FRIB science (lecture 2)

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overview

Lecture 1:

- why reactions?
- reactions and big science questions
- basic concepts in reactions
- production of the exotic nuclei and FRIB

Lecture 2:

- theories for nuclear reactions
- optical model
- Bayesian uncertainty quantification
- emulators for reactions
- hands-on example

Important references

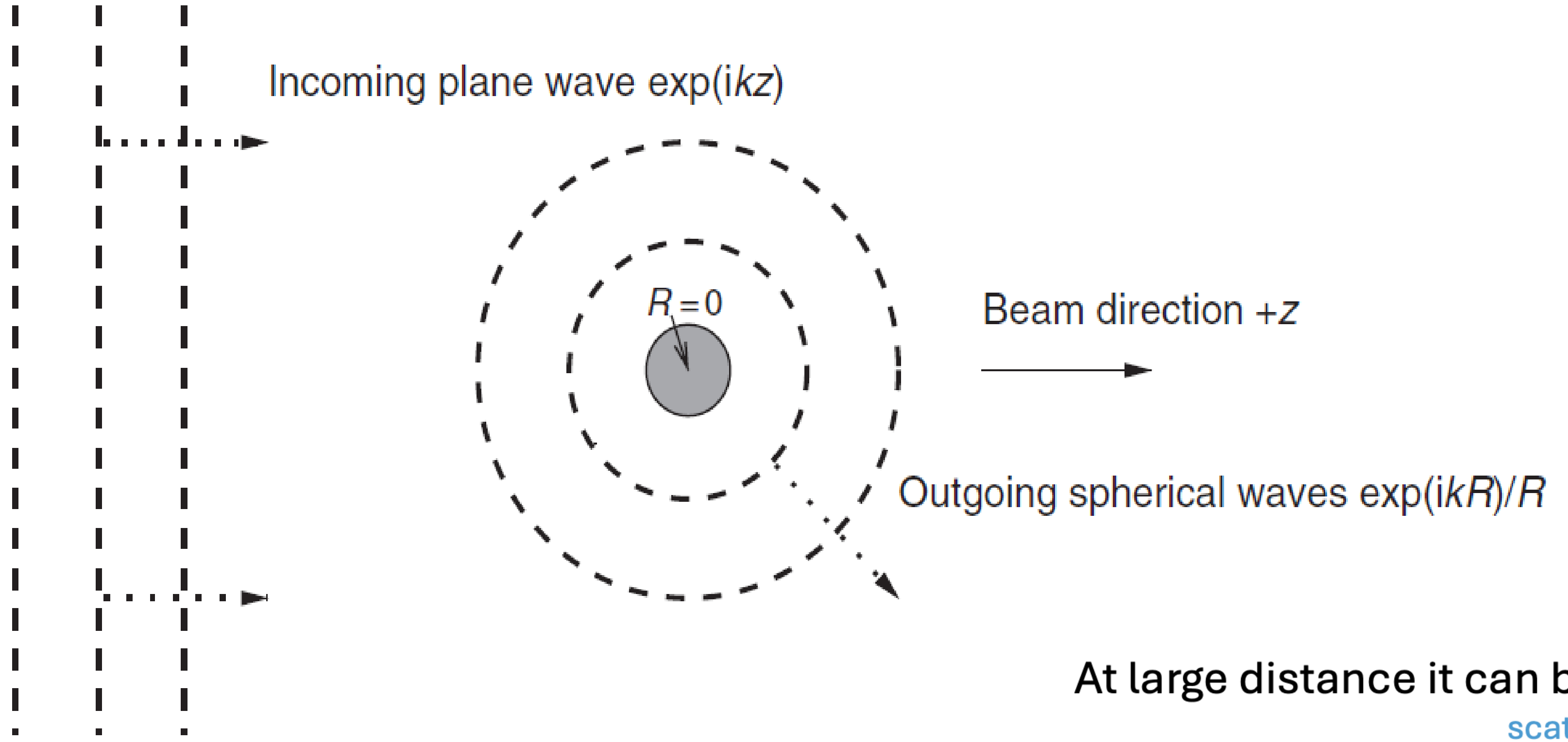
- Thompson and Nunes, Nuclear reactions for astrophysics, Cambridge University Press
- Nuclear Physics Long Range Plan (2015, 2023)

Several pieces borrowed from many places:

- Chloe Hebborn's lectures in nuclear reactions
  XXII Escola de Verão de Física Nuclear Teórica 2025, Niteroi
- Gregory Potel's slides on the optical potential
  Compound Nuclear Reactions Conference 2024
- Pablo Giuliani's slides on reduced order methods
  STREAMLINE collaboration meeting 2026

Single channel scattering

picture for elastic scattering



At large distance it can be shown

Solution for V=0

scattering amplitude

$$\psi^{\text{asym}} = \psi^{\text{beam}} + \psi^{\text{scat}} = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$$

deflection angle

$$\mathbf{j} = \mathbf{v}|\psi|^2$$

Scattering theory: setting up

Incoming beam

$$\psi^{\text{beam}} = A \exp(\mathrm{i}\mathbf{k}_i \cdot \mathbf{R})$$

$$\psi^{\text{beam}} = A\mathrm{e}^{\mathrm{i}k_i z}$$

Incoming flux

$$k = \sqrt{2\mu E/\hbar^2}$$

$$\mathbf{v} = \mathbf{p}/\mu = \hbar\mathbf{k}/\mu$$

$$j_i = v_i |A|^2$$

Scattered wave

$$\psi^{\text{scat}} = Af(\theta,\phi)\frac{\mathrm{e}^{\mathrm{i}k_f R}}{R}$$

Outgoing flux

$$j_f = v_f |A|^2 |f(\theta,\phi)|^2 / R^2$$

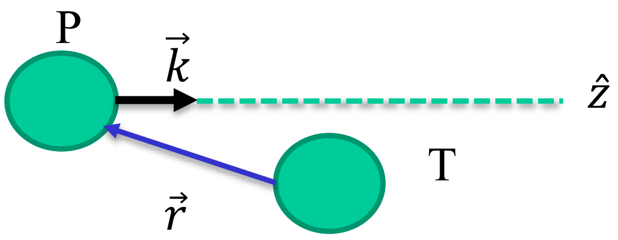
Asymptotic wave

$$\psi^{\text{asym}} = \psi^{\text{beam}} + \psi^{\text{scat}} = A\left[\mathrm{e}^{\mathrm{i}k_i z} + f(\theta,\phi)\frac{\mathrm{e}^{\mathrm{i}k_f R}}{R}\right]$$

Scattering amplitude

From 3D to 1D: Partial wave decomposition

The system is described by the wavefunction $\psi\,(\vec{r})$



Solution of 3-D Schrodinger equation

$$\left[\hat{T} + V - E\right]\psi(r, \theta, \phi) = 0$$

Spherical potential $V(r)$ (here no coulomb)

Partial-wave decomposition: $$\psi(R, \theta) = \sum_{l=0}^{\infty}(2l+1)i^l P_l(\cos\theta)\frac{1}{kr}\phi_l(r)$$

ϕ_l solution of a 1-D Schrodinger equation $$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right) + V(r) - E\right]\phi_l(r) = 0$$

Hebborn, slides for Andre Swieca School 2025

picture for elastic scattering – no target

V(R)=0

Incident plane wave

R=0

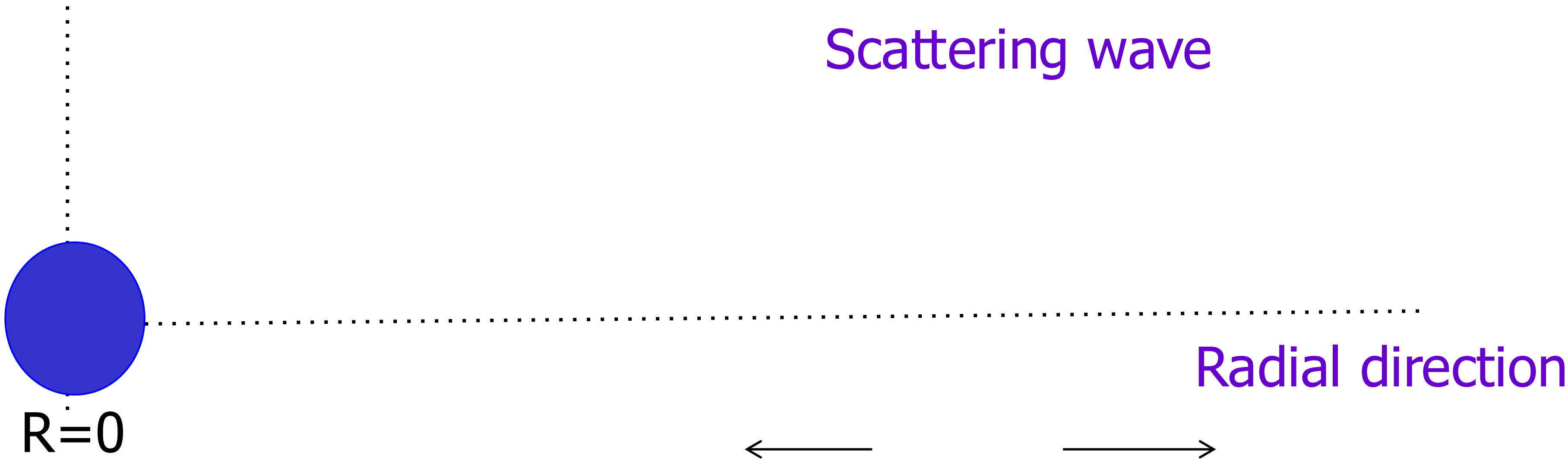
Radial direction

$$e^{ikz} = \sum_{L=0}^{\infty} (2L+1) i^L P_L(\cos\theta) \frac{1}{kR} \frac{i}{2} [H_L^-(0,kR) - H_L^+(0,kR)]$$

incoming ← → outgoing

H=Hankel functions for as e^{-ikR} and e^{+ikR}

picture for elastic scattering – with target

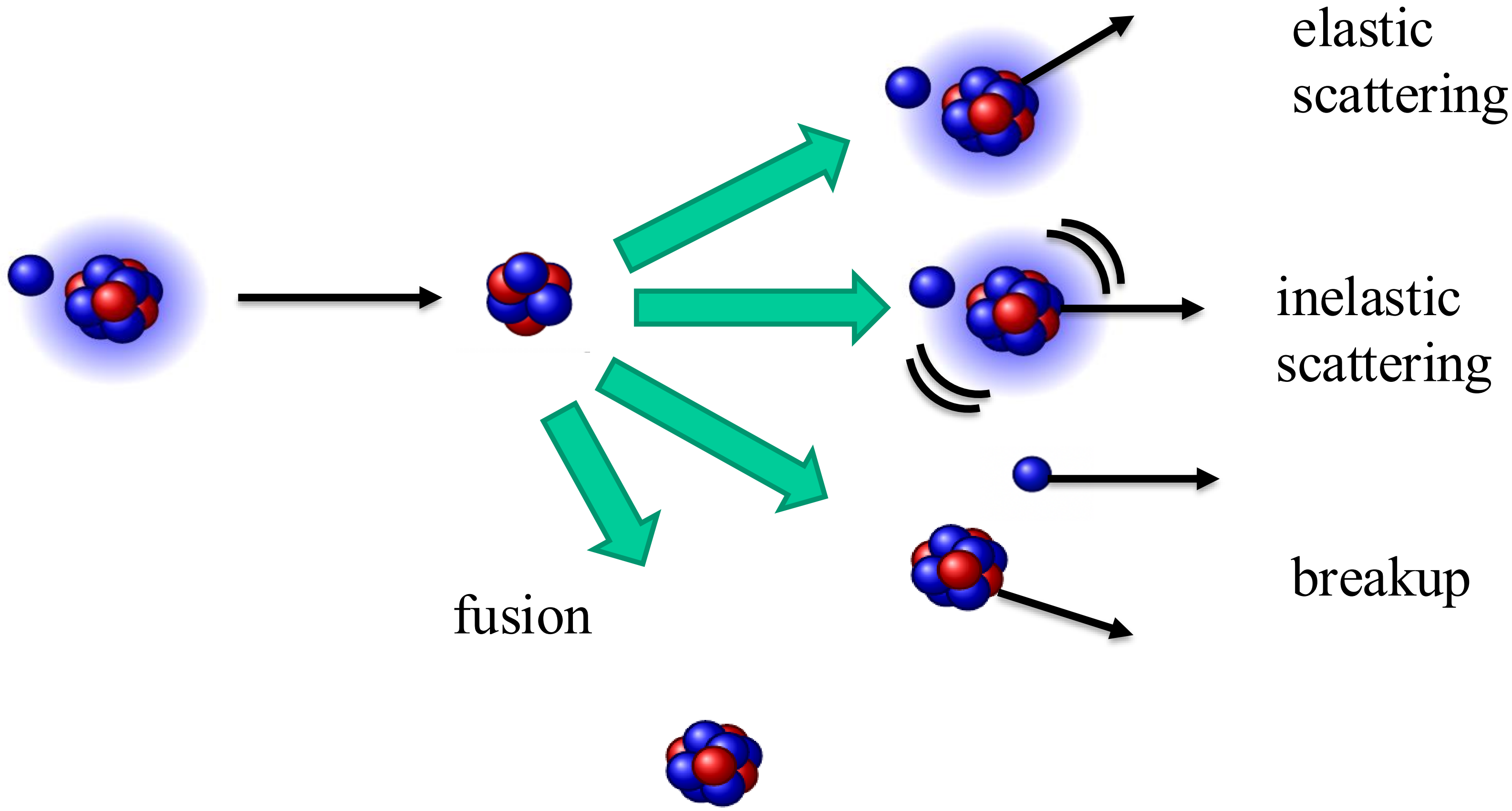


$$\chi_L^{\mathrm{ext}}(R) = A_L \left[H_L^-(0, kR) - \mathbf{S}_L H_L^+(0, kR) \right]$$

incoming outgoing

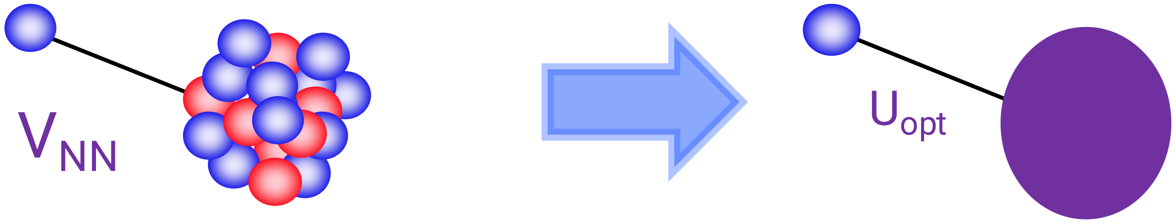
Many possible reaction channels

All these complex processes, take away flux from the elastic channel!



