Hot and Cold Quark-Gluon Matter Under Pressure

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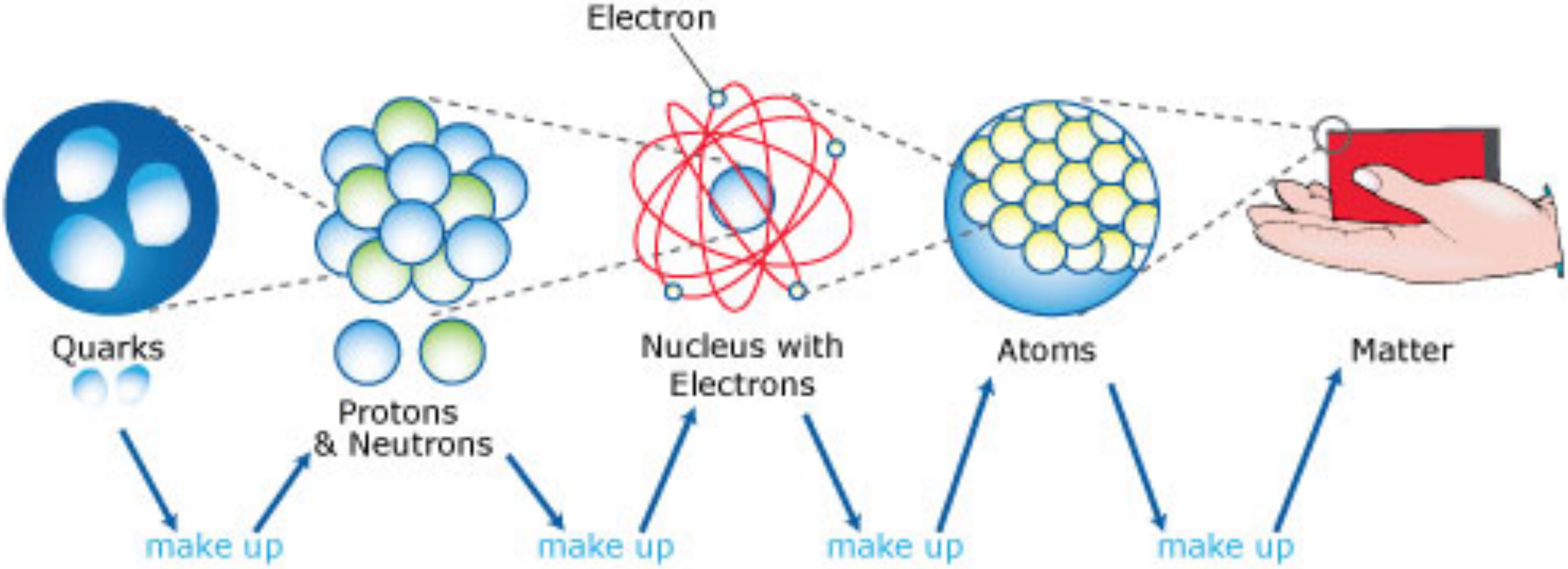
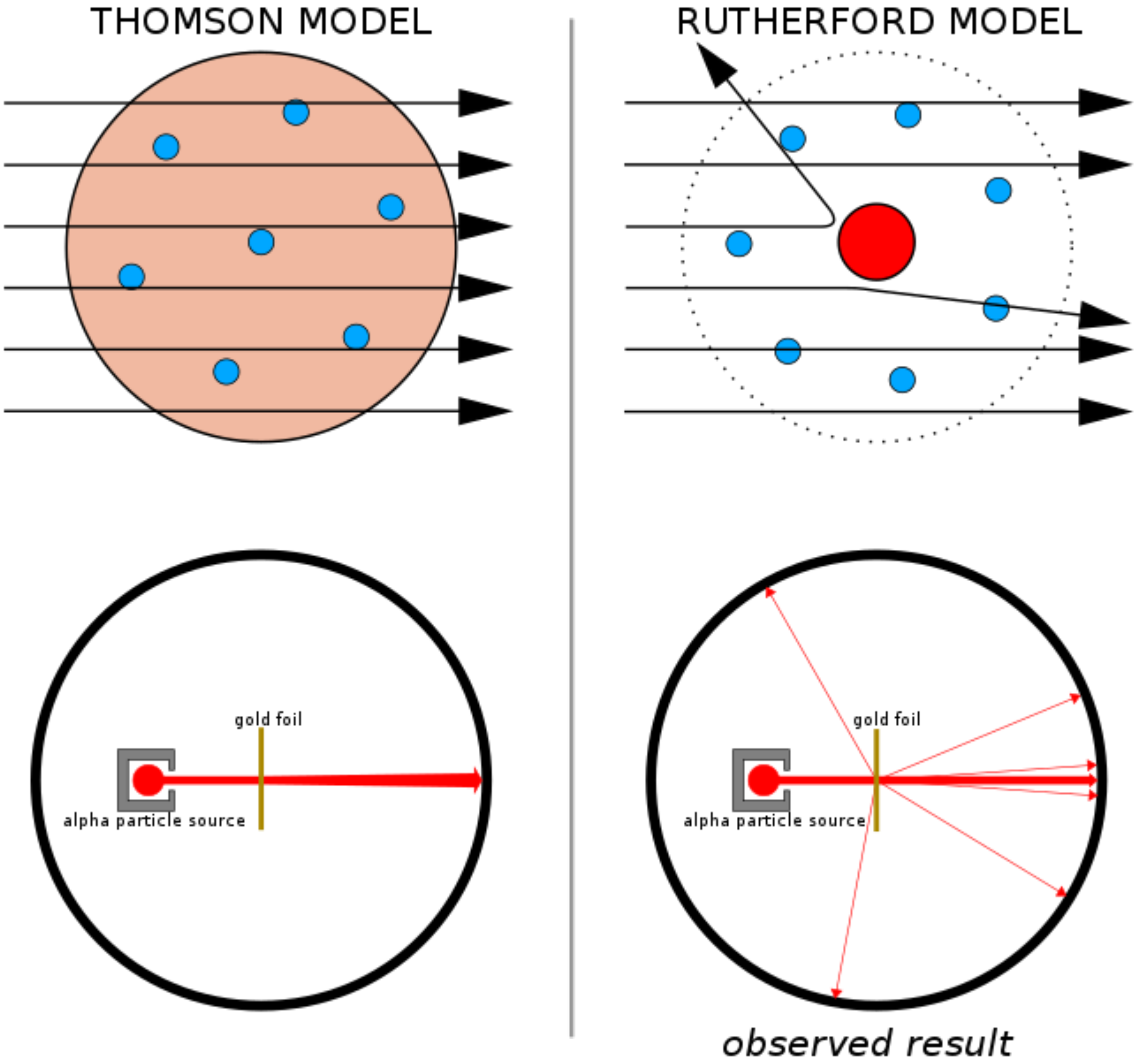

Electron
Quarks
Protons
& Neutrons
Nucleus with
Electrons
Atoms
Matter
make up
make up
make up
make up

What are Atoms Made Of?

- **By $\sim$ 1905, it was known that atoms feature a cloud of negatively charged electrons, and are neutral overall.**
- **Positive charge was thought to be spread throughout the volume of the atom. J. J. Thomson called it "pudding".**
- **Geiger and Marsden's 1909 experiments, whose implications were understood by Rutherford in 1911, contradicted this picture.**
- **They discovered** *atomic nuclei*. **Positively charged;** $1/100,000$ **times smaller than atoms.**
- **How did Geiger, Marsden and Rutherford discover atomic nuclei? What kind of "microscope" did they use? A** *scattering experiment*.

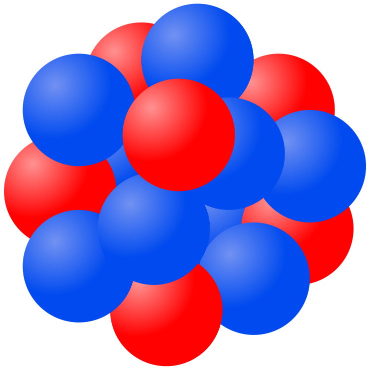
Rutherford Scattering



What are Atoms Made Of?

- **By 1911, atoms known to feature a cloud of electrons (charge -1) around a positively charged** *nucleus*.
  Size of an atom: 10^{-10}**m.**

- **By the 1930s, nuclei of atoms known to be made of protons (charge $+1$) and neutrons (charge 0).**
  Size of a proton or neutron: $\sim 10^{-15}$**m.**



What are Protons Made Of?

- **By 1960s, this was *the* question. And it was very confusing.**

- **Dozens of short-lived "cousins of the proton" had been found. Charge could be $+2$, $+1$, 0, -1, -2. Varying masses. All the same 10^{-15}m size.**

- **Patterns of masses and charges suggested they were made of *quarks*. Up quarks (charge $+2/3$); down quarks (charge $-1/3$) and heavier strange quarks (charge -1/3). Also antiquarks, each with same mass and opposite charge.**

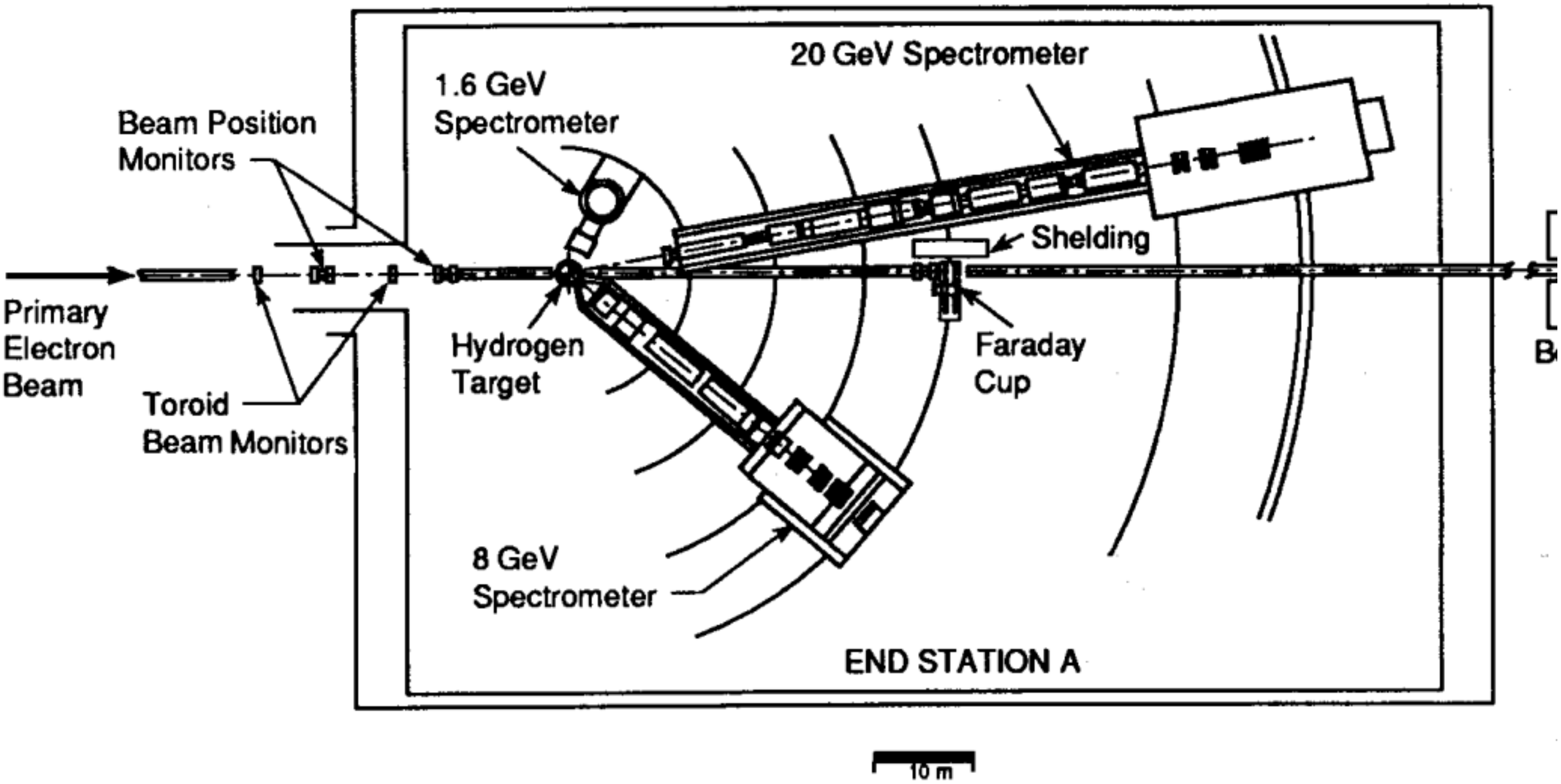
- ***As if* protons made of u,u,d and neutrons made of u,d,d.**

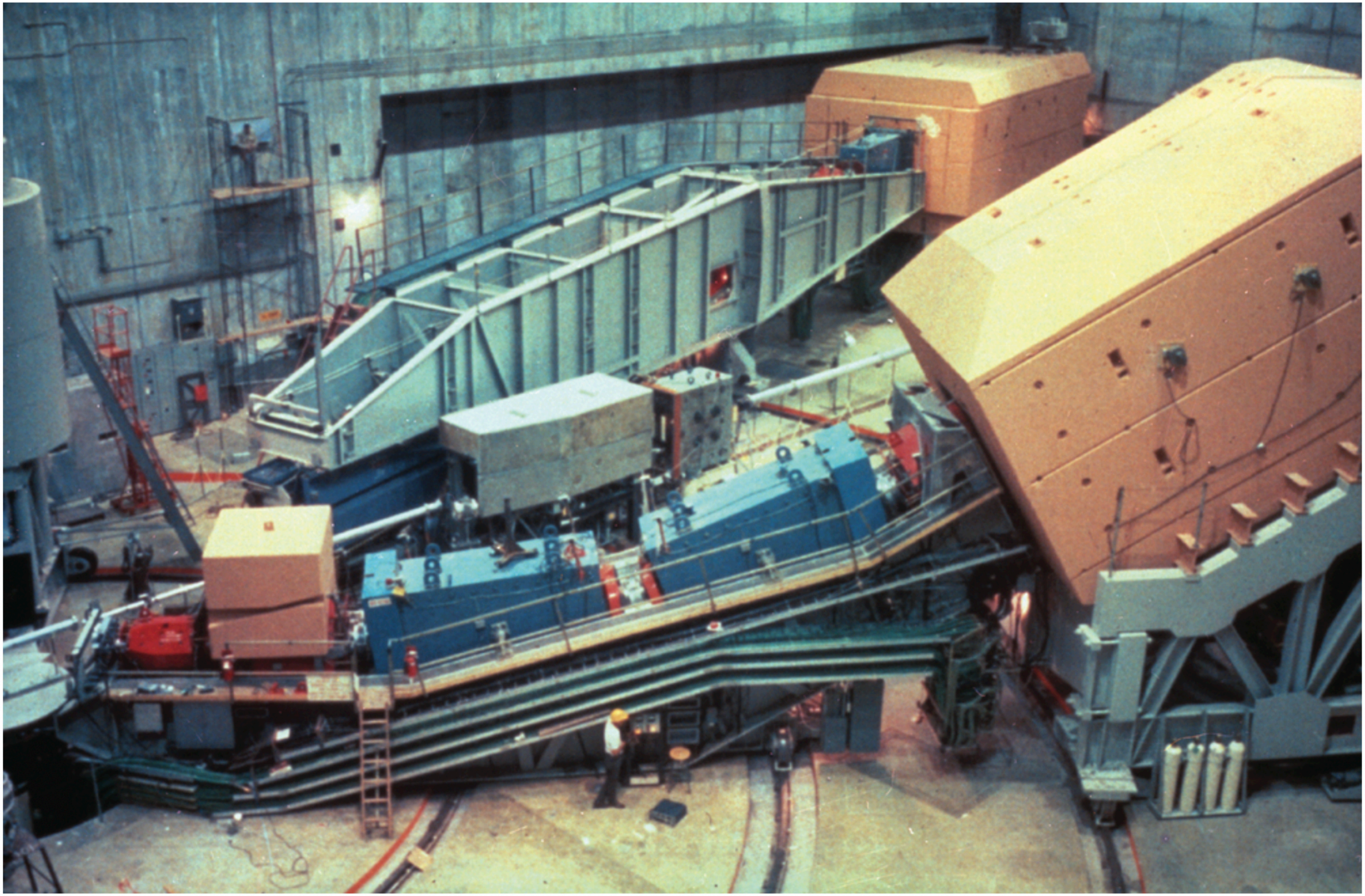
- **Some of the proton's cousins were as if made of three antiquarks, or a quark and an antiquark.**

- **But, only "as if" made of quarks. Nobody had ever succeeded in kicking a quark out of a proton, or finding an isolated quark. And to this day nobody has.**

What are Protons Made Of?

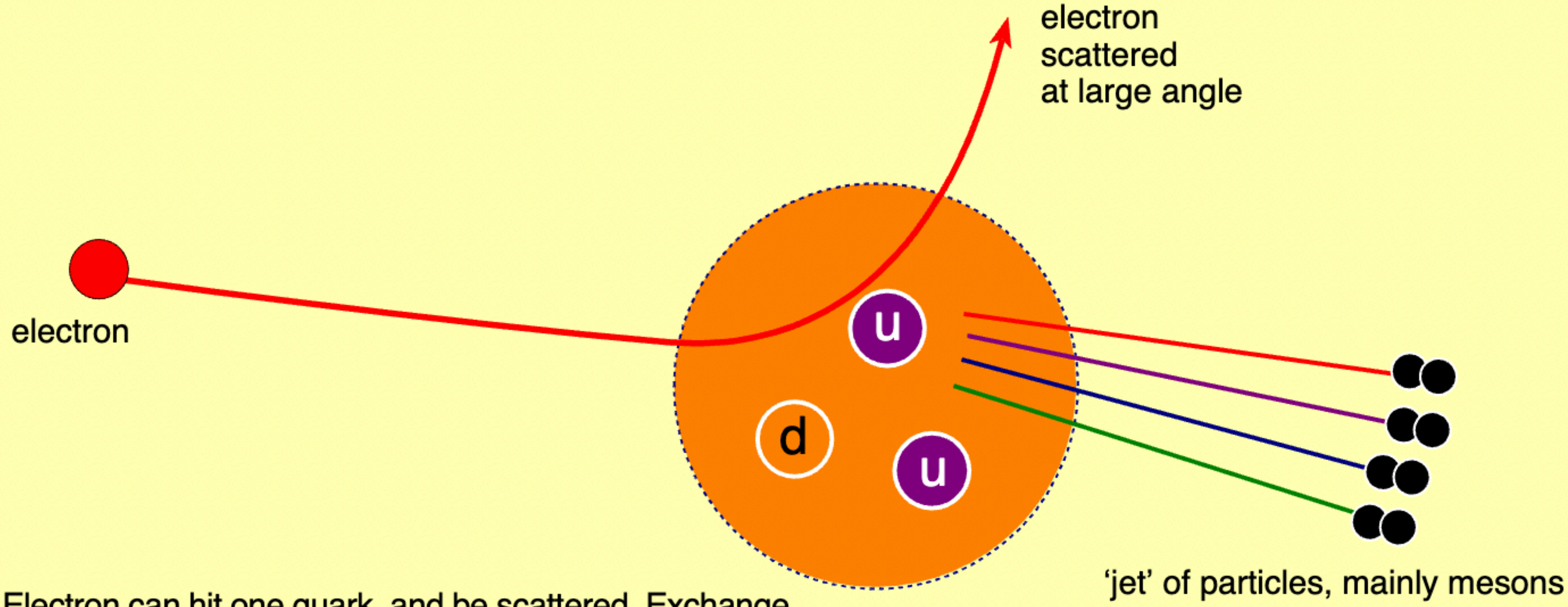
- In 1968-70, Friedman, Kendall and Taylor tried to answer the question “Rutherford style”.
- Aimed the new 3.2km long Stanford Linear Accelerator electron beam at protons...
- ... and looked for large angle scattering!!
- Not what most people were looking at, since it would blast the proton into smithereens. (Smithereens being a jet of many many integer-charged proton-cousins. No quarks.)
- Friedman, Kendall and Taylor looked for large angle scattering anyway, and focused on the scattered electron. Just as Rutherford had done 60 years earlier.


20 GeV Spectrometer
1.6 GeV
Spectrometer
Beam Position
Monitors
Shelding
Primary
Electron
Beam
Hydrogen
Target
Faraday
Cup
B
Toroid
Beam Monitors
8 GeV
Spectrometer
END STATION A
10 m



R. Muffley/SLAC

High energy: deep inelastic scattering



Electron can hit one quark, and be scattered. Exchange of high energy photons leads to the creation of a jet of particles and antiparticles.

What are Protons Made Of?

- In 1968-70, Friedman, Kendall and Taylor *saw* quarks. (They won the Nobel Prize for this in 1988.)
- Quarks inside a proton are just as real as nuclei inside an atom.
- When you bounce a very high energy electron off a quark in a proton, at a large angle, the scattering is *quantitatively* just as if there were point-like quarks in there.
- In a sense this added to the confusion. Quarks *are* real, not just book-keeping. But, why can't anybody kick a quark out of a proton and see it by itself???

What are Protons Made Of?

- In 1973, clarity!!
- Gross, Wilczek and Politzer discovered the laws of nature that govern “QCD”, namely how quarks behave. (They won the Nobel Prize for this in 2004.)
- Analogue of what Galileo, Newton, and Einstein did for gravity, and what Coulomb, Ampere, Faraday, Maxwell, Feynman, Schwinger and Tomonaga did for electricity, magnetism, and light.
- Resolves prior confusion. Quantitative description of all known phenomena involving quarks. A core part of what we now call “the Standard Model” but which really should be called “the Theory of Matter”.
- And, opens new vistas to scientific investigation...

Quarks and Gluons

- Not so different from electrons and photons?
- Electrons have charge. Any charged particles, interact with each other by exchanging photons. Light is made of photons alone, and is governed by Maxwell's equations.
- Quarks have a new kind of charge. Called "color". Electrons are charged; quarks are colored.
- Any colored particles interact with each other by exchanging gluons. Gluons by themselves are governed by equations that look a lot like Maxwell's equations, with one difference. . .
- . . . the gluons are themselves colored. Gluons can exchange gluons.
- This sounds like a technicality, but it's not. Gross, Wilczek and Politzer showed that it has remarkable consequences.

Quarks and Gluons

- **Coulomb force between electrons $\propto \frac{1}{r^2}$ where r is their separation.**

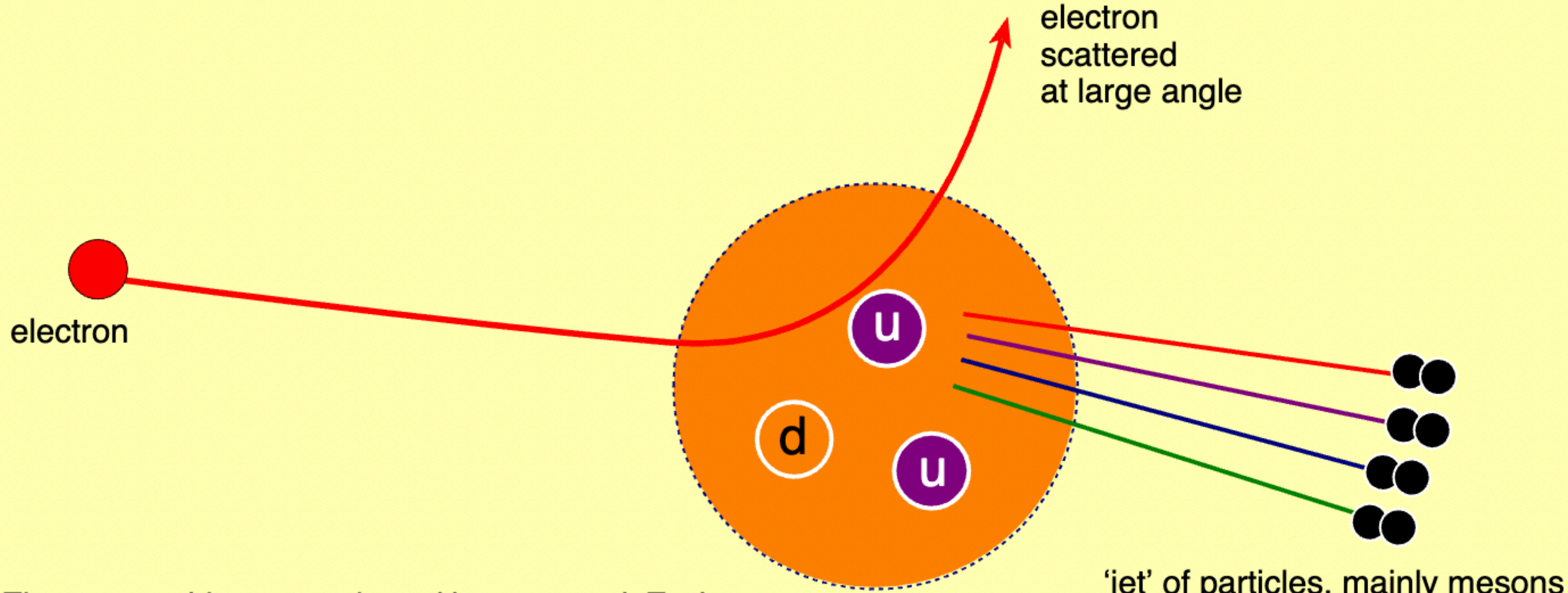
- **As $r \to 0$, force between quarks** *grows more slowly than Coulomb*. **Quarks interact weakly when close to each other.**

- **If you** *could* **pull quarks apart, force between them stops decreasing! When quarks are more than R apart, the force between them is** *much* **stronger than Coulomb. Gluons between them behave more like a rubber band.**

- **Laws that govern quarks encode within them a length scale, R. Forces weaken for $r \ll R$. Forces strong for $r \geq R$. What is R? About 10^{-15}m. The size of a proton!**

- **Consequence: when you kick a quark really hard, it cannot escape by itself. The "rubber band" breaks, and you get a spray, a jet, of color-neutral cousins-of-protons.**

High energy: deep inelastic scattering



Electron can hit one quark, and be scattered. Exchange of high energy photons leads to the creation of a jet of particles and antiparticles.

Quarks and Gluons

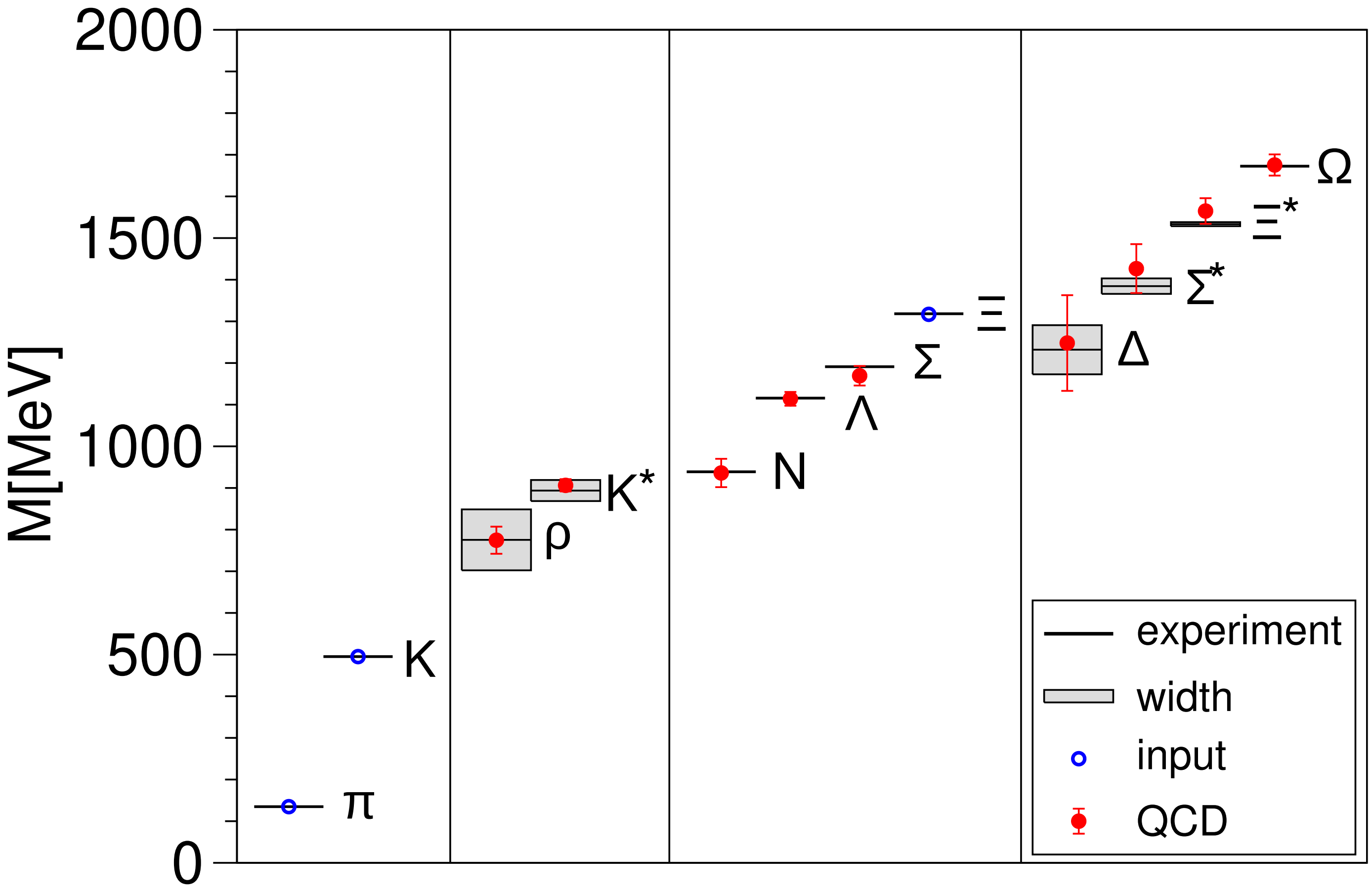
- Laws of QCD which govern quarks and gluons are precise, predictive.

- Immediately understood why all the cousins-of-protons have the same size. Immediately understood why Friedman, Kendall and Taylor were able to see quarks: when you kick them hard you "resolve" the quarks deep inside a proton, where they are weakly interacting.

- As if you are taking a high-resolution photograph with a fast shutter-speed of where one quark is at one moment.

- But the equations we use to describe those laws (in particular if you "look" with slower shutter-speed) are fiendishly difficult to solve in full. For example, it took 35 years, and supercomputers, to calculate the masses of the proton and all of it's cousins from first principles!

Final result for the hadron spectrum 2008



Cold Quark Matter

- **Discovery of fundamental laws of nature opens new vistas for exploration.**
- **Already by 1974, Collins and Perry realized that if you could crush a star's worth of atoms by a factor of more than 10^5, you would crush the protons and neutrons on top of each other, and make** *quark soup*. **(That's what they called it, in 1974.)**
- **Radius of the sun: 7×10^8m. Radius of a** *neutron star*: **around 10^4m.**
- **This is a very exciting area of scientific exploration today. There is increasing evidence that neutron stars with a mass about twice that of the sun may have quark soup cores.**
- **Cold quark soup, under pressure!**
  We shall return to this . . .

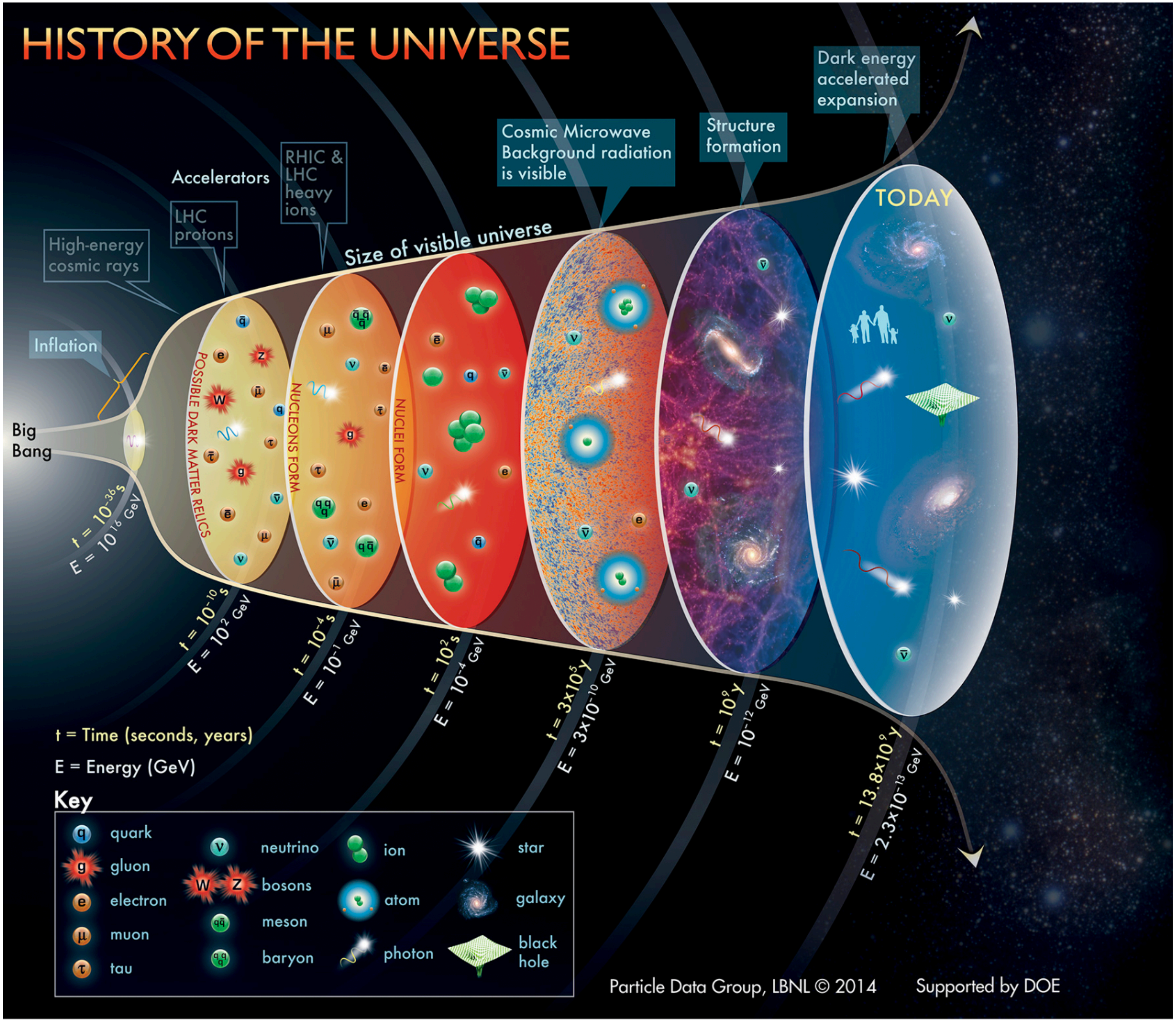
Hot Quark-Gluon Matter

- Discovery of fundamental laws of nature opens new vistas for exploration.

- By the mid 1970's, via the work of many, known that:
  - At high enough temperatures, protons and neutrons cannot exist, matter must be a gas of quarks and gluons. Named "quark-gluon plasma" (QGP); a gas of colored particles.
  - Temperature had to be above 2 *trillion* degrees. (So hot that there are *many* quarks, antiquarks and gluons in the volume of what would be a proton.)
  - The only place/time where such temperatures arose in nature was throughout the universe, for roughly the first 10 microseconds after its birth!!

- All of a sudden, we have a chance to answer: Where did matter come from? How did the first protons form?

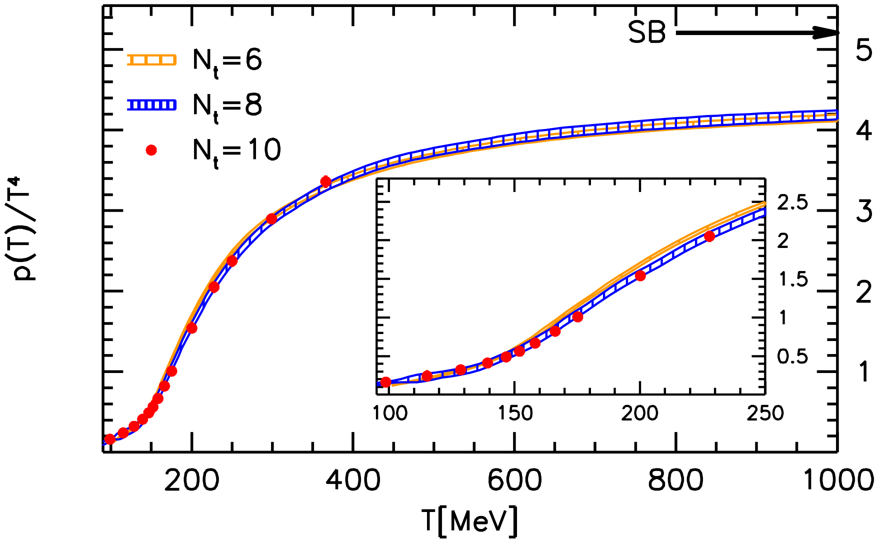
HISTORY OF THE UNIVERSE

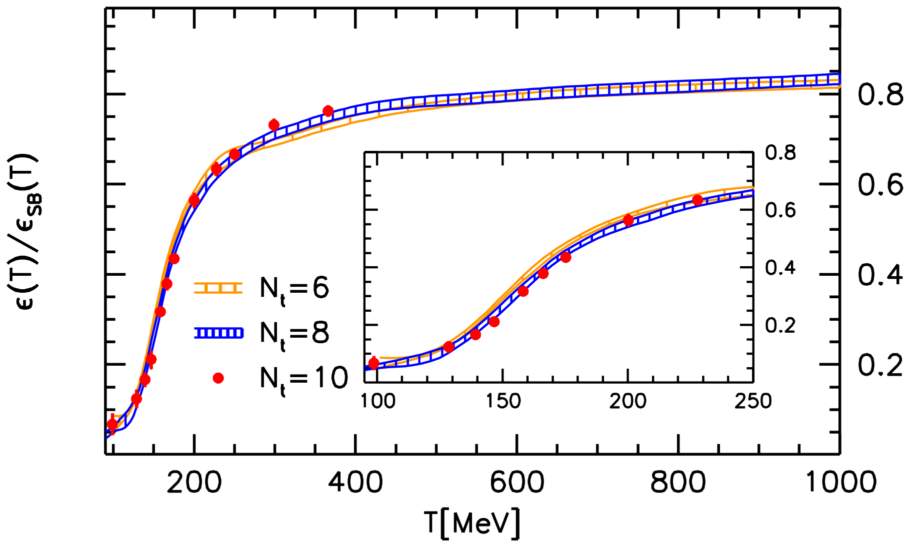


Particle Data Group, LBNL © 2014 Supported by DOE

QGP Thermodynamics

Endrodi et al, 2010





Above $T_{\text{crossover}} \sim$ 150 MeV (1.7 trillion degrees), QCD = QGP. QGP pressure and energy and entropy, can now be calculated reliably.

P(5 trillion degree quark matter)=P(430 MeV quark matter) $=2.5\times10^{31}$ atmospheres!! Hot quark matter, under pressure! $=7\times10^{24}\times$ pressure at the center of the earth. . .

BUT: Although its pressure is almost that of ideal, noninteracting gas, QGP, this stuff is *very* different in its dynamical properties. . .

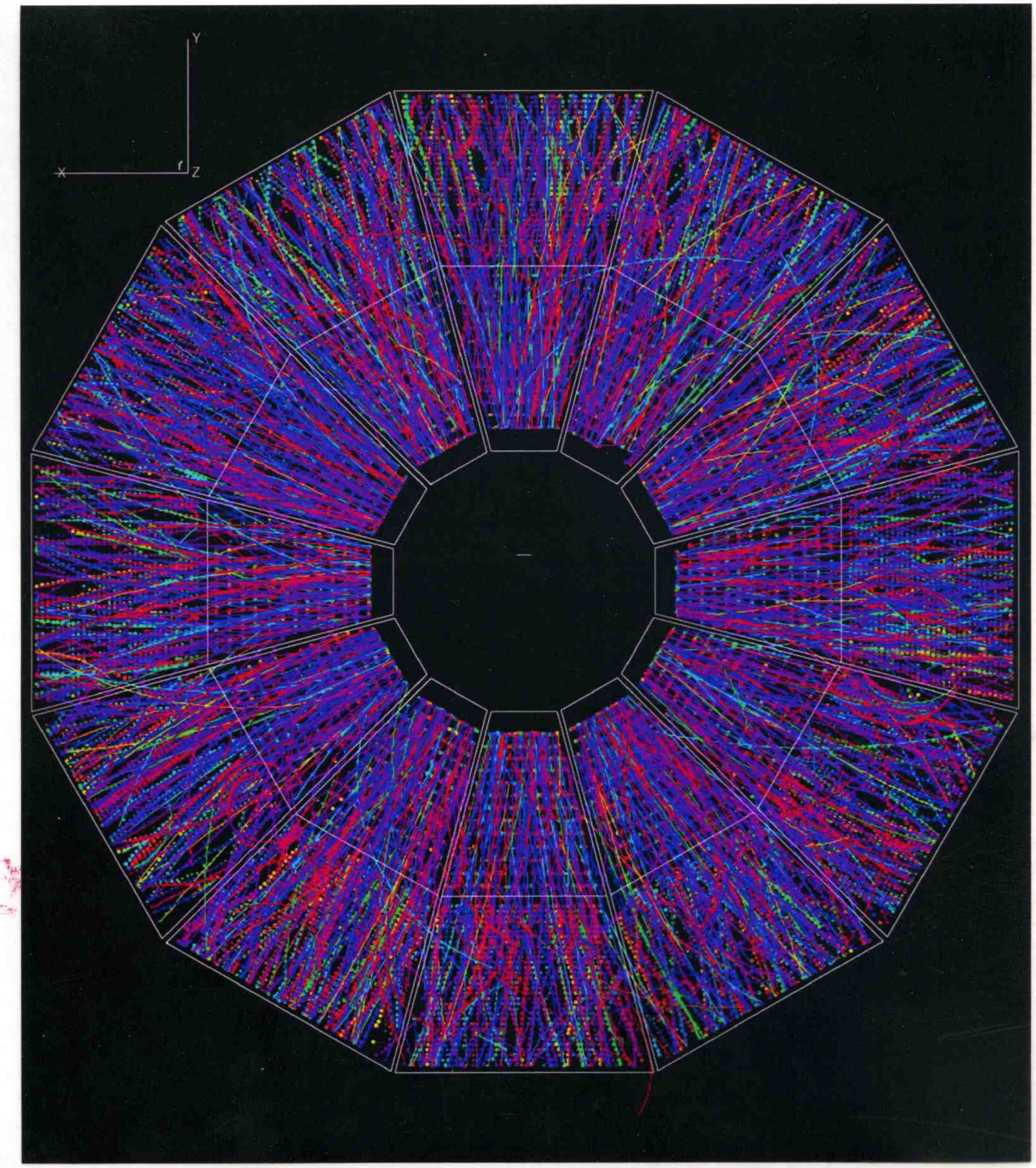
Hot Quark Matter Dynamics

- Understanding what hot quark matter is *actually* like, how it behaves, had to wait until scientists reproduced droplets of it!! By accelerating, and colliding, heavy nuclei to make droplets of hot quark matter.

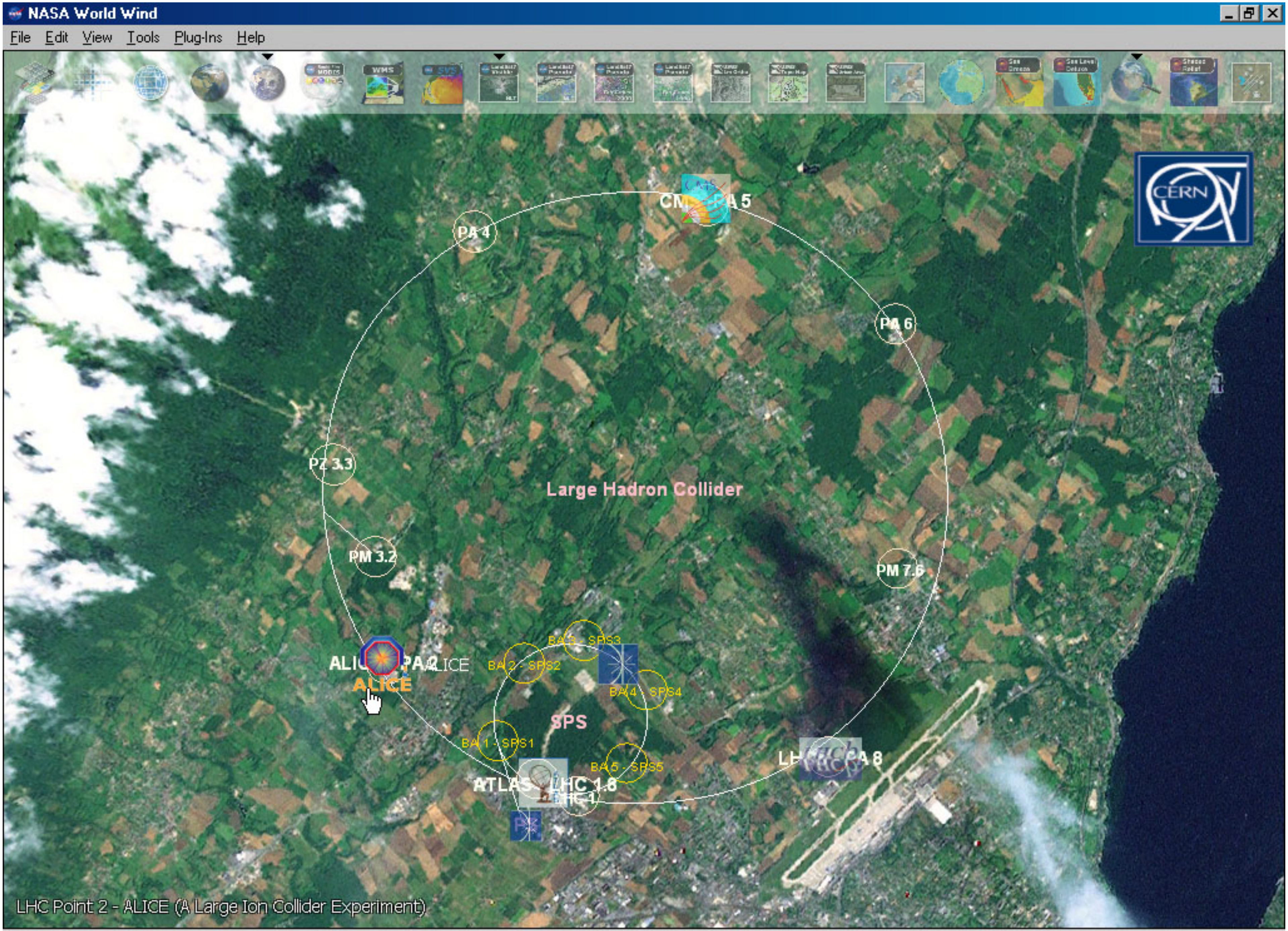
- In 1983, Busza and Goldhaber calculated that collisions with energies > 100 GeV were needed. Almost a factor of 100 greater than accelerators could produce in 1983.

- Since 2000, the Relativistic Heavy Ion Collider, RHIC at Brookhaven National Lab near New York, achieves 200 GeV collisions of Au nuclei. Since 2010, the Large Hadron Collider, LHC at CERN near Geneva, achieves 2700-5000 GeV collisions of Pb nuclei.

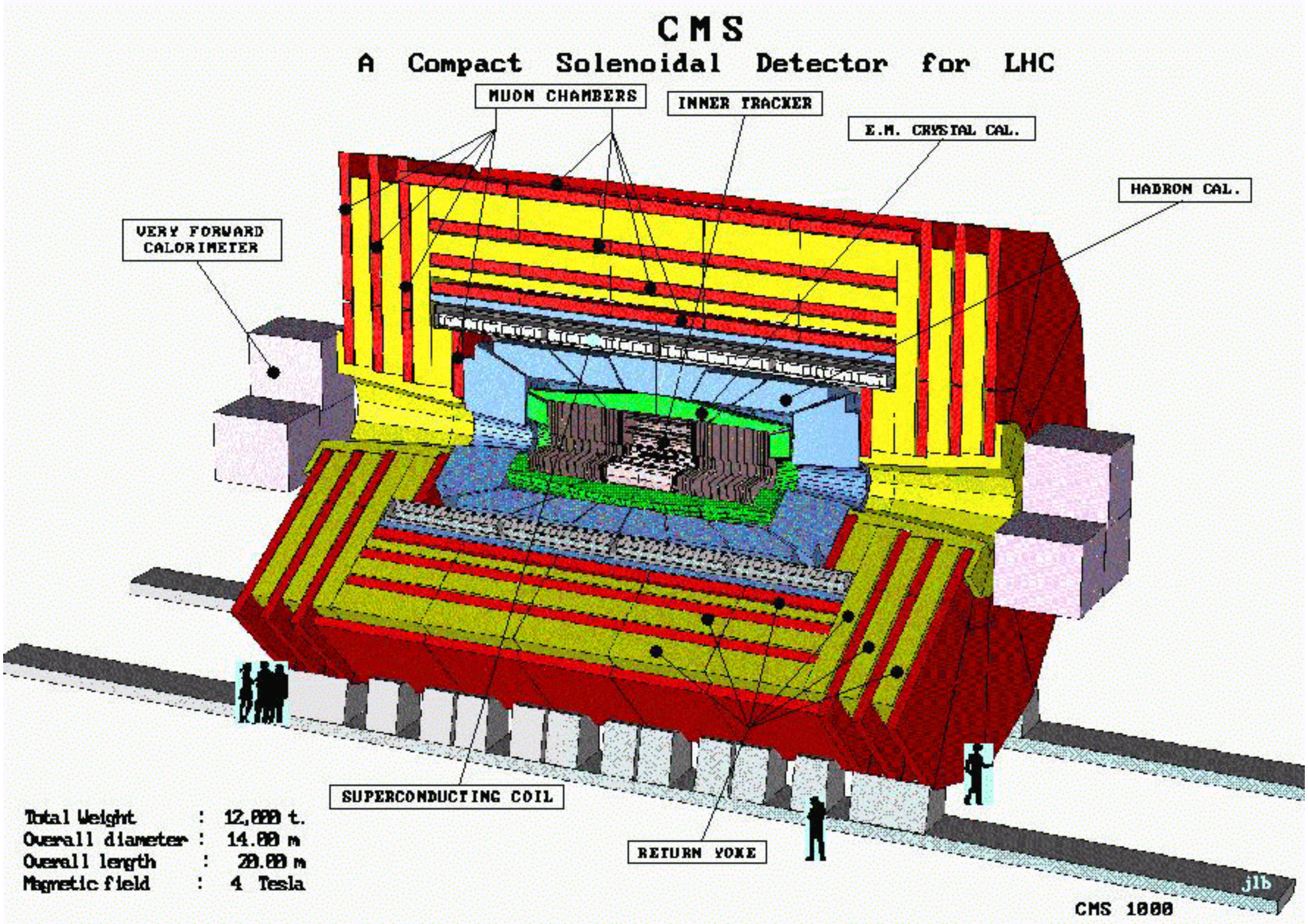
- Recreating “Little Bangs”. Only 10^{-14}m in size. Hot quark matter explodes, expands, cools, forms ordinary protons and it’s cousins within 10^{-22} seconds.

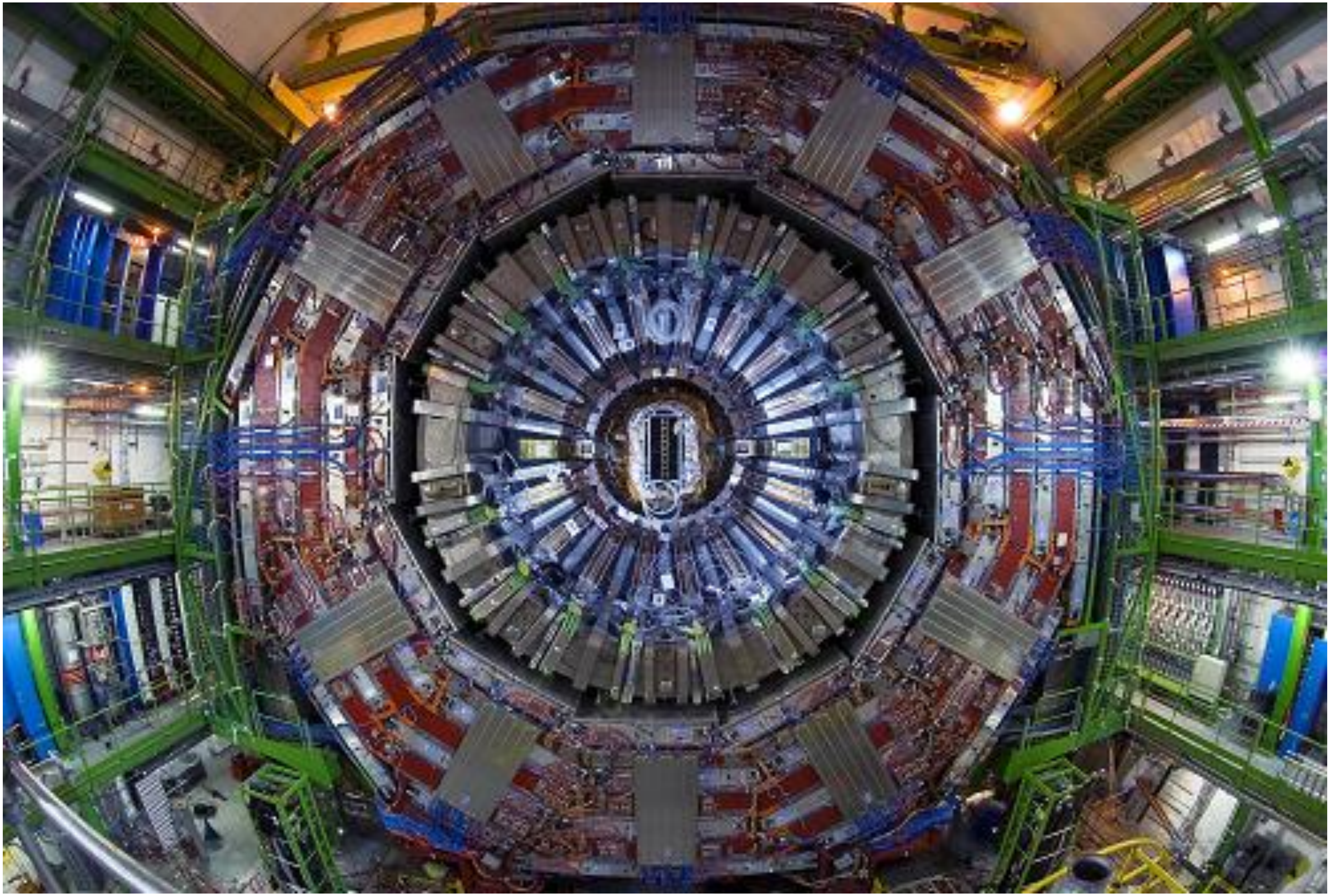


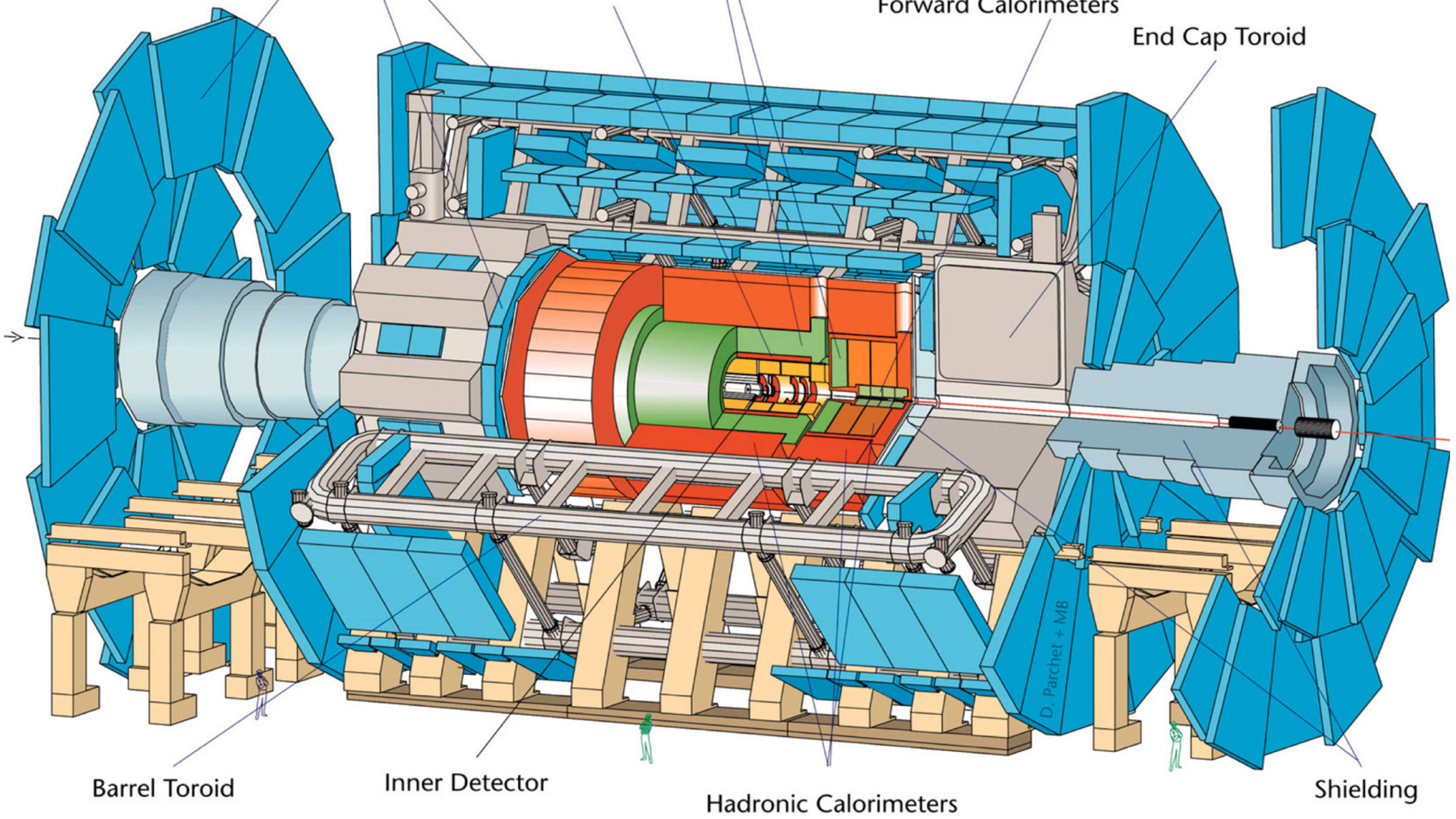
STAR

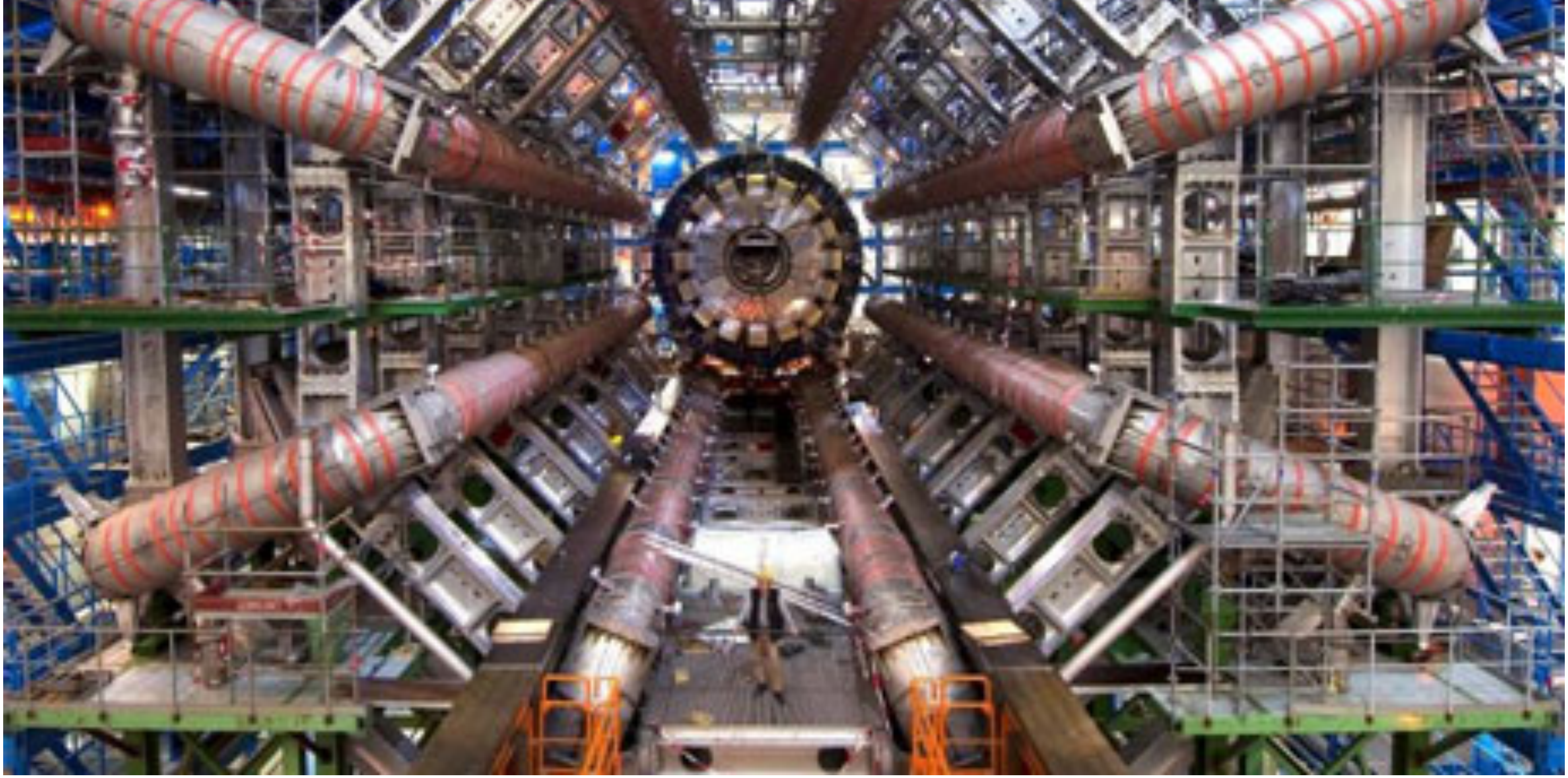


LHC Point 2 - ALICE (A Large Ion Collider Experiment)

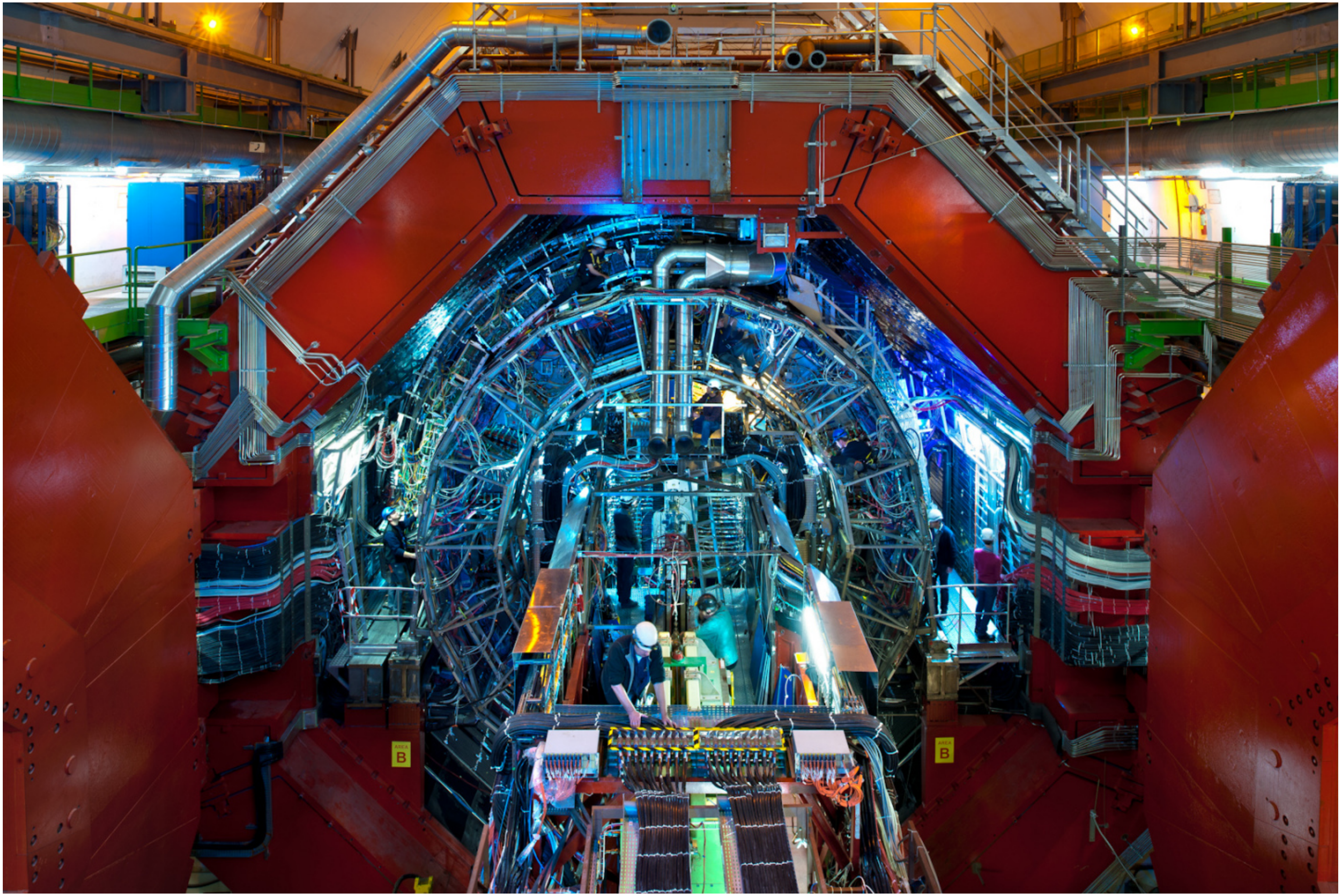

CMS
A Compact Solenoidal Detector for LHC
MUON CHAMBERS
INNER TRACKER
E.M. CRYSTAL CAL.
HADRON CAL.
VERY FORWARD CALORIMETER
SUPERCONDUCTING COIL
RETURN YOKE
Total Weight : 12,000 t.
Overall diameter : 14.00 m
Overall length : 20.00 m
Magnetic field : 4 Tesla
jlb
CMS 1000

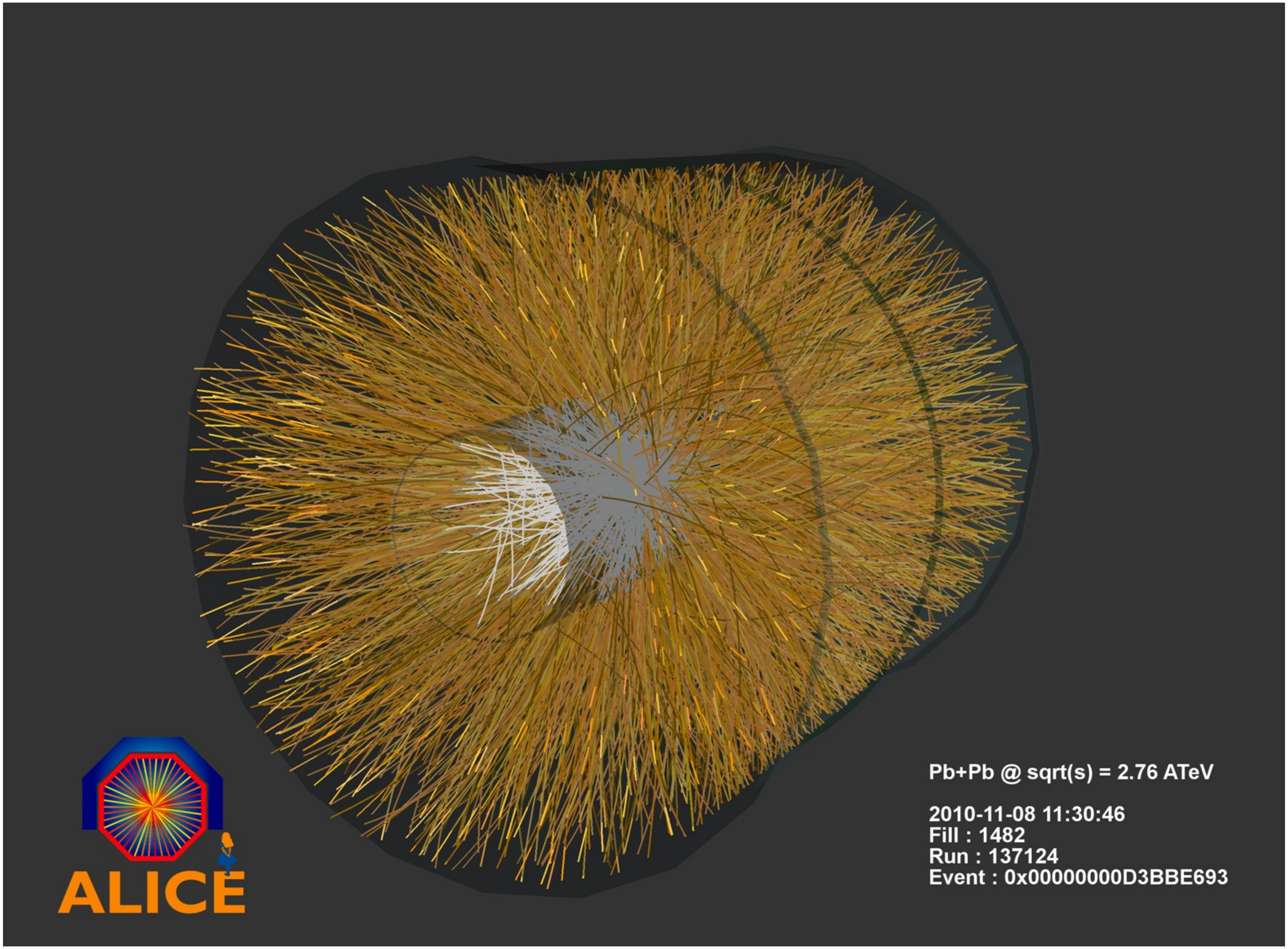



Forward Calorimeters
End Cap Toroid
D. Parchet + MB
Barrel Toroid
Inner Detector
Hadronic Calorimeters
Shielding




LHCb
DANGER
PZ

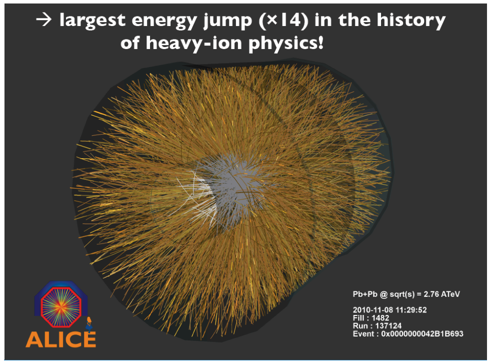

AREA
B
AREA
B

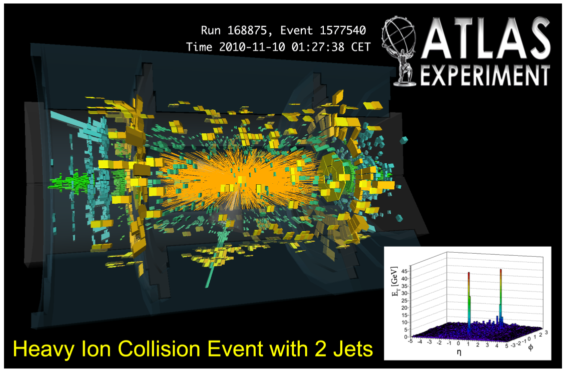


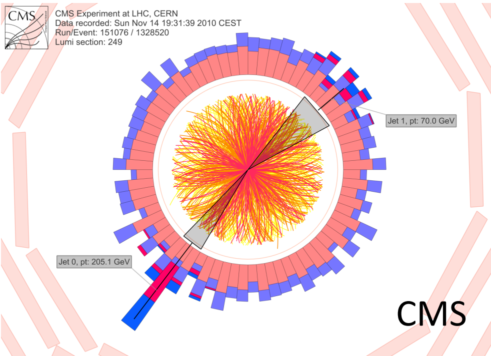
Pb+Pb @ sqrt(s) = 2.76 ATeV

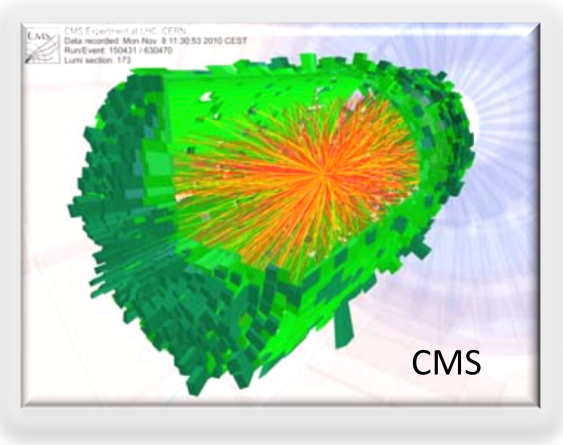
2010-11-08 11:30:46
Fill : 1482
Run : 137124
Event : 0x00000000D3BBE693

Nov 2010 first LHC Pb+Pb collisions


→ largest energy jump (×14) in the history of heavy-ion physics!


Heavy Ion Collision Event with 2 Jets





$\sqrt{s_{NN}}$ = 2760 GeV

Integrated Luminosity = 10 μb^{-1}

Hot Quark Matter Dynamics

- Understanding what hot quark matter is *actually* like, how it behaves, had to wait until scientists reproduced droplets of it!! By accelerating, and colliding, heavy nuclei to make droplets of hot quark matter.
- These collisions really are making little droplets of hot big bang matter!
- But what we see is the debris, after the hot matter has exploded.
- How can we learn about the behavior of the hot matter produced in the collision? What have we learned?

Hot Quark Soup!

- Surprise! By 2005, analyses of RHIC data on how *asymmetric* blobs of hot Big Bang matter expand (explode) revealed that trillions-of-degrees-hot quark matter is a LIQUID!!

- Rutherford looked for “pudding”, and unexpectedly found point-like nuclei in atoms. Friedman, Kendall and Taylor unexpectedly found point-like quarks in protons.
  This surprise has the opposite character! Looking for a hot gas, found a hot liquid.

- Hot quark soup turns out to be *very* liquid-like. Less internal dissipation as it flows (quantified via a parameter called η/s) than in all other known liquids except one.

- The discovery that it is a strongly coupled liquid makes hot quark soup interesting to a broader scientific community. (Much more interesting than a gas.)

- How was this discovery made? Not à la Rutherford...

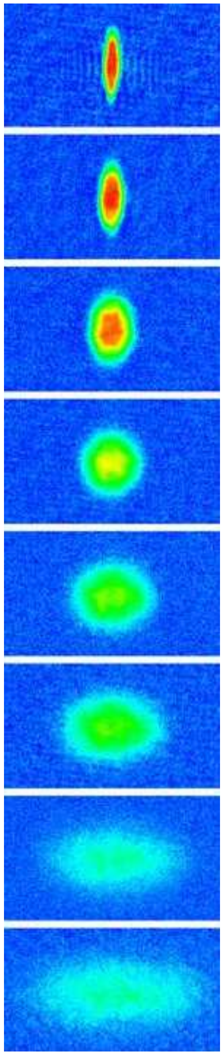
Ultracold Fermionic Atom Fluid

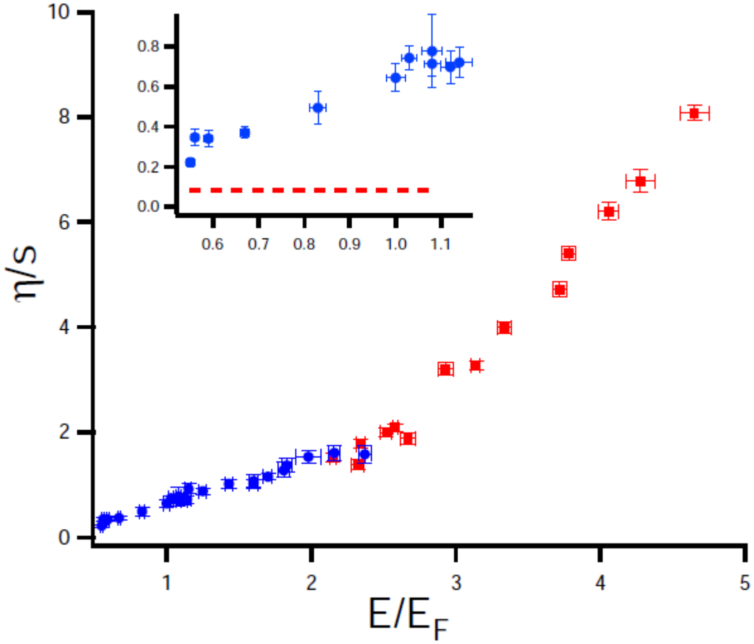
- The one terrestrial fluid with η/s comparably small to that of QGP.
- NanoKelvin temperatures, instead of TeraKelvin.
- Ultracold cloud of trapped atoms, with their interactions tuned to be infinite. A strongly coupled liquid indeed.
- Data on hydrodynamic flow patterns can be used to extract η/s as a function of temperature. . .

Viscosity to entropy density ratio

consider both collective modes (low T)

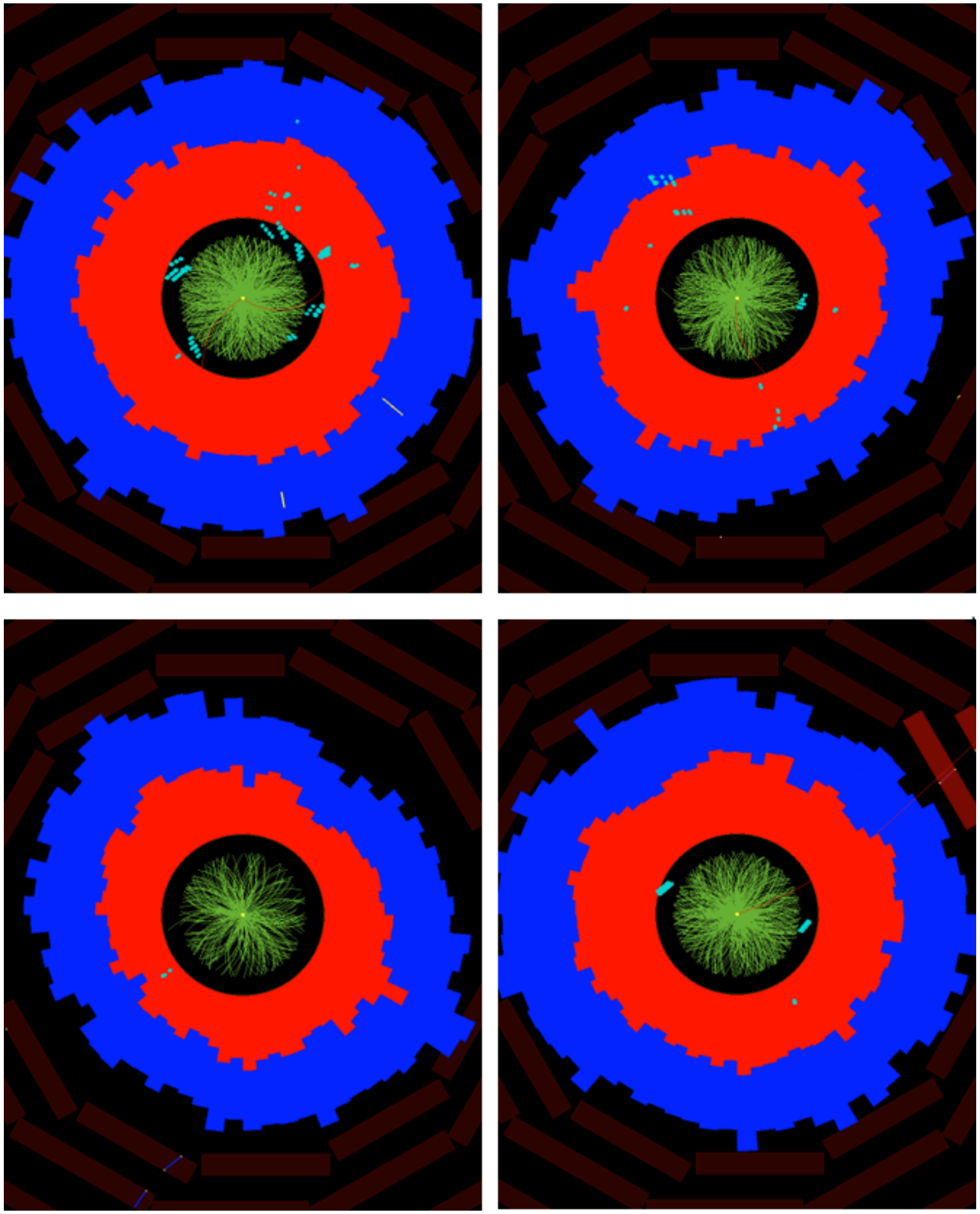
and elliptic flow (high T)





Cao et al., Science (2010)

$$\eta/s \leq 0.4$$



Hot Quark Soup!

- **Hydrodynamic analyses of how asymmetric blobs of hot Big Bang matter (produced in off-center collisions) explode reveal a very liquid-like liquid, with $0.1 < \eta/s < 0.2$.**

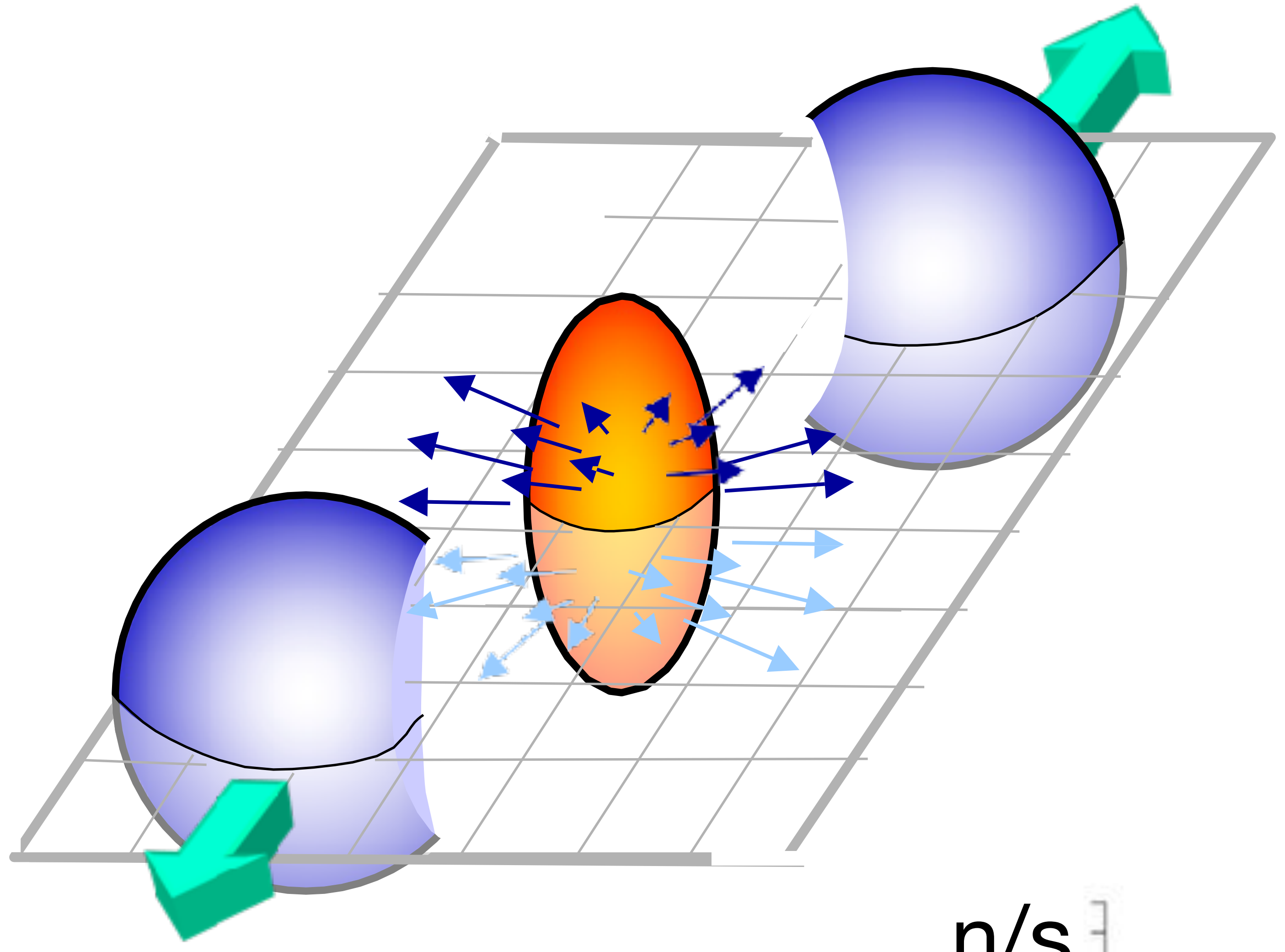
- **η/s is between 1 and 10 for ordinary liquids, and is > 100 for gases. Hot quark soup is "more hydrodynamic than the original hydro".**

- **Quarks and gluons in hot quark soup diffuse, without being confined in protons. Hot quark soup flows. Quarks and gluons are always bumping into each other. Very much *not* a gas; mean free paths so short as to be hard to define.**

- **Quarks and gluons are *not* confined — but also *not* free.**

- **Aside: $\eta/s = 1/(4\pi) \approx 0.08$ for the "hot soup phases" in all known QCD-like theories with strongly coupled phases that are "holograms" of a 4+1-dimensional gravity theory "heated by" a 3+1-dimensional black hole.**

QGP collective flow



QGP collective flow is well established.

