- Medium-range interaction
- Single j-shell favors **oblate** intrinsic shapes
- Mixing with multiple j-shells favors **prolate** intrinsic shapes.
- Deformed mean-field seeks shell gaps, which lowers degeneracy
- All-to-all interaction, $E \sim A(A-1)$

Few valence particles: **pairing wins** ⇒ spherical.
Many valence particles: **deformation wins**.
When this happens, and prolate/oblate depends on shell structure.

Part 1 Summary

- Saturation requires both attraction and repulsion, either from r dependence, or many-body forces
- Shell structure is a generic feature of a bound fermionic system where V is not strong enough to localize the particles
- Pairing is a generic feature of a short-ranged attractive interaction
- Deformation is a generic feature of a medium-ranged attractive interaction
- Treating of the competition between these effects depends on the details of the interaction and the system