Effective interactions and flux loss

The effective interactions between projectile-target needs to incorporate these effects.

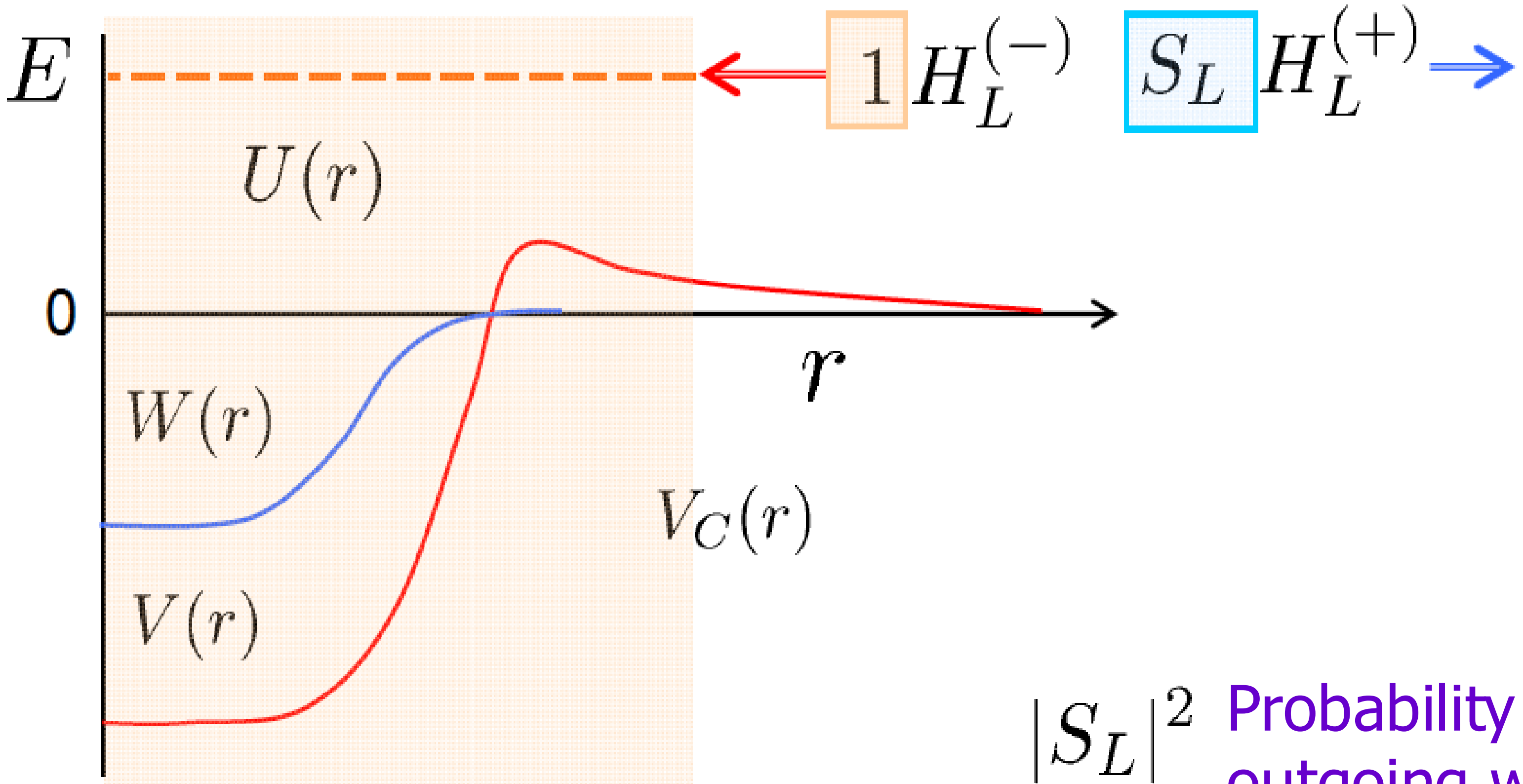


$U_{opt} = V(R) + iW\,(R)$

Imaginary term is responsible for the absorbtion of flux away from the elastic

The meaning of the S-matrix

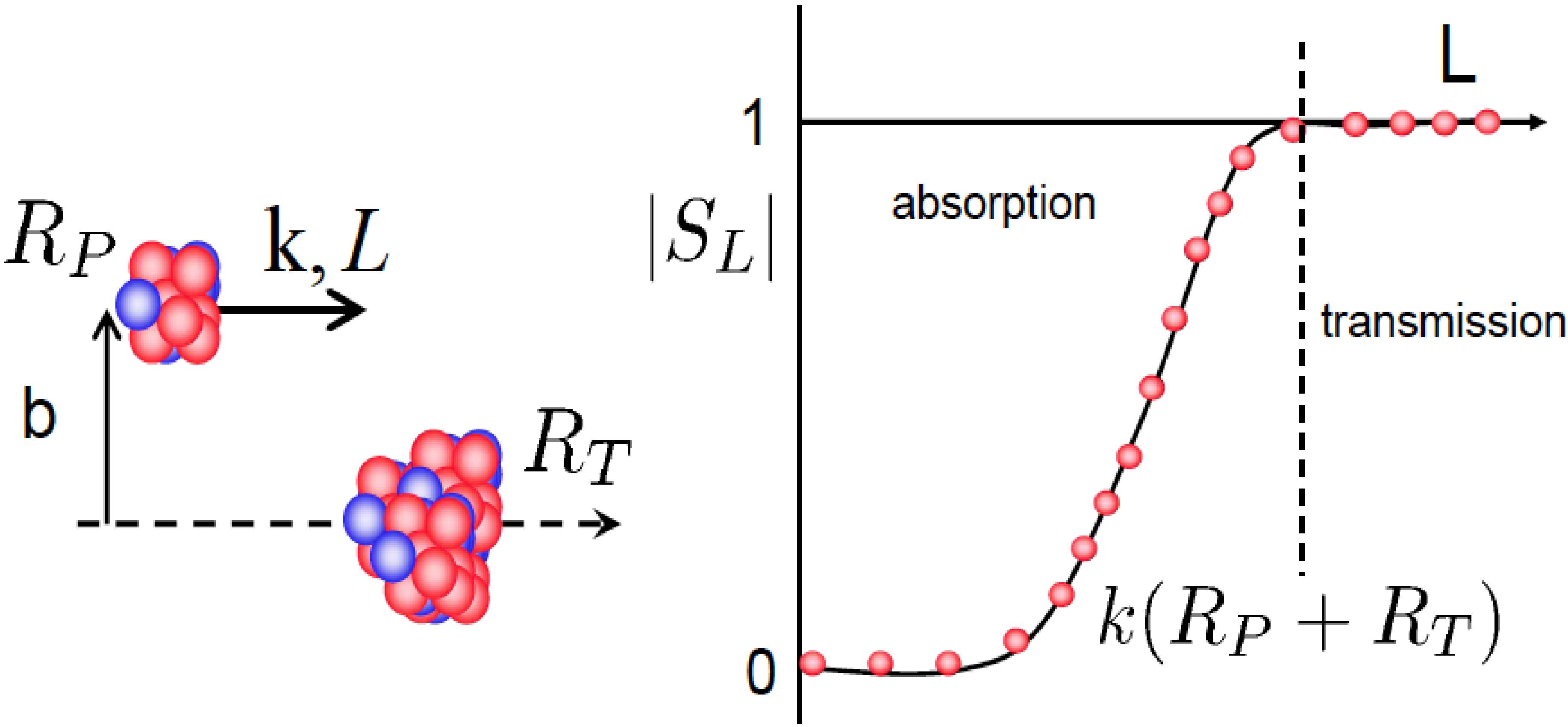
$$\chi_L^{\text{ext}}(R) = A_L\left[H_L^-(0,kR) - \mathbf{S}_L H_L^+(0,kR)\right]$$



$|S_L|^2$ Probability of outgoing wave

$(1-|S_L|^2)$ Probability of getting absorbed

Scattering and angular momentum



In the classical approximation L=kb:

When L>k(R_p+R_t) all the flux goes through

When L<k(R_p+R_t) flux gets absorbed

S-matrix and the scattering amplitude

Relation between the 3D picture

$$\psi^{\text{asym}}(R,\theta) = e^{ikz} + f(\theta)\frac{e^{ikR}}{R}$$

And the radial picture (through partial wave decomposition)

$$\psi(R,\theta) \overset{R>R_n}{\rightarrow} \frac{1}{kR}\sum_{L=0}^{\infty}(2L+1)i^L P_L(\cos\theta)A_L[H_L^-(0,kR) - \mathbf{S}_L H_L^+(0,kR)]$$

Elastic cross section

$$f(\theta) = \frac{1}{2ik}\sum_{L=0}^{\infty}(2L+1)P_L(\cos\theta)(\mathbf{S}_L - 1)$$

$$\sigma(\theta) \equiv \frac{d\sigma}{d\Omega} = \left|\frac{1}{2ik}\sum_{L=0}^{\infty}(2L+1)P_L(\cos\theta)(\mathbf{S}_L - 1)\right|^2$$

Phase shifts

The wavefunction at large distances can be equivalently described with a phase shift:

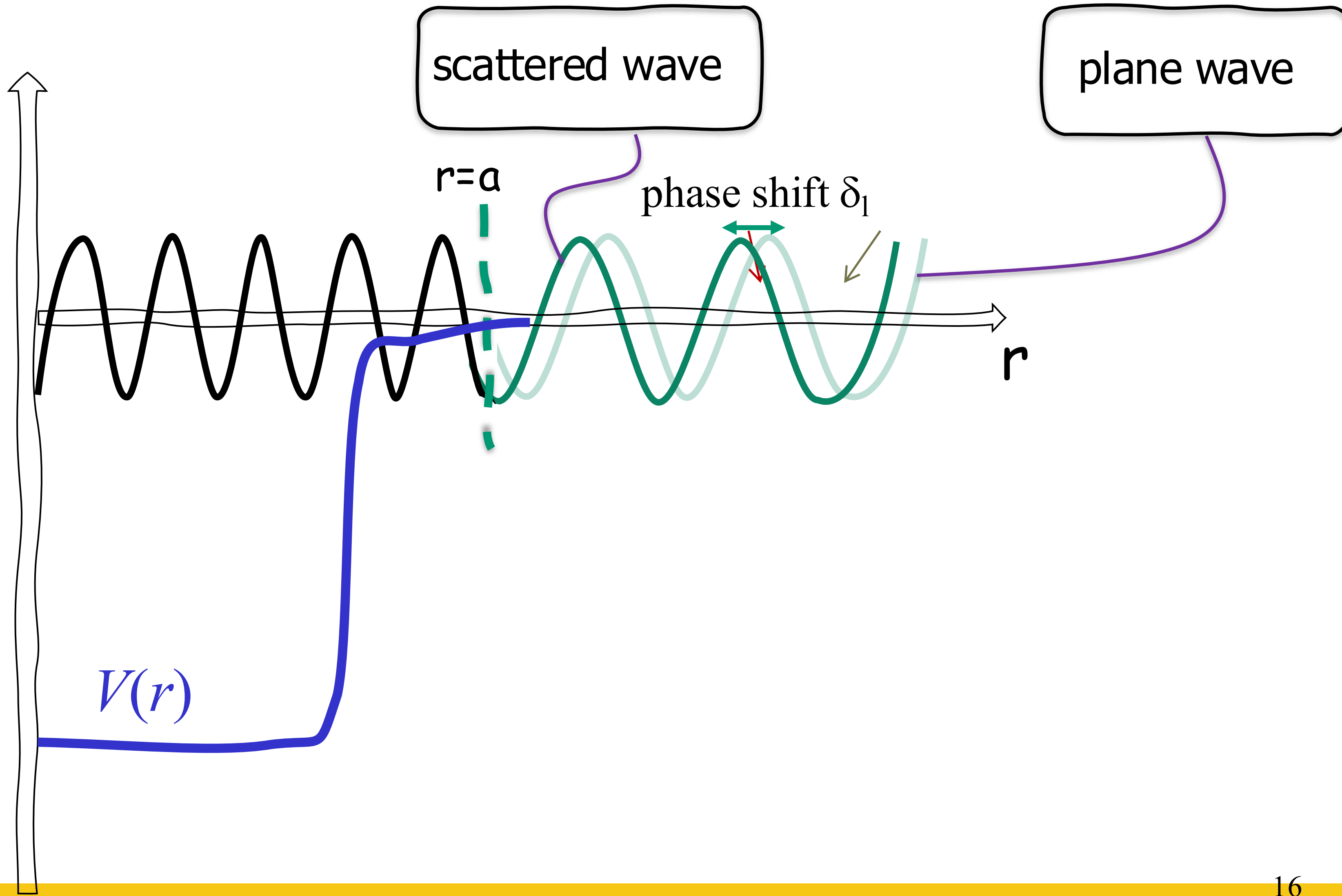
$$\chi_L^{\text{ext}}(R) = A_L \left[H_L^-(0, kR) - \mathbf{S}_L H_L^+(0, kR) \right]$$

$$\boxed{\mathbf{S}_L = e^{2i\delta_L}}$$

$$\chi_L^{\text{ext}}(R) \to e^{i\delta_L}[\cos\delta_L \sin(kR - L\pi/2) + \sin\delta_L \cos(kR - L\pi/2)]$$