Fireball shape is characterized by eccentricity: **ε_2**

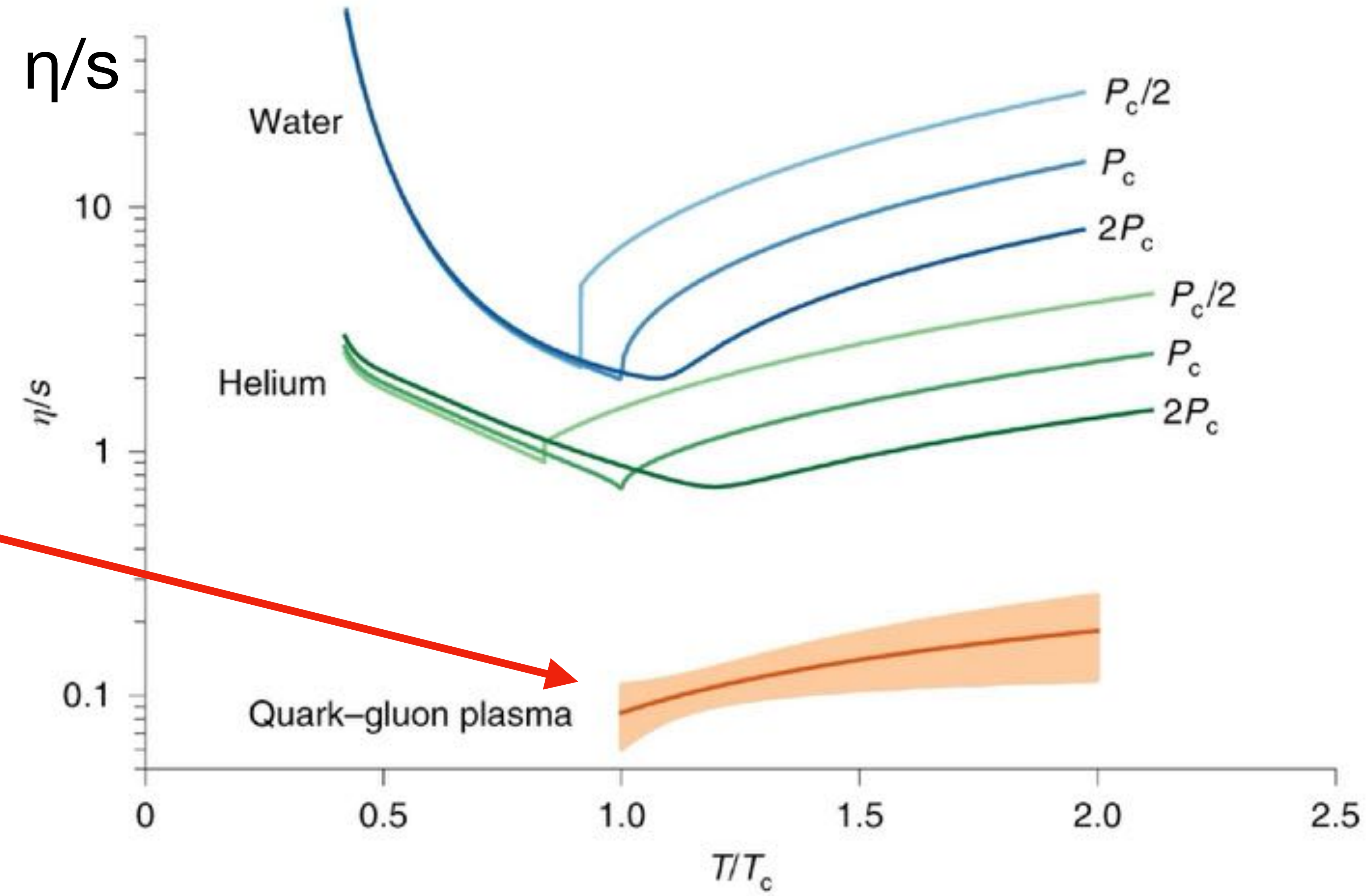
Hydrodynamical expansion: $\varepsilon_2 \rightarrow$ collective flow $v_2(p_T)$

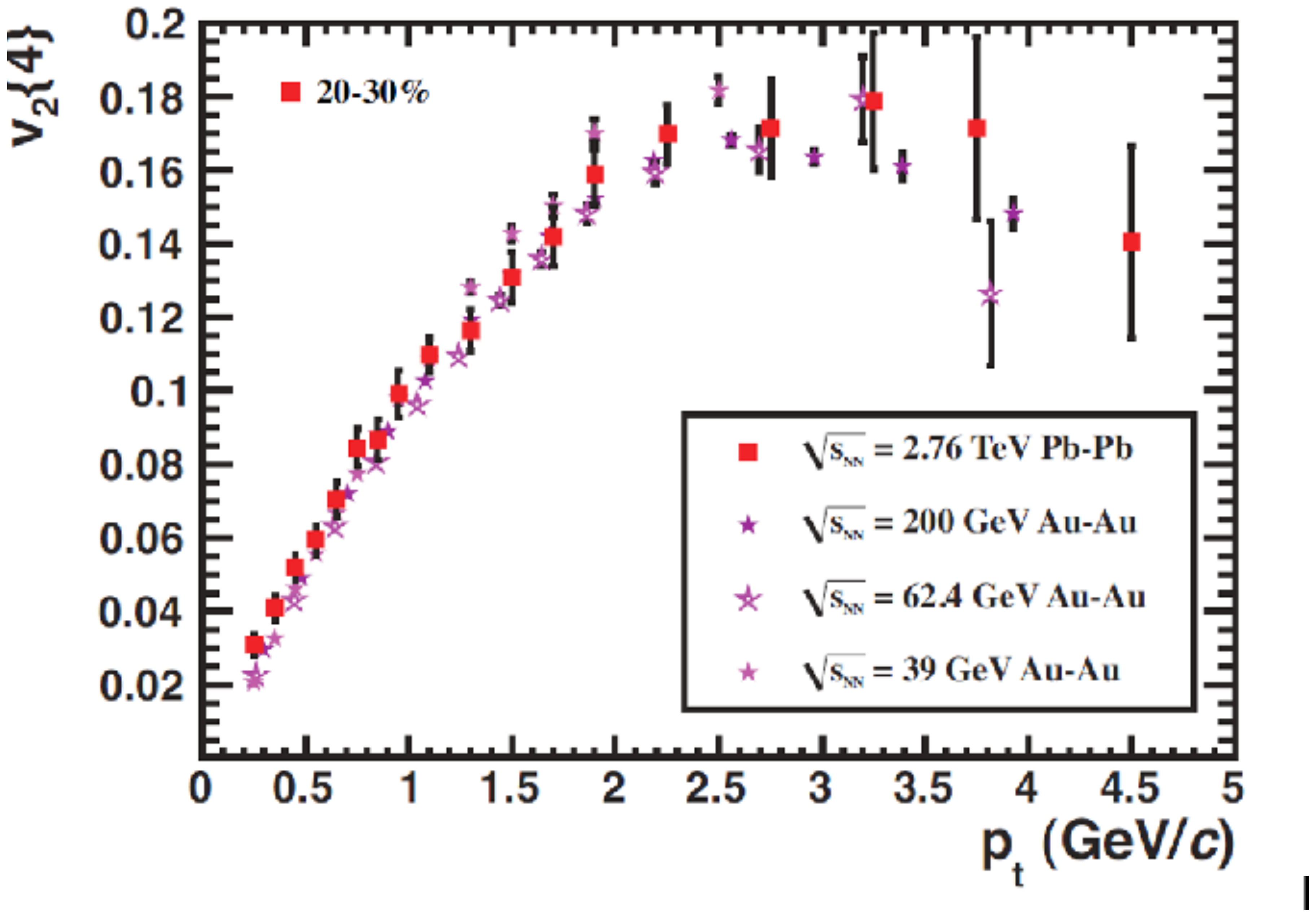
Ratio v_2/ε_2 is **very** sensitive to the viscosity of the QGP.

Main result:

$$\frac{\eta}{s} \approx 0.1 - 0.15$$

Most “perfect” fluid!

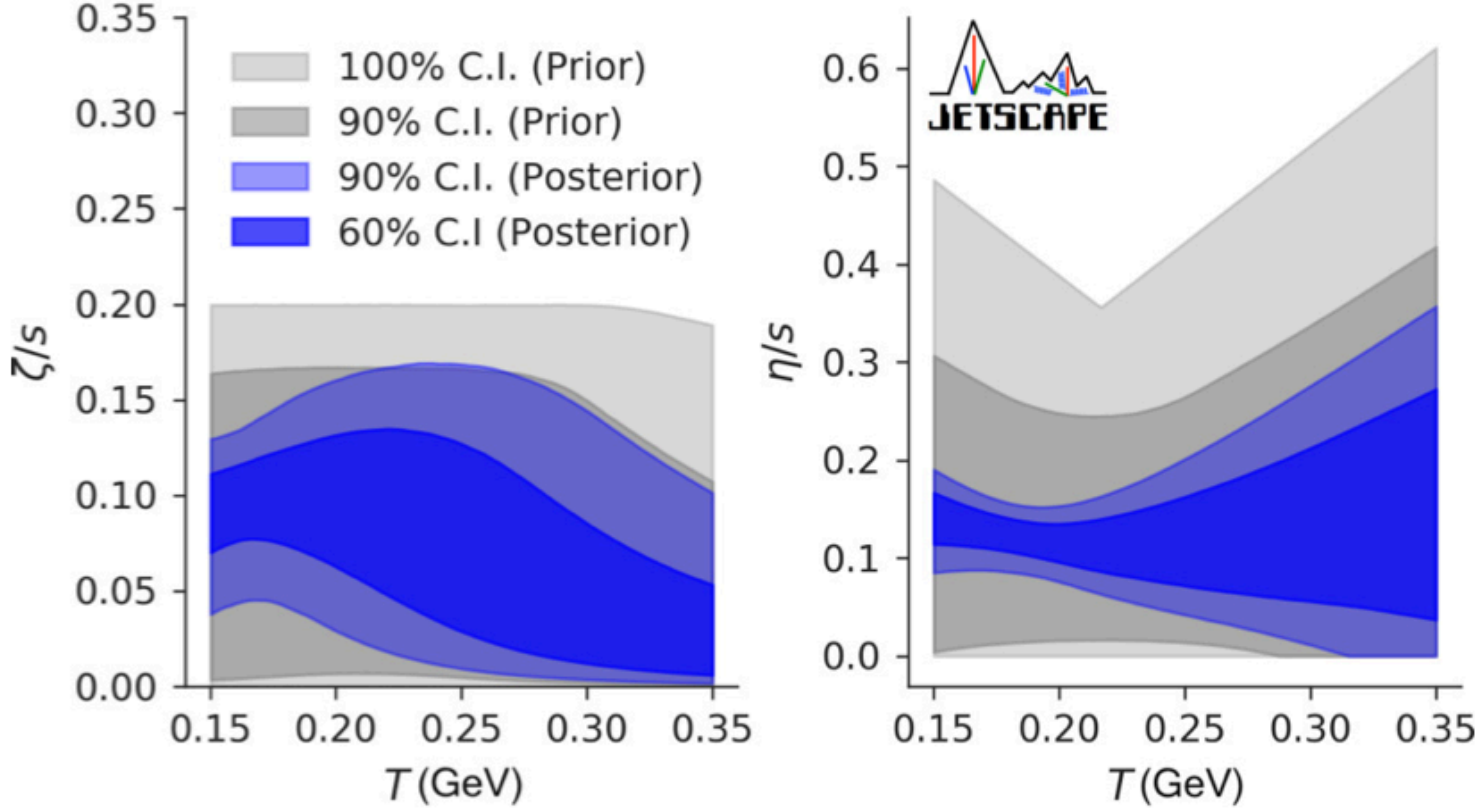




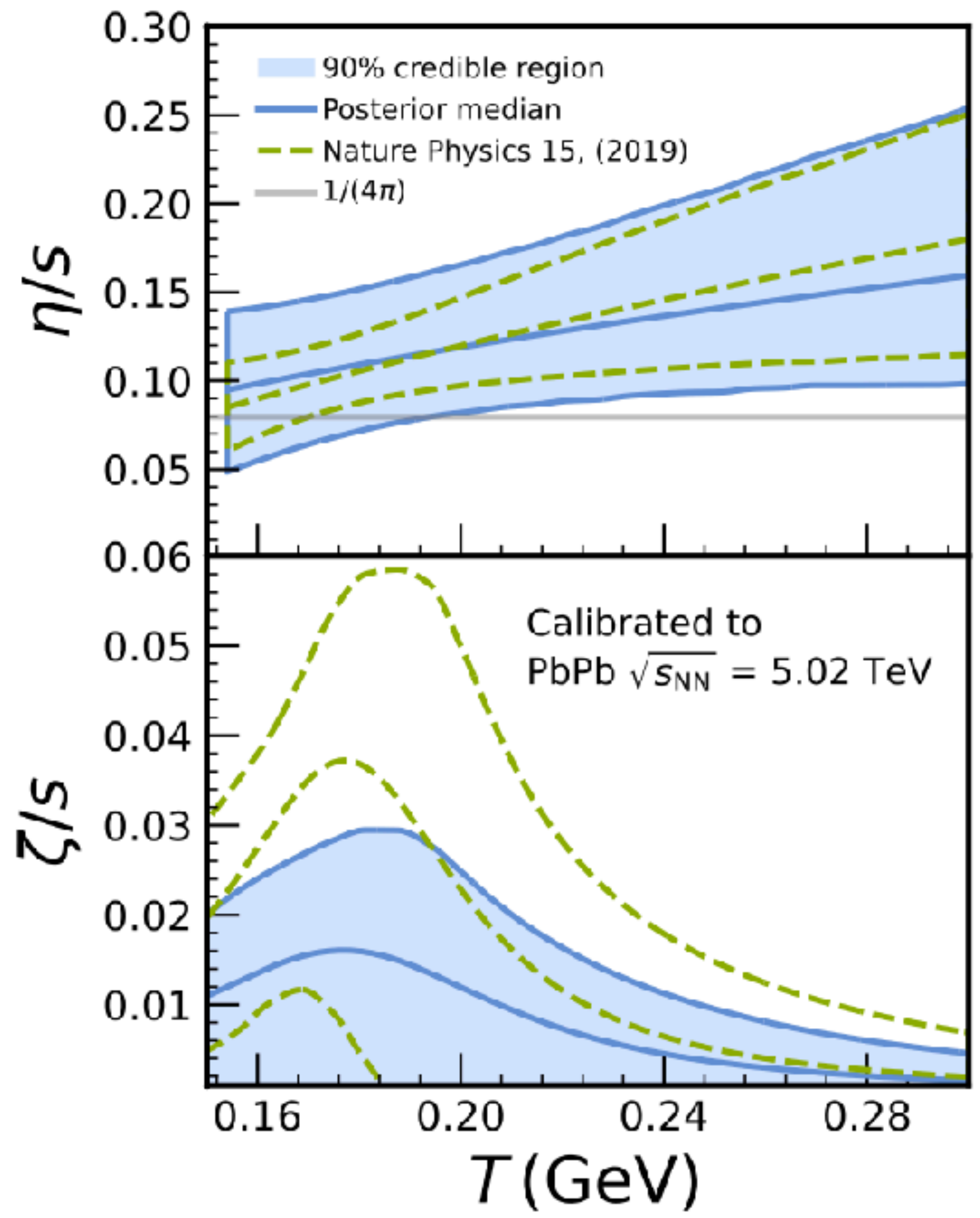
Bulk EoS: Non-equilibrium

Combined Bayesian analysis of Au+Au (RHIC) and Pb+Pb (LHC)

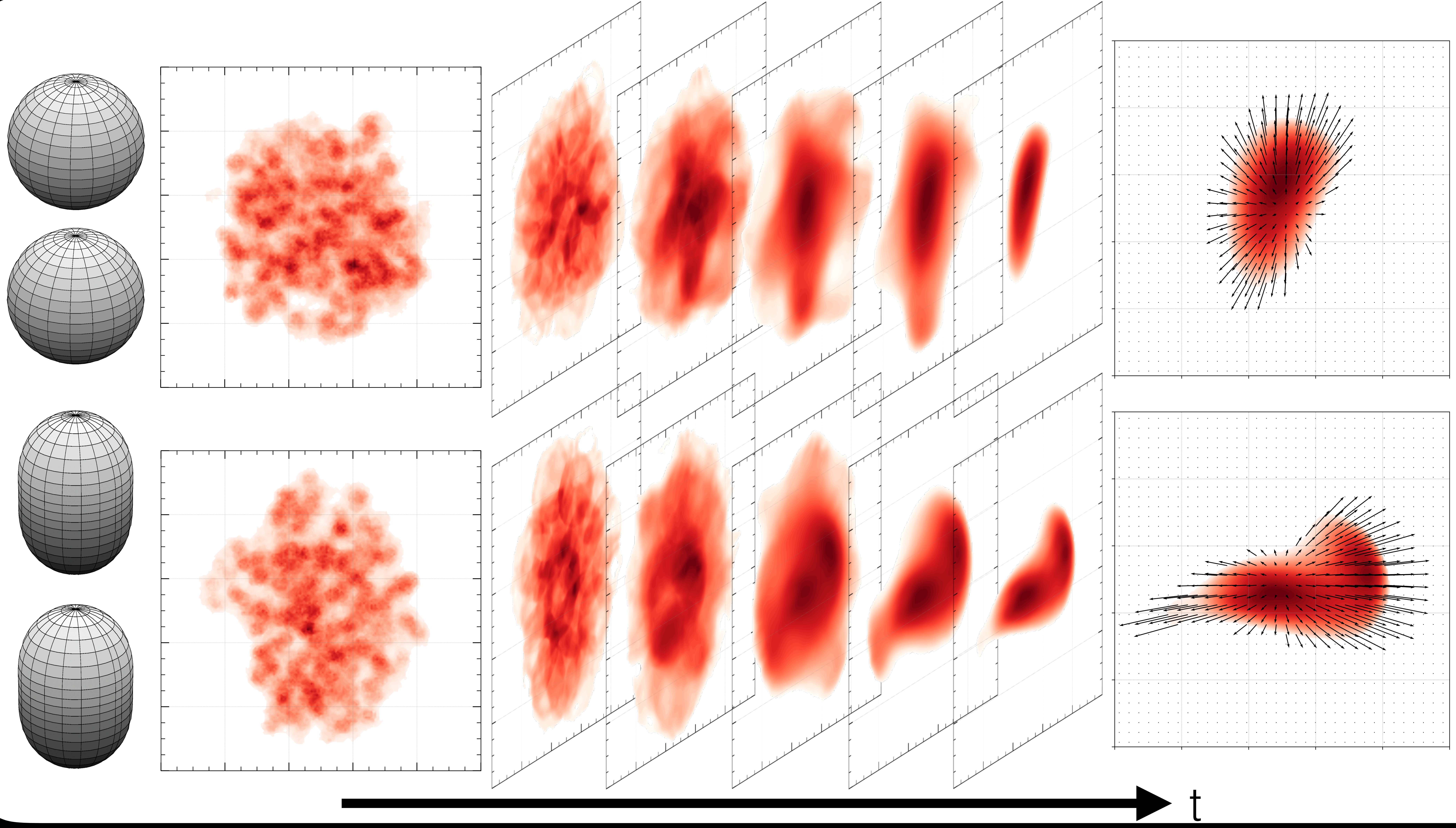
JETSCAPE, Phys. Rev. C **103** (2021) 054904



Parkkila, Onnerstad & Kim,
Phys. Rev. C **104** (2021) 054904



MAPPING FLOW BACK TO INITIAL-STATE GEOMETRY

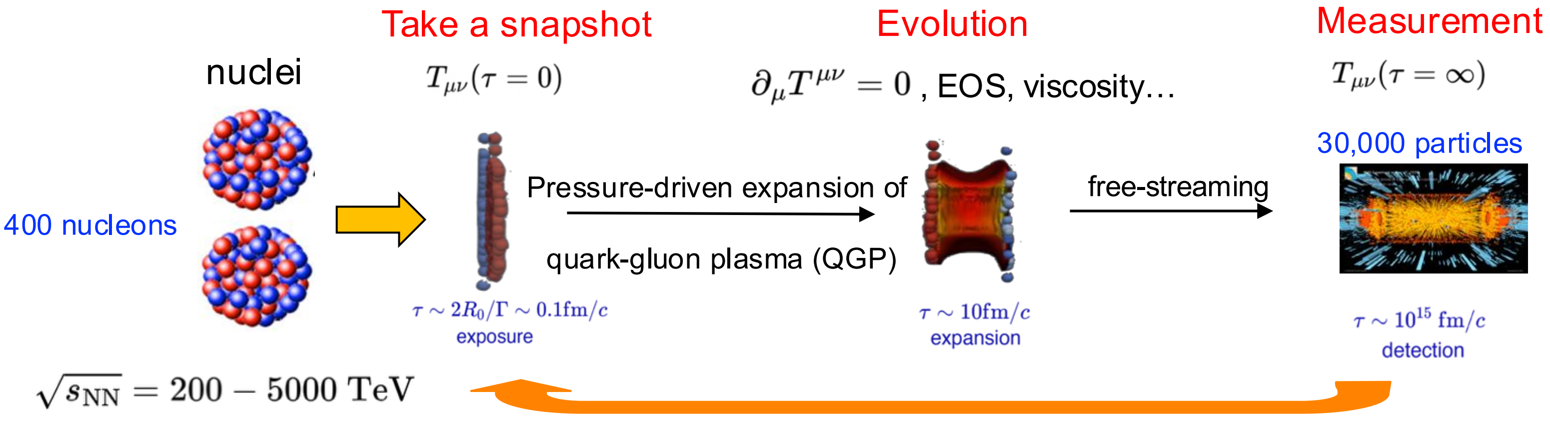


Take observable ratios:

$$\frac{O_{X+X}}{O_{Y+Y}}$$

to cancel effects from QGP evolution

High-energy imaging method



Take a snapshot

Evolution

Measurement

nuclei

$T_{\mu\nu}(\tau=0)$

$\partial_\mu T^{\mu\nu}=0$, EOS, viscosity…

$T_{\mu\nu}(\tau=\infty)$

30,000 particles

400 nucleons

Pressure-driven expansion of quark-gluon plasma (QGP)

free-streaming

$\tau \sim 2R_0/\Gamma \sim 0.1\mathrm{fm}/c$
exposure

$\tau \sim 10\mathrm{fm}/c$
expansion

$\tau \sim 10^{15}\ \mathrm{fm}/c$
detection

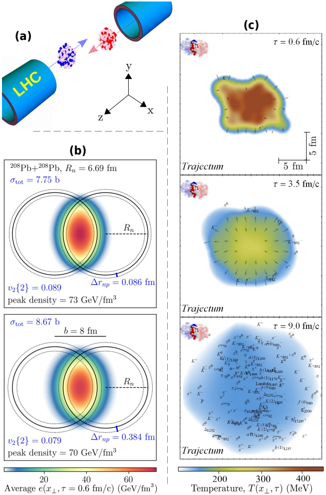
$\sqrt{s_{\mathrm{NN}}} = 200-5000\ \mathrm{TeV}$

Image inferred after destruction

Exploiting liner response: $V_n \propto \mathcal{E}_n \qquad \frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp}$

- Map moments of final-state observables back to the moments of initial geometry, order by order. This reverse-engineering procedure is what we call imaging-by-smashing.
- Not just a scientific curiosity. The ability to do imaging requires a solid understanding of the QGP dynamics. So every successful structure extraction also serves as an independent validation.

How to measure neutron skin?

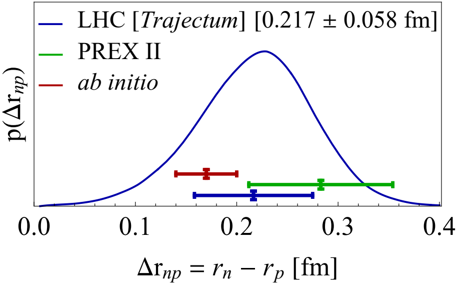


- To measure the neutron skin, we need the distributions of protons and neutrons inside the nucleus.
    - The proton distribution distribution is well-known from electron scattering.
- Several different methods are in use for the neutron distribution:
    - Polarized electron scattering off ^{208}Pb (PREX).
    - Photon tomography of ^{197}Au (STAR).
- Heavy ion collisions provide a completely orthogonal method.
    - Sensitive to the total matter distribution inside the nucleus.
    - Purely gluonic measurement.

[Giacalone, GN, van der Schee, 2305.00015; STAR, 2204.01625; PREX, 2102.10767]

Bayesian analysis result using LHC data

- Resulting posterior for Δr_{np} is compatible with PREX II and ab initio nuclear theory.
- Slightly stronger constraint than PREX II ($\Delta r_{np} = 0.283 \pm 0.071$).
- Result is in principle improvable with better Bayesian analyses.
  - May be hard to do in practice.
  - The current analysis already took 2M CPUh.



[Giacalone, GN, van der Schee, 2305.00015; PREX, 2102.10767; Hu et al., *Nat. Phys.* **18**, 1196–1200 (2022)]

Hot Quark Soup!

- **Hydrodynamic analyses of how asymmetric blobs of hot Big Bang matter (produced in off-center collisions) explode reveal a very liquid-like liquid, with $0.1 < \eta/s < 0.2$.**

- **η/s is between 1 and 10 for ordinary liquids, and is > 100 for gases. Hot quark soup is "more hydrodynamic than the original hydro".**

- **Quarks and gluons in hot quark soup diffuse, without being confined in protons. Hot quark soup flows. Quarks and gluons are always bumping into each other. Very much *not* a gas; mean free paths so short as to be hard to define.**

- **Quarks and gluons are *not* confined — but also *not* free.**

- **Aside: $\eta/s = 1/(4\pi) \approx 0.08$ for the "hot soup phases" in all known QCD-like theories with strongly coupled phases that are "holograms" of a 4+1-dimensional gravity theory "heated by" a 3+1-dimensional black hole.**

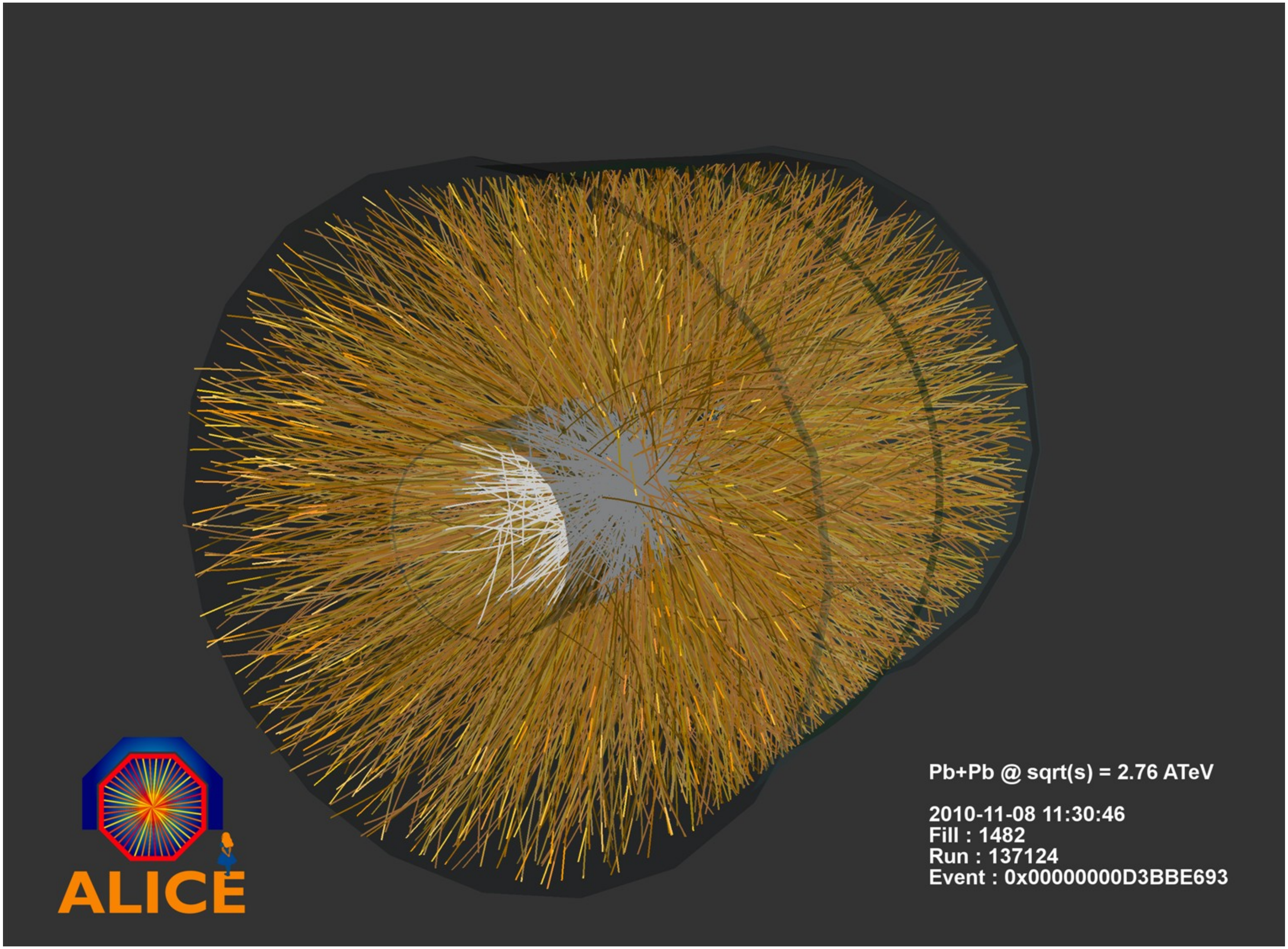
Hot Quark Soup, Under Pressure

- What does hot quark soup, with a pressure greater than 2×10^{31} atmospheres, do??

- If it fills the entire universe, it expands “slowly”. It takes $\sim 10-20$ microseconds to cool, and fall apart into a mist of protons. (That continues to expand, and form structure, over the next 13 billion years.)

- If you make a little droplet of the stuff, with nothing around it, it explodes!

- And, the shape of the explosion tells you about the shape of the droplet, and about its liquidness.


Pb+Pb @ sqrt(s) = 2.76 ATeV

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Run : 137124
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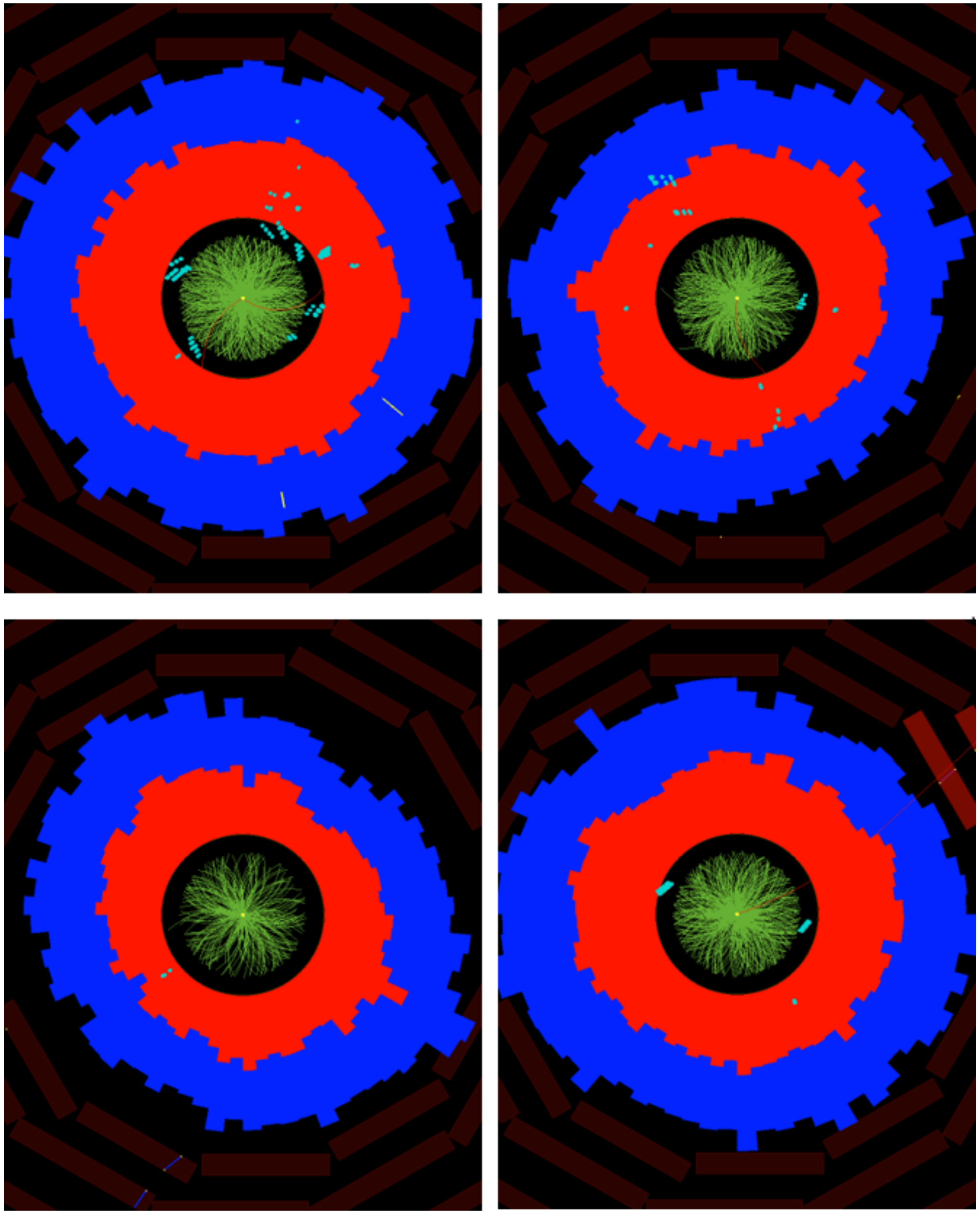
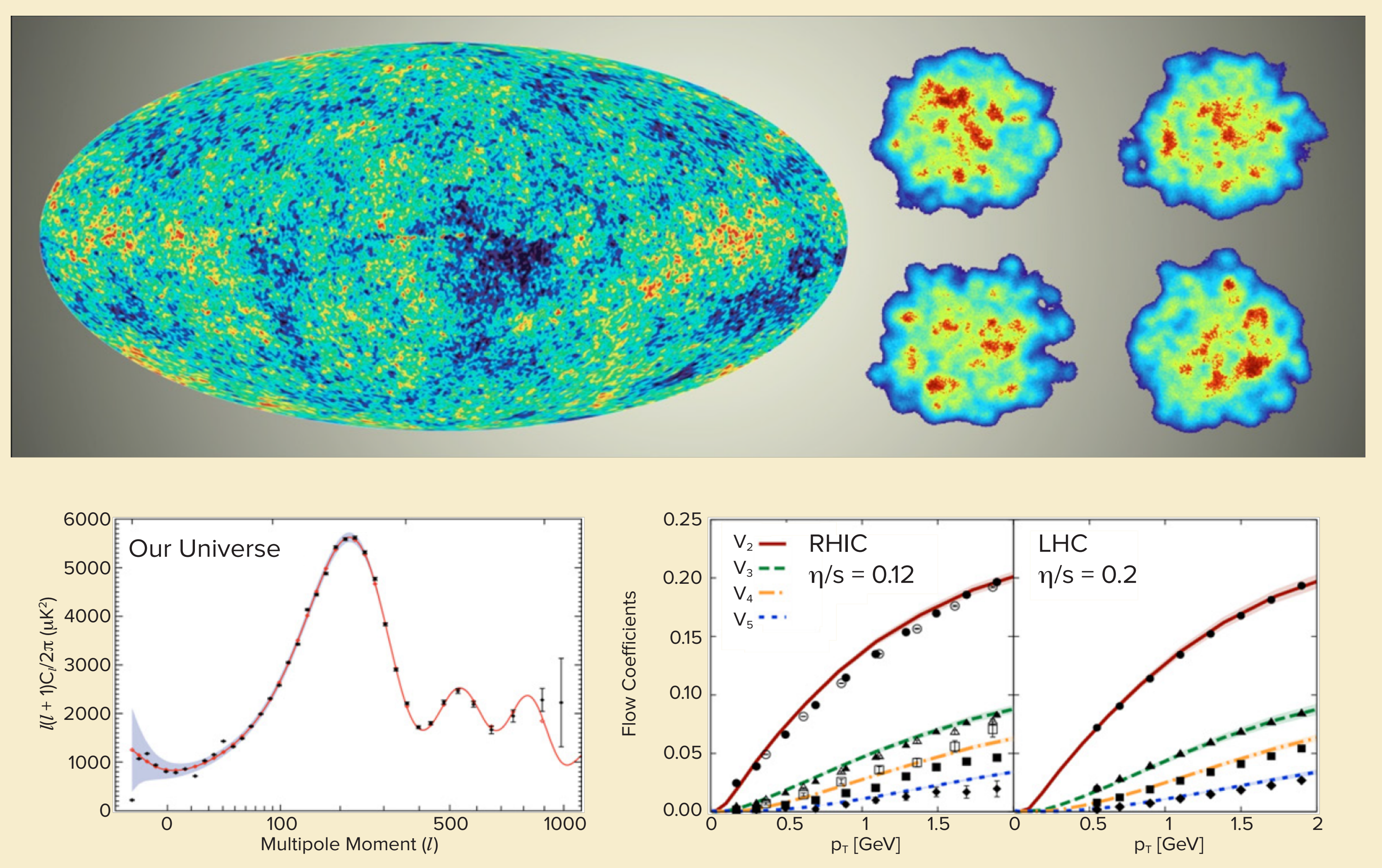
ALICE




Our Universe
6000
5000
4000
3000
2000
1000
0
$l(l + 1)C_l/2\pi$ (μK^2)
0
100
500
1000
Multipole Moment (l)
V_2
V_3
V_4
V_5
RHIC
$\eta/s = 0.12$
LHC
$\eta/s = 0.2$
Flow Coefficients
0.25
0.20
0.15
0.10
0.05
0.00
0
0.5
1
1.5
0
0.5
1
1.5
2
p_T [GeV]
p_T [GeV]

Hot Quark Soup cf Cosmic Background Photons

- In cosmology, initial-state quantum fluctuations, processed by hydrodynamics, appear in data as c_ℓ's. From the c_ℓ's, learn about initial fluctuations, and about the matter that filled the universe 400,000 years after the Big Bang, eg its proton/photon ratio.

- In heavy ion collisions, initial state quantum fluctuations, processed by hydrodynamics, appear in data as v_n's. From v_n's, learn about initial fluctuations, and about the hot quark soup that filled the microseconds-old universe — eg its η/s.

- Cosmologists have a huge advantage in resolution: c_ℓ's up to $\ell \sim$ thousands. But, they have only one "event"!

- Heavy ion collisions only up to v_6 at present. But they have billions of events. And, they can do controlled variations of the initial conditions, to understand systematics.

Beyond Quasiparticles

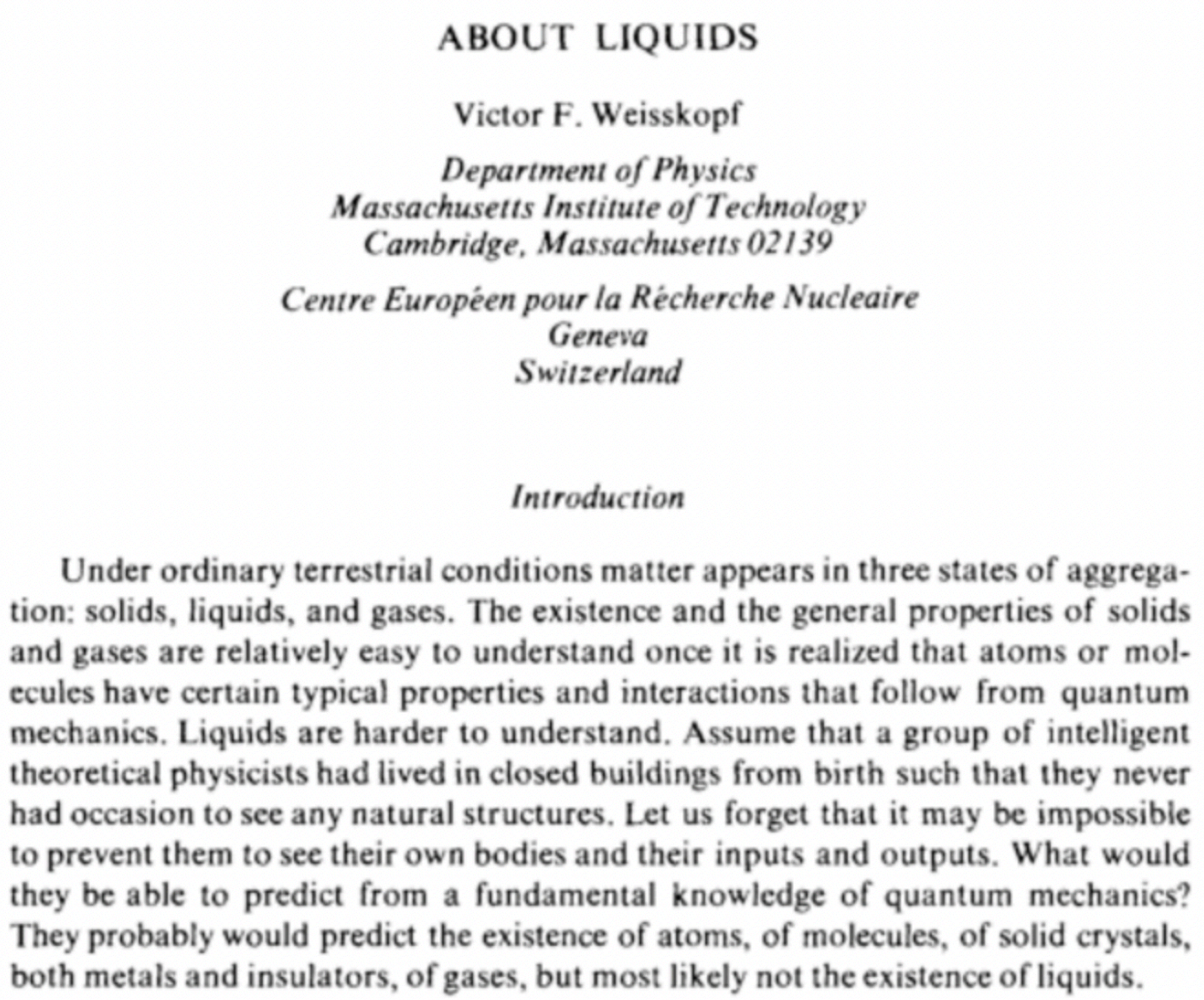
- **QGP at RHIC & LHC, unitary Fermi "gas", gauge theory plasmas with holographic descriptions are all strongly coupled fluids with no apparent quasiparticles.**

- **In QGP, with η/s as small as it is, there can be no 'transport peak', meaning no self-consistent description in terms of quark- and gluon-quasiparticles. [Q.p. description self consistent if $\tau_{\rm qp} \sim (5\eta/s)(1/T) \gg 1/T$.]**

- **Other "fluids" with no quasiparticle description include: the "strange metals" (including high-T_c superconductors above T_c); quantum spin liquids; matter at quantum critical points;... Among the grand challenges at the frontiers of condensed matter physics today.**

- **In all these cases, after discovery two of the central strategies toward gaining understanding are *probing* and *doping*. To which we will turn...**

About Liquids. . .



ABOUT LIQUIDS

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Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

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Geneva
Switzerland

Introduction

Under ordinary terrestrial conditions matter appears in three states of aggregation: solids, liquids, and gases. The existence and the general properties of solids and gases are relatively easy to understand once it is realized that atoms or molecules have certain typical properties and interactions that follow from quantum mechanics. Liquids are harder to understand. Assume that a group of intelligent theoretical physicists had lived in closed buildings from birth such that they never had occasion to see any natural structures. Let us forget that it may be impossible to prevent them to see their own bodies and their inputs and outputs. What would they be able to predict from a fundamental knowledge of quantum mechanics? They probably would predict the existence of atoms, of molecules, of solid crystals, both metals and insulators, of gases, but most likely not the existence of liquids.

Viki Weisskopf was right, in his 1977 homage to liquids, and to Edward Purcell. All the physicists on planet earth had “a fundamental knowledge” of the laws that govern quarks and gluons from 1973 onward. They did not predict that hot quark soup was a liquid!

Heavy Ion Collisions: What Next?

By recreating droplets of the matter that filled the microseconds-old universe in ultrarelativistic heavy ion collisions, we have discovered a liquid that, as far as we now know, is:

- The first liquid that ever existed; the "original liquid"...
- The liquid from which the protons and neutrons in today's universe formed, as the liquid fell apart into mist.
- At a few trillion degrees, the hottest liquid that has ever existed.
- The earliest complex form of matter.
- The most liquid liquid that has ever existed, with a specific viscosity $\eta/s \sim 0.1$.
- In a sense the simplest form of complex matter, namely in the sense that it is "close" to the fundamental degrees of freedom of the standard model.

All great discoveries pose new challenges...

What Next?

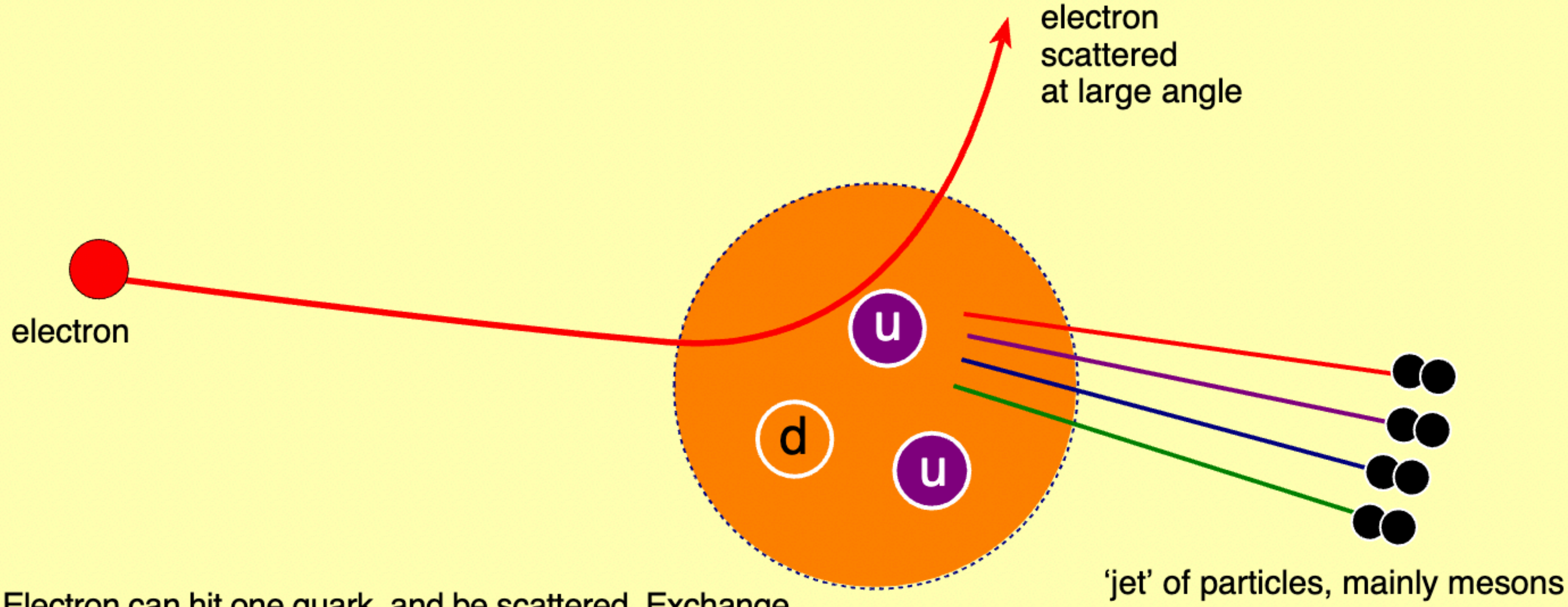
Two kinds of What Next? questions for the coming decade...

- A question that one asks after the discovery of any new form of complex matter: What is its phase diagram? For high temperature superconductors, for example, phase diagram as a function of temperature and doping. Same here! For us, doping means excess of quarks over antiquarks, rather than an excess of holes over electrons.
- A question that we are privileged to have a chance to address, after the discovery of "our" new form of complex matter, which is so close to the fundamentals: How does the strongly coupled liquid emerge from laws governing quarks and gluons? Maybe answering this question could help to understand how strongly coupled matter emerges in other contexts.

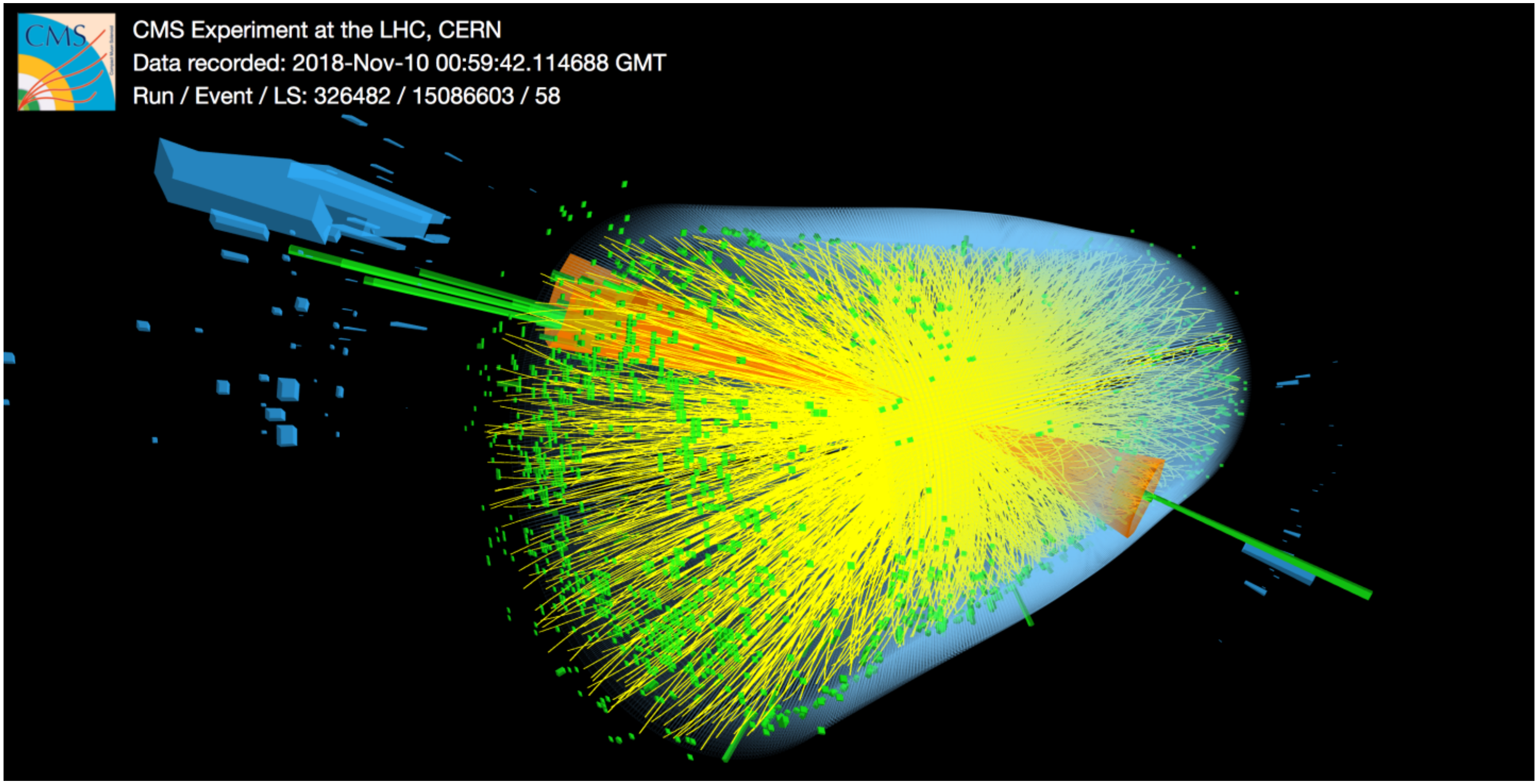
Probing the Original Liquid

- The question How does the strongly coupled liquid emerge from the fundamental laws governing quarks and gluons? is one of today's most active research frontiers.
- Seeing the inner workings of hot quark soup.
- First step to seeing what they are doing is we need to "see" the individual quarks and gluons that make up the liquid. Need a high-resolution, fast shutter-speed, look at one quark or gluon at one moment.
- Need to do for hot quark soup what Rutherford did for atoms and Friedman, Kendall and Taylor did for protons.
- Need to probe the liquid, see how the liquid responds, *and watch how the probe scatters*.
- Can't bring a drop of Big Bang matter from Geneva to Stanford to image it with an electron beam; it only lives for 10^{-22} seconds! Have to use a probe made in the same collision that makes the drop of hot quark soup. Jets!

High energy: deep inelastic scattering



Electron can hit one quark, and be scattered. Exchange of high energy photons leads to the creation of a jet of particles and antiparticles.


CMS Experiment at the LHC, CERN
Data recorded: 2018-Nov-10 00:59:42.114688 GMT
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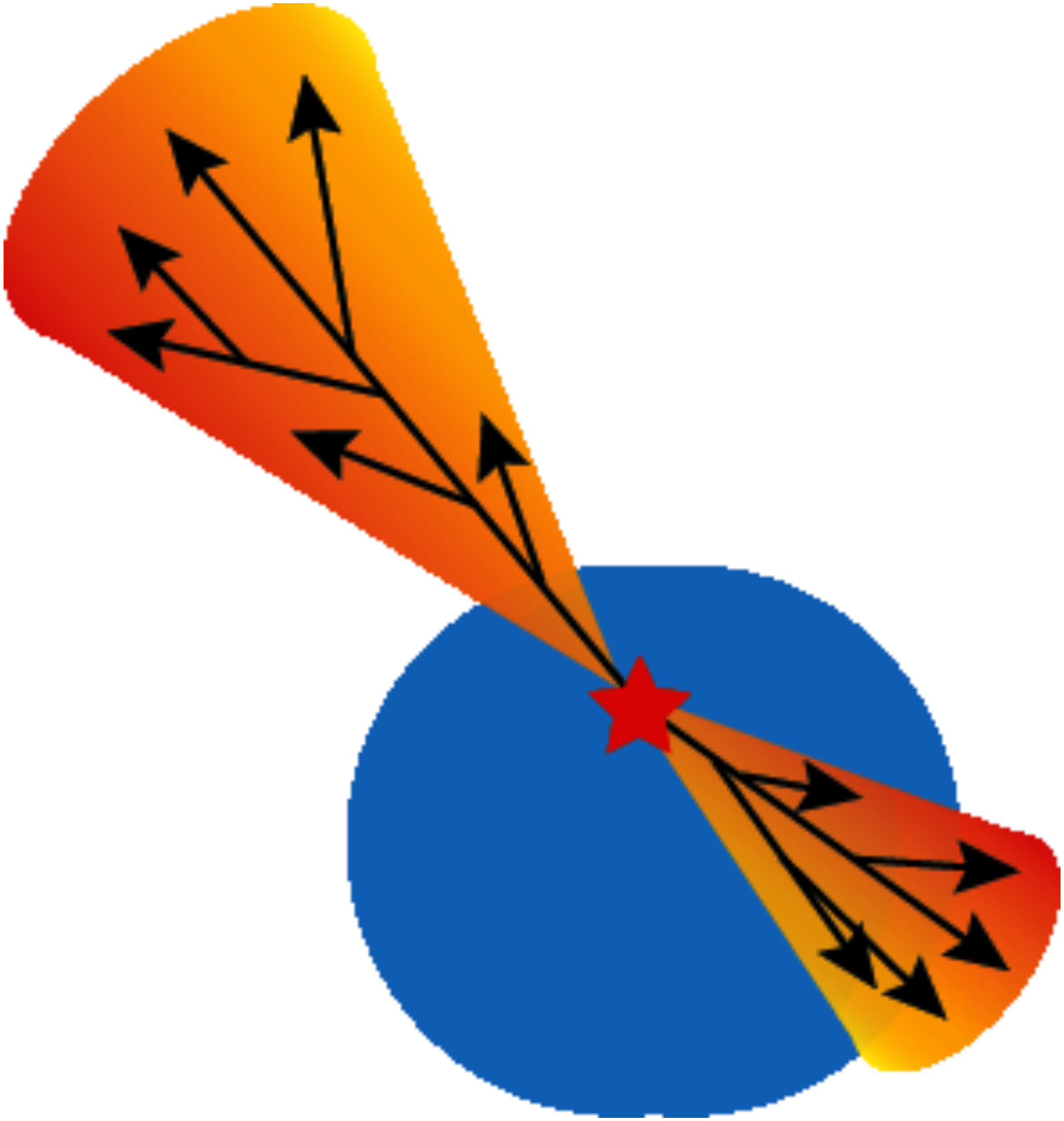
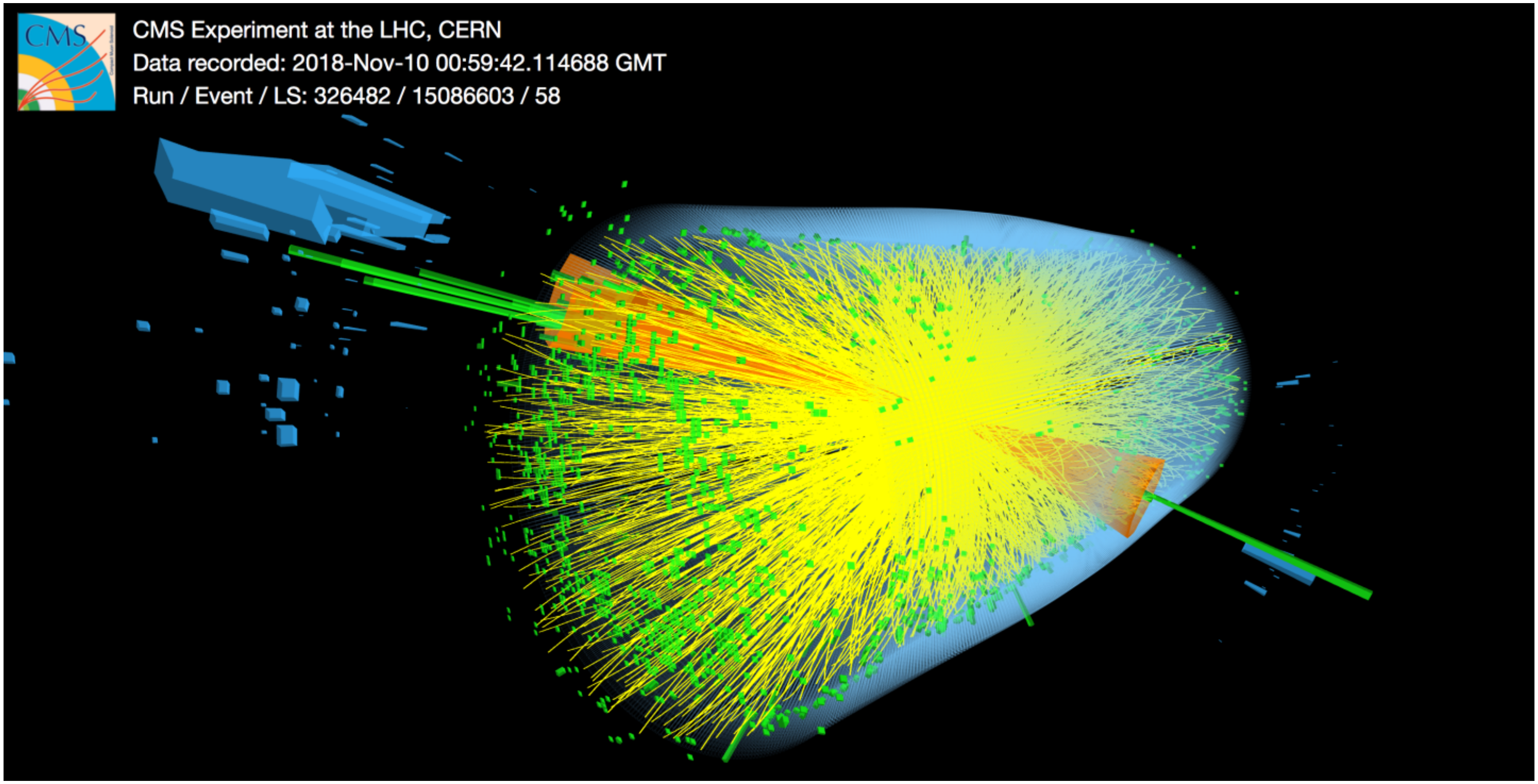


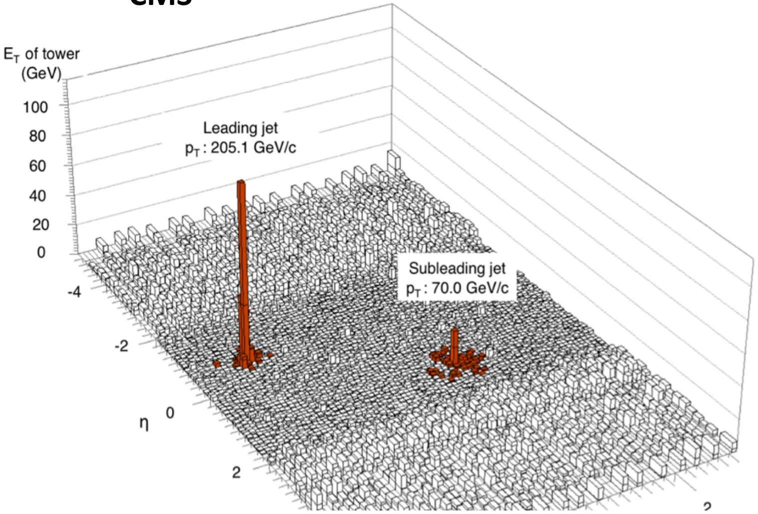

CMS Experiment at the LHC, CERN
Data recorded: 2018-Nov-10 00:59:42.114688 GMT
Run / Event / LS: 326482 / 15086603 / 58

Why Jets?

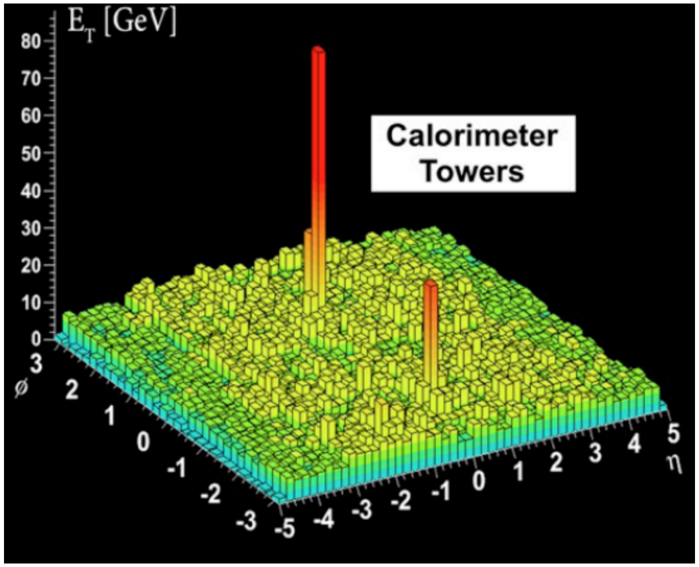
- The remarkable utility of hydrodynamics, eg. in describing the dynamics of small lumps in the initial state in heavy ion collisions, tells us that to see the inner workings of hot quark soup, namely to see how the liquid is put together from quarks and gluons, we will need probes with fine resolution.
- Jets in heavy ion collisions provide best chance for scattering off a droplet of hot Big Bang matter to see its inner workings à la Rutherford.
- Jets in heavy ion collisions *also* offer best chance of watching how the droplet responds. Jets leave a wake in the droplet of liquid. Can we see how this wake ripples and dissipates? Jets are our best shot at seeing this, too.
- → not easy to decode the wealth of info that jets contain! Need high statistics LHC and sPHENIX data; and need to use today's data to build baseline of understanding.

Jet Quenching, in brief

CMS



ATLAS



Jet quenching discovered @ RHIC; @ LHC, seen instantly!

- 200+ GeV jets lose many tens of GeV passing through the liquid QGP. This is well established.
- Lost energy turns into a wake, which becomes many soft particles, spread widely around the jet.
- To see the high energy quarks and gluons in a jet scatter off the quarks and gluons in the soup need more sophisticated measurements, now being defined, developed, planned.

Jets as Probes of QGP

- Theorists taking key steps. . .
- Disentangling jet modification from jet selection.
- Calculations of the dynamics of jet wakes in droplets of QGP, identification of new experimental observables, and predictions that will enable experimental measurements to "see" the particles coming from these wakes.
- Showing that hot quark soup (QGP) *can* respond to substructure within jets.
- Identifying those jet substructure observables that *are* sensitive to scattering of jet quarks/gluons off QGP quarks/gluons, "seeing" the latter à la Rutherford, and are *not* sensitive to particles coming from the wake.
- Next several years will be the golden age of jet physics: sPHENIX, LHC runs 3 and 4, new substructure observables. *Many* theory advances, and analyses of today's data, whet our appetite for the feast to come.
- We shall learn about the microscopic structure of QGP, and the dynamics of rippling QGP.

How you can learn from a model

- There are things you can do with a model (here, the Hybrid Model) that you cannot do with experimental data. (Eg, turn physical effects off and on) . . .
- . . . but that nevertheless teach us important lessons for how to look at, and learn from, experimental data.
- TODAY: finding jet observables that are sensitive to wakes that jets make in the soup.
- TODAY: finding jet observables that are sensitive to how individual branches in a jet lose energy to the soup
- TODAY: finding jet observables that can see jet partons scattering off QGP partons, à la Rutherford

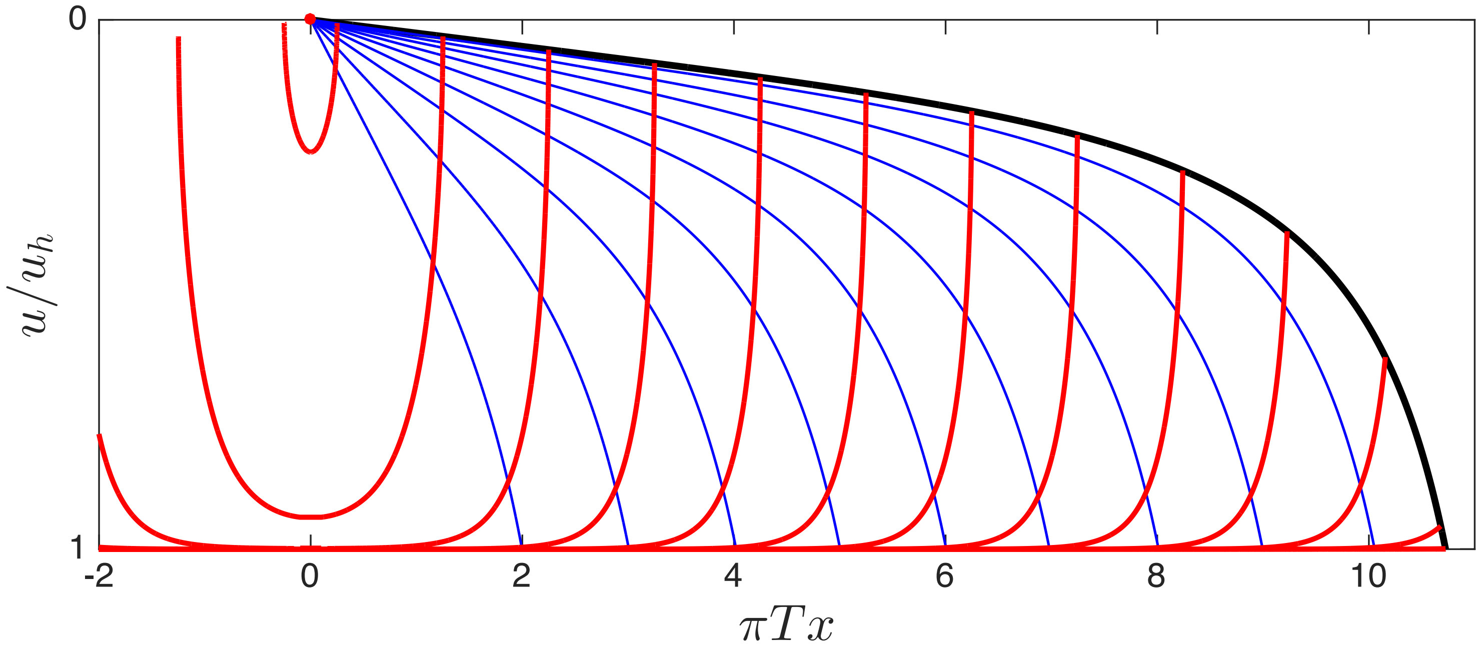
A Hybrid Approach

Casalderrey-Solana, Gulhan, Milhano, Pablos, KR, 2014,15,16; Hulcher, DP,KR, '17; JCS,ZH,GM,DP,KR, '18; JCS,GM,DP,KR, '19; JCS,GM,DP,KR, Yao, '20; ZH, DP, KR, '20; Bossi, Kudinoor, Moult, DP, Rai, KR, '24; AK, DP, KR, '25; Beraudo, Du Plessis, DP, KR, '25; ZH, AK, DP, KR, '25

- **Hard scattering and fragmentation — jet formation — described well by traditional weakly coupled QCD calculational methods.**
- **The medium itself is a strongly coupled liquid, with no apparent weakly coupled description. Describe the dynamics of the droplet with hydrodynamics. And, the energy the jet loses seems to quickly become one with the medium.**
- **Try a hybrid approach. Shower forms á la PYTHIA. Every parton in the shower loses energy á la dE/dx for quarks in a strongly coupled liquid in a strongly coupled cousin of QCD — with one parameter fit to data. Jet wakes with hydrodynamics. And, describe rare, hard, large-angle scattering of a parton in the jet of a parton in the soup à la Rutherford (traditional QCD calculation).**

Quenching a Light Quark "Jet"

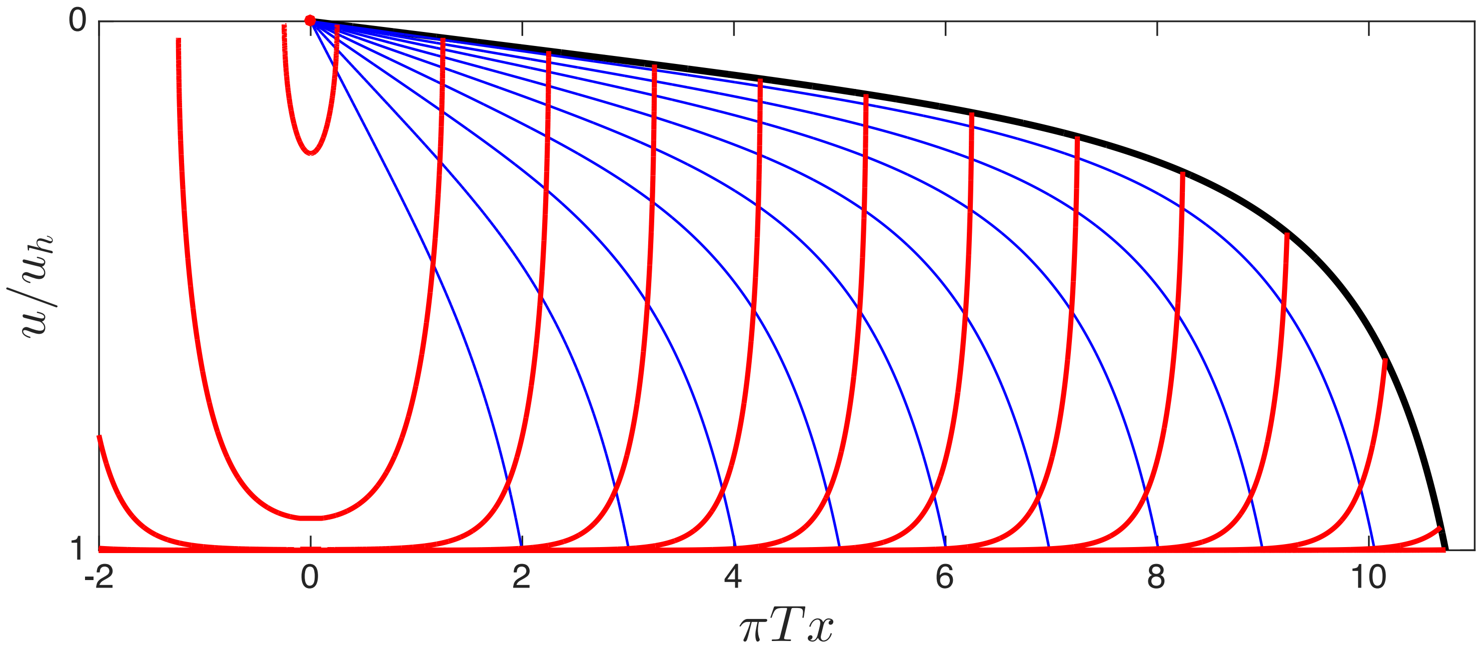
Chesler, Rajagopal, 1402.6756, 1511.07567



- Take a highly boosted light quark and shoot it through strongly coupled plasma...
- A fully geometric characterization of energy loss. Which is to say a new form of intuition. Energy propagates along the blue curves, which are null geodesics in the bulk. When one of them falls into the horizon, that's energy loss! Precisely equivalent to the light quark losing energy to a hydrodynamic wake in the plasma.

Quenching a Light Quark "Jet"

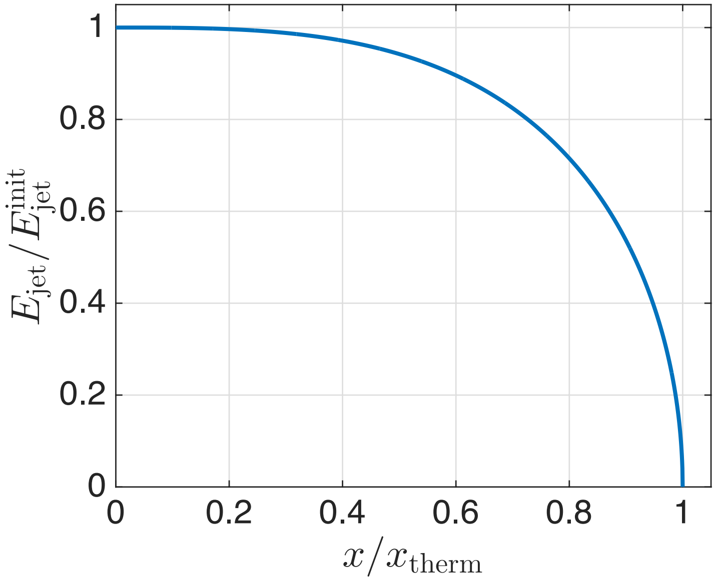
Chesler, Rajagopal, arXiv:1402.6756, 1511.07567



- **Can try to interpret this object as a toy model for a jet.**
- **Depth into the bulk ↔ transverse size of the gauge theory object being described.**
- **Thus, downward angle into the bulk ↔ opening angle.**
- **This calculation describes a "jet" with some initial $\theta_{\text{jet}}^{\text{init}} \propto$ initial downward angle of the endpoint.**

Quenching a Light Quark “Jet”

Chesler, Rajagopal, 1402.6756, 1511.07567



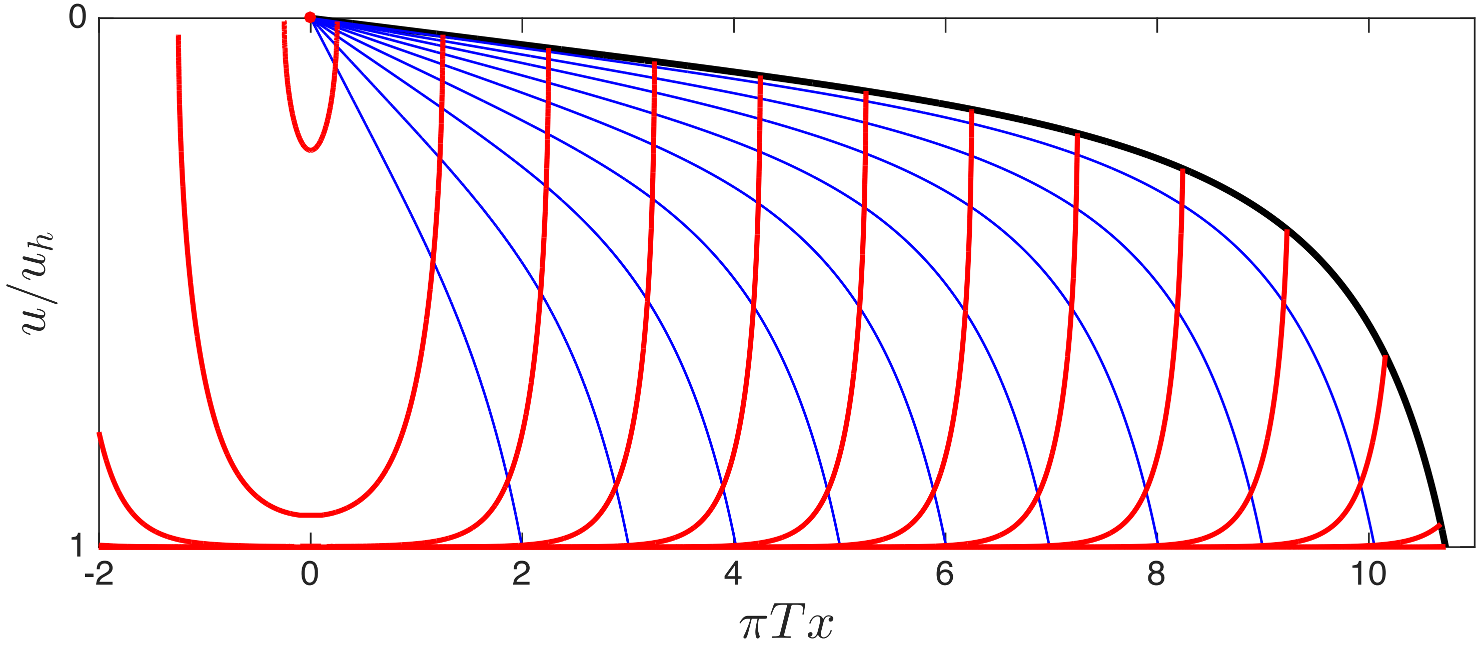
We compute E_{jet} analytically, by integrating the energy flowing into hydrodynamic modes, and showing its equivalence to that falling into the horizon. Geometric derivation of analytic expression for dE_{jet}/dx

$$\frac{1}{E_{\text{jet}}^{\text{init}}}\frac{dE_{\text{jet}}}{dx} = -\frac{4x^2}{\pi x_{\text{therm}}^2}\frac{1}{\sqrt{x_{\text{therm}}^2 - x^2}}$$

where $T x_{\text{therm}} = \mathcal{C}(E_{\text{jet}}^{\text{init}}/(\sqrt{\lambda}T))^{1/3}$ where $\mathcal{C}$ is $\mathcal{O}(1)$.

Quenching a Holographic Jet

Chesler, Rajagopal, arXiv:1511.07567

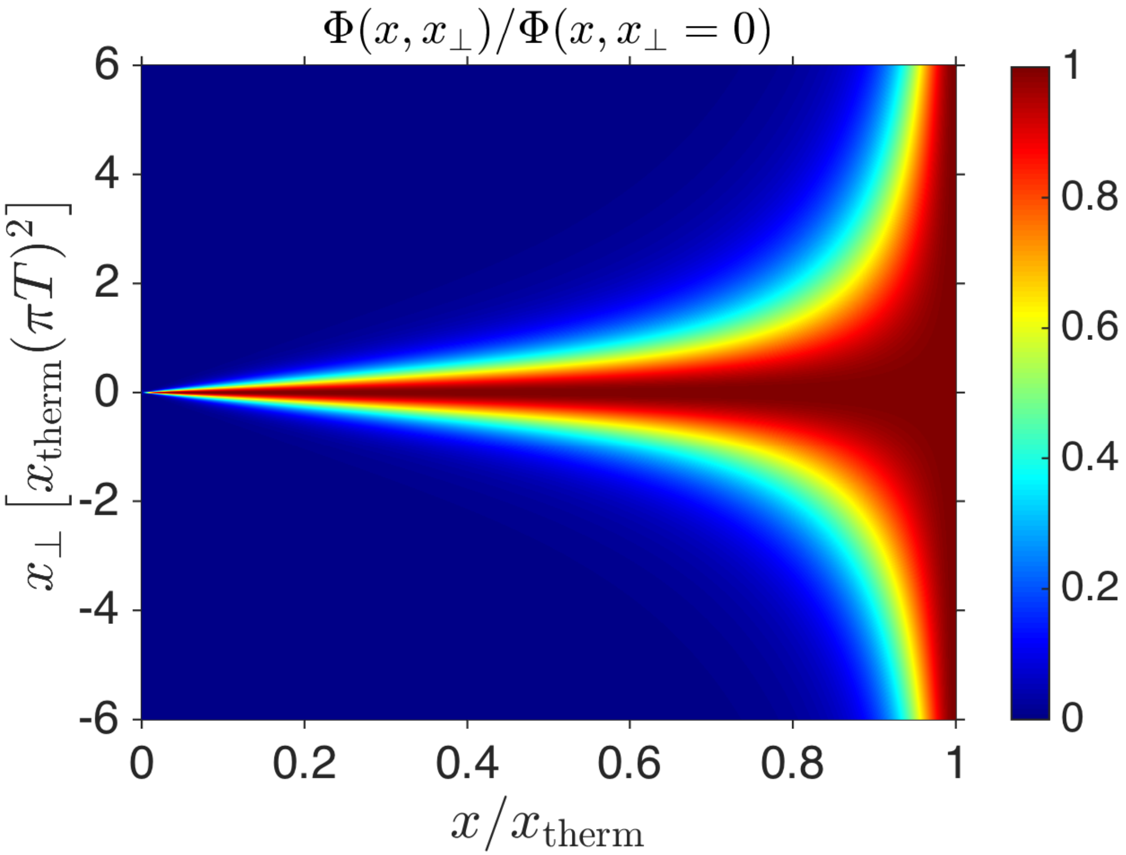


Two immediate, inescapable, qualitative consequences, of geometric origin when described holographically:

- First, every jet broadens in angle as it propagates through the strongly coupled plasma. θ_{jet} increases as E_{jet} decreases.

Holographic “Jet” Energy Loss

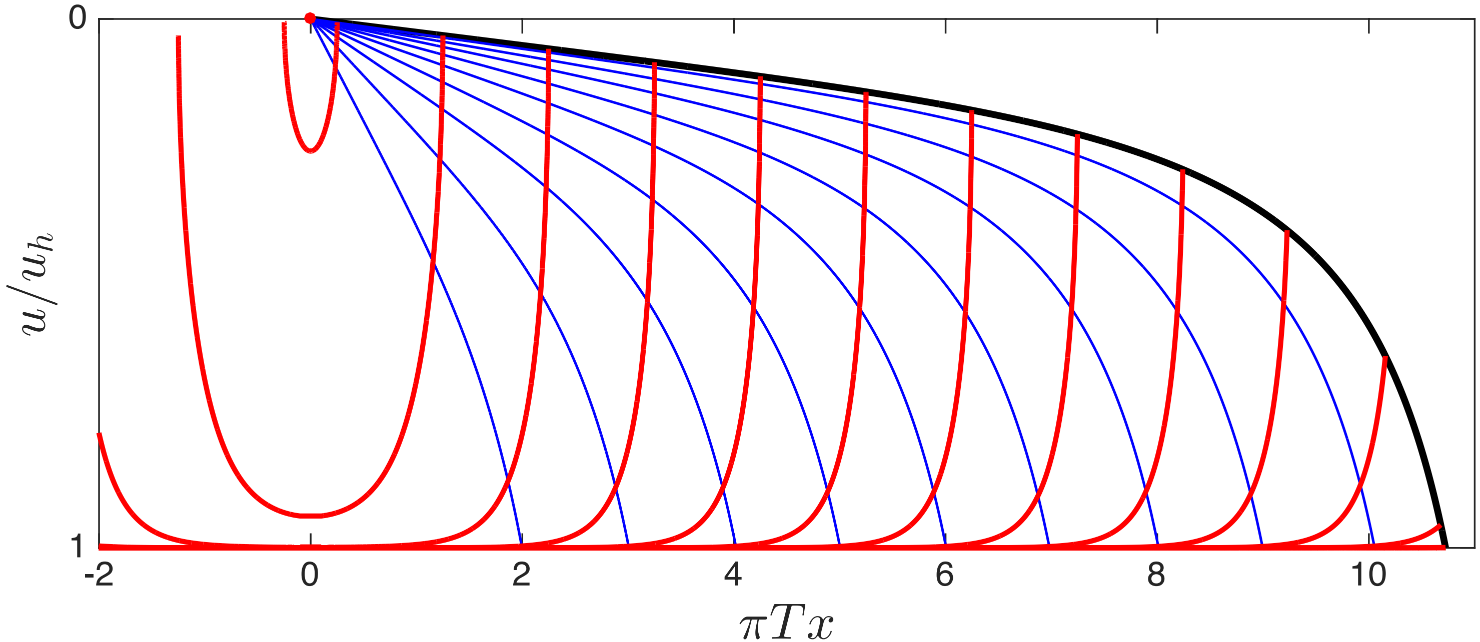
Chesler, Rajagopal, arXiv:1511.07567



- **First, every jet broadens in angle as it propagates through the strongly coupled plasma. $\theta_{\rm jet}$ increases as $E_{\rm jet}$ decreases. (What is plotted here is energy flux, renormalized at every x so loss of energy is not visible. Plot is for the small $\theta_{\rm jet}^{\rm init}$ limit.)**

Holographic "Jet" Energy Loss

Chesler, Rajagopal, arXiv:1511.07567



Two immediate, inescapable, qualitative consequences, of geometric origin when described holographically:

- **Second, jets with smaller initial $\theta_{\text{jet}}^{\text{init}}$ have a longer x_{therm}. They lose their energy more slowly, over a longer distance. (In fact, $Tx_{\text{therm}} \propto 1/\sqrt{\theta_{\text{jet}}^{\text{init}}}$.)**
- **That is, for jets with the same $E_{\text{jet}}^{\text{init}}$ that travel through the same plasma, those with larger $\theta_{\text{jet}}^{\text{init}}$ will lose more energy.** *A lesson that applies to real jets!* **As we shall see.**

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Casalderrey-Solana, Gulhan, Milhano, Pablos, KR, 2014,15,16; Hulcher, DP,KR, '17; JCS,ZH,GM,DP,KR, '18; JCS,GM,DP,KR, '19; JCS,GM,DP,KR, Yao, '20; ZH, DP, KR, '20; Bossi, Kudinoor, Moult, DP, Rai, KR, '24; AK, DP, KR, '25; Beraudo, Du Plessis, DP, KR, '25; ZH, AK, DP, KR, '25

- **OR: try a hybrid approach. Shower forms á la PYTHIA. Every parton in the shower loses energy á la dE/dx for quarks in a strongly coupled liquid in a strongly coupled cousin of QCD — with one parameter fit to data. Jet wakes with hydrodynamics. And, describe rare, hard, large-angle scattering of a parton in the jet of a parton in the soup à la Rutherford (traditional QCD calculation).**
- **Upon fitting one parameter, *lots* of data described well. Fitted parameter is reasonable: x_{therm} (energetic parton thermalization distance) 3-4 times longer in QGP than in $\mathcal{N} = 4$ SYM plasma at same T.**
- **Then add: momentum broadening; wake in the plasma; resolution effects; Molière scattering – turning each off/on all while looking at jet substructure observables.**

Implementation of Hybrid Model

Casalderrey-Solana, Gulhan, Milhano, Pablos, KR, 1405.3864,1508.00815

- **Jet production and showering from PYTHIA.**
- **Embed the PYTHIA parton showers in hydro background. (2+1D hydro from Heinz and Shen.)**
- **Between one splitting and the next, each parton in the branching shower loses energy according to**

$$\frac{1}{E_{\rm in}}\frac{dE}{dx} = -\frac{4x^2}{\pi x_{\rm therm}^2}\frac{1}{\sqrt{x_{\rm therm}^2 - x^2}}$$

  where $x_{\rm therm} \equiv E_{\rm in}^{1/3}/(2\kappa_{\rm sc}T^{4/3})$ **with** $\kappa_{\rm sc}$ **one free parameter that to be fixed by fitting to one experimental data point. (**$\kappa_{\rm sc} \simeq 1.5$ **in** $\mathcal{N}=4$ **SYM; fitted** $\kappa_{\rm sc} \simeq 0.4$ **means** $x_{\rm therm}$ **is longer in QGP than in** $\mathcal{N}=4$ **SYM plasma with same** T**.)**
- **Turn energy loss off when hydrodynamic plasma cools below a temperature that we vary between 145 and 170 MeV. (This, plus the experimental error bar on the one data point, becomes the uncertainty in our predictions.)**
- **Reconstruct jets using anti-**k_T**.**

Perturbative Shower … Living in Strongly Coupled QGP

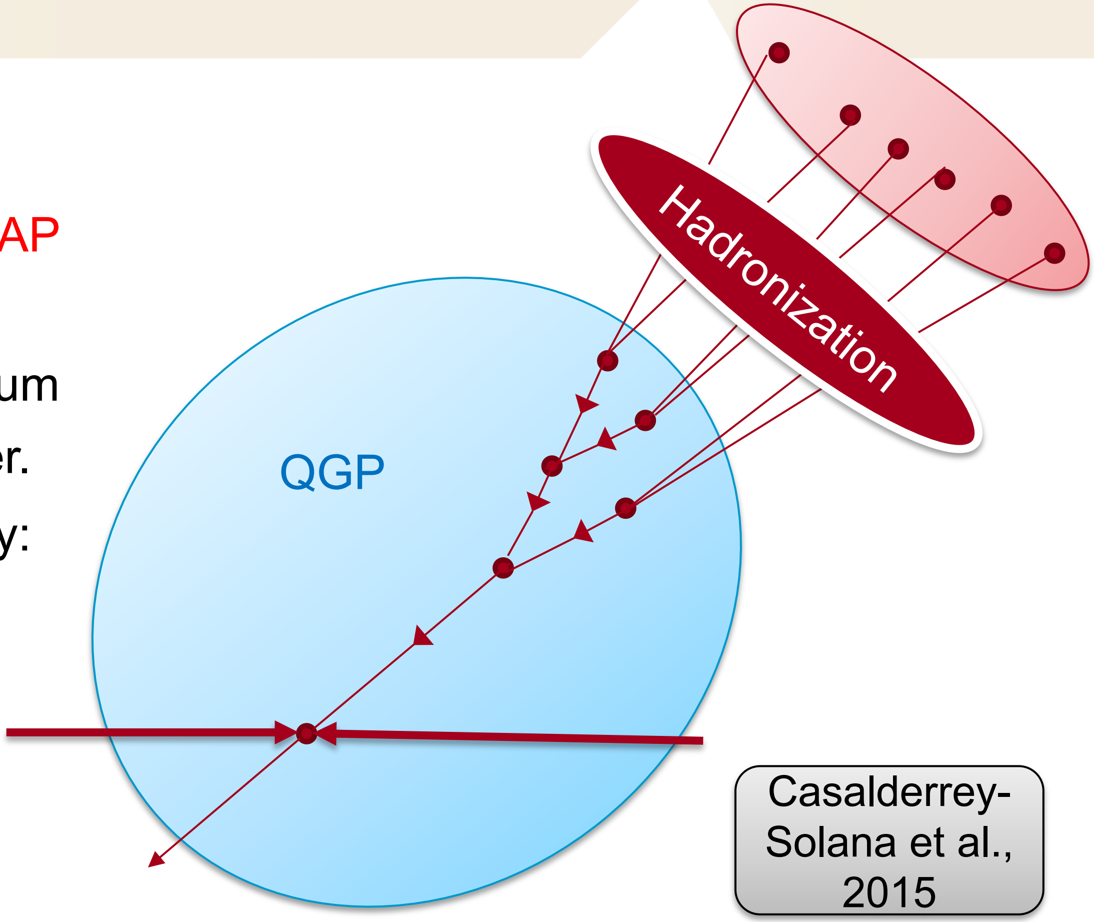
- High Q^2 parton shower up until hadronization described by DGLAP evolution (PYTHIA).
- For QGP with $T \sim \Lambda_{QCD}$, the medium interacts strongly with the shower.
    - Energy loss from holography:

$$\frac{1}{E_{in}}\frac{dE}{dx} = -\frac{4}{\pi}\frac{x^2}{x_{stop}^2}\frac{1}{\sqrt{x_{stop}^2 - x^2}}$$

O(1) fit const.

$$x_{stop} = \frac{1}{2\kappa_{sc}}\frac{E_{in}^{\frac{1}{3}}}{T^{\frac{4}{3}}}$$

$$\tau = \frac{2E}{Q^2}$$



Casalderrey-Solana et al., 2015

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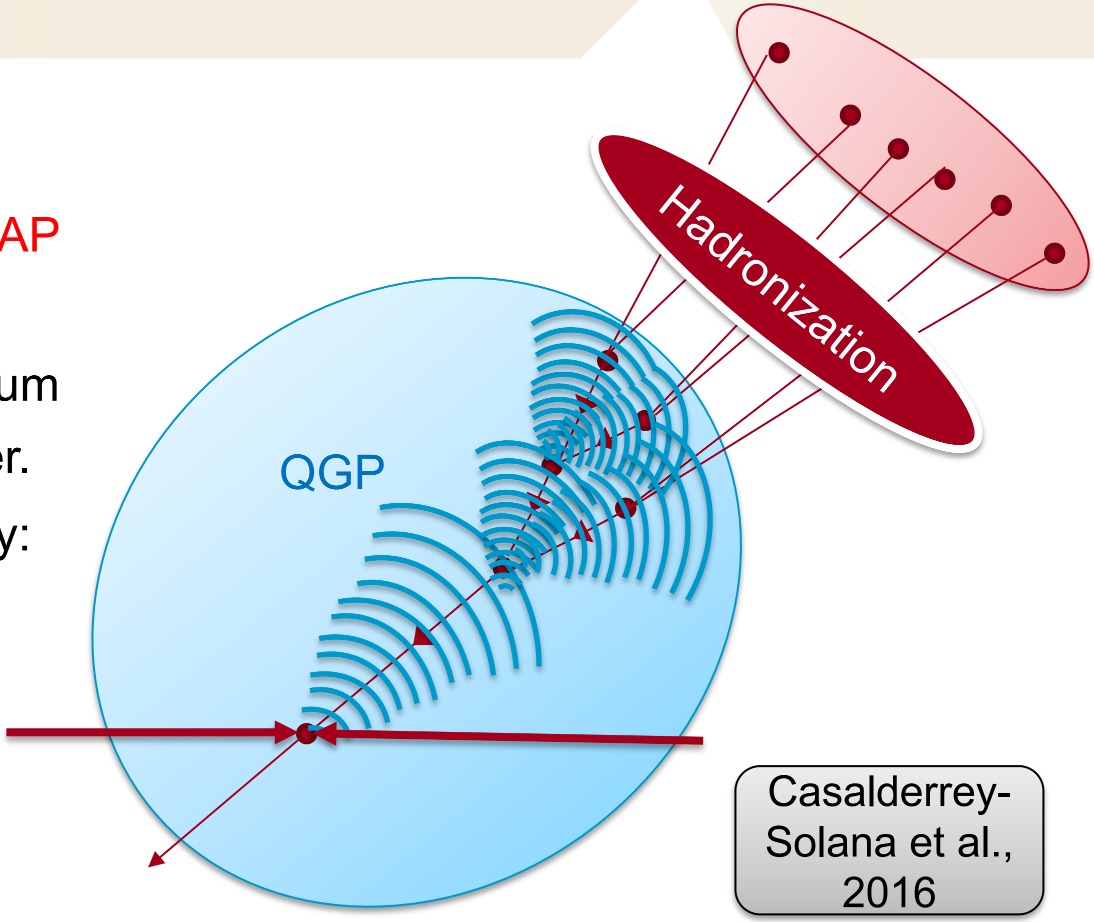

Casalderrey-Solana et al., 2016

Energy and momentum conservation → deposit hydrodynamic wake in QGP liquid

$$\frac{d\Delta N}{p_T dp_T d\phi dy} = \frac{1}{(2\pi)^3}\int \tau dx dy d\eta_s m_T \cosh(y-\eta_s)\left[f\left(\frac{u^\mu p_\mu}{T_f+\delta T}\right) - f\left(\frac{u_0^\mu p_\mu}{T_f}\right)\right]$$

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Visualizing Jet Wakes

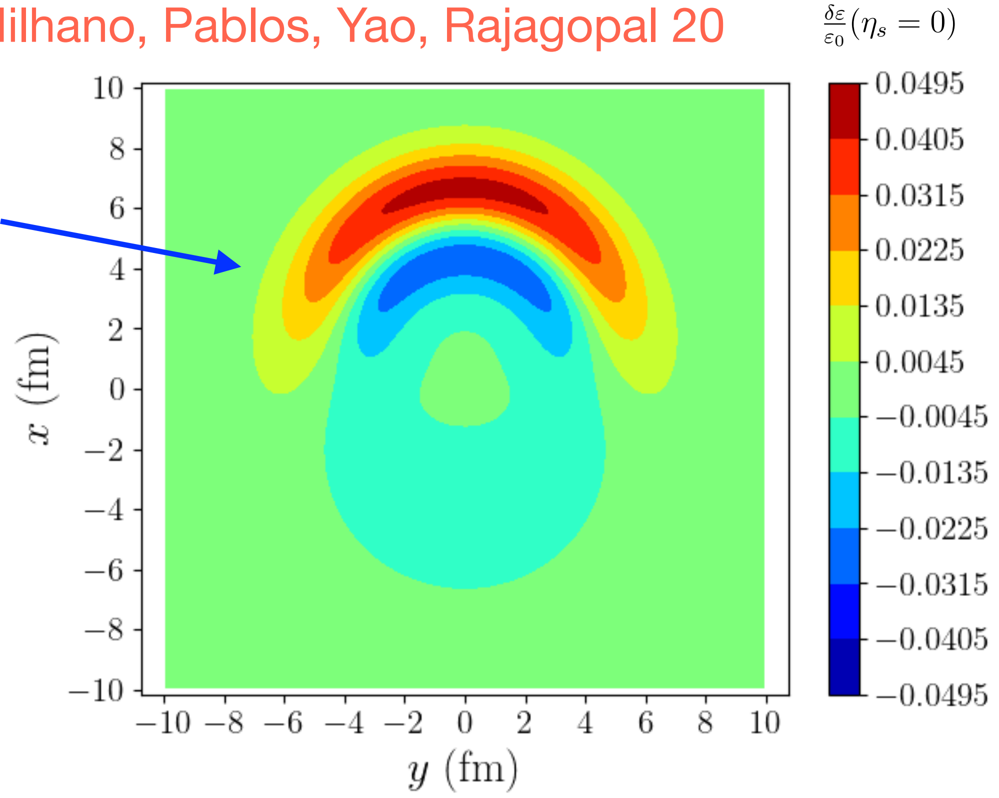
- Three of the recent examples of observables that we can use to visualize jet wakes, in different ways. . .
- In all cases, use Hybrid Model to assess sensitivity to jet wakes by turning them off and on.
- TODAY: Yen-Jie Lee's CMS talk at Hard Probes 2024 (now published).
- Wake(s) of large-radius jets with two skinny subjets. . .
  Kudinoor, Pablos, KR 2501.18683
- Energy-energy-energy three-point correlators. . .
  Bossi, Kudinoor, Moult, Pablos, Rai, KR 2407.13818

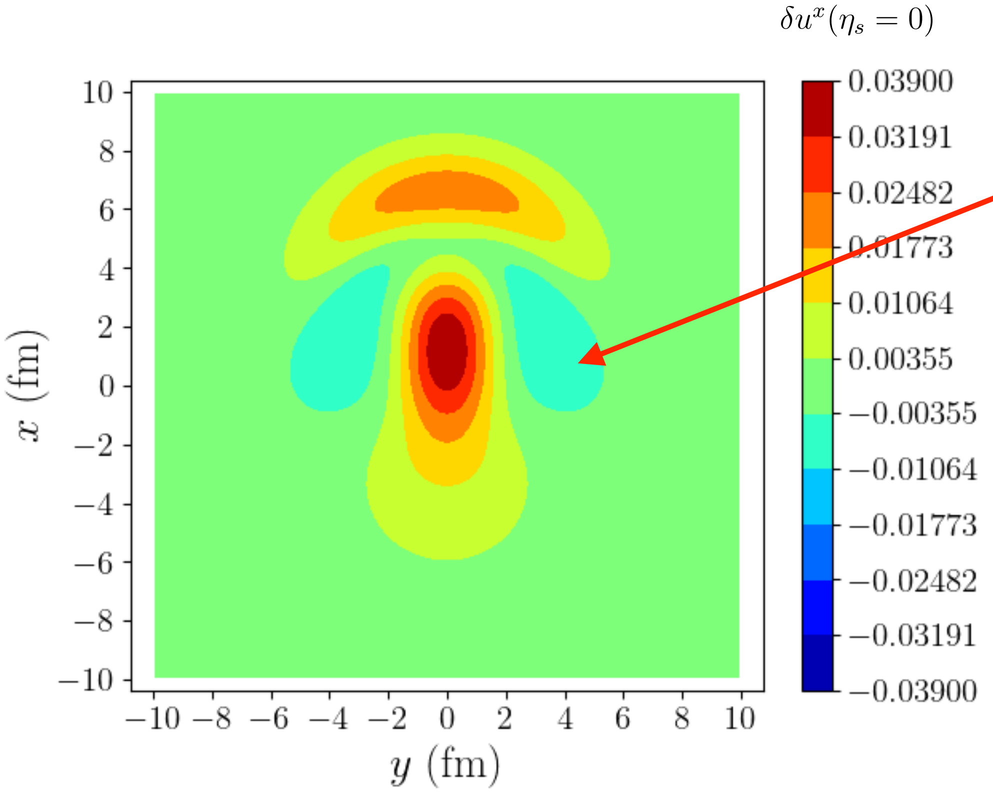
Response without Transverse Flow

- Building block: perturbation on-top of Bjorken flow

JCS, Milhano, Pablos, Yao, Rajagopal 20

Sound waves


$\frac{\delta\varepsilon}{\varepsilon_0}(\eta_s = 0)$


$\delta u^x(\eta_s = 0)$

Diffusion wake

- Sound waves ⇒ take energy away from jet
- Diffusion wake

  ⇒lost momentum becomes moving fluid along the jet path
- On average:

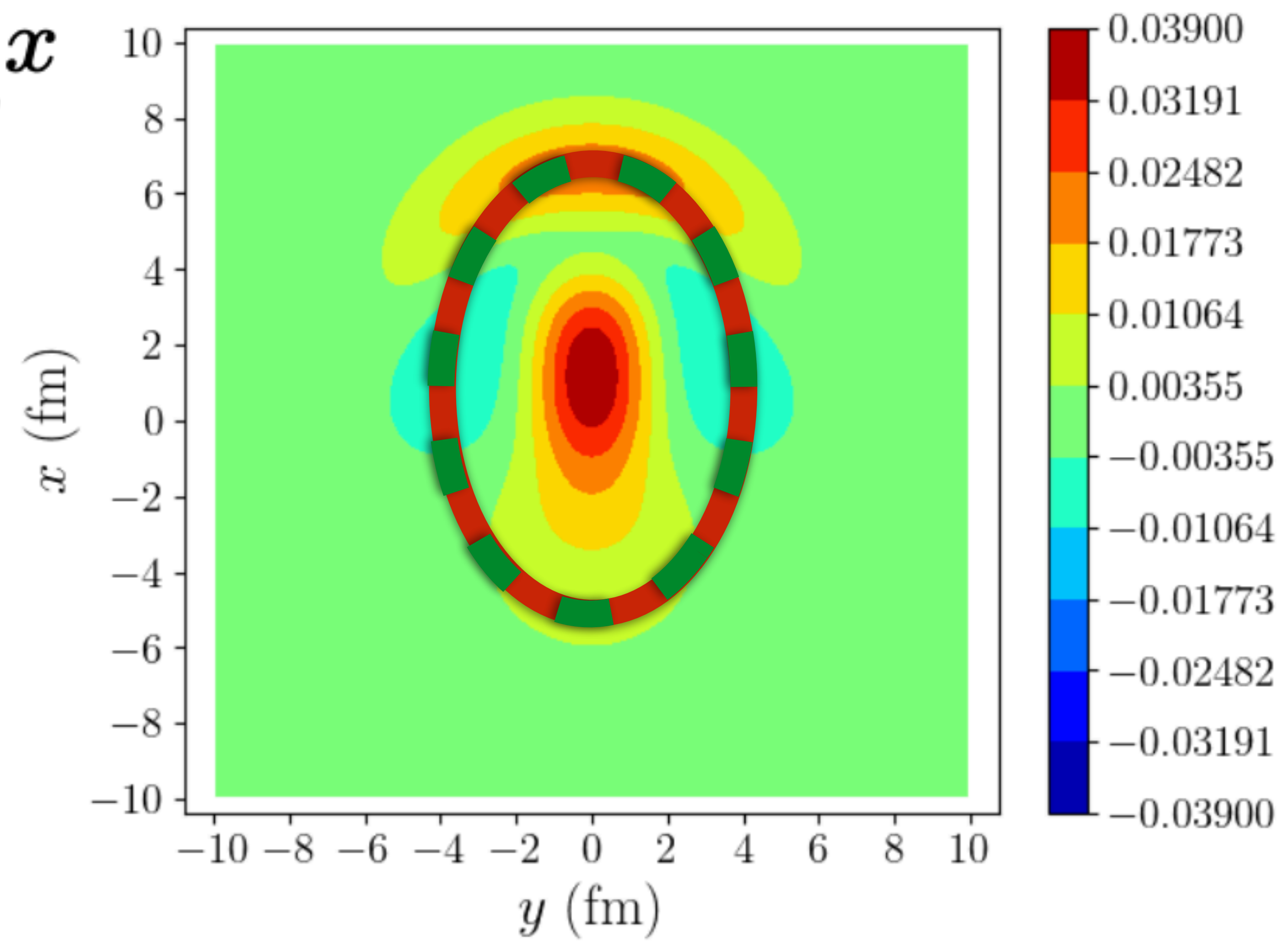
JCS, Teaney and Shuryak 05

diffusion wake dominates over sound waves in particle production

See for Yang, He, Chen, Ke, Pang and Wangattempts to disentangle Mach and wake in COLBT

Dragging the QGP

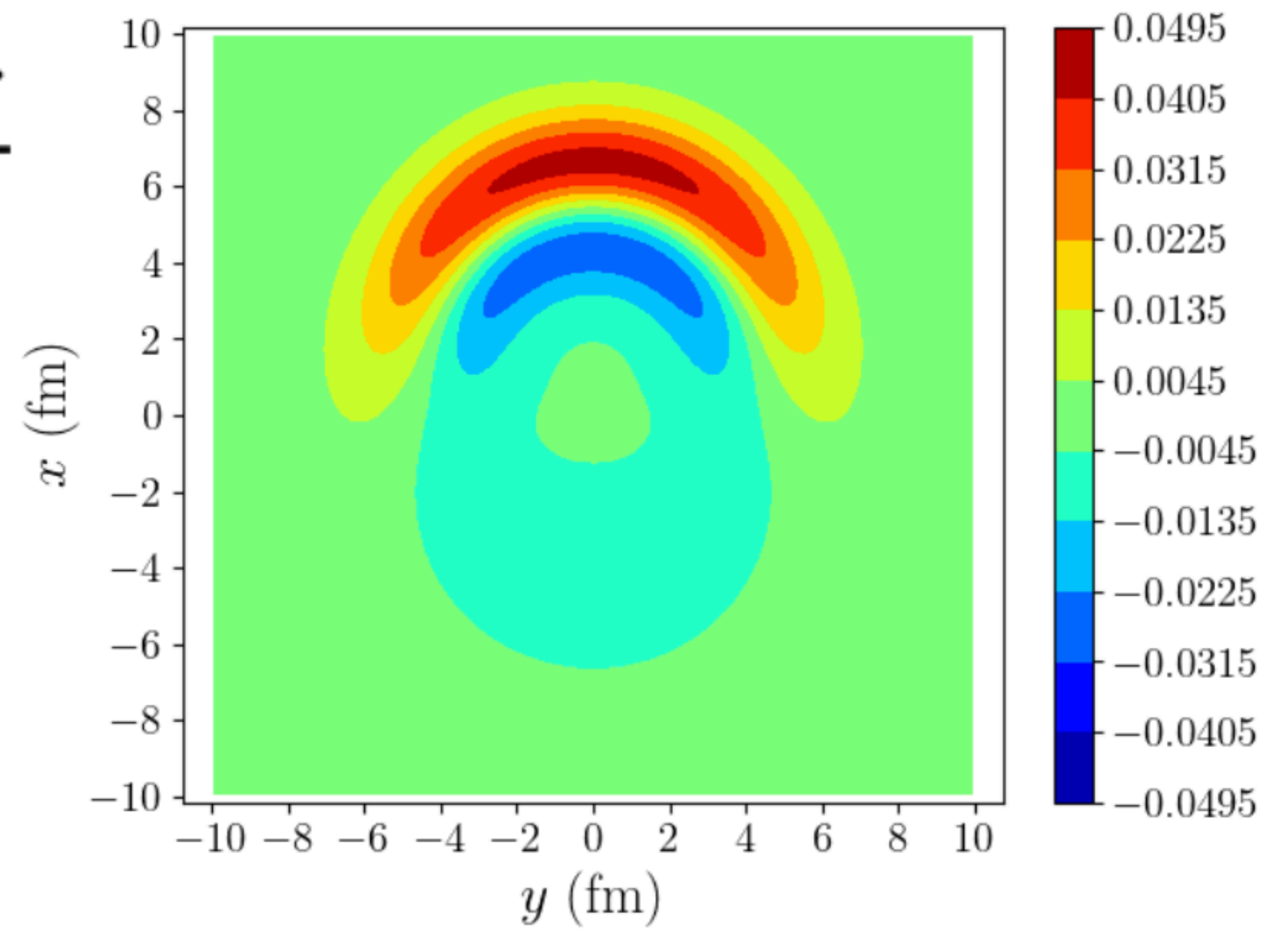
δu^x



Increase particle production in jet direction, **decrease** in opposite direction (boosted fluid cells).

Cooper-Frye

$\frac{\delta\varepsilon}{\varepsilon_0}$



Increase particle production isotropically.

With respect to unperturbed background

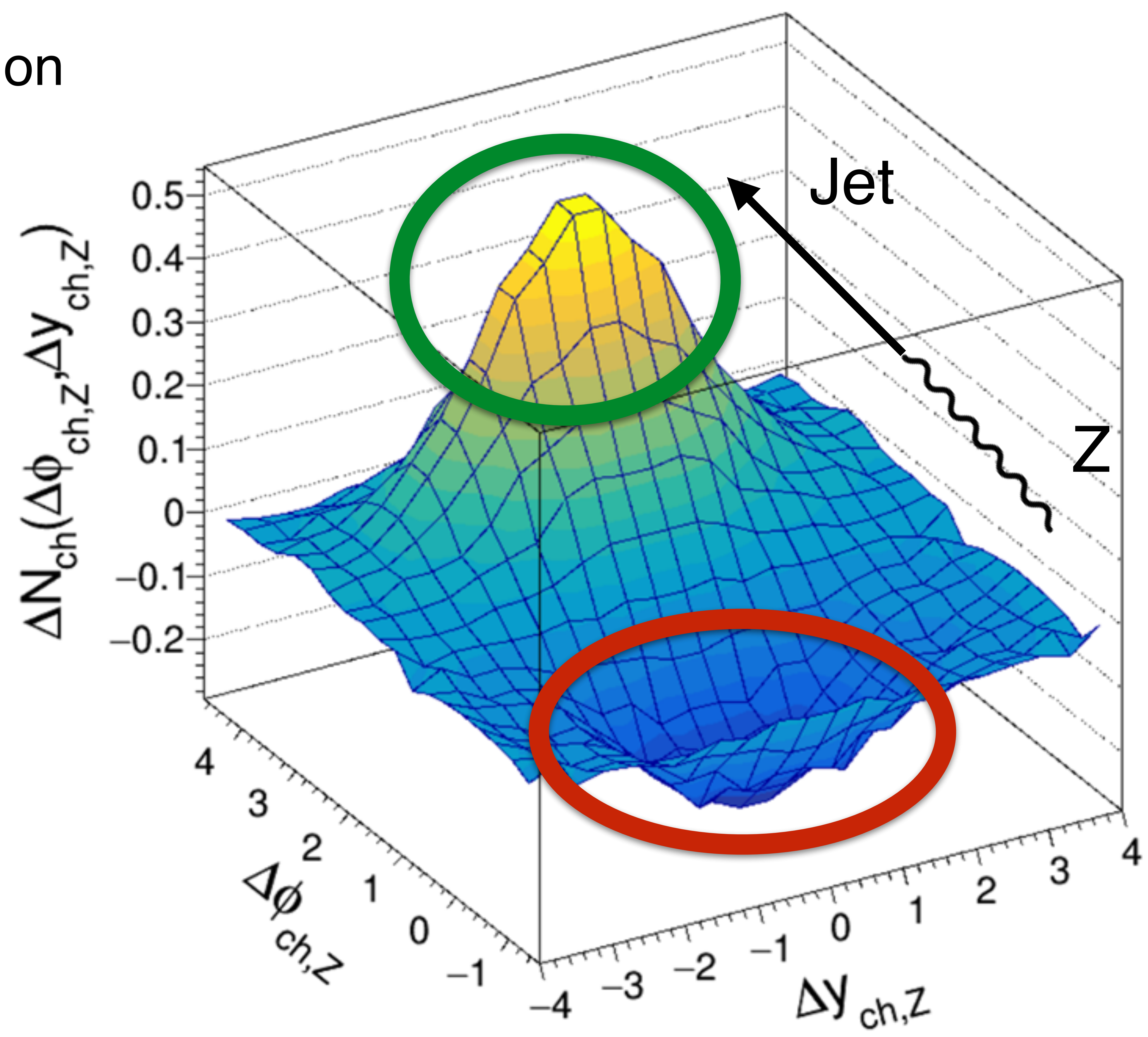
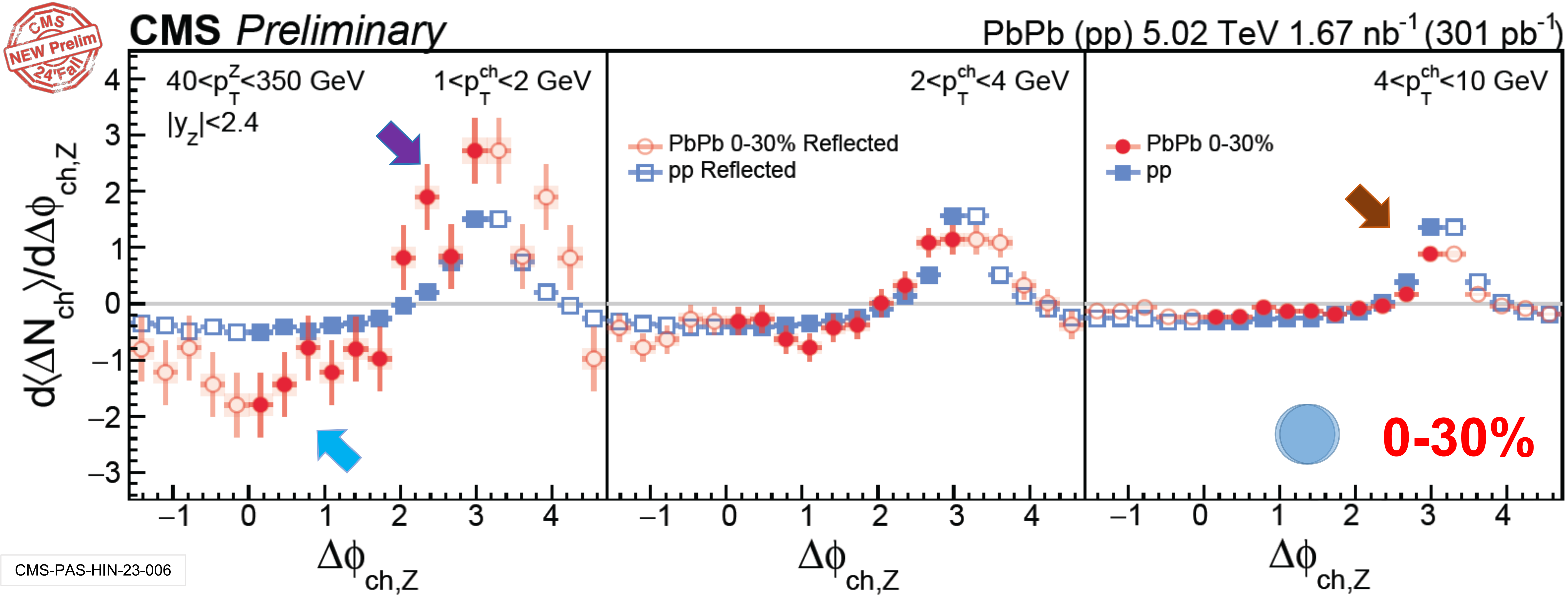


Fig. from Yen-Jie's slides

Azimuthal Angle Distributions in pp and 0-30% PbPb



CMS-PAS-HIN-23-006

Low Charged Hadron p_T → High Charged Hadron p_T

PbPb: Clear depletion in **Z^0 side ($\Delta\phi=0$)** and enhancement in **jet side ($\Delta\phi=\pi$)**

PbPb: Effect reduced in the intermediate p_T region (2-4 GeV)

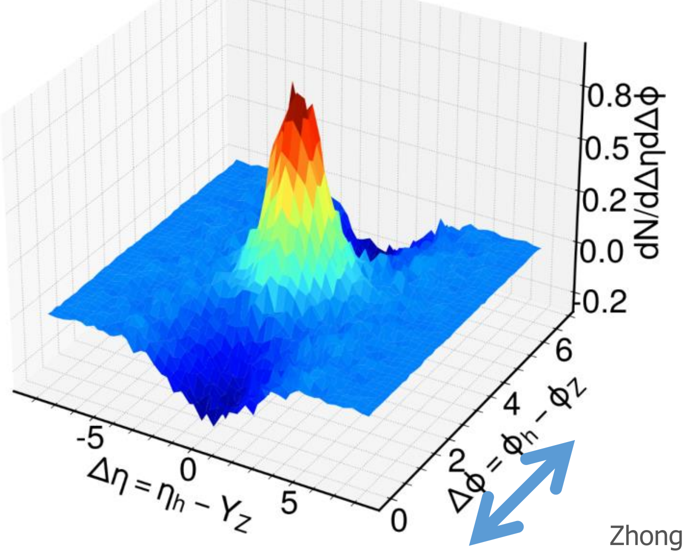
PbPb: **Jet side peak** ($\Delta\phi=\pi$) reduced due to jet quenching at high hadron p_T

Azimuthal Angle Distribution in 0-30% PbPb vs. Theory

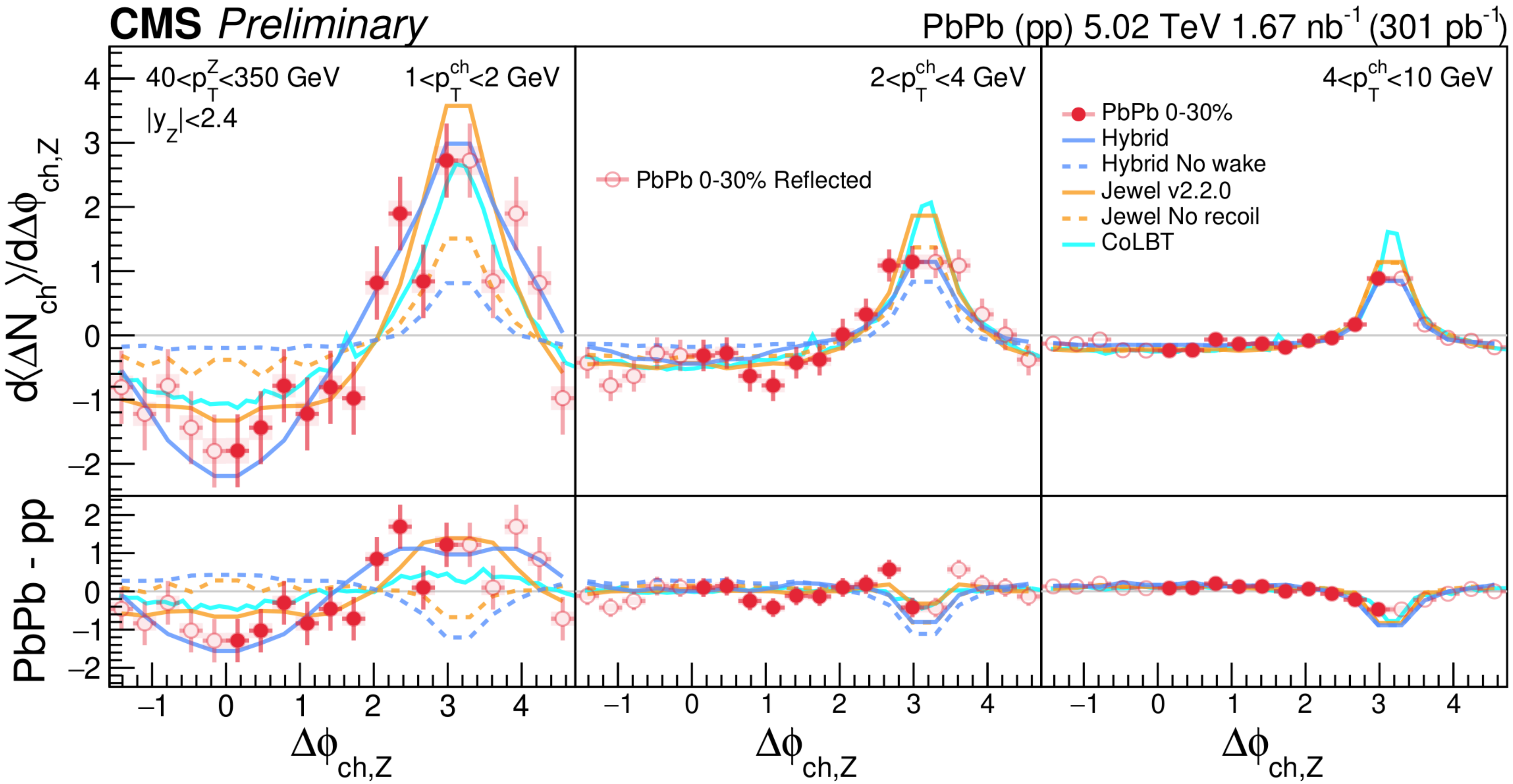
- **Hybrid without wake** and **Jewel without recoil** (dashed lines) underpredict magnitude at low hadron p_T

- **Hybrid with wake**, **Jewel with recoil** and **CoLBT with wake** (solid lines) agree better with the data with hadron p_T < 4 GeV

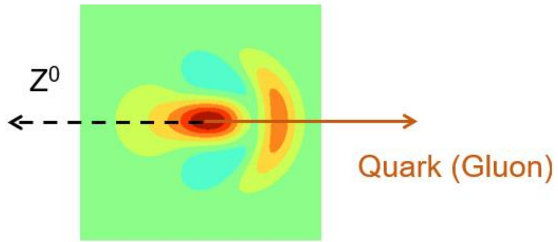
CoLBT Z^0+hadron



Zhong Yang, Xin-Nian Wang

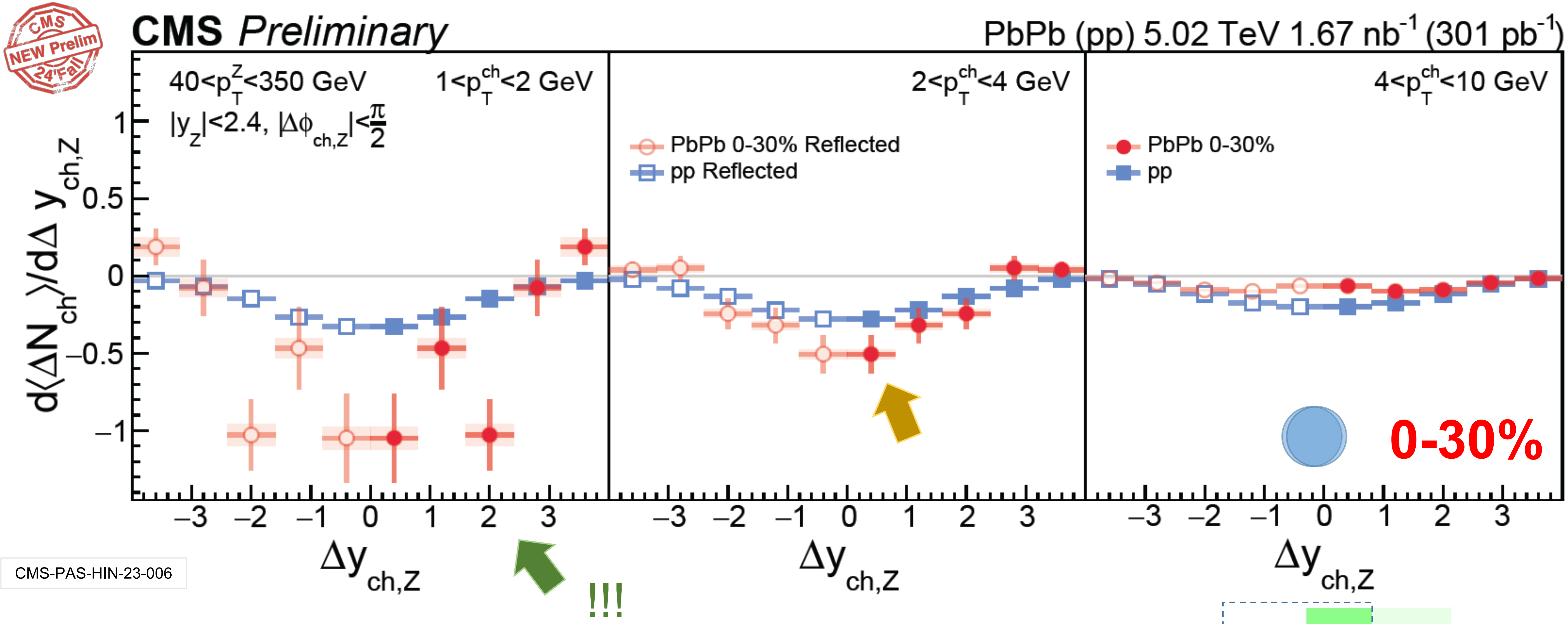


CMS-PAS-HIN-23-006



CMS NEW Prelim 24'Fall

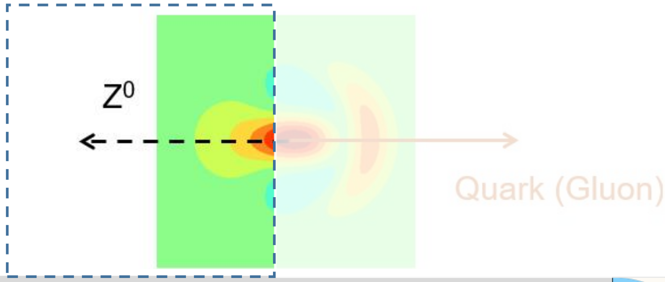
Rapidity Distributions in pp and 0-30% PbPb



CMS-PAS-HIN-23-006

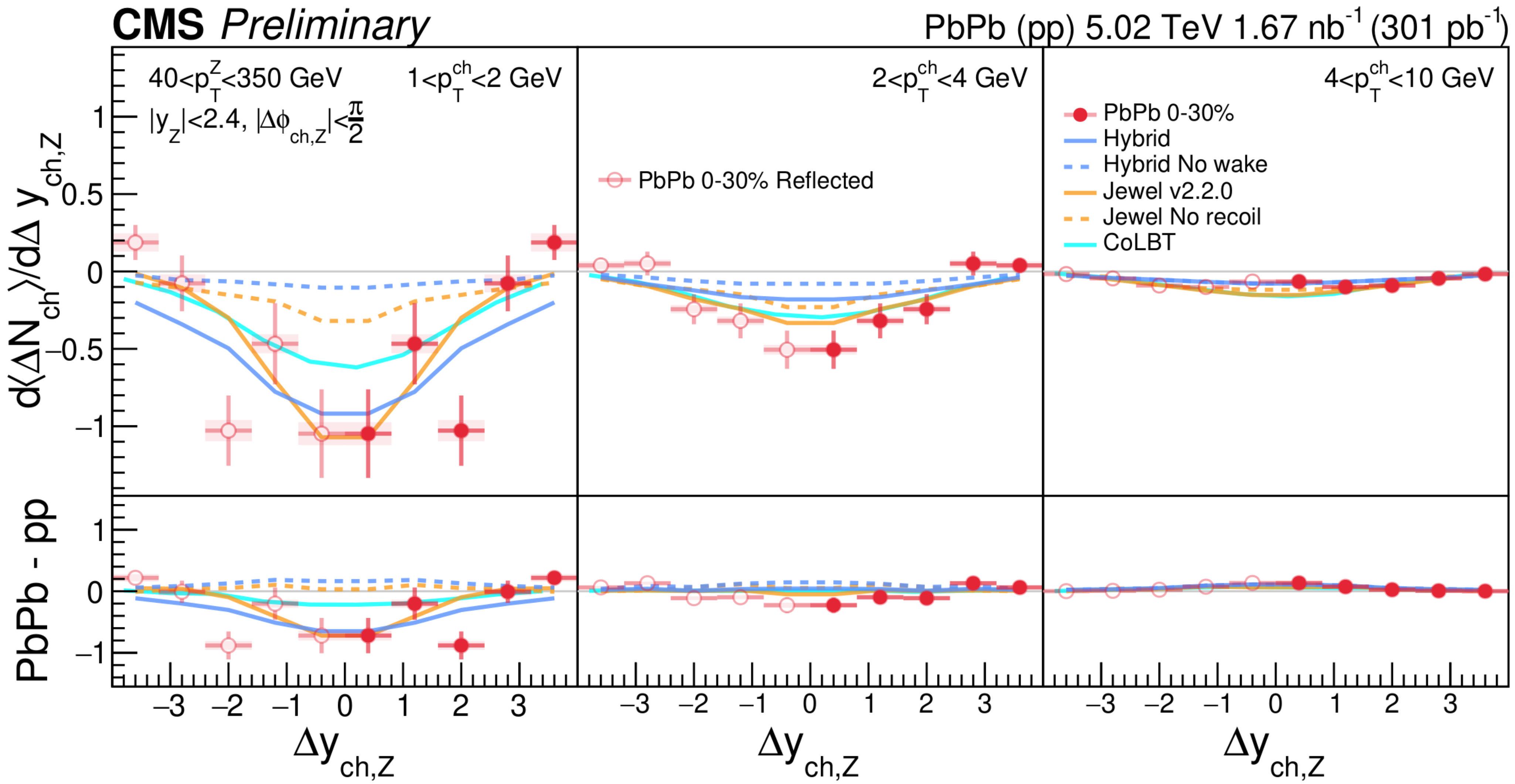
PbPb: Clear depletion around the Z (Δy=0) and the effect reduces at higher Δy

PbPb: Effect reduced in the intermediate p_T region (2-4 GeV)



Rapidity Distribution in 0-30% PbPb vs. Theory

- **Hybrid without wake** and **Jewel without recoil** (dashed lines) underpredict magnitude at low hadron p_T
- **Hybrid with wake**, **Jewel with recoil** and **CoLBT** (solid lines) agree better with data

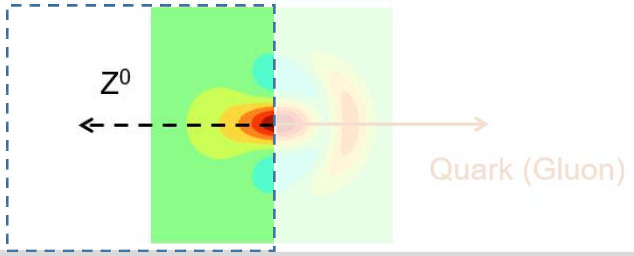


CMS-PAS-HIN-23-006

HP 2024
RIDING THE WAKE

With Δy and Δϕ spectra at low charged hadron p_T:

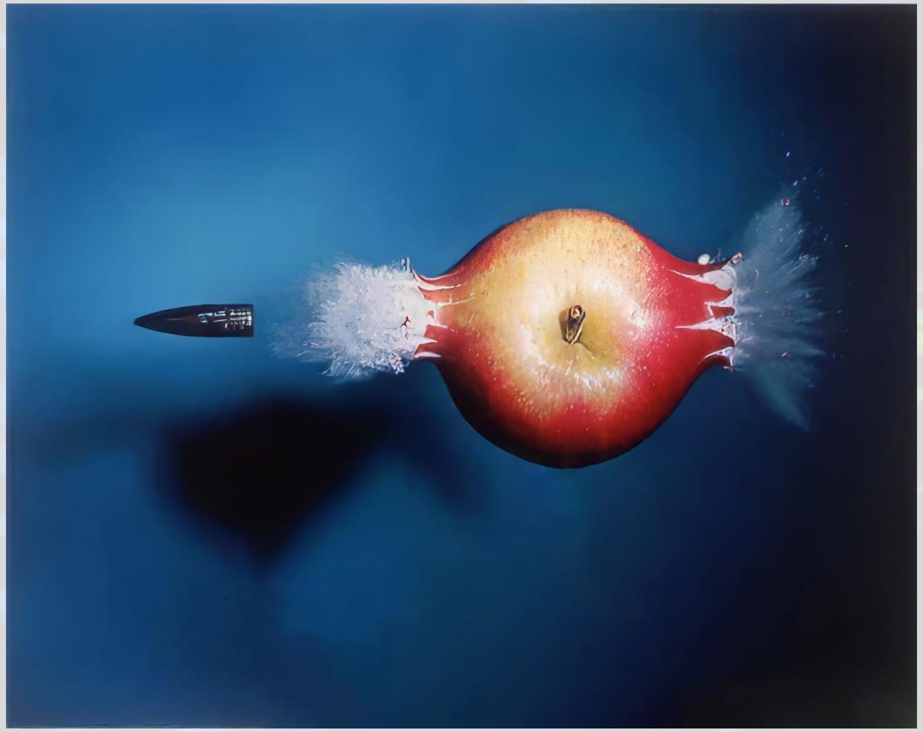
The first evidence of negative QGP wake!

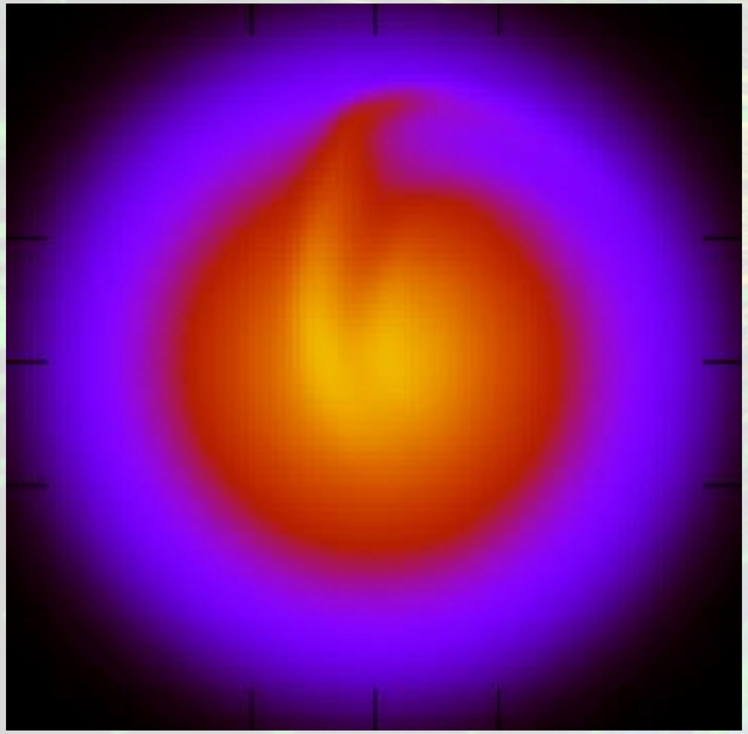


CMS NEW Prelim 24'Fall

CMS Experiment at the LHC, CERN
Data recorded: 2011-Dec-01 14:35:39.907994 GMT
Run / E

Implication and Outlook

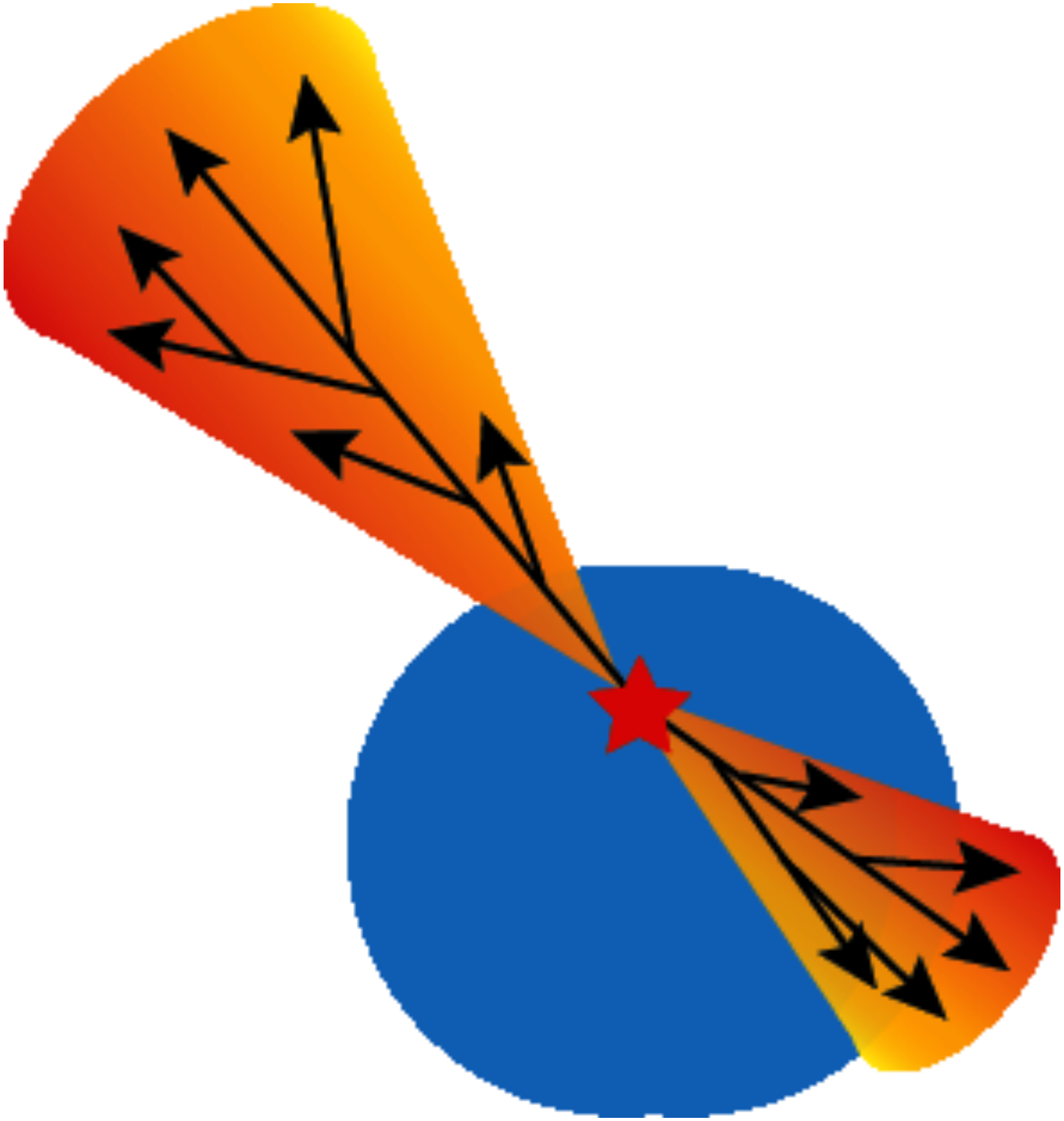




Unambiguous evidence of the **QGP wake** created by a fast moving quark

Jets as Probes of QGP

- Theorists taking key steps. . .
- Disentangling jet modification from jet selection.
- Calculations of the dynamics of jet wakes in droplets of QGP, identification of new experimental observables, and predictions that will enable experimental measurements to "see" the particles coming from these wakes.
- Showing that hot quark soup (QGP) *can* respond to substructure within jets.
- Identifying those jet substructure observables that *are* sensitive to scattering of jet quarks/gluons off QGP quarks/gluons, "seeing" the latter à la Rutherford, and are *not* sensitive to particles coming from the wake.
- Next several years will be the golden age of jet physics: sPHENIX, LHC runs 3 and 4, new substructure observables. *Many* theory advances, and analyses of today's data, whet our appetite for the feast to come.
- We shall learn about the microscopic structure of QGP, and the dynamics of rippling QGP.

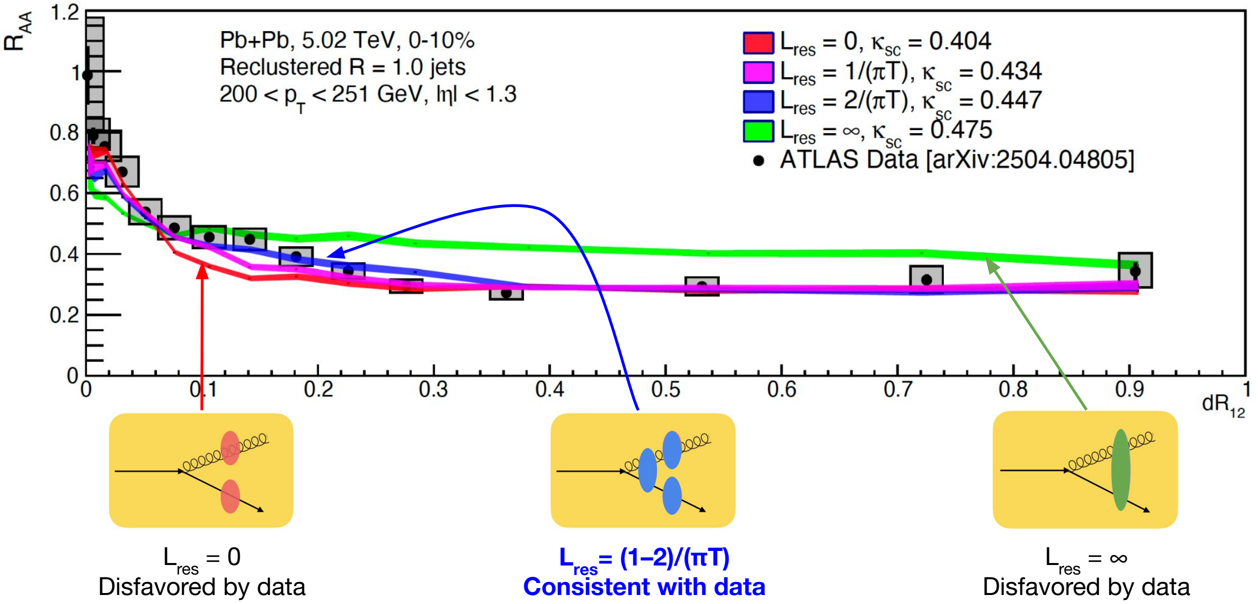


QGP Resolution Length

- Can the QGP resolve individual jet partons within a parton shower? Or does it “see” and entire jet as if it were one object? Or, something in between?
- Two partons in a jet lose energy as if they were one if they are closer together than $L_{\rm res}$. What is the value of $L_{\rm res}$ for QGP?
- If we are to have any chance whatsoever of using jets to probe the microscopic structure of QGP, namely of seeing jet partons scatter off short-length-scale quark-like and gluon-like quasiparticles in the QGP, we have to first show that $L_{\rm res}$ is $\sim 1/T$ or smaller.
- This has recently been accomplished. ATLAS data: 2504.04805; Kudinoor, Pablos, KR 2509.08881
- Data disfavors $L_{\rm res} = \infty$ and disfavors $L_{\rm res} = 0$.
- Data favors $L_{\rm res} \sim (1-2)/(\pi T)$, which is $\lesssim 1/T$. At $T = 300$ MeV, $L_{\rm res} = 0.2 - 0.4$ fm. Very promising.

Partons Inside Jets as Probes?

Kudinoor, Pablos, KR, 2509.08881



Two partons in a jet lose energy as if they were one if they are closer together than $L_{\rm res}$. What is value of $L_{\rm res}$?

To have any chance of using jets to probe microscopic structure of QGP, have to show $L_{\rm res}$ is $\sim 1/T$ or smaller.

Recent ATLAS data favors $L_{\rm res} \sim (1-2)/(\pi T) \lesssim 1/T$. At $T = 300$ MeV, $L_{\rm res} \sim 0.2 - 0.4$ fm. Very promising.

Jets as Probes of QGP

- Model calculations are enabling key steps. . .
- Disentangling jet modification from jet selection.
- Calculations of the dynamics of jet wakes in droplets of QGP, identification of new experimental observables, and predictions that will enable experimental measurements to “see” the particles coming from these wakes.
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- Next several years will be the golden age of jet physics: sPHENIX, LHC runs 3 and 4, new substructure observables. *Many* theory advances, and analyses of today’s data, whet our appetite for the feast to come.
- We shall learn about the microscopic structure of QGP, and the dynamics of rippling QGP.

Jets as Probes

- Goal long established: see microscopic particulate structure of liquid QGP at short length scales; see jet partons scatter off QGP quasiparticles; “Rutherford”; “DIS”.
- Are we there? Getting close. QGP sees jet substructure!
- PbPb and AuAu collisions: parton energy loss and jet wakes. Interesting in their own right. Substantial theory advances, recent and coming. Substantial experimental advances, recent and coming. Eg unambiguous evidence for jet wakes. Jet wakes can teach us about how QGP equilibrates.
- New jet substructure observables: Soft Drop splitting angle R_g. Lund plane. Energy correlators. From HEP, but we are putting them to better uses!
- Oxygen-oxygen collisions may be perfect arena in which to see hard scattering of jet partons off QGP quasiparticles.
- Major advances in experiment to come... sPHENIX, LHC Runs 4&5

Add Molière Scattering . . .

- QGP, at length scales $\mathcal{O}(1/T)$, is a strongly coupled liquid. Flow, and jet observables sensitive to parton energy loss, are well-described (eg in hybrid model) in such a fluid, without quasiparticles.
- At shorter length scales, probed via large momentum-exchange, asymptotic freedom → quasiparticles matter.
- High energy partons in jet showers *can* probe particulate nature of QGP. Eg via power-law-rare, high-momentum-transfer, large-angle, Molière scattering
- "Seeing" such scattering is first step to probing microscopic structure of QGP.
- What jet observables are sensitive to effects of high-momentum-transfer scattering? To answer, need to turn it off/on.
- Start from Hybrid Model – in which any particulate effects are definitively off! Add Molière, and look at effects. . .

Molière Scattering

arXiv: 1808.03250 [D'Eramo, et al.], 2208.13593 [Hulcher, et al.], 2603.08776 [Hulcher, et al.]

- Asymptotic freedom ⇒ QGP is particulate when probed at large momentum transfer

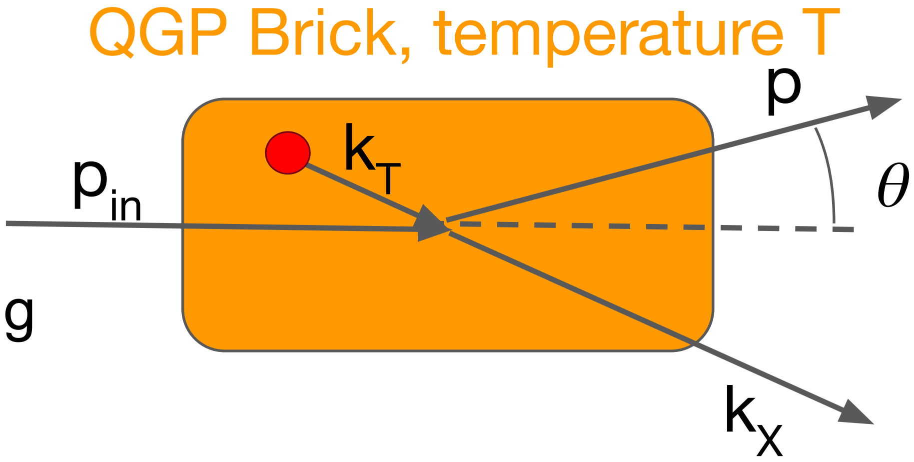
- **Jet partons trigger Molière scattering: Rare, large angle, 2 → 2 scatterings, with high momentum exchange**

- Implemented in Hybrid Model with pQCD scattering amplitudes, while requiring

$$|t|, |u| > am_D^2 \ , \ a = 10$$

t and u are Mandelstam variables, m_D is the Debye mass

$$m_D^2 \propto g^2 T^2$$



Gaussian Broadening vs Large Angle Scattering

Elastic scatterings of exchanged momentum $\sim m_D$

→ Gaussian broadening due to multiple soft scattering

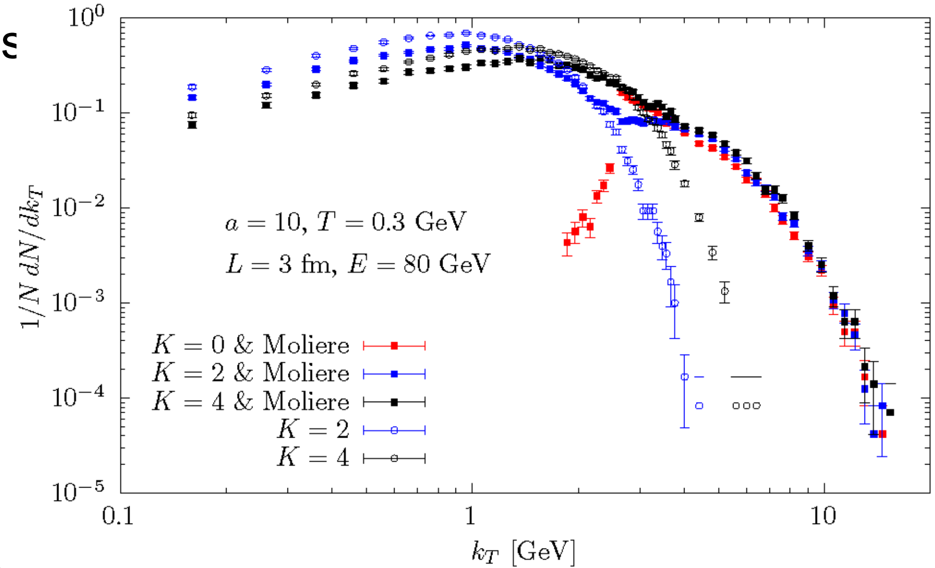
At strong coupling, holography predicts Gauss broadening without quasi-particles (eg: N=4 SYM)

$$P(k_\perp) \sim \exp\left(-\frac{\sqrt{2}k_\perp^2}{\hat{q}L^-}\right) \qquad \hat{q} = \frac{\pi^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\sqrt{\lambda}T^3$$

Adding this in hybrid model (C-S et al 2016) yielded little effect on jet observables.

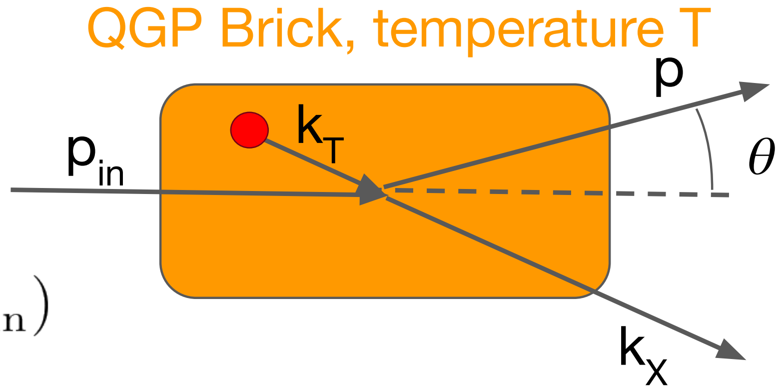
Today, Bayesian inference from hadron R_{AA} data indicates $P(k_\perp) \sim K\, T^3$ with $K \sim 2 - 4$. This need not have anything to do with quasiparticles.

- Add Moliere scattering with momentum exchanges $> m_D$; here, $a = 10$ and 80 GeV incident jet parton

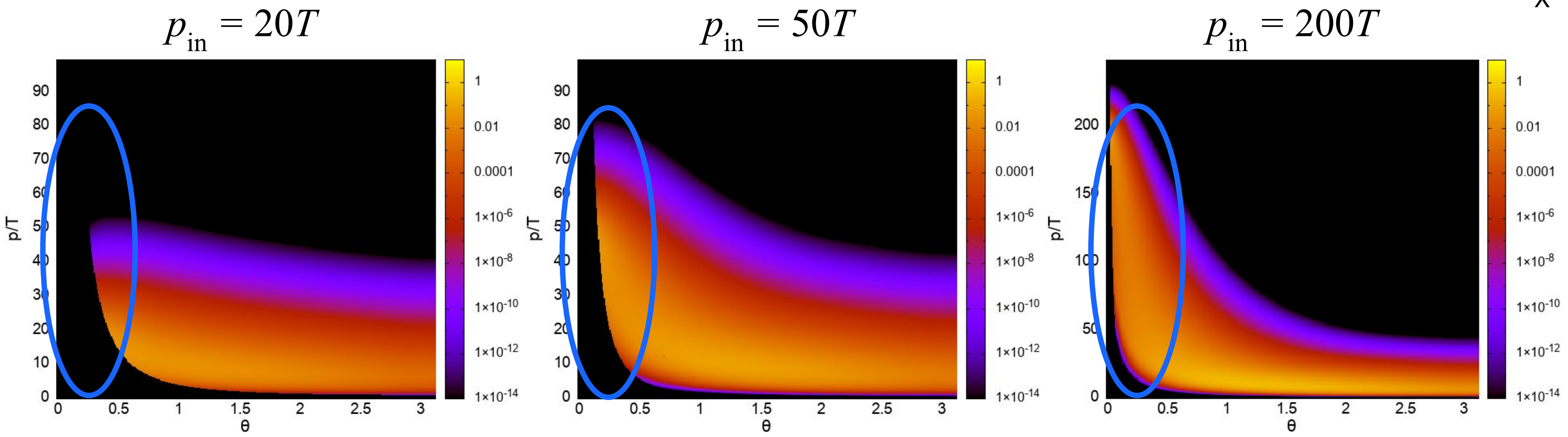


Molière Scattering Probability

arXiv: 2603.08776 [Hulcher et. al]


QGP Brick, temperature T

Gluon Molière scattering differential probability $F^{G\to\text{all}}(p, \theta; p_{\text{in}})$


$p_{\text{in}} = 20T$

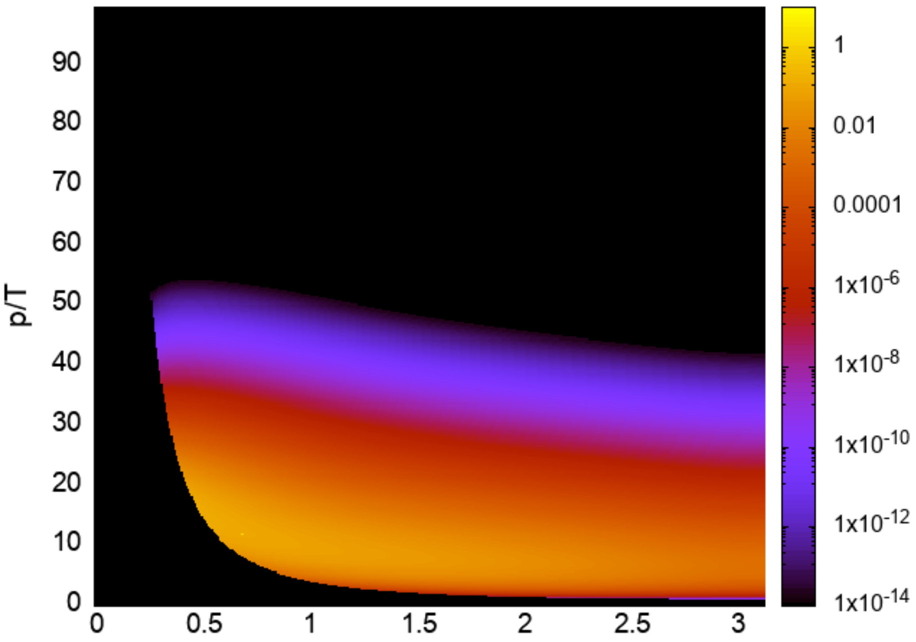
$p_{\text{in}} = 50T$

$p_{\text{in}} = 200T$

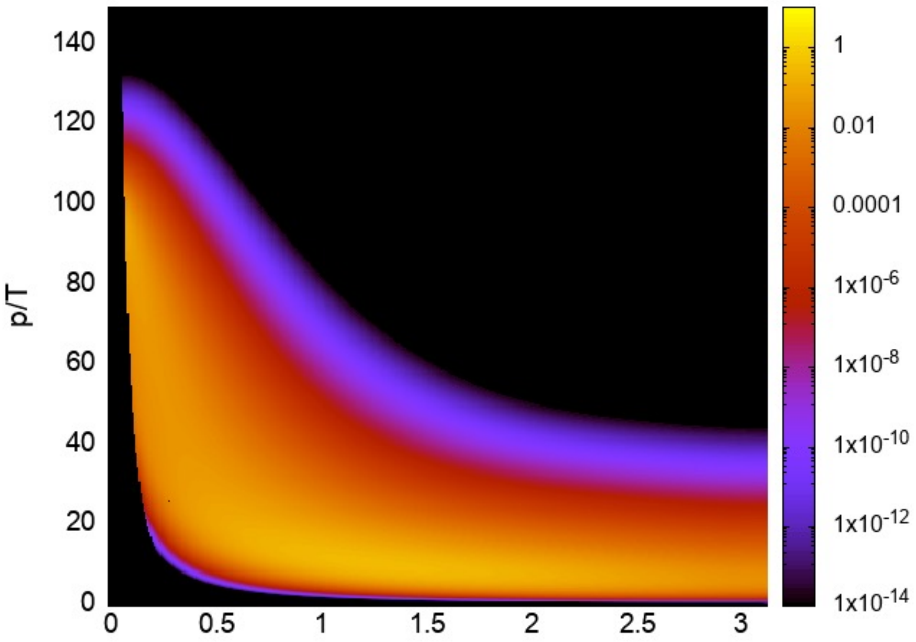
Small p_{in}: small angle scatterings are forbidden due to $|t|$ and $|u|$ constraints

Large p_{in}: the phase space opens up, typical scattering angle decreases

Results (for a QGP brick)



Incoming gluon, $p_{in} = 20T, \mathrm{L} = 15/T$



Incoming gluon, $p_{in} = 100T, \ \mathrm{L} = 15/T$

- Excluding $\tilde{u} > 10\, m_D^2$ not a simple curve on this plot, but effects visible
- Restricting to $\tilde{u}, \tilde{t} > 10\, m_D^2$ excludes soft scatterings; justifies assumptions made in amplitudes; avoids double counting. Can vary where to set this cut…
- Analytical results → fast to sample
- Apply at every time step, to every rung, in every shower, in Hybrid Model Monte Carlo…. And, if a scattering happens, two subsequent partons then lose energy a la Hybrid

Perturbative Shower … Living in Strongly Coupled QGP

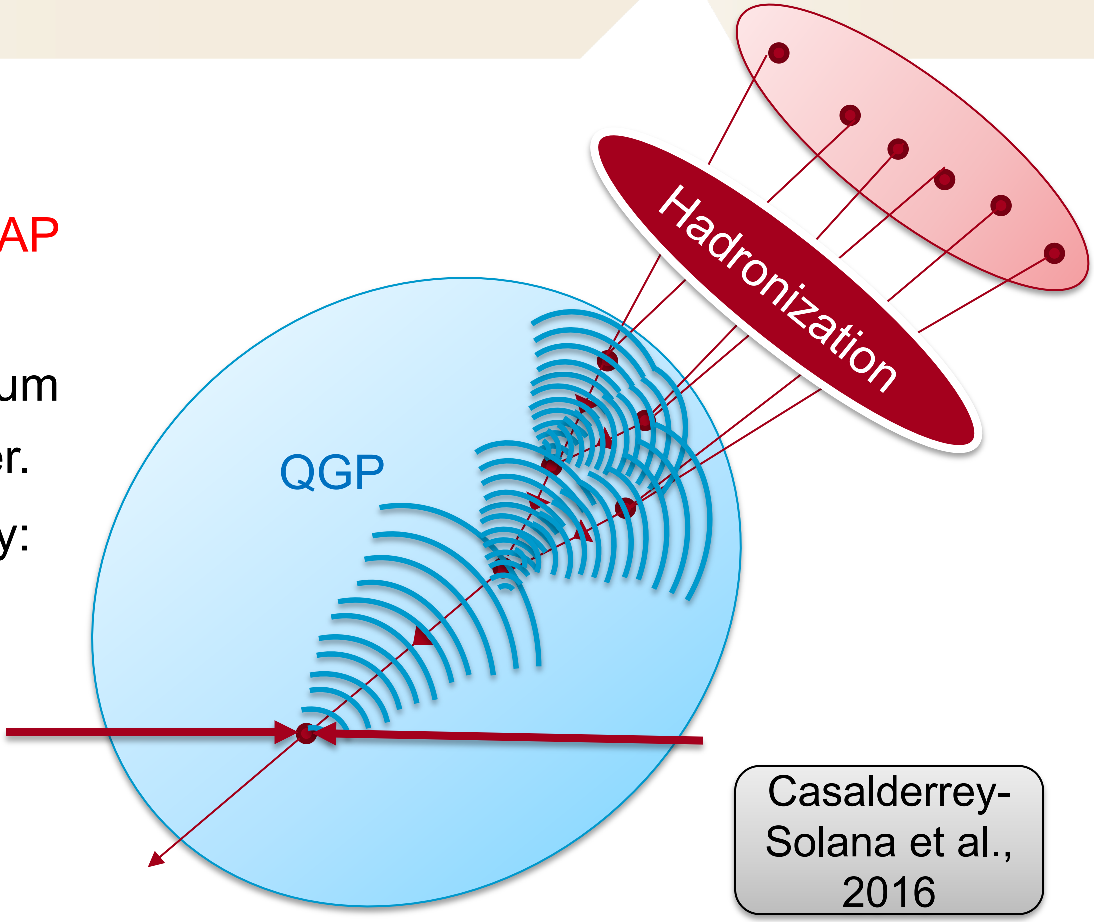
- High Q^2 parton shower up until hadronization described by DGLAP evolution (PYTHIA).
- For QGP with $T \sim \Lambda_{QCD}$, the medium interacts strongly with the shower.
  - Energy loss from holography:

$$\frac{1}{E_{in}}\frac{dE}{dx} = -\frac{4}{\pi}\frac{x^2}{x_{stop}^2}\frac{1}{\sqrt{x_{stop}^2 - x^2}}$$

$$x_{stop} = \frac{1}{2\kappa_{sc}}\frac{E_{in}^{\frac{1}{3}}}{T^{\frac{4}{3}}}$$

$$\tau = \frac{2E}{Q^2}$$

O(1) fit const.



Casalderrey-Solana et al., 2016

Energy and momentum conservation → deposit hydrodynamic wake in QGP liquid

$$\frac{d\Delta N}{p_T dp_T d\phi dy} = \frac{1}{(2\pi)^3}\int \tau dx dy d\eta_s m_T \cosh(y - \eta_s)\left[f\left(\frac{u^\mu p_\mu}{T_f + \delta T}\right) - f\left(\frac{u_0^\mu p_\mu}{T_f}\right)\right]$$

Adding Moliere Scattering to Hybrid Model

- High Q^2 parton shower up until hadronization described by DGLAP evolution (PYTHIA).
- For QGP with $T \sim \Lambda_{QCD}$, the medium interacts strongly with the shower.
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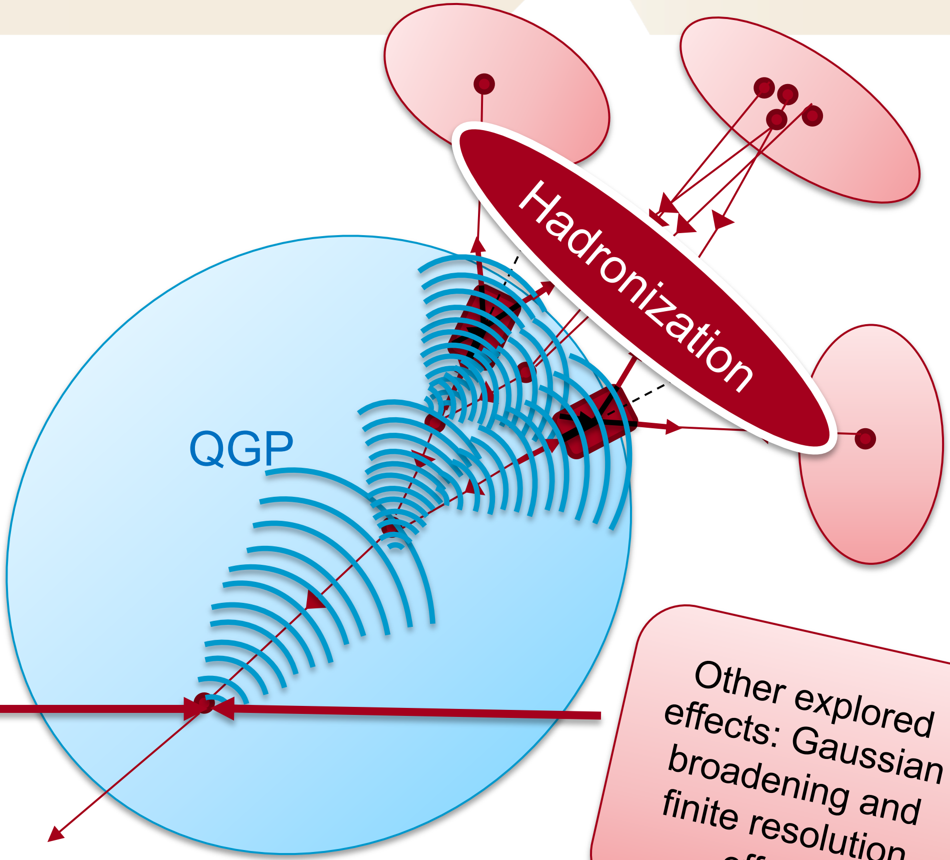


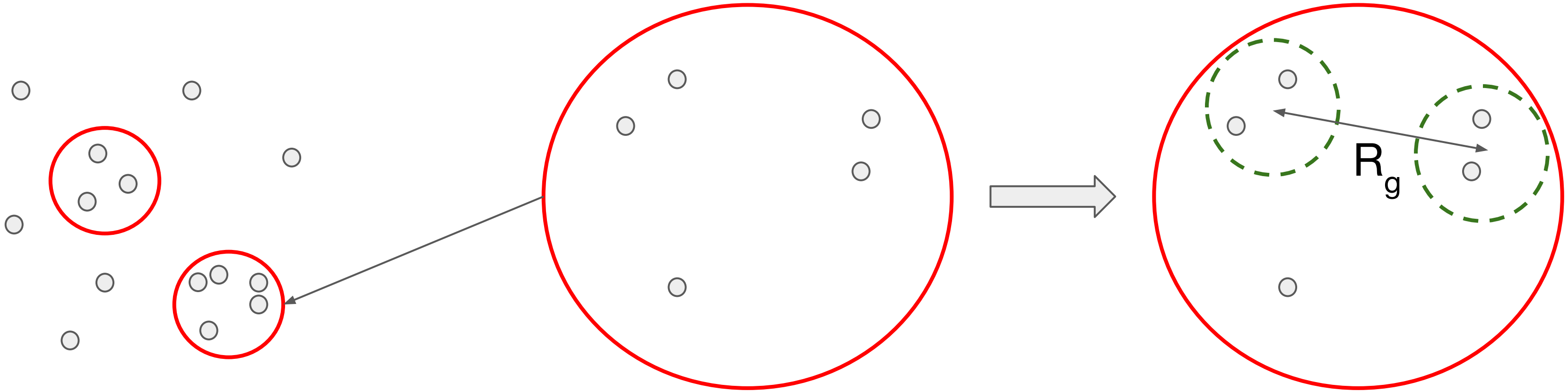
Other explored effects: Gaussian broadening and finite resolution effects

Energy and momentum conservation → activate hydrodynamic modes of plasma

$$\frac{d\Delta N}{p_T dp_T d\phi dy} = \frac{1}{(2\pi)^3}\int \tau dx dy d\eta_s m_T \cosh(y - \eta_s)\left[f\left(\frac{u^\mu p_\mu}{T_f + \delta T}\right) - f\left(\frac{\mu_0^\mu p_\mu}{T_f}\right)\right]$$

Probing Molière Scatterings using Soft Drop

arXiv:1402.2657 [Larkoski, Marzani, Soyez, Thaler]



Anti-k_t jets

Look inside each jet

Re-cluster the jet while dropping/grooming away the soft prongs

R_g is the separation between the first two prongs which satisfy

$$z_{\text{subleading}} > z_{\text{cut}} \left(\frac{R_g}{R}\right)^{\beta}$$

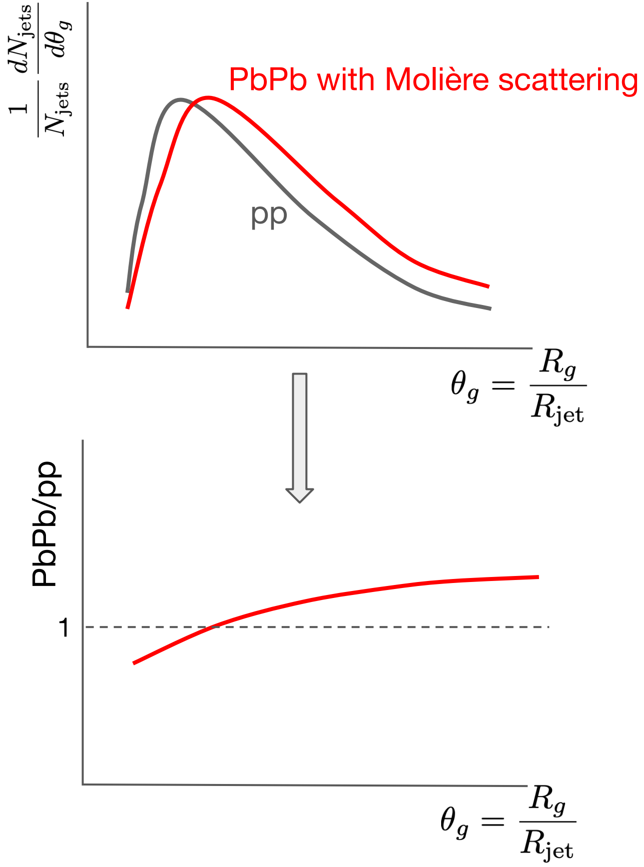
R_g measures the angle of the first hard splitting in the jet

Scale by jet radius →

$$\theta_g = \frac{R_g}{R_{\text{jet}}}$$

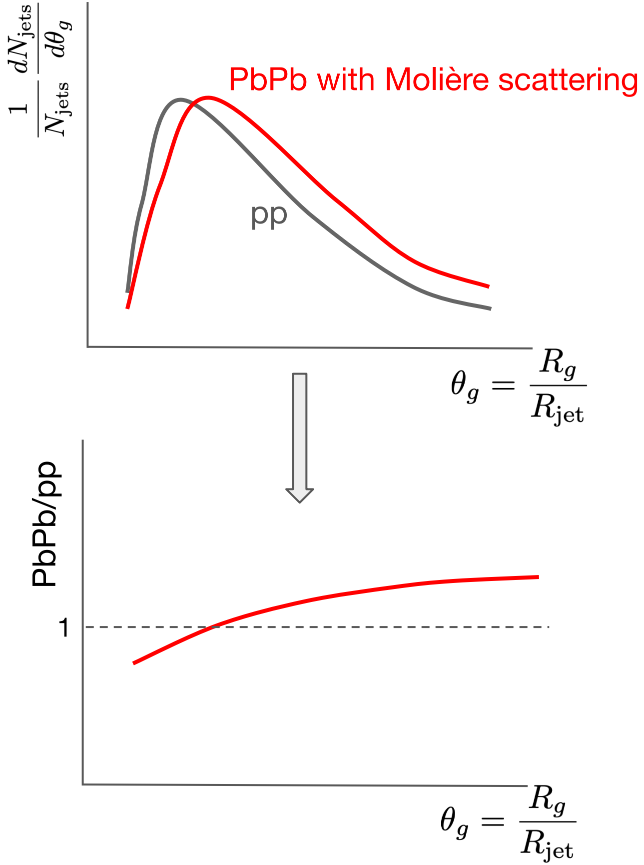
Soft Drop Angle in PbPb Collisions

Naive Expectation

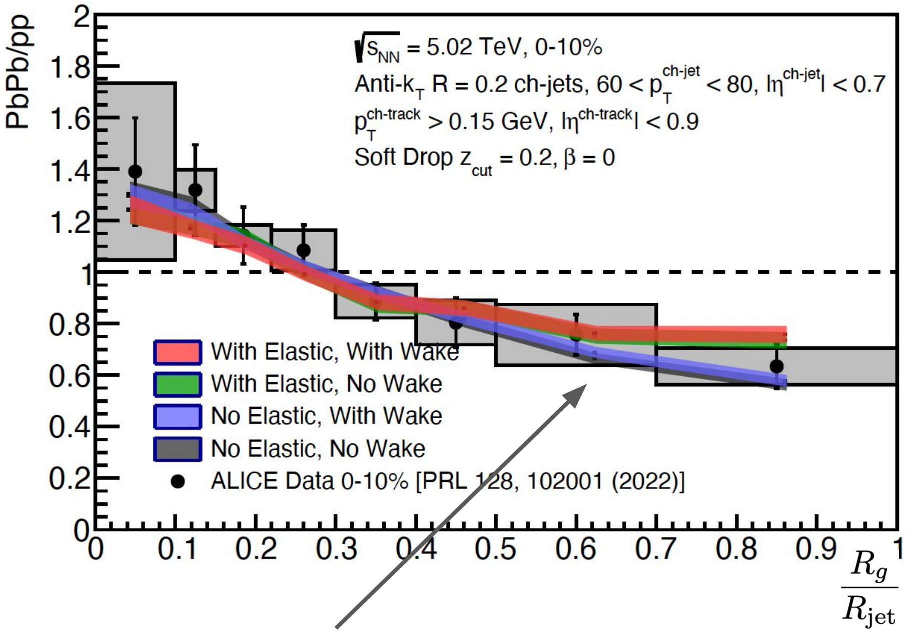


Soft Drop Angle in PbPb Collisions

Naive Expectation

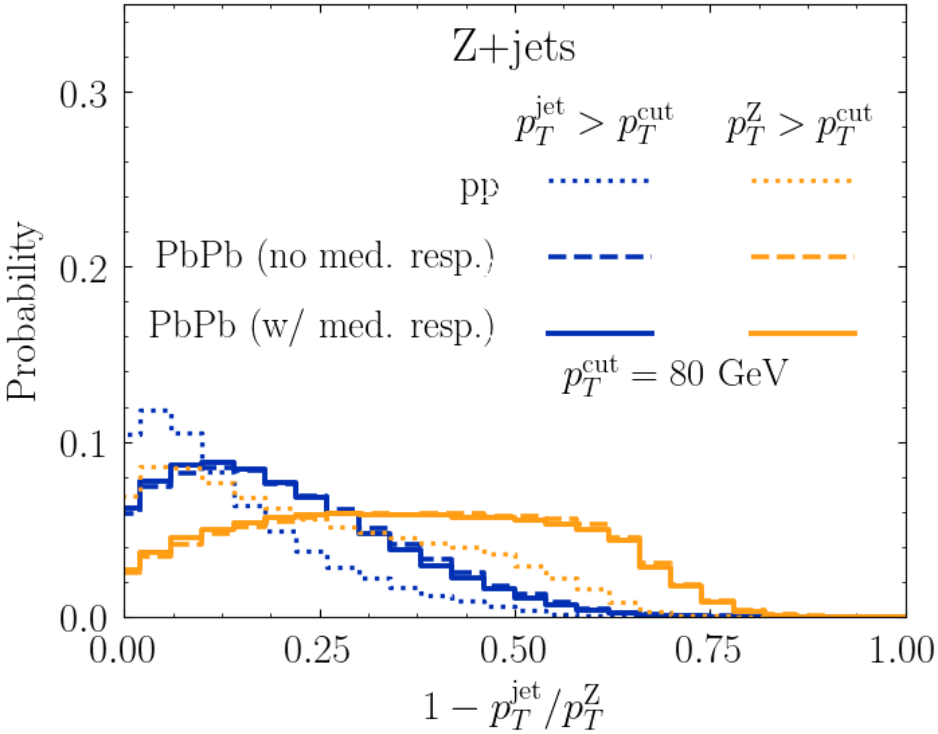


Reality



PbPb jet sample is biased towards those that have lost the least energy, which tend to be narrower

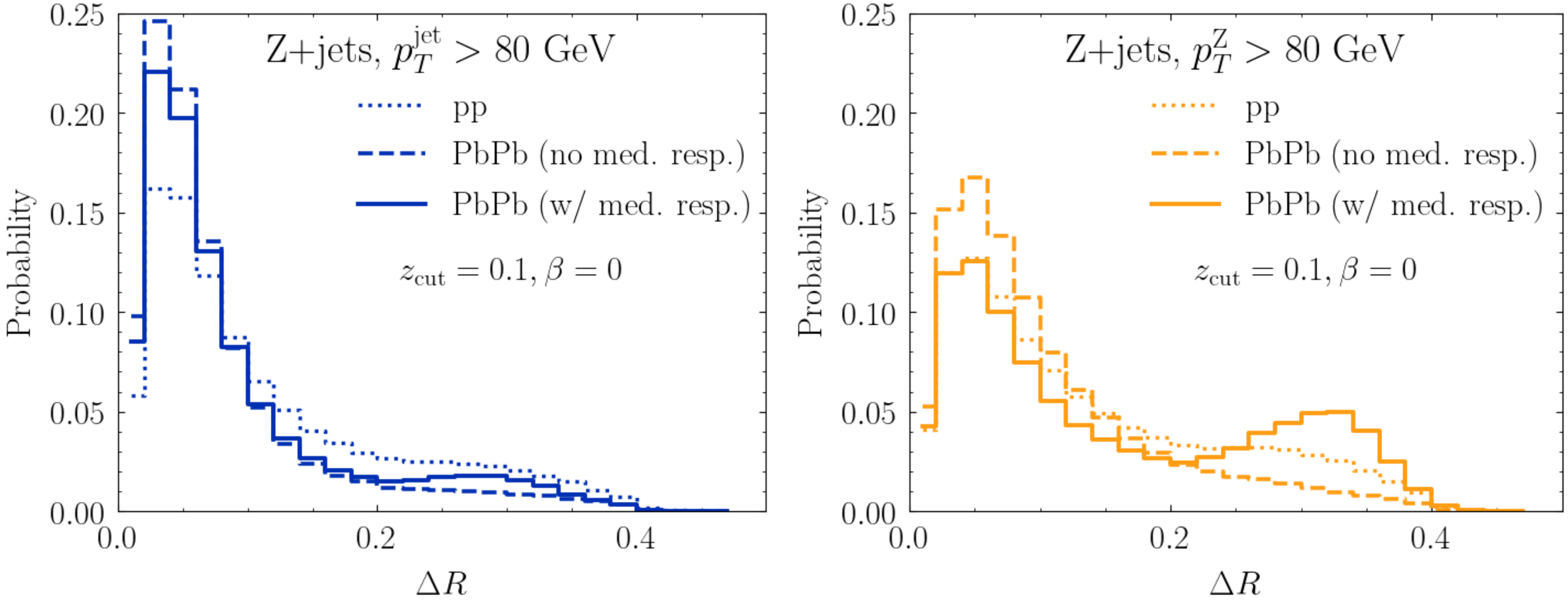
Disentangling Jet Modification from Selection



Orange: $p_T^Z > 80$ GeV; $p_T^{\rm jet} > 30$ GeV

Blue: $p_T^{\rm jet} > 80$ GeV; $p_T^Z > 30$ GeV — jet selection biases toward those jets that lose less energy

Disentangling Jet Modification from Selection

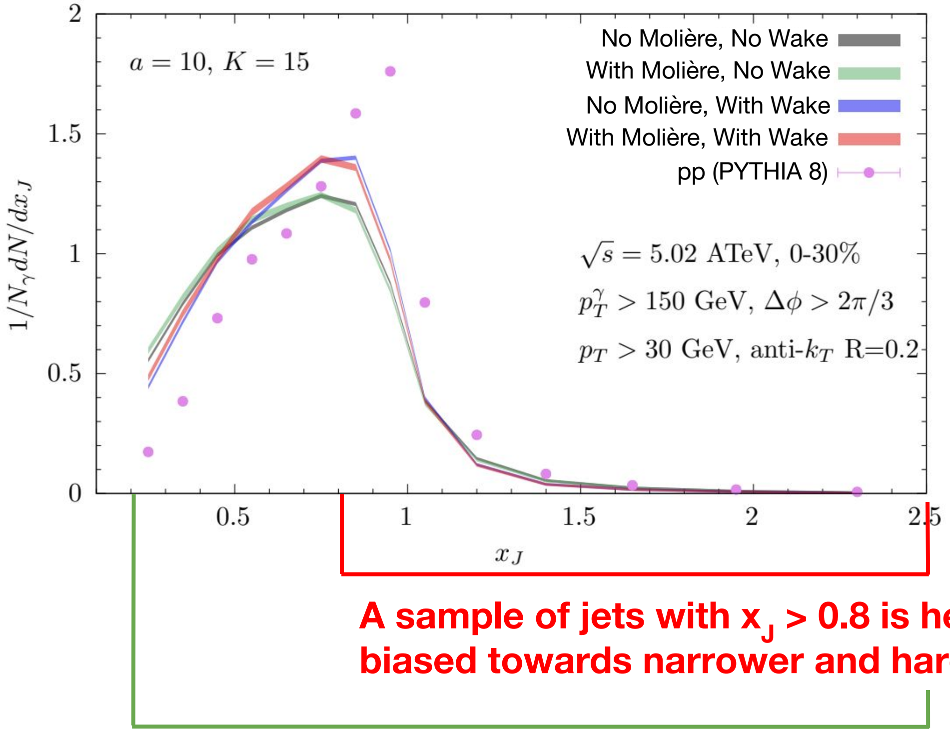


Orange: $p_T^Z > 80$ GeV; $p_T^{\mathrm{jet}} > 30$ GeV. See jet modification.