$$= e^{i\delta_L} \sin(kR + \delta_L - L\pi/2).$$

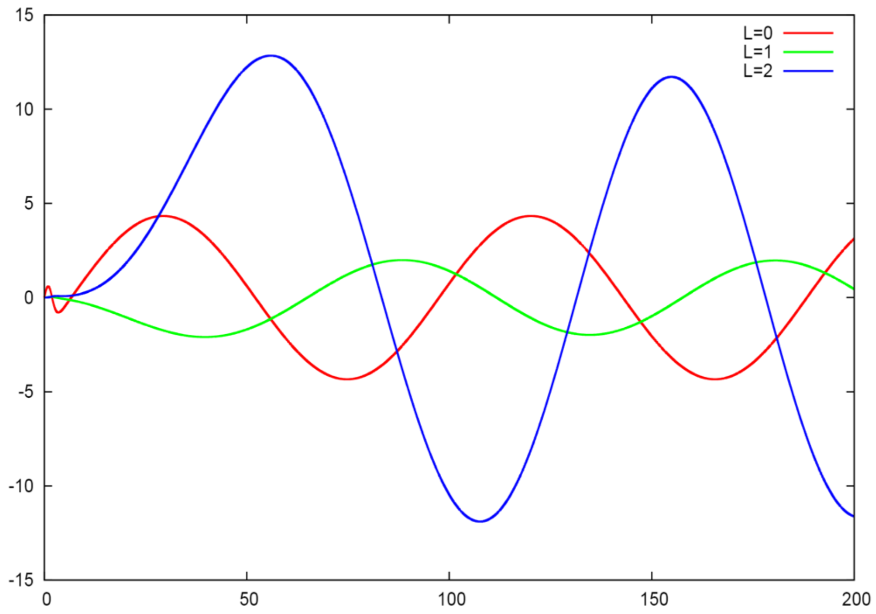
Phase shifts



Phase shifts

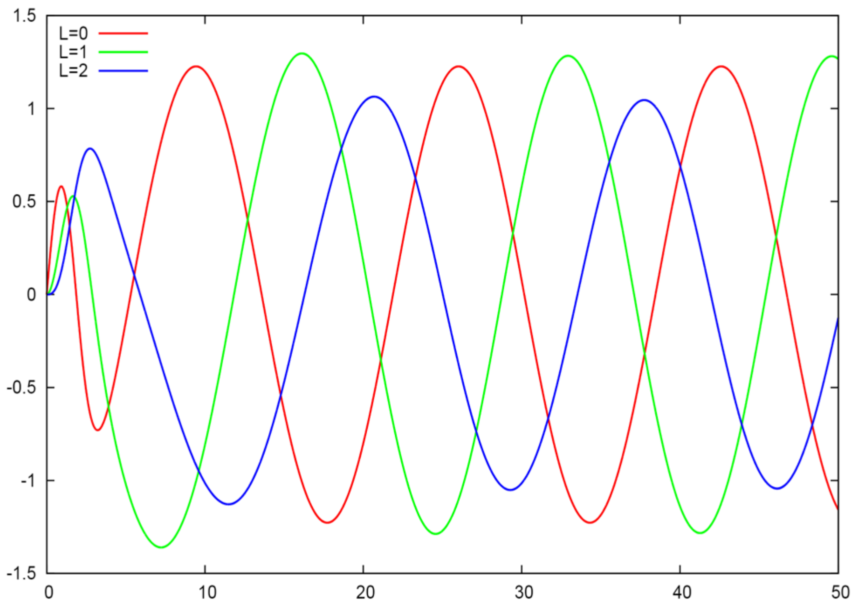
$$\chi_L^{\text{ext}}(R) \to e^{i\delta_L}[\cos\delta_L \sin(kR - L\pi/2) + \sin\delta_L \cos(kR - L\pi/2)]$$
$$= e^{i\delta_L}\sin(kR + \delta_L - L\pi/2).$$

wavefunction n+10Be for E=0.1 MeV



Radius (fm)

wavefunction n+10Be for E=3.0 MeV



Radius (fm)

$$k = \sqrt{2\mu E/\hbar^2}$$

QUIZ TIME

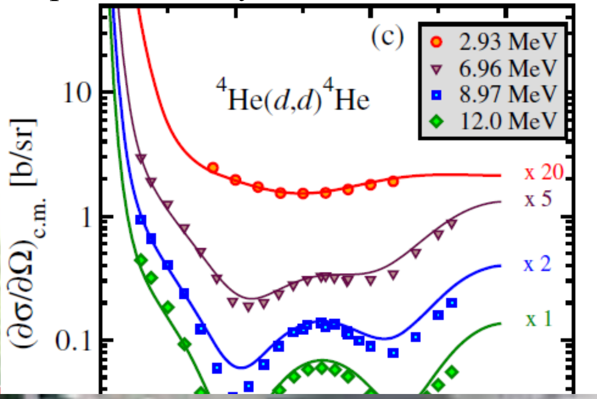
If the $|S|^2=1$,
how much flux was lost from the elastic channel?

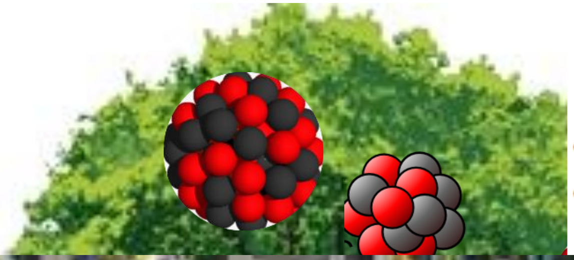
a) 0%
b) 100%
c) not enough information

Theories for reactions beyond elastic scattering

Ab-initio reactions for the lightest nuclei

Hupin et al., Phys. Rev. Lett. **114**, 212502





hpc at LLNL

Cannot use ab-initio for reactions with heavy nuclei

$^{6}Li(d,p)^{7}Li$

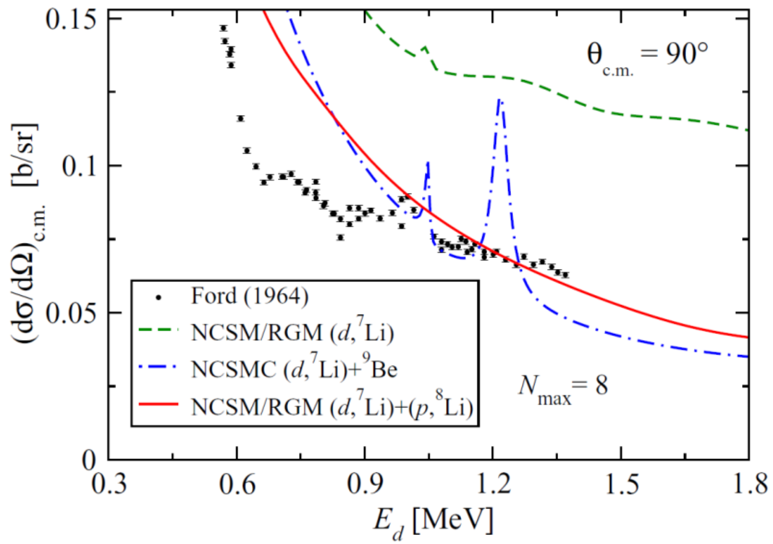


FIG. 7. Computed $^{7}Li(d,d)^{7}Li$ differential cross sections in the c.m. frame at the deuteron scattering angle of 90° as function of the kinetic energy of deuterons in the laboratory system, compared to the experimental data of Ref. [39]. The three sets of theoretical curves correspond to calculations within the $(d,{}^{7}Li)$ NCSM-RGM (green dashed line), $(d,{}^{7}Li) + {}^{9}Be$ NCSMC (blue dash-dotted line), and $(d,{}^{7}Li) + (p,{}^{8}Li)$ NCSM-RGM (red solid line) model spaces.

PRC93, 054606 (2016)

$^{132}Sn(d,p)^{133}Sn$

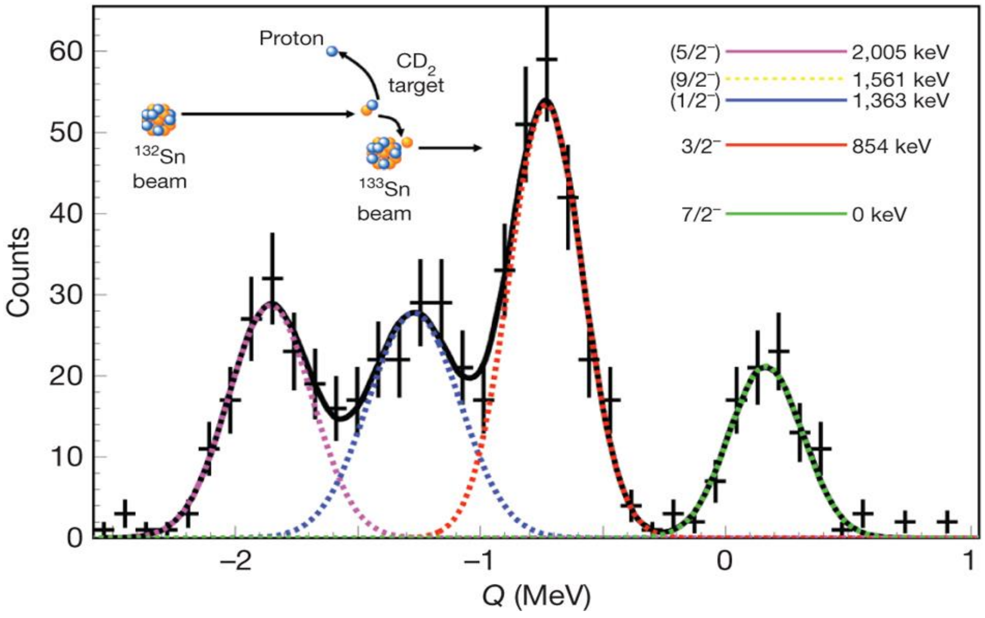
$^{59}Cu(d,n\gamma)^{60}Zn^{*}$

$^{95}Mo(d,p\gamma)\ ^{96}Mo^{*}$

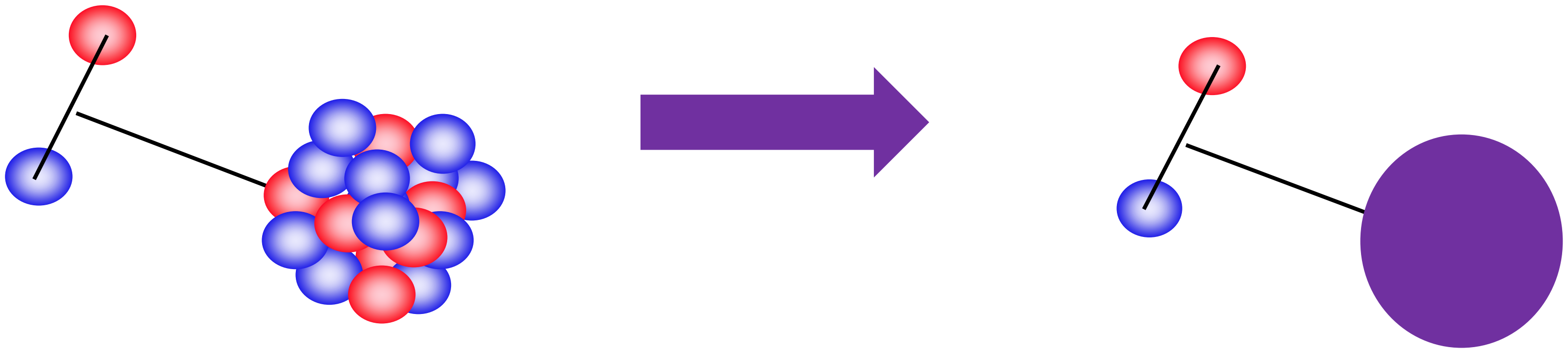
....?

Here is the challenge…

- A complex many-body problem
- Scattering boundary conditions
- Huge sensitivity to thresholds
- Large Coulomb (infinite-range) interactions
- Specific clustering



Reduction from many-body to few-body

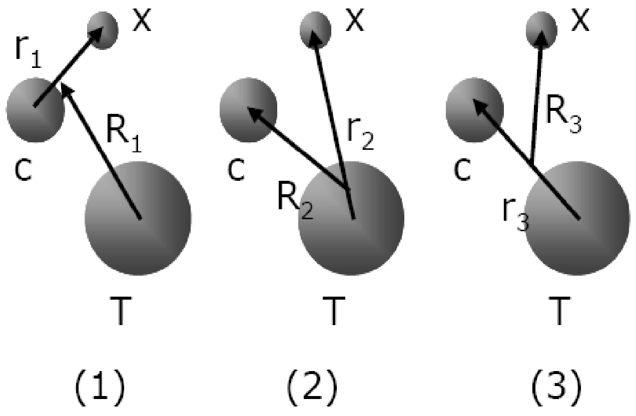


- ❑ isolating the important degrees of freedom in a reaction
- ❑ effective nucleon-nucleus interactions (or nucleus-nucleus) usually referred to as **optical potentials**

→ main cause of uncertainty

Solving the few-body scattering problem

Faddeev Formalism for reactions



$$
\begin{aligned}
(E - T_1 - V_{xc})\Psi^{(1)} &= V_{xc}(\Psi^{(2)} + \Psi^{(3)}) \\
(E - T_2 - V_{ct})\Psi^{(2)} &= V_{ct}(\Psi^{(3)} + \Psi^{(1)}) \\
(E - T_3 - V_{tx})\Psi^{(3)} &= V_{tx}(\Psi^{(1)} + \Psi^{(2)})
\end{aligned}
$$

Still challenging today due to Coulomb!