Blue: $p_T^{\mathrm{jet}} > 80$ GeV; $p_T^Z > 30$ GeV — jet selection biases toward those jets that lose less energy. These jets are skinnier. And the bias is toward less jet modification.

Use Photon-Tagged Jets

2603.08776 [Hulcher, Kudinoor, Pablos, Rajagopal]



A sample of jets with $x_J > 0.8$ is heavily biased towards narrower and harder jets

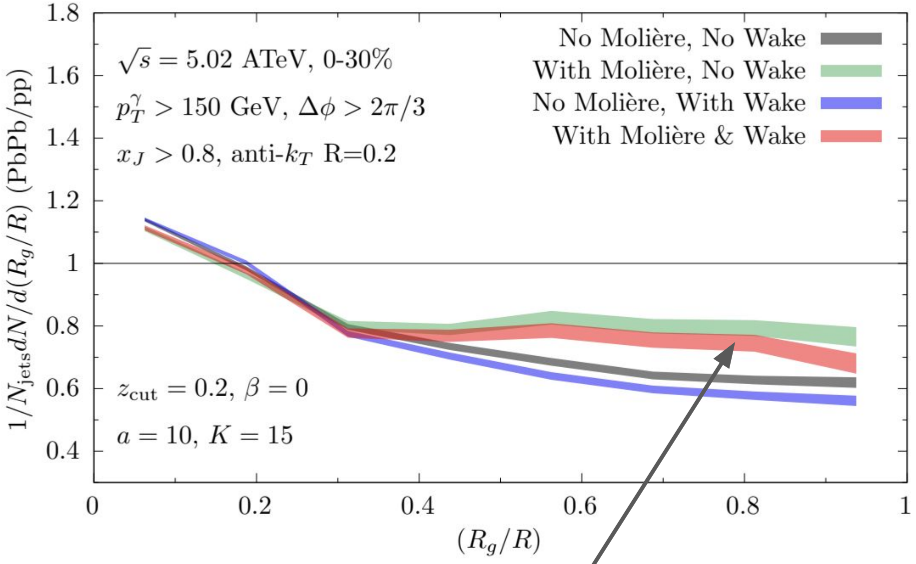
A sample of jets with $x_J > 0.2$ is less biased

- Photons do not interact strongly with QGP
- Selecting on p_T^γ lets us select on the initial energy scale of the scattering
- $x_J = p_T^{jet} / p_T^\gamma$ estimates the amount of energy lost by the recoiling jet

Soft Drop Angle in Photon-Tagged Jets with R = 0.2

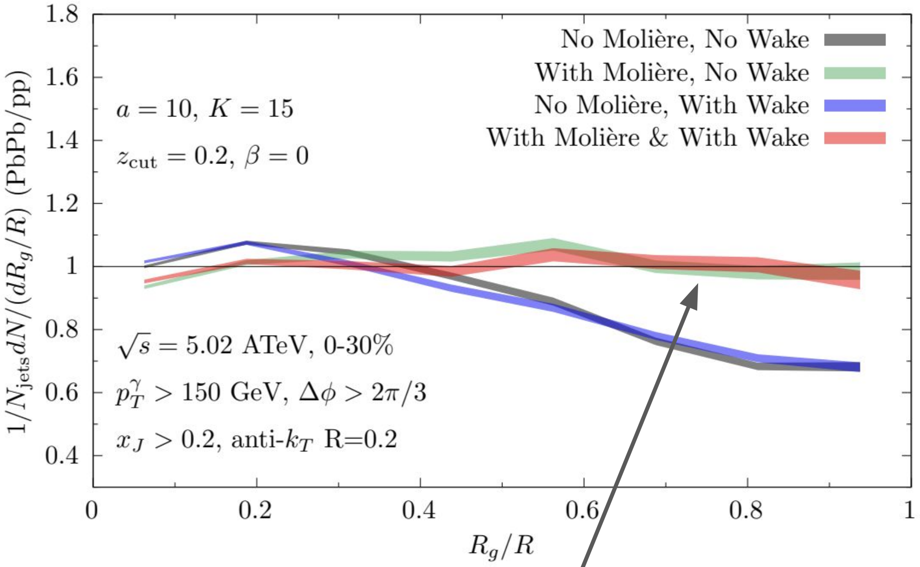
2603.08776 [Hulcher, Kudinoor, Pablos, Rajagopal]

$x_J > 0.8$ (More Biased)



Molière scattering increases the number of jets with large R_g, but not enough to overcome selection bias effects

$x_J > 0.2$ (Less Biased)

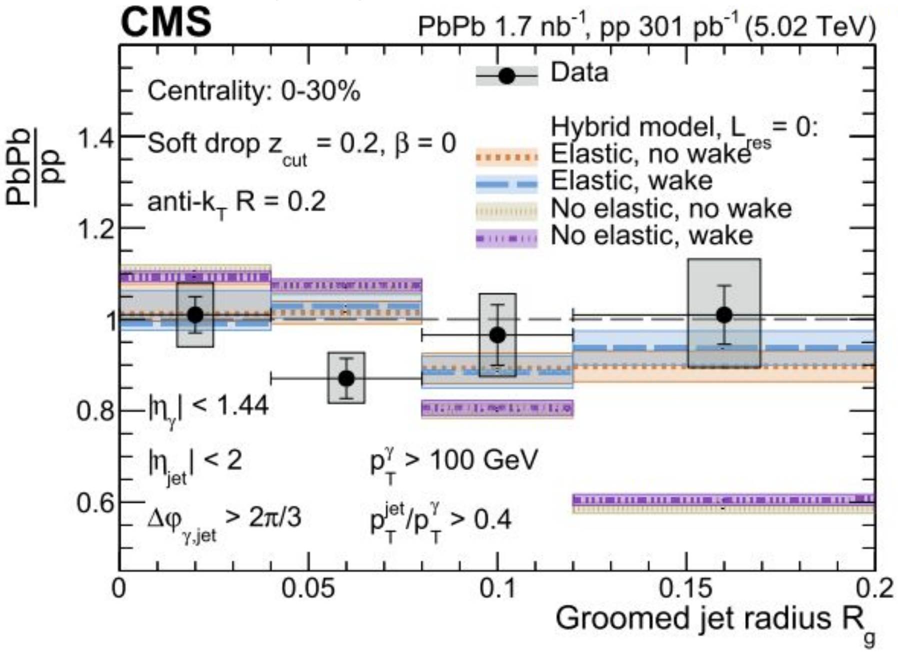


Reduce selection bias ⇒ More detectable Molière scattering signal

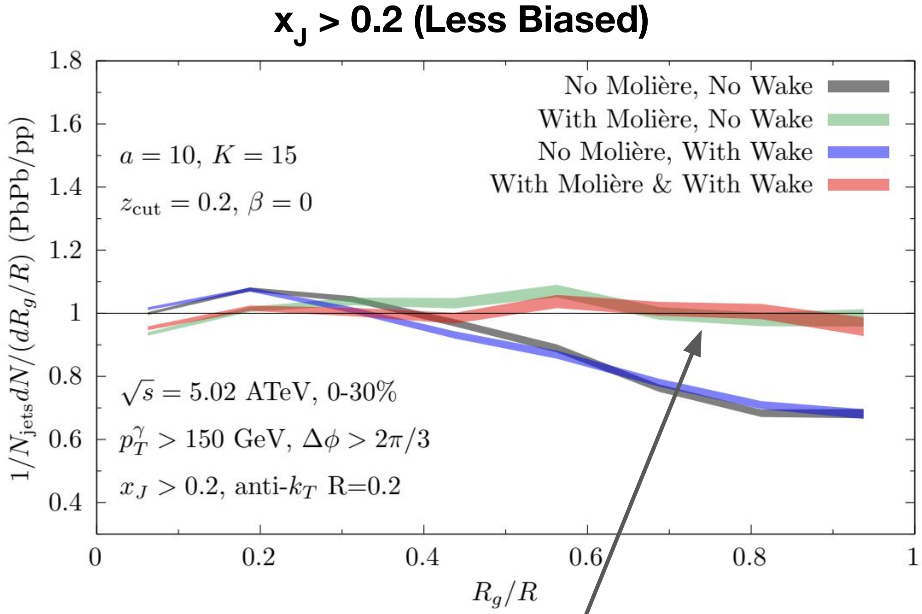
Soft Drop Angle in Photon-Tagged Jets with R = 0.2

2603.08776 [Hulcher, Kudinoor, Pablos, Rajagopal]

PLB 861 (2025) 139088

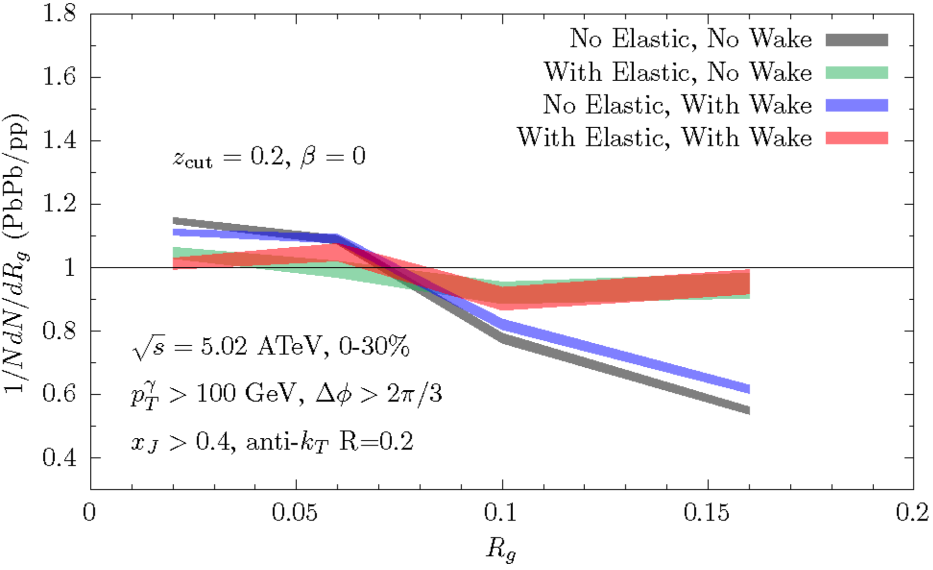


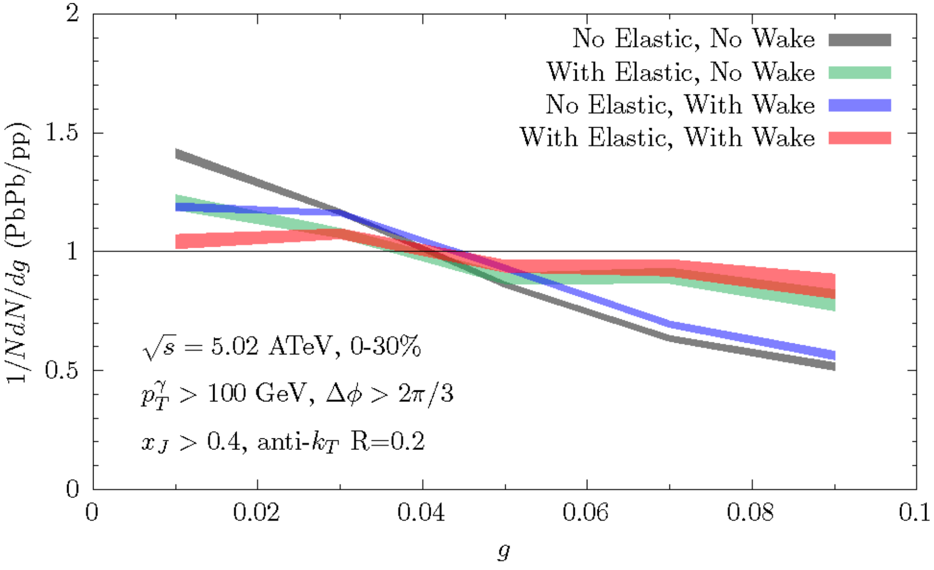
Relatively flat ratio around unity is also seen in CMS data!


x_J > 0.2 (Less Biased)

Reduce selection bias ⇒ More detectable Molière scattering signal

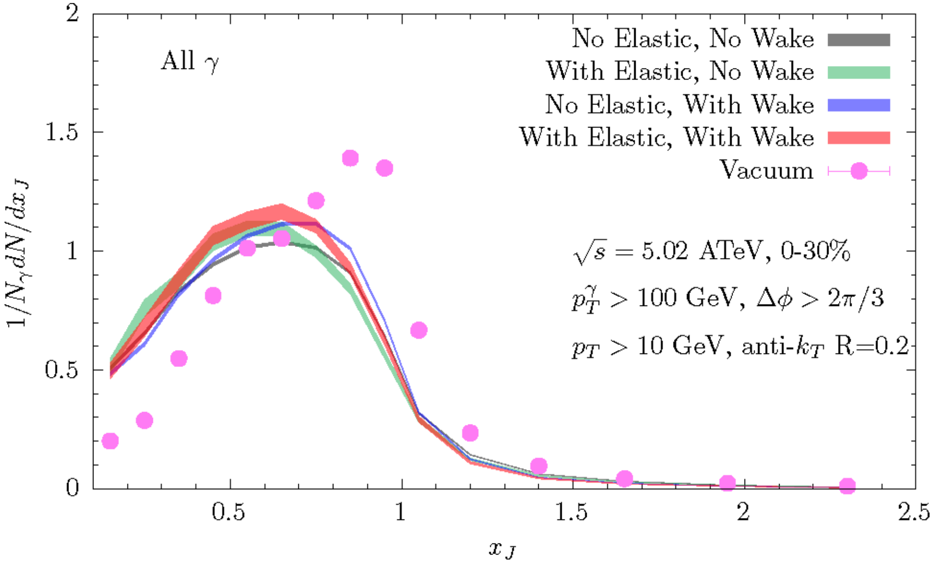
Gamma-Jet Observables: R_g and Girth, with x_J>0.4



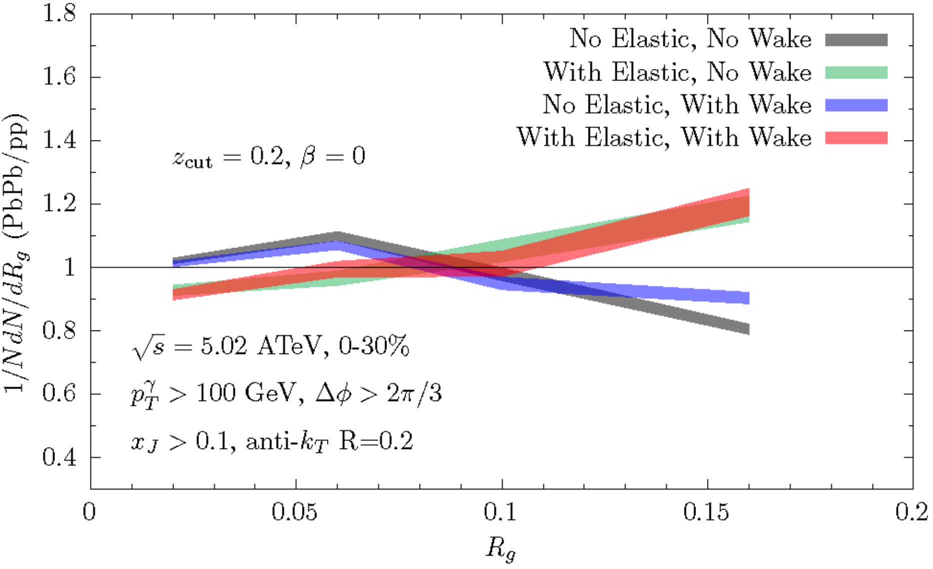


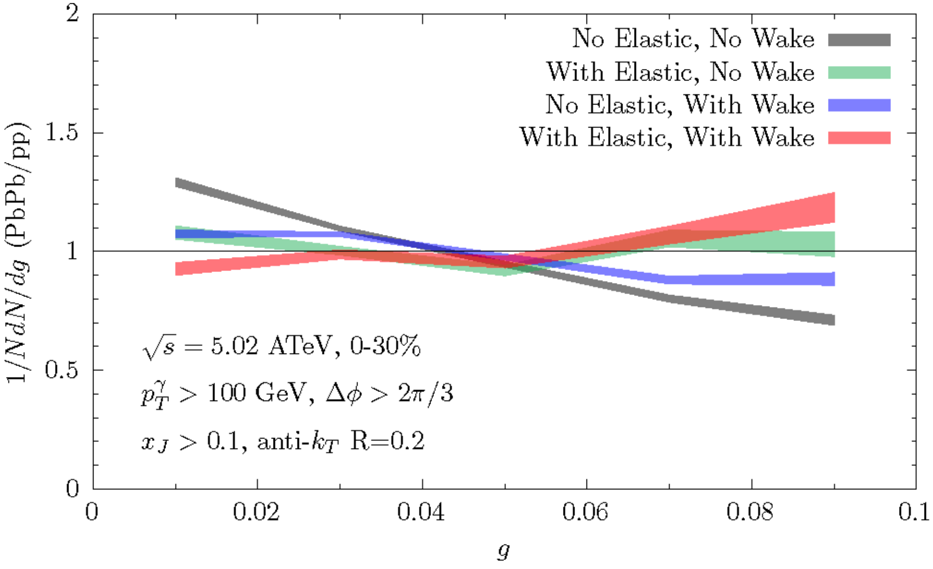
All show much less sensitivity to wake: R=0.2; Moliere scattering effects are very much dominant.

But why is R_{AA} below 1? Selection bias! With x_J>0.4 selection, missing too many of the most modified jets.



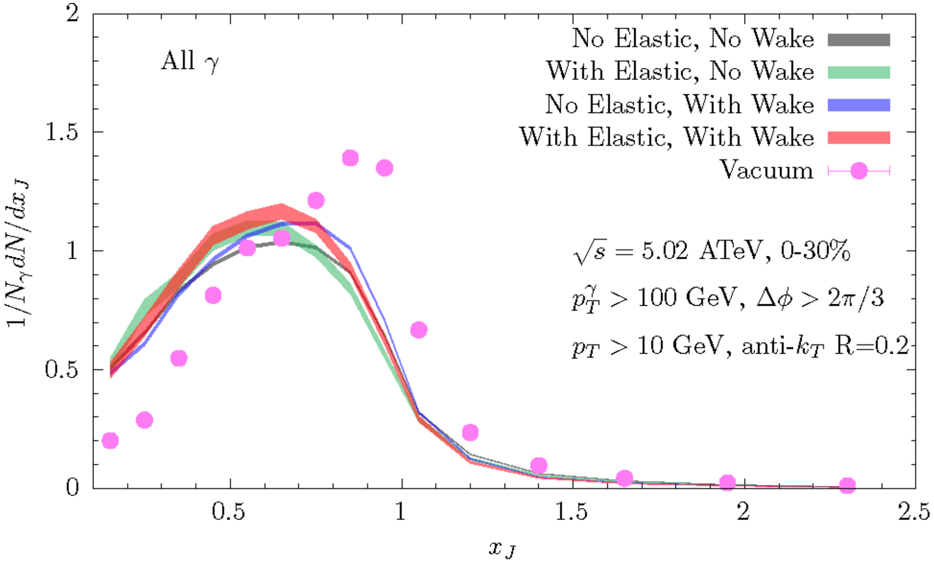
Gamma-Jet Observables: R_g and Girth, with x_J>0.1



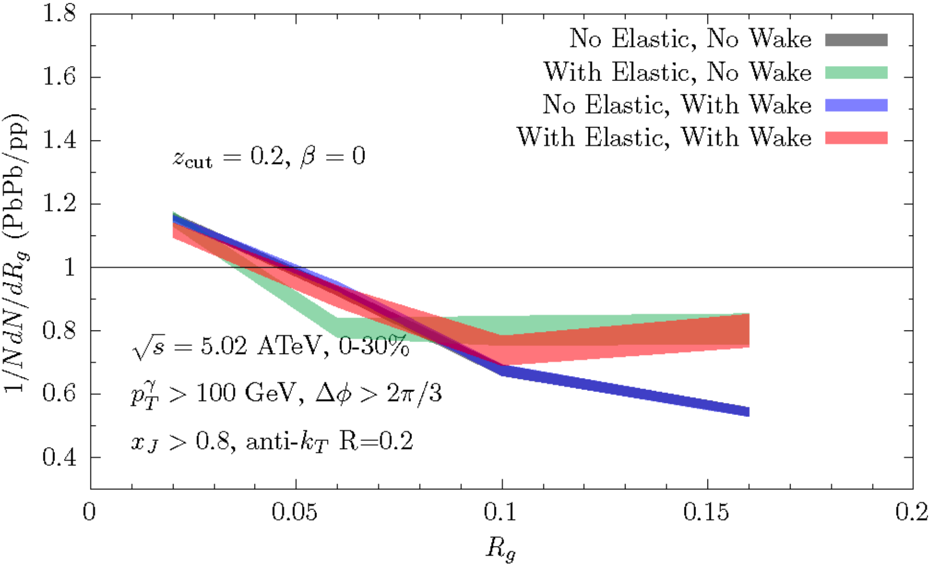


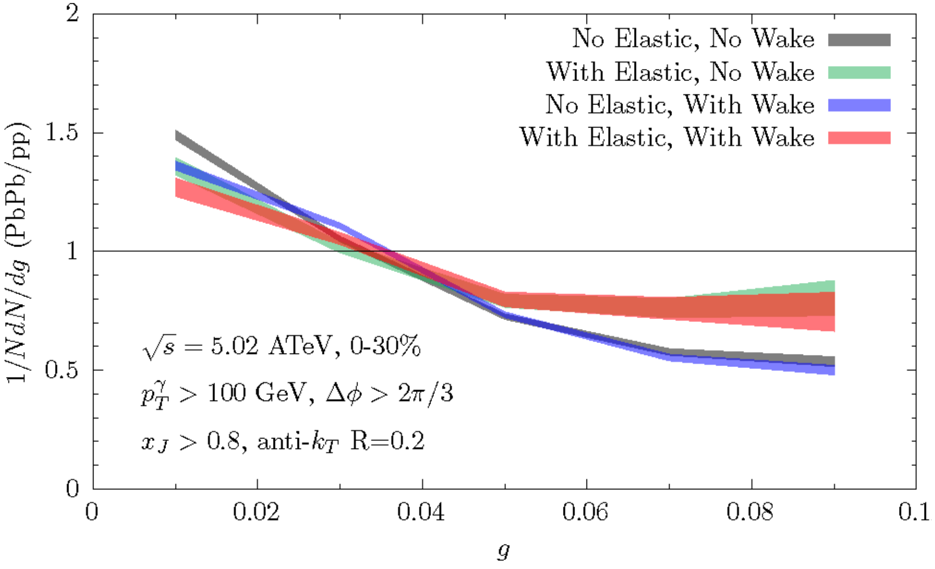
On previous slides, Rg and Girth with xJ>0.4: missing the most modified jets. Here, xJ>0.1. Moliere scattering important, and causes R_{AA} >1.

Selection bias reduced (cf Brewer+Brodsky+KR); some effects of wake visible.



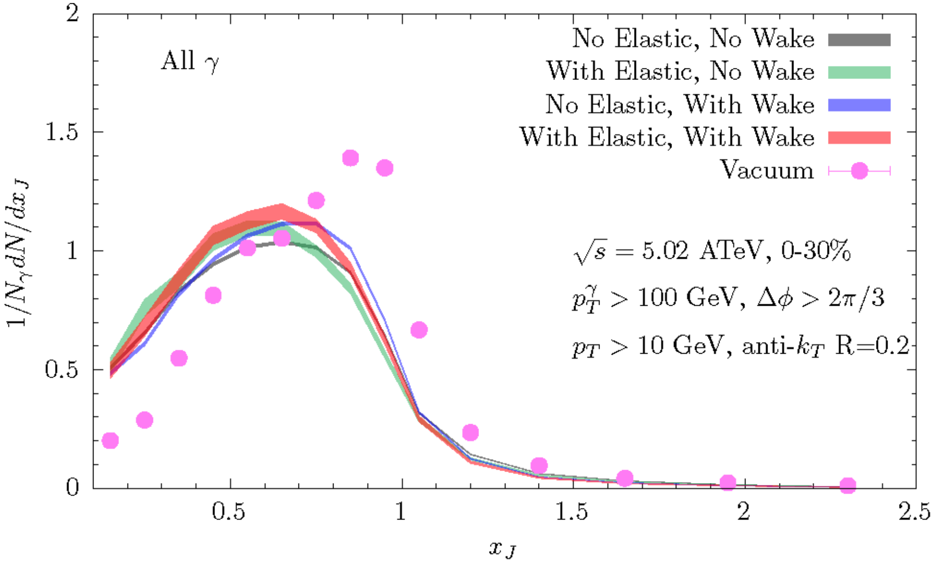
Gamma-Jet Observables: R_g and Girth, with $x_J>0.8$



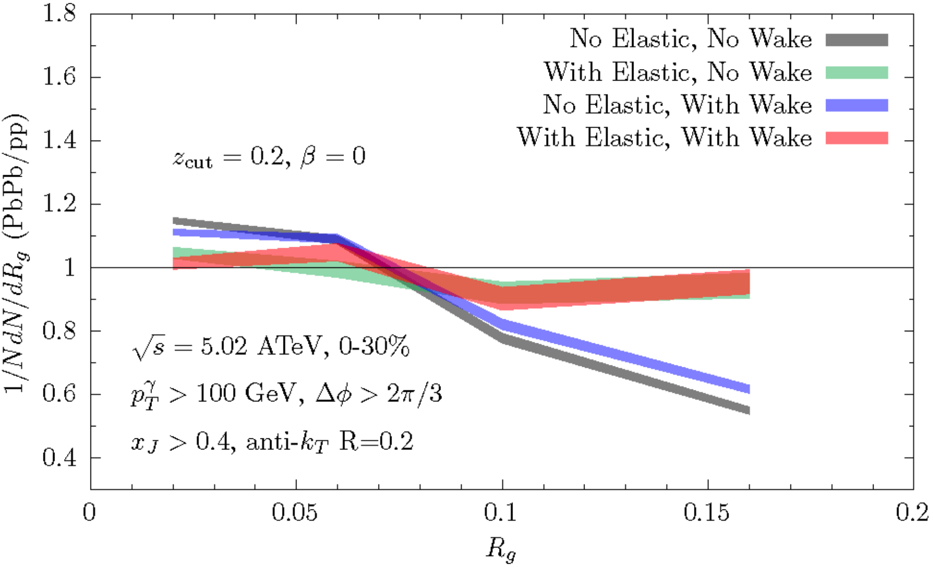


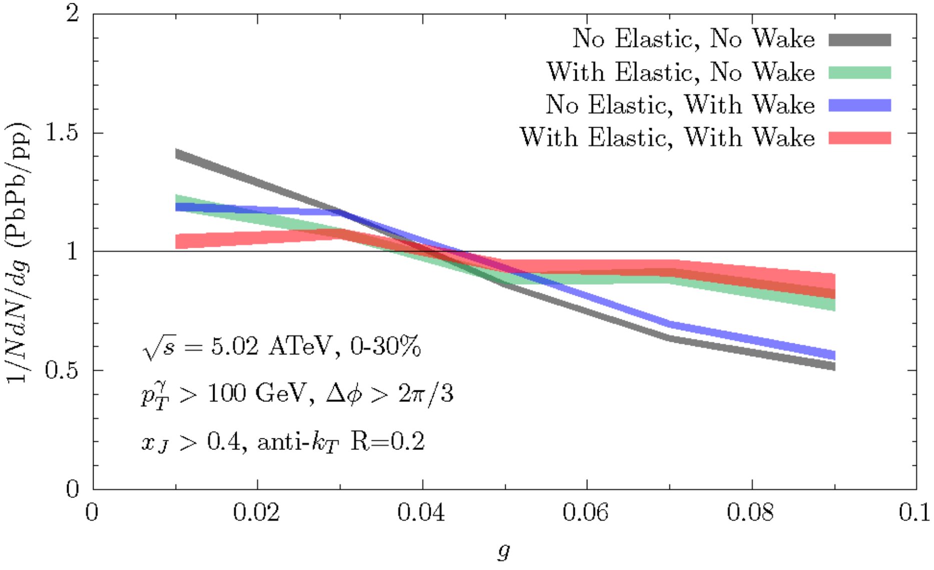
On previous slides, Rg and Girth with xJ>0.4: missing the most modified jets. Here, xJ>0.8. Selection bias increased.

Moliere scattering still important, and but selection bias so strong that it does not yield R_{AA} >1.



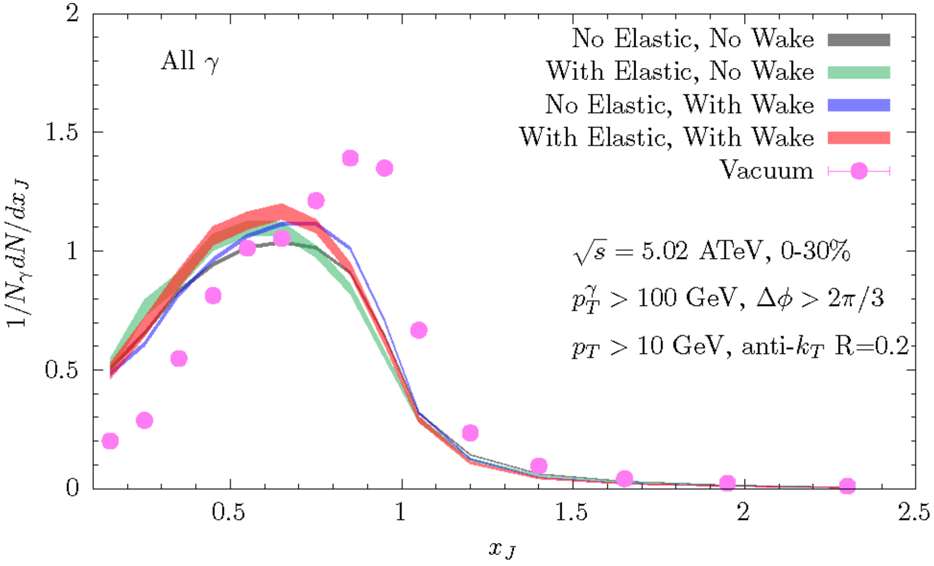
Gamma-Jet Observables: R_g and Girth, with x_J>0.4





All show much less sensitivity to wake: R=0.2; Moliere scattering effects are very much dominant.

But why is R_{AA} below 1? Selection bias! With x_J>0.4 selection, missing too many of the most modified jets.



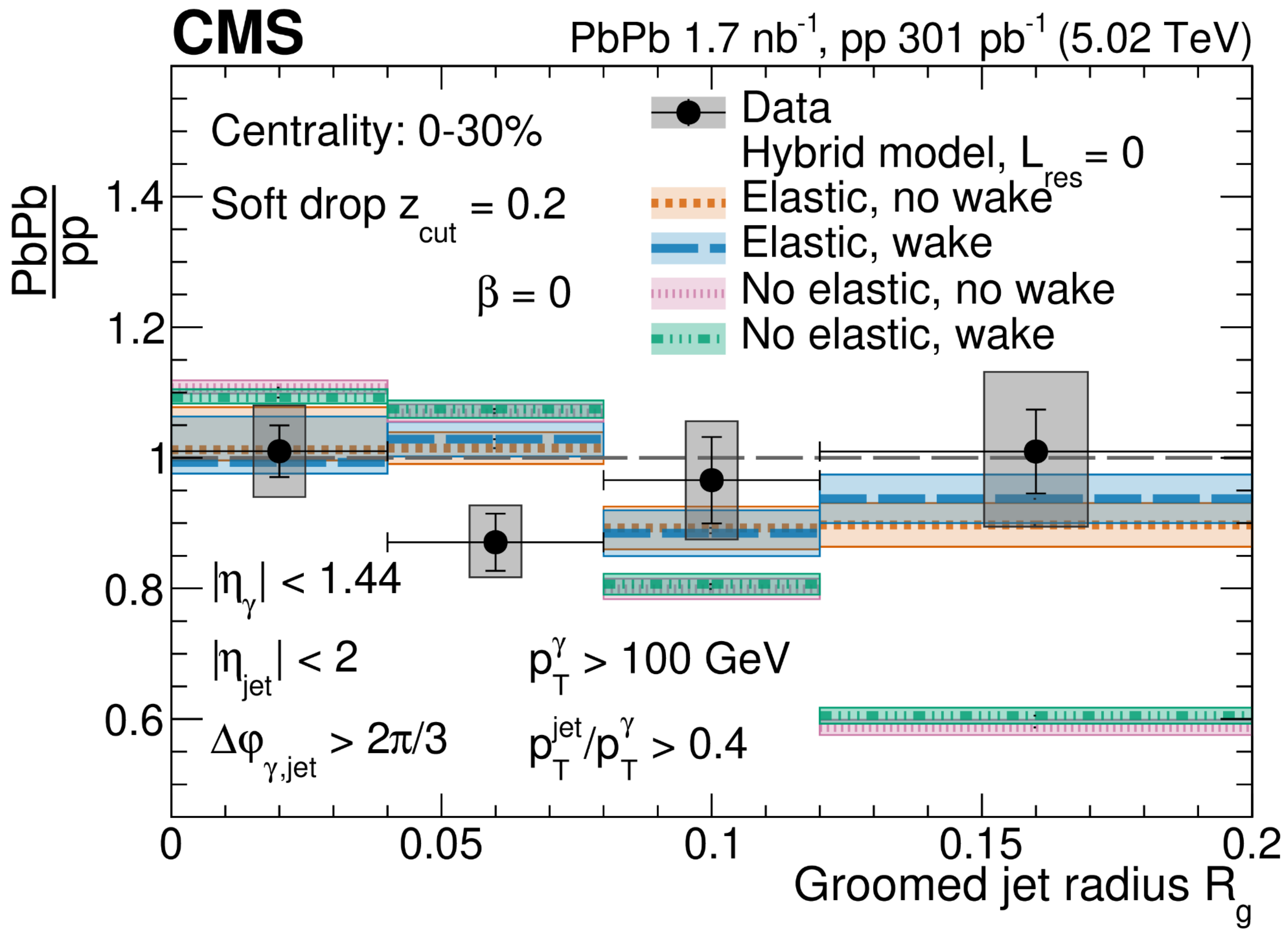
Summary

- Groomed jet radius and girth measured in γ+jet events in pp and PbPb

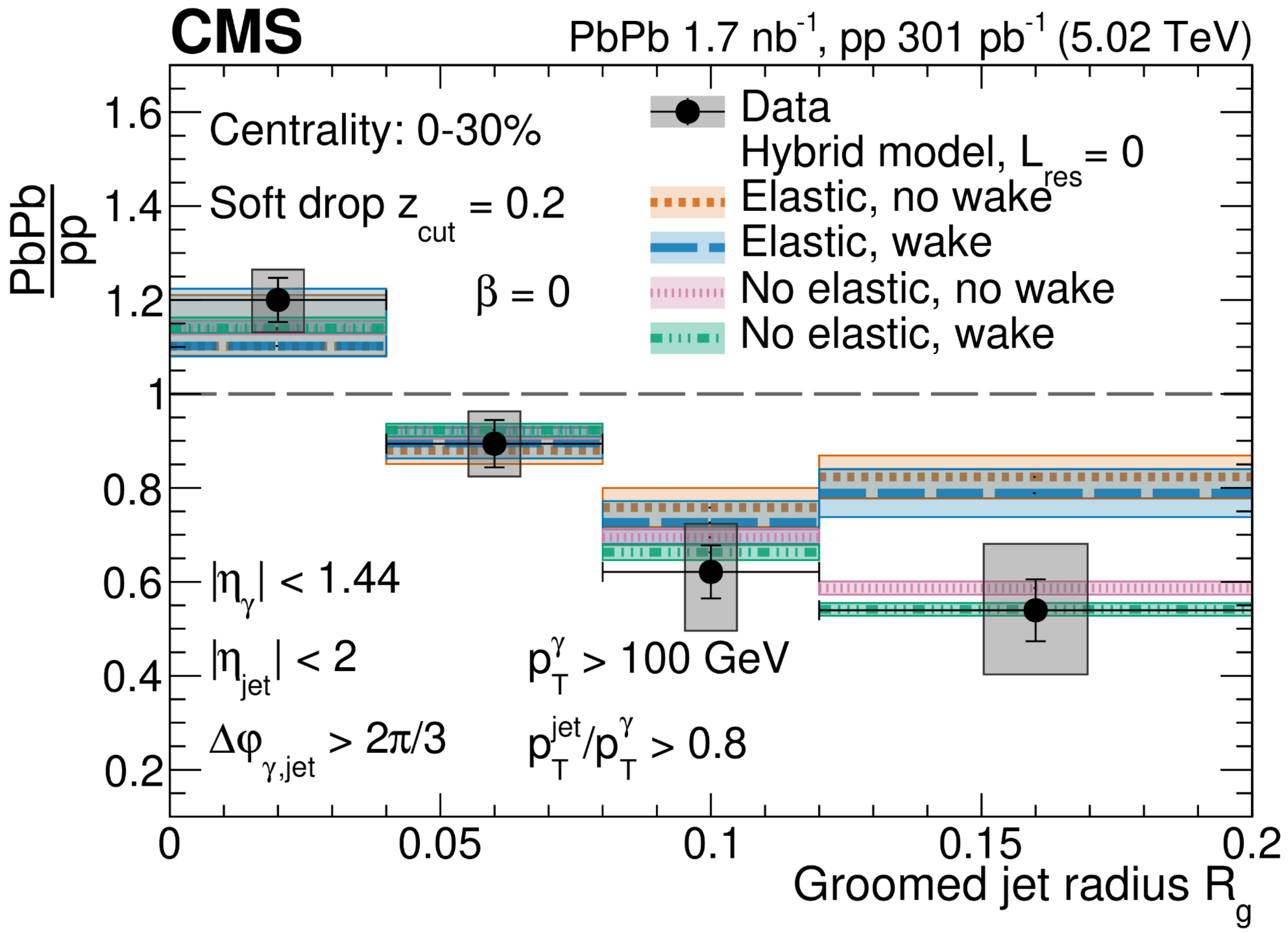
‣Leading recoil jet from p_T > 100 GeV photons studied for two selections:

$x_{\gamma j}$ > 0.4 (w/ quenched jets):
no narrowing observed

$x_{\gamma j}$ > 0.8 (less quenched jets):
narrowing is restored



CMS, arXiv:2405.0273

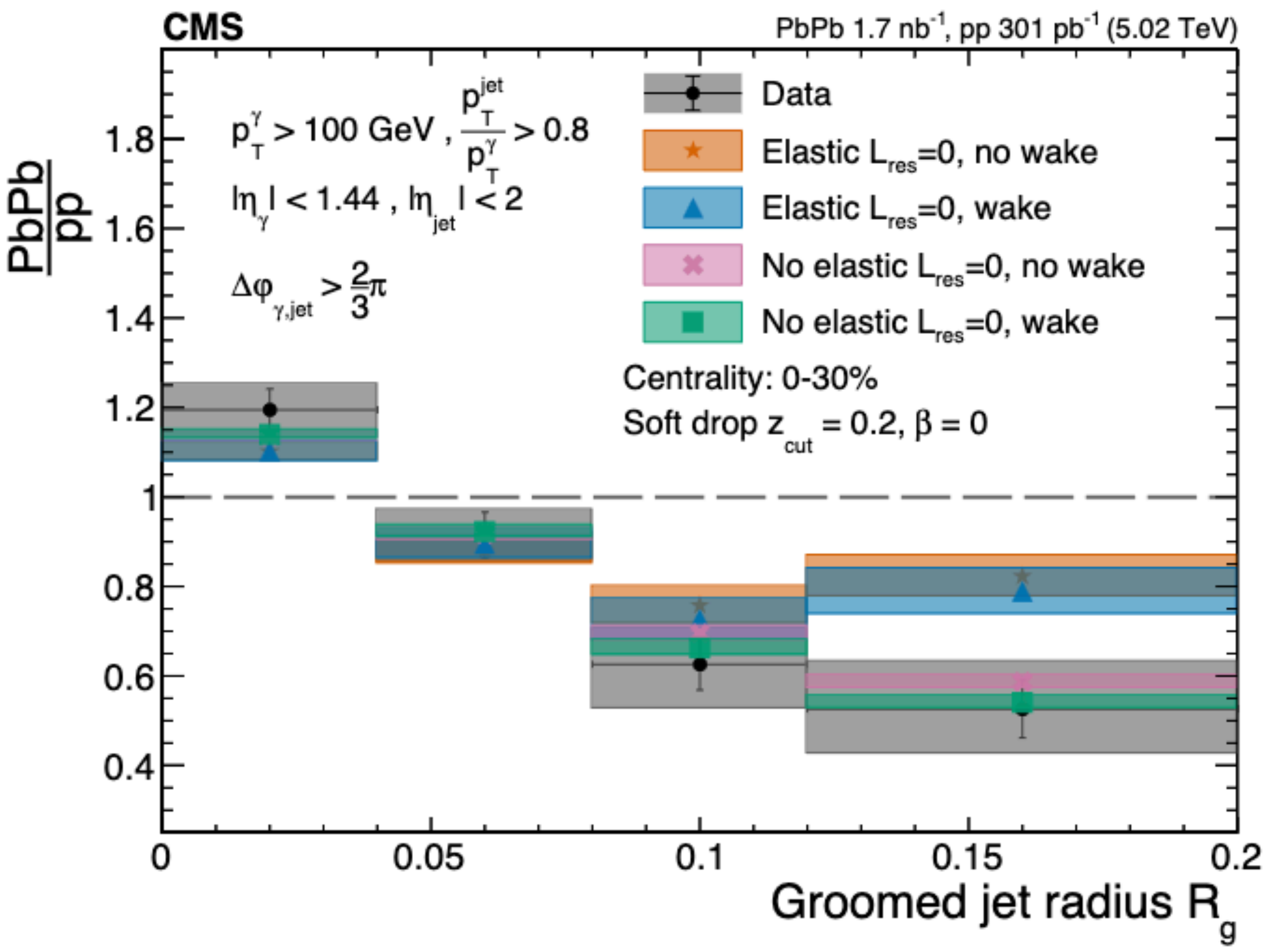


lesson 4: it seems all we see is survivor bias

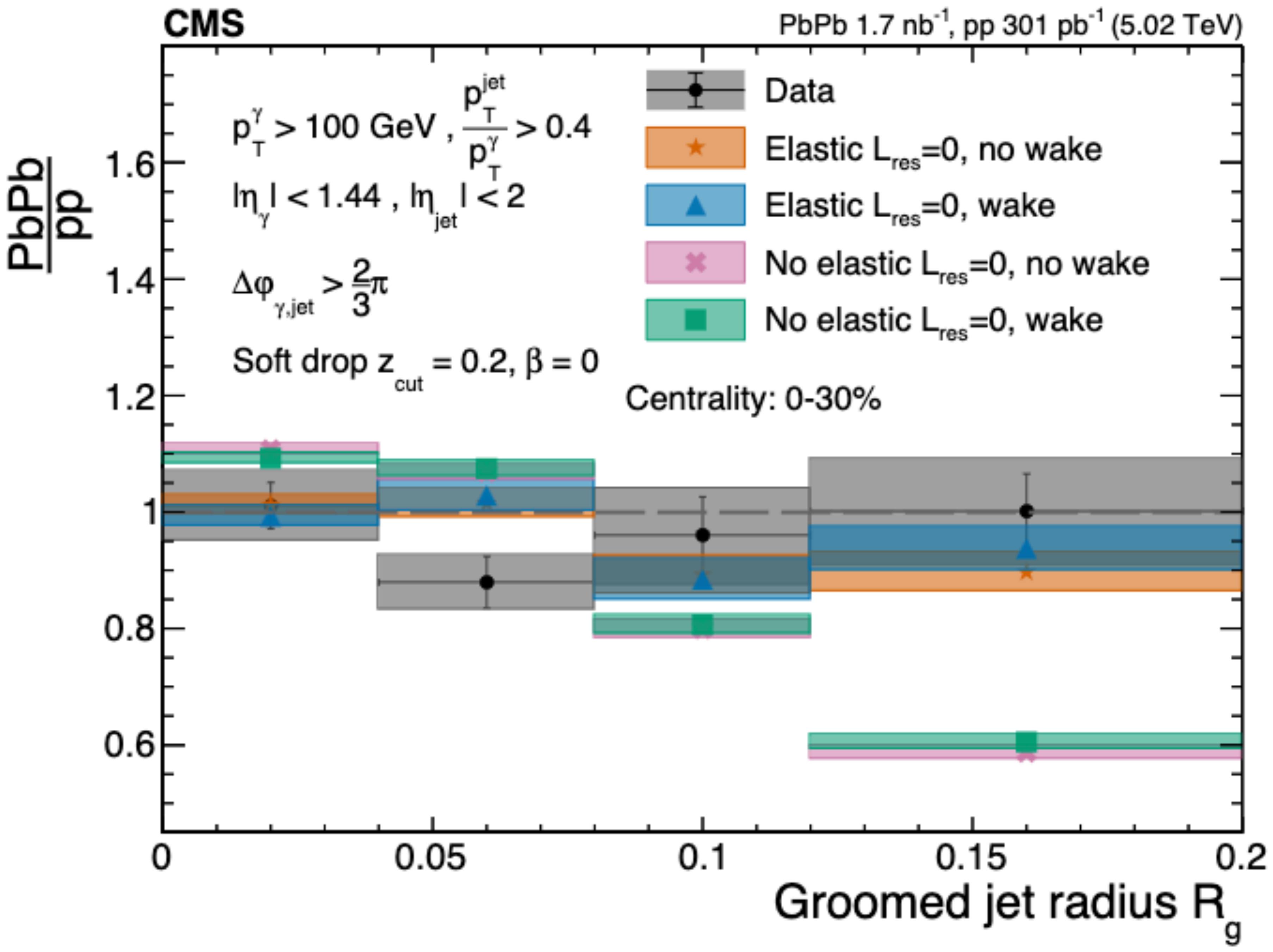
γ-jet substructure: suppression of the survivor bias

PbPb

less quenched xJ>0.8



more quenched xJ>0.4



Comparison to the Hybrid model ***(Rajagopal et al, JHEP 10 (2014) 019)***

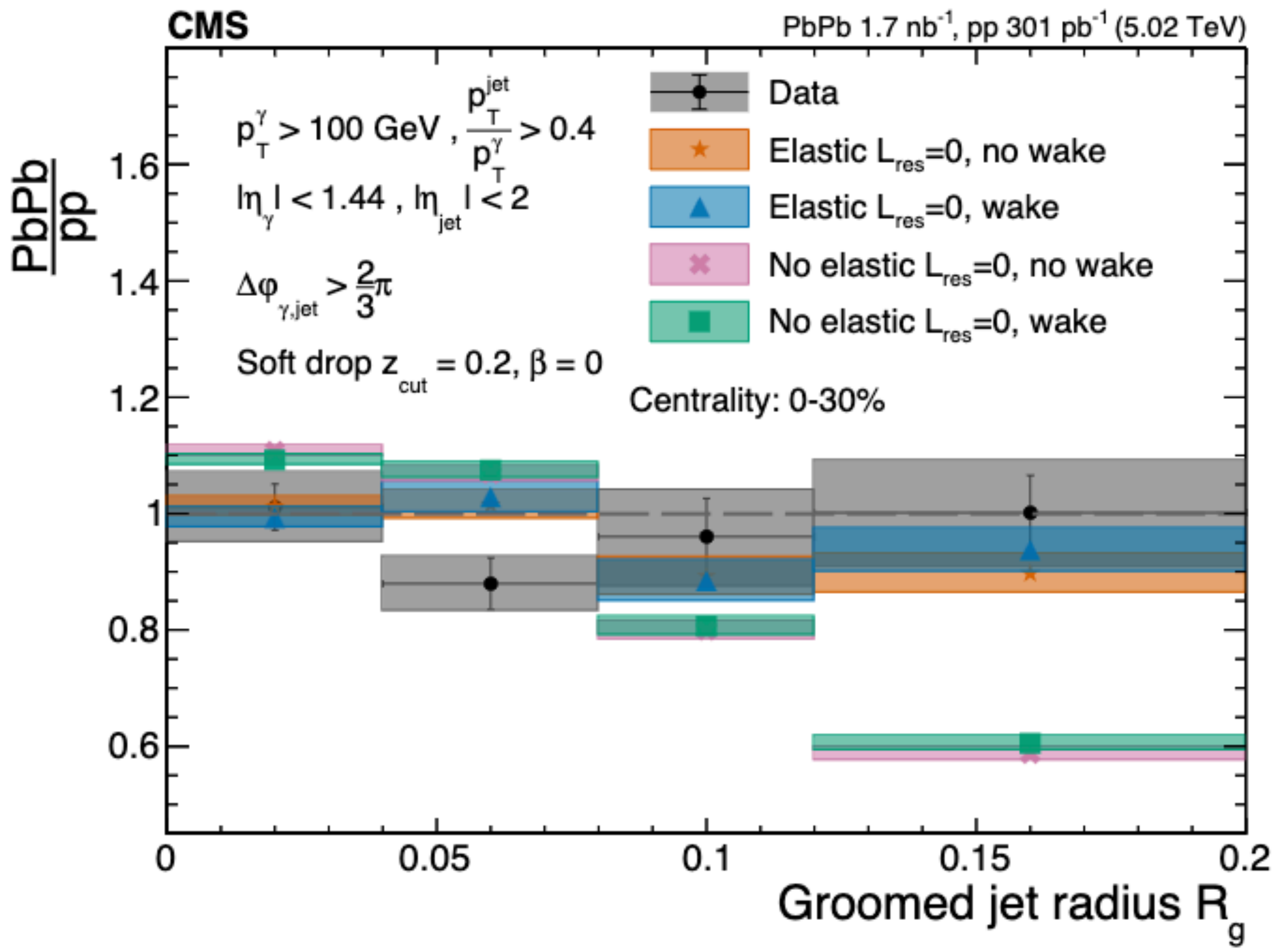
Not a single set of parameters describes the differential data consistently
Great constraining power of the data

small-R suppresses nonperturbative effects like the wake!

CMS-PAS-HIN-23-001

γ-jet substructure, prospects

xJ>0.4



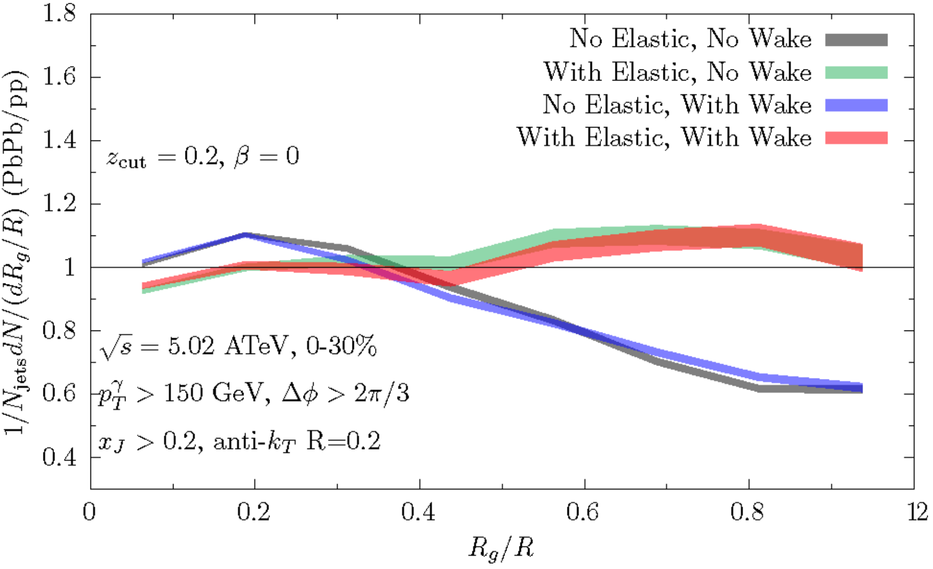
The survivor bias can be fully suppressed when $x_J \to 0$
(the model has a strong survivor bias down to xJ=0.1)

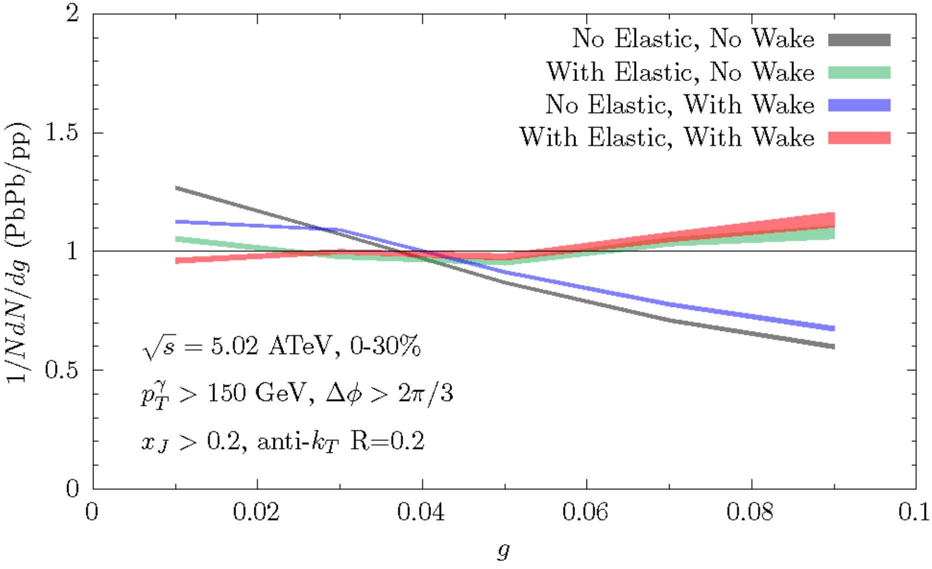
Since low jet p_T is limited by detector effects, such zero bias limit can be achieved by increasing the energy of the photons

Ideally, **simultaneous measurement of x_J and substructure** ,current results are statistically limited

CMS-PAS-HIN-23-001

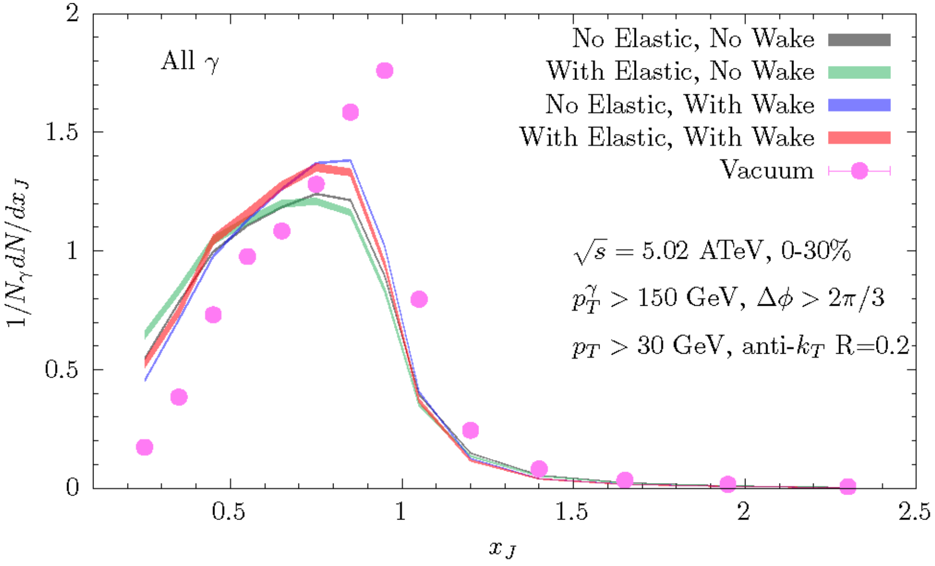
Gamma-Jet Observables with p_T^{γ} >150 GeV: R_g and Girth, with x_J>0.2



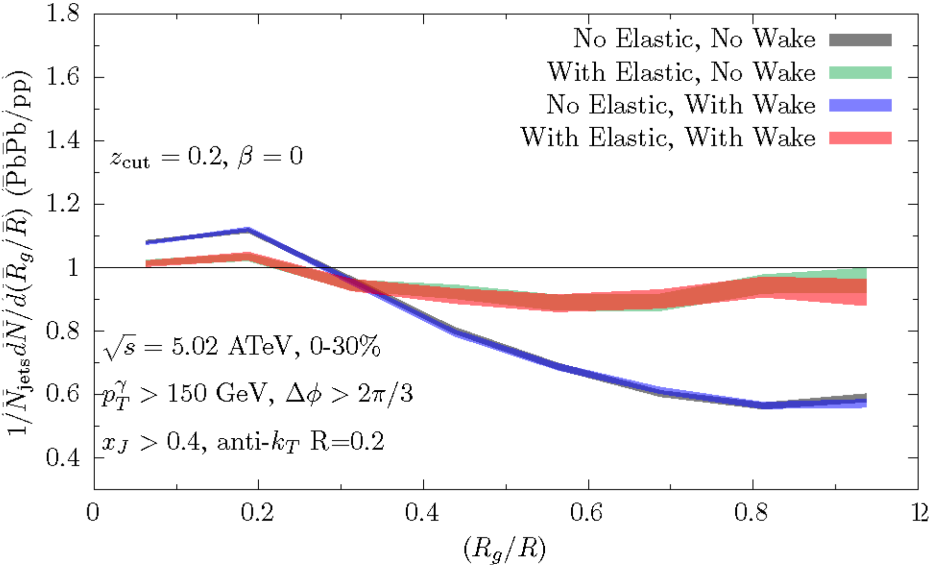


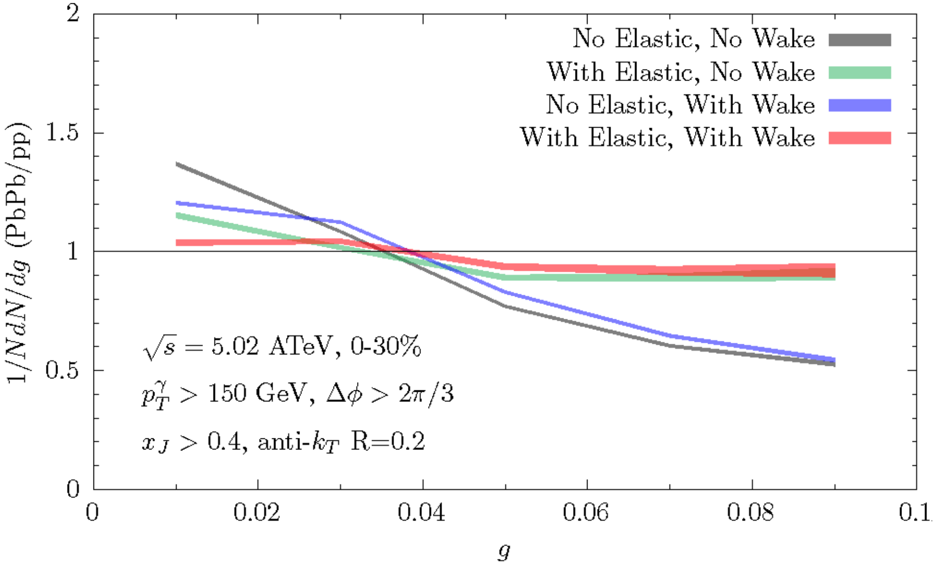
On previous slides, p_T^{γ} >100 GeV; here, p_T^{γ} >150 GeV. Means x_J>0.2 corresponds to $p_T^{\rm jet}$>30 GeV. And, no need to go down to x_J>0.1.

Moliere effects substantial; selection bias reduced; wake effects negligible.



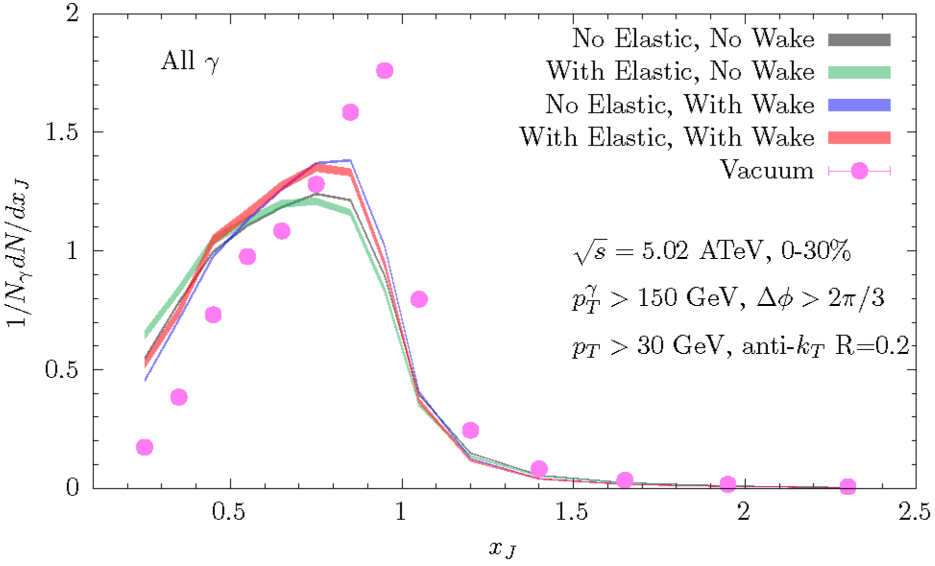
Gamma-Jet Observables with p_T^γ >150 GeV: R_g and Girth, with x_J>0.4



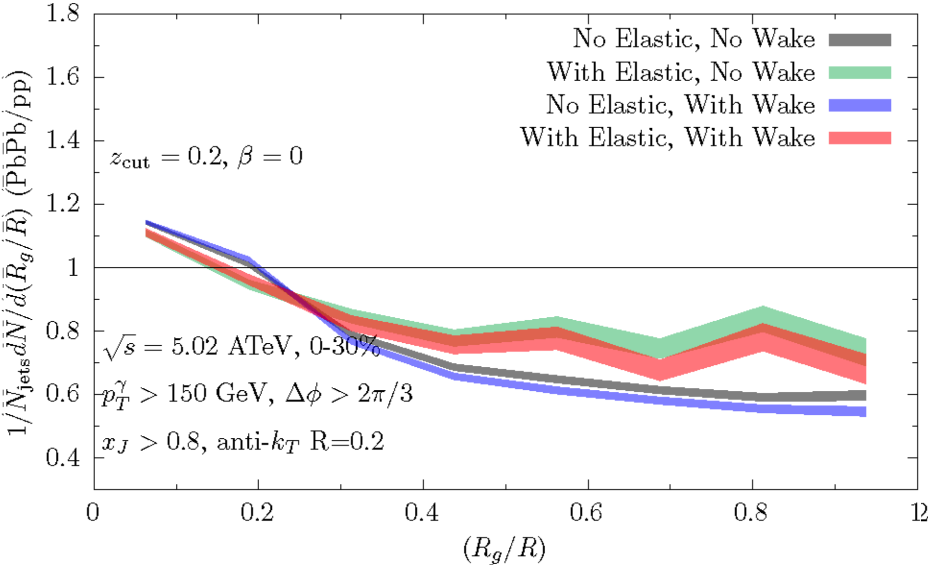


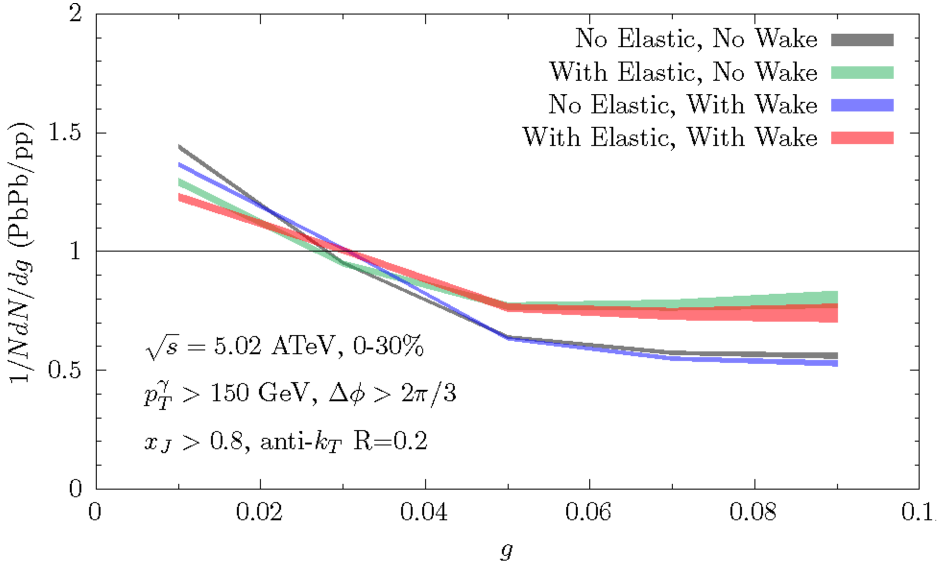
On previous slides, p_T^γ >100 GeV; here, p_T^γ >150 GeV.
Means x_J>0.4 corresponds to p_T^{jet}>60 GeV.

Moliere effects substantial; selection bias significant; wake effects negligible.



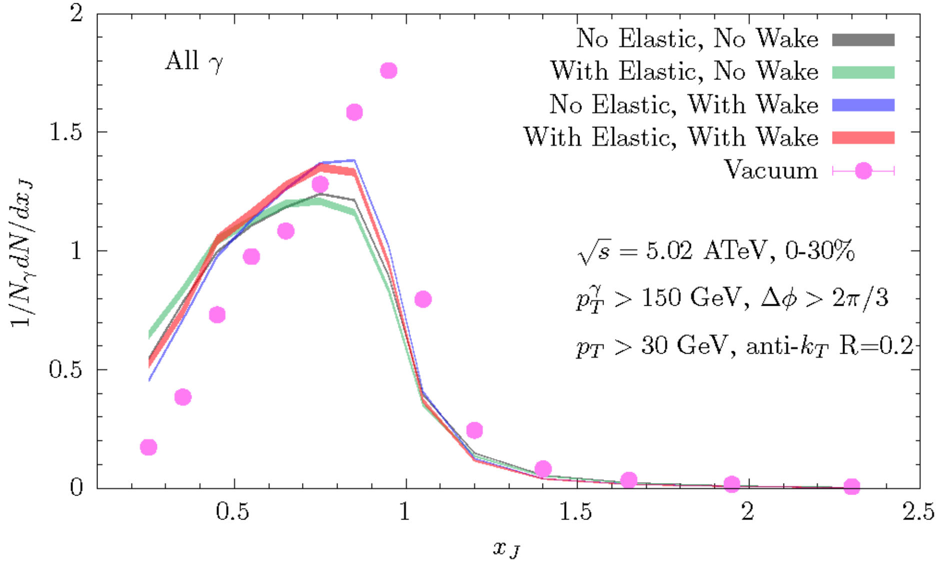
Gamma-Jet Observables with p_T^γ >150 GeV: R_g and Girth, with x_J>0.8



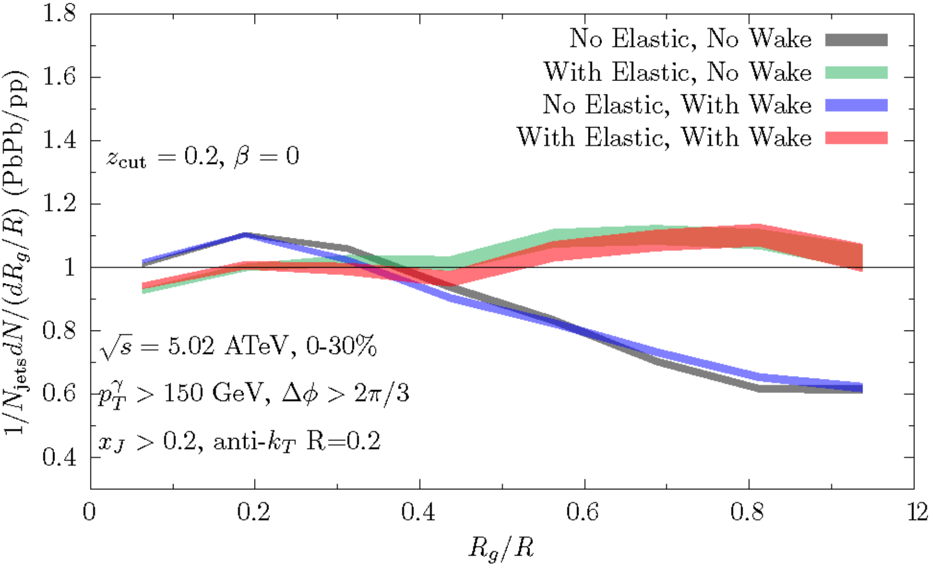


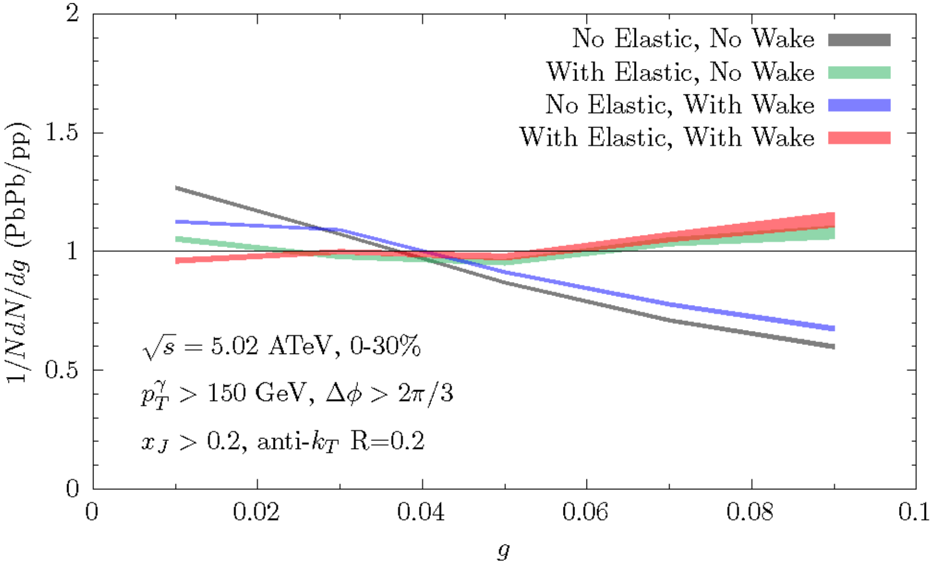
On previous slides, p_T^γ >100 GeV; here, p_T^γ >150 GeV. Means x_J>0.8 corresponds to p_T^{jet}>120 GeV.

Moliere effects substantial; selection bias dominant; wake effects negligible.



Gamma-Jet Observables with p_T^γ >150 GeV: R_g and Girth, with x_J>0.2





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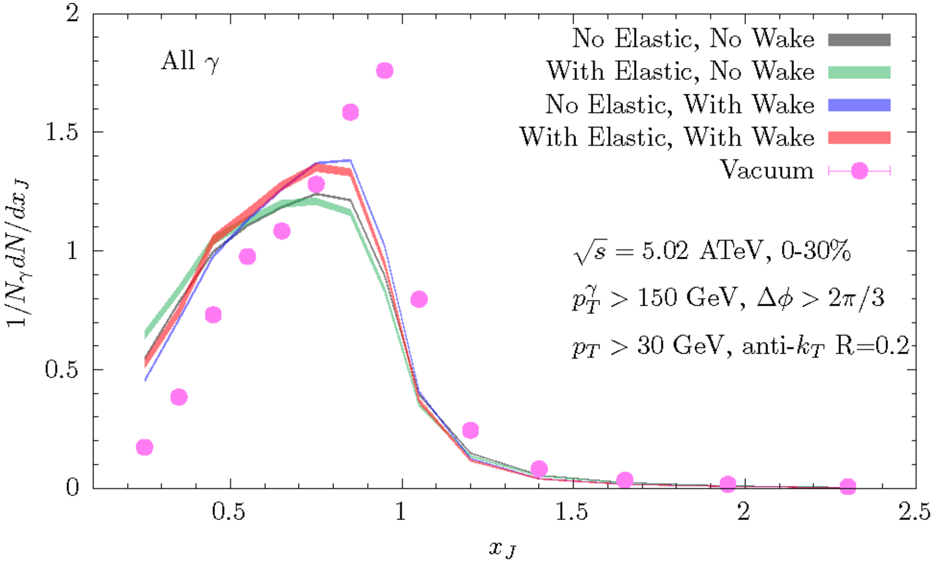
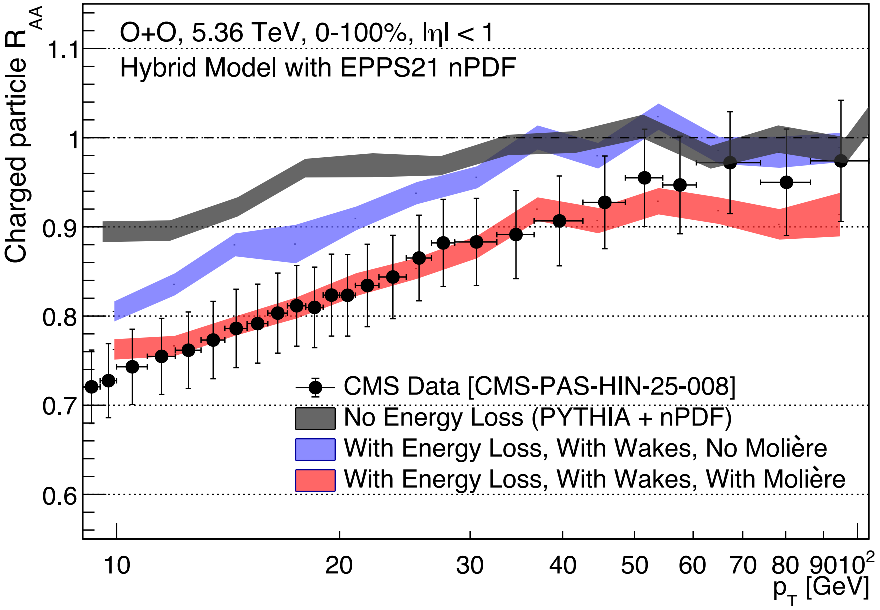


Why Look for Molière Scattering in O+O Collisions

- The challenge in Pb+Pb collisions: how to discern effects of Molière scattering (makes jets sprout an extra prong, and so get wider) given the confounding effect of energy loss + selection bias (jets you select in Pb+Pb are those that lose less energy, which are the narrower jets).
- First O+O collisions @LHC in July 2025. And, the last RHIC run, in February 2026, was an O+O run with sPHENIX collecting 181% of the data they hoped for.
- First show that jets do lose energy in O+O collisions. Then, because they lose *less* energy than in Pb+Pb, see that Molière scattering has distinctive observable consequences!
- "Seeing" such scattering is first step to probing microscopic structure of QGP.

Jets as Probes

Kudinoor. Lin. Pablos. KR, 2603.23596



Earliest hard probes measurements in OO collisions from ALICE, ATLAS, CMS shown in December. sPHENIX and STAR data coming soon — tremendous OO run at RHIC.

Effects of large momentum transfer, $2 \to 2$ elastic scattering — Molière scattering — linear with in-medium path length L, so relatively more important in OO than PbPb.

First hadron $R_{\rm AA}$ and dijet asymmetry data consistent with importance of Molière scattering…
… but this is a model DEpendent comparison.

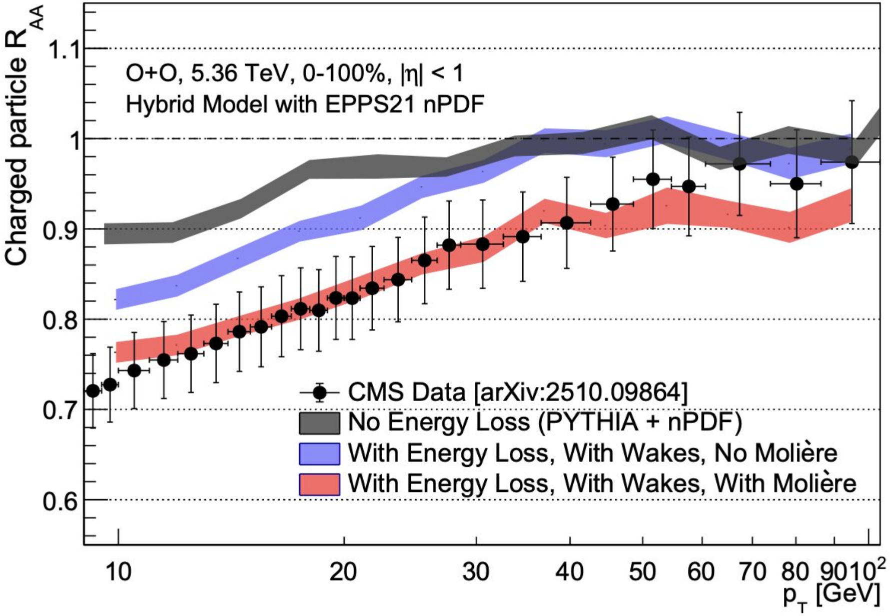
Jet Partons Lose Less Energy in OO Collisions

2603.23596 [Kudinoor, Lin, Pablos, Rajagopal]

Droplets of QGP are smaller in OO than in PbPb.

- Jet partons lose less energy in OO collisions than they do in PbPb collisions. **This means less selection bias.**

- Energy loss due to Molière scattering scales as L; strongly coupled energy loss scales as L^3. **So, Molière scattering is a relatively more important effect in OO. It is needed for the Hybrid Model to explain the CMS OO data!**



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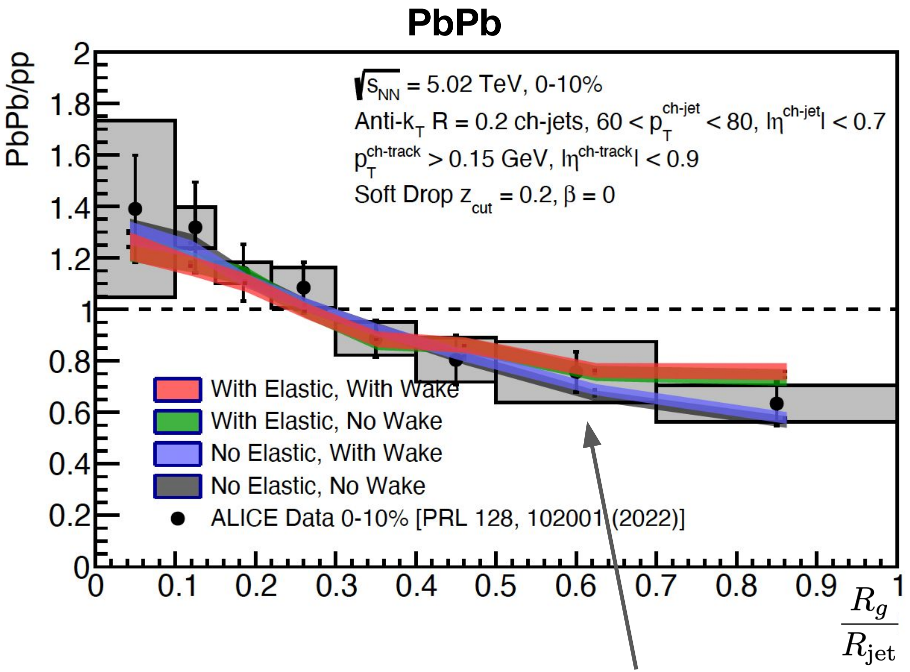
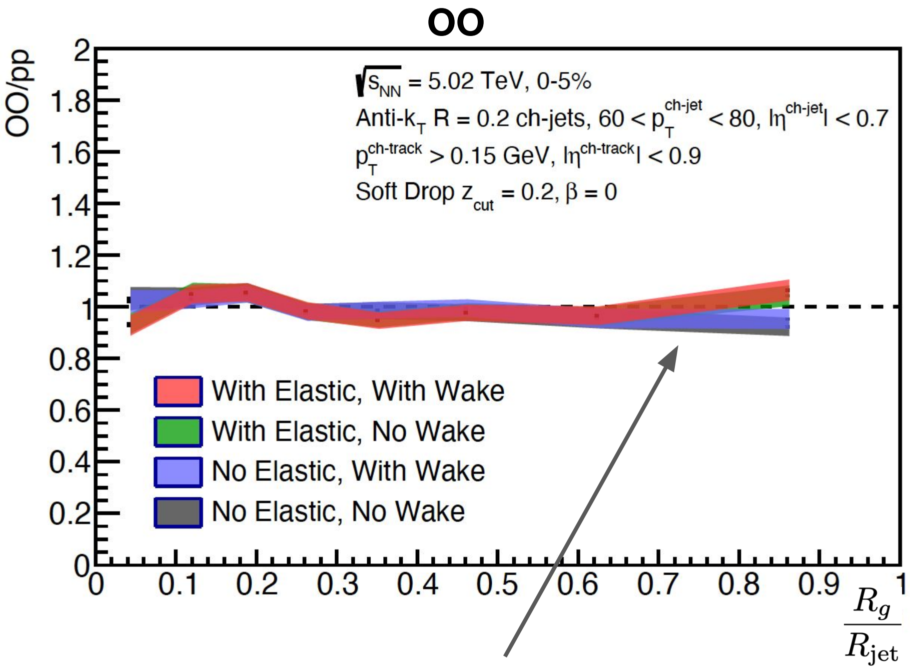
Soft Drop Angle for R = 0.2 Jets





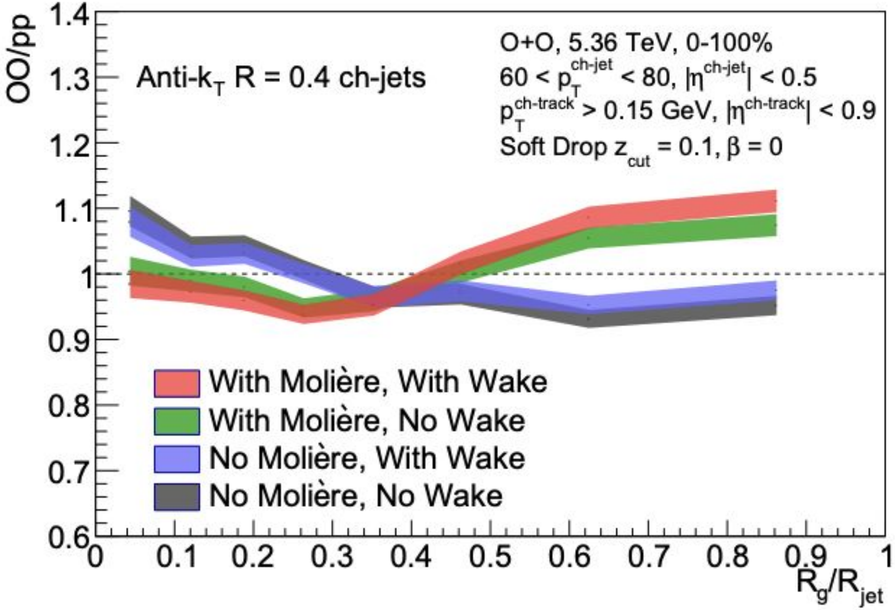
PbPb jet sample is biased towards those that have lost the least energy, which tend to be narrower

Effects of selection bias and jet energy loss are minimal ⇒ Flatter OO/pp ratio

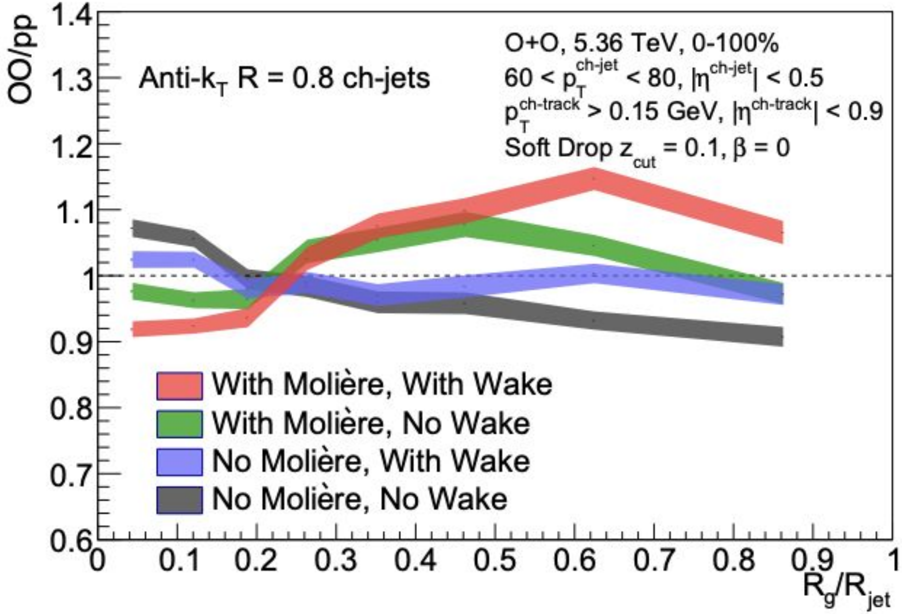
Soft Drop Angle for R = 0.4 and R = 0.8 Jets

2603.23596 [Kudinoor, Lin, Pablos, Rajagopal]

OO Jets with R = 0.4



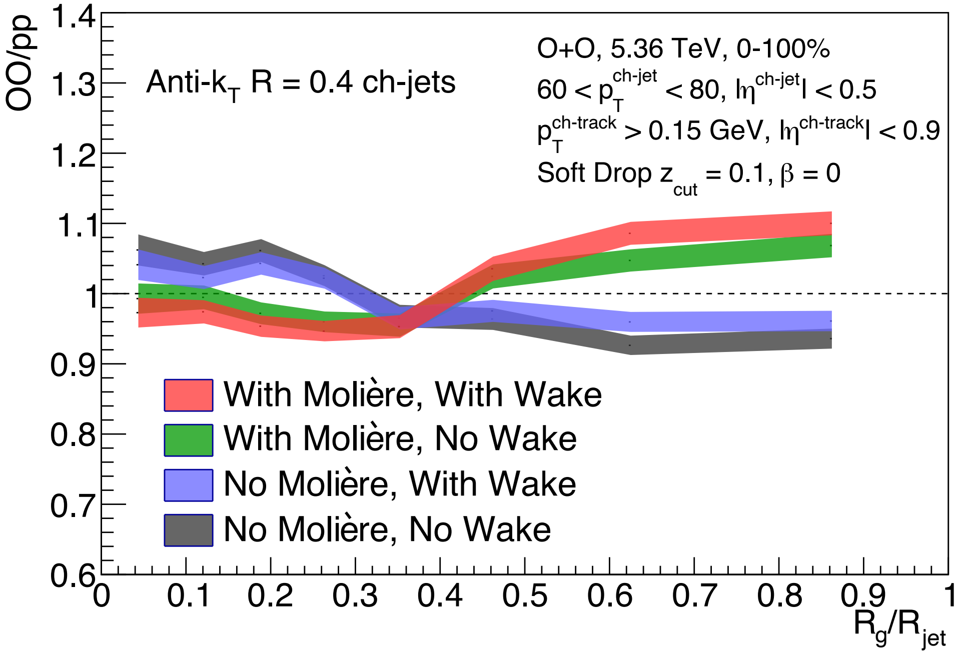
OO Jets with R = 0.8



Distinctive consequence of Molière scatterings:
More jets with $R_g > 0.2$ in OO collisions than in pp collisions

Jets as Probes

Kudinoor, Lin, Pablos, KR, 2603.23596



R_g is a *groomed* measure of angular separation between semi-hard structures within jets. Just what we need.