4N bound state

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

Method	S wave	P wave	D wave
FY	85.71	0.38	13.91
CRCGV	85.73	0.37	13.90
SVM	85.72	0.368	13.91
HH	85.72	0.369	13.91
NCSM	86.73	0.29	12.98
EIHH	85.73(2)	0.370(1)	13.89(1)

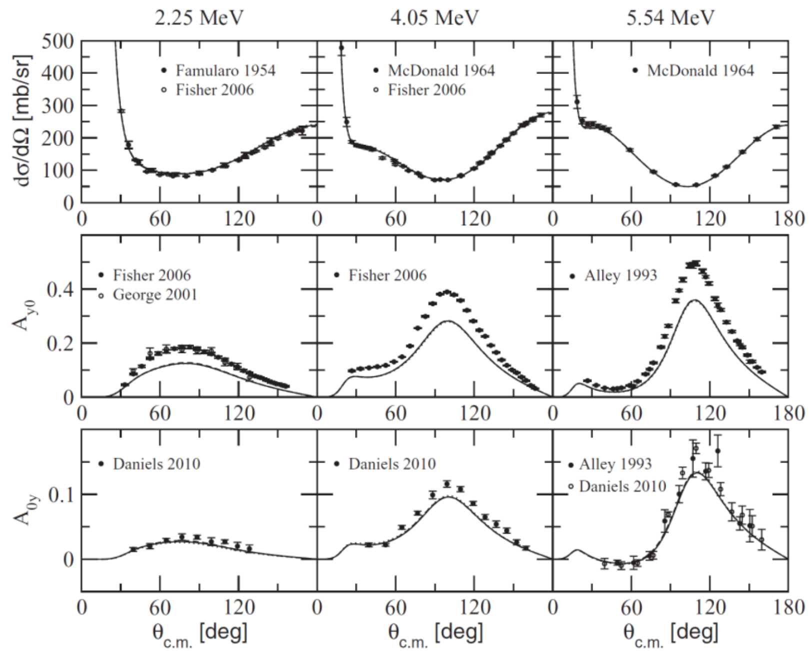
H. Kamada, et al, PRC 64, 044001 (2001)

Solving the few-body scattering problem

n-^{3}He scattering

	I-N3LO	AV18	
E_n	σ_t	σ_t	
1.0	1.77	1.80	AGS
	1.77	1.78	HH
	1.81	1.81	FY
2.0	2.13	2.12	AGS
	2.13	2.10	HH
	2.19	2.13	FY
3.5	2.38	2.33	AGS
	2.38	2.32	HH
	2.41	2.33	FY
6.0	1.97	1.93	AGS
	1.97	1.93	HH
	1.97	1.92	FY

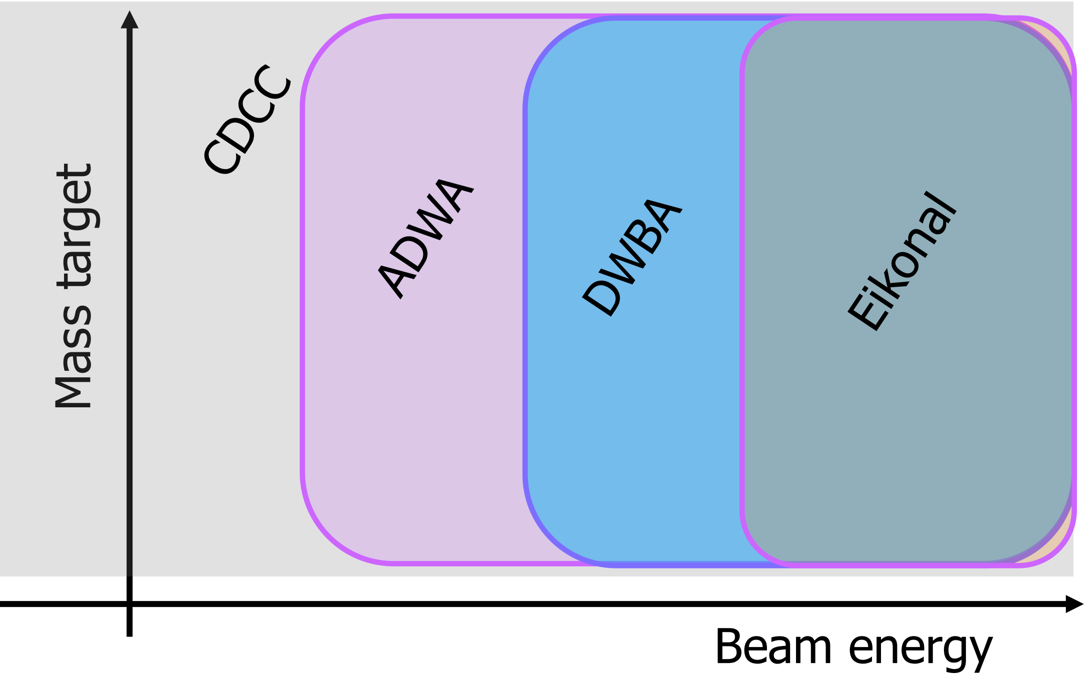
p-^{3}He scattering (N3LO)



Viviani et al., Phys. Rev. C 84, 054010 (2011)

For the lighter systems, the dynamics can be solved exactly (Coulomb is weak)

Theories for reactions



Eikonal (semi-classical straightline trajectories +adiabatic approx.)

DWBA - Distorted Wave Born Approximation (pertubative 1-step)

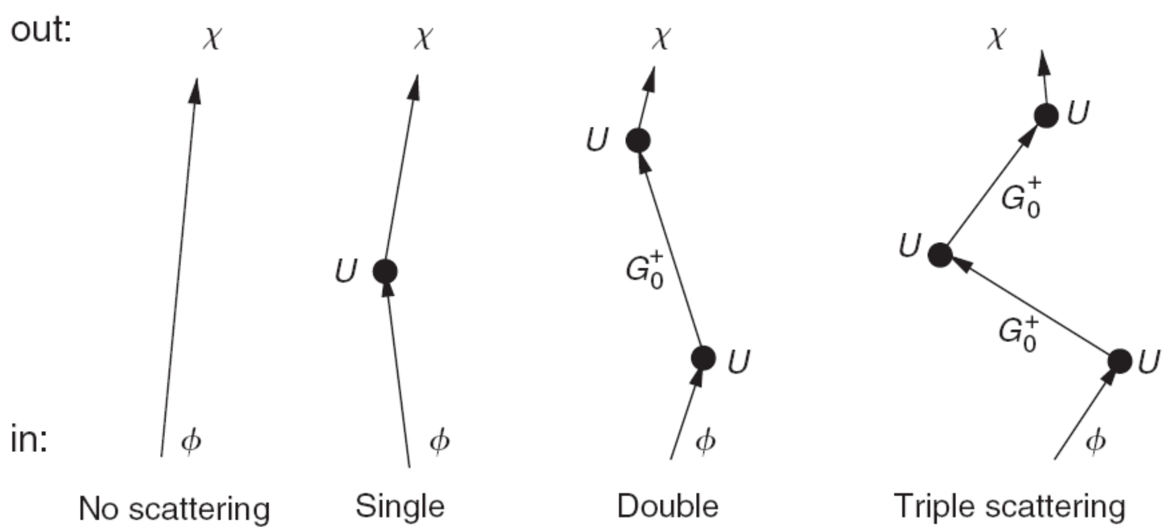
ADWA – Adiabatic wave approximation (QM and non-perturbative)

CDCC – Continuum Discretized Coupled Channel (fully coupled but only one Jacobi component)

Theories for reactions: DWBA

Distorted Wave Born Approximation is based on a pertubative expansion;
it includes quantum effects and is often truncated to first order.

$$\Psi = \phi + G_0 V \phi + G_0 V G_0 V \phi + G_0 V G_0 V G_0 V \phi + \ldots.$$



$$T = -\frac{2\mu}{\hbar^2 k}(< \phi|V|\phi > + < \phi|V G_0 V|\phi > + < \phi|V G_0 V G_0 V|\phi > + \ldots.)$$

Theories for reactions: DWBA

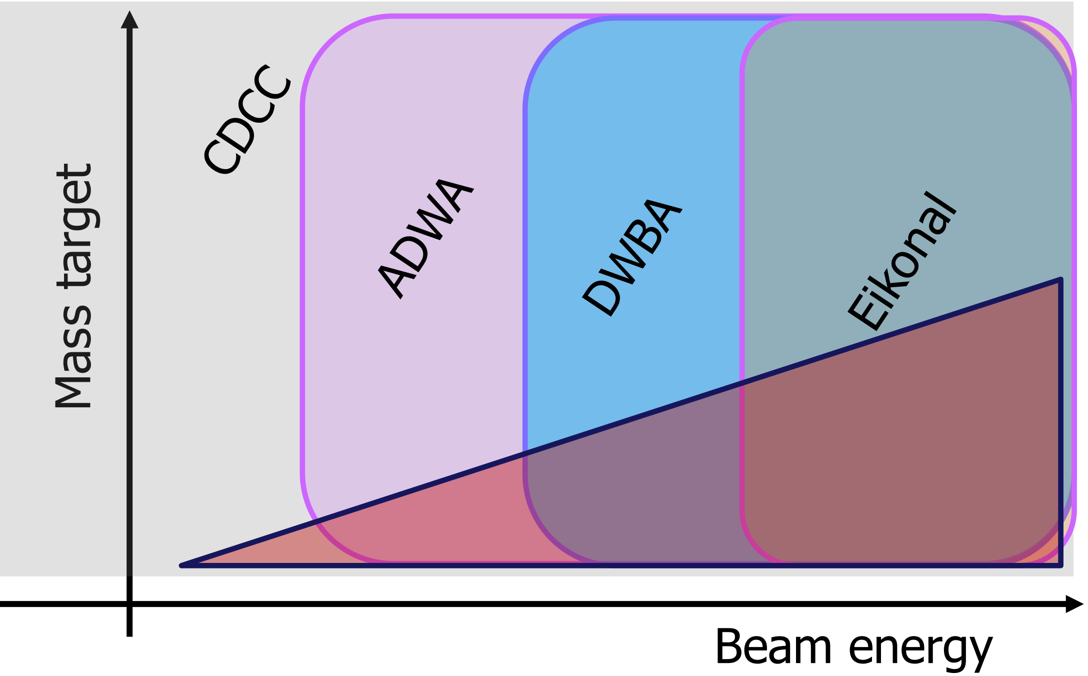
1-step DWBA is only good if the interaction in the operator is weak.

What you treat exactly (in the bra/ket) and what you treat perturbatively (in the operator) matters!

$$\mathbf{T}^{(1+2)} = \mathbf{T}^{(1)} - \frac{2\mu}{\hbar^2 k}\left[\langle\chi^{(-)}|U_2|\chi\rangle + \langle\chi^{(-)}|U_2\hat{G}_1 U_2|\chi\rangle + \cdots\right].$$

U_1 can be strong but, if U_2 is weak, we might expect this series to converge

Theories for reactions



Eikonal (semi-classical straightline trajectories +adiabatic approx.)

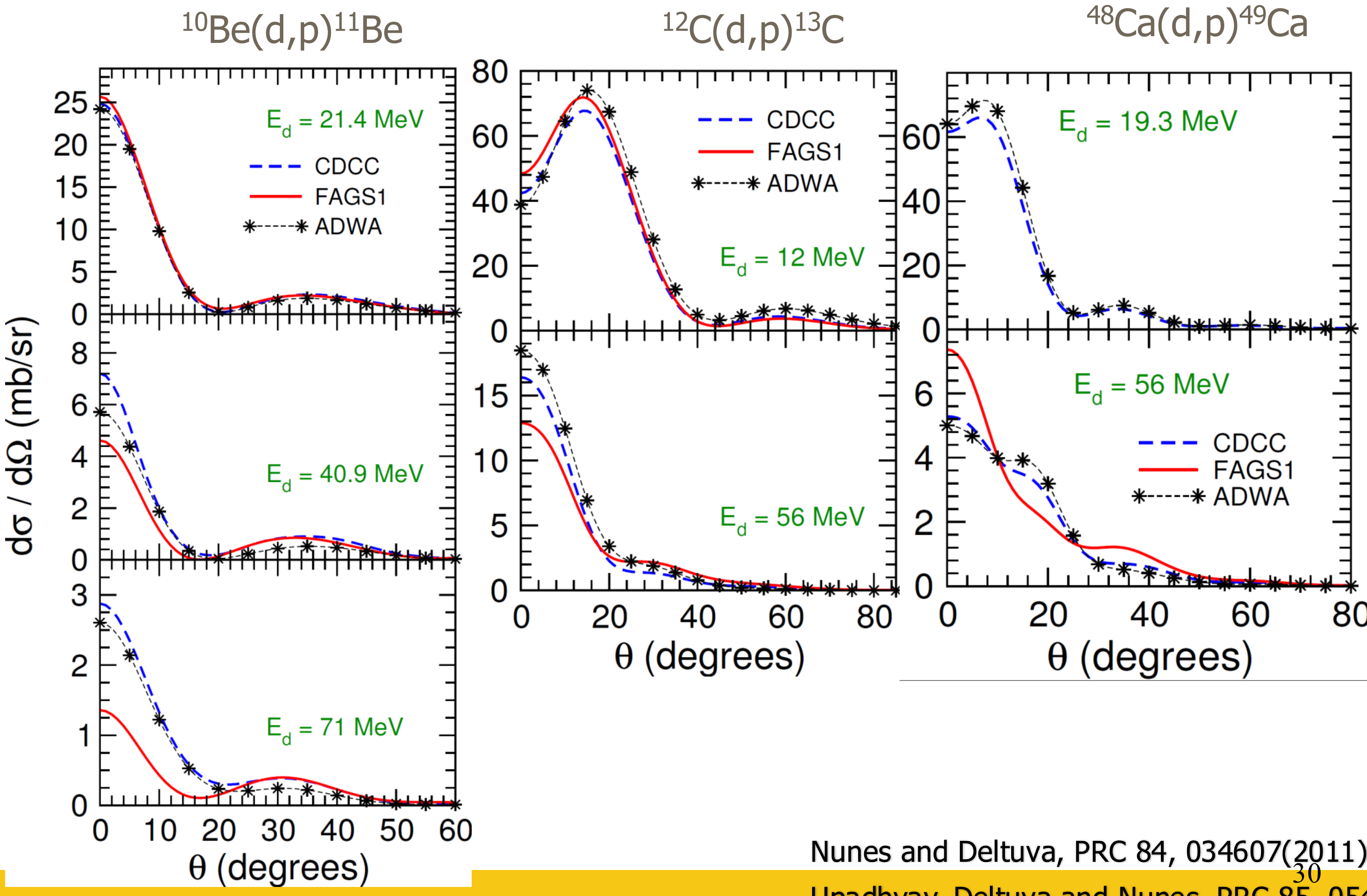
DWBA - Distorted Wave Born Approximation (pertubative 1-step)

ADWA – Adiabatic wave approximation (QM and non-perturbative)

CDCC – Continuum Discretized Coupled Channel (fully coupled but only one Jacobi component)

Faddeev: 3-body dynamics exact, all rearrangement channels treated on equal footing, uses screening to deal with long-range Coulomb interaction.
Limitations for low energy and high Z!