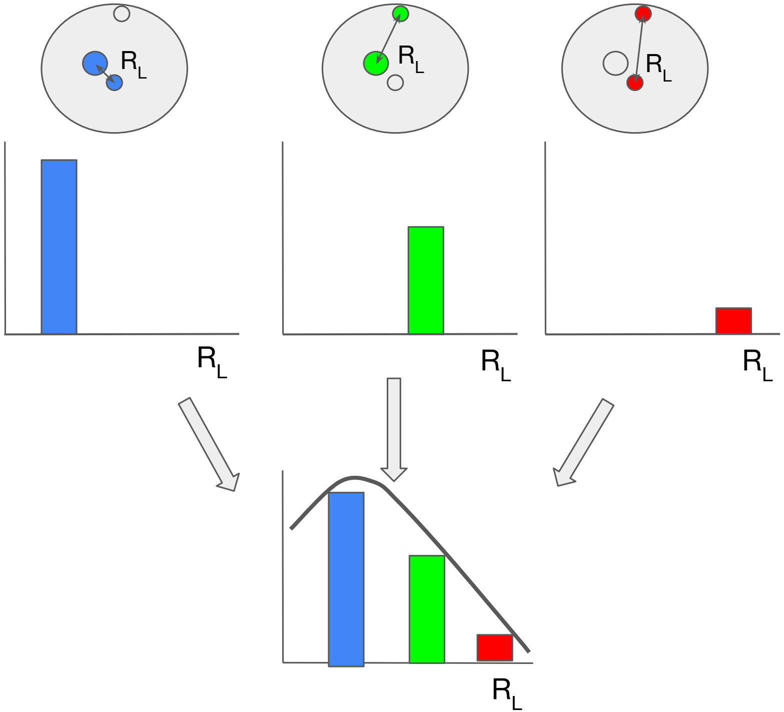
Grooming greatly reduces effects of jet wakes. But, in PbPb, very large confounding effects from selection bias due to energy loss. Not here!

Seeing OO/pp ratio of R_g above unity, in experimental data, would be a distinctive, model-independent, signature of hard scattering of jet partons off QGP.

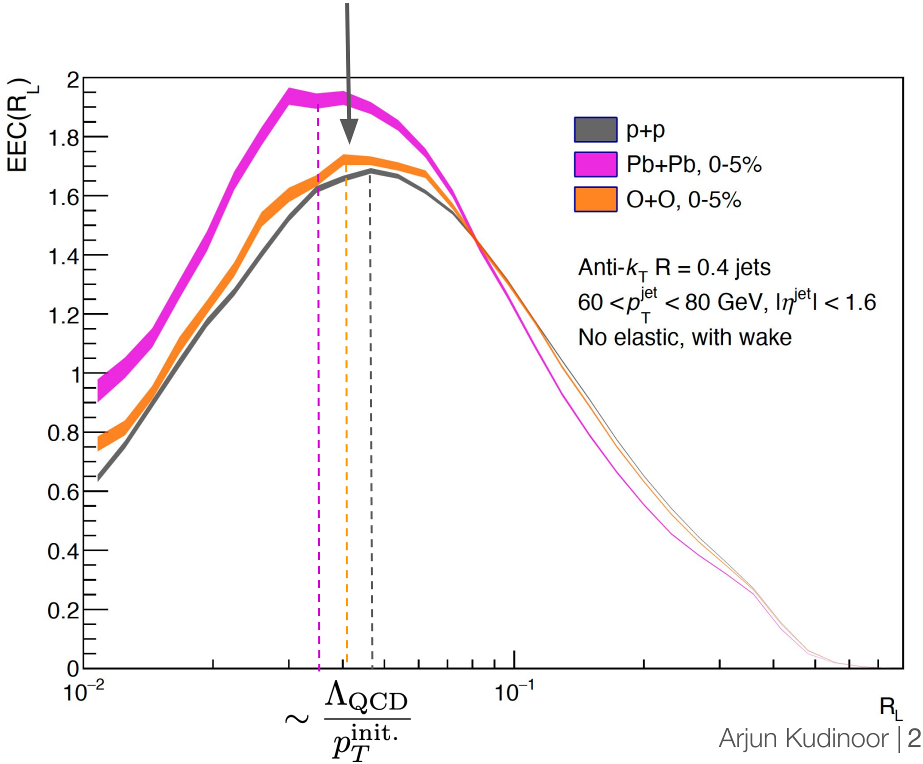
Energy-Energy Correlators

2409.12687 [ALICE]
2503.19993 [CMS]
2409.07514 [Andres et al.]

$$\mathrm{EEC}(R_L) = \sum_{i_1, i_2 \in \mathrm{jet}} \int dR_L \frac{p_T^{i_1} p_T^{i_2}}{p_{T,\mathrm{jet}}^2} \delta(R_L - \Delta\hat{R}_L)$$

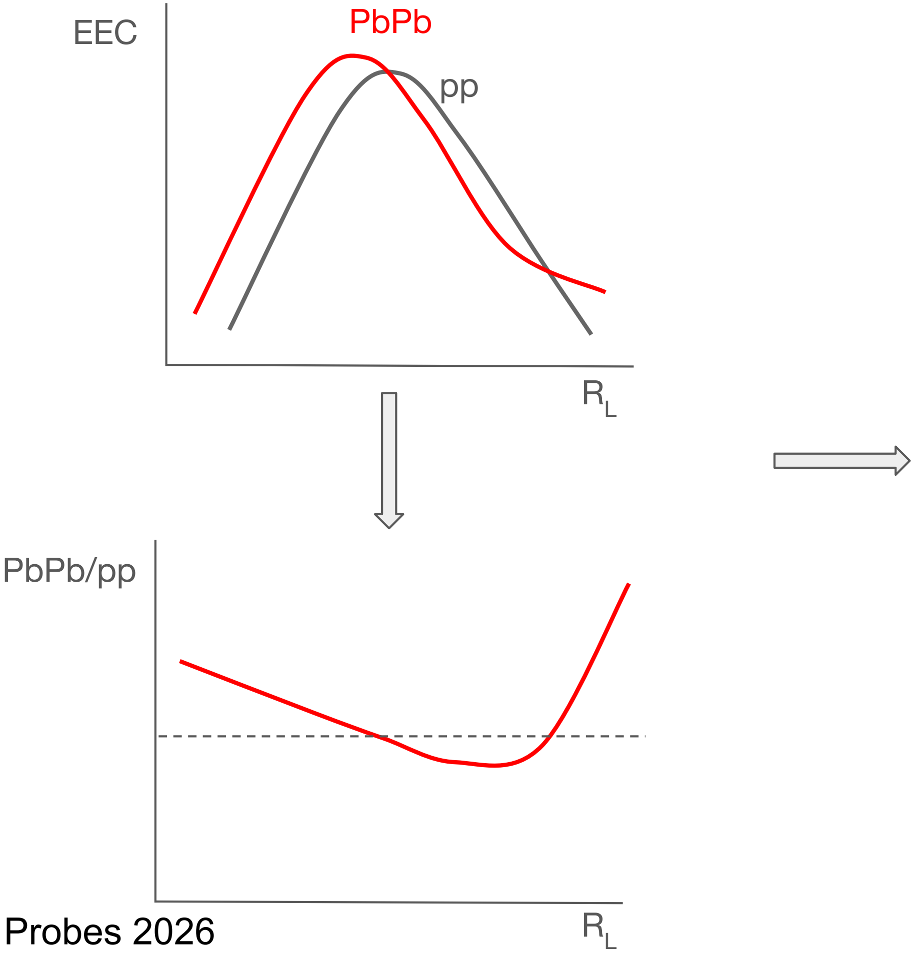


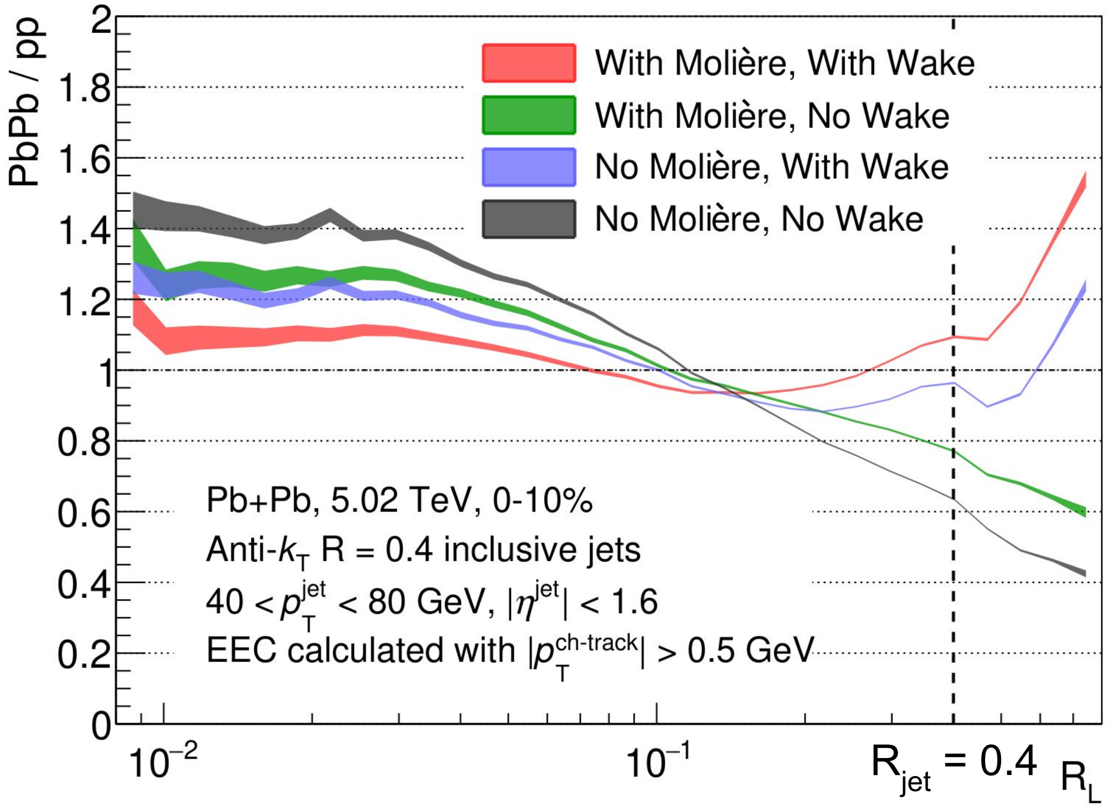
Jet energy loss
⇒ EEC shifts left



EECs in PbPb Collisions

In PbPb, wake dominates large angle correlations
It is difficult to disentangle Molière scattering from the wake

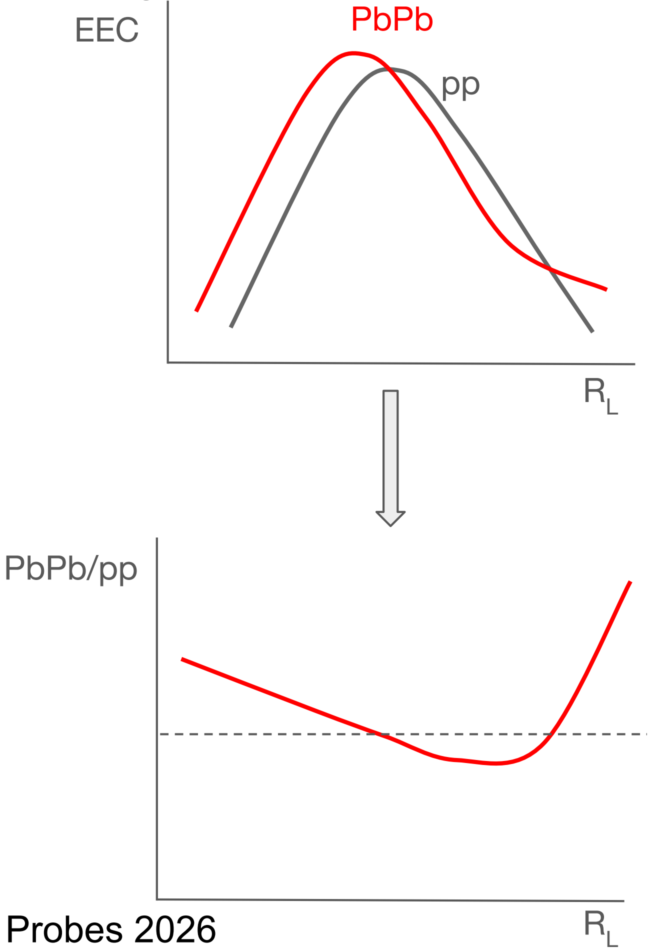




Why OO? EECs in PbPb Collisions

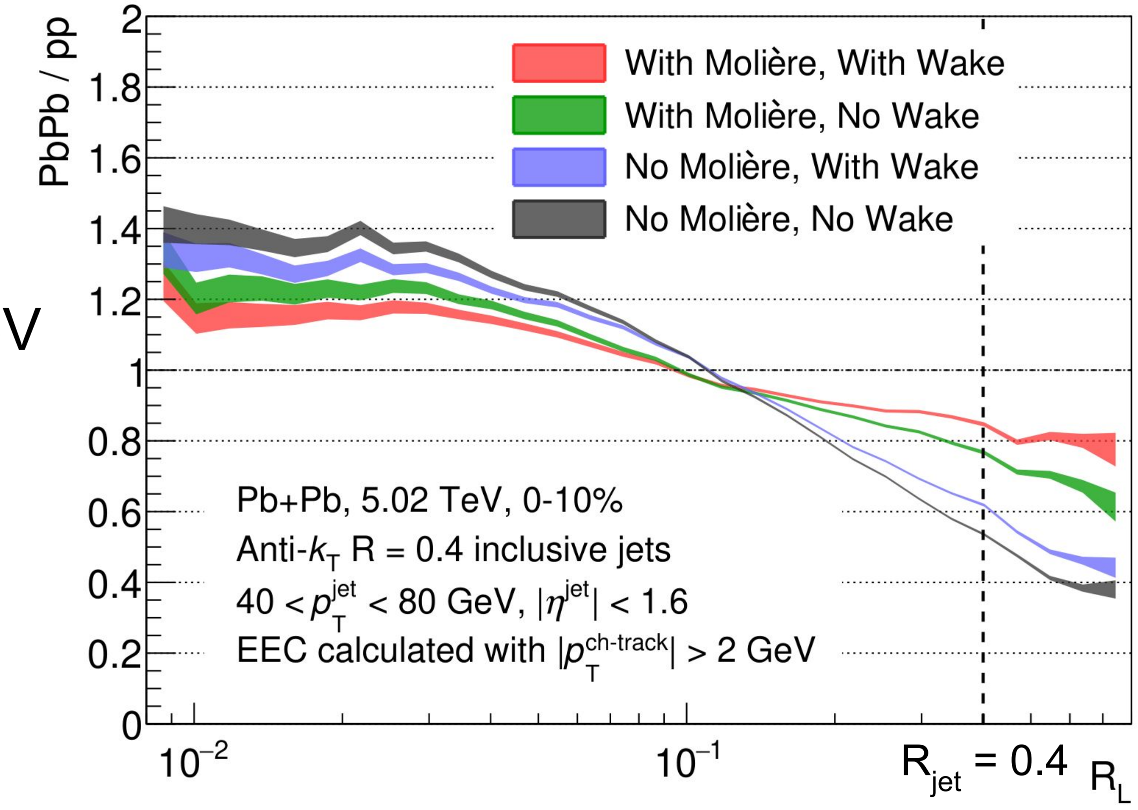
Apply track cut $p_T > 2$ GeV to remove the soft wake and isolate Molière scattering

However, selection bias in PbPb competes with Molière scattering, unambiguous experimental evidence of Molière scattering is hard to see



Track cut

$p_T > 2$ GeV

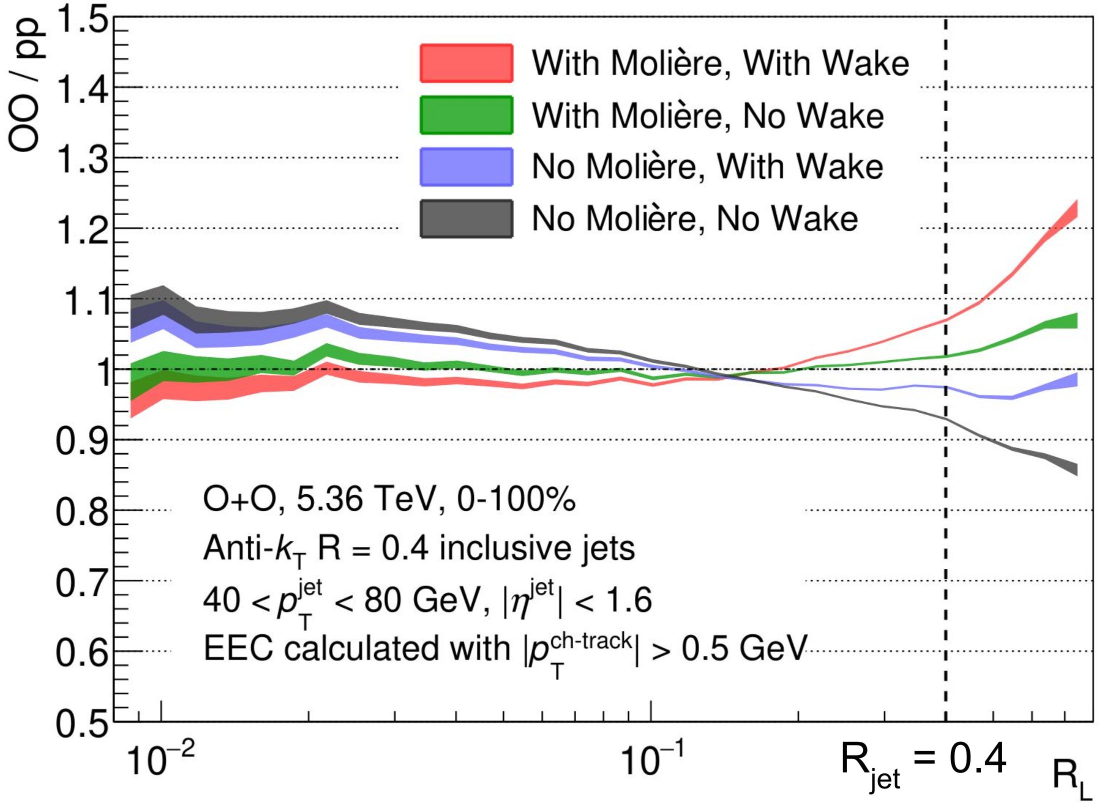


Charged Hadron EECs of R = 0.4 Jets

Jet reconstruction & EEC calculation scheme: [Phys.Lett.B 866 (2025) 139556]

OO/pp ratio shows relative increase/decrease of EEC due to medium effects

- Reduced selection bias ⇒ OO/pp ratios of EEC go above one at large R_L

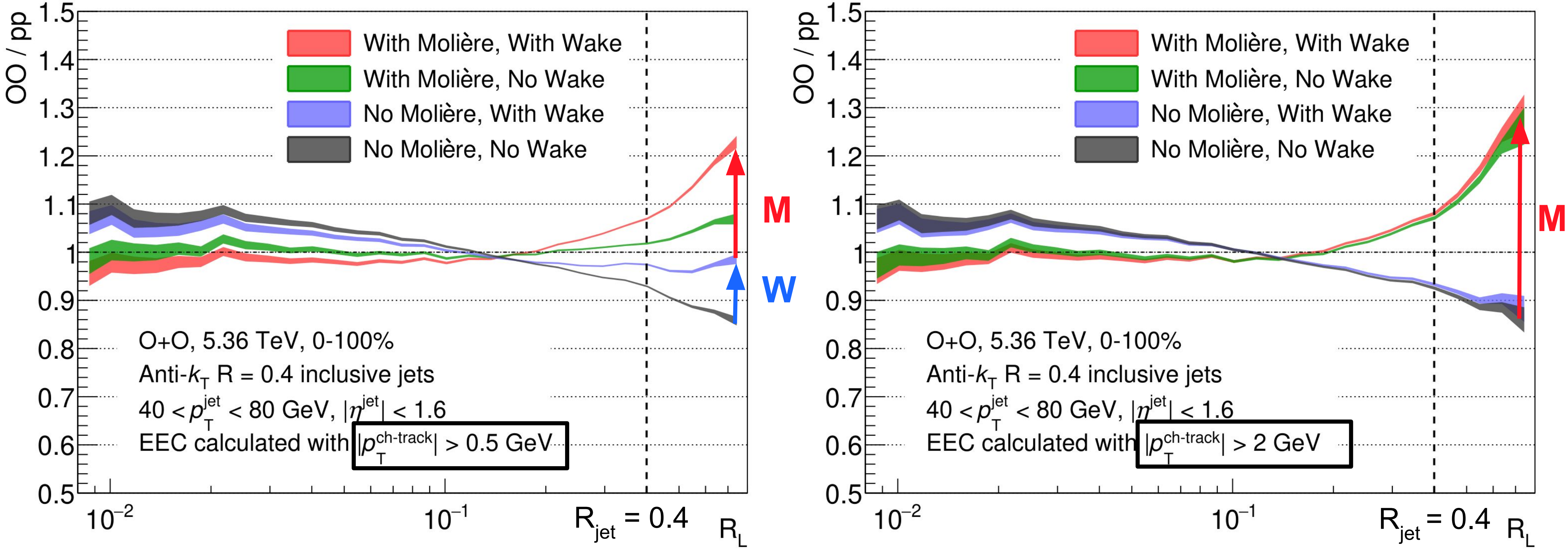


2603.23596 [Kudinoor, AL, Pablos, Rajagopal]

Charged Hadron EECs of R = 0.4 Jets

W: Wake ; **M:Molière**

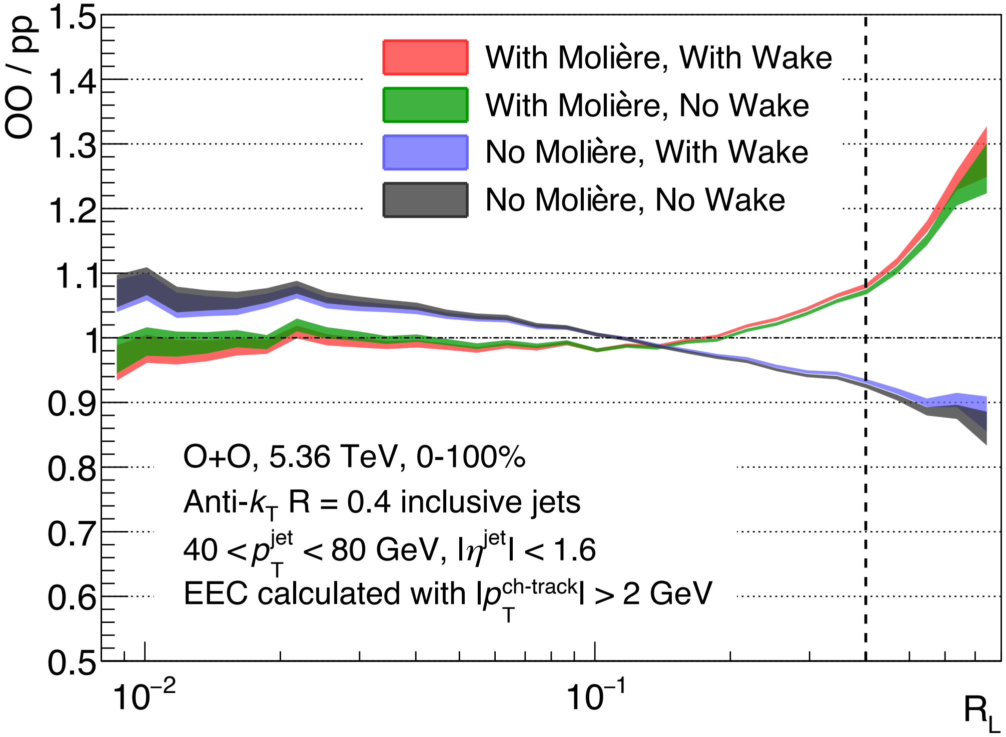
- Reduced selection bias ⇒ OO/pp ratios of EEC go above one at large R_L
- Large angle contribution of Molière scattering and wake are of comparable magnitude
- Imposing ch-track cut p_T > 2 GeV removes wake particles



[2603.23596](https://arxiv.org/abs/2603.23596) [Kudinoor, AL, Pablos, Rajagopal]

Jets as Probes

Kudinoor, Lin, Pablos, KR, 2603.23596



Energy-energy correlator (EEC) is also well-suited for the task — *if* "groomed" by imposing a track cut, $p_T^{\text{track}} > 2$ GeV.

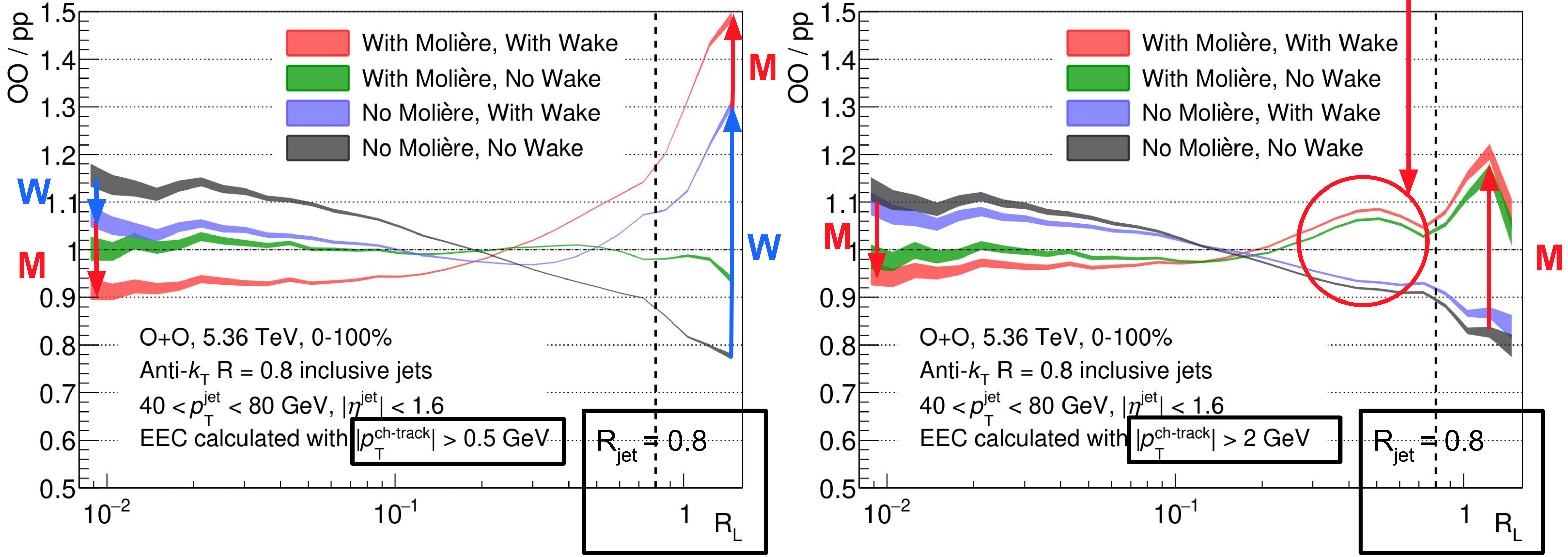
Little effect of jet wakes. Confounding effects of energy loss (smaller in OO) overwhelmed by effects of hard scattering.

Seeing OO/pp ratio of EEC above unity, in experimental data, would be a distinctive, model-independent, signature of hard scattering of jet partons off QGP.

Charged Hadron EECs of R = 0.8 Jets

W: Wake ; **M:Molière**

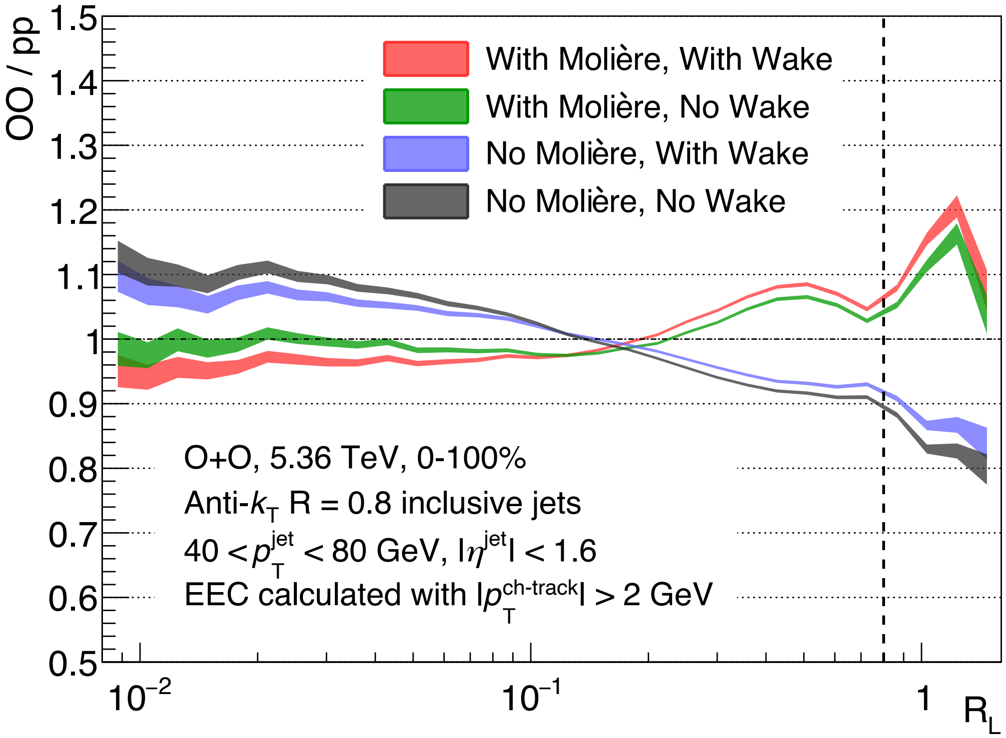
- Small angle: Molière scattering broaden the jet core ⇒ EEC ratio is suppressed to below 1
- Large angle: hard track cut **reveals Molière scattering as a bump**
- $R_L > R_{jet}$: excludes jet core, EEC is dominated by wake–wake correlation



2603.23596 [Kudinoor, AL, Pablos, Rajagopal]

Jets as Probes

Kudinoor, Lin, Pablos, KR, 2603.23596



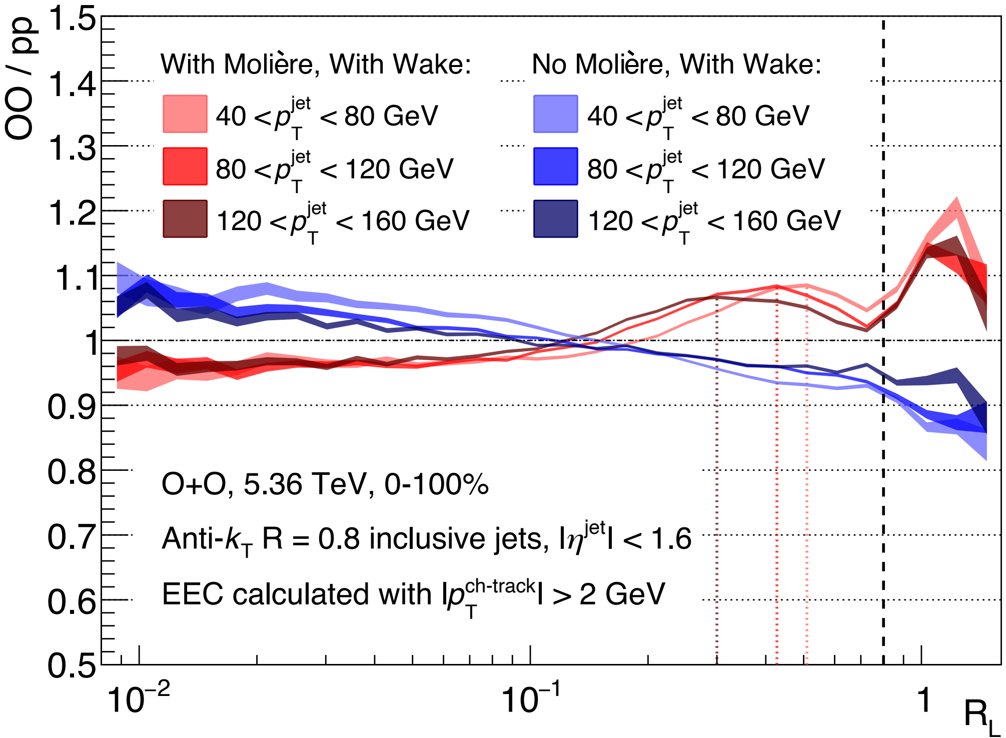
Energy-energy correlator (EEC) is also well-suited for the task — *if* "groomed" by imposing a track cut, $p_T^{\text{track}} > 2$ GeV.

In OO collisions, can make these measurements in wider jets with $R_{\text{jet}} = 0.8$. Reveals a "bump" at $R_L \sim 0.5$!

Is this the angular scale of deflections due to Molière scattering? If so, seeing this bump in the OO/pp EEC ratio in experimental data, would be of exceptional interest!

Jets as Probes

Kudinoor, Lin, Pablos, KR, 2603.23596



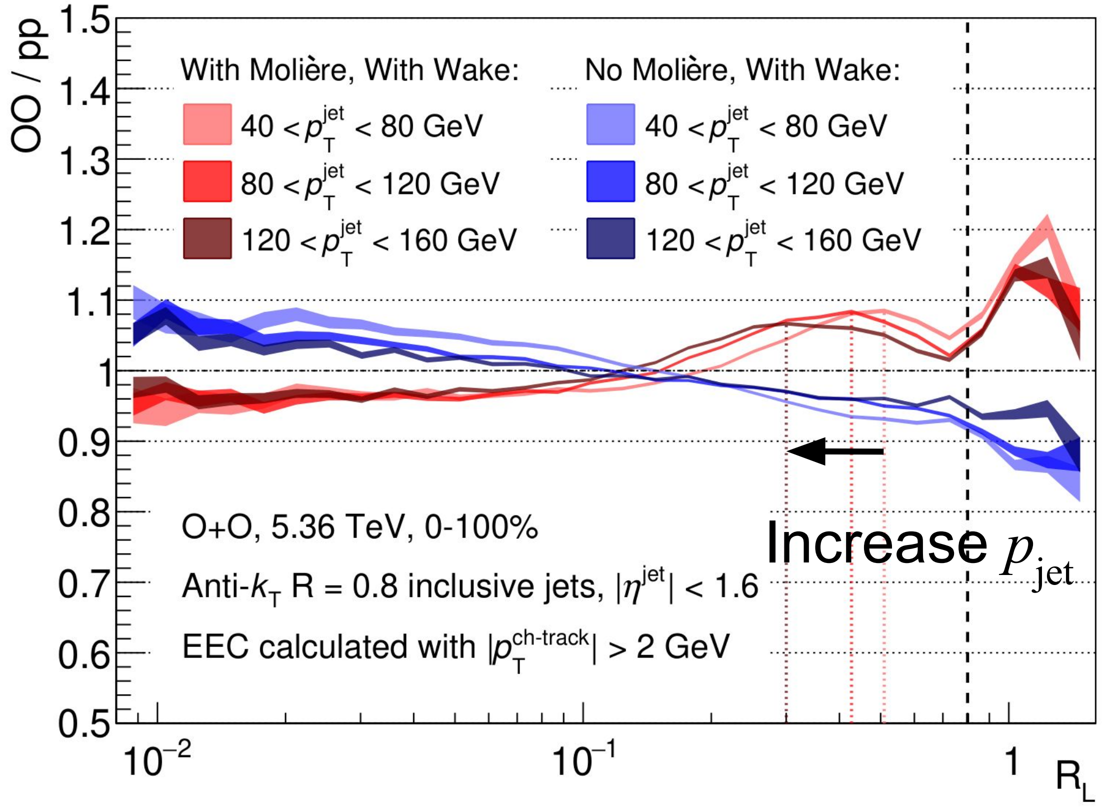
The bump moves toward smaller R_L when you increase jet p_T — what you expect if this is the angular scale of hard scattering of jet partons off QGP quasiparticles!

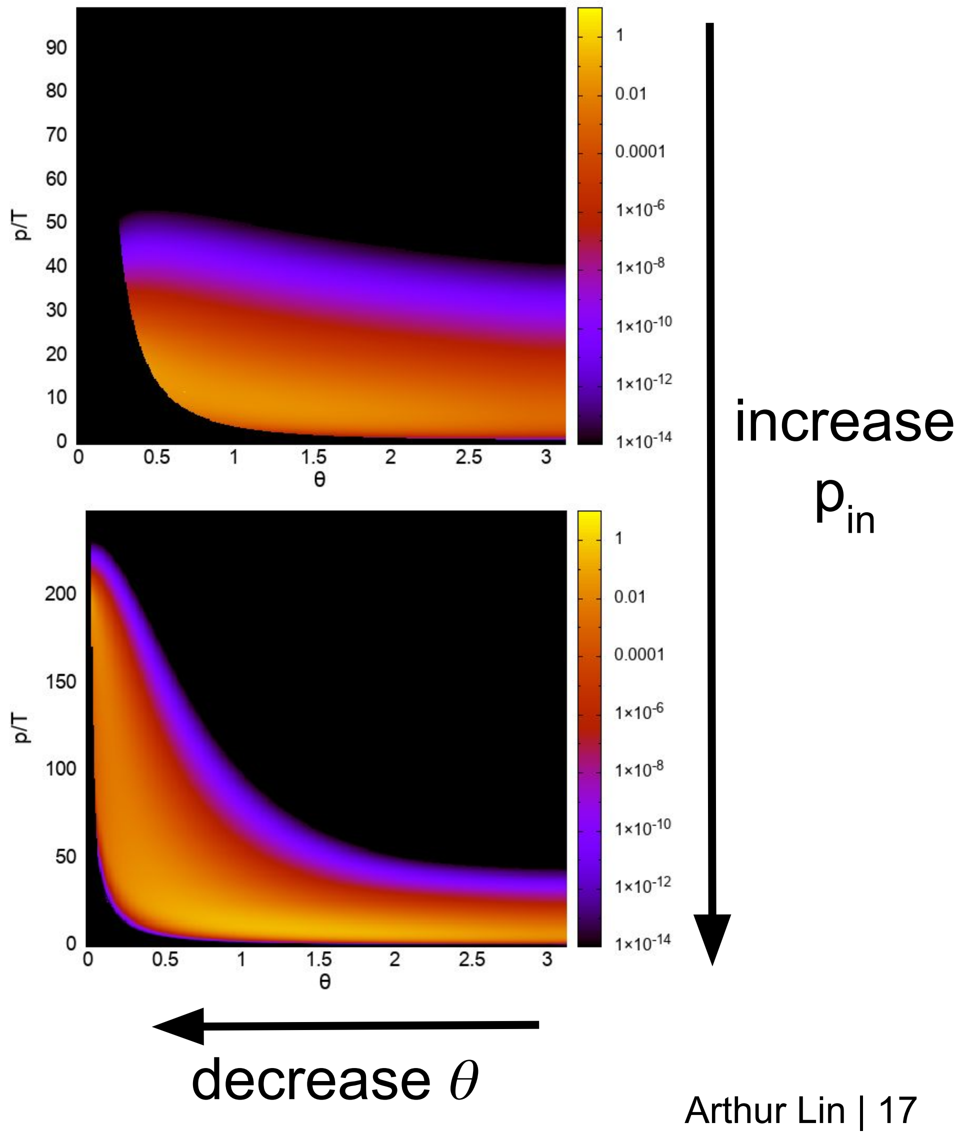
Seeing this bump in the OO/pp EEC ratio in experimental data, at an $0.2 < R_L < R_{\text{jet}} \sim 0.8$, and seeing its jet-$p_T$-dependence, would be a *third* distinctive, model-independent, signature of hard scattering of jet partons off QGP.

Bump on EEC: Typical Angle of Molière Scattering

Bump moves to the left as jet momentum increases
Experimental measurement of this feature would be solid evidence of Molière scattering!

arXiv: 2603.08776 [Hulcher et. al]





2603.23596 [Kudinoor, AL, Pablos, Rajagopal]

Jets as Probes

- **Goal in sight! See microscopic particulate structure of liquid QGP at short length scales; see jet partons scatter off QGP quasiparticles. To get there we need . . .**
- **Measurement of jet substructure observables in OO and PbPb from both RHIC and LHC. If QGP@RHIC more strongly coupled, hard scattering of jet partons off QGP quasiparticles should be less apparent.**
- **OO collisions may be perfect arena for seeing hard scattering of jet partons off QGP. PbPb collisions** *are* **perfect arena for studying jet wakes, the response of the QGP.**
- **Luminosity in LHC Runs 4&5. Both PbPb and OO. 10x the current OO statistics will make a substantial impact.**
- **Advances in both theory and modeling.**
- **Need Bayesian analyses of jet and heavy flavor data together, to constrain parameters governing jet quenching, resolution,** *scattering off QGP quasiparticles, microscopic properties of QGP*, **drag, diffusion, hadronization.**

Jets as Probes of QGP

- Model calculations are enabling key steps...
- Disentangling jet modification from jet selection.
- Calculations of the dynamics of jet wakes in droplets of QGP, identification of new experimental observables, and predictions that will enable experimental measurements to “see” the particles coming from these wakes.
- Showing that hot quark soup (QGP) *can* respond to substructure within jets.
- Identifying those jet substructure observables that *are* sensitive to scattering of jet quarks/gluons off QGP quarks/gluons, “seeing” the latter à la Rutherford, and are *not* sensitive to particles coming from the wake.
- Next several years will be the golden age of jet physics: sPHENIX, LHC runs 3 and 4, new substructure observables. *Many* theory advances, and analyses of today’s data, whet our appetite for the feast to come.
- We shall learn about the microscopic structure of QGP, and the dynamics of rippling QGP.

Jets as Probes of QGP

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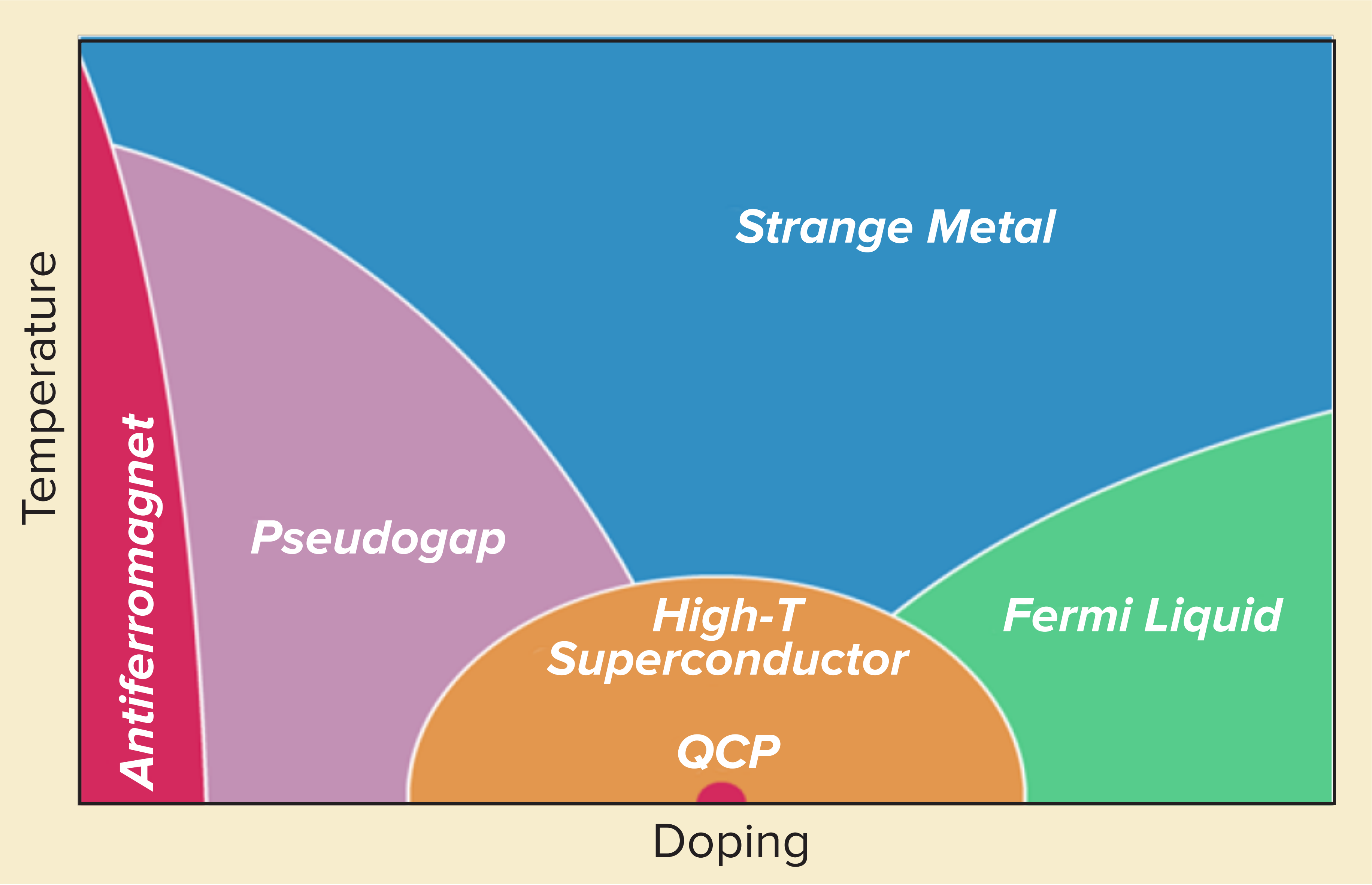
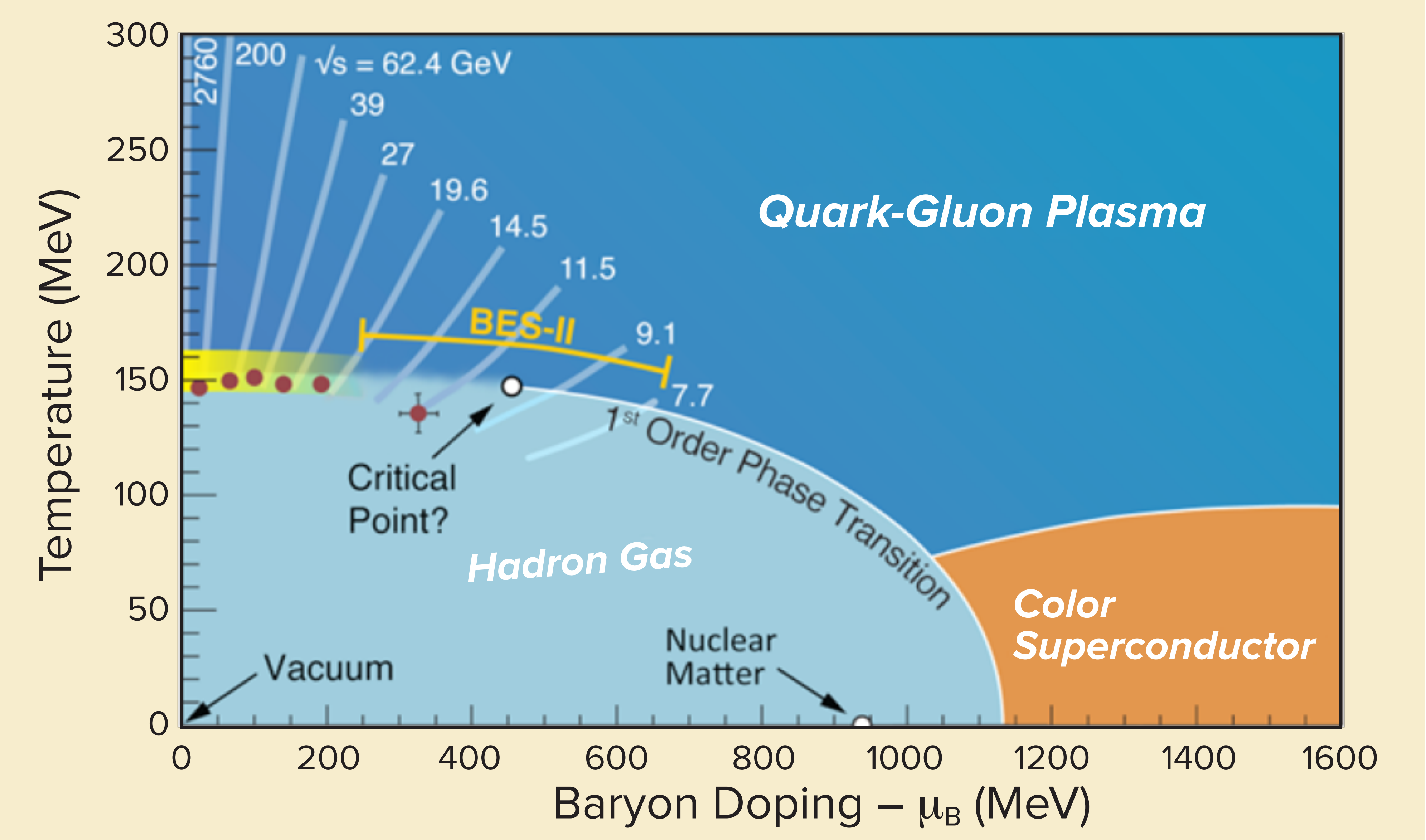
What Next?

Two kinds of What Next? questions for the coming decade...

- A question that one asks after the discovery of any new form of complex matter: What is its phase diagram? For high temperature superconductors, for example, phase diagram as a function of temperature and doping. Same here! For us, doping means excess of quarks over antiquarks, rather than an excess of holes over electrons.
- A question that we are privileged to have a chance to address, after the discovery of “our” new form of complex matter, which is so close to the fundamentals: How does the strongly coupled liquid emerge from laws governing quarks and gluons? Maybe answering this question could help to understand how strongly coupled matter emerges in other contexts.


Temperature
Antiferromagnet
Pseudogap
Strange Metal
High-T
Superconductor
QCP
Fermi Liquid
Doping



300
250
200
150
100
50
0
Temperature (MeV)
2760
200
√s = 62.4 GeV
39
27
19.6
14.5
11.5
BES-II
9.1
7.7
Quark-Gluon Plasma
1st Order Phase Transition
Critical Point?
Hadron Gas
Color Superconductor
Nuclear Matter
Vacuum
0
200
400
600
800
1000
1200
1400
1600
Baryon Doping – μB (MeV)

Search for a Possible Critical Point

- How does the thermodynamics of the transition from QGP to hadronic matter as it cools change as QGP is doped with an excess of quarks over antiquarks? Is there a critical point in the region of the QCD phase diagram that heavy ion collisions can explore, or do all collisions that make QGP only explore a crossover in the phase diagram?
- This remains a big question, but we are *much* closer to answering it than we were a few years ago.
- Now know, from STAR data and from lattice QCD calculations, that for $\mu_B \lesssim 400 - 450$ MeV, there is a crossover, no critical point.
- STAR fixed target collision data coming soon; CBM data coming later this decade; will explore up to $\mu_B \sim 600$ MeV. Heavy ion collisions likely do not reach the crossover/transition beyond this μ_B — we are close to answering the question as posed above.

Search for a Possible Critical Point

- I am going to start with slides pulled from talks that I gave in 2008 at an INT workshop, at the Critical Point and Onset of Deconfinement (CPOD) 2009 conference, to the STAR collaboration in 2009, and at the KITP in 2009.

- This will give you a good sense of what we knew, and were looking for, as we set out to search for the QCD critical point at RHIC.

- I will then show slides from talks (by others, including from STAR) from the CPOD 2024 and CPOD 2026 conferences.

THE SEARCH FOR THE QCD CRITICAL POINT

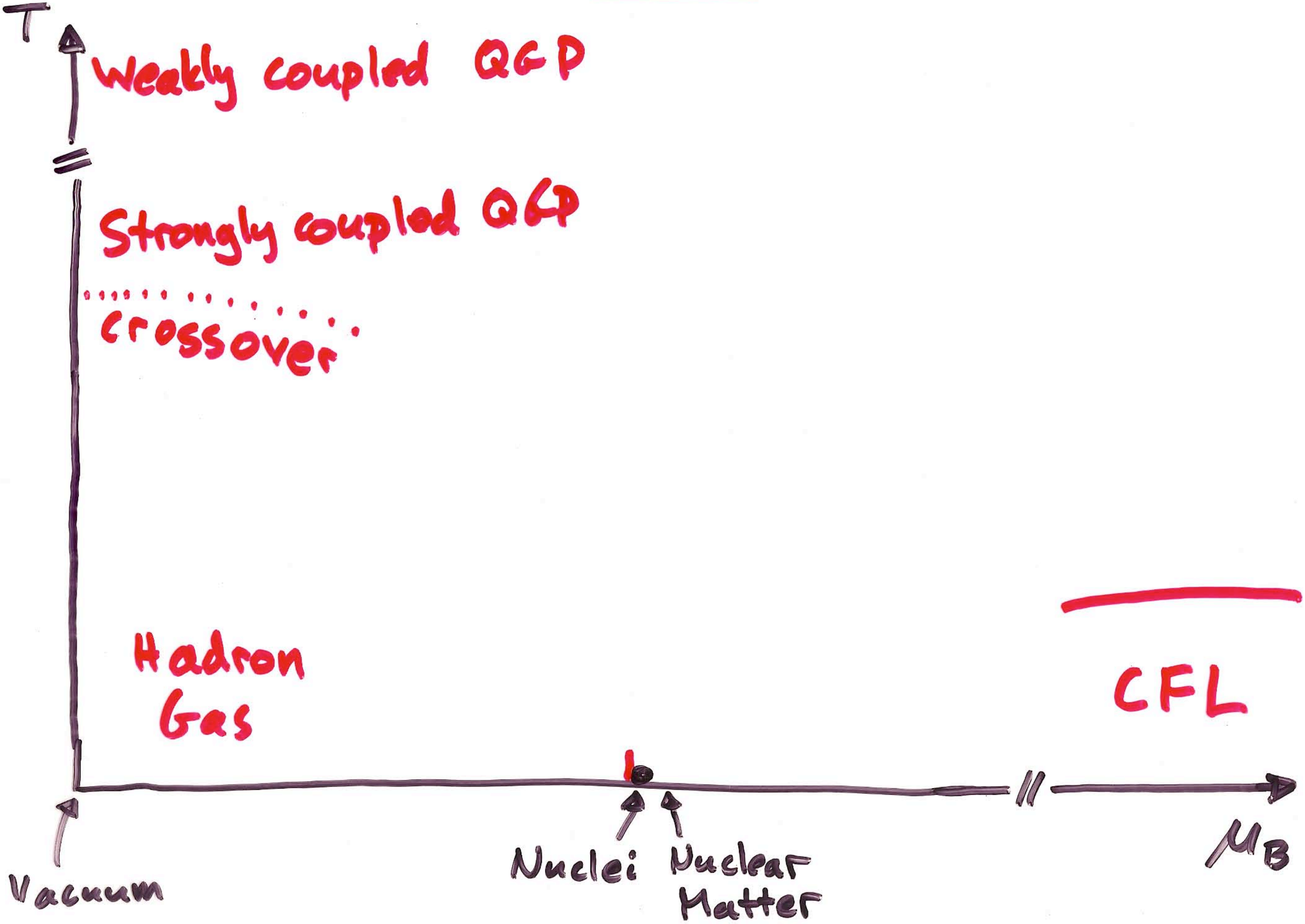
USING LATTICE QCD CALCULATIONS

AND HEAVY ION COLLISION EXPERIMENTS

KRISHNA RAJAGOPAL
(MIT)

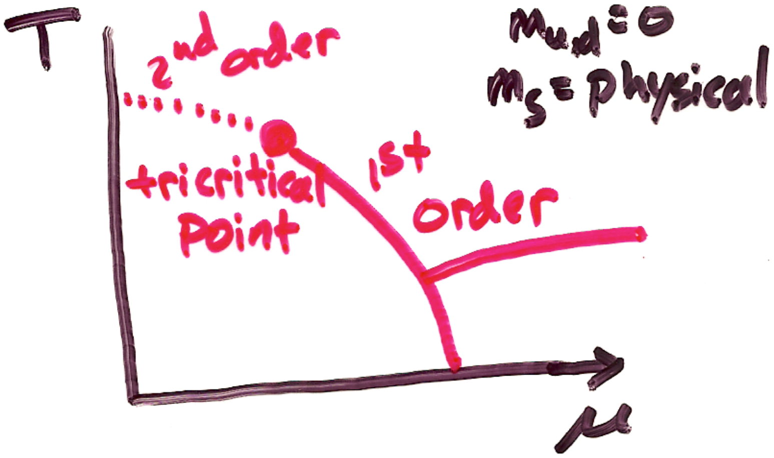
INT, Seattle. 8/11/08

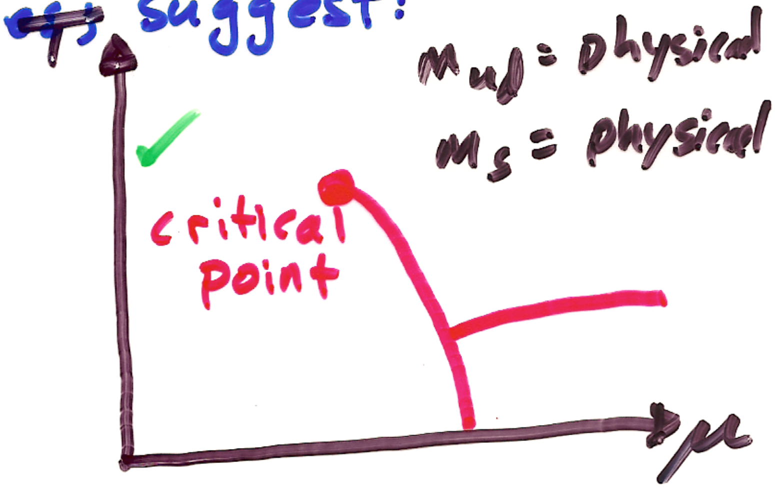
WHAT WE KNOW, SO FAR

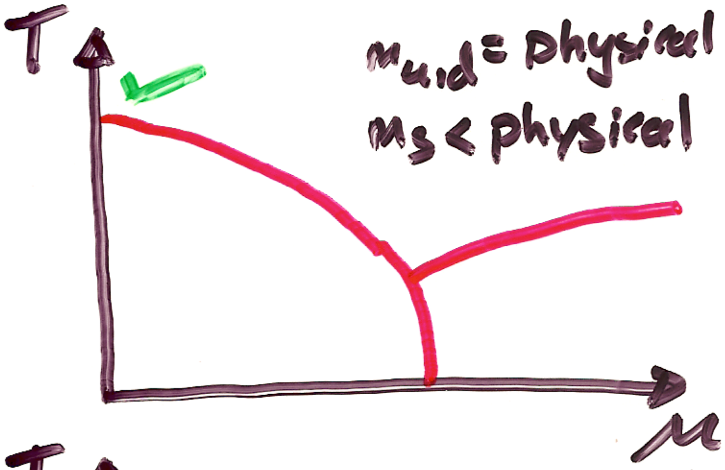


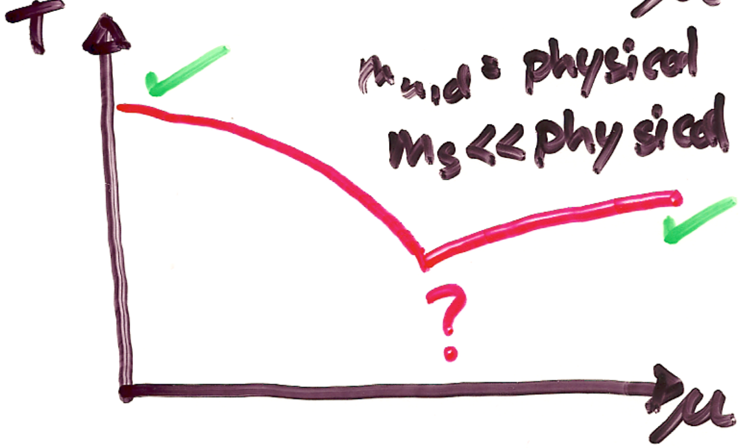
WHY EXPECT A CRITICAL POINT?

- Models; lattice QCD calculations at $\mu=0$ with varying quark masses, suggest:







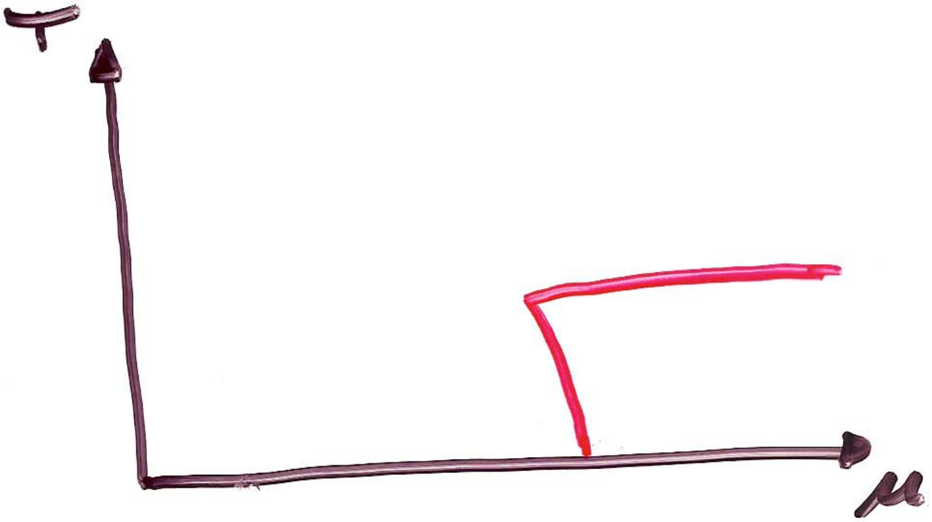


✓ : known

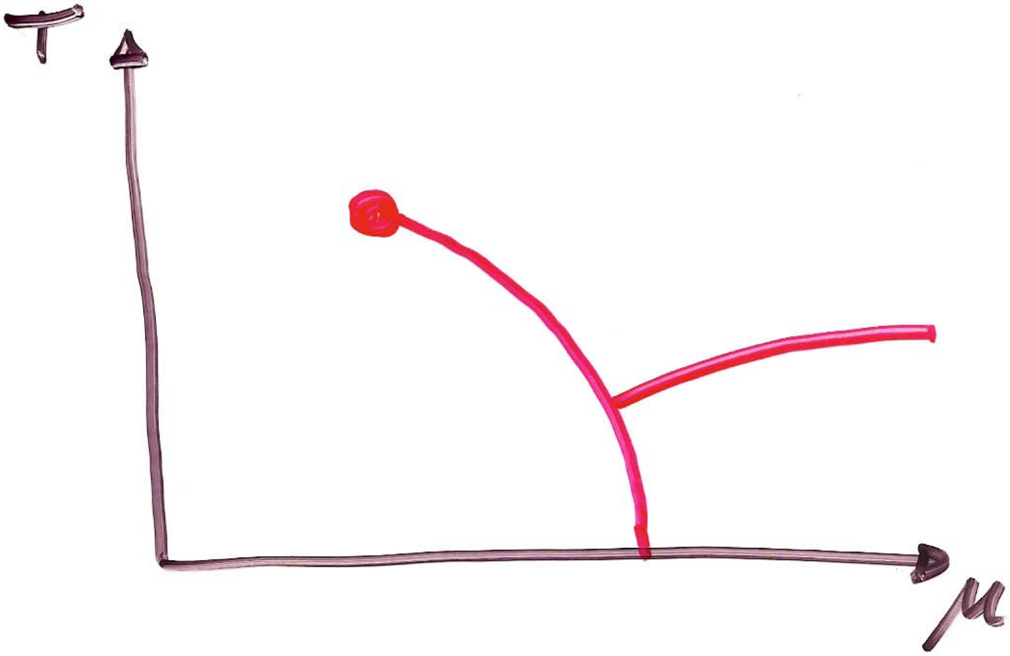
- Universality class of the QCD critical point is known. (ISING)
- Experiments, and lattice calculations with $T\neq 0, \mu\neq 0$, needed to locate it.

WARNING

Nothing we know precludes pushing the critical point so far to the right that:



although models and some lattice calculations favor



LOCATING THE CRITICAL POINT....

... either via lattice calculations, or via detection of its signatures in heavy ion collision experiments,

would add a point and a line to the known QCD phase diagram.

- A qualitative leap in our understanding of QCD in the interior of its phase diagram, currently terra incognita.
- An opportunity for RHIC to write another new chapter in any future book on QCD.

LATTICE RESULTS

- via reweighting (Fodor & Katz)

  $\mu_0 = 360 \pm 40\,(\text{stat})\ \text{MeV}$

- via calculating $dm_c/d\mu$ (de Forcrand + Philipsen)

  $\frac{\mu_0}{T_0} > 3 \rightarrow \mu_0 > 500\ \text{MeV}$

- via radius of convergence of Taylor expansion

  $\frac{\mu_0}{T_c(\mu=0)} = 1.7 \pm .1\,(\text{stat})$ (Gavai + Gupta)

  $\rightarrow 250 < \mu_0 < 400\ \text{MeV}$

  (with a "very naive" estimate of systematics)

  $\mu_0 / T_c(\mu=0) > 1.5$ (RBC- Bielefeld)

- STILL SYSTEMATICS DOMINATED

  Nevertheless,

ONE CLEAR LESSON

Lattice calculations provide strong indications, via <u>all</u> algorithms employed to date, that:

$$\boxed{\mu_0 > 200\ \text{MeV}}$$

LOCATING THE CRITICAL POINT

Location still uncertain:

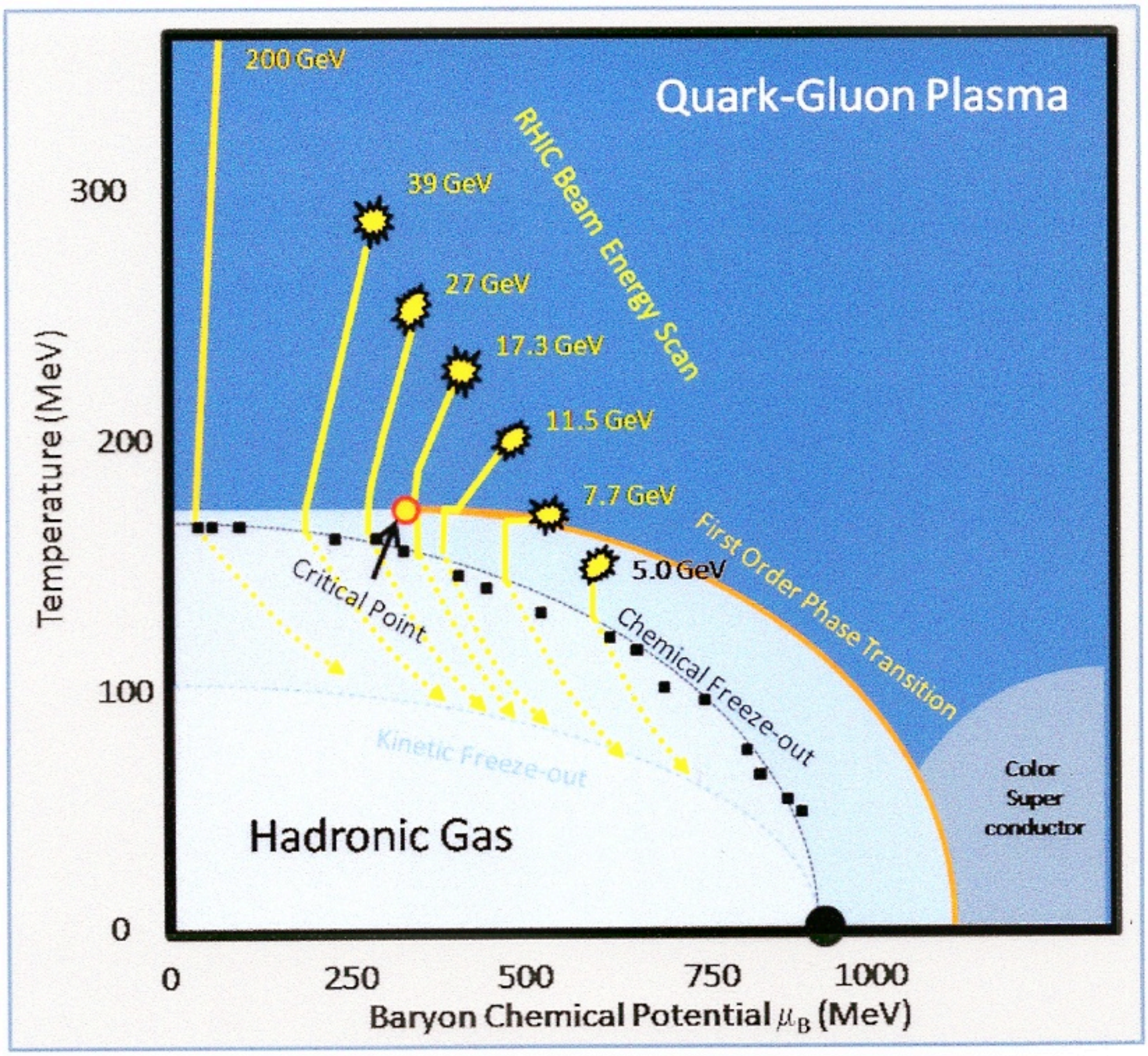
$$\frac{\mu_B^{\text{critical point}}}{T_c(\mu=0)} \sim \underset{\substack{\text{Fodor}\\ \text{Katz}}}{2}\,,\ \underset{\substack{\text{Philipsen}\\ \text{deForcrand}}}{>\mathcal{O}(3)}\,,\ \underset{\substack{\text{Gavai}\\ \text{Gupta}}}{\sim 1.7}\,,\ \overset{\substack{\text{RBC}\\ \text{-BI}\\ \downarrow}}{\gtrsim 2}$$

- gaining confidence will require "crawling towards the continuum limit", and several methods agreeing.
- If $\mu_B^{c.p.} < 3T$, this ↑ will happen
- If $\mu_B^{c.p.} > 3T$, all methods should come to agree on this. But, barring an unforeseen algorithmic breakthrough, unlikely that lattice calculations will locate it with confidence.

In the race between lattice calculations and experimental searches to locate the critical point, the lattice team is running strongly but not yet threatening to end the race.

So, lets turn to experimental searches....

SEARCHING FOR THE CRITICAL POINT



Decreasing $\sqrt{s}$: decreases T and increases μ at which collision equilibrates, "landing on the phase diagram".

⇒ increases μ at which the trajectory followed by the cooling plasma crosses the transition or crossover.

NB: location of ● in fig. is merely illustrative – we don't know where ● is!

SIGNATURES OF THE CRITICAL POINT

In those collisions that pass near the critical point as they cool, find long wavelength oscillations of a mode that is a linear combination of σ (ie fluctuations couple to $\pi\pi$ and pp) and baryon number.

Fujii Ohtani; Son Stephanov

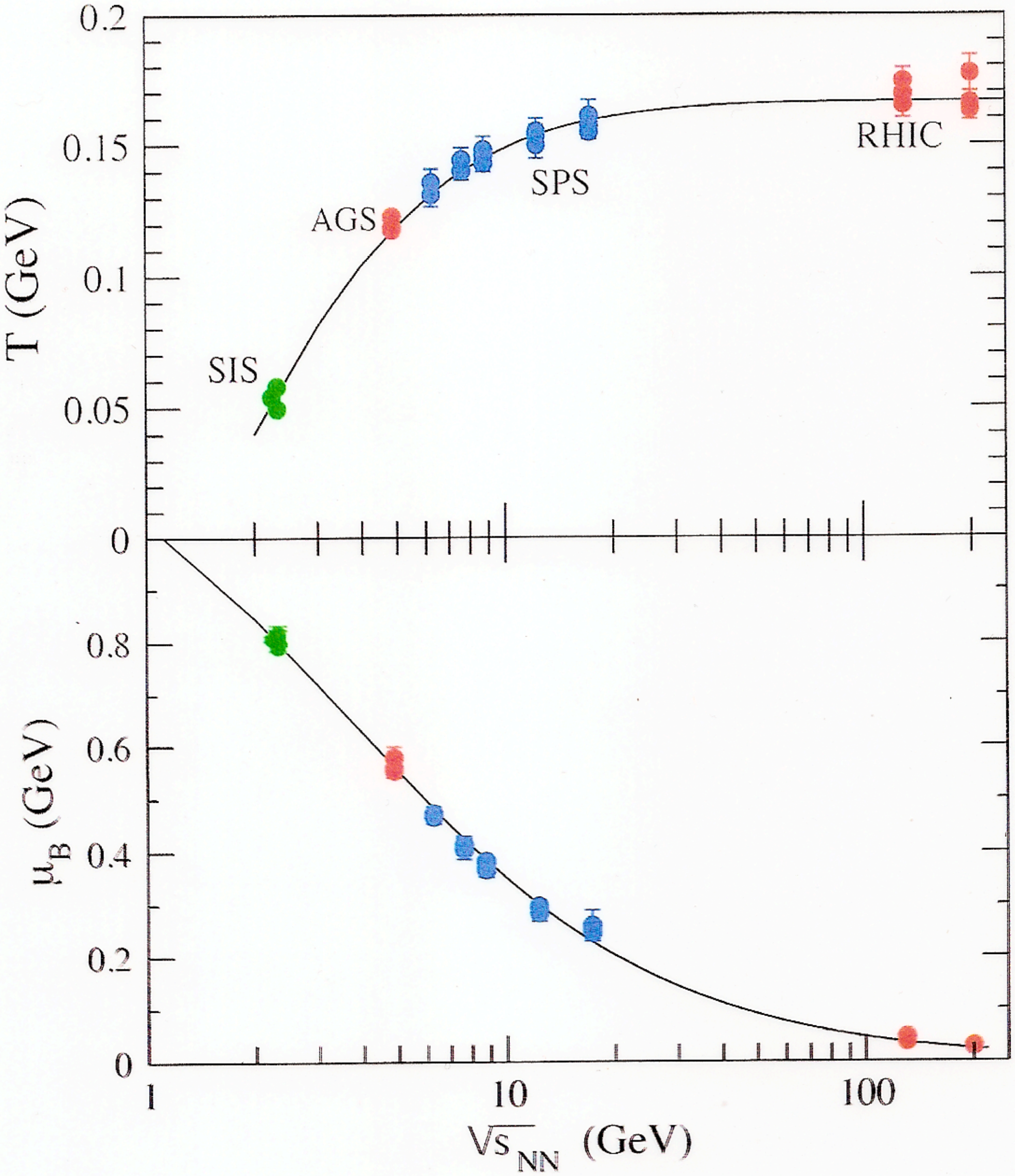
The longer the correlation length ξ gets, the bigger the signatures.

Signatures are event-by-event fluctuations of specific observables, calculable in magnitude in terms of ξ. Stephanov KR Shuryak

- Vary μ by varying $\sqrt{s}$
- Search for enhancement of these fluctuations in a window in $\sqrt{s}$, ie μ
- Analogue of critical opalescence
- Long wavelength fluctuations $\Rightarrow$ effects greatest at low $p_\perp$.

Examples .....   But, first :

CHEMICAL FREEZEOUT T & μ



Parametrization from Cleymans et al, 2005

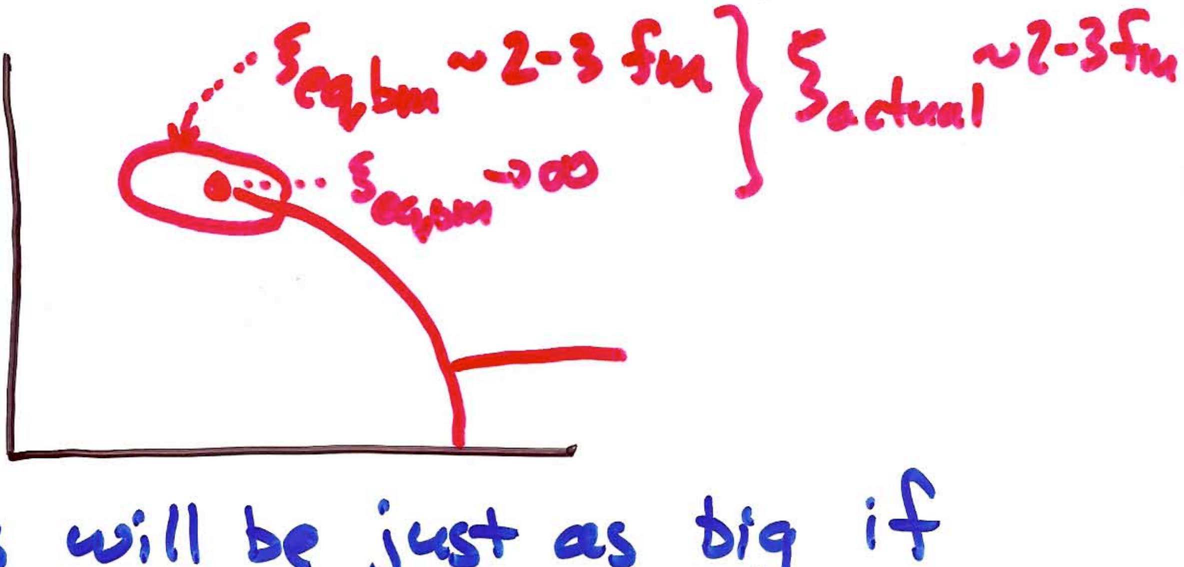
HOW LARGE CAN ξ GET?

HOW CLOSE TO ● NEED WE BE?

- Obviously ξ limited by finite size of system. But, turns out that finite time is a more severe limitation.

Berdnikov KR; Asakawa Nonaka

- Finite time spent in critical region means that even if equilibrium value of ξ is much larger, actual ξ won't grow bigger than 2-3 fm.
- Means no need to hit ● precisely.



Signatures will be just as big if you pass anywhere in ⬭. No bigger, even if you hit ●.

SIGNATURES OF CRITICAL POINT

- Decreasing $\sqrt{s}$ → Increasing μ
- Vary $\sqrt{s}$, and hence μ, and look for nonmonotonic enhancement (rise and then fall) of:

  i) Event-by-event fluctuations of mean p_T of low p_T pions
  ii) Event-by-event fluctuations of net proton number ($N_p - N_{\bar{p}}$)
  iii) Event-by-event fluctuations of particle ratios involving pions and/or protons
  iv) Kurtosis of the N_p or $(N_p - N_{\bar{p}})$ event-by-event distribution

  ⋮

In all cases, the enhancement should be greater at lower p_T.

KURTOSIS OF EVENT-BY-EVENT DISTRIBUTION OF (NET) PROTON NUMBER

Stephanov; "a direct consequence of discussions" at Aug 2008 INT workshop

Critical fluctuations couple to $\pi\pi$, pp
→ event-by-event fluctuations in their multiplicities, multiplicity ratios, p_T, that are $\propto \xi^2$

Higher moments of the event-by-event distributions receive effects that are more sensitive to ξ.

Skewness $\propto \xi^{4.5}$

Kurtosis $\propto \xi^7$!!!

The prefactors work out particularly nicely for kurtosis of proton distribution, but Stephanov makes predictions for π & p, skewness & kurtosis.

DEFINITIONS

N = # of protons (or π, or $p - \bar{p}$) in 1 event

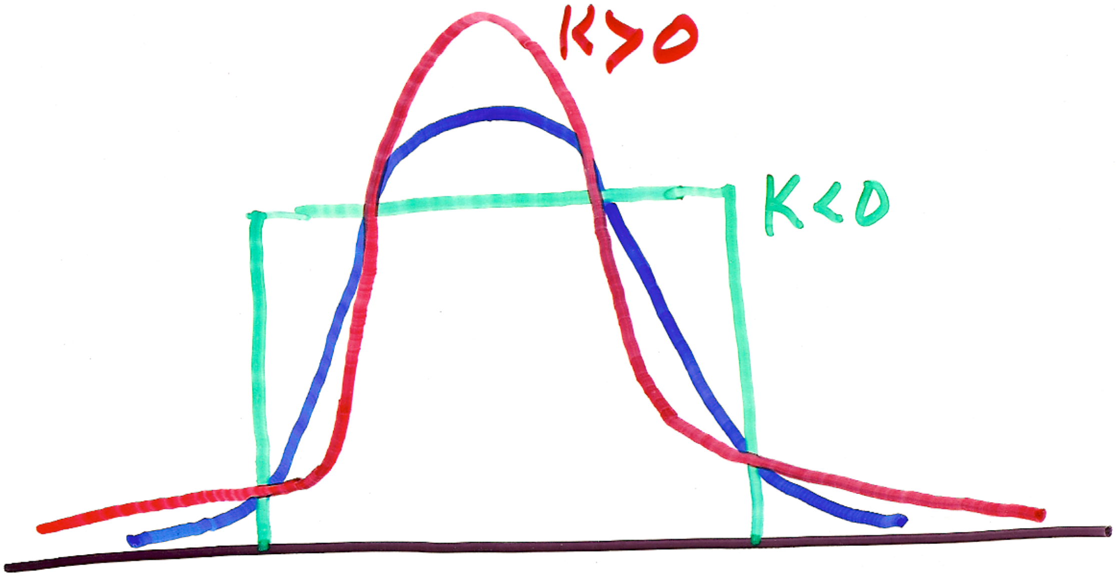
$\bar{N} \equiv \langle N \rangle$ = mean

$\delta N \equiv N - \bar{N}$ in one event

$\langle (\delta N)^2 \rangle$ = variance

$$K^{eff} \equiv \underbrace{K}_{\text{Kurtosis} \sim \frac{1}{\bar{N}}} \underbrace{\langle (\delta N)^2 \rangle}_{\text{Variance} \sim \bar{N}} \equiv \frac{\langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2}{\langle (\delta N)^2 \rangle}$$

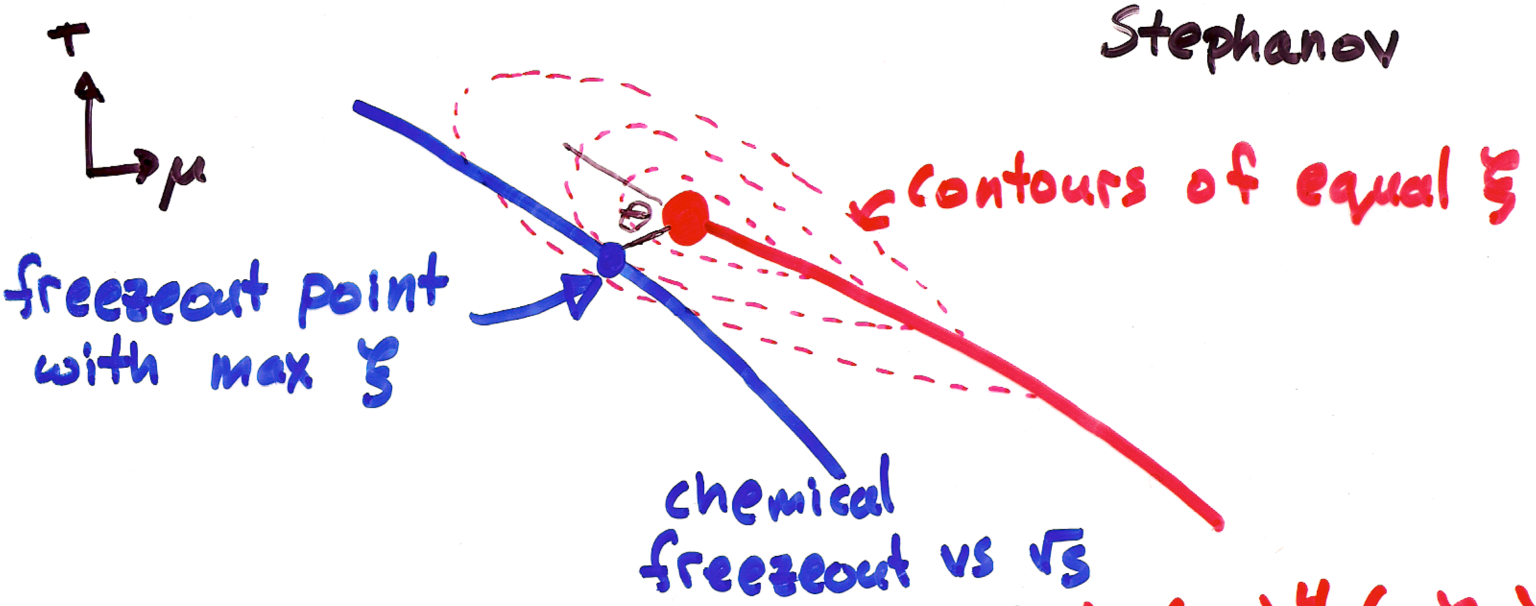
⌈In lattice QCD literature, K^{eff} is called κ_4/κ_2 or χ_4/χ_2. It is calculated for N = baryon number, & N = charge.⌋



Gaussian: $K=0$

EFFECT OF CRITICAL FLUCTUATIONS

Stephanov



$$K^{\text{eff}}\underbrace{\left(\frac{\bar{N}}{\langle(\delta N)^2\rangle}\right)}_{\simeq 1} = \underbrace{23}_{!}\underbrace{\left(\frac{2\tilde{\lambda}_3^2-\tilde{\lambda}_4}{50}\right)\left(\frac{g}{10}\right)^4}_{\mathcal{O}(1)}\underbrace{\left(\frac{\xi}{1\,\text{fm}}\right)^7}_{!!!!}$$

$\tilde{\lambda}_3, \tilde{\lambda}_4$: Obscure but <u>universal</u> constants that depend on θ. Known for Ising ●.

g: σpp coupling. $g \sim m_p/f_\pi \sim 10$

So, how big is the background? Theory suggests $\mathcal{O}(1)$, but better to determine it experimentally....

PROTON NUMBER KURTOSIS

- Predicted effect of proximity to ● is:
  - large
  - strongly ξ-dependent, ∴ $\sqrt{s}$-dependent
  - larger at lower p_T

  → How you know its ● if you see it.
- Cannot be washed out between chemical & kinetic freezeout
- Background measured, and small, at $\sqrt{s} = 62$ & 200.
- Error bars ~ ±.1 with 5M events STAR B.U.R
- Calculations done for $\bar{N} \to \infty$. Finite $\bar{N}$ and acceptance corrections need to be assessed. (Latter minimal for STAR)
- Analogous analyses can be done for skewness, and for pions. Many signatures will be in play if/when ● found.

WHAT RANGE OF $\sqrt{s}$, ie μ_B

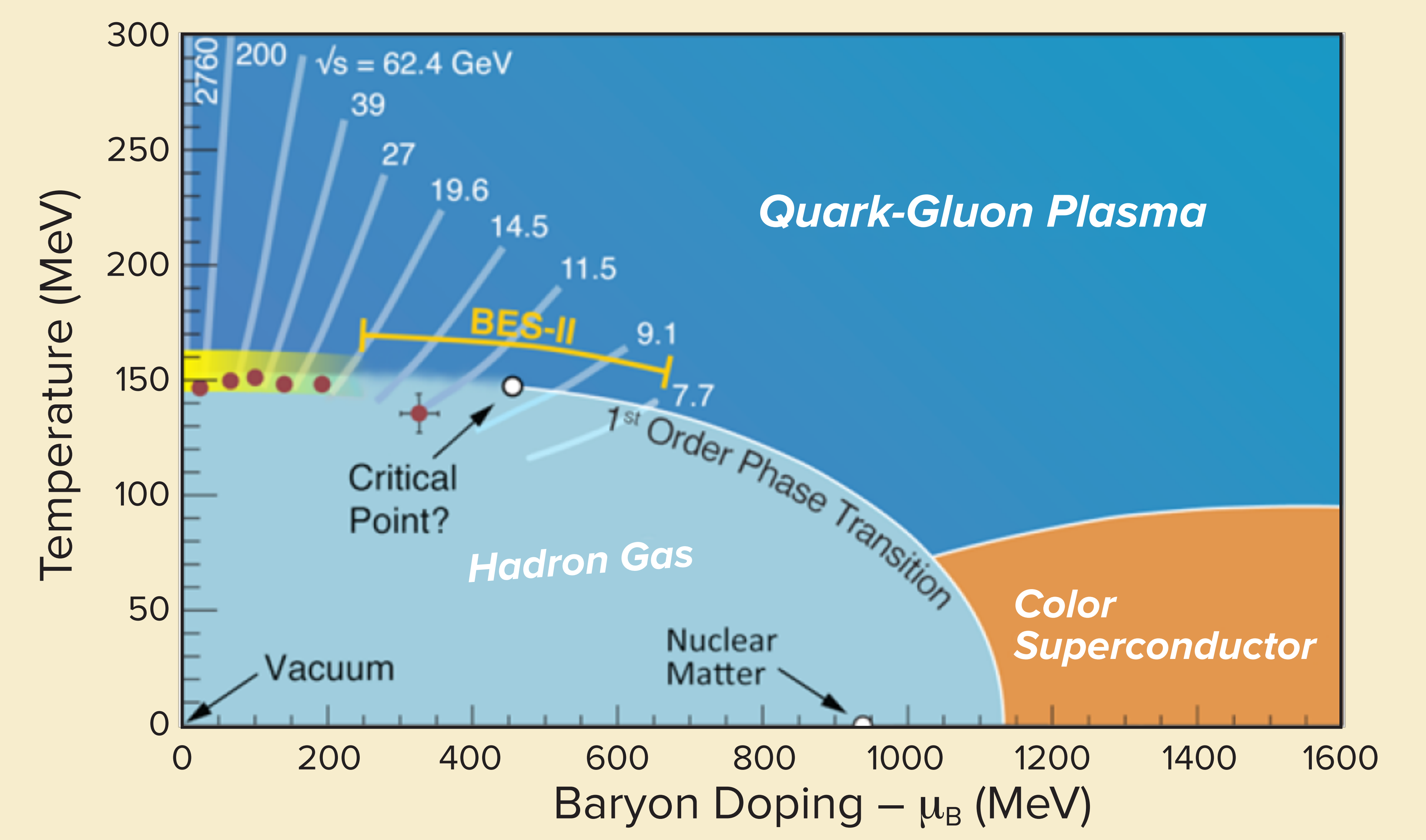
- RHIC should, and can, explore $\mu_B < 500$ MeV
- Want to test NA49 observation of K/π fluctuations at $\mu_B \sim 400-450$ MeV
- If $\mu_B^{\bullet} < 3T_c \sim 500$ MeV, plausibly the different lattice calculations will converge as each improves. If $\mu_B^{\bullet} > 500$ MeV, quantitative comparison with theory will be hard.
- If $\mu_B^{\bullet} > 500$ MeV, also tough to find experimentally. (Low $T_{freezeout}$; equilibration ???)
- A scan with steps $\lesssim 100$ MeV apart in μ_B should allow to make discoveries.
- In the vicinity of a discovery, will want μ_B's spaced by ~ 50 MeV.

A "STRAW MAN" CHOICE OF ENERGIES

	$\sqrt{s}$ (AGeV)	μ_B (MeV)*	10 hr days per 10^6 events†
	5	550	20
largest K/π fluctuations	6.27	480	9
	7.62	425	5
	9.4	365	3
	12.3	300	1
	18	220	0.4
	24	170	0.2
	36	120	0.1
done	60	75	
	130	40	
	200	25	

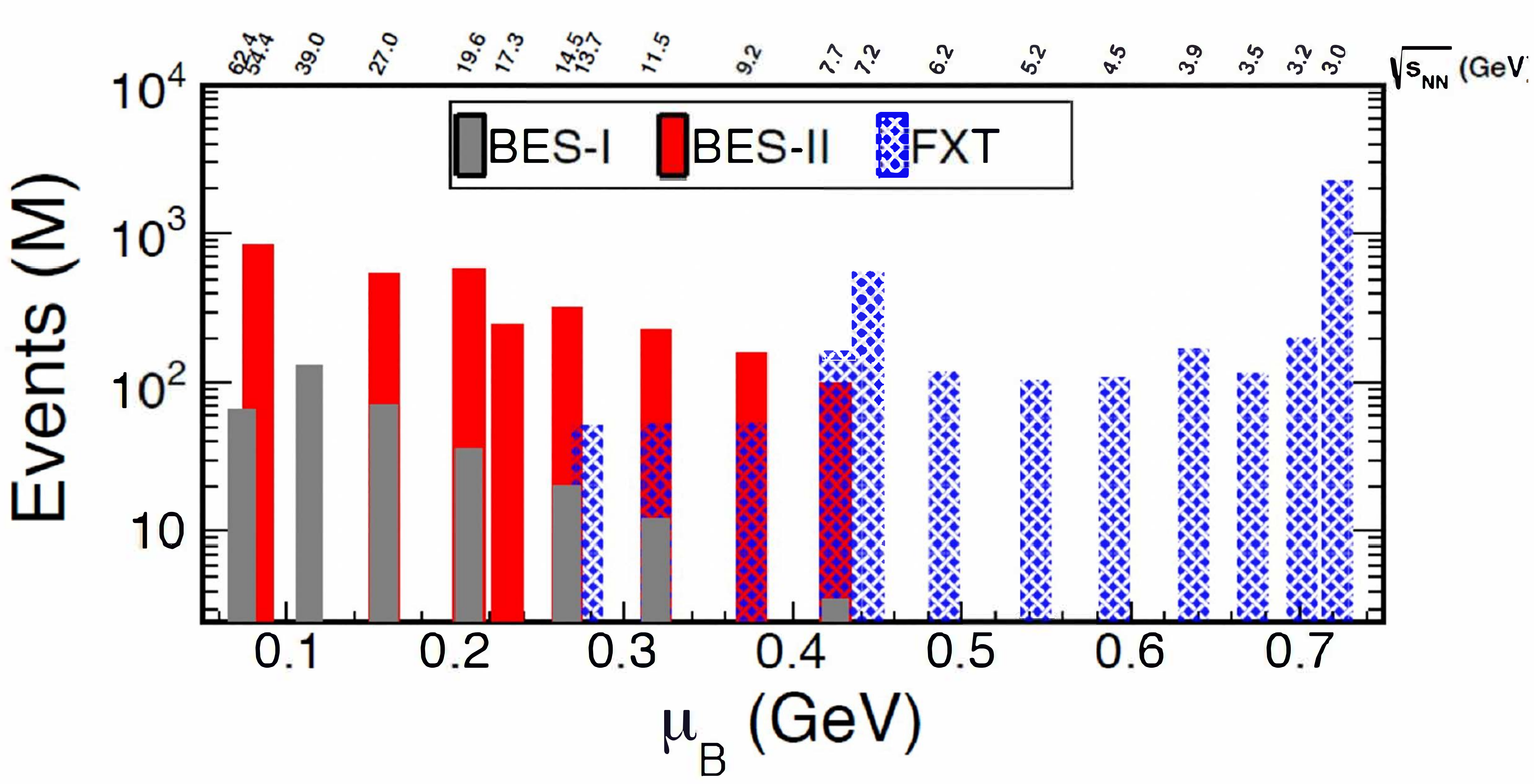
* from Cleymans et al's 2005 empirical fit to compilation of data

† from Roser's "guidance" luminosity vs. $\sqrt{s}$ curve



Temperature (MeV)
300
250
200
150
100
50
0
Baryon Doping – μ_B (MeV)
0
200
400
600
800
1000
1200
1400
1600
2760
200
√s = 62.4 GeV
39
27
19.6
14.5
11.5
9.1
7.7
BES-II
Quark-Gluon Plasma
Critical Point?
1st Order Phase Transition
Hadron Gas
Color Superconductor
Vacuum
Nuclear Matter

RHIC BES II Data Taken...

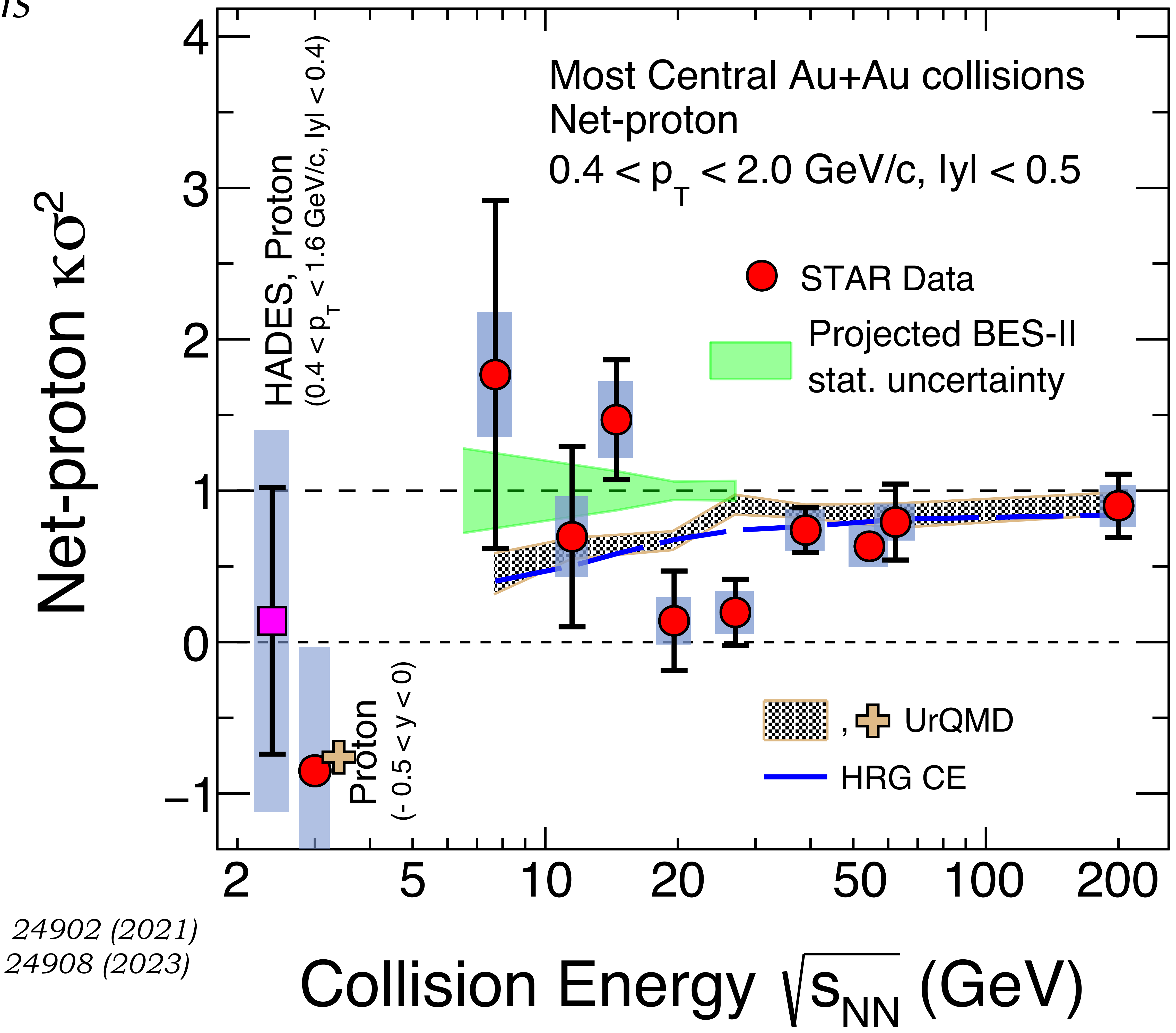


EXPERIMENTAL SEARCH FOR CP: BES SCAN AT STAR-RHIC

Phase I of BES program (BES-I): Au+Au collisions

J. Cleymans, et. al, PRC. 73, 034905 (2006)

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	μ_B (MeV)
200	220	25
62.4	43	75
54.4	550	85
39	92	112
27	31	156
19.6	14	206
14.5	14	262
11.5	7	316
7.7	2.2	420
3.0	140	750

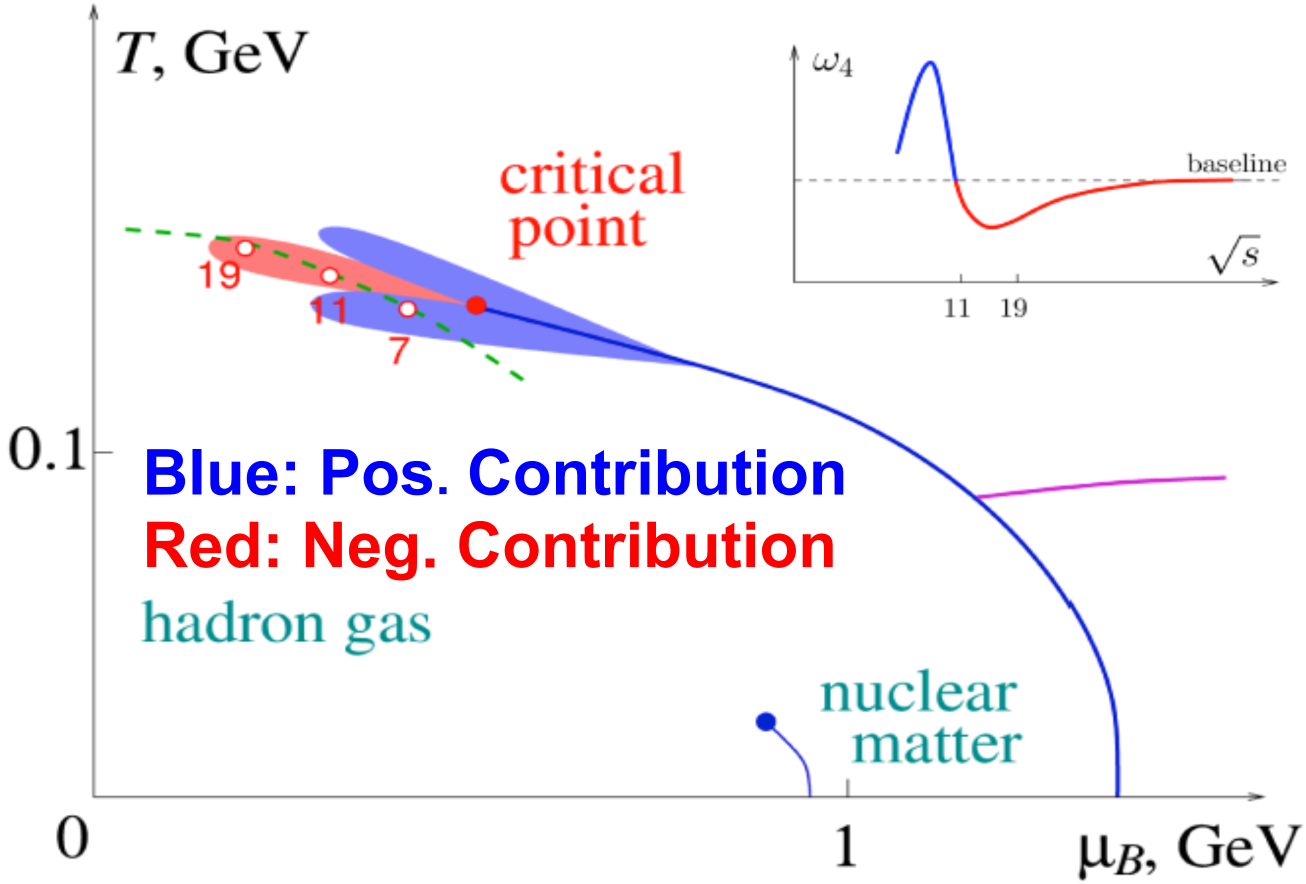


STAR : PRL 127, 262301 (2021), PRC 104, 24902 (2021)
: PRL 128, 202302 (2022), PRC 107, 24908 (2023)
HADES: PRC 102, 024914 (2020)

Observed hint of non-monotonic trend in BES-I (3 σ): consistent with model expectation with a CP

Robust conclusion require confirmation from precision measurement from BES-II.

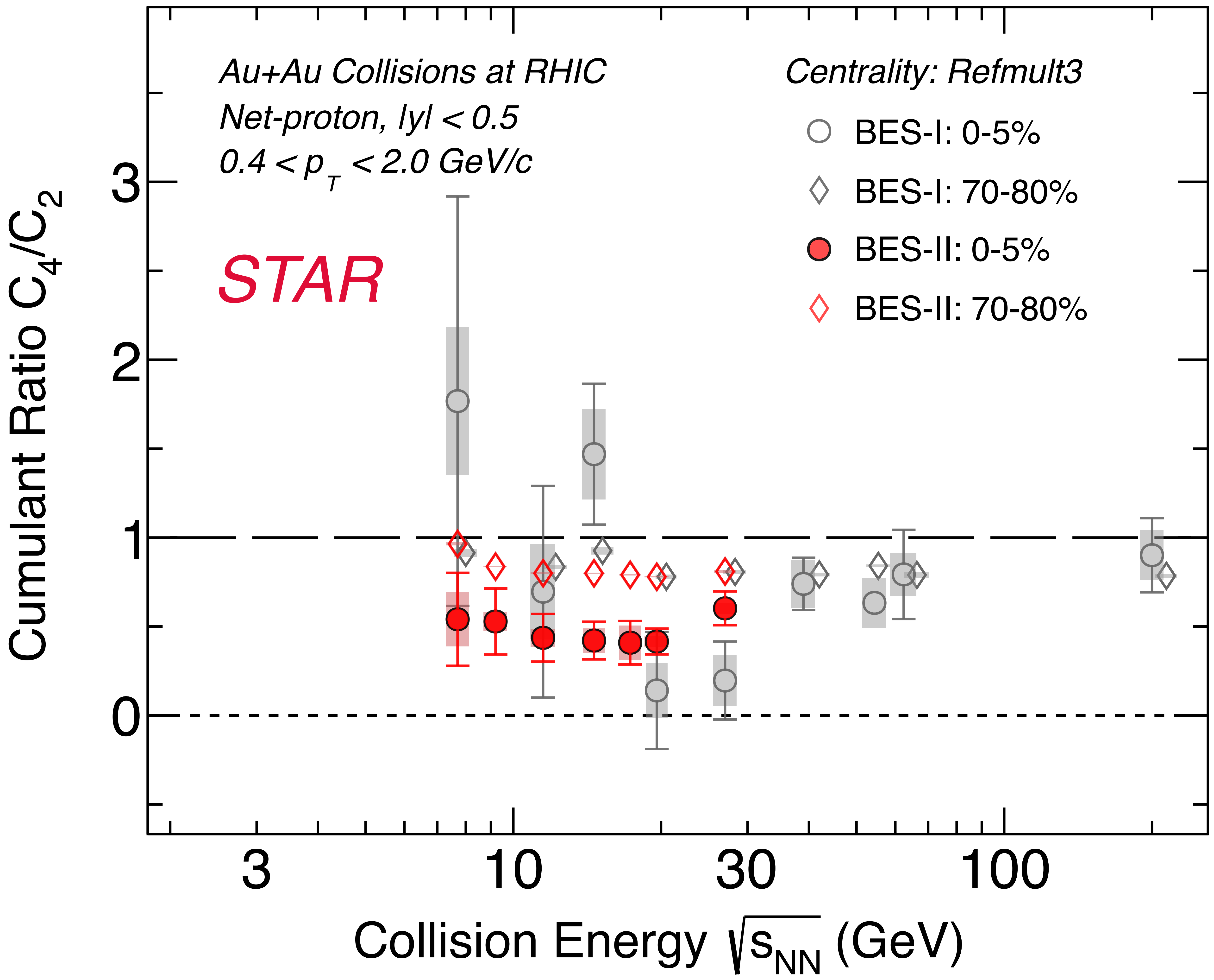
Extend reach to even lower collision energies with FXT energies



Blue: Pos. Contribution

Red: Neg. Contribution

ENERGY DEPENDENCE OF C_4/C_2: COMPARISON WITH BES-I



Deviation between BES-II and BES-I data

$\sqrt{s_{NN}}$ (GeV)	0-5%	70-80%
7.7	1.0σ	0.9σ
11.5	0.4σ	1.3σ
14.6	2.2σ	2.5σ
19.6	0.7σ	0.0σ
27	1.4σ	0.2σ

- BES-II results consistent with BES-I within uncertainties.

QCD phase structure from Lattice QCD

Jana N. Guenther
Alexander Adam, Szabolcs Borsanyi, Zoltan Fodor, Piyush Kumar, Paolo Parotto, Attila Pasztor, Ludovica Pirelli, Claudia Ratti, Volodymyr Vovchenko and Chik Him Wong

April 13th 2026

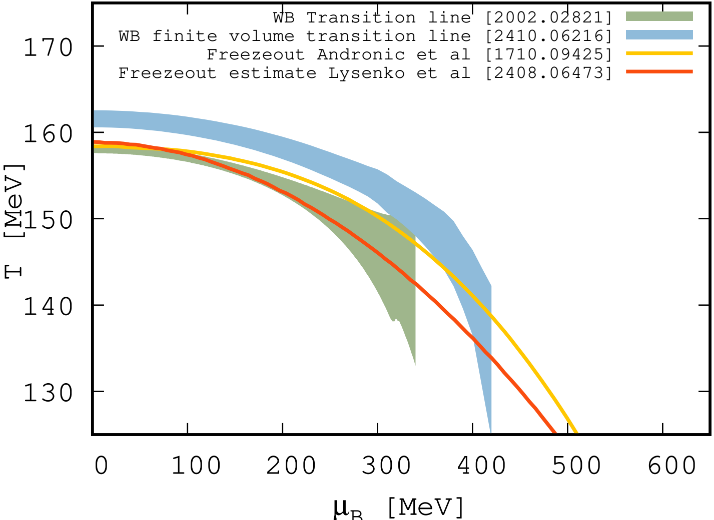
BERGISCHE UNIVERSITÄT WUPPERTAL

WB collaboration

BMW collaboration

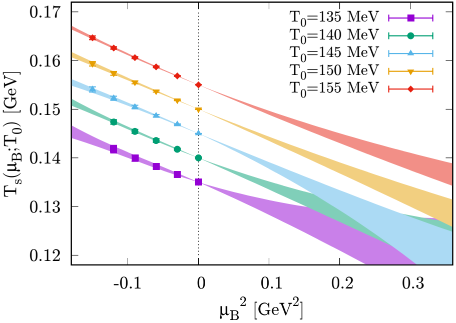
NRW-FAIR

Exclusion on the Critical End Point [Borsanyi:2025dyp]



Extrapolating to $\mu_B > 0$

For each chemical potential, we determine the temperature $T_s(\mu_B, T_0)$ at which the entropy has the desired value $s(T_0)$.



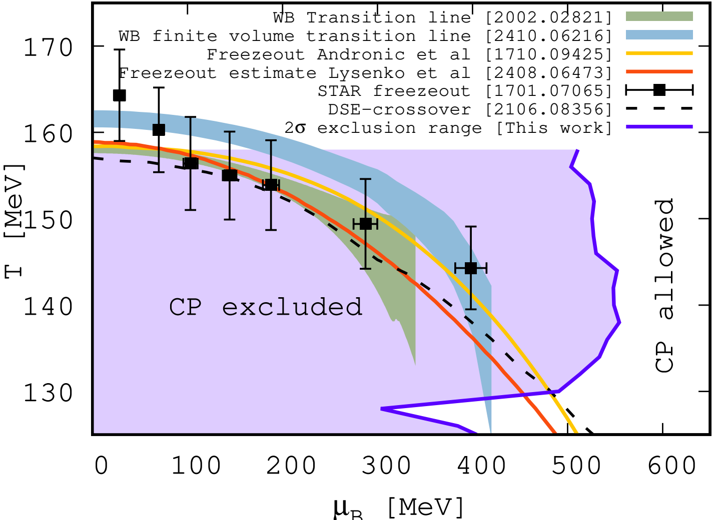
The function $T_s(\mu_B, T_0)$ then yields the contour stemming from T_0:

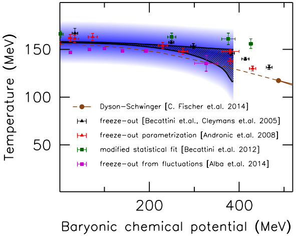
$$T_s(\mu_B^2, T_0) = \frac{T_0 + a\mu_B^2}{1 + b\mu_B^2}$$

$$T_s(\mu_B^2, T_0) = T_0 + a\mu_B^2 + b\mu_B^4$$

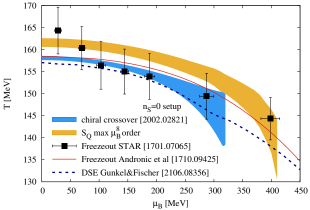
Question where is $\frac{\mathrm{d}T_s}{\mathrm{d}T_0} = 0$?

Exclusion on the Critical End Point [Borsanyi:2025dyp]

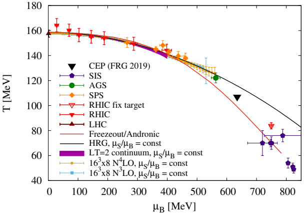




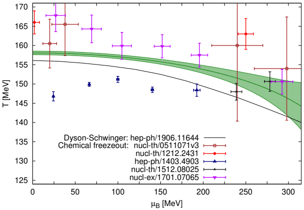
[Bellwied:2015rza]



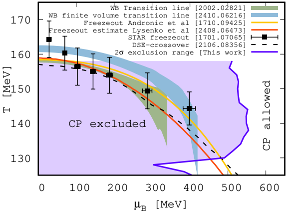
[Borsanyi:2024xrx]



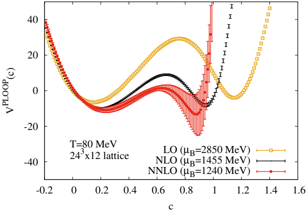
[Borsanyi:2025kiv]



[Borsanyi:2020fev]

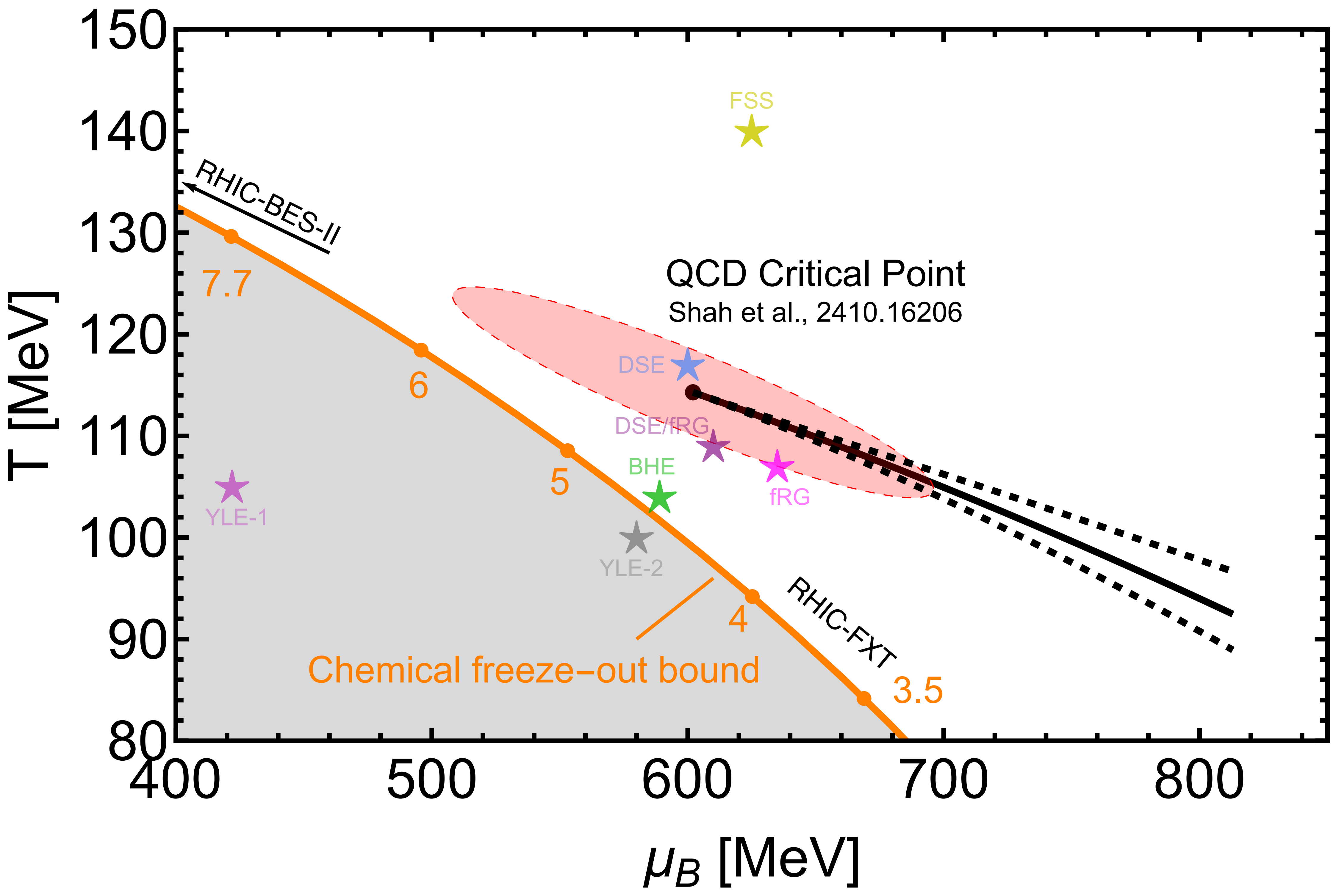


[Borsanyi:2025dyp]



Summary of CP Estimations

- Clustering of critical point location estimates that also fall within limits set by lattice QCD



Results on CP limits/estimations at CPOD 2026:

J. Günther, Mon
W.-J. Fu, Mon
J. Jahan, Tues
Y. Huang, Wed

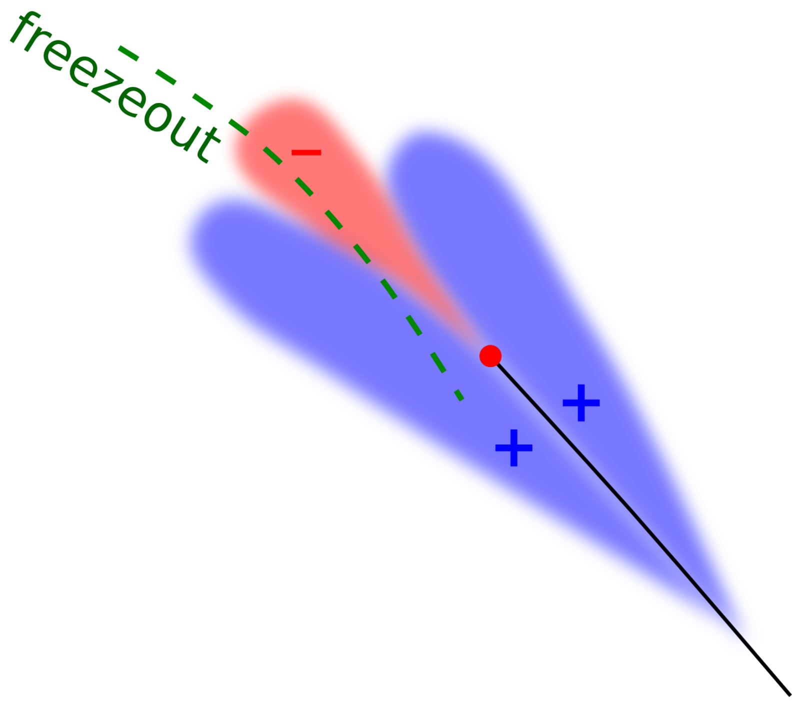
See also: SQM 2026 indico

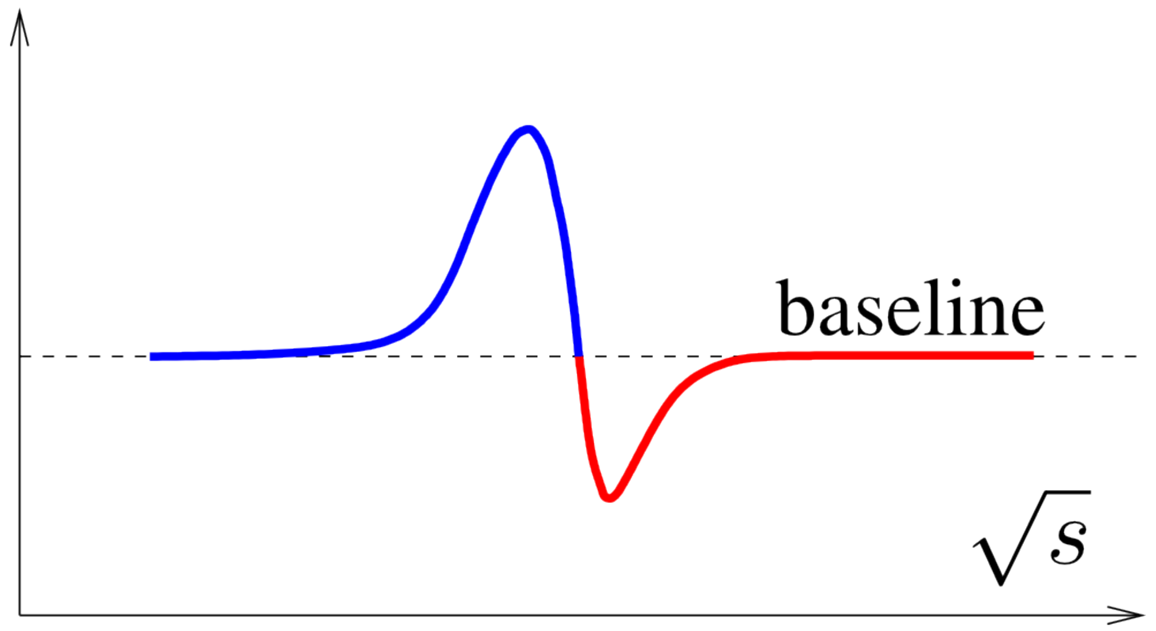
V. Vovchenko and V. Koch, J.Subatomic Part.Cosmol. (2025)

Historical Theoretical View of the QCD CP

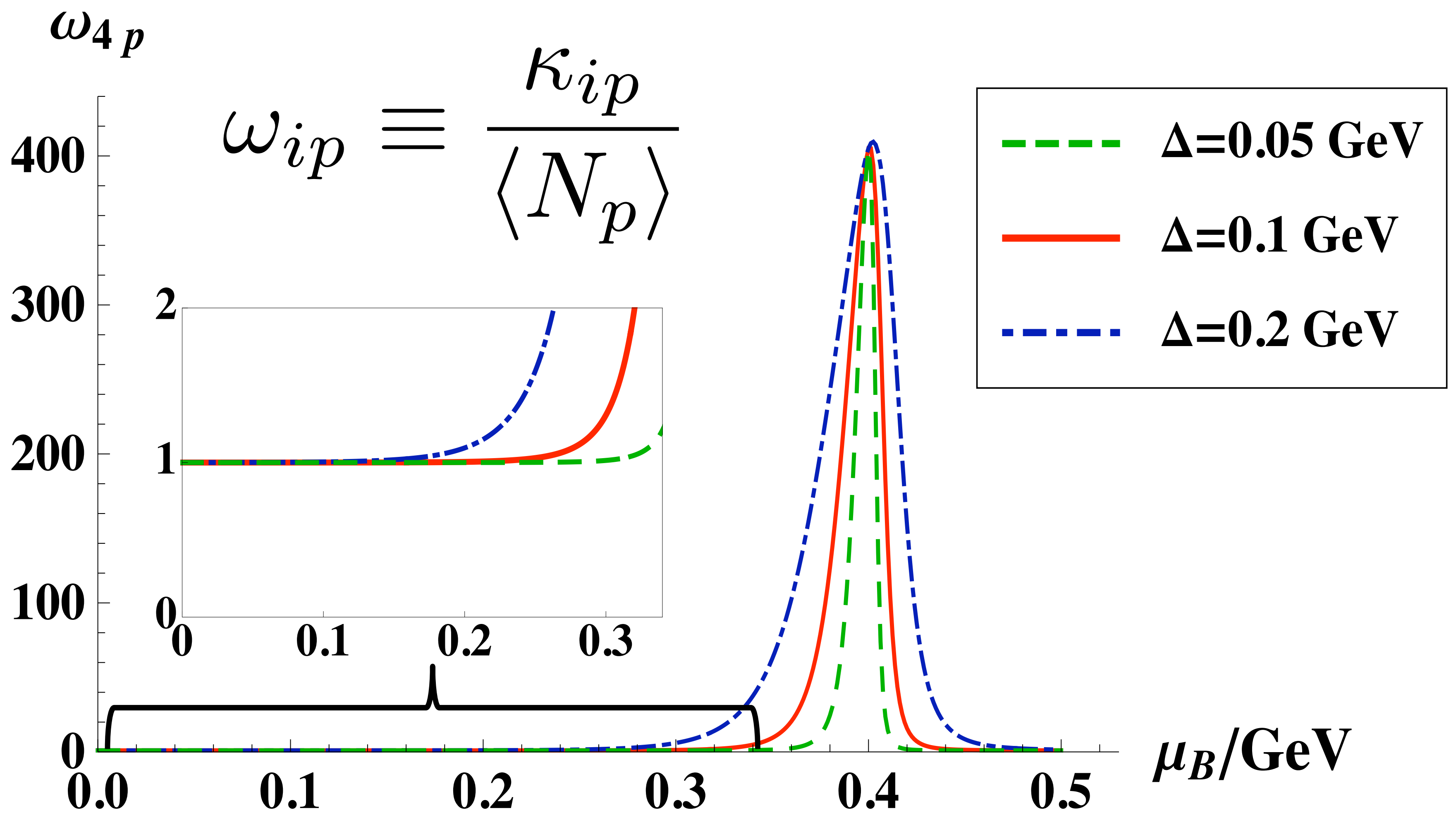
- Fluctuations serve as critical signal (diverging ξ):
  - Higher order susceptibilities diverge with higher power of the correlation length, $\kappa_4 \propto \xi^7$
  - Susceptibilities are derivatives of EoS:

$$\chi_n^B \equiv \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$$





- Relate baryon fluctuations to experimentally observable proton fluctuations



M. Stephanov, K. Rajagopal and E. Shuryak, PRD (1999)
M. Stephanov, PRL (2009) & PRL (2011)
C. Athanasiou, K. Rajagopal, M. Stephanov, PRD (2010)

Ingredients to Study Critical Point Effect

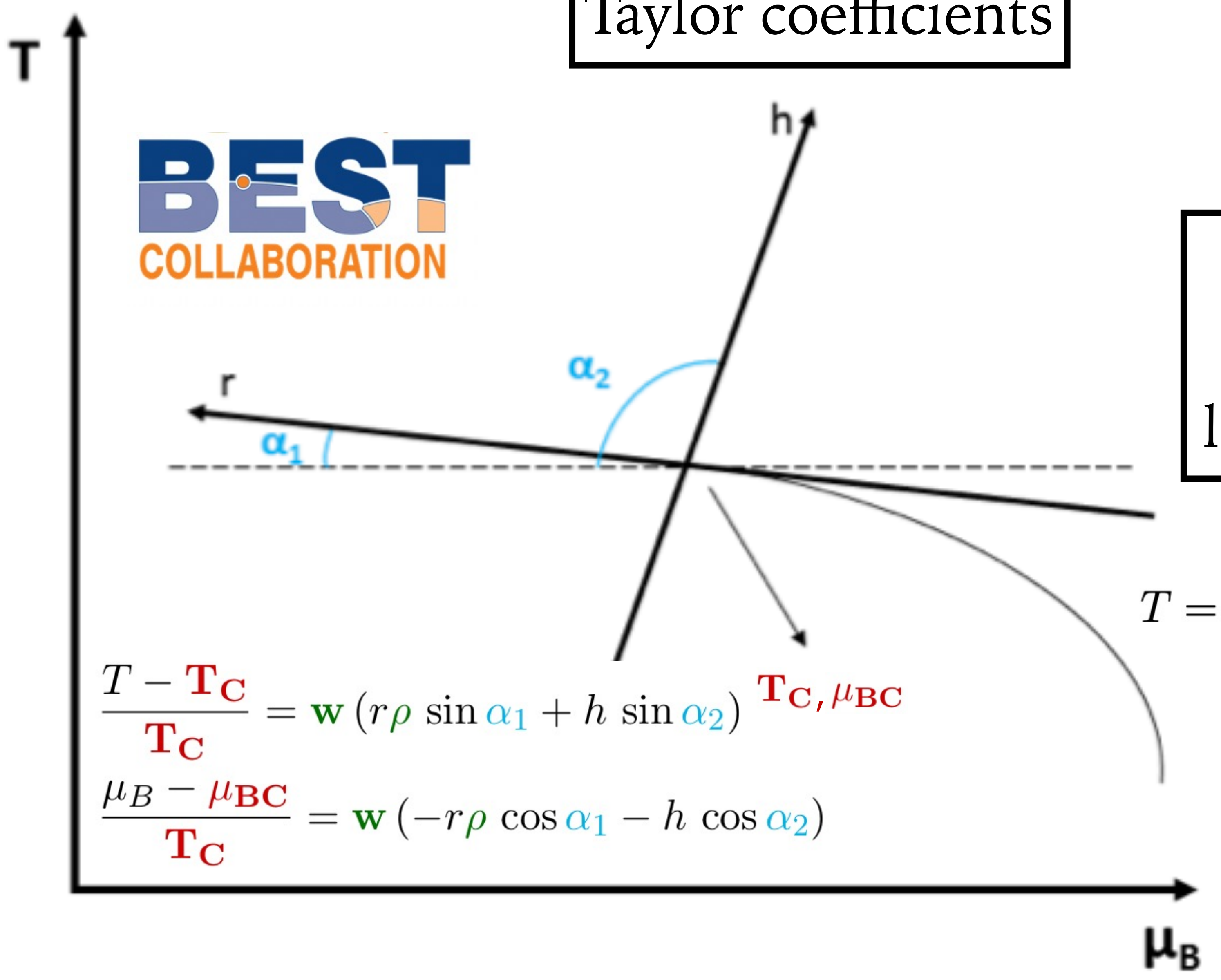
… in Equilibrium

- Equation of state is crucial input for simulations: important first step from BEST - EoS with critical features

$$P(\mu,T) = P^{\mathrm{reg}}(\mu,T) + P^{\mathrm{sing}}(\mu,T)$$

Matched to lattice QCD Taylor coefficients

3D Ising model



$$\frac{T - T_C}{T_C} = w\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)$$

$$\frac{\mu_B - \mu_{BC}}{T_C} = w\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

4 free parameters with CP along lattice chiral PT line

$$T = T_0 + \kappa T_0\left(\frac{\mu_B}{T_0}\right)^2 + O(\mu_B^4)$$

$$\alpha_1 = \tan^{-1}\left(2\frac{\kappa}{T_0}\mu_{BC}\right)$$

P. Parotto et al, PRC (2020)

- Need a method of computing particle correlators: Maximum Entropy method
  - Least-biased determination of particle multiplicity fluctuations consistent with hydrodynamic correlations

$$\hat{\Delta} G_{A_1 \dots A_k} = \hat{\Delta}\mathcal{H}_{a_1 \dots a_k} \prod_{i=1}^{k} P_{A_i}^{a_i}$$

Irreducible relative cumulant (IRC)

(Particle) Gas

Hydro (fields)

Matching conditions projectors

M. Pradeep and M. Stephanov, PRL (2023)

Quantifying Critical Fluctuation Signatures

- Extract critical contribution to proton multiplicity fluctuations from an Ising-like critical point
  - Focus on deviations from Hadron Resonance Gas background of EoS close to critical point where singular part dominates

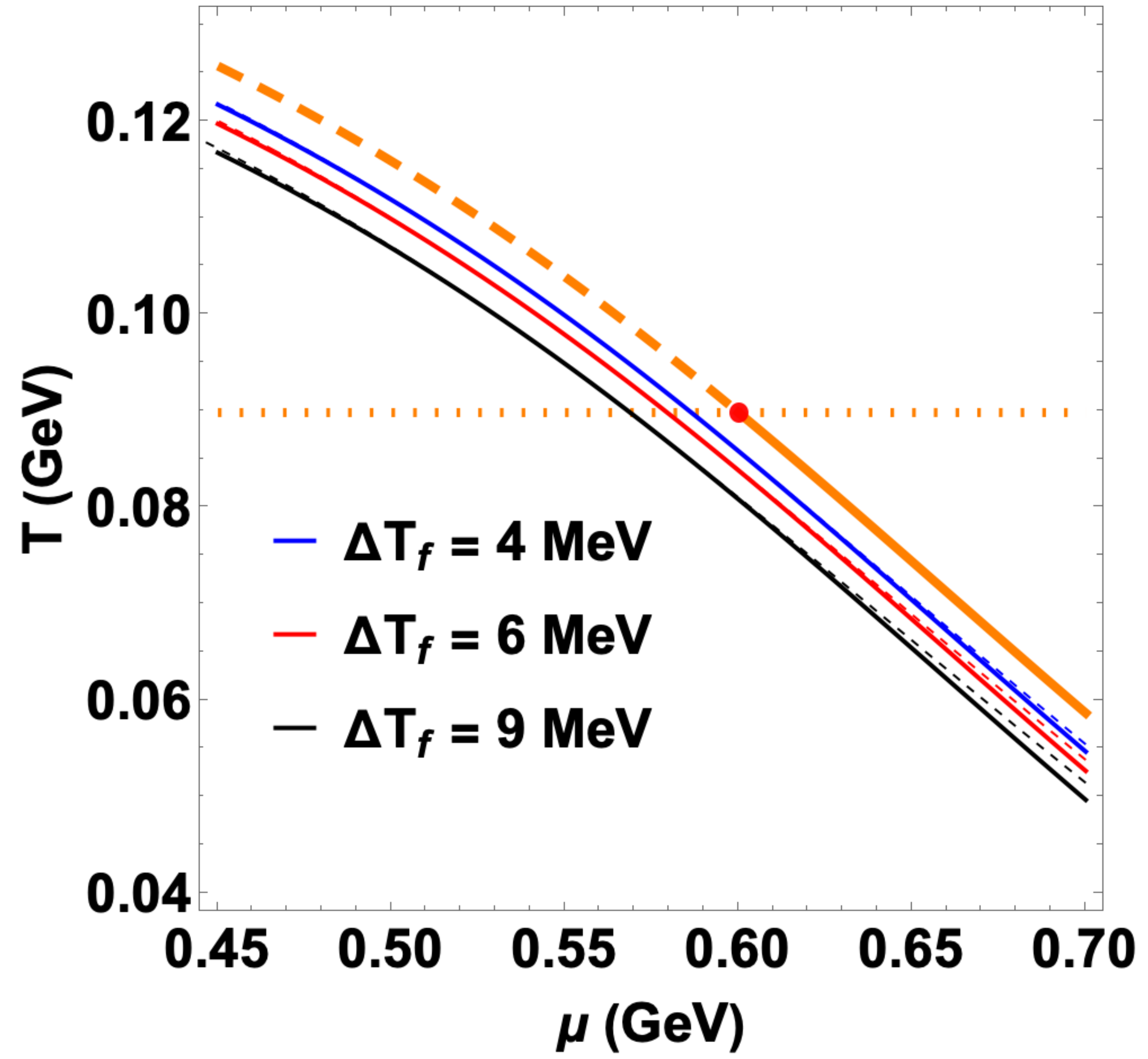
$$P(\mu,T) = \underbrace{P^{\rm reg}(\mu,T)}_{\boxed{\text{HRG}}} + \underbrace{P^{\rm sing}(\mu,T)}_{\boxed{\text{3D Ising model}}}$$

  - Use quadratic map for Ising transition line

M. Kahangirwe et al, PRD (2024)

  - Calculate multiplicity fluctuations along freeze-out lines

$$T_f(\mu_B) = T_{\rm crossover}(\mu_B) - \Delta T_f$$



***J.M. Karthein,** K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, PRD (2026, Editors' Suggestion)*

Maximum Entropy Method

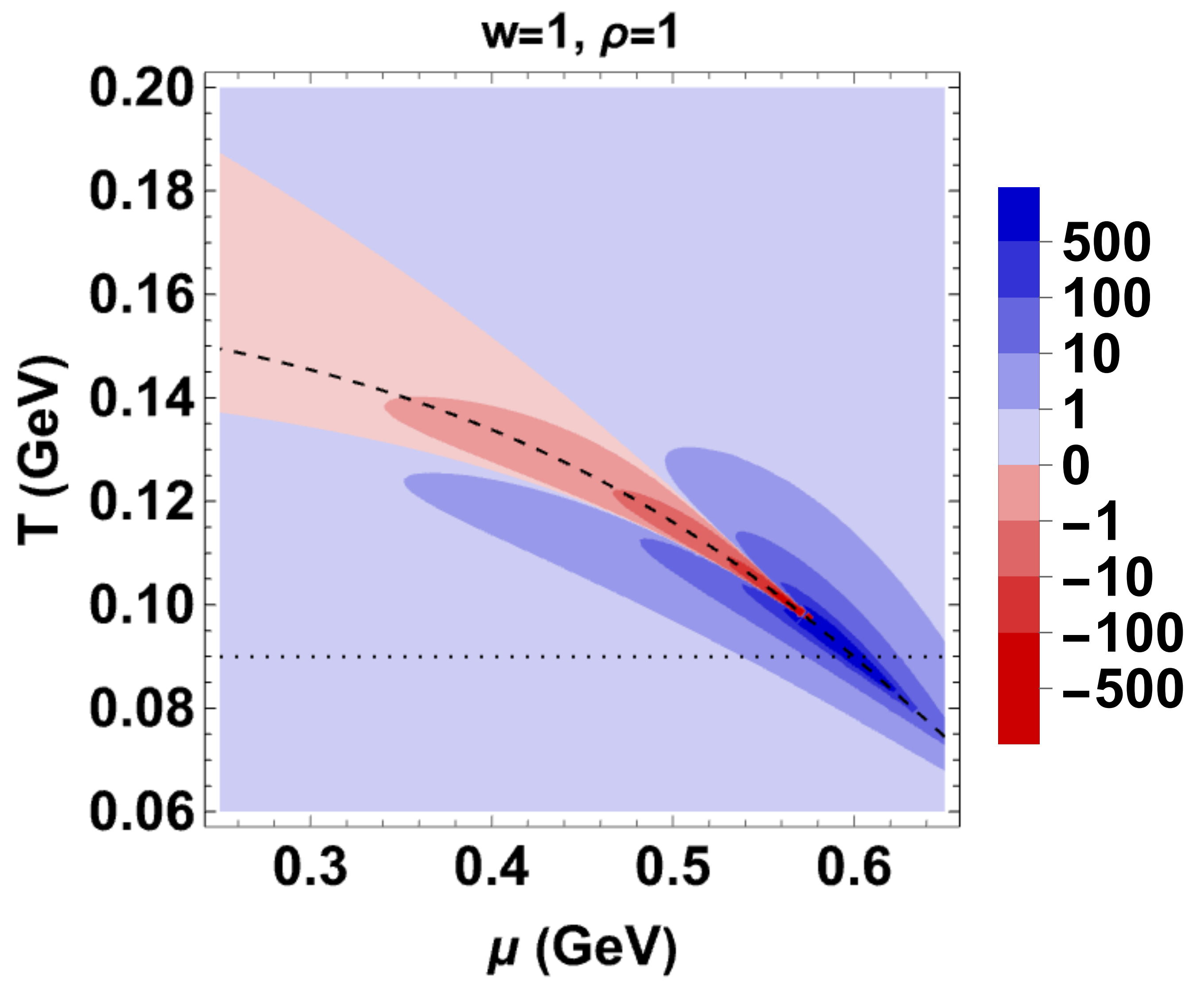
- Utilize maximum entropy freeze-out procedure to calculate proton factorial cumulants & study the influence of the unknown EoS parameters
  - Determine the particle fluctuations (G) from only the input of the EoS and by matching to the hydrodynamic description (H)

$$\hat{\Delta} G_{A_1\ldots A_k} = \hat{\Delta}\mathcal{H}_{a_1\ldots a_k} \prod_{i=1}^{k} P_{A_i}^{a_i}$$

$\widehat{\Delta}$: critical contribution (subtract background HRG EoS) to irreducible relative cumulants (subtract lower order cumulants)

$P_{A_k}^{a_n}$: contribution of particle A to conserved quantity a in the hydrodynamic cell

EoS input (here k=4): $\Delta H_{kn} \equiv \langle \delta n^k \rangle$



$\mu_c = 600\,\mathrm{MeV}, \alpha_2 = 0^{\circ}\,(\alpha_1 = 16.6^{\circ}, \mathrm{T_c} = 89\,\mathrm{MeV})$

__J.M. Karthein,__ K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, PRD (2026, Editors' Suggestion)

Proton Factorial Cumulants from Max Ent

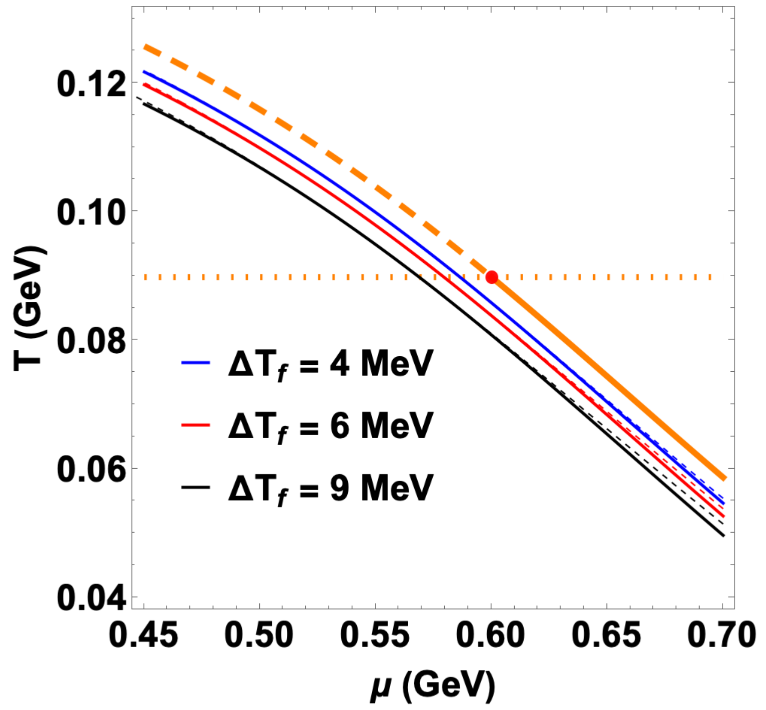
- ➤ Critical contribution to fourth order proton factorial cumulants in equilibrium for range of EoS parameters and freeze-out scenarios

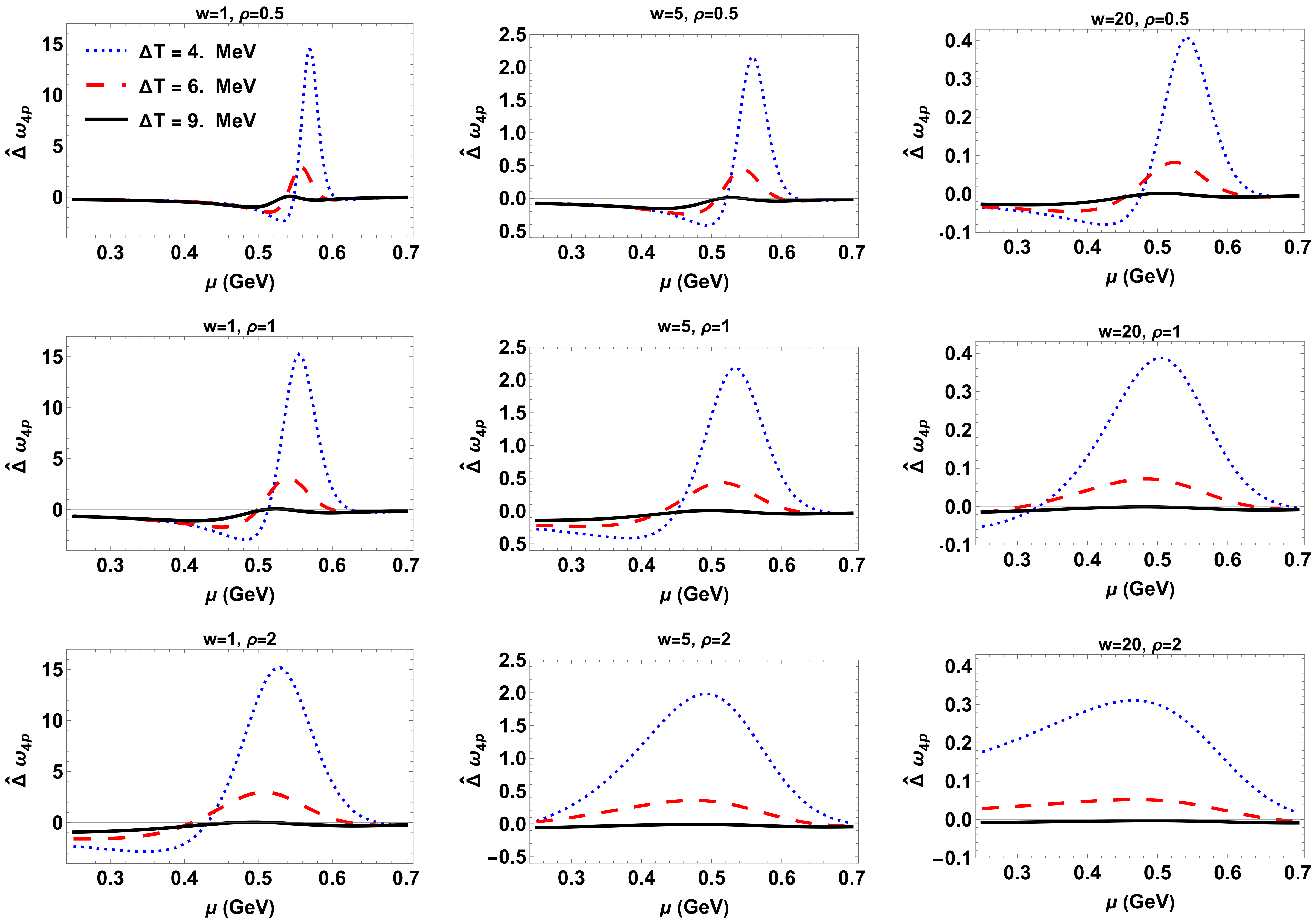
Normalized proton factorial cumulants:

$$\widehat{\Delta}\omega_{kp} = \frac{\widehat{\Delta}H_{a_1\ldots a_k}P_p^{a_1}\ldots P_p^{a_k}}{\langle N_p\rangle}$$

$$= \frac{\kappa_{4p}}{\kappa_{1p}}$$

Along freeze-out curves:





J.M. Karthein, *K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, PRD (2026, Editors' Suggestion)*

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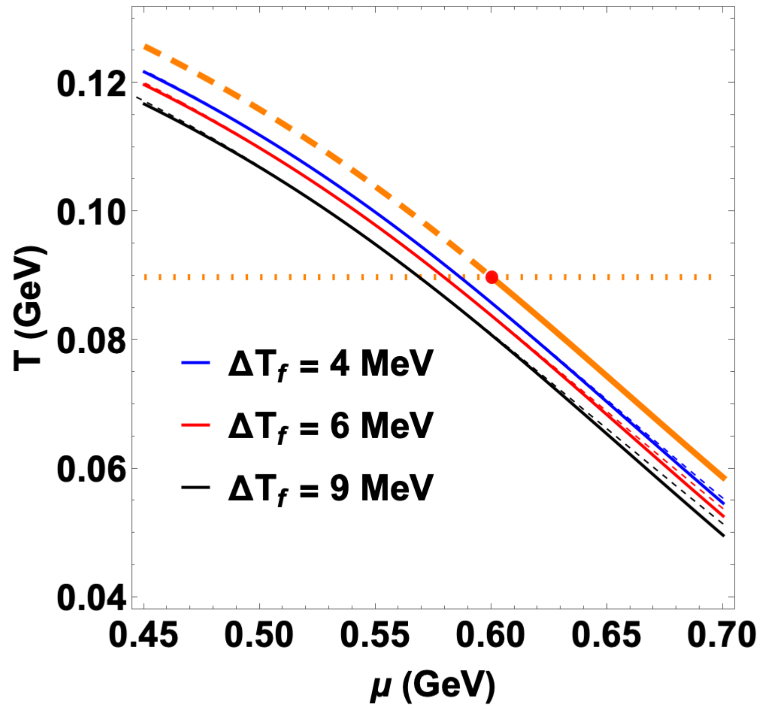

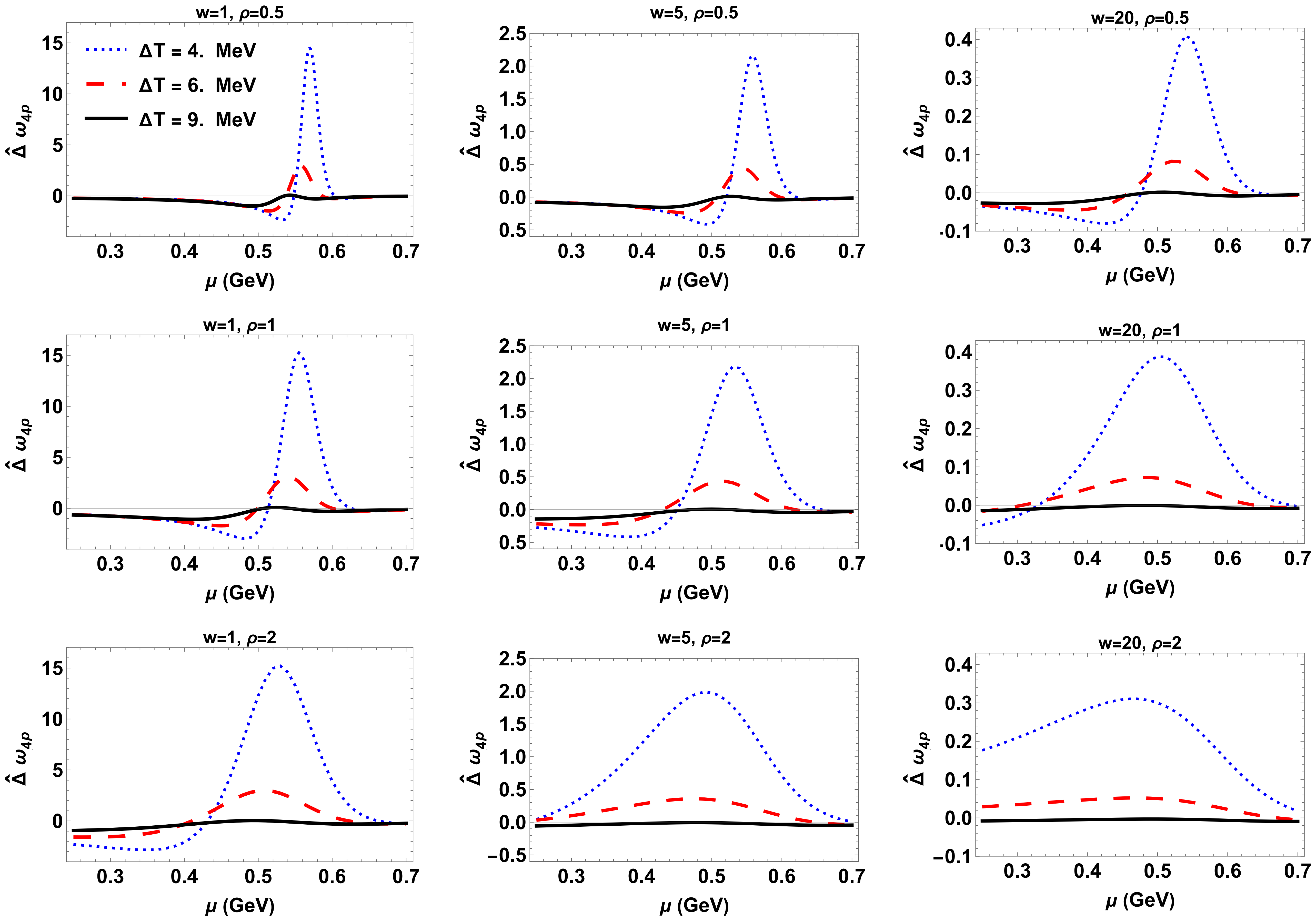

Increasing ρ
increases peak width

__J.M. Karthein,__ K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, PRD (2026, Editors' Suggestion)

Proton Factorial Cumulants from Max Ent

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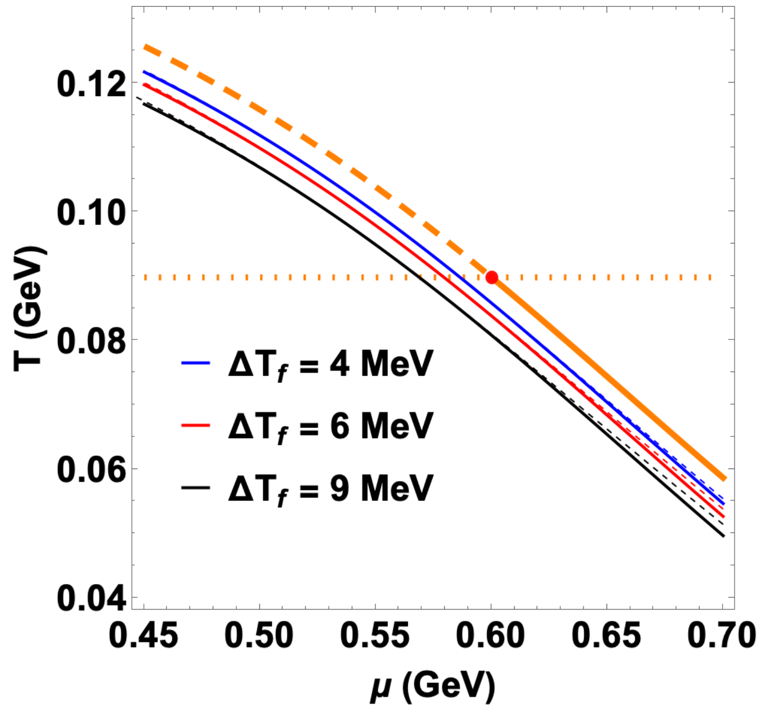
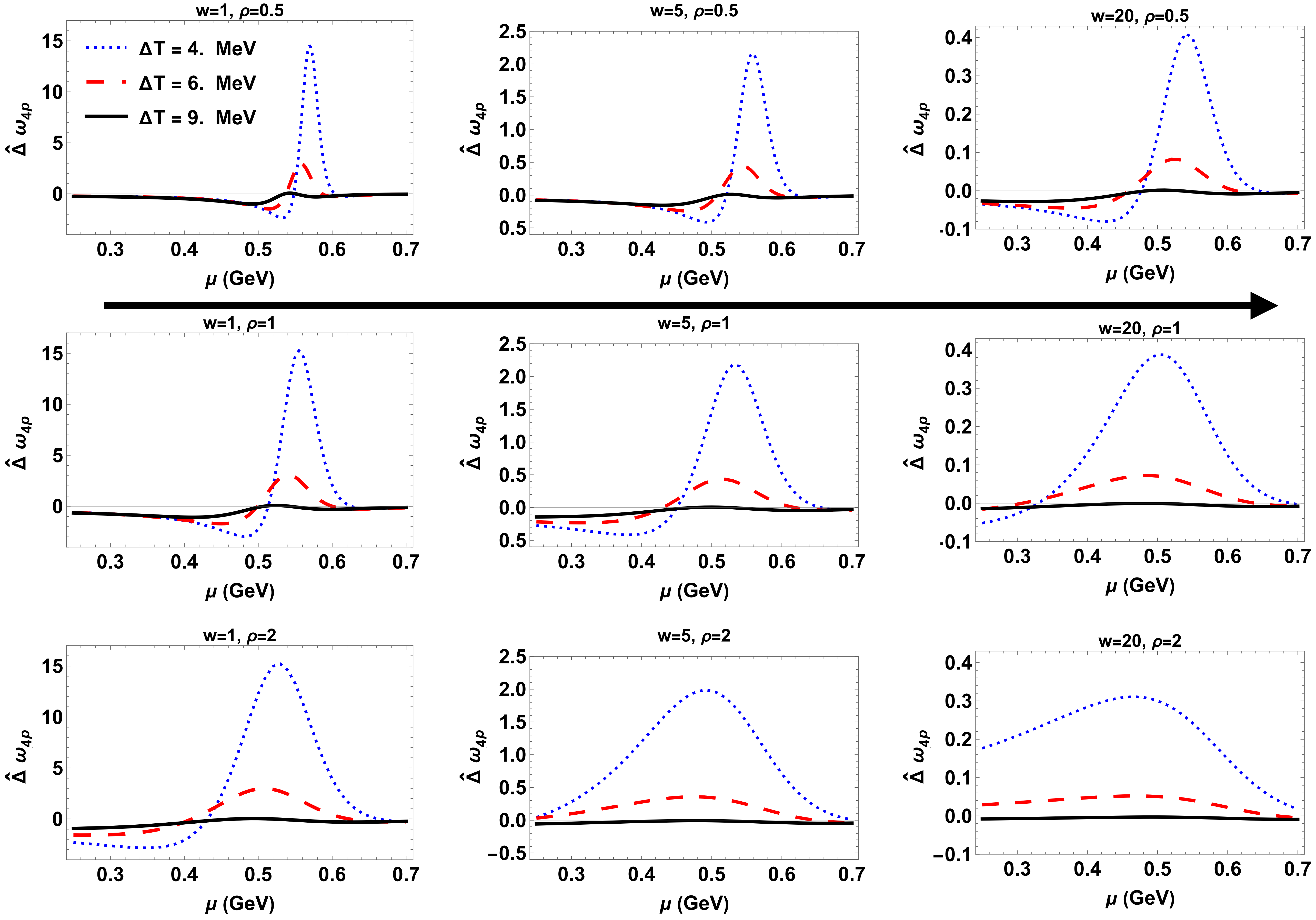
$$= \frac{\kappa_{4p}}{\kappa_{1p}}$$

Along freeze-out curves:





Increasing *w* reduces peak height

J.M. Karthein, K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, PRD (2026, Editors' Suggestion)

Proton Factorial Cumulants from Max Ent

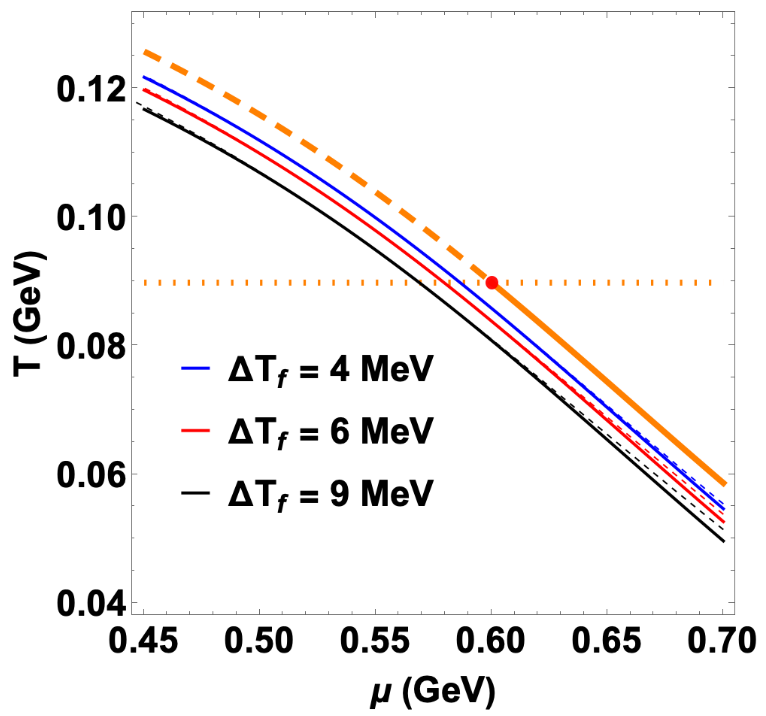
- ➤ Critical contribution to fourth order proton factorial cumulants in equilibrium for range of EoS parameters and freeze-out scenarios

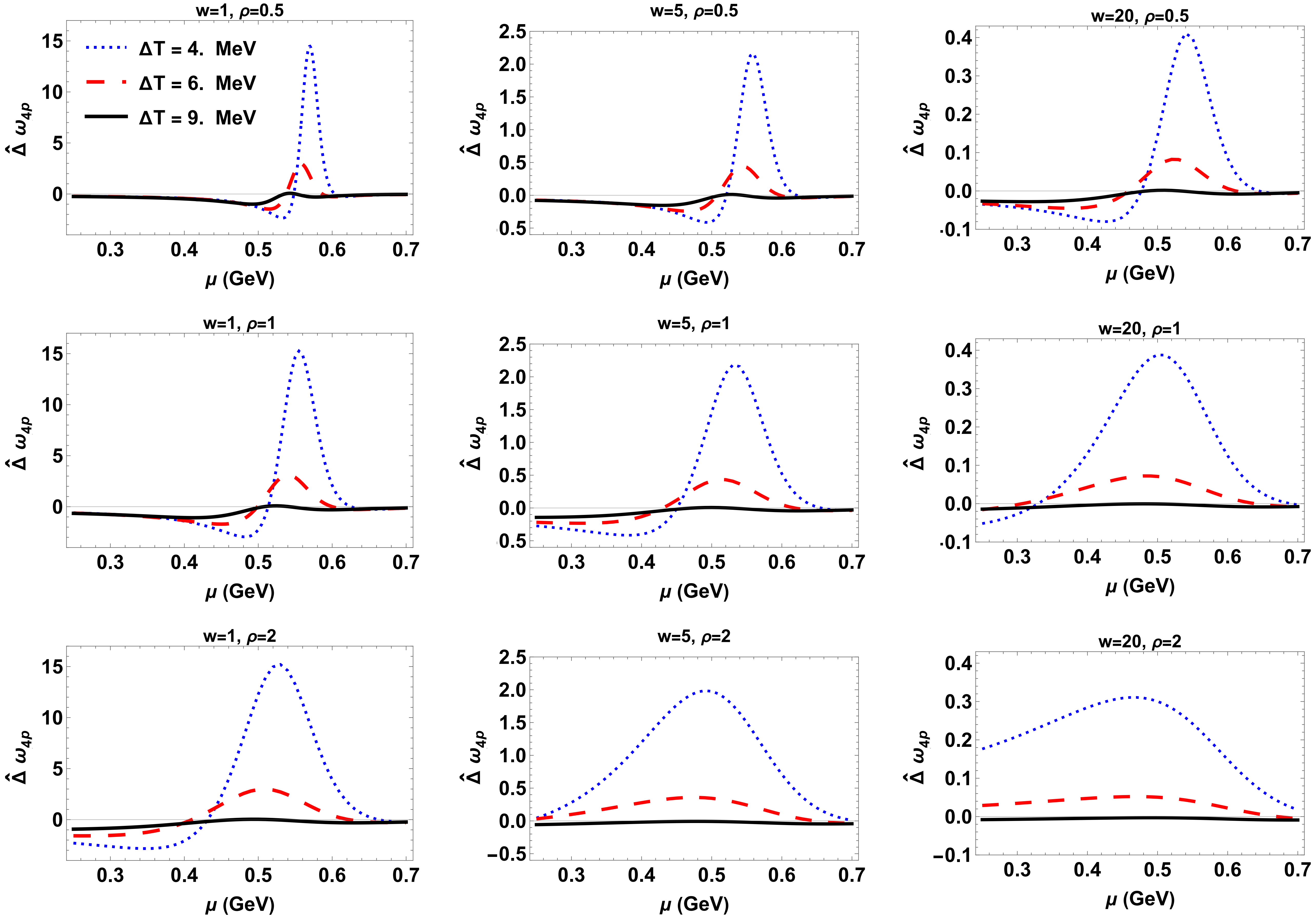
Normalized proton factorial cumulants:

$$\widehat{\Delta}\omega_{kp} = \frac{\widehat{\Delta}H_{a_1\ldots a_k}P_p^{a_1}\ldots P_p^{a_k}}{\langle N_p\rangle}$$

$$= \frac{\kappa_{4p}}{\kappa_{1p}}$$

Along freeze-out curves:





Scaling of magnitude:

$$\hat{\Delta}\omega_p^k \sim w^{-1.2}\Delta T_f^{1.2-k} \sim \xi^{k(5-\eta)/2-3}$$

Uncertainty in ΔT_f:

factorial cumulants can vary by order of magnitude

__J.M. Karthein,__ K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, PRD (2026, Editors' Suggestion)

Future Work: Critical Effects Beyond Equilibrium

- ➤ Direct protons
  - ✓ Qualitative results persist with resonance feed-down (Appendix B)
- ➤ Current background EoS is HRG
  - ➡ Extend to more realistic EoS constrained by lattice
  - ➡ Compare to non-critical baseline

P. Braun-Munzinger, B. Friman et al, NPA (2021)
V. Vovchenko, V. Koch & C. Shen, PRC (2022)

- ➤ Assumption of thermal equilibrium
  - ➡ Full Hydro+ simulations

M. Stephanov and Y. Yin, PRD (2018)
K. Rajagopal, G. Ridgway et al, PRD (2020)
M. Pradeep, K. Rajagopal et al, PRD (2022)

- ➤ Simultaneous particlization and chemical freeze-out
  - ➡ Couple to hadronic afterburner

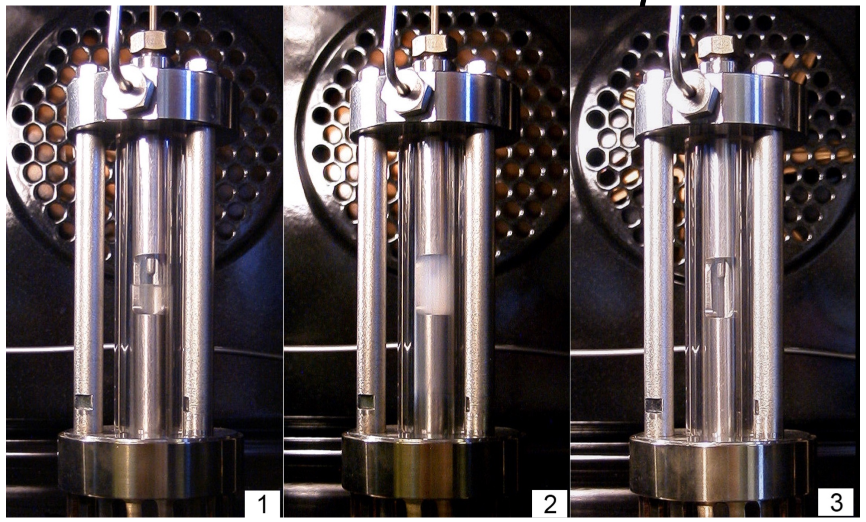
Critical effects seen to survive in SMASH:
J. Hammelman, M. Bluhm et al, PRC (2024)

***J.M. Karthein,** K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, PRD (2026, Editors' Suggestion)*

Experimental Observable - Cumulants

At critical point with an infinite system
- correlation length should diverge
- susceptibilities should diverge

ethane near critical point

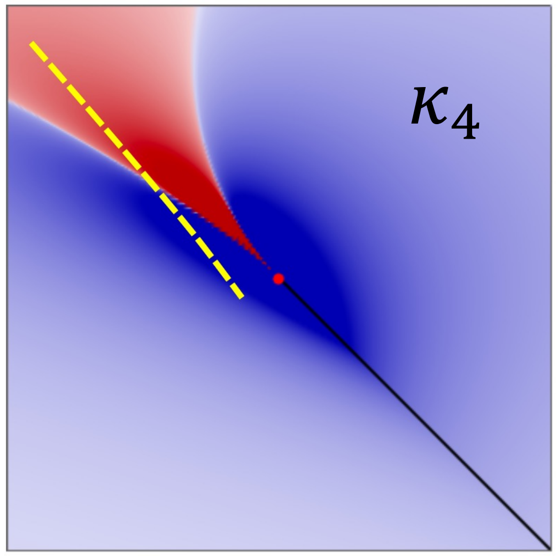


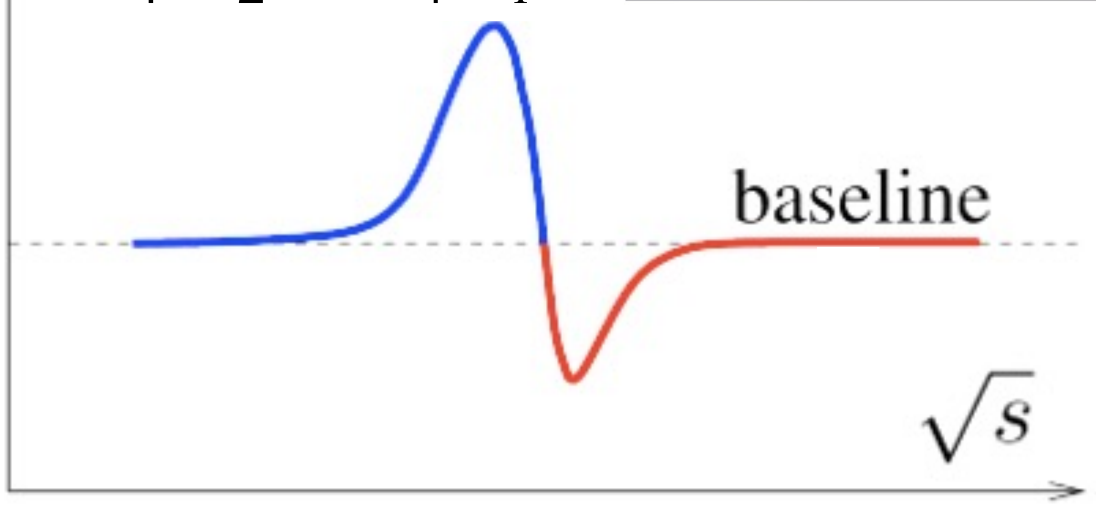
Experimental Observables:

Moments of conserved quantities: net-***B***, net-***S***, net-***Q*** etc.
- directly related to the susceptibility ratios
  - calculable from Lattice QCD
- sensitive to correlation lengths

$$\chi_n^B = -\frac{1}{3^n}\frac{\partial^n f/T^4}{\partial \hat{\mu}_q^n}$$

$$\begin{aligned}
\text{mean}: \mathrm{M} = \langle N \rangle &= C_1, \\
\text{variance}: \sigma^2 = \langle (\delta N)^2 \rangle &= C_2, \\
\text{skewness}: \mathrm{S} = \langle (\delta N)^3 \rangle / \sigma^3 &= C_3 / C_2^{3/2}, \\
\text{kurtosis}: \kappa = \langle (\delta N)^4 \rangle / \sigma^4 - 3 &= C_4 / C_2^2.
\end{aligned}$$





M. A. Stephanov, PRL 102 (2009) 032301

Notation:

	STAR	ALICE/HADES
Cumulants:	$C_{1,2,...}$	$\kappa_{1,2,...}$
Factorial Cumulants:	$\kappa_{1,2,...}$	$C_{1,2,...}$

Motivation >> Observables

N: Event-by-event multiplicity

$\delta N = N - \langle N \rangle$

- ☐ $C_1 = \langle N \rangle$
- ☐ $C_2 = \langle \delta N^2 \rangle$
- ☐ $C_1 = \langle \delta N^3 \rangle$
- ☐ $C_1 = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2$

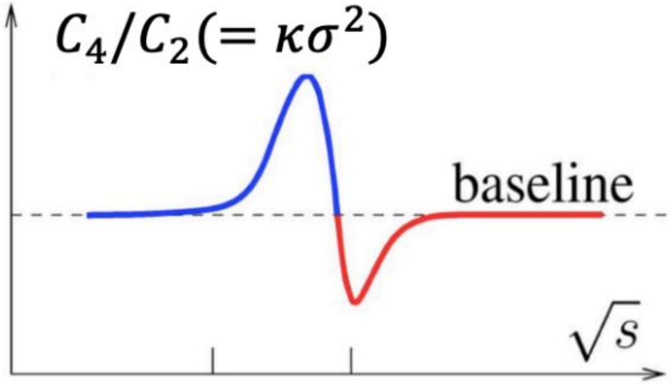
Cumulants

1) Related to correlation length ξ, $C_2 \sim \xi^2$ and $C_4 \sim \xi^7$
2) ξ diverges at CEP
3) Higher order: more sensitive

- ☐ $FC_1 = C_1$
- ☐ $FC_2 = -C_1 + C_2$
- ☐ $FC_3 = 2C_1 - 3C_2 + C_3$
- ☐ $FC_4 = -6C_1 + 11C_2 - 6C_3 + C_4$

Factorial cumulants

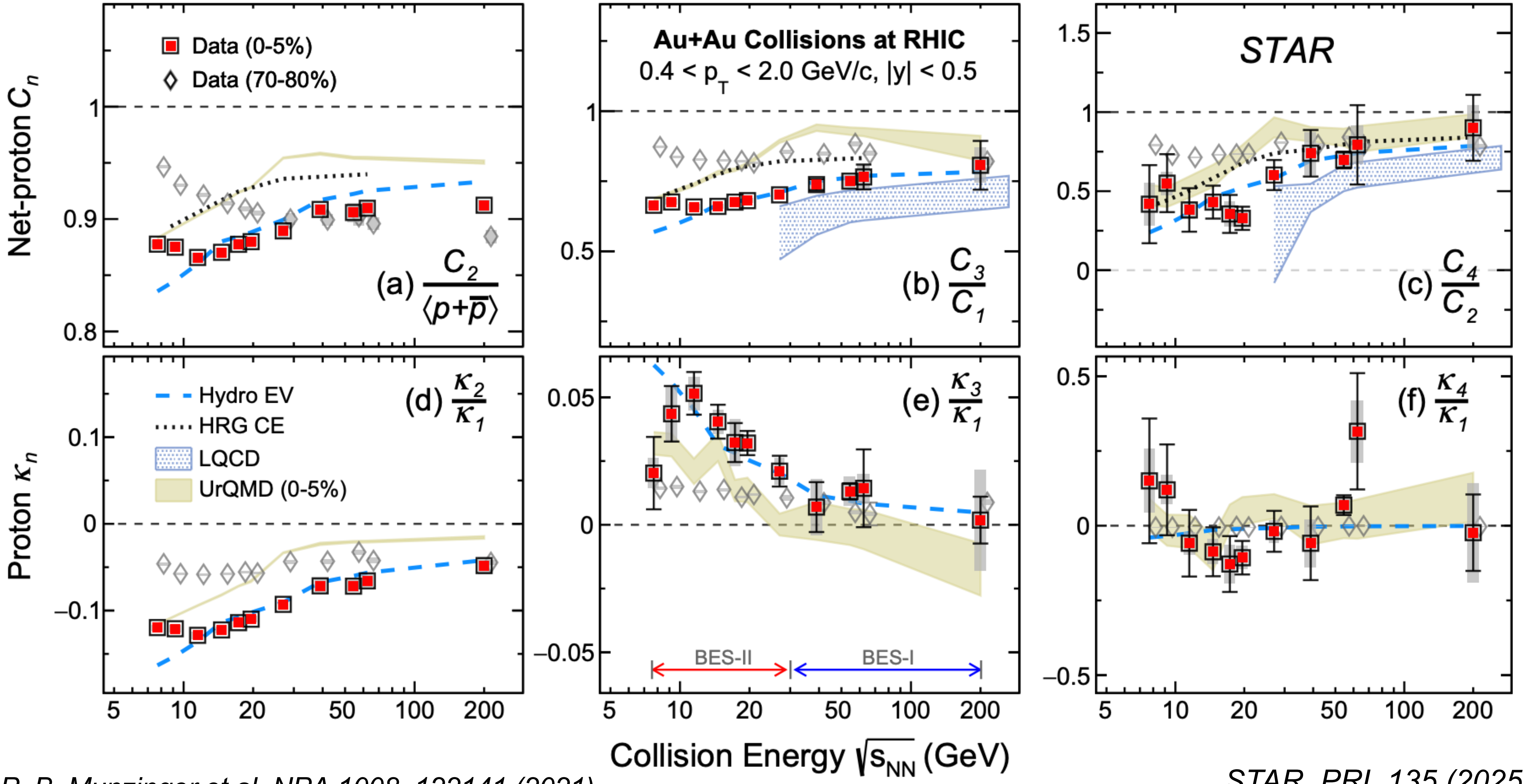
1) Can be expressed by linear combinations ordinary cumulant
2) Directly capture and are sensitive to genuine multi-particle correlations



Non-monotonic energy dependence of C_4/C_2 of conserved quantity indicates the existence of critical region.[1]

[1] M. A. Stephanov. Phys.Rev.Lett. 107 (2011) 052301

Net-proton Cumulant Ratios from RHIC BES-II



HRG CE: P. B. Munzinger et al, NPA 1008, 122141 (2021)
Hydro EV: V. Vovchenko et al, PRC 105, 014904 (2022)
LQCD: D. Bollweg et al, PRD 110, 054519 (2024)

STAR, PRL 135 (2025) 142305

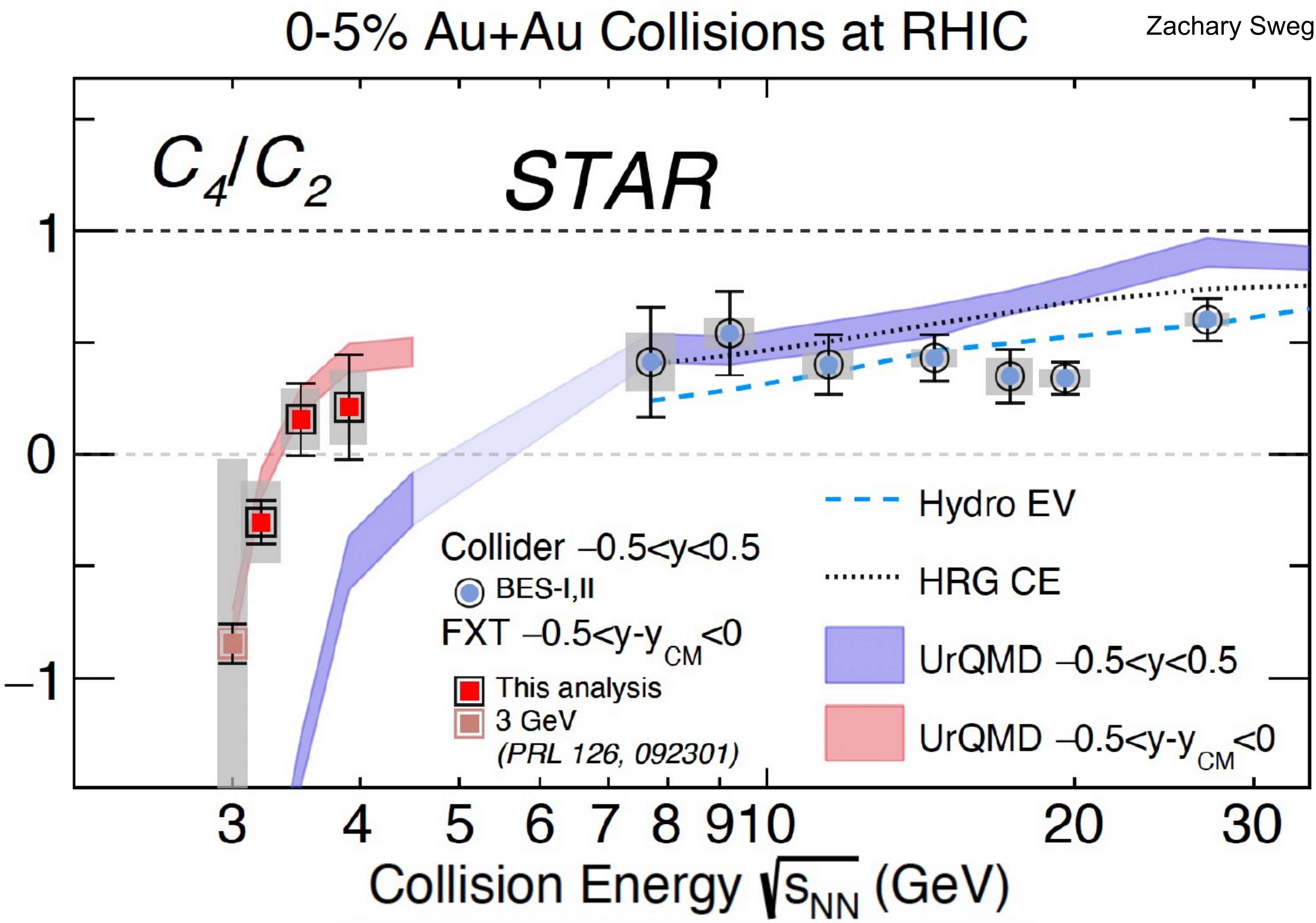
In 0-5% central Au+Au collisions

1) Net-proton C_4/C_2 shows deviation from non-CP references at ~ 20 GeV!