Comparing theories for reactions



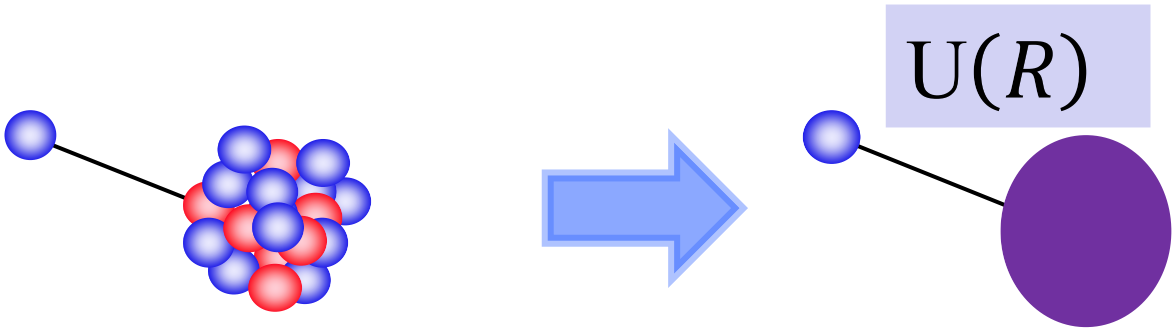
Nunes and Deltuva, PRC 84, 034607(2011)

Upadhyay, Deltuva and Nunes, PRC 85, 054621

The optical potential

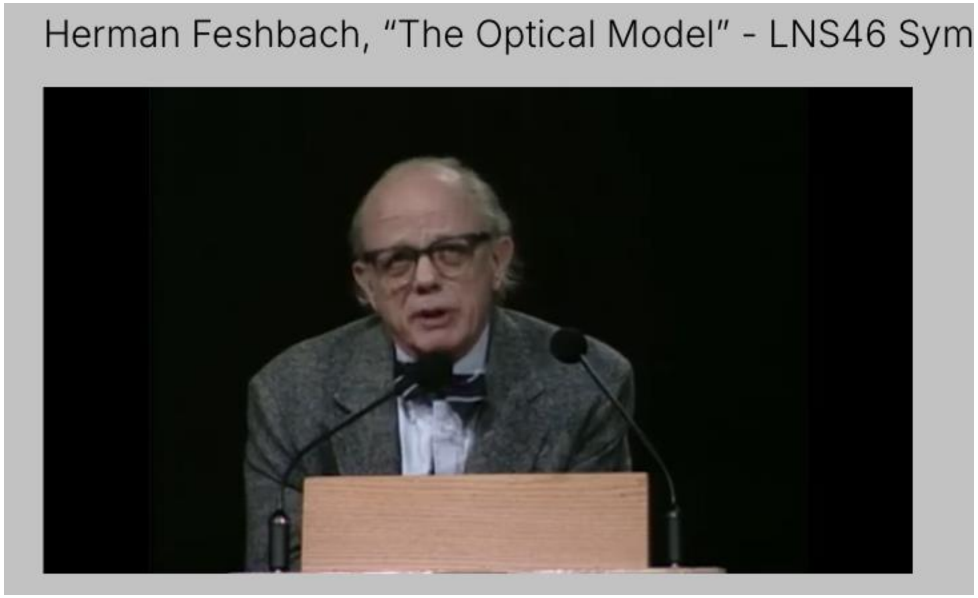
We have methods to solve the few-body scattering problem, but…

how do we know the Hamiltonian?

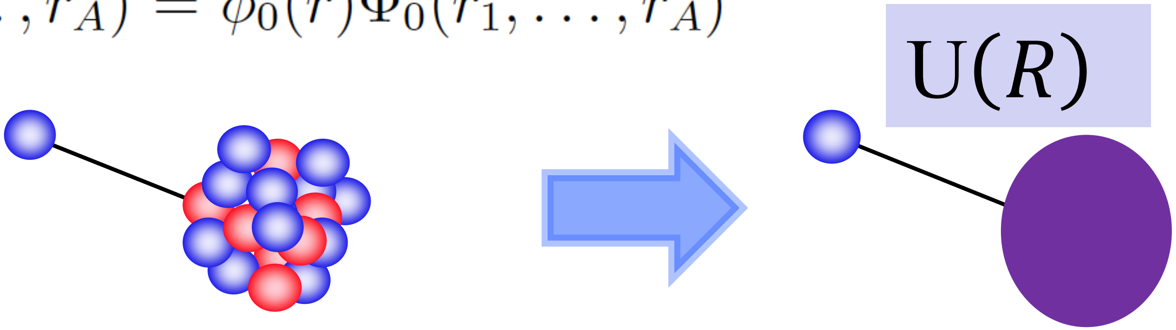


The optical potential

Herman Feshbach Talk
at the 46th anniversary of
MIT’s Laboratory for Nuclear
Science



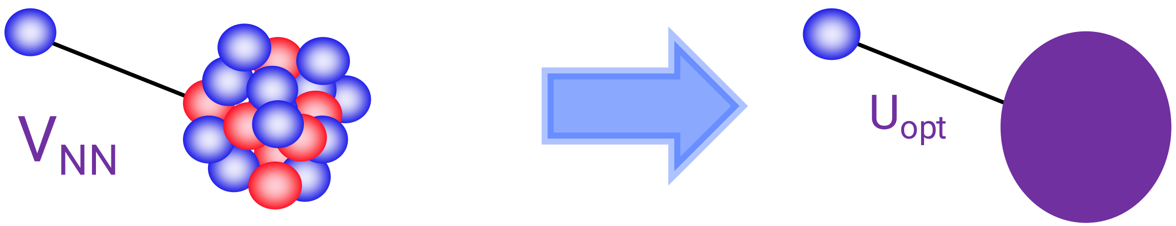
$$P\Psi(\vec{r},\vec{r}_1,\ldots,\vec{r}_A) = \phi_0(\vec{r})\Phi_0(\vec{r}_1,\ldots,\vec{r}_A)$$



https://infinite.mit.edu/video/herman-feshbach-“-optical-model”-lns46-symposium-matter-particles-5141992/

Effective interactions and flux loss

The effective interactions between projectile-target needs to incorporate these effects.



This the projection of the many-body scattering problem on the ground state:

$$P\Psi(\vec{r},\vec{r}_1,\ldots,\vec{r}_A) = \phi_0(\vec{r})\Phi_0(\vec{r}_1,\ldots,\vec{r}_A)$$

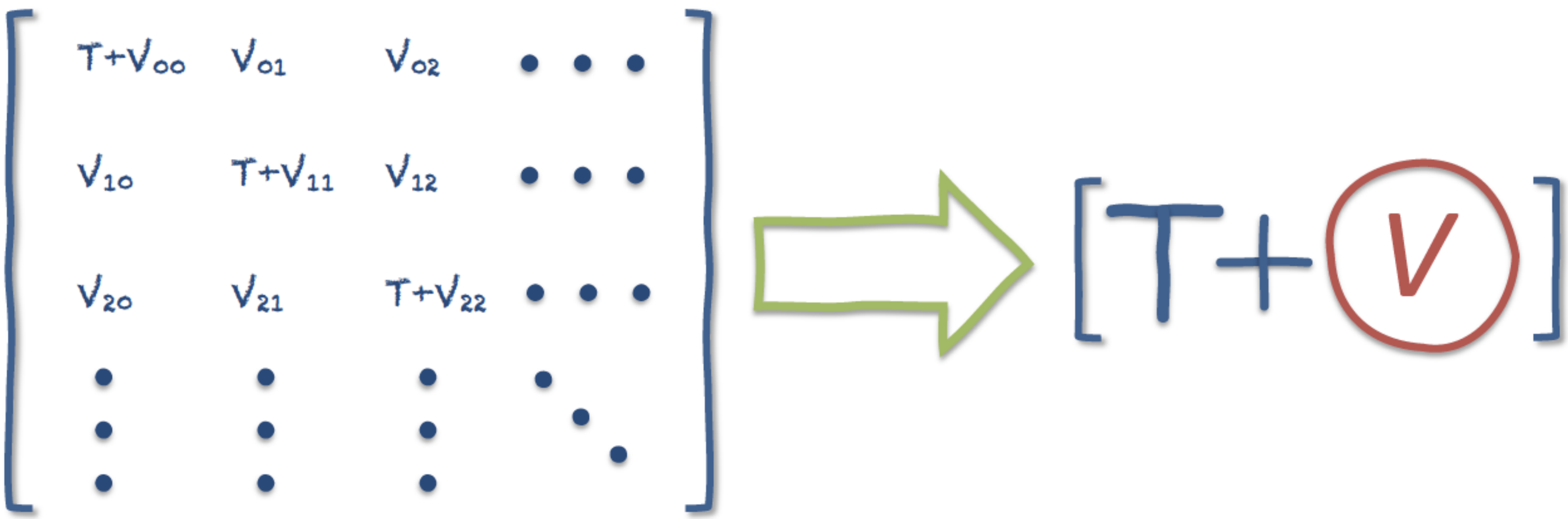
End up with a single-channel scattering equation with potential:

$$V_{\text{opt}} = \mathcal{V}_{00} + \sum_{j,k\neq 0} \mathcal{V}_{0j} \frac{1}{E - H_{jk} + i\eta} \mathcal{V}_{k0}$$

Hebborn, Nunes, et al., JPG 50, 060501 (2023)

The optical potential

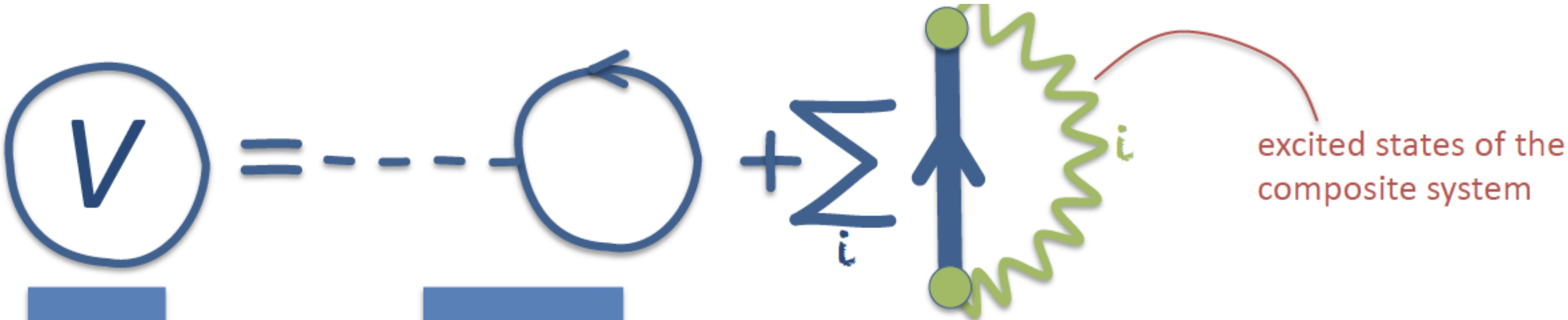
It results from the projection of the many-body Hamiltonian onto the elastic channel



- The "optical reduction" transforms a many-body operator into a one-body operator
- It is a well-defined, in principle exact, mathematical operation

The optical potential

It accounts for the complex structure of the nucleus



full optical potential (OP)

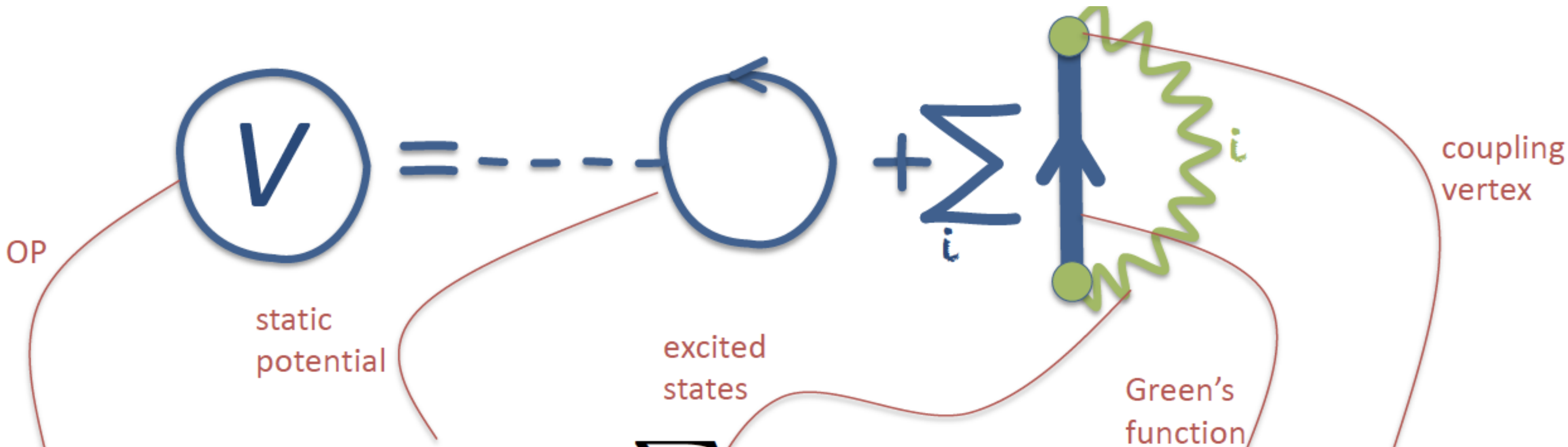
static, real, energy independent (mean field) potential

energy-dependent, complex, dynamical polarization potential

slides from Gregory Potel

The optical potential

It accounts for the complex structure of the nucleus

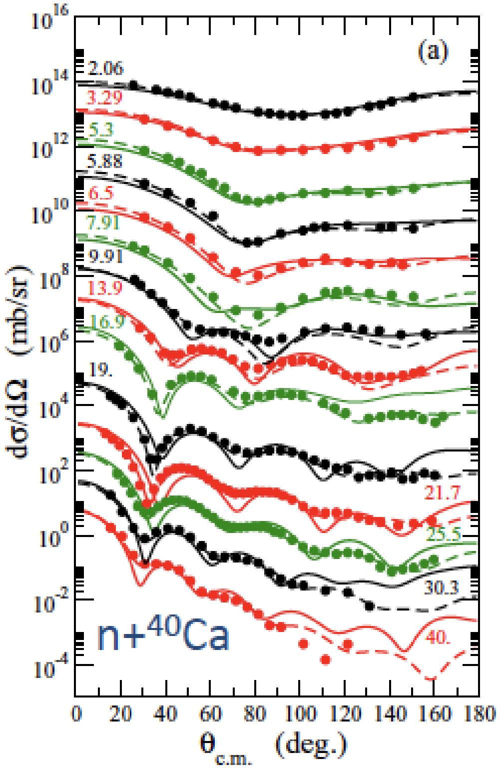


$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r}) G(E - E_i, \mathbf{r}, \mathbf{r}') U_{i0}(\mathbf{r}')$$