2) Lower order net-p cumulant ratios and proton factorial cumulant ratios show deviations from non-CP models too, especially below 11.5 GeV.

Proton C_4/C_2 from FXT Energies

Zachary Sweger, QM25



Mapping the QCD Phase Diagram

- **STAR and RHIC have done as promised. High statistics data, mapping the $\mu_B \leq 420$ MeV region.**
- **No evidence for a critical point in this region of the phase diagram. A significant experimental result.**
- **Theorists with parametrized equations of state will use new STAR data to constrain parameters.**
- **STAR Fixed Target (FXT) data coming soon. Measurements of these observables from $\sqrt{s} = 3$ GeV up to 4.5 GeV. STAR FXT acceptance limited above $\sqrt{s} = 4.5$ GeV.**
- **For discussion, but not today:**
  - **STAR collider data motivate exploring** $420 \text{ MeV} < \mu_B \lesssim 600$ **MeV, meaning** $7.7 \text{ GeV} > \sqrt{s} \gtrsim 4$ **GeV.**
  - **Several recent lattice-based theoretical explorations point to this region also.**
  - **STAR FXT will give us a good look at fluctuations in** $4.5 \text{ GeV} > \sqrt{s} > 3 \text{ GeV}$ **collisions, but what is the best option for** $7.7 \text{ GeV} > \sqrt{s} > 4.5 \text{ GeV}$ **collisions?**

Cold Quark Soup, Under Pressure

- **No way *we* can squeeze (i.e. crush) nuclei into quark matter without heating them up to trillions of degrees. So, where to find cold quark soup?**

- **At the centers of the heaviest neutron stars! Heaviest known neutron star has a mass 2.08 ± 0.14 times that of the sun, and a radius of only 11-15 km!**

- **The weight of the star crushes the matter at its center to 4-5 times the density of atomic nuclei, and to a pressure ~ 0.2 GeV/fm^3 $\sim 3 \times 10^{29}$ atmospheres.**

- **This matter (cold quark soup?) under pressure does *not* explode. Held in by the weight of all the outer layers of the neutron star.**

- **1.4 solar mass neutron stars are made of neutrons all the way in. Growing evidence that 2.0 solar mass neutron stars have cold quark soup in their cores!**

X-ray

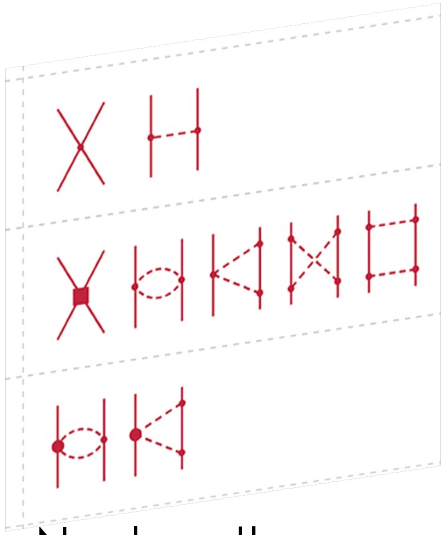
NICER, M-R measurements

Gravitational waves

LIGO, tidal deformability

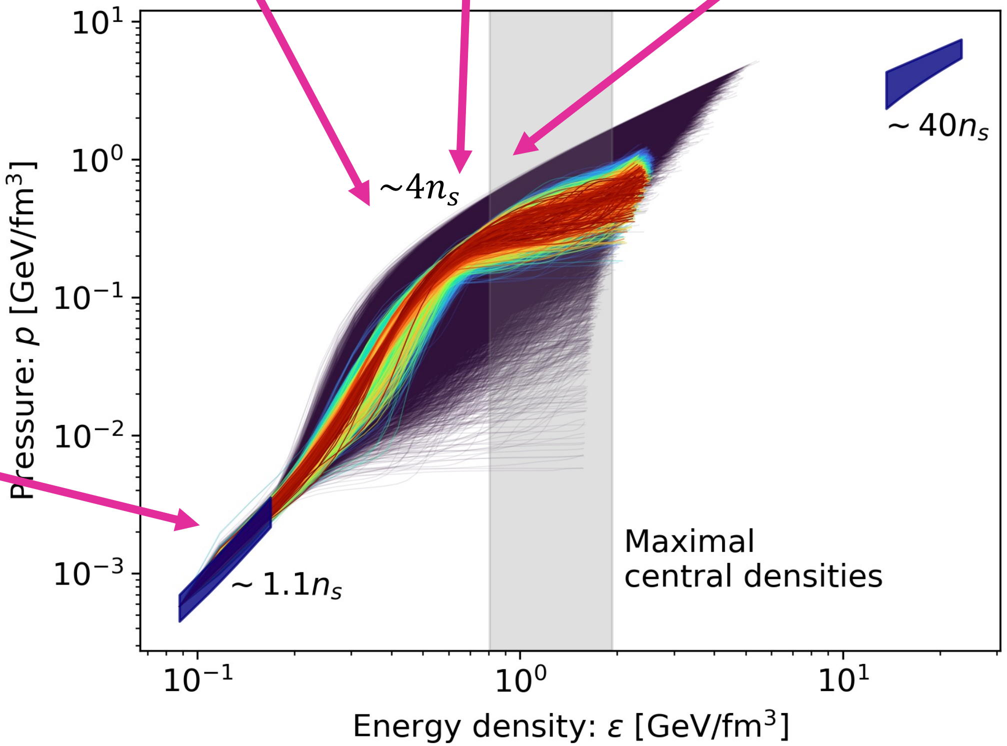
Radio astronomy

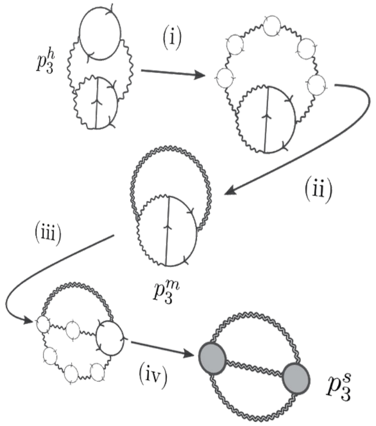
Pulsar timing, NS masses



Nuclear theory

Tews et al. PRL 110 (2013)
Hebeler, Lattimer et.al. APJ 773 (2013)
Drischler, Furnstahl et.al. PRL 125 (2020)
Keller, et al, PRL 130, 072701 (2023)



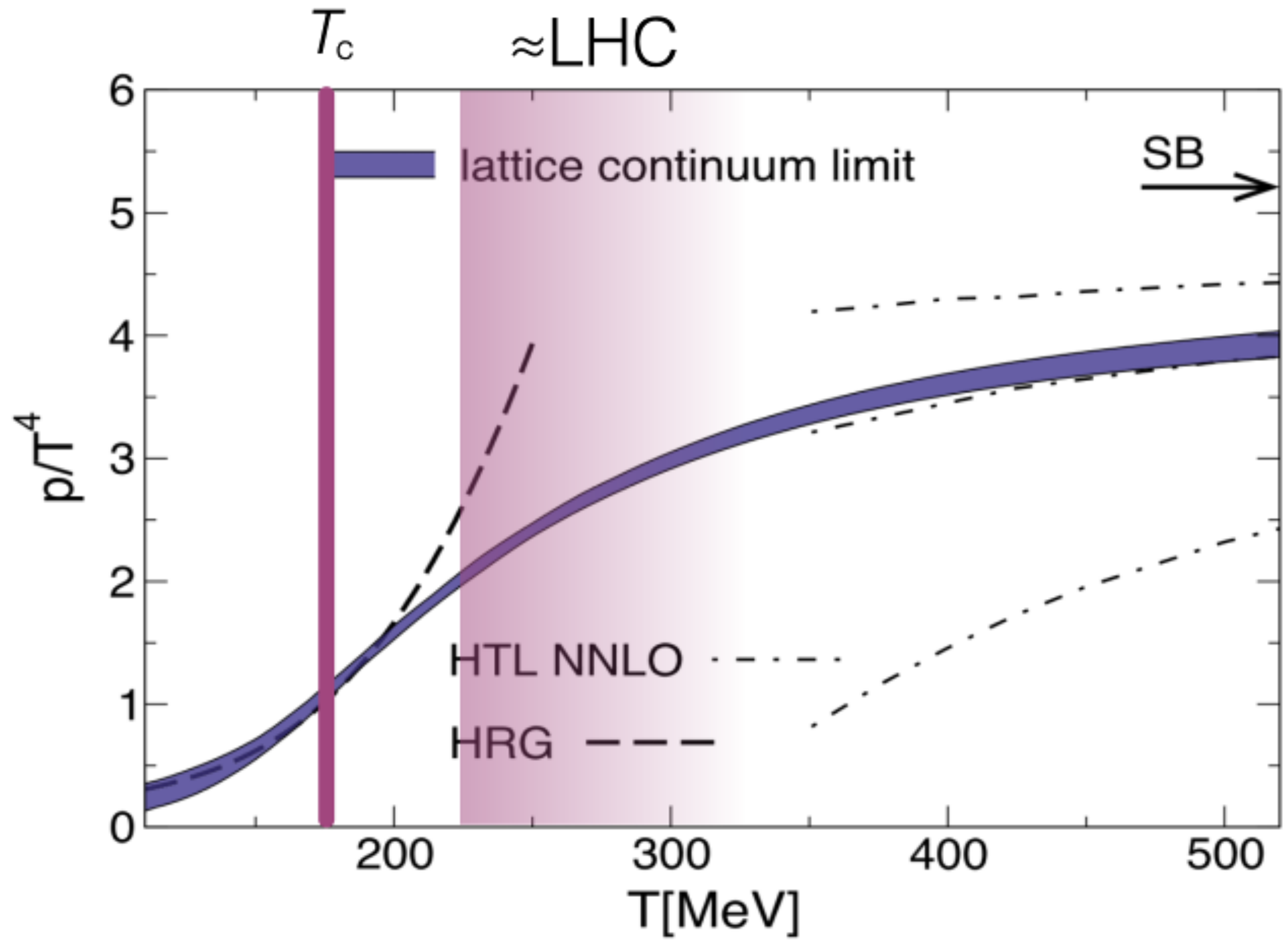


pQCD computations of Quark Matter phase

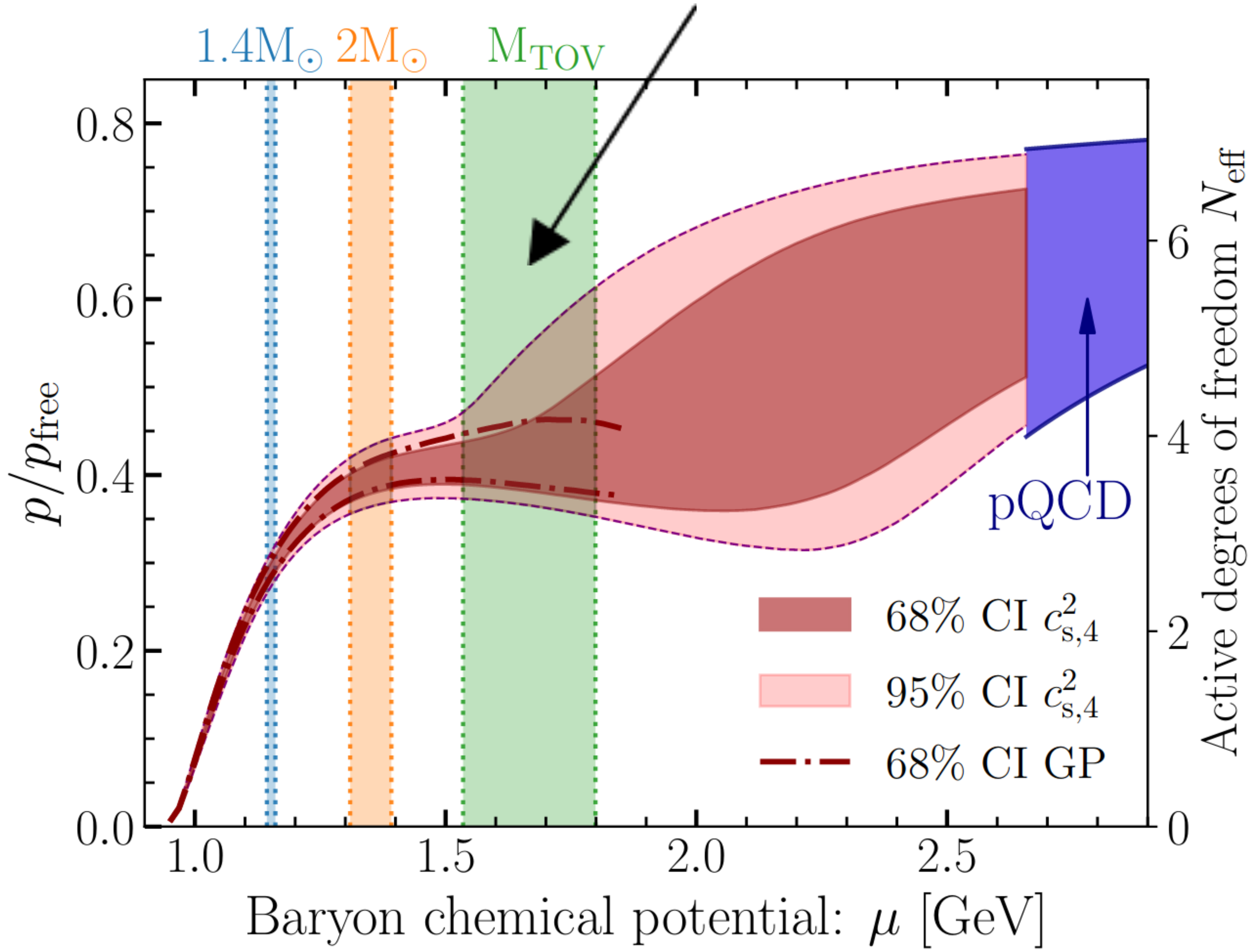
AK, Romatschke, Vuorinen, PRD 81 (2010)
Gorda, AK et al. PRL 121 (2018)
Gorda, AK et al. PRL 127 (2021)
Gorda, AK et al. PRD 107 (2023)
Gorda, Paatelainen et al. PRL131, (2023)
Talk by Paatelainen

Hebeler, Lattimer, et al., ApJ. 773, 11 (2013)
AK, Fraga, et al., ApJ. 789, 127 (2014)
…
…
Gorda, Komoltsev, AK, ApJ. (2023)

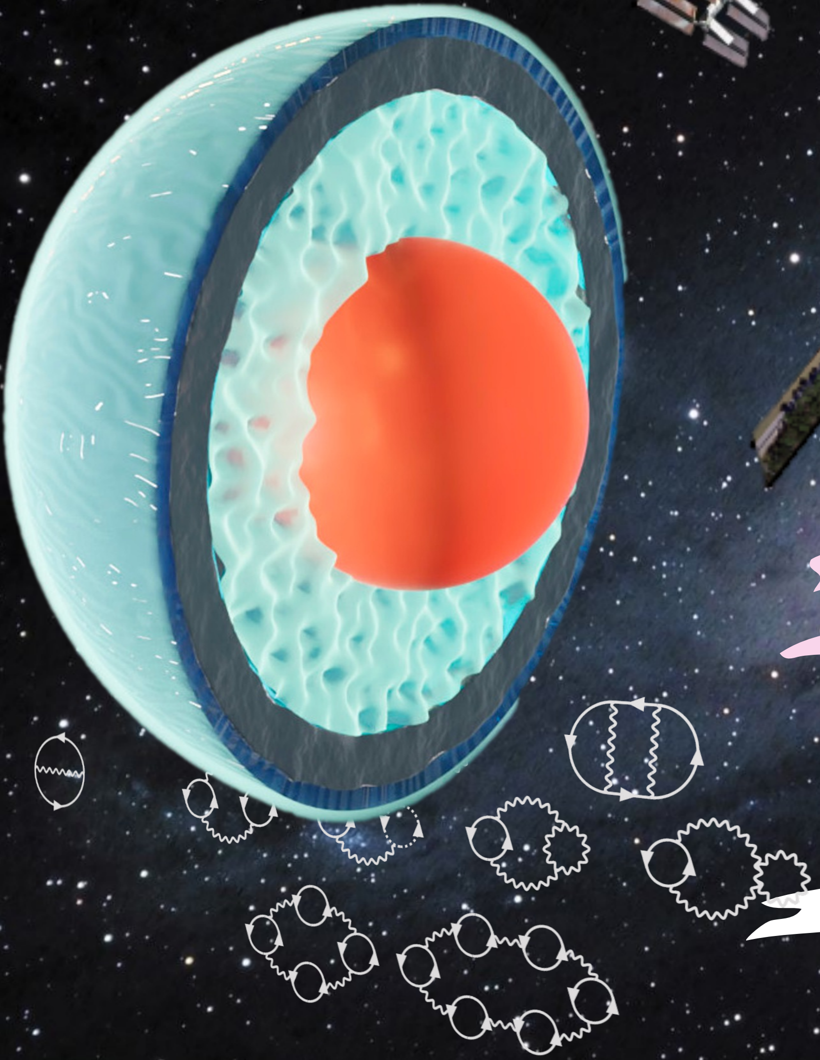
Number of Degrees of freedom consistent with deconfined quark matter



Maximum central density



Annala, TG+, Nat. Comm. 14 (2023)
Annala, TG+, Nat. Phys. 16 (2020)





Astrophysical Equation-of-state Constraints on the Color-Superconducting Gap

ALEKSI KURKELA,
KRISHNA RAJAGOPAL,
RACHEL STEINHORST

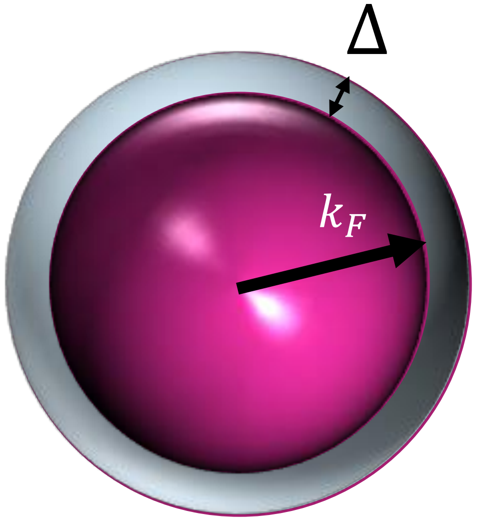
PHYS.REV.LETT. 132 (2024)

Color-super conductivity in Quark Matter

- If there is quark matter, it is most likely superconducting
  - At high enough densities, Color-Flavor-Locked pattern

Alford, Rajagopal, Wilczek, PLB 422 (1998)
Review: Alford, Rajagopal, Schaefer, Schmitt, Rev Mod Phys 80 (2008)

$$\left\langle \psi_i^{\alpha} C\, \gamma_5\, \psi_j^{\beta} \right\rangle = \Delta\, \epsilon^{\alpha\beta A} \epsilon_{ijA}$$



- The **gap** Δ difficult to compute:
  - Asymptotically, ***ab-initio***: $\Delta \approx 845 \frac{\mu}{g^5} e^{-\frac{20.9}{g}} \ll \mu$

Son, Phys. Rev. D 59, 094019 (1999)
Schäfer, Wilczek, PRD 60 (1999)

Pisarski, Rischke, PRD 61 (2000)
Brown, Liu, Ren, PRD 61 (2000)

  - At NS densities, **models**: $\Delta \approx 20\text{ MeV} - 250\text{ MeV}$, even $\Delta \approx$ 300MeV

Rapp, Schäfer, Shuryak, Velkovsky, 81 (1998)
G. W. Carter and D. Diakonov, PRD 60, (1999)
Baym, Hatsuda, Kojo, et al. Rept. Prog. Phys. 81, (2018)
Leonhardt, Pospiech, Schallmo, et al. PRL 125 (2020), …

Alford, Rajagopal, Wilczek, NPB 537 (1999)
Berges and Rajagopal, NPB 538 (1999)
Wang and Rischke, PRD 65 (2002)
Brown, Liu, Ren, PRD 61 (2000),

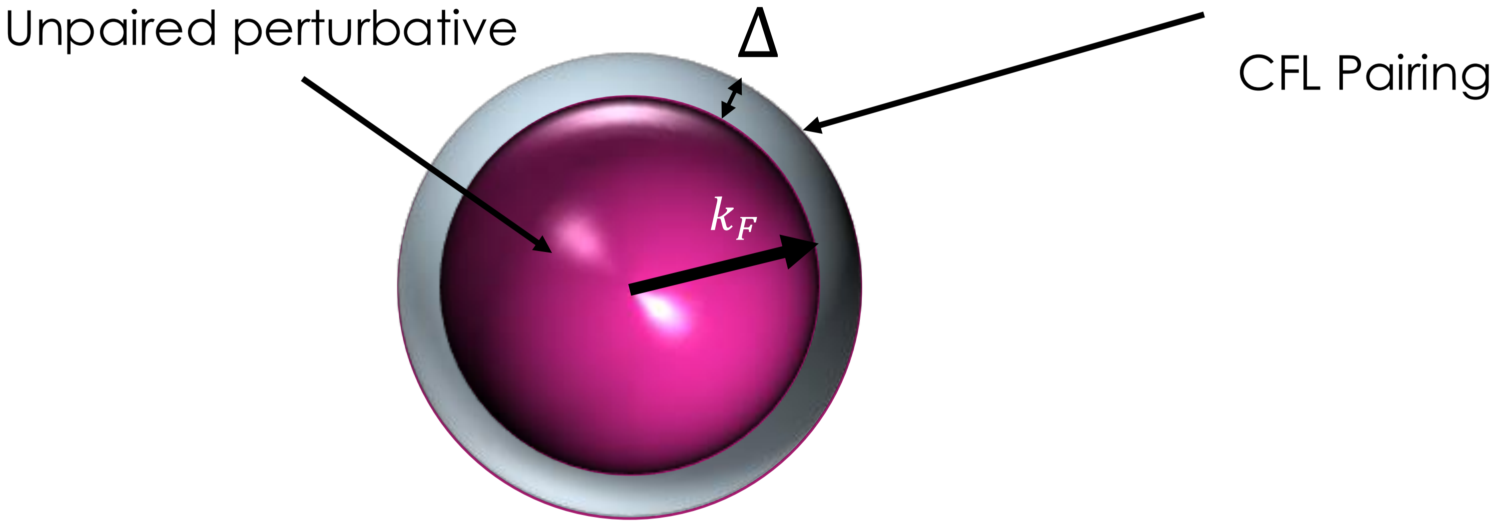
Braun, Schallmo, PRD 105 (2022)

How to **empirically** measure the gap Δ using Neutron Stars?

Color-superconducting thermodynamics

- Pairing affects transport strongly. A small positive correction the pressure
- At perturbative densities:

$$p_H \approx \frac{3}{4\,\pi^2}\,\mu^4(1+c_1g^2+c_2g^4+\ \dots)+\frac{3}{\pi^2}\Delta^2\mu^2$$

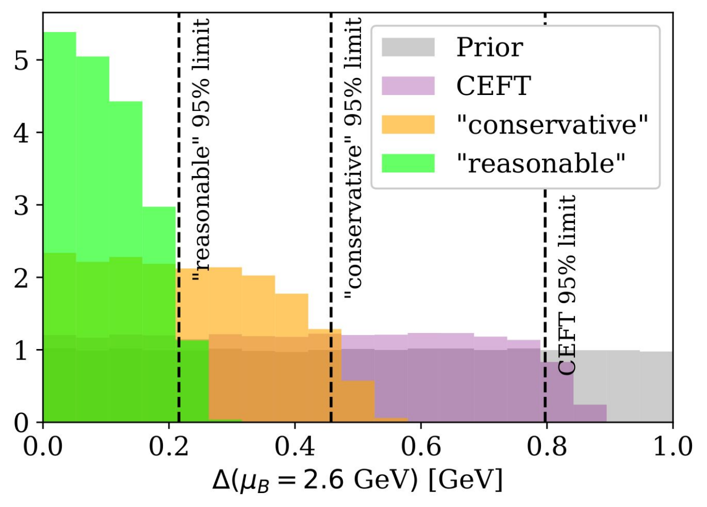


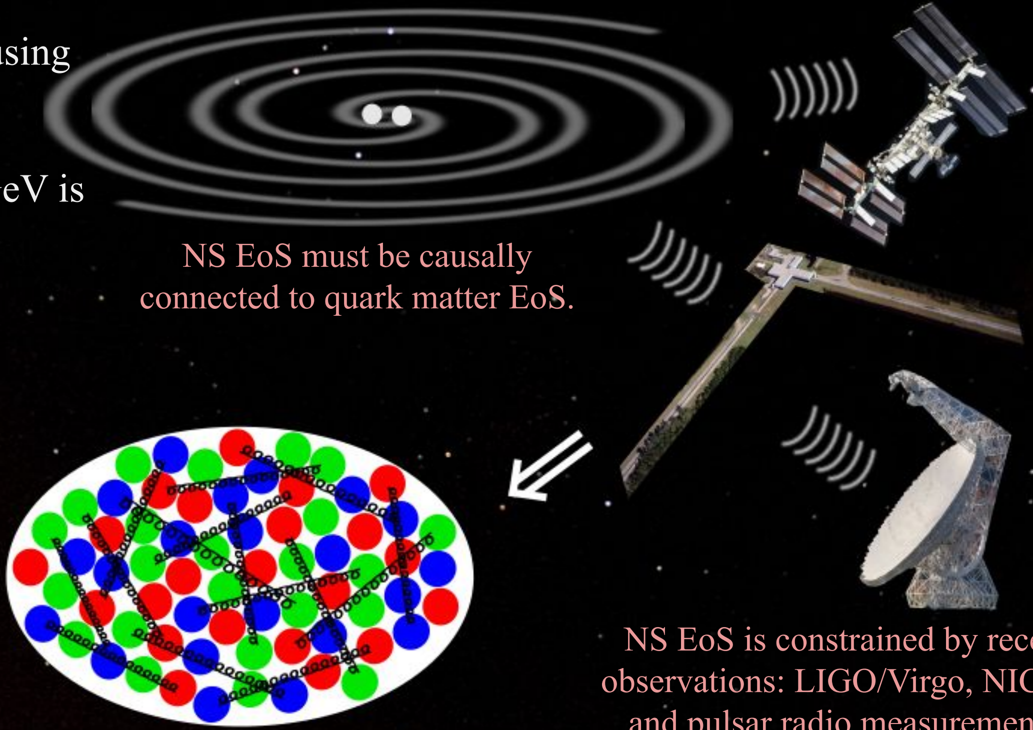
- Here: Constrain gap at **perturbative** densities $\Delta(\mu_B = 3\mu = 2.6\ \mathrm{GeV})$ using NS observations

Astrophysical Equation-of-State (EoS) Constraints on the Color-Superconducting Gap

[Kurkela, Rajagopal, Steinhorst] arXiv:2401.16253

- First constraint of quark matter properties using astrophysical observations of neutron stars!
- Constraint of $\Delta_{CFL} < 216$ MeV at $\mu_B = 2.6$ GeV is within striking distance of theory estimates



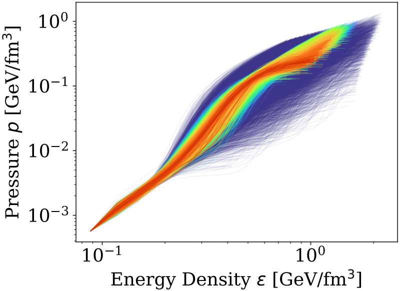


NS EoS must be causally connected to quark matter EoS.

Cooper pairs form in cold dense quark matter, affecting the EoS.

NS EoS is constrained by recent observations: LIGO/Virgo, NICER, and pulsar radio measurements.

Setup for Bayesian analysis



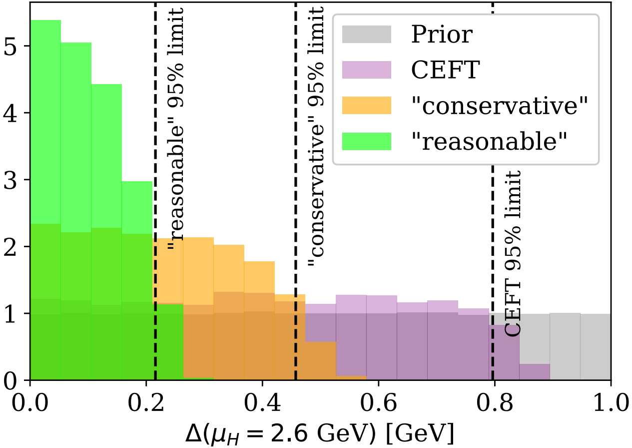
$$P(\Delta|\mathrm{data}) = \int P(\Delta|\mathrm{EoS})P(\mathrm{EoS}|\mathrm{data})$$

$$P(\Delta|\mathrm{EoS}) = \frac{P(\mathrm{EoS}|\Delta)P(\Delta)}{P(\mathrm{EoS})}$$

We consider two scenarios:

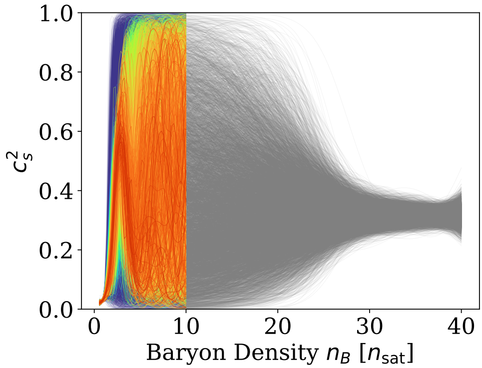
- **Conservative:** $c_s^2 \leq 1$, infer NS EoS up to no more than 2.1 $M_\odot$.
- **Reasonable:** $c_s^2 \leq 1/2$, infer NS EoS up to most massive stable NS.

Result: An Empirical Constraint on Non-Perturbative Physics!



Kurkela, Rajagopal, Steinhorst arXiv:2401.16253

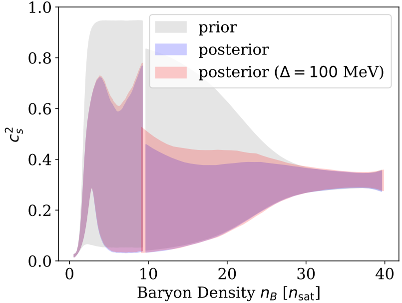
Another reasonable approach...

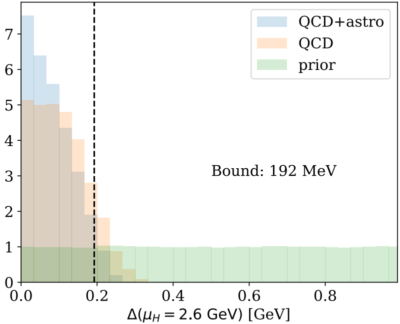


Another way to infer the EoS is to weight based on how many ways there are to connect a given low-energy EoS to QCD, and vice versa.
We weight based on how well (ε, p) matches at fixed $n_s = 9.5$.

Komoltsev et al arXiv:2312.14127

Another reasonable approach...





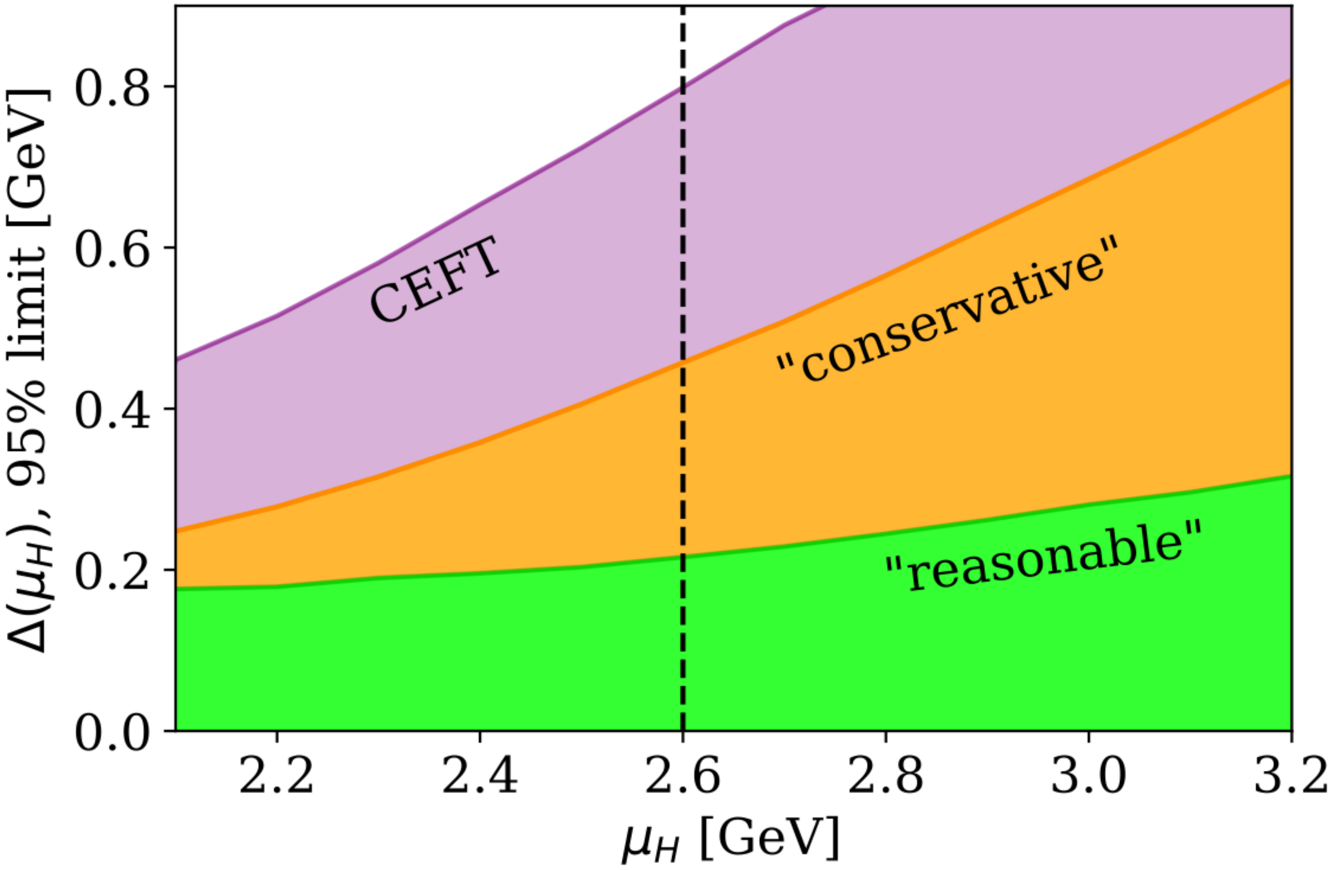
We see that the assumption of $c_s^2 \leq 1/2$ is indeed reasonable. Matching the ensembles yields a slightly stronger constraint!

Kurkela, Rajagopal, RS (in progress)

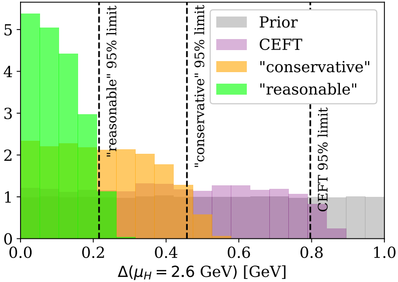
Density dependence of the gap

For low densities,
bound gets tighter

but pQCD less reliable

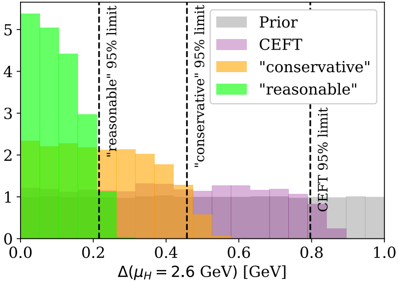


Conclusions



- We find the first empirical constraint of the color superconducting gap using astrophysical observations.
- With reasonable assumptions, we constrain the value of Δ to $\lesssim 200$ MeV - close to phenomenologically relevent values!

Conclusions



- We find the first empirical constraint of the color superconducting gap using astrophysical observations.
- With reasonable assumptions, we constrain the value of Δ to $\lesssim 200$ MeV - close to phenomenologically relevent values!

Conclusions:

- The requirement of **stability**,**causality** and **consistency** connect pQCD and neutron stars
- Here, first **empirical** constraints on gap using **astrophysical** observations
- With **reasonable** assumptions, gaps of $\mathbf{\Delta > 216\ MeV}$ are excluded
  - Starting to constrain phenomenologically relevant values Braun, Schallmo, PRD 105 (2022)
  - New observations and improved QCD calculations will lead to stronger constraints
  - Strong motivation for N3LO pQCD. Possible even lower limit? Talks Paatelainen, Gorda

- Inclusion of the gap makes QCD constraint more stringent.
  For inference **Δ = 0 is the most conservative** choice

Cold Quark Soup, Under Pressure

- No way *we* can squeeze (i.e. crush) nuclei into quark matter without heating them up to trillions of degrees. So, where to find cold quark soup?

- At the centers of the heaviest neutron stars! Heaviest known neutron star has a mass 2.08 ± 0.14 times that of the sun, and a radius of only 11-15 km!

- The weight of the star crushes the matter at its center to 4-5 times the density of atomic nuclei, and to a pressure ~ 0.2 GeV/fm^3 $\sim 3 \times 10^{29}$ atmospheres.

- This matter (cold quark soup?) under pressure does *not* explode. Held in by the weight of all the outer layers of the neutron star.

- 1.4 solar mass neutron stars are made of neutrons all the way in. Growing evidence that 2.0 solar mass neutron stars have cold quark soup in their cores!

Recall Jets as Probes of QGP. . .

- What I described previously was outstanding progress toward seeing *individual* quasiparticles, at short length scales, in QGP. And the path to doing so.

- The path to addressing the question: How can we use jets to see the inner workings of QGP?

- But, we *know* that QGP is a strongly coupled liquid. That means that short-range quasiparticle-quasiparticle correlations are strong and important.

- Could we *ever* see how the quasiparticle right next to the quasiparticle that a jet parton just struck and scattered off responds? Could we ever see strong *Short-Range Correlations* in QGP?

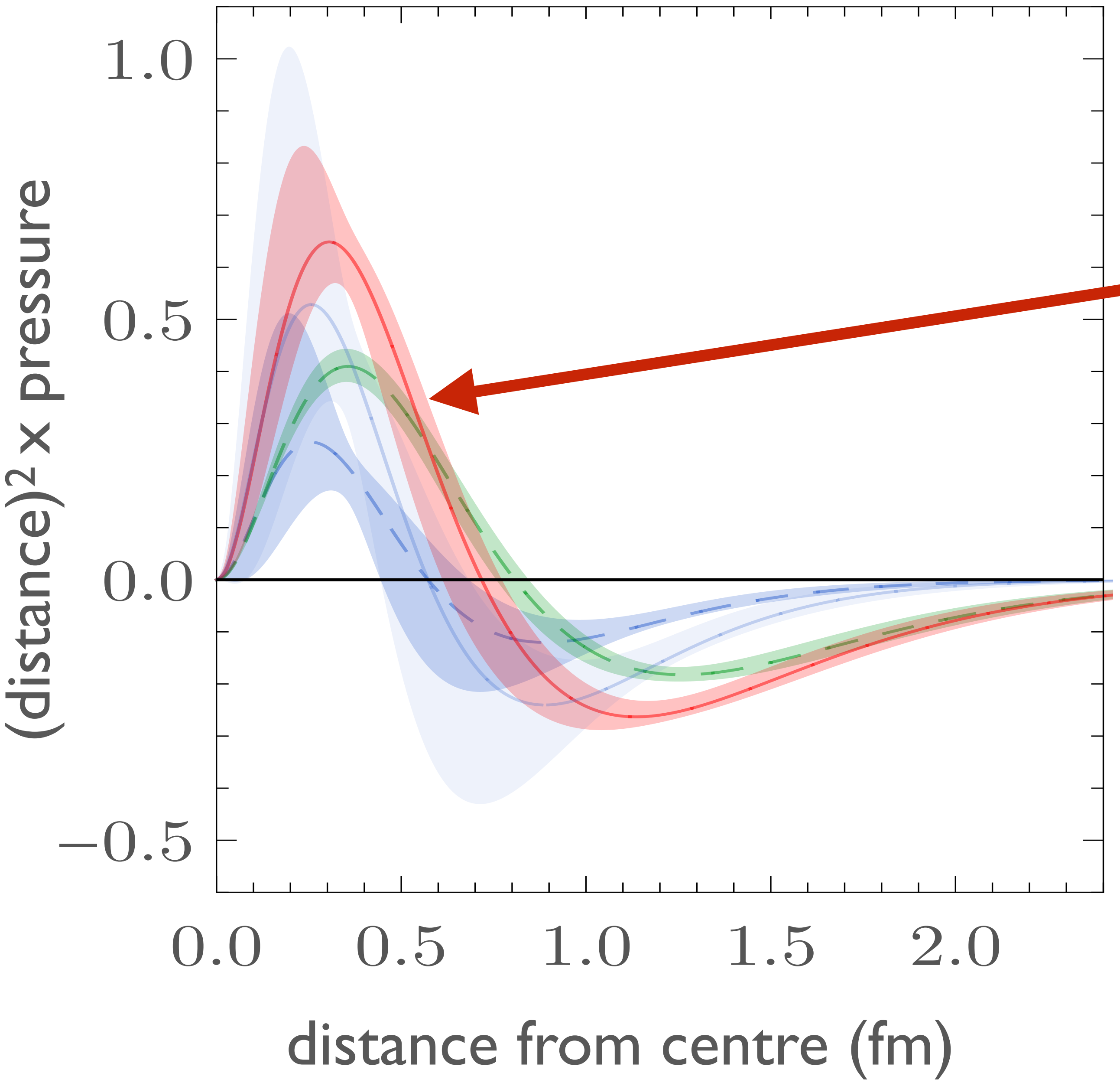
- This would take our investigation of the microscopic structure of QGP to a new level . . .

- Surely impossible. Or is it?

Quark-Gluon Matter Under Pressure

- **Where else do we find quark-gluon matter under pressure?**
- **In a proton!! Protons form from tiny drops of QGP, 10-20 microseconds after the big bang, or 10$^{-22}-10^{-21}$ seconds after a HIC. What is the pressure inside a proton?**
- **Answer: $\sim$ (0.07-0.7) GeV/fm^3 $\sim (1.1-11)\times 10^{29}$ atmospheres!** [Extracted from SIDIS data (Burkert, Elouadrhiri, Girod, '18) and from lattice calculations (Shanahan, Detmold '18; Pefkou, Hackett, Shanahan '21, '23)]
- **This is pressure of QGP at $T=T_p\sim 155-200$ MeV, at or just above T at which it falls apart into a mist of protons!**
- **Remarkably, protons have kept this high interior pressure since they formed.**
- **Even more remarkably, protons don't explode! That is the essence of confinement — which squeezes/compresses a tiny bit of 155 MeV QGP into each proton at its birth.**

Pressure inside the proton



Total pressure including **gluon** (lattice QCD) and **quark** (experiment) contributions

- Total pressure by combining theory + experiment
- Peak pressure near centre $\sim 10^{35}$ Pascal, greater than estimated for neutron stars
- Gluons extend radial pressure distribution

[V. D. Burkert et al, Nature 557 (2018)]

[*Phys.Rev.Lett.* 122, 072003 (2019), *Phys.Rev.D* 99, 014511 (2019), *Phys.Rev.D* 108, 114504 (2023), *Phys.Rev.Lett.* 132, 251904 (2024)]

Quark-Gluon Matter Under Pressure

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- **Answer: $\sim$ (0.07-0.7) GeV/fm^3. This is the pressure of QGP at $T = T_p \sim 155 - 200$ MeV, at or just above T at which it falls apart into a mist of protons!**

- **Note: spherical droplet of QGP with $V_p \equiv 938$ MeV$/\varepsilon(T_p)$, $\varepsilon(T)$ from lattice QCD, and radius 0.86 fm / 0.49 fm has $T_p =$ 155/200 MeV, ie. ε and pressure of a proton.**

- **Remarkably, protons have kept this high interior pressure since they formed.**

- **Even more remarkably, protons don't explode! That is the essence of confinement — which squeezes/compresses a tiny bit of 155 MeV QGP into each proton at its birth.**

Entropy of Protons through a QGP Lens

- **If pressure in a proton is pressure of hot quark-gluon plasma with $T = T_p \sim 155 - 200$ MeV...**
- **What about entropy? The hot QGP from which one proton forms after the Big Bang has $S_{\text{thermal}} \sim (5.2 - 7.3)k_B$. Isn't the entropy of a proton zero? What about the second law of thermodynamics?**
- **Quantum state of a proton has *entanglement entropy*. Three estimates, each qualitative, with different uncertainties: $(7-8)k_B$; $(7-8)k_B$; $(5-9)k_B$** [Kharzeev, KR, arXiv:2605.19058]
- **Thermal entropy from Big Bang QGP → entanglement entropy inside protons → released as thermal entropy (and more entropy produced) when nuclei collide.**
- **Confinement (the essence of confinement?) as the rearrangement of thermal entropy into entanglement entropy!**

Entropy of Protons through a QGP Lens

Kharzeev, KR 2605.19058

- **One estimate (that I won't describe today) extrapolates from past work quantifying longitudinal entanglement entropy of struck parton in DIS with the other partons.**
- **Second estimate (that I also won't describe today) is an attempt to use basic facts that we know about protons to estimate the entanglement entropy of the three valence quarks, by tracing over all the rest.**
- **Both give similar rough estimate:** $S_{\text{entanglement}}/k_B \sim 7-8$, **with uncertainties hard to quantify, but different.**
- **Third estimate starts by taking seriously the notion that a proton has an internal effective temperature** $T_p \ldots$. **Although** $\rho = |p\rangle\langle p|$ **has** $\text{Tr}\rho \ln \rho = 0$, **if we decompose the Hilbert space into resolved (infrared) and unresolved (ultraviolet) sectors, the reduced density matrix** $\rho_{\text{IR}} = \text{Tr}_{\text{UV}}|p\rangle\langle p|$ **can take an approximately thermal form.**

Entropy of Protons through a QGP Lens

Kharzeev, KR 2605.19058; with thanks to Mark Srednicki

- **A somewhat formal argument, that does not yield a quantitative estimate...**
- **If $E_p = \mathrm{Tr}\rho_p H$, where the reduced density matrix of the proton takes the thermal form**

$$\rho_p = \frac{1}{Z(T_p)} e^{-H/T_p} \ , \qquad Z(T_p) = \mathrm{Tr} e^{-H/T_p}$$

where T_p is some effective internal temperature characterizing the entanglement spectrum of the quantum state of the proton, not the temperature of an external bath, then

- **formally, equating $E_p = \mathrm{Tr}\rho_p H = -\partial/\partial\beta \ln Z$ serves to fix T_p in terms of E_p.**
- **Then, $S/k_B = -\mathrm{Tr}\rho_p \ln \rho_p$. Voilà!**

Entropy of Protons through a QGP Lens

Kharzeev, KR 2605.19058

- **A somewhat informal argument, that yields an estimate** . . .
- **A proton has a Hagedorn density of excited states with invariant mass** M**:** $\rho_p^H(M) = A_p \exp(M/T_H)/(M^2+M_r^2)^{5/4}$ **where a fit to masses of** > 3000 **hadrons gives** $M_r = 0.5$ GeV**,** $T_H = 0.1672$ **GeV, and** $A_p < A = 0.4735$ **GeV**$^{3/2}$**.**
- **Take the idea of** T_p **seriously: probability that proton is found in an excited state with mass** M **is**

$$p(M) = \frac{1}{Z}\rho_p(M)\exp\left(-\frac{M}{T_p}\right) \qquad Z = \int_{M_p}^{\infty} dM\,\rho_p^H(M)\exp\left(-\frac{M}{T_p}\right)$$

and

$$S/k_B = -\int_{M_p}^{\infty} dM\,\rho_p^H(M)\frac{p(M)}{\rho_p^H(M)}\ln\left(\frac{p(M)}{\rho_p^H(M)}\right) = \int_{M_p}^{\infty} dM\,p(M)\left(\frac{M}{T_p} + \ln Z\right)$$

Entropy of Protons through a QGP Lens

Kharzeev, KR 2605.19058

- $\rho_p^H(M) = A_p \exp(M/T_H)/(M^2 + M_r^2)^{5/4}$ **with** $M_r = 0.5$ GeV, $T_H = 0.1672$ **GeV, and** $A_p < A = 0.4735$ **GeV**$^{3/2}$**.**

- **Take the idea of** T_p **seriously: probability that proton is found in an excited state with mass** M **is**

$$p(M) = \frac{1}{Z}\rho_p(M)\exp\left(-\frac{M}{T_p}\right) \qquad Z = \int_{M_p}^{\infty} dM\,\rho_p^H(M)\exp\left(-\frac{M}{T_p}\right)$$

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- **Note:** Z **is finite as long as** $T_p < T_H$**. Note:** S **depends on** A_p **only logarithmically; take** $A_p = A/10$**; lack of knowledge of** $\ln A_p/A$ **is a source of uncertainty.**

- **For** $T_p = 155/160/165$ **MeV, we find** $S/k_B = 5.3/6.3/8.5$ **!!!**

- **Third estimate:** $S \sim (5-9)k_B$**.**

Entropy of Protons through a QGP Lens

Kharzeev, KR 2605.19058

- The agreement between the Gibbs entropy of the drop of QGP from which a proton forms at freezeout and three very different estimates of the internal (entanglement) entropy of a proton could be a quadruple coincidence.
- Or, will poking at this coincidence — and the questions it raises — advance our understanding of confinement, hadron structure, QGP, and the QCD crossover transition?
- How does idea that at the QCD transition Gibbs entropy of QGP rearranges into quantum entanglement entropy within hadrons help us to understand freezeout? Entropy production in collisions? Confinement?

Quark-Gluon Matter, Under Pressure

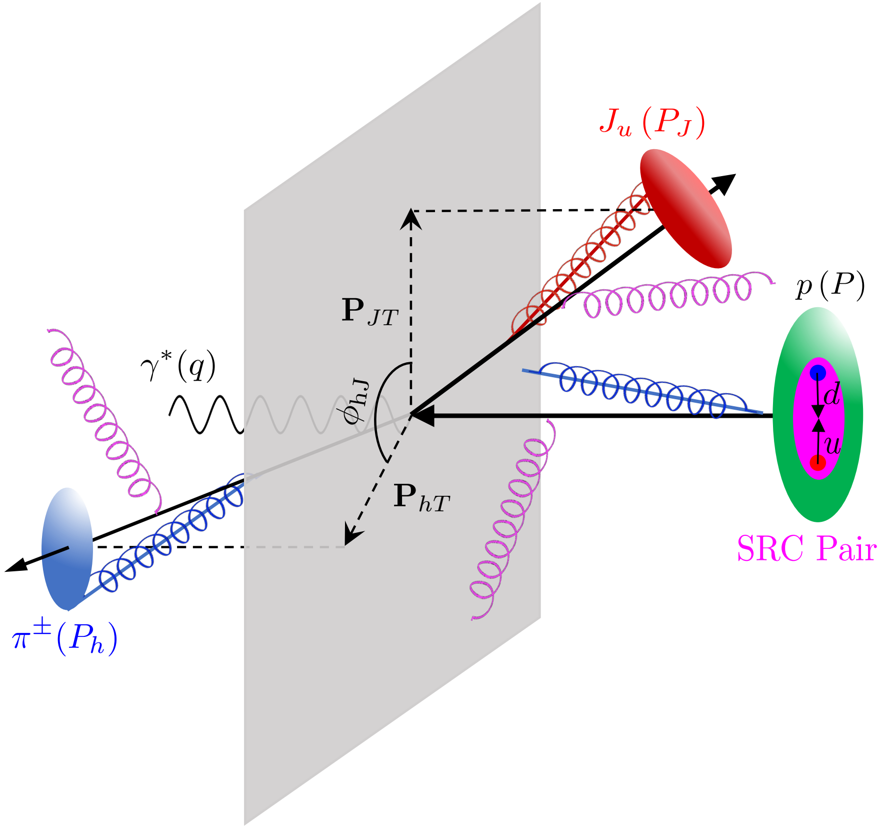
- **Protons form from tiny drops of QGP. The internal pressure, energy density,** *and entropy* **of a proton are comparable to that of the drop of QGP from which it formed.**

- **What further lessons can we learn about the interior of a proton from hot QGP? The same strong correlations that make QGP a liquid, and that give the proton its entanglement entropy, can be directly measured at the Electron-Ion Collider!!** [Peng, KR, Terry, 2606.17133]

- **QGP at temperatures $\sim T_p$ is a strongly coupled fluid: strong correlations between the momenta of fluid cells that are near each other in position space. Strong** *Short-Range Correlations*. **Macroscopic manifestation: strongly coupled liquid QGP. Can we see the microscopic mechanism, directly, within protons?? How can** *partonic SRCs* **be measured at the EIC?** [Peng, KR, Terry, 2606.17133]

An EIC Measurement...

[Peng, KR, Terry, 2606.17133]



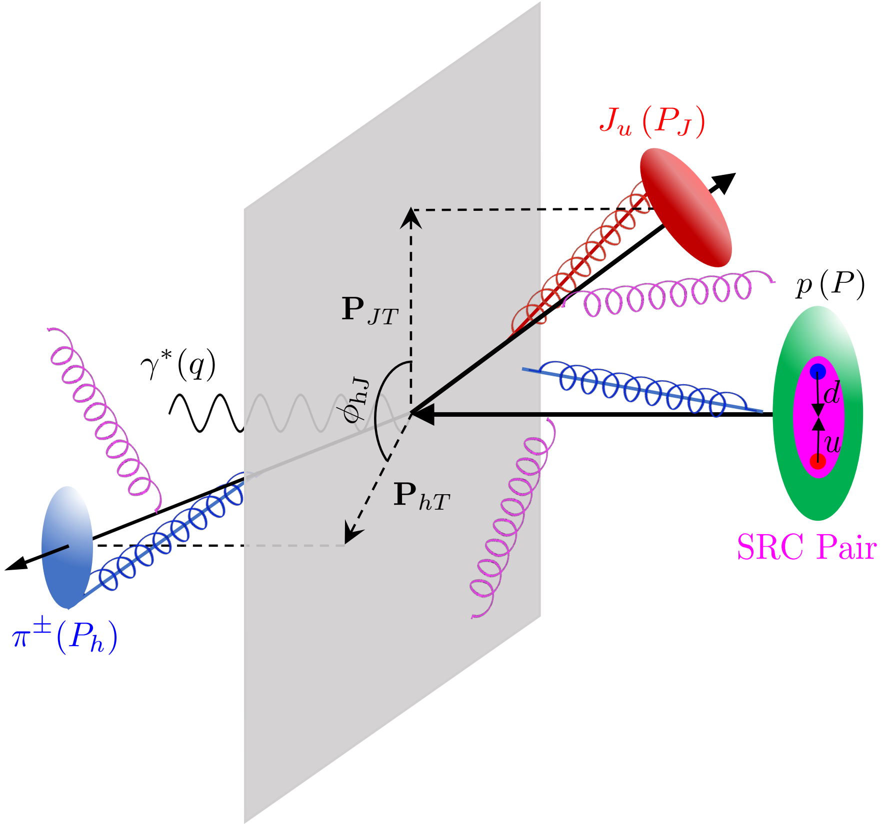
Incident electron (via virtual photon) kicks a u quark from the proton.

If all you measure is the scattered electron and the jet, all you can learn about is (generalized) parton distribution functions.

So, measure the blue hadron too!

An EIC Measurement…

[Peng, KR, Terry, 2606.17133]



Measure the correlations in ϕ (azimuthal angle around the virtual-photon direction) between…

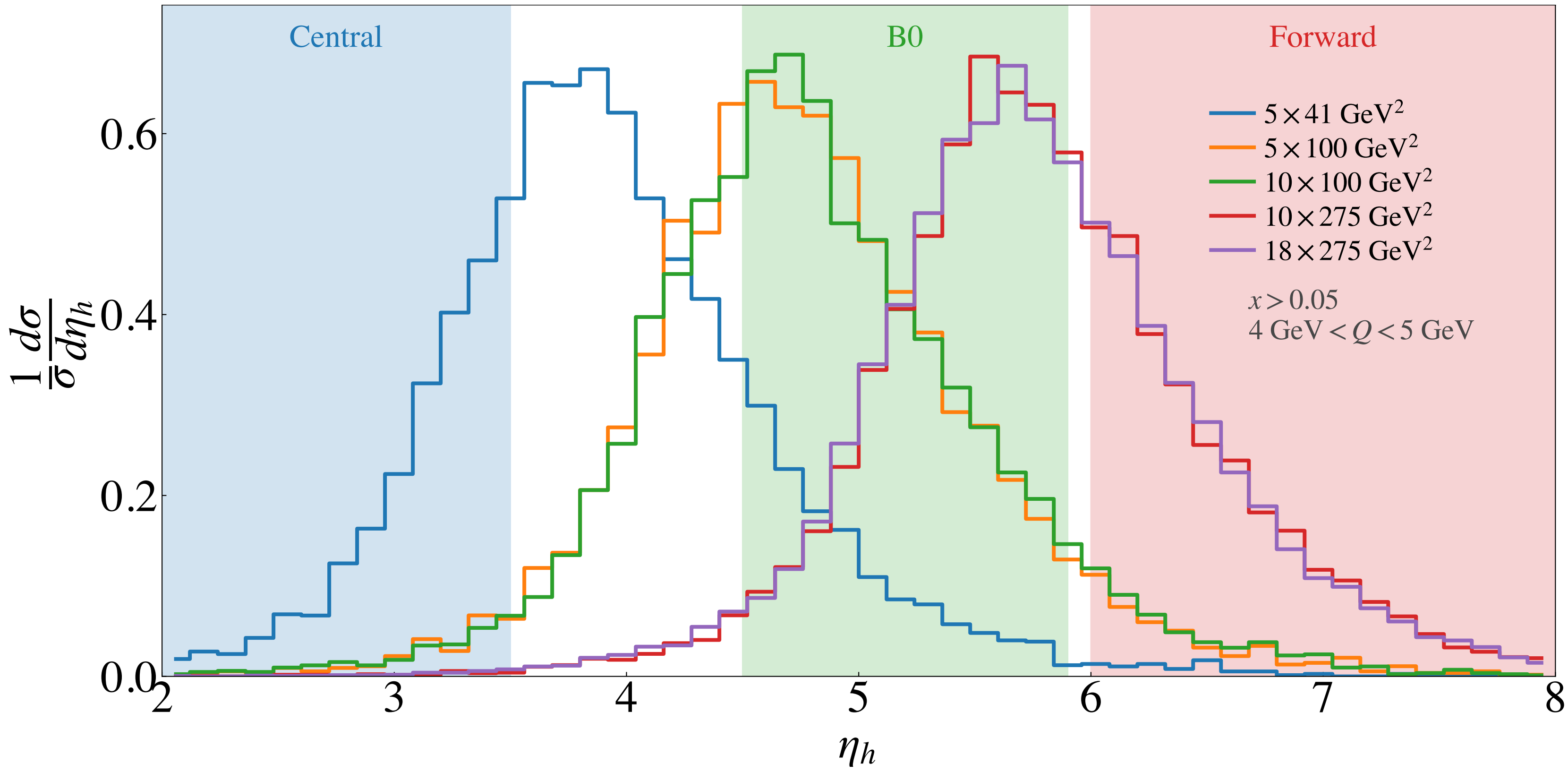
… the u-jet (jet with leading π^+) that the virtual photon kicked …

… and a π^- (containing a d-quark that the u was strongly correlated with).

Direct access to strong Diquark Short Range Correlations inside a proton.
Which is to say inside quark-gluon matter under pressure.

An EIC Measurement. . .

[Peng, KR, Terry, 2606.17133]

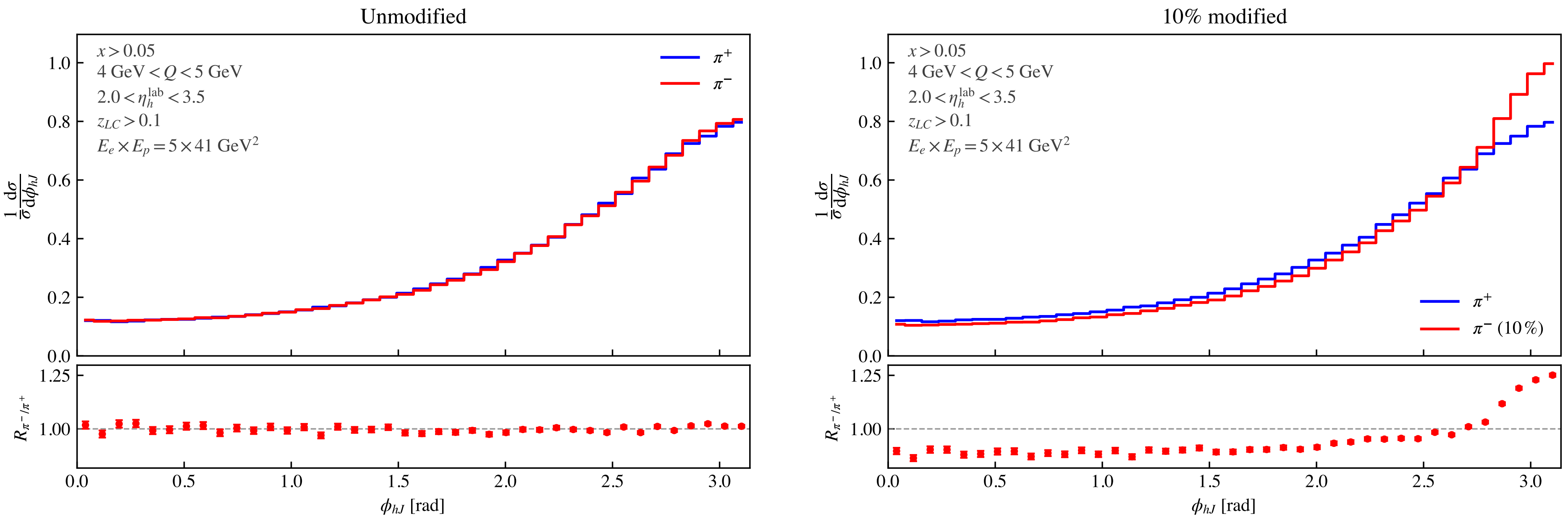


EIC is *designed* to measure the scattered electron and hence Q. EIC is *deisgned* to measure the jet that the virtual photon kicked.

PYTHIA simulation shows that $\pi^{+/-}$ from fragments of the proton can be measured in the central detector in EIC collisions with $E_e = 5$ or 10 GeV and $E_p = 41$ GeV. Also makes clear that B0 detector is important for larger E_p, if it can identify $\pi^{+/-}$. On next slides, $E_p = 41$ GeV and $\eta_h < 3.5$.

An EIC Measurement. . .

[Peng, KR, Terry, 2606.17133]

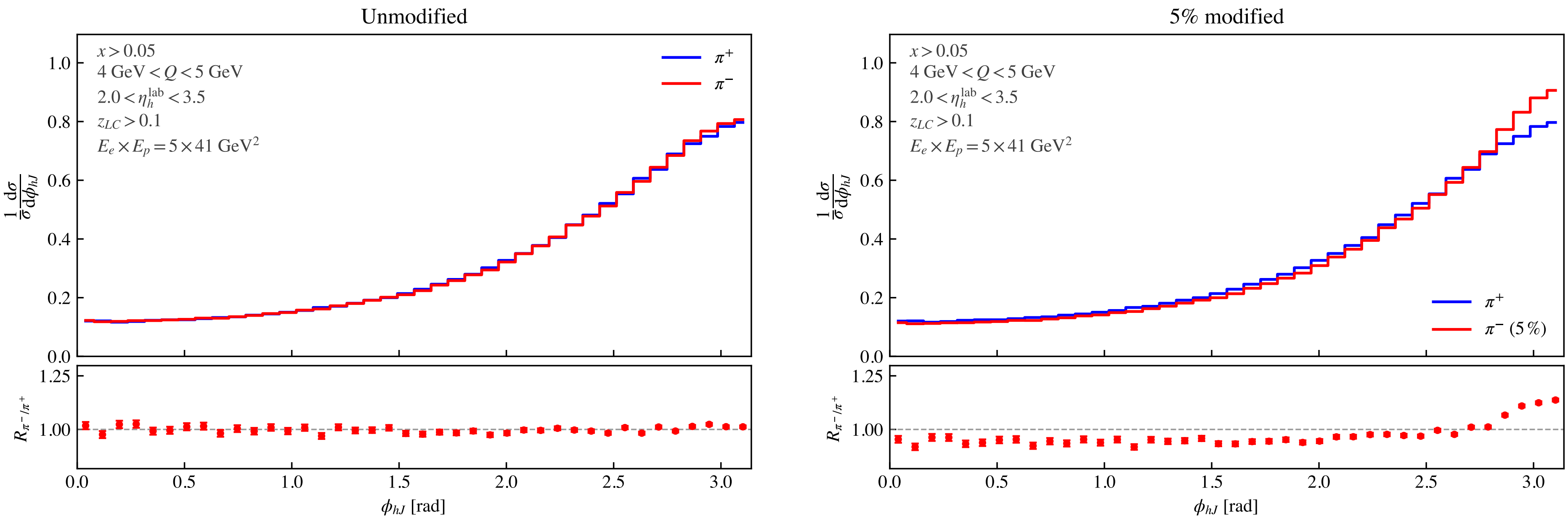


Left panel: PYTHIA, unaltered. $\phi_{hJ} \equiv \phi_h - \phi_J$. J **is** u**-jet.** ϕ_{hJ} **distribution** $\sim$ **same for** π^+ **and** π^-**: no diquark SRC in protons in PYTHIA. Shape of** ϕ_{hJ} **distribution comes from momentum conservation.**

Right panel: we have altered 10% of the events in the PYTHIA simulation. Crude model of effects of ud**-diquark SRCs in proton. Only** π^- **affected in simulation;** u**-jet;** d **from proton remnant** $\rightarrow \pi^-$**. Signal is** π^-/π^+ **ratio above one around** $\phi_{hJ} \sim 180^\circ$**. Nb: this crude model is not a prediction; an illustration of how to look for partonic SRCs with the EIC.**

An EIC Measurement. . .

[Peng, KR, Terry, 2606.17133]



Left panel: PYTHIA, unaltered. $\phi_{hJ} \equiv \phi_h - \phi_J$. J is u-jet. ϕ_{hJ} distribution $\sim$ same for π^+ and π^-: no diquark SRC in protons in PYTHIA. Shape of ϕ_{hJ} distribution comes from momentum conservation.

Right panel: we have altered 5% of the events in the PYTHIA simulation. Crude model of effects of ud-diquark SRCs in proton. Only π^- affected in simulation; u-jet; d from proton remnant $\rightarrow \pi^-$. Signal is π^-/π^+ ratio above one around $\phi_{hJ} \sim 180°$. Nb: this crude model is not a prediction; an illustration of how to look for partonic SRCs with the EIC.

Probing Quark-Gluon Matter Under Pressure

- Where else do we find quark-gluon matter under pressure? In a proton!!
- Protons form from tiny drops of QGP. The internal pressure, energy density, *and entropy* of a proton are comparable to that of the drop of QGP from which it formed.
- What lessons can we learn about the interior of a proton from hot QGP?
- Macroscopic volume of QGP at $T \sim T_p$ is a strongly coupled fluid: strong correlations between momenta of fluid cells near each other in position space. Microscopic implication: strong *Short-Range Correlations*. How can partonic SRCs be measured, directly??
- Seeing strong parton-parton correlations would show that protons cannot be described one parton at a time by (generalized) PDFs. As important to our image of the proton as seeing that QGP is a strongly coupled liquid.

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Quark-Gluon Matter, Under Pressure

- The universe was filled with hot quark soup for the first 10-20 microseconds after the Big Bang.
- LHC and RHIC heavy ion collisions recreate droplets of this hot quark soup, which promptly explode! From the shape of these Little Bangs, learn that hot quark soup is a very liquid-like liquid.
- New measurements of jets in heavy ion collisions reveal wakes that jets create in droplets of hot quark soup and, soon, will allow us to probe its microscopic structure.
- Cold quark soup is found at the centers of the heaviest neutron stars. Almost as high pressure, but the weight of the star prevents it from exploding.
- Quark soup inside a proton has a higher pressure than in a neutron star! "QCD confines" ↔ QCD describes that protons *don't* explode. We are just beginning to learn the lessons that hot quark soup teaches us about protons.

Understanding Quark-Gluon Matter Rippling Far and Wide

- Connections to astrophysics of neutron stars and their mergers, condensed matter physics of strongly correlated materials, ultracold atom physics, holography and string theory, were appreciated a decade ago and are even stronger now.
- New connections to nuclear shapes & structure.
- We have long known that we need insights from proton structure physics. Now starting to see how insights from QGP open paths to new understanding of quark-gluon matter in protons.
- Connections to new techniques from quantum field theory, quantum information theory, and new observables from high energy physics — put to new and fruitful uses — are apparent now in ways that were not the case til recently.

Our Journey has taken us From Protons to Hot Big Bang Soup . . . and Back

- Rutherford saw nuclei in atoms. Friedman, Kendall and Taylor saw quarks in protons, as was understood by Gross, Wilczek and Politzer fifty years ago. . .
- Understanding the laws that govern quarks opened the way quark matter. Cold quark soup in neutron stars? Hot quark matter filled the universe in its first microseconds. And is being reproduced and studied today!
- Hot quark matter turns out to be a very liquid-like liquid, not a gas at all. The simplest form of complex matter.
- This decade, we can repeat Rutherford's strategy yet again, to see the inner workings of hot Big Bang soup.
- Next decade, the descendants of Friedman, Kendall and Taylor will train an extraordinary new camera on the proton – and maybe turn the vision that protons are the littlest droplets of hot quark soup from metaphor into science.

Rapid Equilibration?

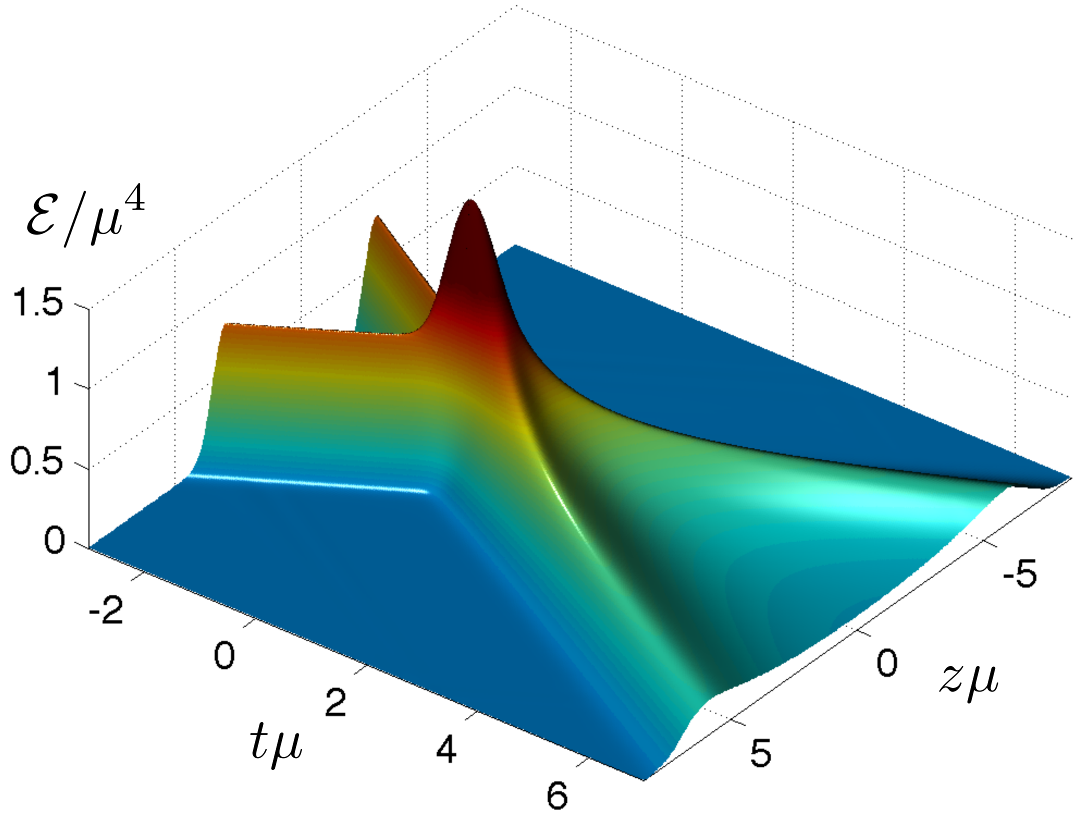
- Agreement between data and hydrodynamics can be spoiled either if there is too much dissipation (too large η/s) or if it takes too long for the droplet to equilibrate.

- Long-standing estimate is that a hydrodynamic description must already be valid only 1 fm/c after the collision.

- This is the time it takes light to cross a proton, and was long seen as *rapid equilibration*.

- But, is it really? How rapidly does equilibration occur in a strongly coupled theory?

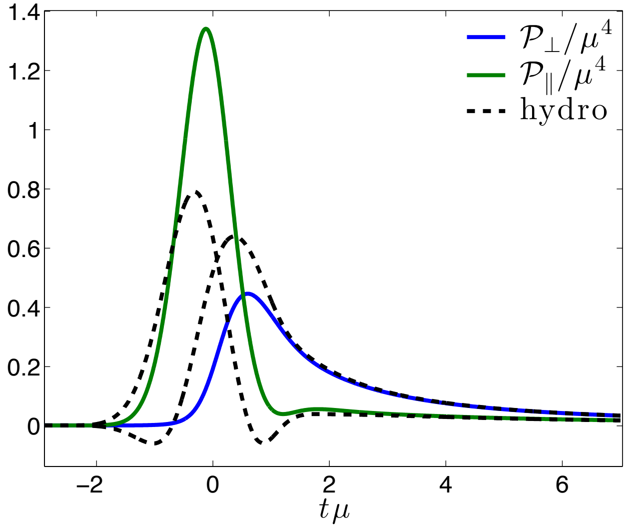
Colliding Strongly Coupled Sheets of Energy



Hydrodynamics valid ~ 3 sheet thicknesses after the collision, i.e. ~ 0.35 fm after a RHIC collision. Equilibration after ~ 1 fm need not be thought of as rapid. Chesler, Yaffe 1011.3562; generalized in C-S,H,M,vdS 1305.4919; CY 1309.1439 **Similarly 'rapid' hydrodynamization times ($\tau T \lesssim 0.7-1$) found for *many* initial conditions.** 1103.3452, 1202.0981, 1203.0755, 1304.5172.

This was the best answer we had circa 2015.

Anisotropic Viscous Hydrodynamics



Hydrodynamics valid so early that the hydrodynamic fluid is not yet isotropic. 'Hydrodynamization before isotropization.' An epoch when first order effects (spatial gradients, anisotropy, viscosity, dissipation) important. Hydrodynamics with entropy production.

This has now been seen in very many strongly coupled analyses of hydrodynamization. Janik et al., Chesler et al., Heller et al., ...

Could have been anticipated as a possibility without holography. But, it wasn't — because in a weakly coupled context isotropization happens first.

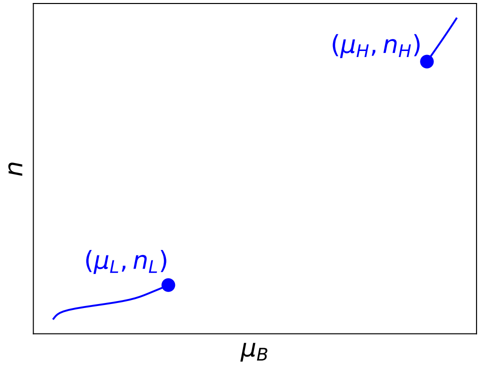
2024 Updates to 2015 Intro

- **Much more complete understanding now of how hydrodynamization happens in kinetic theory. A weakly coupled picture, applied at intermediate coupling. Hydrodynamization in 1 fm/c is no longer surprising in kinetic theory.** Berges, Heller, Kurkela, Mazeliauskas, Paquet, Schlichting, Spalinski, Strickland, Teaney, Zhu...

- **We had a qualitative, intuitive, understanding of how it can happen on this timescale at strong coupling in 2015. Now we have a qualitative, intuitive, understanding in kinetic theory also:** *adiabatic hydrodynamization.* Brewer, Yan, Yin; Brewer, Scheihing-Hitschfeld, Yin; KR, Scheihing-Hitschfeld, Steinhorst...

- **Quantification! including uncertainty quantification. Via work of** ***many*** **experimentalists and theorists, we now have more, and more precise, experimental data that, together with improved theoretical modeling, are driving Bayesian determinations, by multiple groups, of the "shape" of the fluid at the time of hydrodynamization, and key properties of QGP and their temperature dependence.**

Causality Constraint



Any equation of state must be:

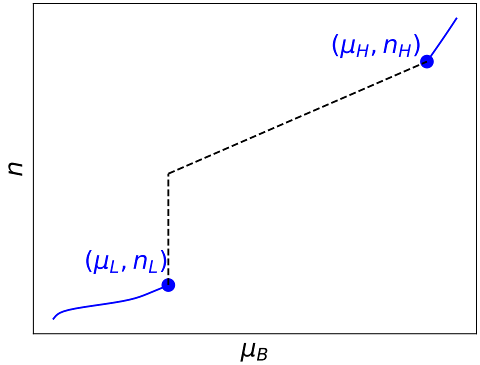
- **Stable:** monotonic & single-valued
- **Consistent:**

$$p_H = p_L + \int_{\mu_L}^{\mu_H} n(\mu) d\mu$$

- **Causal:**

$$1 \leq c_s^{-2} = \frac{d \log n}{d \log \mu}$$

Causality Constraint



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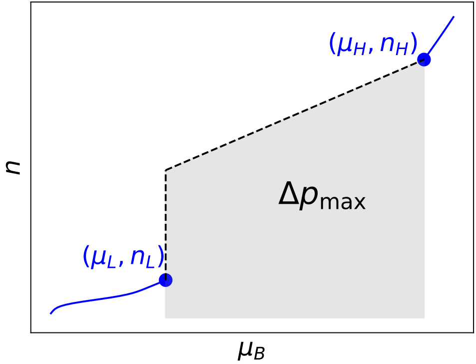

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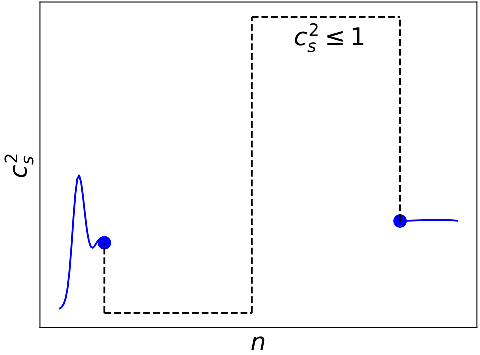
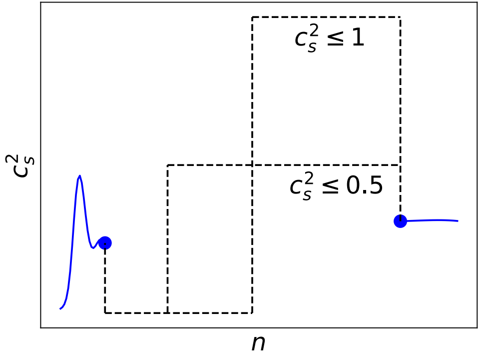
If $\Delta p_{\max} < p_H - p_L$, the low-energy and high-energy EoSs contradict each other.

Komoltsev, Kurkela arXiv:2111.05350

Maximal-Pressure Equation of State



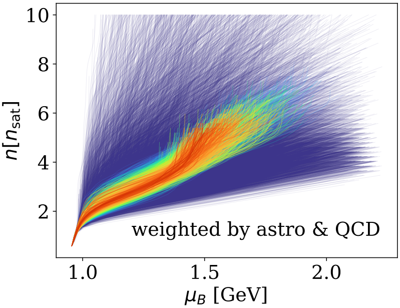
Maximal-Pressure Equation of State



Clearly, the maximal-pressure EoS is not reasonable.

While $c_s^2 \leq 1$ is conservative, we might also consider e.g. $c_s^2 \leq 0.5$.

QCD Inputs Can Constrain NS EoS!



PSR J0348+0432
PSR J1624-2230
PSR J0740+6620 (NICER)
GW170817

Gorda, Komoltsev, Kurkela arXiv:2204.11877

Of the many possible ways to draw the NS EoS:

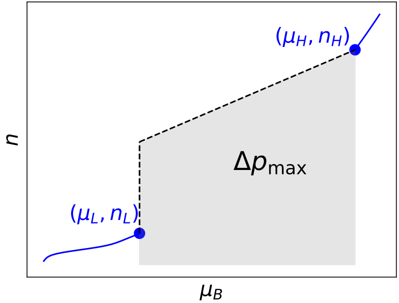
- Determine likelihood using astrophysical measurements:

$$P(\mathrm{EoS}|\mathrm{astro})$$

- Reject those which are inconsistent with QCD:

$$P(\mathrm{EoS}|\mathrm{QCD}) = 0 \text{ or } 1$$

Can NS measurements constrain QCD?



If non-perturbative physics increases p_H at fixed μ_H, it will be more difficult to connect (μ_L, n_L, p_L) to (μ_H, n_H, p_H).

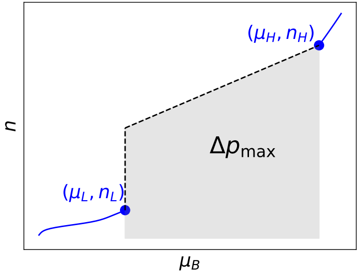
(CFL) Color Superconductivity

- Sufficiently dense, cold quarks form Cooper pairs.
- This increases the pressure by

$$\delta p_{\mathrm{CFL}} = \frac{\Delta^2 \mu_B^2}{3\pi^2}$$

  where Δ is the superconducting gap
- At weak coupling, $\Delta/\mu \propto g^{-5} e^{\frac{-3\pi^2}{\sqrt{2}g}}$, while models give estimates ranging between 20 and 300 MeV
- Using causality constraints, we should be able to empirically constrain Δ.

How large can the gap be?



If $p_{\mathrm{pQCD}} + p_{\mathrm{CFL}} - p_L > \Delta p_{\mathrm{max}}$, the gap is too large to be causally consistent!

Setting $p_{\mathrm{pQCD}} + p_{\mathrm{CFL}} - p_L = \Delta p_{\mathrm{max}}$ and solving for Δ, we have

$$\Delta_{\mathrm{max}}(\mu_H)^2 = \frac{3\pi^2}{\mu_L^2}\left[\frac{n_{\mathrm{pQCD}}(\mu_H)}{2\mu_H}\left(\mu_H^2 - \mu_L^2\right) - \left(p_{\mathrm{pQCD}}(\mu_H) - p_L\right)\right].$$

This is a constraint for Δ at μ_H.

Back of the envelope estimate...