- The computed OP is energy dependent, non-local, complex, and dispersive
- The OP verifies the Kramers-Kronig dispersion relations between the real and the imaginary part

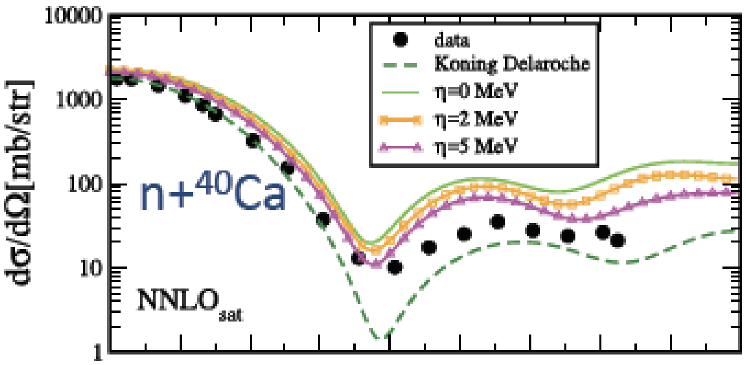
The optical potential from microscopic theory

RPA calculation with added imaginary part



Blanchon *et al*. PRC **91** 014612 (2015)

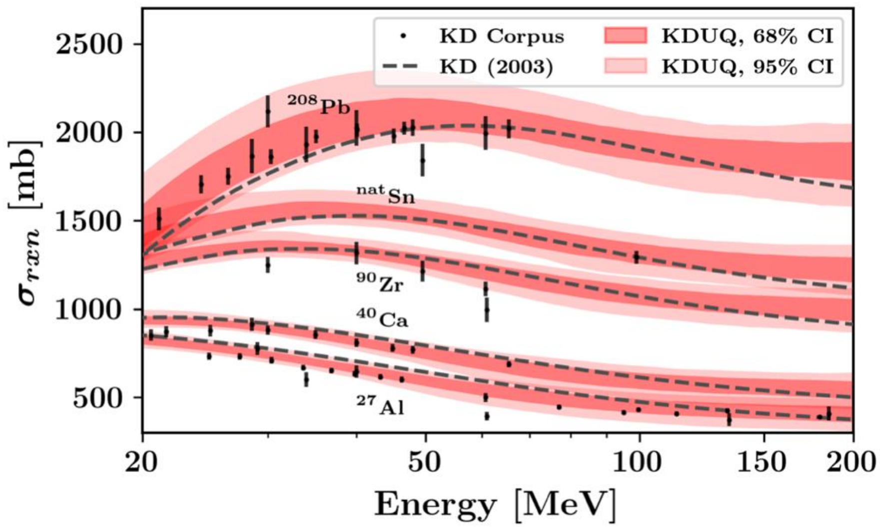
coupled-cluster ab initio with non-zero η parameter



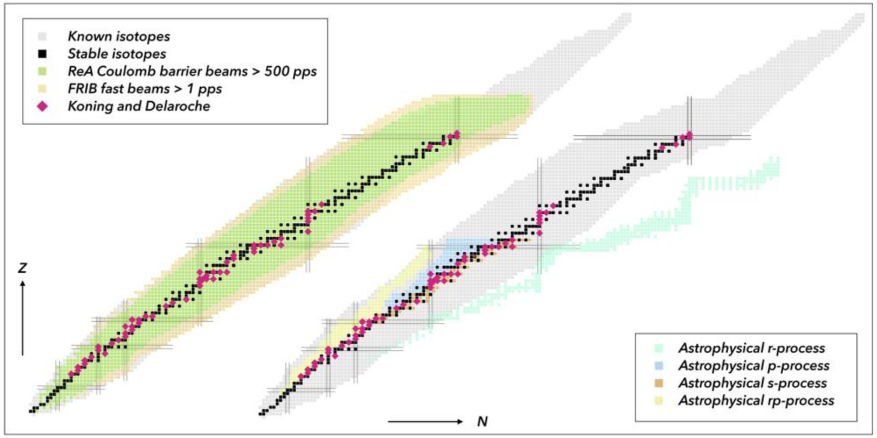
Rotureau *et al*. PRC **98** 044625 (2018)

The optical potential from experimental data

- Mostly fit to a large body of elastic data



Pruitt, Rahman, Escher, PRC107 014602 (2023)



Whitepaper: JPG 50 060501 (2023)

The optical potential from experimental data

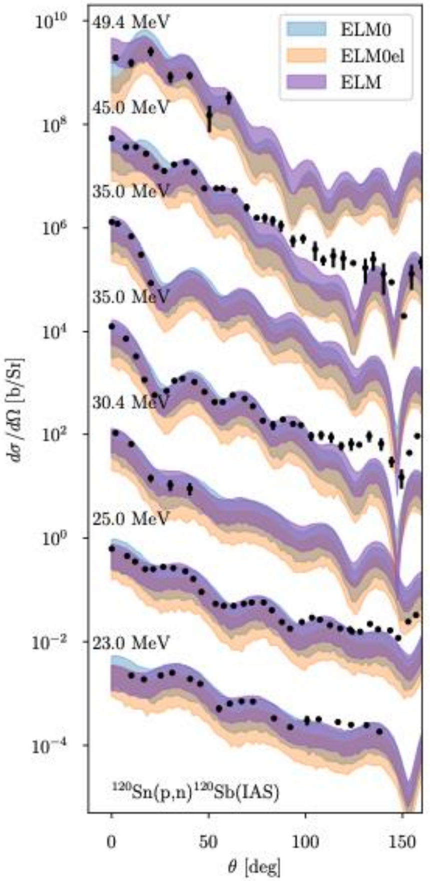


FIG. 4: 68% credible intervals for the predictive posterior of the latent truth for ^{120}Sn(p,n)^{120}Sb(IAS) from each model calibration, with experimental data in black: ELM (purple), ELM0 (orange), ELM0el (blue). Each successive energy is offset by a factor of 30 for visibility.

Including charge-exchange in the large set of data considered (e.g. The East Lansing Model)

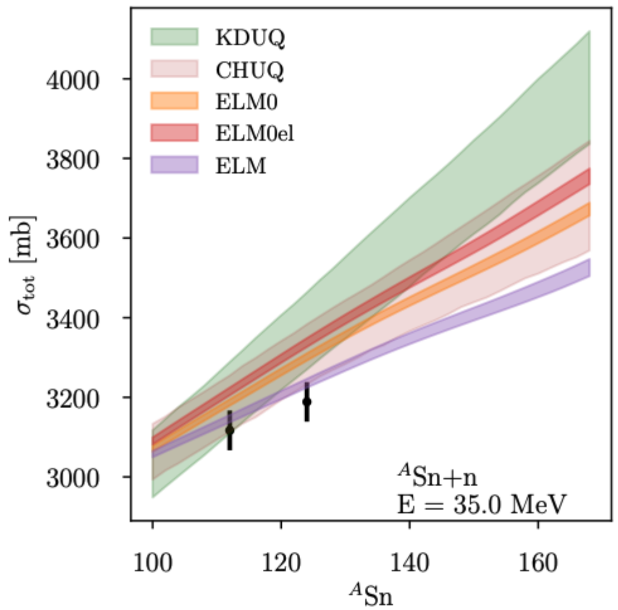


FIG. 6: Neutron total cross sections on Sn isotopes from A =100 to 170: 68% credible intervals predictions when extrapolating in asymmetry for the three models from this work – ELM (purple), ELM0 (orange), ELM0el (blue) – compared to CHUQ (pink) and KDUQ (green) from [26]. Experimental data for ^{112}Sn and ^{124}Sn from [31].

Beyers and Nunes, submitted PRL 2026

Bayesian analysis for reactions and emulators

Bayesian statistics

Thomas Bayes (1701–1761)

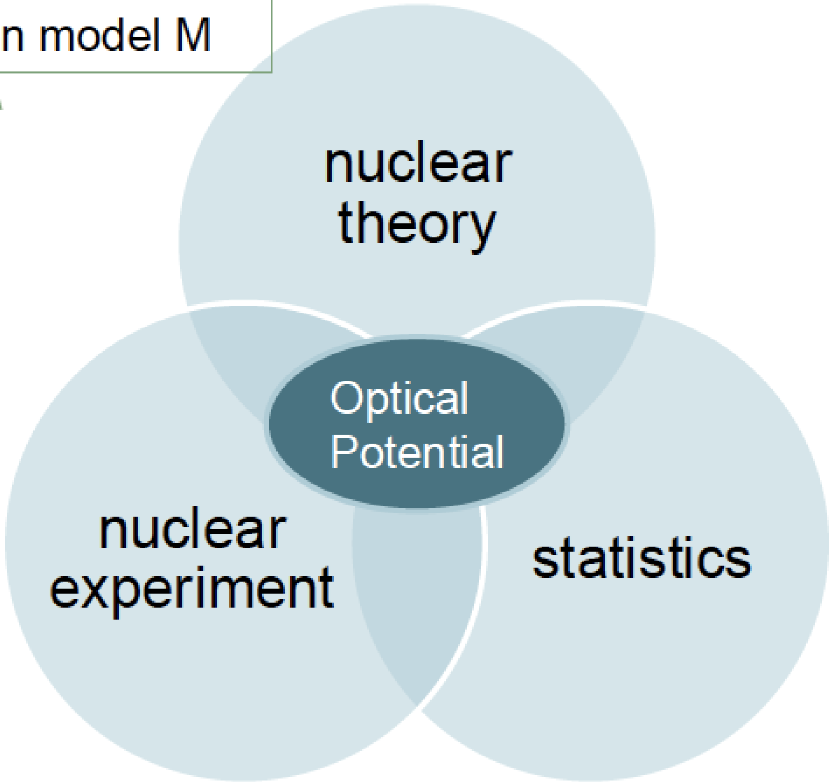
likelihood: assess the probability of observing D given model M

prior distribution of parameters H given model M

$$p(H|D,M) = \frac{p(D|H,M)\,p(H|M)}{p(D|M)}$$

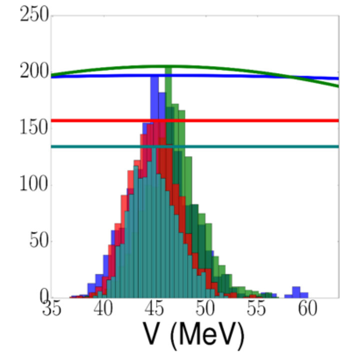
Bayesian evidence

posterior distributions: information updated after seeing data D

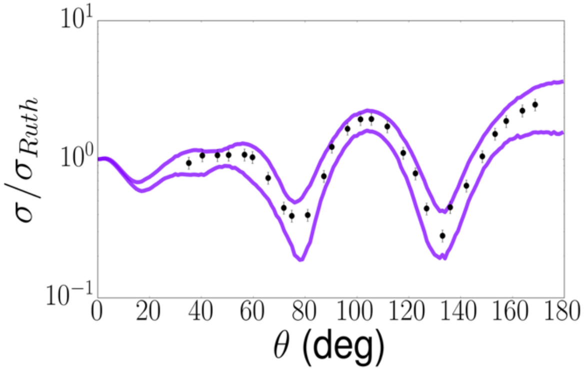


Bayesian analysis

- Calibration of the model M
  (parameter posterior distributions)

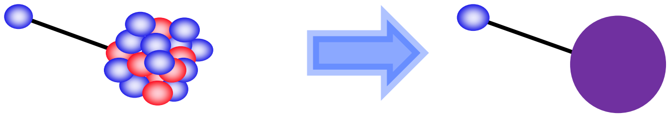


- Estimation of uncertainty in predictions
  (credible intervals on observables)



- Model assessment
  (comparison between models and with data)
- Model mixing
  (admixture between models with different strengths)
- Experimental design
  (What is the optimum measurement
  that adds information?)