- If we use only nuclear physics (CEFT) equation of state ($\mu_L \approx 1$ GeV), plugging in reasonable numbers gives

$$\Delta_{\max}(2.6\ \mathrm{GeV}) \sim 880\ \text{MeV}$$

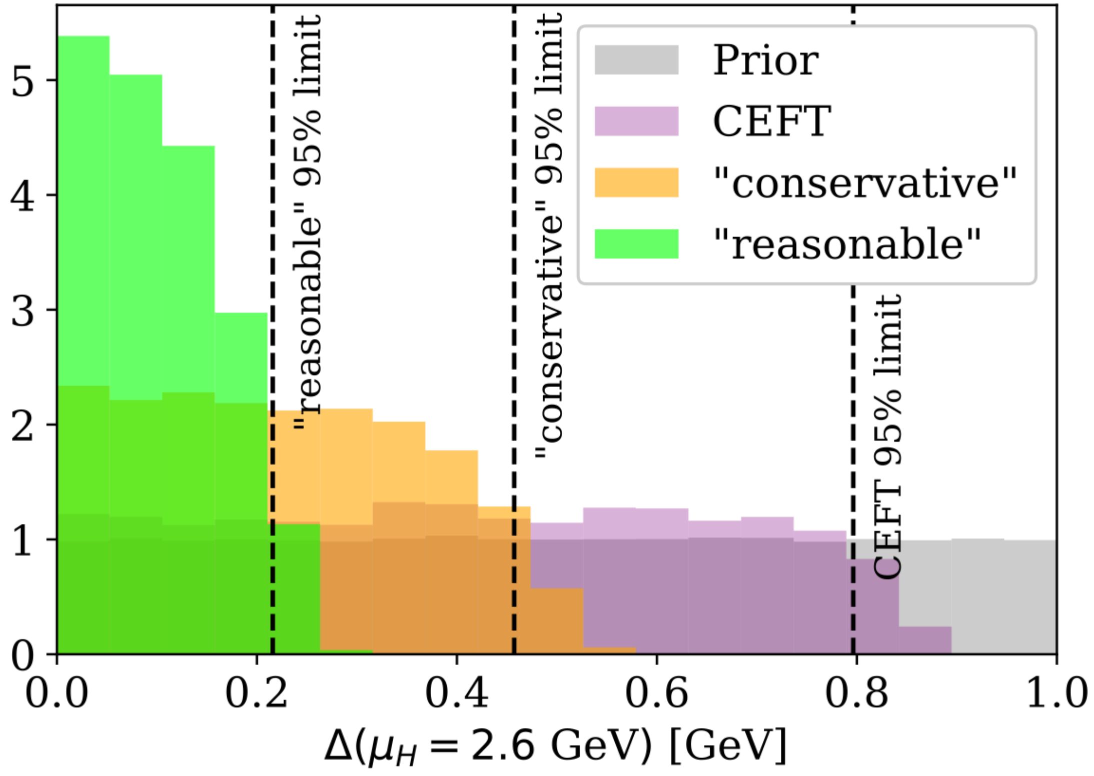
- If we instead use a reasonable guess for the EoS of a 2.1 $M_{\odot}$, ($\mu_L \approx 1.5$ GeV), we get

$$\Delta_{\max}(2.6\ \mathrm{GeV}) \sim 440\ \text{MeV}$$

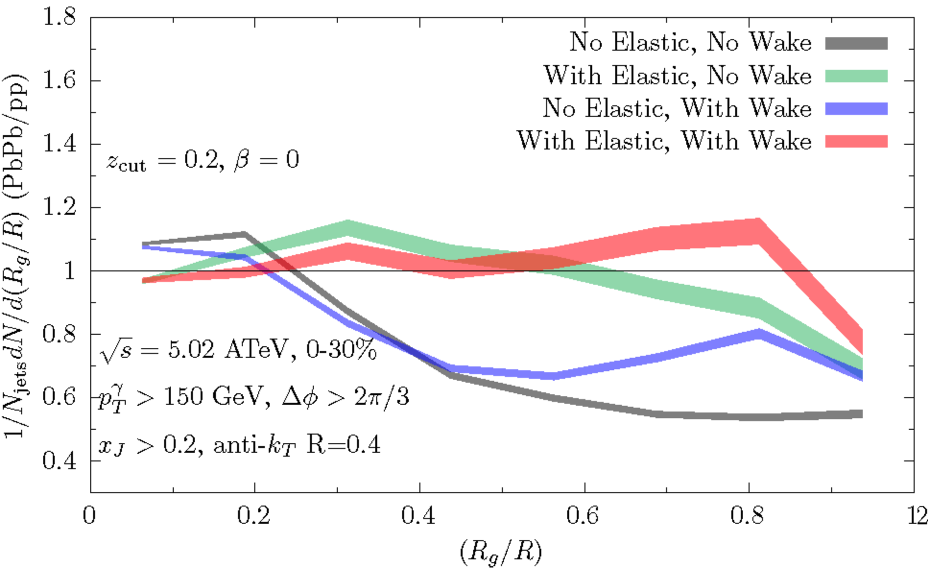
Bayesian framework:

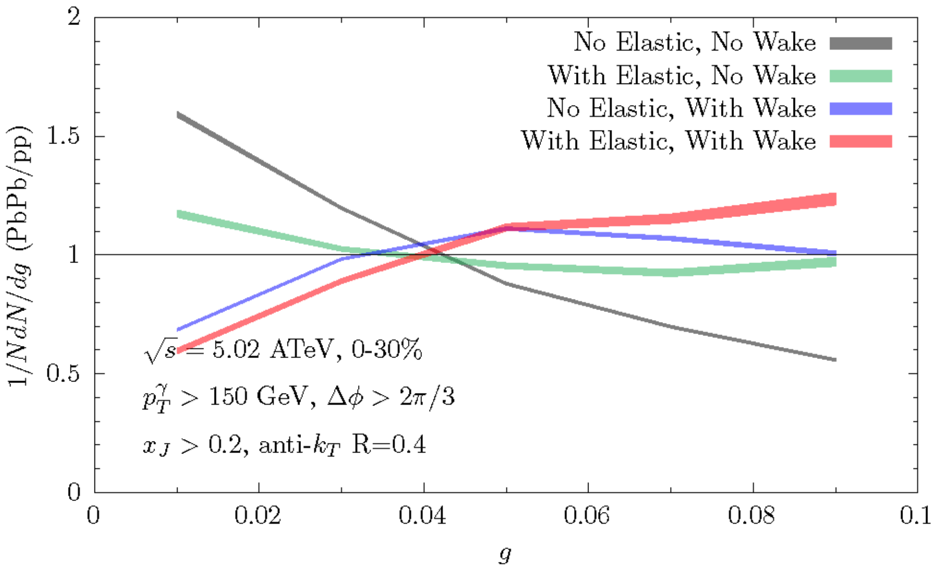
- Observational and theoretical error estimation
- Less unreasonable assumptions about interpolation
    - “**Conservative**”: $c_s^2 < 1$, Inferred EOS up to 2.1 $M_\odot$
    - “**Reasonable**”: $c_s^2 < 0.5$, Inferred EOS up to M_{TOV}

$$\Delta_{max}^{95\%}(2.6\text{GeV}) \approx 216\text{ MeV}$$



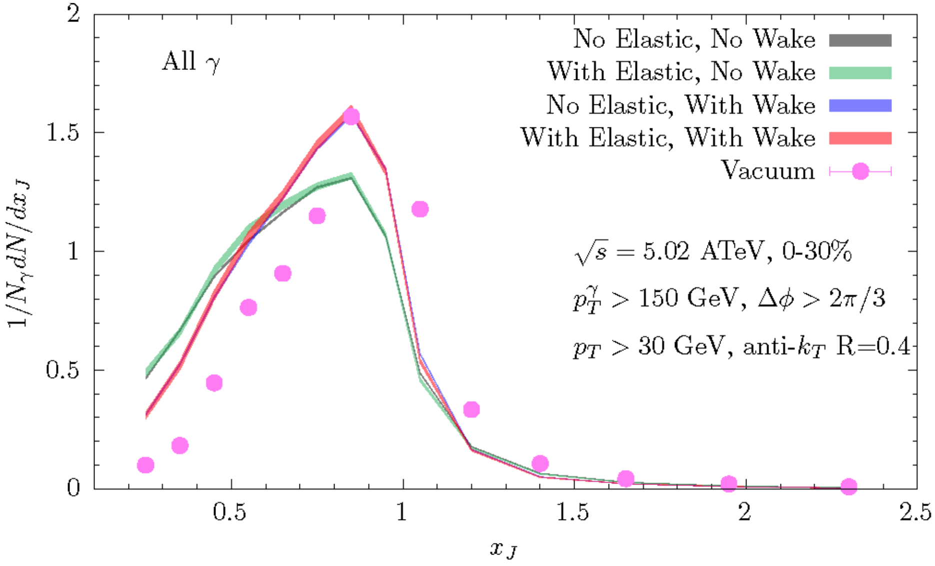
Gamma-Jet Observables with p_T^γ >150 GeV and R=0.4: R_g and Girth, with x_J>0.2



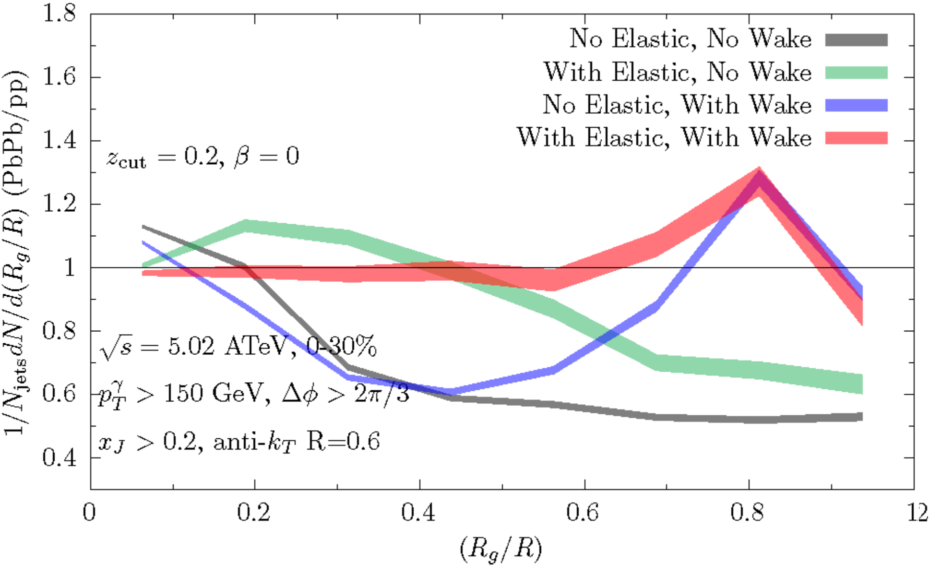


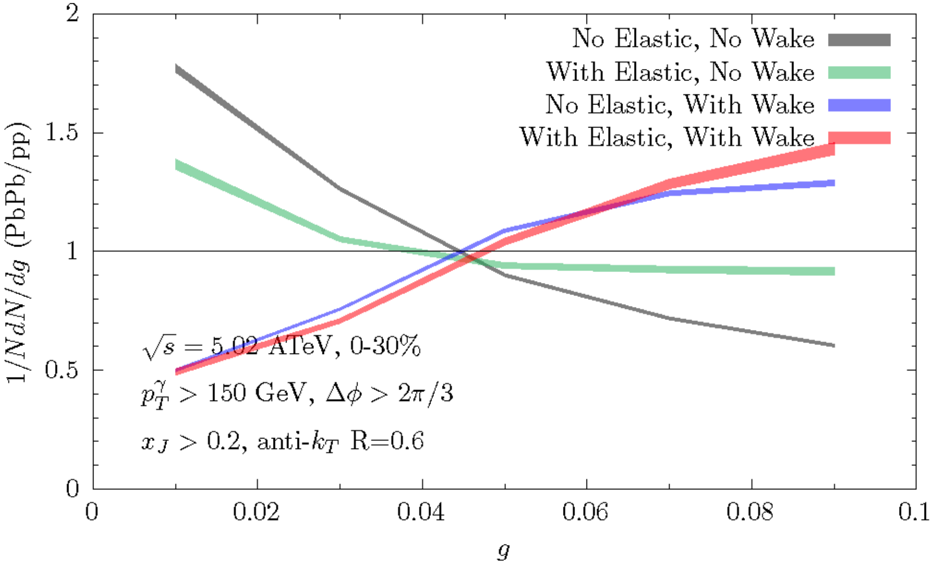
On previous slides, p_T^γ >150 GeV with R=0.2. Here, R=0.4, so that we can “catch” more wake, with little selection bias.

Moliere effects substantial; selection bias reduced; wake effects significant.



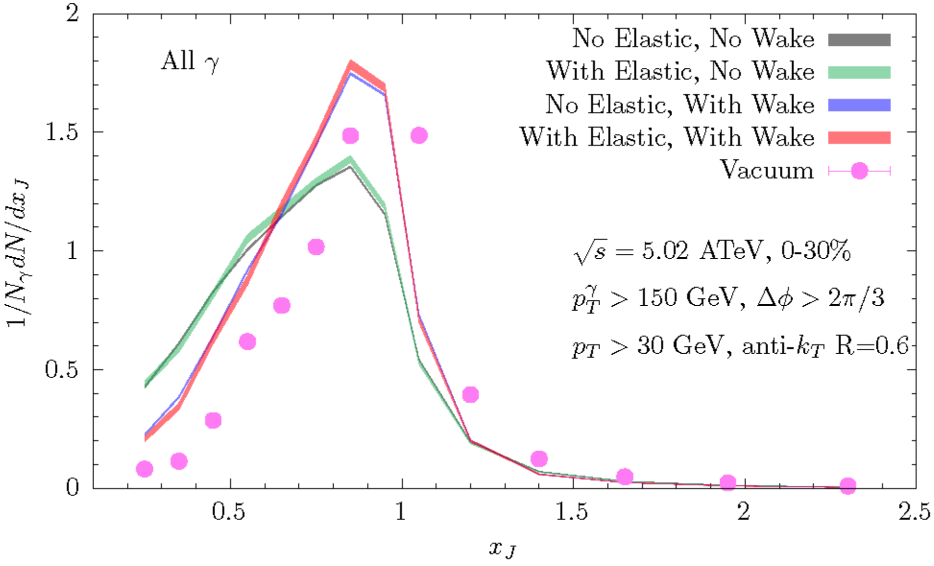
Gamma-Jet Observables with p_T^γ >150 GeV and R=0.6: R_g and Girth, with x_J>0.2



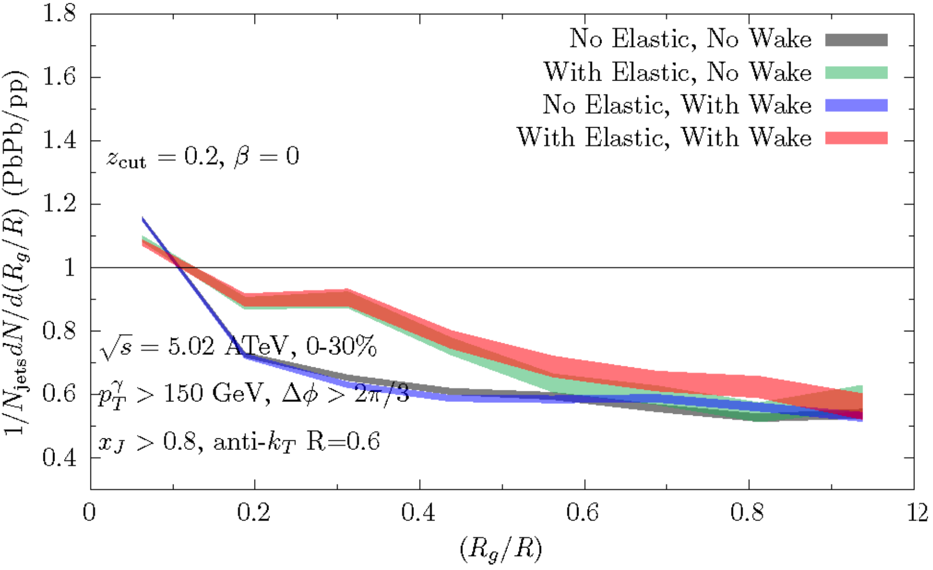


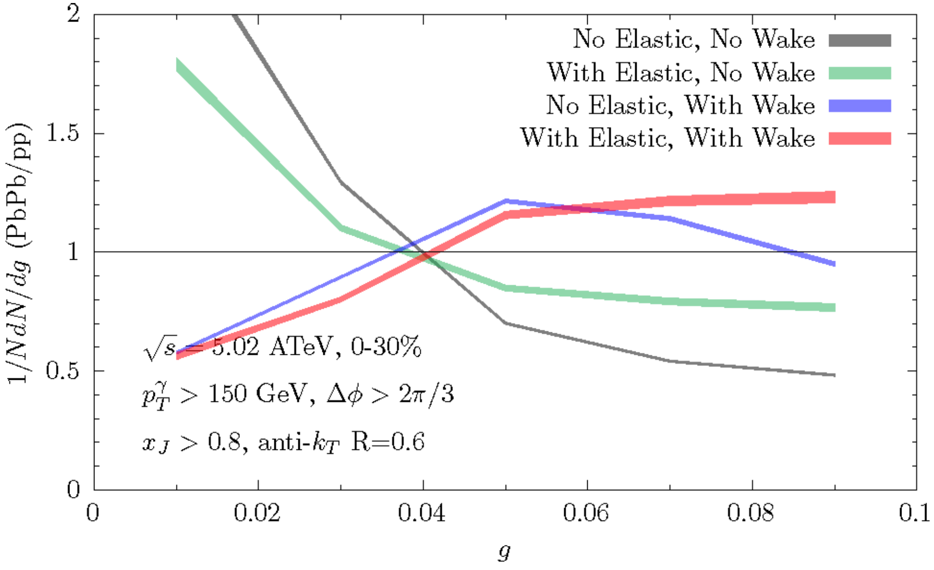
On previous slides, p_T^γ >150 GeV with R=0.2. Here, R=0.6, so that we can "catch" *even more* wake, with little selection bias.

Moliere effects substantial; selection bias reduced; wake effects enormous, and as in Brewer+Brodsky+KR.



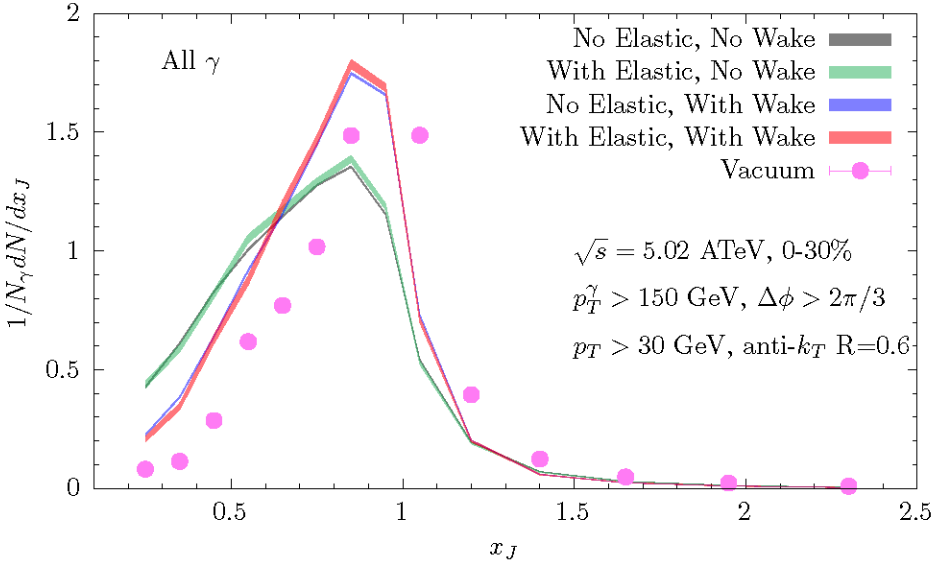
Gamma-Jet Observables with $p_T^\gamma > 150$ GeV and R=0.6: R_g and Girth, with $x_J > 0.8$





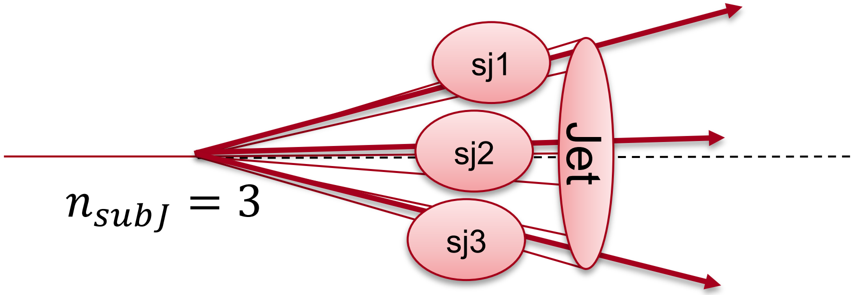
On previous slides, $p_T^\gamma > 150$ GeV with R=0.2. Here, R=0.6. But, we've turned the selection bias back ON.

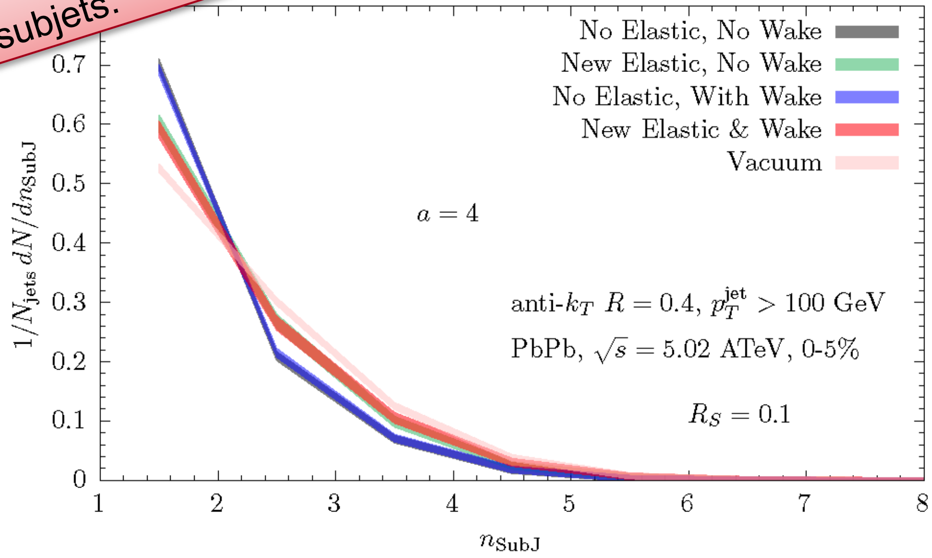
Moliere effects still substantial; selection bias dominant; wake effects *greatly reduced*, as in Brewer+Brodsky+KR.



Inclusive Jets within Inclusive Jets: Inclusive Subjets

1. Reconstruct jet with R=0.6
2. Recluster each jet's particle content into subjets with R=0.15





Increase in number of subjets.

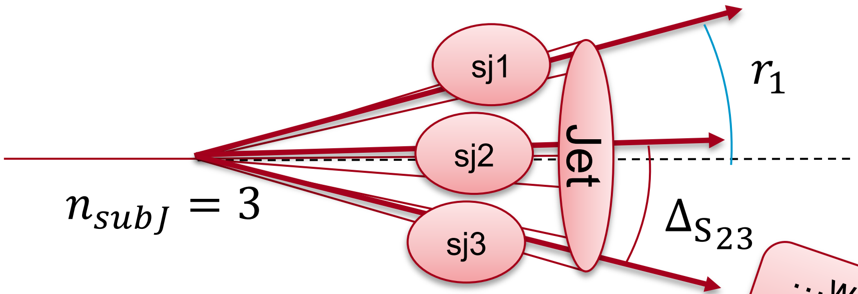
Moliere scattering visible as increase in number of subjets; no such effect coming from wake at all.

Moliere scattering also yields more separated subjets…

These observables are directly sensitive to "sprouting a new subjet" the intrinsic feature of Moliere scattering which makes it NOT just a bit more wake.

Inclusive Subjets

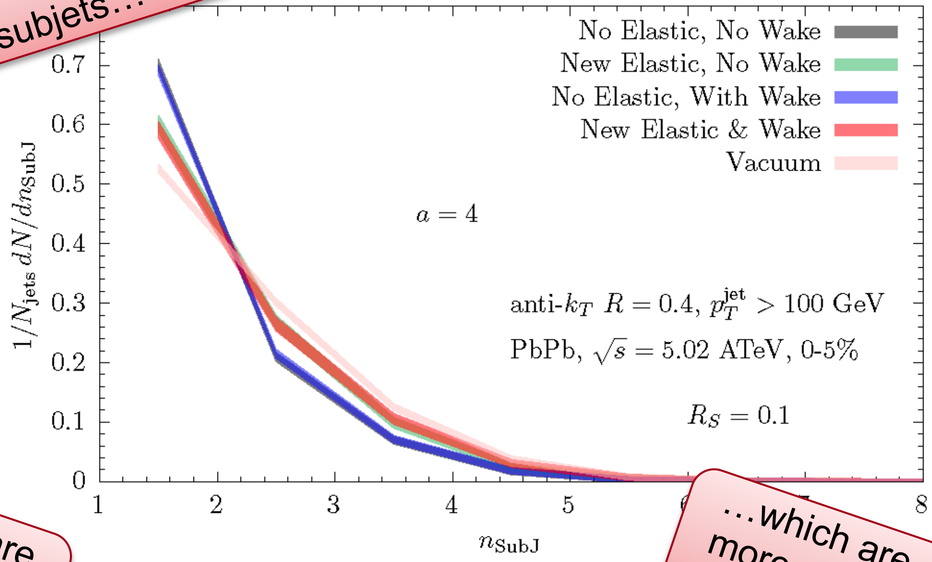
1. Reconstruct jet with R=0.4
2. Recluster each jet’s particle content into subjets with R=0.1

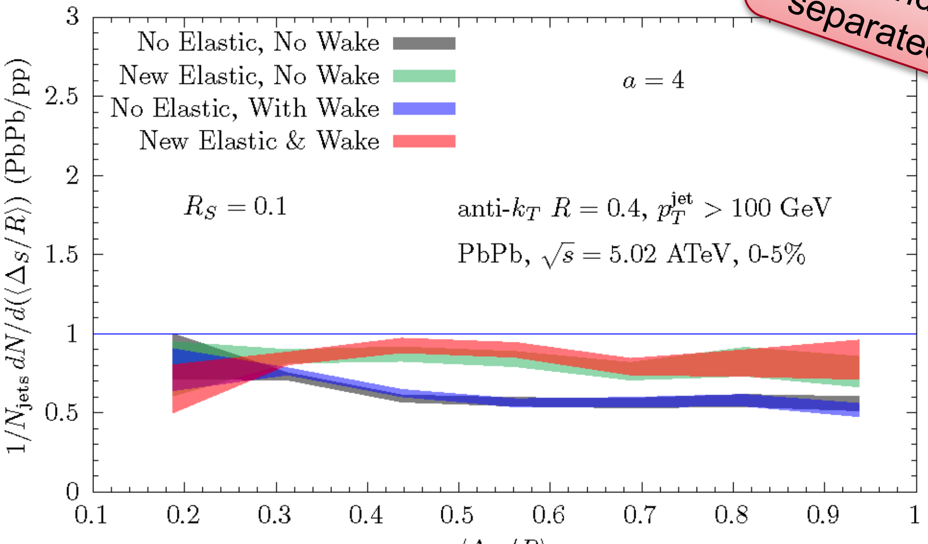


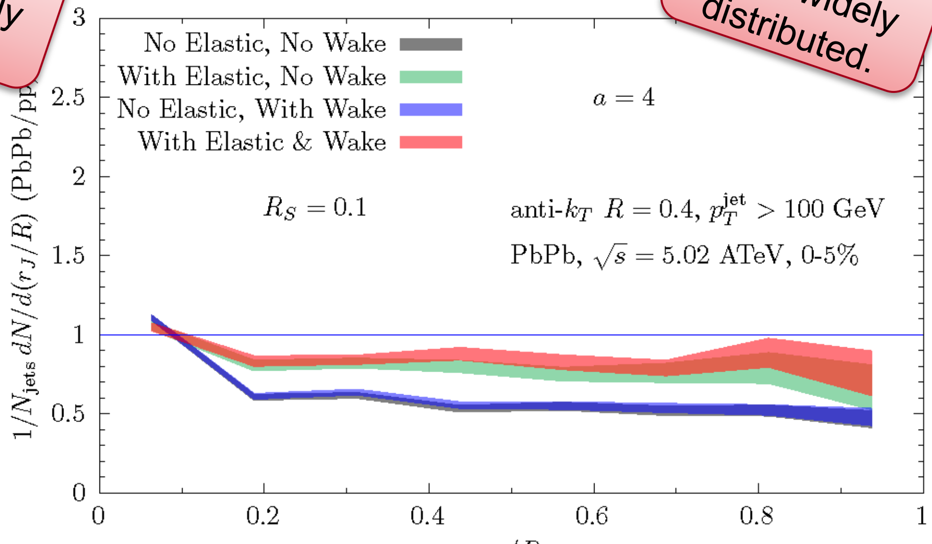
Increase in number of subjets…

…which are more widely separated.

…which are more widely distributed.







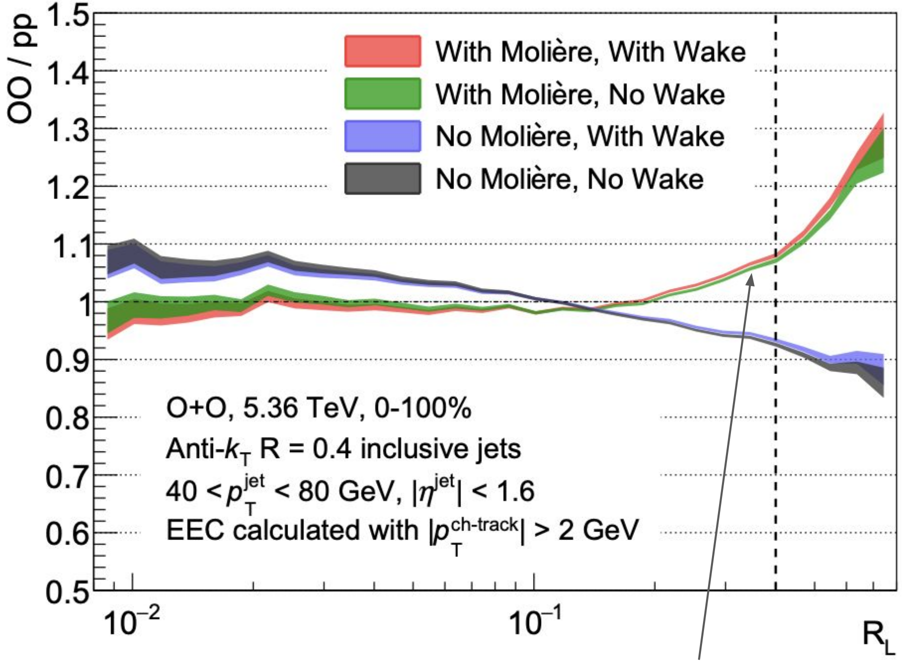
Conclusions

- Studied the effect of elastic Moliere scattering of jet partons off medium partons on jet observables in the perturbative regime.
- For "overall shape observables" (jet shapes; FF) effects of Moliere scattering are similar to, and smaller than, effects of wake.
- Grooming helps, by grooming away the soft particles from the wake. Effects of Moliere scattering dominate the modification of several groomed observables (R_g, Leading k_T, Girth, WTA axis angle.)
- R_g and girth observables in γ+jet events can be "engineered" to reduce (or enhance) selection bias by selecting with x_J > a low (or high) threshold. When selection bias is reduced, Moliere scattering yields $R_{AA}>1$.
- R_g and girth observables in γ+jet events can be "engineered" to remove (or highlight) effects of the wake by choosing small R (or large R with x_J > a low threshold).
- Modification of inclusive subjet observables (number, and angular spread, of subjets) are especially sensitive to the presence of Moliere scatterings. These observables are unaffected by the wake. They reflect what it is that makes the effects of scattering different from those of the wake.
- Subjet and γ+jet observables may also be influenced by other ways in which jet shower partons "see" particulate aspects of the QGP. That's great!
- Acoplanarity observables that we have investigated to date show little sensitivity to Moliere scattering; significant sensitivity to the wake in many cases.

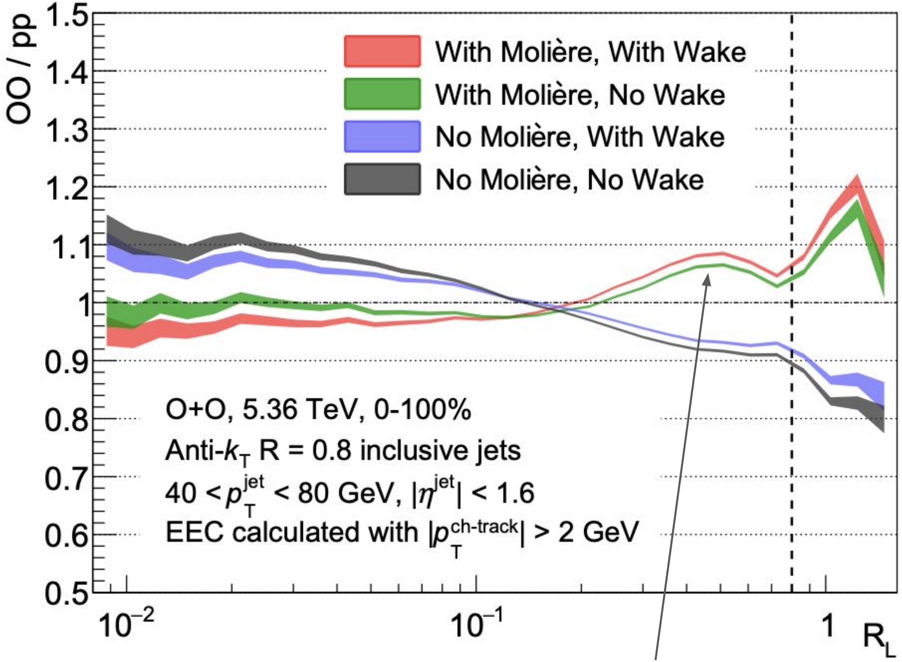
EECs in OO Collisions with $p_T^{\text{ch-track}} > 2$ GeV

2603.23596 [Kudinoor, Lin, Pablos, Rajagopal]

R = 0.4 Jets



R = 0.8 Jets



Minimal selection bias ⇒ **Visible enhancement of the EEC at large R_L due to large-angle Molière scattering**

Typical angle of Molière scattering…
See Arthur Lin's talk to learn more!

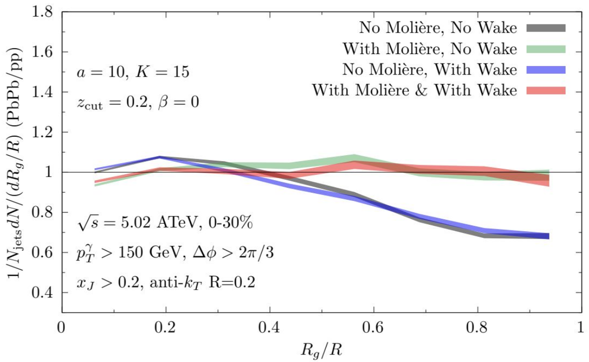
Key Takeaways

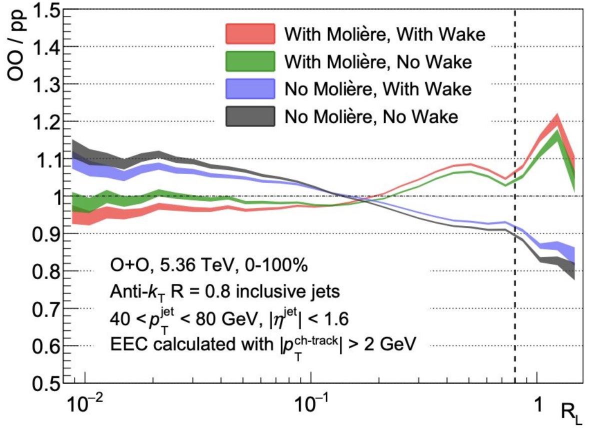
- Quasiparticles in QGP can be revealed by rare, hard, large-angle Molière scatterings between jet partons and the medium.

- Molière scatterings can be a dominant effect in select observables like Soft Drop R_g , jet girth *g,* and EECs

- The confounding effects of selection bias due to jet energy loss can be minimized in
  - **Photon-tagged jet measurements in PbPb collisions**
  - **Inclusive jet measurements in OO collisions**

- Detecting Molière scattering experimentally: **An opportunity to reveal the microscopic structure of QGP and to develop and test theoretical descriptions of it** (analogous to PDFs in DIS)



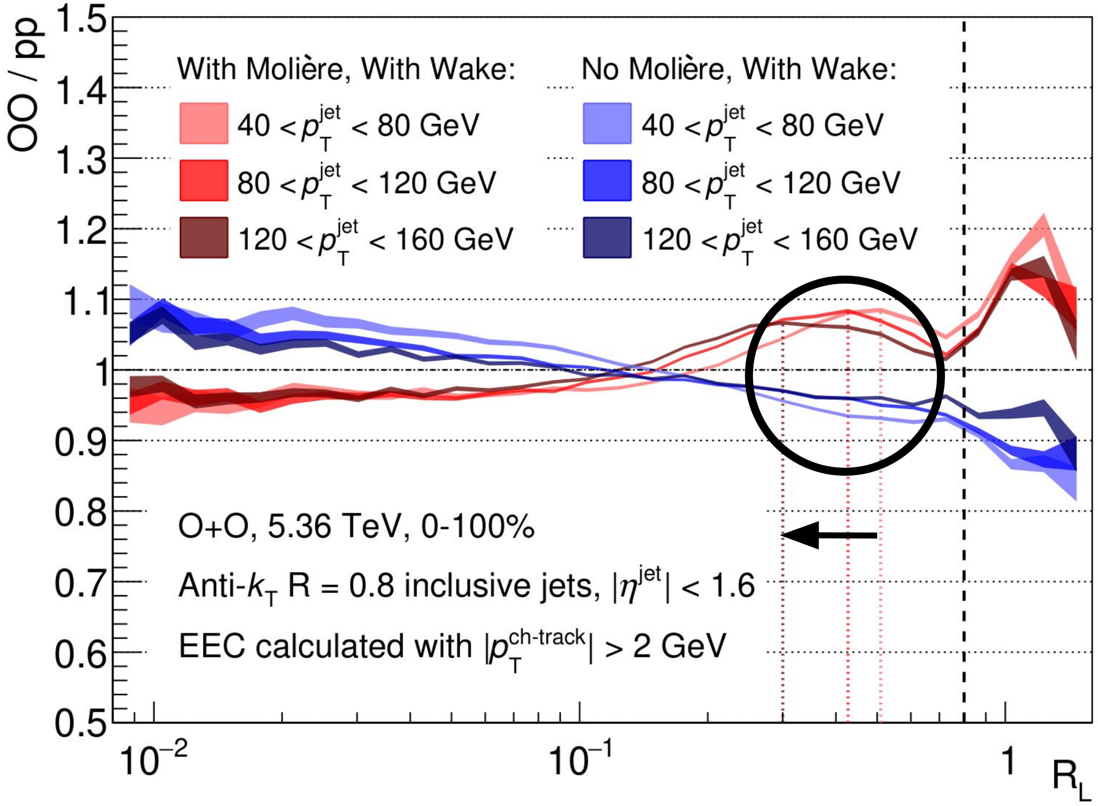


[2603.08776](https://arxiv.org/abs/2603.08776)
Molière in PbPb

[2603.23596](https://arxiv.org/abs/2603.23596)
Molière in OO

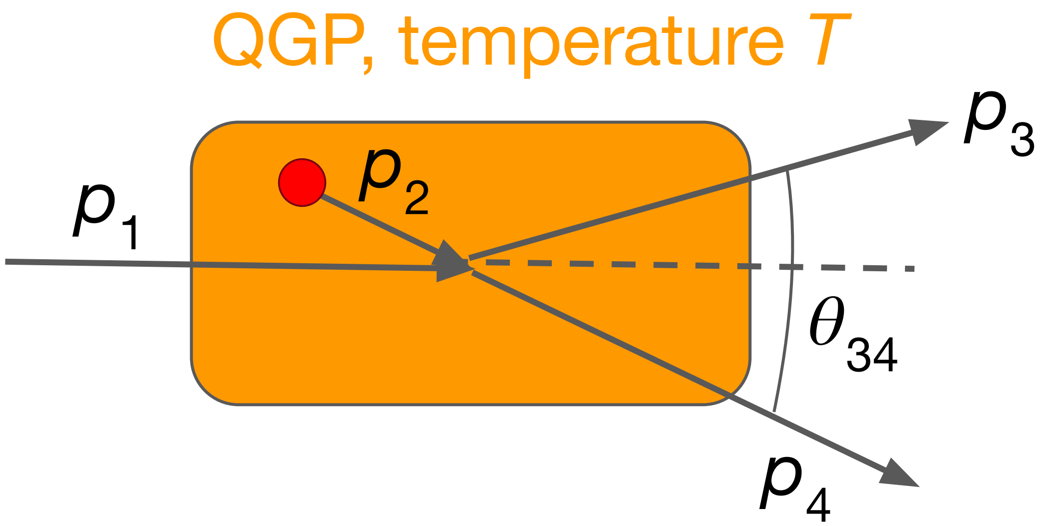
We observed Molière scattering as a bump in EEC. Can we understand its location more systematically?

Let's investigate!



Simplified Picture: EEC of a Single Molière Scattering

- Fixed p_1 : a high momentum parton in a jet that triggers Molière scattering
- Thermal p_2 : sample p_2 from Boltzmann distribution with fixed temperature T
- The cross section corresponding to outgoing momenta p_3 and p_4 are calculated
- EEC is the energy product weighted differential cross section



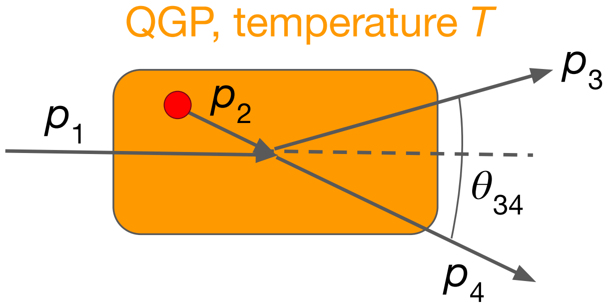
Kinematic cuts: $|t|, |u| > am_D^2 \; , \; m_D^2 \propto g_s^2 T^2$

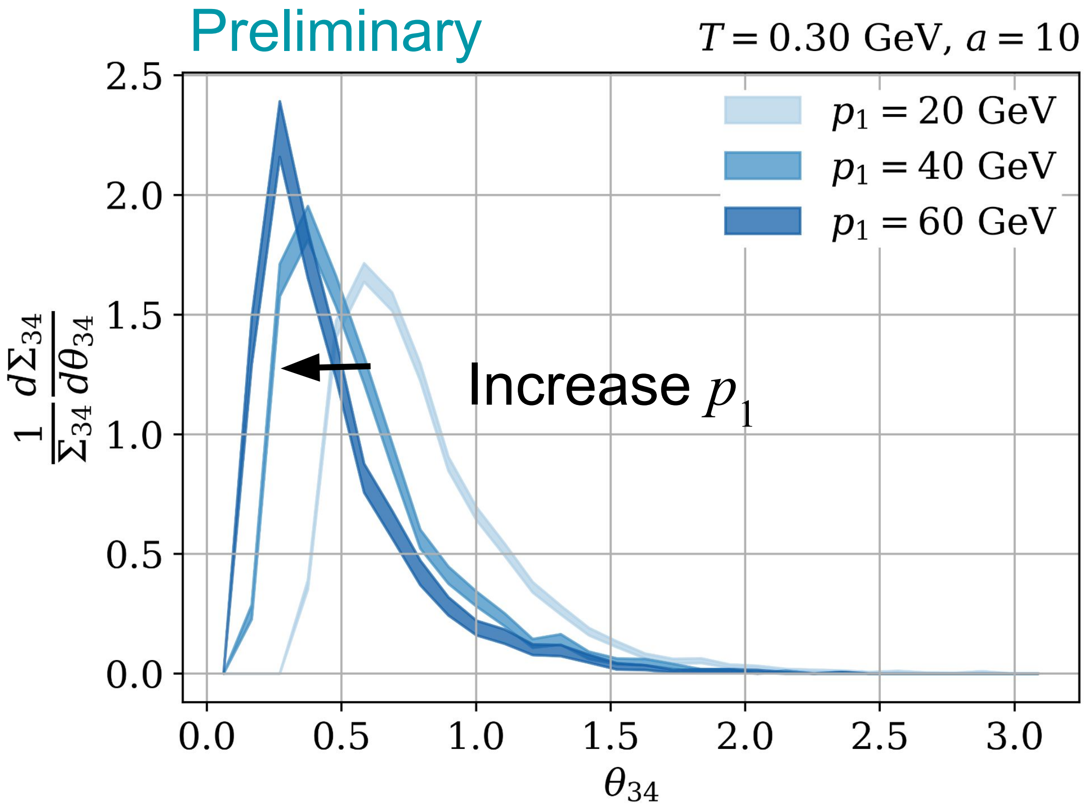
$$\frac{d\Sigma_{34}}{d\chi} = \int d\sigma_{12\to34} \frac{E_3 E_4}{Q^2} \delta(\cos\chi - \cos\theta_{34})$$

Normalized EEC of a Single Molière Scattering

Kinematic constraints: $|t|, |u| > a m_D^2\ ,\ m_D^2 \propto g_s^2 T^2$

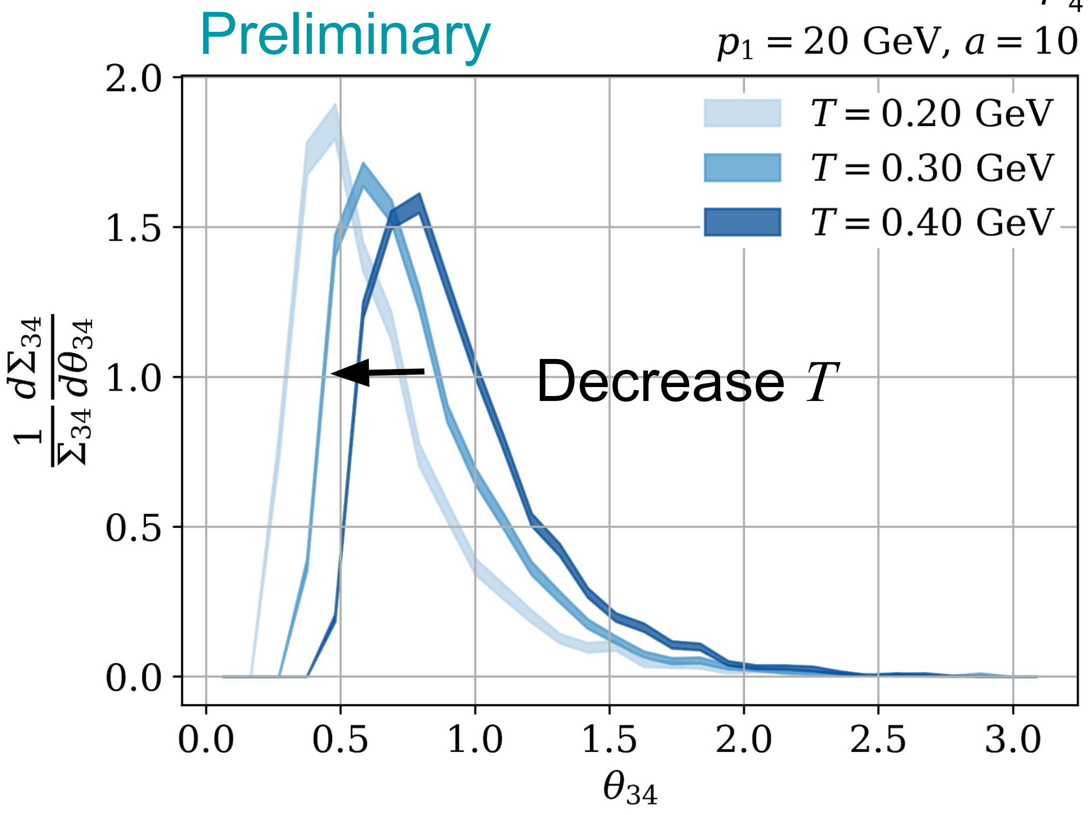
Bump moves to smaller angle as p_1 increases or as T decreases




Preliminary

$T = 0.30$ GeV, $a = 10$

Increase p_1


Preliminary

$p_1 = 20$ GeV, $a = 10$

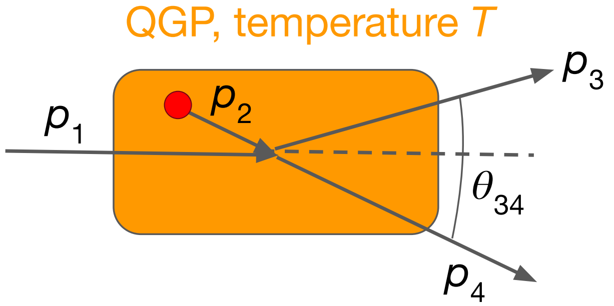
Decrease T

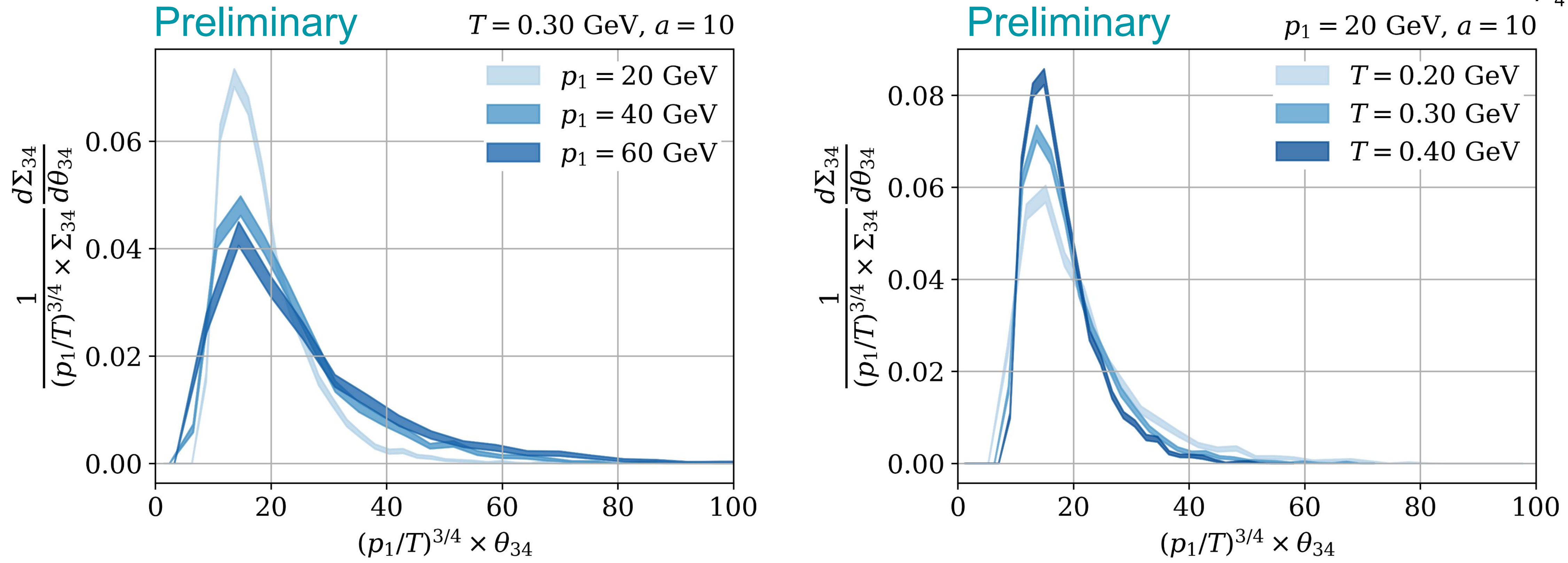
Normalized EEC of a Single Molière Scattering

Kinematic constraints: $|t|, |u| > a m_D^2 \ , \ m_D^2 \propto g_s^2 T^2$

Could this be scaling like $\theta_{\text{bump}} \propto (T/p_1)^{3/4}$?

Reminiscent of 1407.0293 [Kurkela and Wiedemann], although they were not looking at EEC

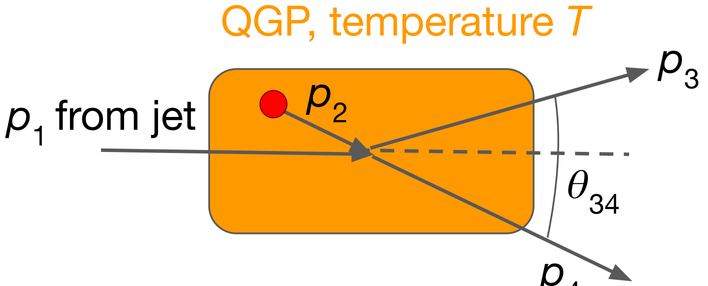


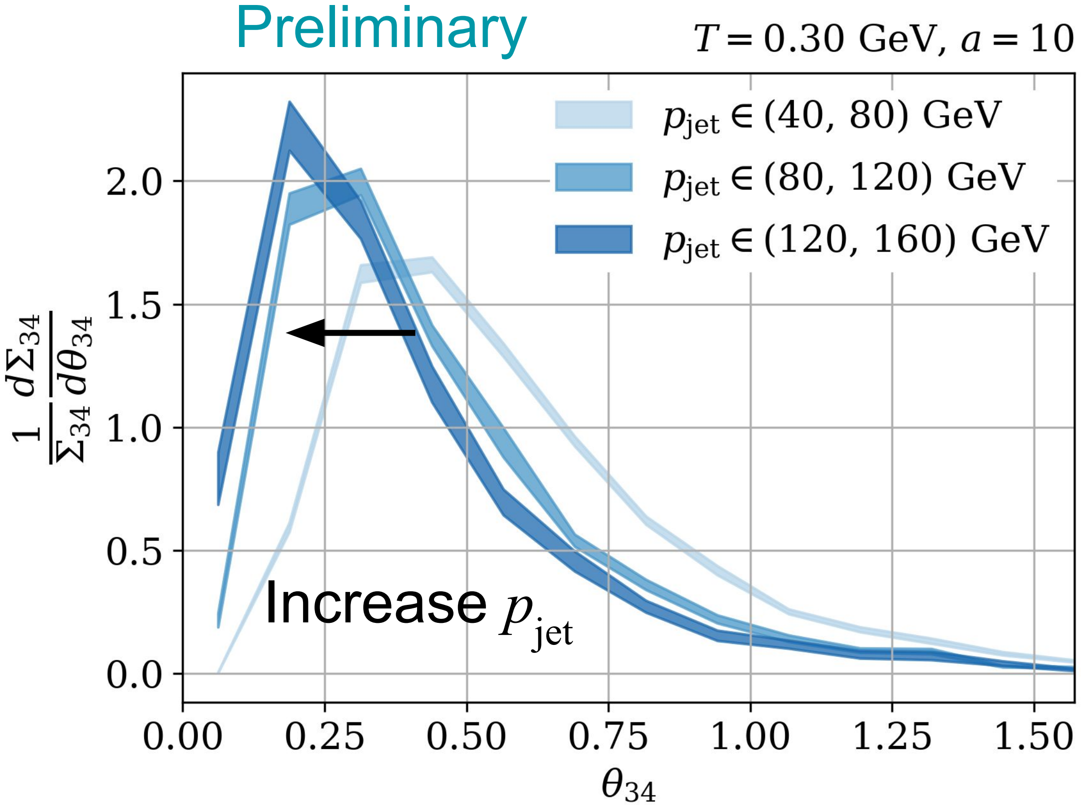


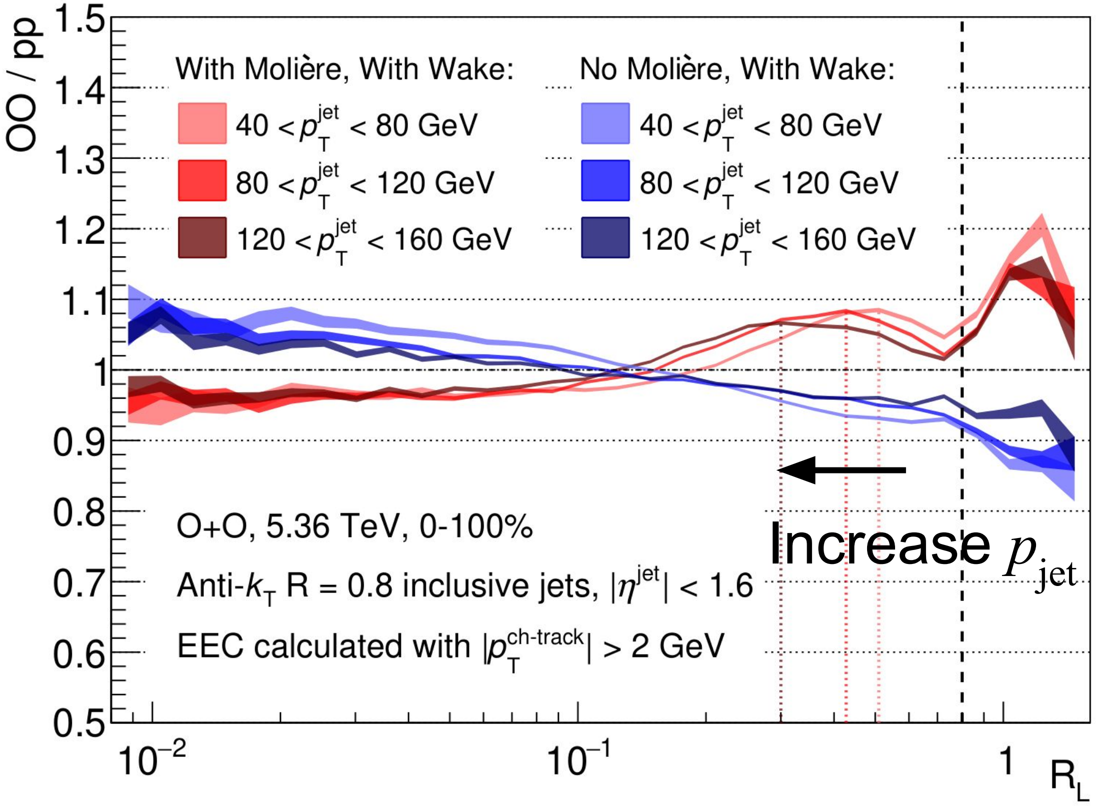
Normalized EEC of a Single Molière Scattering in Jet

Instead of considering a fixed incident p_1, identify p_1 as p_T of the highest p_T parton in a sample of jets within a specified range of p_T^{jet}

Kinematic constraints: $|t|, |u| > a m_D^2 \ , \ m_D^2 \propto g_s^2 T^2$





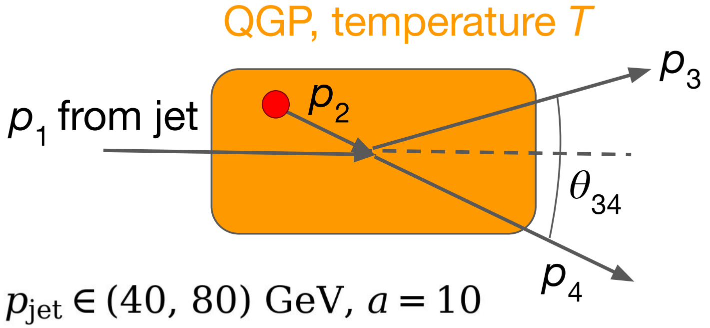


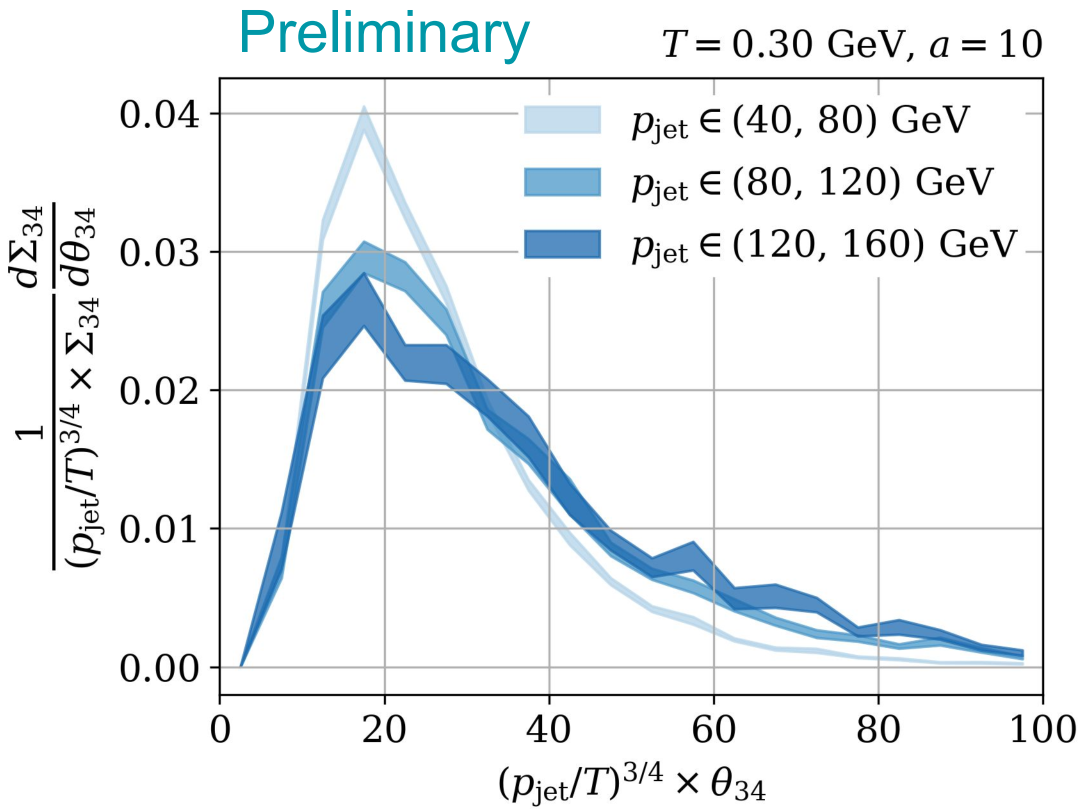
2603.23596 [Kudinoor, AL, Pablos, Rajagopal]

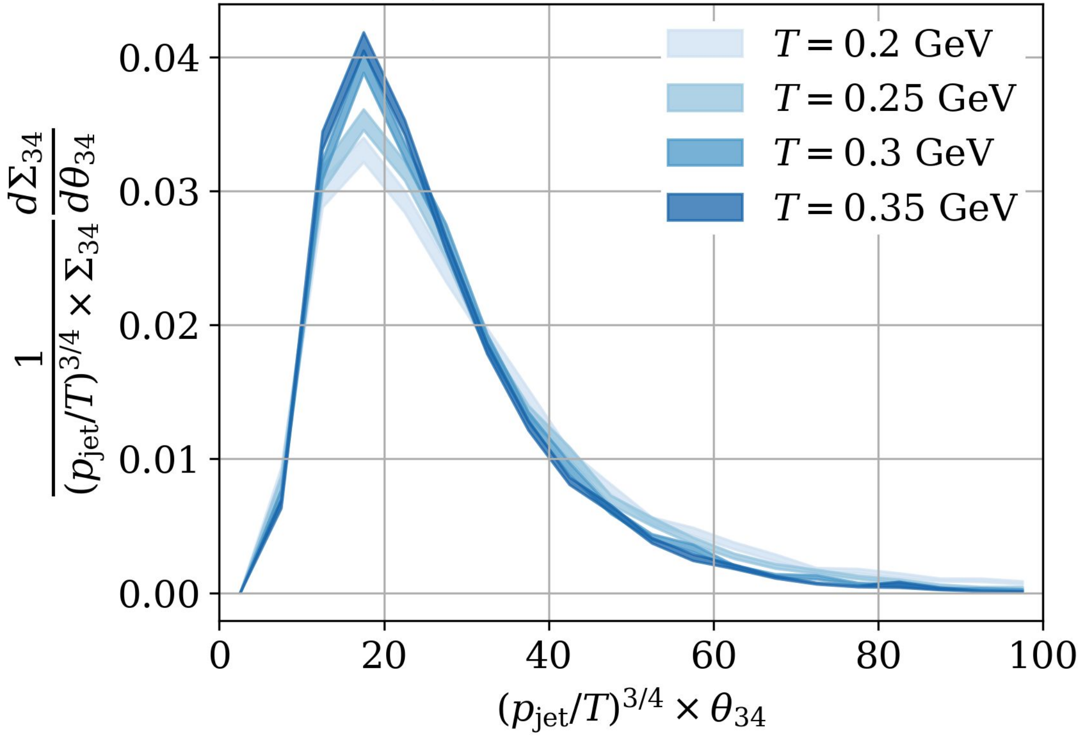
Normalized EEC of a Single Molière Scattering in Jet

Kinematic constraints: $|t|, |u| > a m_D^2 \ , \ m_D^2 \propto g_s^2 T^2$

Scaling looks similar to fixed p_1 result: $\theta_{\mathrm{bump}} \propto (T/p_{\mathrm{jet}})^{3/4}$





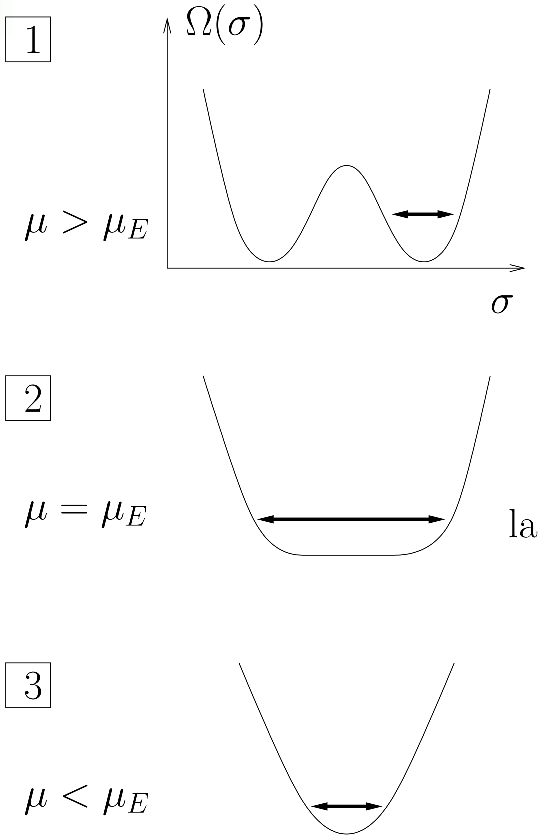


Conclusions

- Jet partons lose less energy in OO collisions, reducing selection bias, yet jet-induced wake and Molière scattering still significantly modify EECs
- Jet-induced wake enhances correlation of soft particles at very large angles, they can be removed by imposing a ch-track cut at 2 GeV
- High momentum jet partons trigger **Molière scattering off QGP quasiparticles**, resulting in a **bump of the EEC ratio** at large angles. **Measuring the bump experimentally would provide compelling evidence of Molière scattering and a big step forward in understanding the microscopic structure of QGP**
- We calculated EEC of a single Molière scattering triggered by highest momentum parton in a jet, and found that the bump location shifts with parton energy in the same way as it shifts with jet energy in Hybrid Model
- We can't wait to see this bump, and its systematic behavior, in data! And, let's see this investigated in other models too…

2603.23596

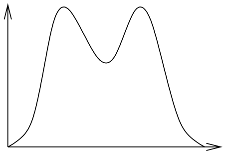
Critical mode and equilibrium fluctuations

- 

$$\boxed{\bar\psi\psi - \langle\bar\psi\psi\rangle \equiv \sigma}$$

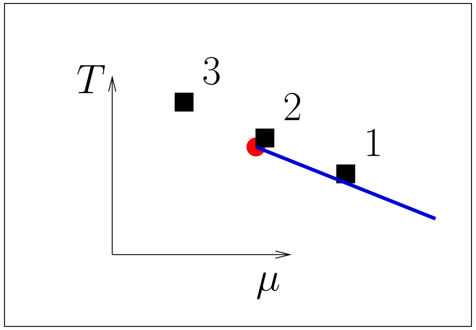
$$\langle\sigma^2\rangle \sim (\Omega'')^{-1}$$

think $e^{-\Omega}$:



$$(\Omega'')^{-1} \to \infty$$

large equilibrium fluctuations



- Magnitude of fluctuation and correlation length:

$$\langle\sigma(\boldsymbol{x})\sigma(\mathbf{0})\rangle \sim \begin{cases} e^{-|\boldsymbol{x}|/\xi} & \text{for} \quad |\boldsymbol{x}| \gg \xi \\ 1/|\boldsymbol{x}|^{1+\eta} & \text{for} \quad |\boldsymbol{x}| \ll \xi \end{cases}$$

$$\langle\sigma_{\mathbf{0}}^2\rangle = \int d^3\boldsymbol{x}\langle\sigma(\boldsymbol{x})\sigma(\mathbf{0})\rangle \sim \xi^{2-\eta}$$

critical singularity is a *collective* phenomenon

- σ or n_B or T^{00}? Because they mix, only *one* linear combination is critical.

Relation between σ fluctuations and observables

Consider example: fluctuations of multiplicity of pions (or protons).

- Free gas: $n^0_{\boldsymbol p}$ – fluctuating occupation number of momentum mode $\boldsymbol p$.
  Ensemble (event) average $\langle n^0_{\boldsymbol p}\rangle = f_{\boldsymbol p}$ and

$$n^0_{\boldsymbol p} = f_{\boldsymbol p} + \delta n^0_{\boldsymbol p}; \quad \langle \delta n^0_p \delta n^0_{\boldsymbol k}\rangle = f'_{\boldsymbol p}\delta_{\boldsymbol{pk}}; \qquad f_{\boldsymbol p} = (e^{\omega_{\boldsymbol p}/T} \mp 1)^{-1};\ f'_{\boldsymbol p} \equiv f_{\boldsymbol p}(1 \pm f_{\boldsymbol p}).$$

- Couple these particles to σ field: $G\sigma\pi\pi$ (or $g\sigma\bar{N}N$).
  Think of $m^2 \equiv m_0^2 + 2G\sigma$ as “fluctuating mass”. Then

$$\delta n_{\boldsymbol p} = \delta n^0_p + \frac{\partial f_{\boldsymbol p}}{\partial m^2} 2G\sigma = \delta n^0_p + \frac{f'_{\boldsymbol p}}{\omega_{\boldsymbol p}} \frac{G}{T}\sigma$$

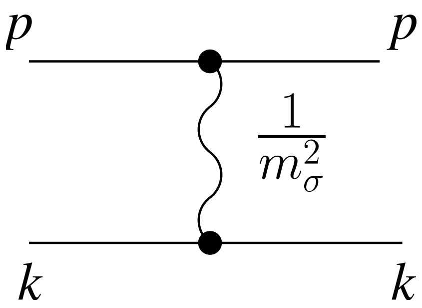
- Using $\langle \delta n^0_p \sigma\rangle = 0$ and $\langle \sigma^2\rangle = (T/V)\xi^2$.

$$\langle \delta n_p \delta n_{\boldsymbol k}\rangle = f'_{\boldsymbol p}\delta_{\boldsymbol{pk}} + \frac{1}{VT}\frac{f'_{\boldsymbol p}}{\omega_{\boldsymbol p}}\frac{f'_{\boldsymbol k}}{\omega_{\boldsymbol k}} G^2 \xi^2 .$$

More formal derivation: PRD65:096008,2002

4-point function

- The 2-particle correlator measures 4-point function at $q = 0$ (for $p \neq k$). Singularity appears at $q = 0$ due to vanishing σ screening mass $m_\sigma \to 0$. (i.e., $\xi = 1/m_\sigma \to \infty$).



$$\langle \delta n_p \delta n_k \rangle_\sigma = \frac{1}{T} \frac{f_p(1+f_p)}{\omega_p} \frac{f_k(1+f_k)}{\omega_k} \frac{G^2}{m_\sigma^2} .$$

Check: $\langle \delta n_p \delta n_k \rangle = \langle n_p n_k \rangle - \langle n_p \rangle \langle n_k \rangle > 0$ — as in attraction. Attraction lowers the energy of a pair (making it more likely) by $\langle H_{\text{interaction}} \rangle \sim$ forward scattering amplitude.

- Consider baryon number susceptibility, which should diverge: $\chi_B \sim \xi^{2-\eta}$

$$\chi_B \sim \langle \delta B \delta B \rangle_\sigma = \langle (\delta N_p - \delta N_{\bar{p}} + \delta N_n - \delta N_{\bar{n}})^2 \rangle_\sigma = \langle \delta N_p \delta N_p \rangle_\sigma + \ldots$$

Each term on r.h.s. is $\sim \frac{1}{m_\sigma^2}$, $\Rightarrow$ $\langle \delta B \delta B \rangle \sim 1/m_\sigma^2 = \xi^2$.

- - It is enough to measure protons $\langle \delta N_p \delta N_p \rangle$ (Hatta, MS, PRL91:102003,2003)

Higher moments (cumulants) of fluctuations

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

Ω – effective potential:

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right]. \qquad \Rightarrow \qquad \xi = m_\sigma^{-1}$$

- Moments of zero-momentum mode $\sigma_0 \equiv \int d^3x\, \sigma(x)/V$.

$$\kappa_2 = \langle\sigma_0^2\rangle = \frac{T}{V}\,\xi^2\,; \qquad \kappa_3 = \langle\sigma_0^3\rangle = \frac{2\lambda_3 T^2}{V^2}\,\xi^6\,;$$

$$\kappa_4 = \langle\sigma_0^4\rangle_c \equiv \langle\sigma_0^4\rangle - \langle\sigma_0^2\rangle^2 = \frac{6T^3}{V^3}\left[2(\lambda_3\xi)^2 - \lambda_4\right]\xi^8\,.$$

- Tree graphs. Each zero-momentum propagator gives m_σ^{-2}, i.e., ξ^2.

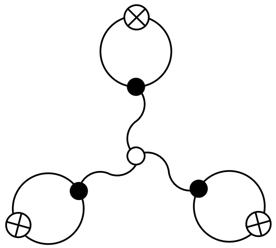
Moments of *observables*

- Use multiplicity for an example. Since multiplicity is just the sum of all occupation numbers, and thus

$$\delta N = \sum\nolimits_{\boldsymbol{p}} \delta n_{\boldsymbol{p}},$$

the cubic moment (skewness) of the pion multiplicity distribution is given by

$$\langle (\delta N)^3 \rangle = \sum\nolimits_{\boldsymbol{p}_1} \sum\nolimits_{\boldsymbol{p}_2} \sum\nolimits_{\boldsymbol{p}_3} \langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle \,, \qquad \text{where } \sum\nolimits_{\boldsymbol{p}} = V \int d^3\boldsymbol{p}/(2\pi)^3.$$

- 

$$\langle \delta n_{\boldsymbol{p}_1} \delta n_{\boldsymbol{p}_2} \delta n_{\boldsymbol{p}_3} \rangle_\sigma = \frac{2\lambda_3}{V^2 T} \left(\frac{G}{m_\sigma^2} \right)^3 \frac{v_{\boldsymbol{p}_1}^2}{\omega_{\boldsymbol{p}_1}} \frac{v_{\boldsymbol{p}_2}^2}{\omega_{\boldsymbol{p}_2}} \frac{v_{\boldsymbol{p}_3}^2}{\omega_{\boldsymbol{p}_3}}$$

$$v_{\boldsymbol{p}}^2 = \bar{n}_{\boldsymbol{p}}(1 \pm \bar{n}_{\boldsymbol{p}})$$

Similarly for $\langle (\delta N)^4 \rangle_c$.

- Since $\langle (\delta N)^3 \rangle$ scales as V^1 it is convenient to normalize it by the mean total multiplicity $\bar{N}$ which scales similarly. Thus we define

$$\omega_3(N) \equiv \frac{\langle (\delta N)^3 \rangle}{\bar{N}}$$

Moments of observables contd.

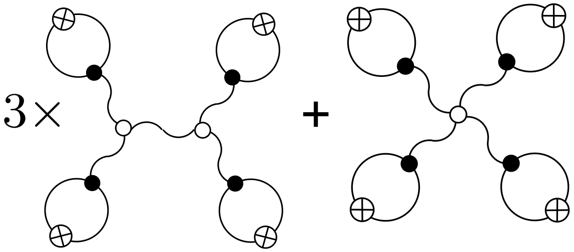
... and find

$$\omega_3(N)_\sigma = \frac{2\lambda_3}{T}\frac{G^3}{m_\sigma^6}\left(\int_{\boldsymbol{p}}\frac{v_{\boldsymbol{p}}^2}{\omega_{\boldsymbol{p}}}\right)^3\left(\int_{\boldsymbol{p}}\bar{n}_{\boldsymbol{p}}\right)^{-1}.$$

- Similarly, for

$$\omega_4(N) \equiv \frac{\langle(\delta N)^4\rangle_c}{\bar{N}}$$

from



we find

$$\omega_4(N)_\sigma = \frac{6}{T}\left[2\frac{\lambda_3^2}{m_\sigma^2} - \lambda_4\right]\frac{G^4}{m_\sigma^8}\left(\int_{\boldsymbol{p}}\frac{v_{\boldsymbol{p}}^2}{\omega_{\boldsymbol{p}}}\right)^4\left(\int_{\boldsymbol{p}}\bar{n}_{\boldsymbol{p}}\right)^{-1}.$$

Scaling, λ_n

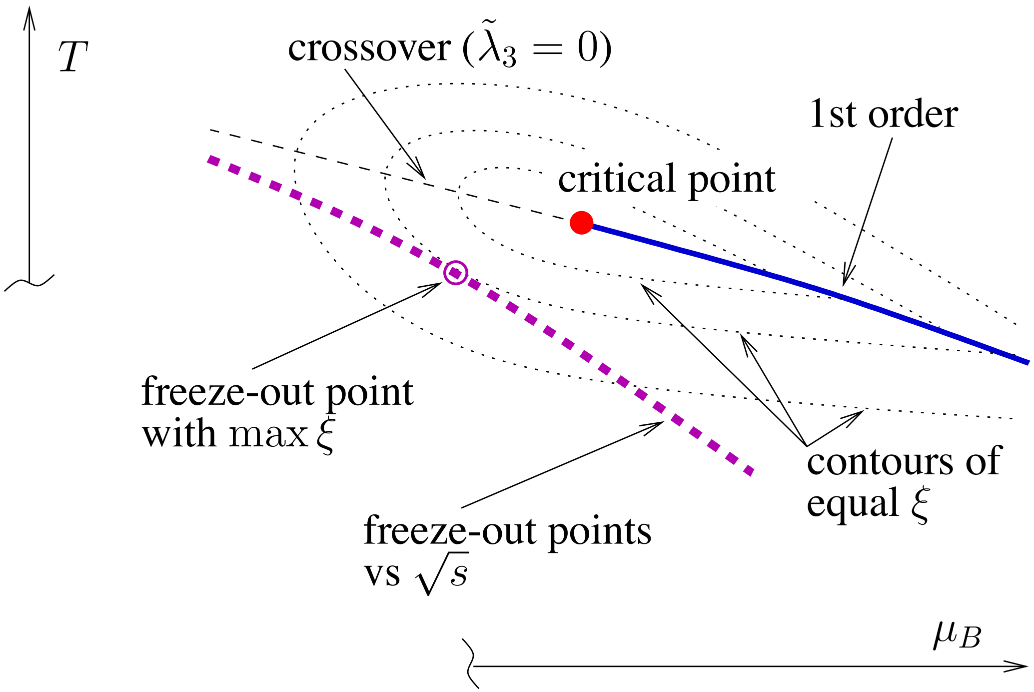
- Scaling requires that both λ_3 and λ_4 vanish with a power of ξ given by:

$$\lambda_3 = \tilde\lambda_3 T\cdot(T\xi)^{-3/2}, \qquad \text{and} \quad \lambda_4 = \tilde\lambda_4\cdot(T\xi)^{-1}, \qquad\qquad (\eta \ll 1)$$

(because $[(\nabla\sigma)^2] = 3 \;\Rightarrow\; [\sigma] = 1/2$ and $\Rightarrow\; [\lambda_n] = 3 - n/2$)

Dimensionless couplings $\tilde\lambda_3$ and $\tilde\lambda_4$ are universal, and for the Ising universality class they have been measured on the lattice.

- λ_3 is nonzero:



Estimates

Pions (top SPS):

$$\omega_3(N_\pi)_\sigma \equiv \frac{\langle(\delta N_\pi)^3\rangle}{\bar{N}_\pi} \approx 1. \left(\frac{\tilde{\lambda}_3}{4.}\right)\left(\frac{G}{300\text{ MeV}}\right)^3\left(\frac{\xi}{3\text{ fm}}\right)^{9/2}$$

$$\omega_4(N_\pi)_\sigma \equiv \frac{\langle(\delta N_\pi)^4\rangle_c}{\bar{N}_\pi} \approx 12. \left(\frac{2\tilde{\lambda}_3^2-\tilde{\lambda}_4}{50.}\right)\left(\frac{G}{300\text{ MeV}}\right)^4\left(\frac{\xi}{3\text{ fm}}\right)^{7}$$

Protons (top SPS):

$$\omega_3(N_p)_\sigma \equiv \frac{\langle(\delta N_p)^3\rangle}{\bar{N}_p} \approx 3. \left(\frac{\tilde{\lambda}_3}{4.}\right)\left(\frac{g}{10.}\right)^3\left(\frac{\xi}{1\text{ fm}}\right)^{9/2}$$

$$\omega_4(N_p)_\sigma \equiv \frac{\langle(\delta N_p)^4\rangle_c}{\bar{N}_p} \approx 23. \left(\frac{2\tilde{\lambda}_3^2-\tilde{\lambda}_4}{50.}\right)\left(\frac{g}{10.}\right)^4\left(\frac{\xi}{1\text{ fm}}\right)^{7}$$

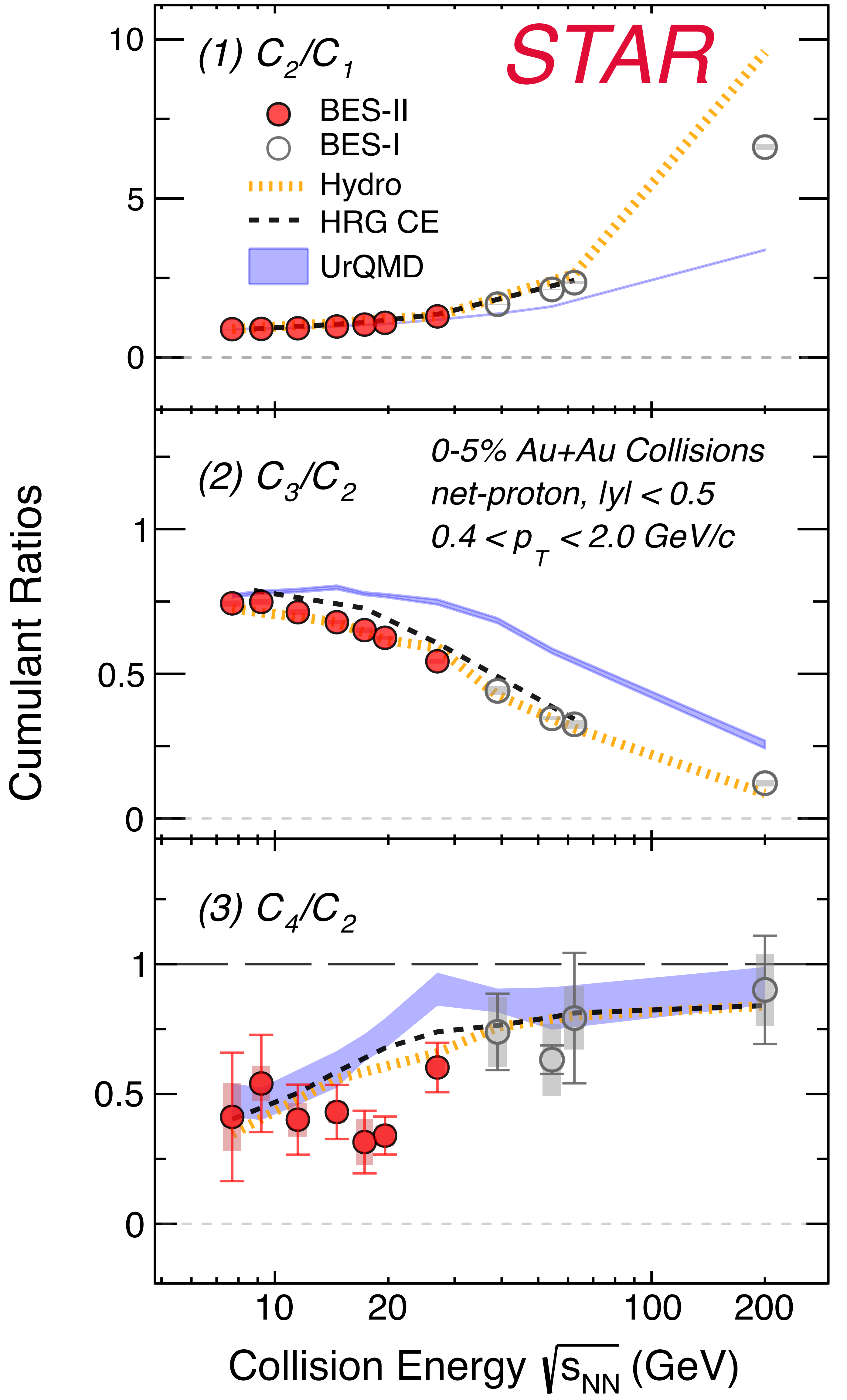
Notes:

- Strong dependence on ξ, compared to $\omega_2 \sim \xi^2$.
- Significant uncertainty due to G, g.
- Crosscheck: same exponents as baryon number cumulants from scaling/universality:

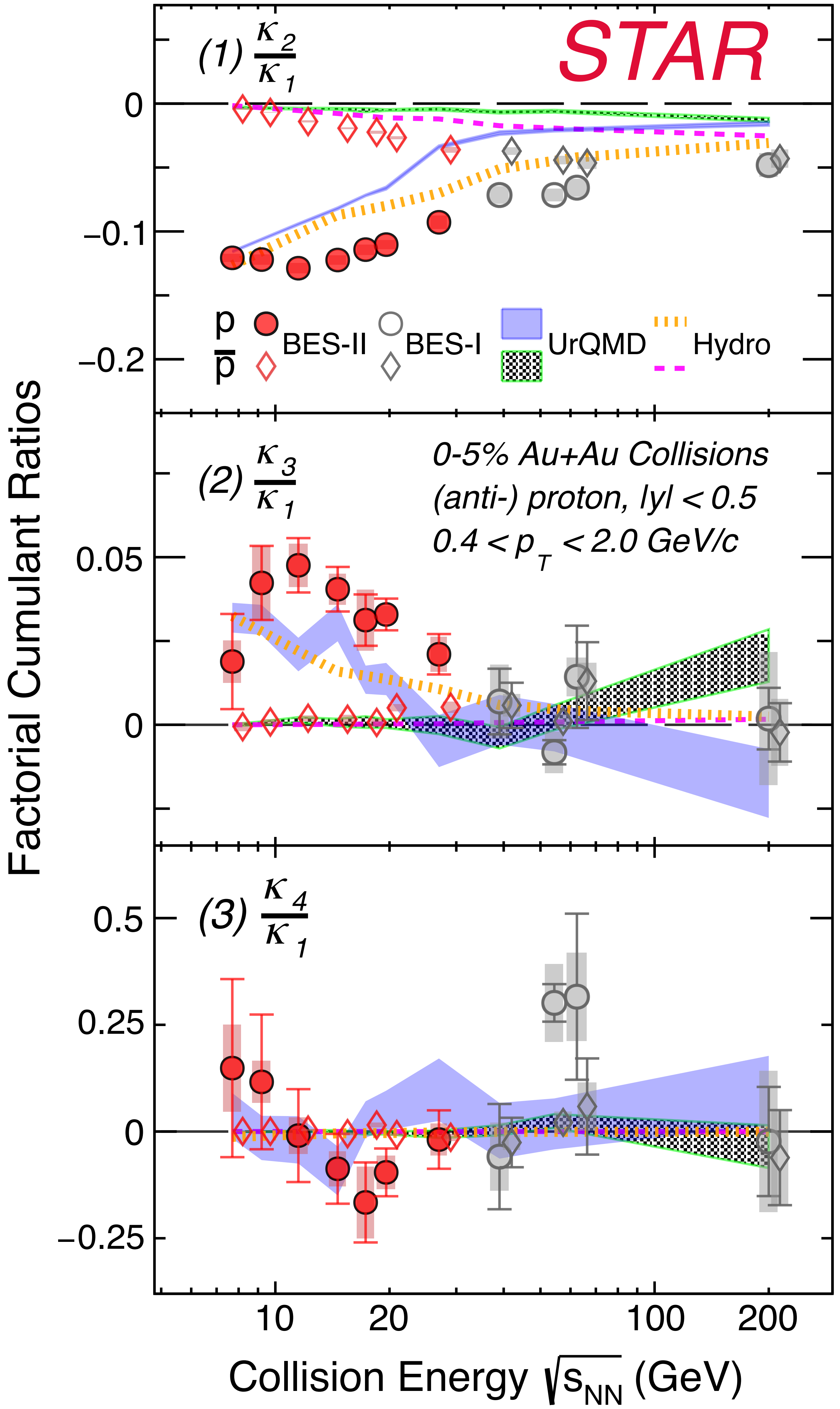
$$\langle(\delta N_B)^k\rangle_c = VT^{k-1}\frac{\partial^k P(T,\mu_B)}{\partial\mu_B^k} \sim \xi^{k(5-\eta)/2-3}. \qquad (\eta \ll 1)$$

ENERGY DEPENDENCE: MODEL COMPARISON

Net-proton cumulant ratios



Proton/antiproton factorial cumulant ratios



1. Smooth variation vs $\sqrt{s_{NN}}$ in C_2/C_1 and C_3/C_2 observed. C_4/C_2 decreases with decreasing $\sqrt{s_{NN}}$.

2. Non-CP models used for comparison:

A. Hydro: Hydrodynamical model

B. HRG CE: Thermal model with canonical treatment of baron charge

C. UrQMD: Hadronic transport model
(All models include baryon number conservation)

3. Proton factorial cumulant ratios deviates from poisson baseline at 0.
Antiproton κ_3/κ_1, κ_4/κ_1 closer to 0.

4. Qualitative trend described by model. Quantitative differences exist b/w data and non-CP model.

HRG CE: P. B Munzinger et al, NPA 1008, 122141 (2021)
Hydro: V. Vovchenko et al, PRC 105, 014904 (2022)