Physical model: optical model



$$[T+U_{opt}(R)-E]\psi=0$$

The model has a set of parameters

$$U_{opt}(R)= V\, f(R, r, a) + W\, f(R, r_w, a_w) + W_s\, f(R, r_s, a_s) + V_{so} + V_C$$

We use previous OP parameterizations to set the priors
(typically wide priors to allow process to be data driven)

- use MCMC to sample parameter space

Statistical model

Data: elastic scattering angular distributions/polarizations/total xs

- real exp data with evaluated errors
- mock data calculated using KD with 10% errors

$$\chi^2 = \sum_{i=1}^{N} \frac{[\sigma_{\rm exp}(\theta_i) - \sigma_{\rm th}(\theta_i, x)]^2}{[\Delta\sigma_{\rm exp}(\theta_i)]^2}$$

Likelihood:

- No correlations and errors normally distributed

$$p(D|H,M) = \exp[-\chi^2/2]$$

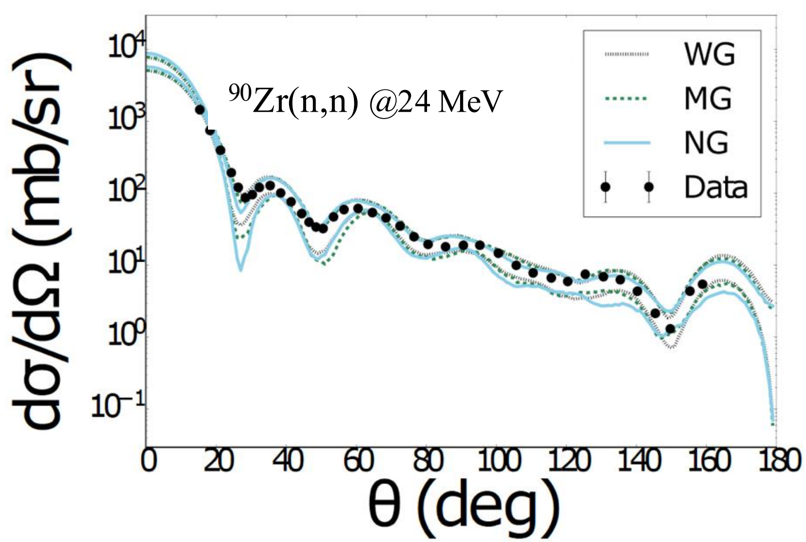
- Include correlations effectively by tempering the likelihood (equivalent to inflating errors)

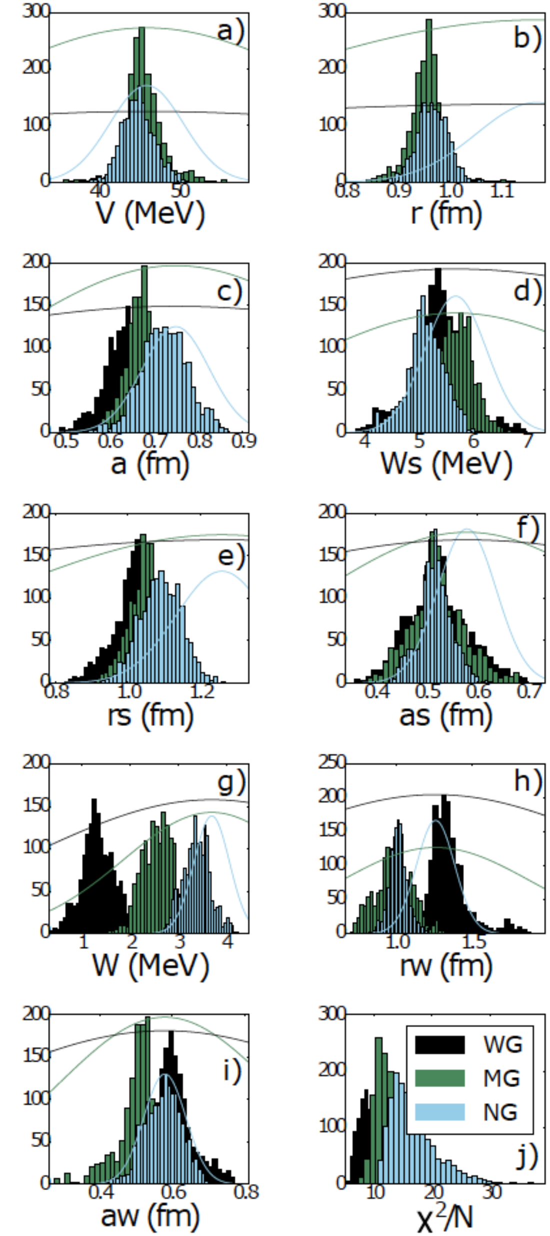
$$p(D|H,M) = \exp[-\chi^2/\ (2\lambda)$$

What prior to use?

Priors encapsulate our prior knowledge
(e.g. a previous global parameterization)

Use gaussian distributions on parameters
How wide should these be?

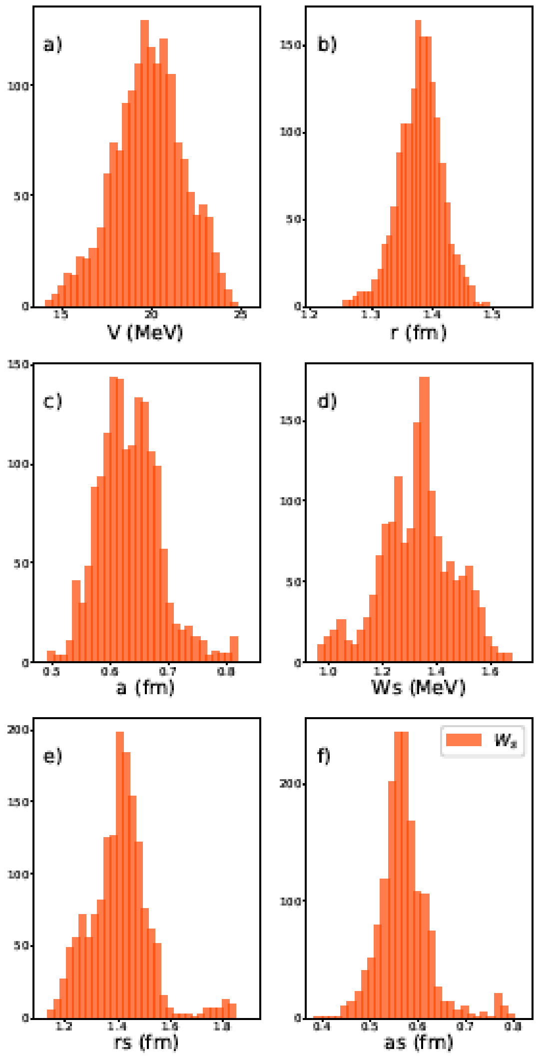




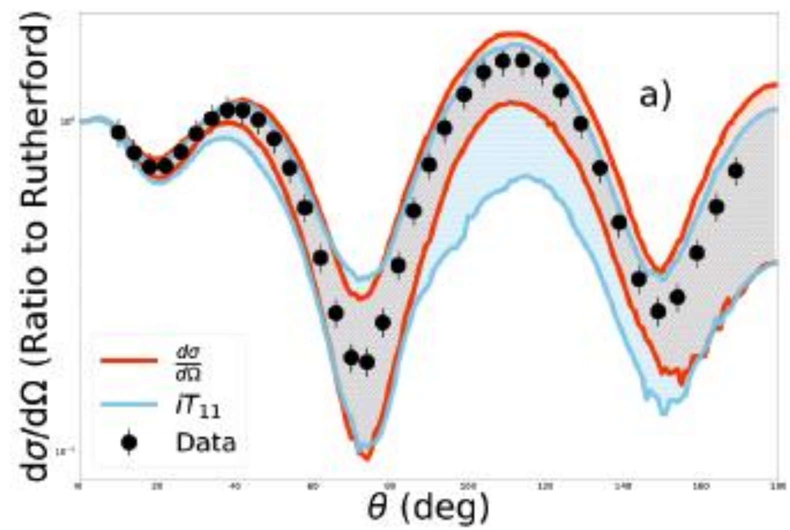
Lovell and Nunes PRC (2018)

FIG. 2: (Color online) Comparison of the posterior distributions (histograms) resulting from various prior distributions (corresponding solid lines) for a wide Gaussian (WG), medium Gaussian (MG), and narrow Gaussian (NG) as defined in Table II for ^{90}Zr(n,n)^{90}Zr at 24.0 MeV.

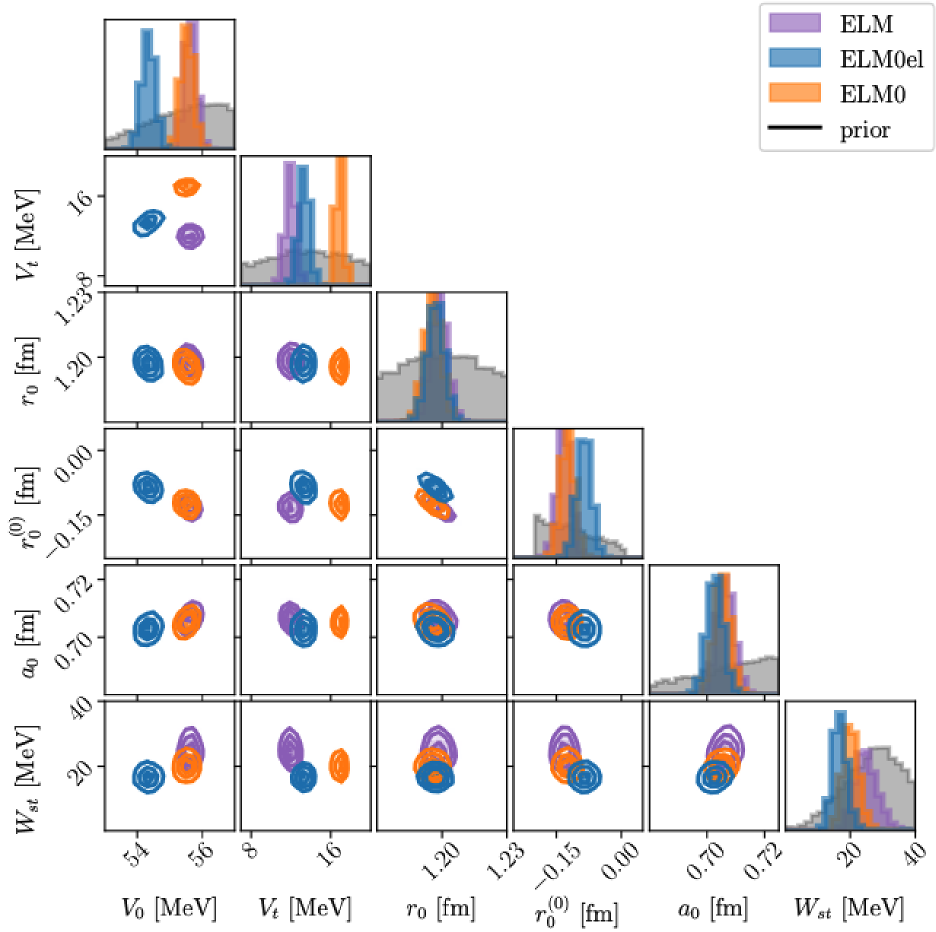
Bayesian: parameter posterior distributions



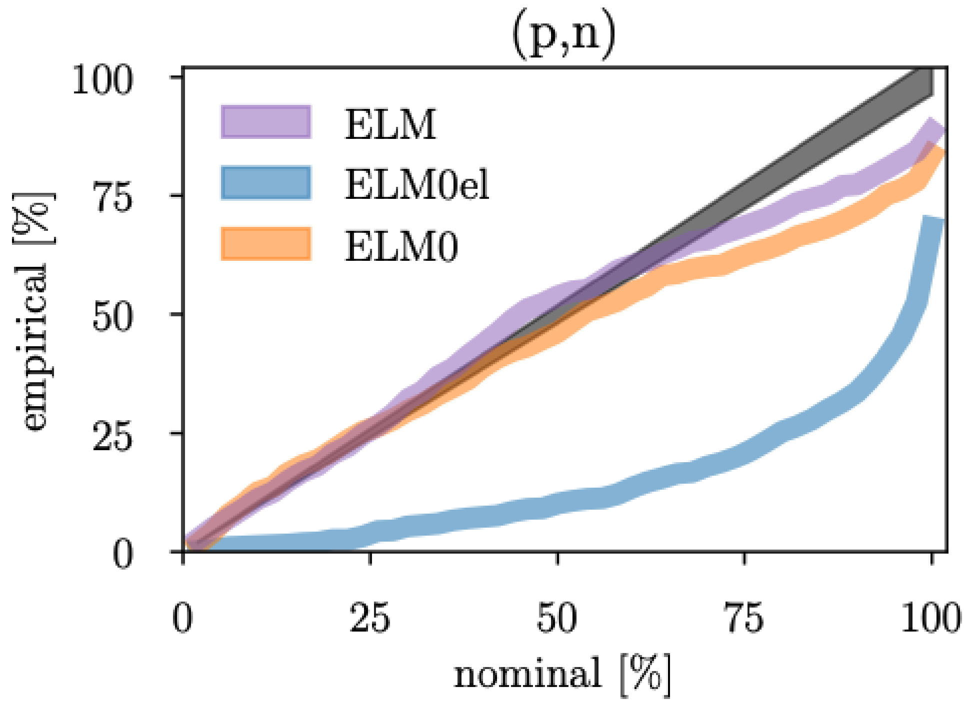
Create 95% confidence intervals for observable



Bayesian analysis for global OP calibrations



Bayesian analysis: empirical coverage

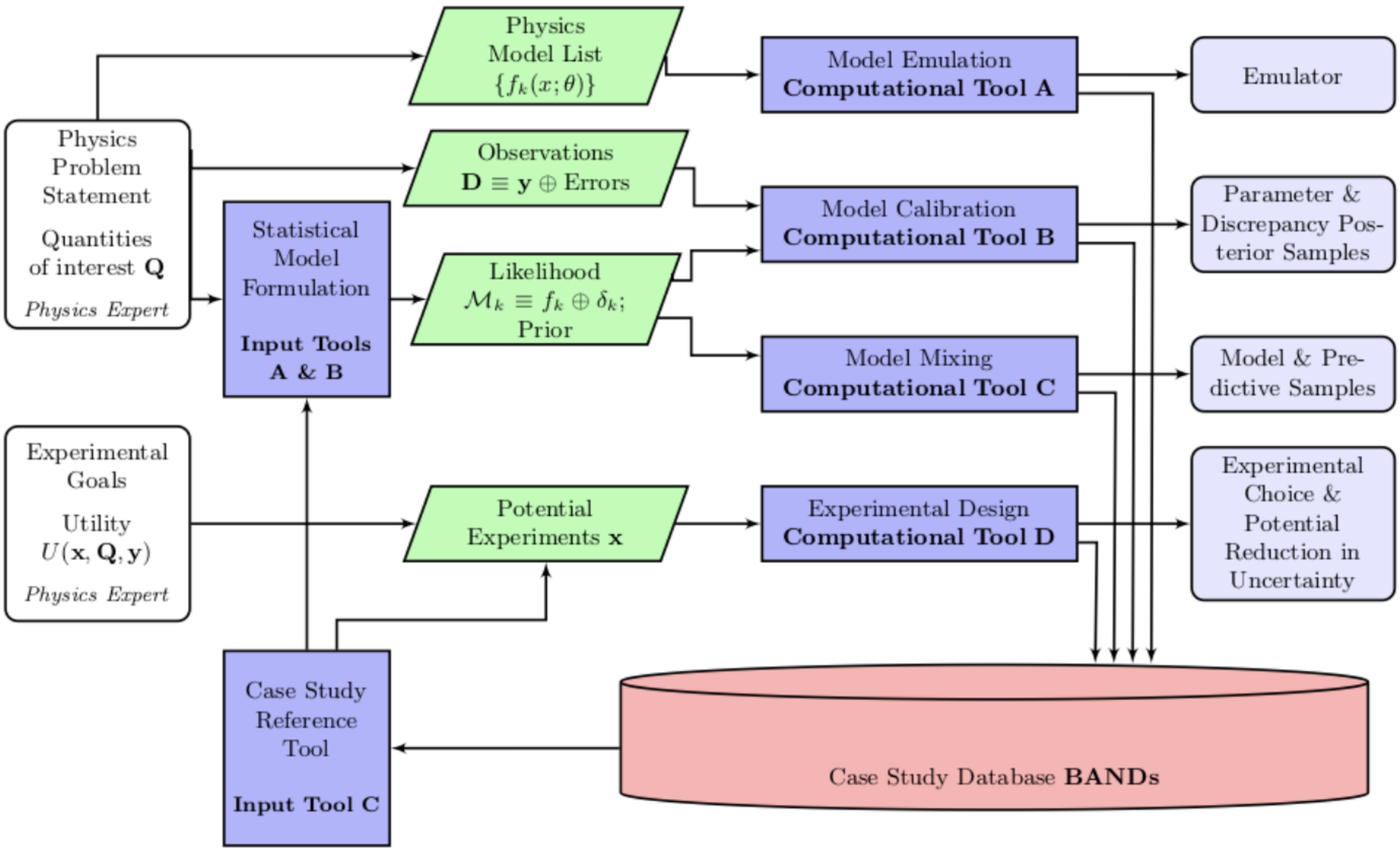


BAND

Bayesian Analysis of Nuclear Dynamics

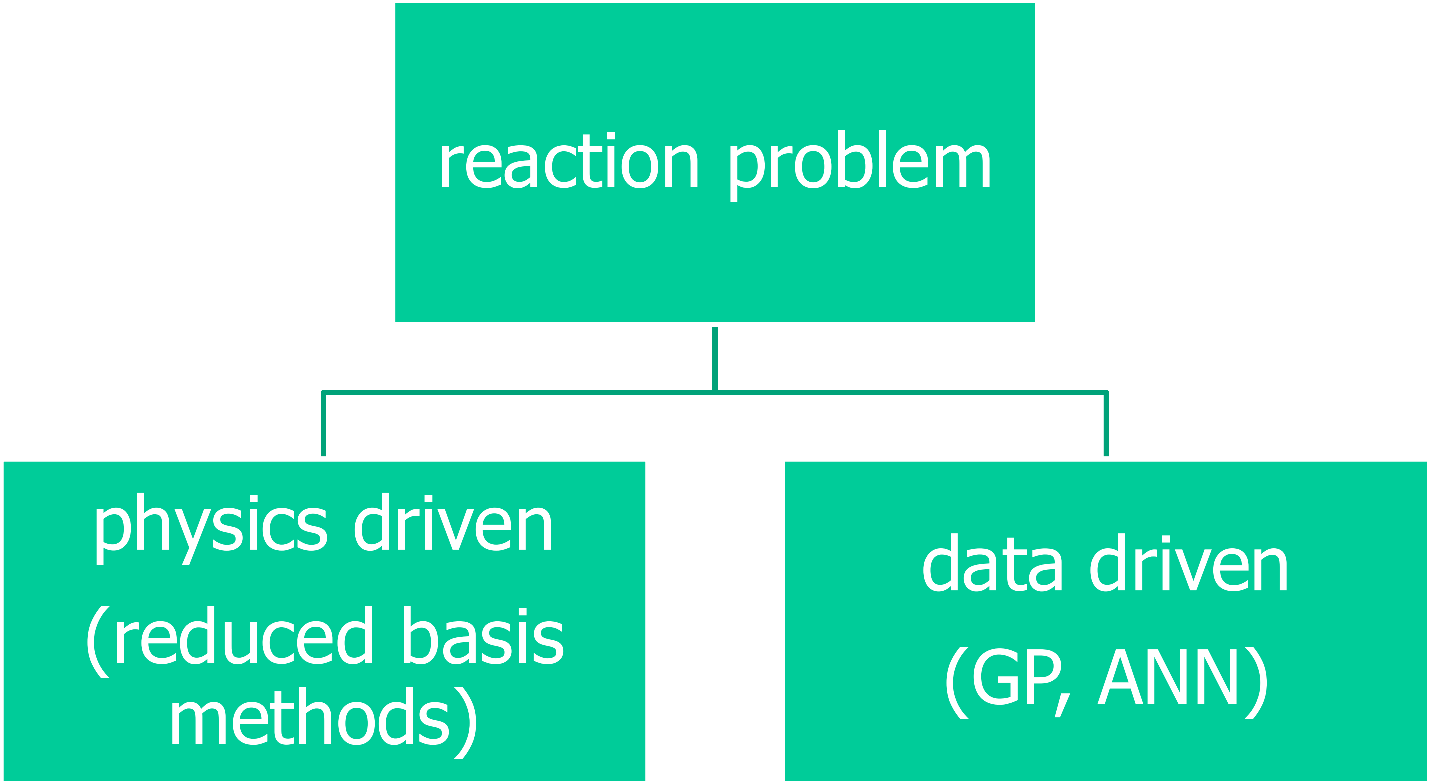
NSF CSSI program

https://bandframework.github.io

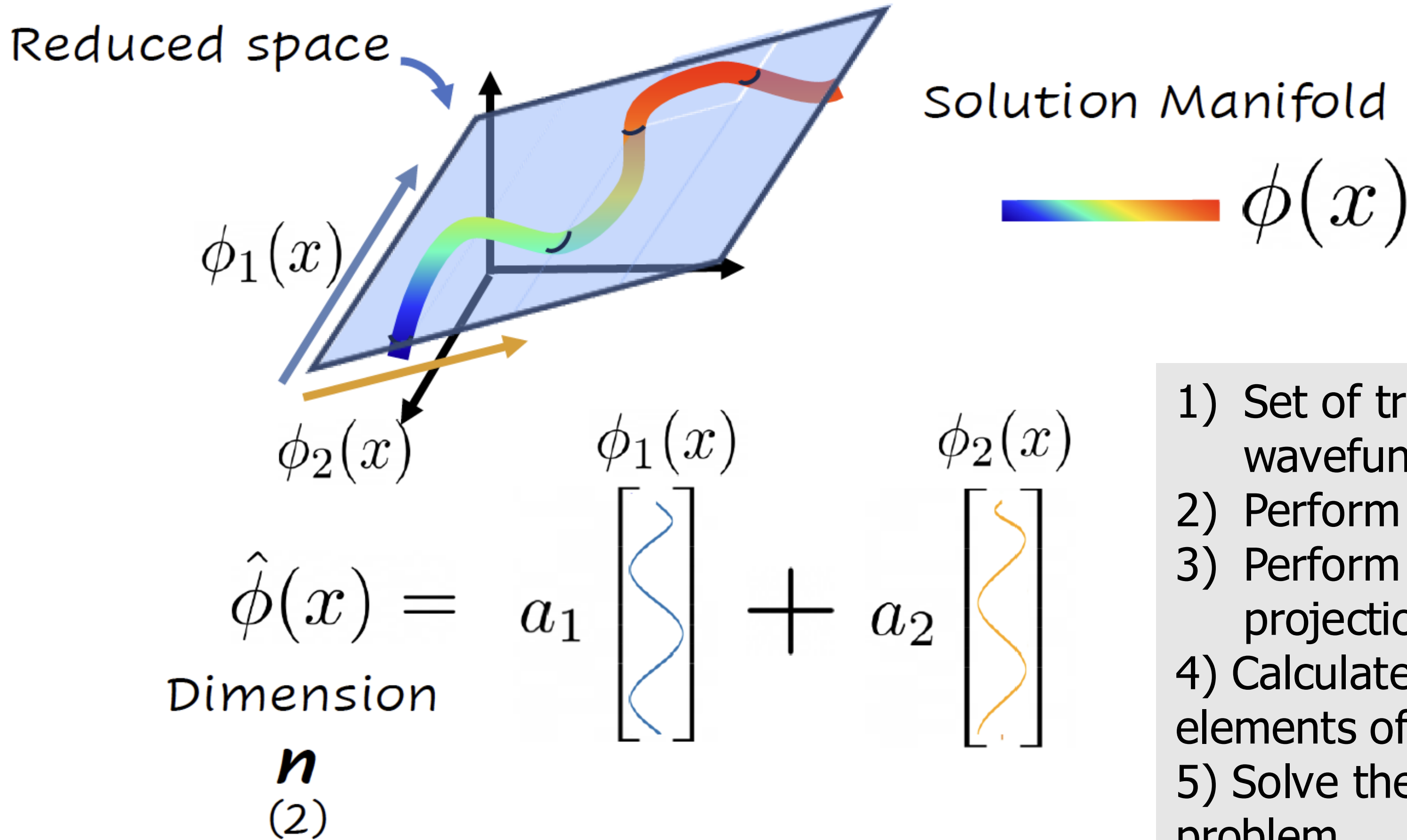


Emulators for nuclear reactions

An emulator is a fast and efficient replacement for a complex physics model



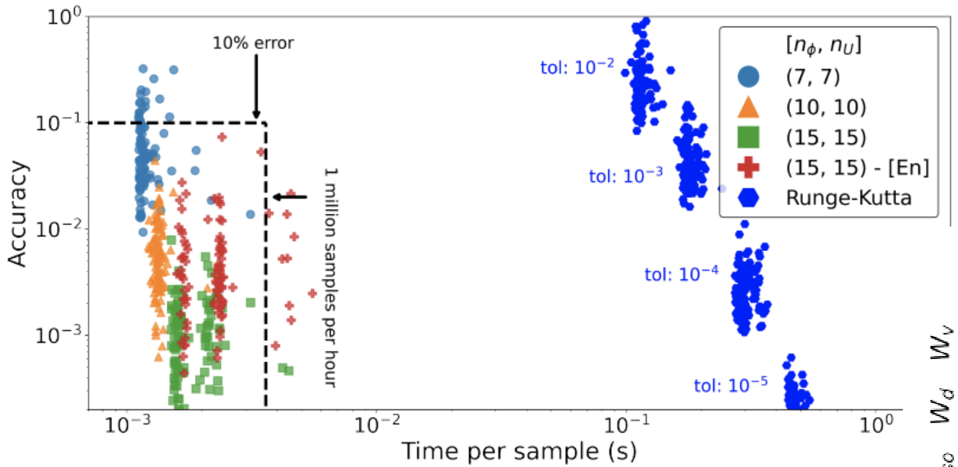
RBM Emulator Approximation

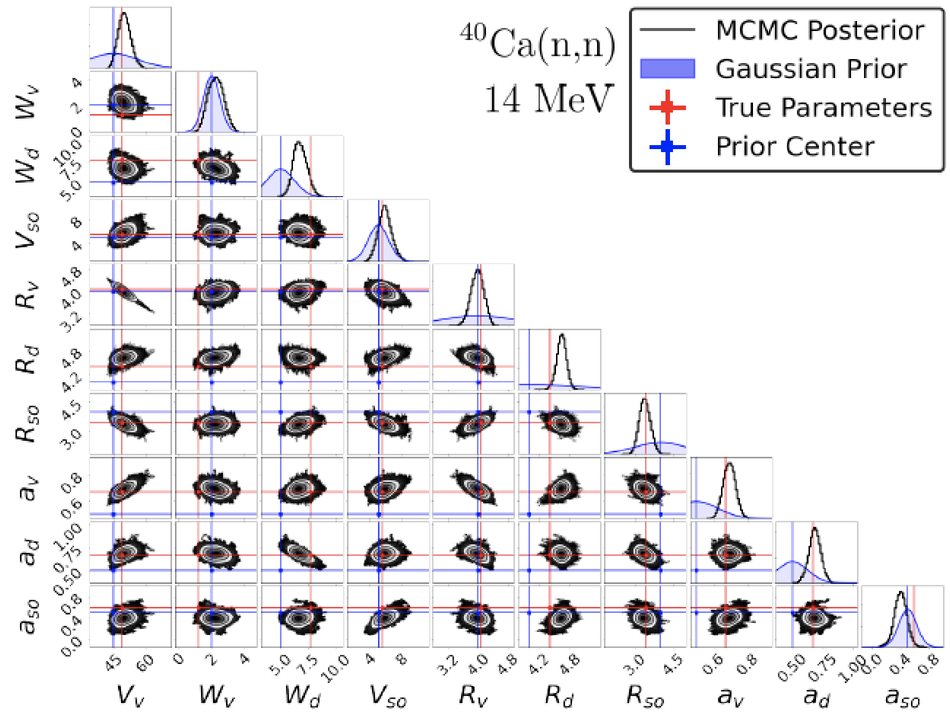


1) Set of training wavefunctions
2) Perform PCA
3) Perform Galerkin projection
4) Calculate matrix elements off-line
5) Solve the linear algebra problem

A. Quarteroni et al, Reduced Basis Methods for Partial Differential Equations: An Introduction, Vol. 92 (Springer, 2015)

Physics Driven Emulator
ROSE: Reduced Order Scattering Emulator

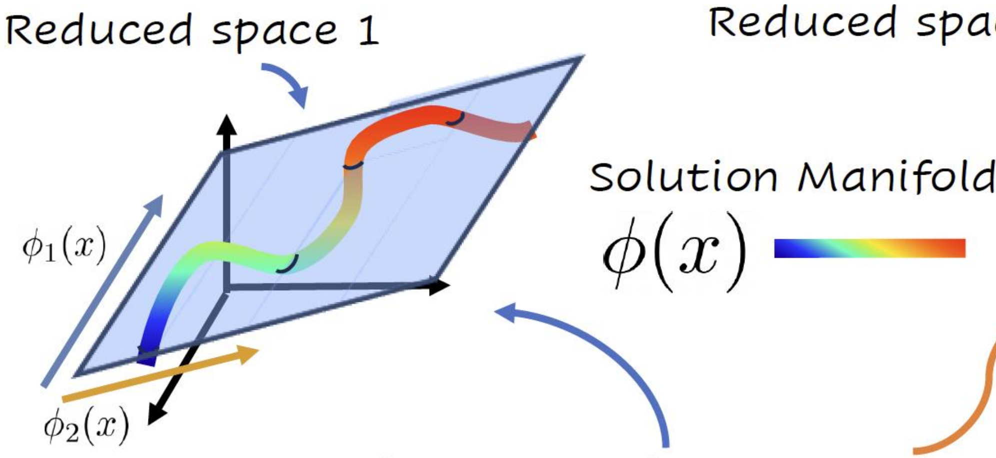


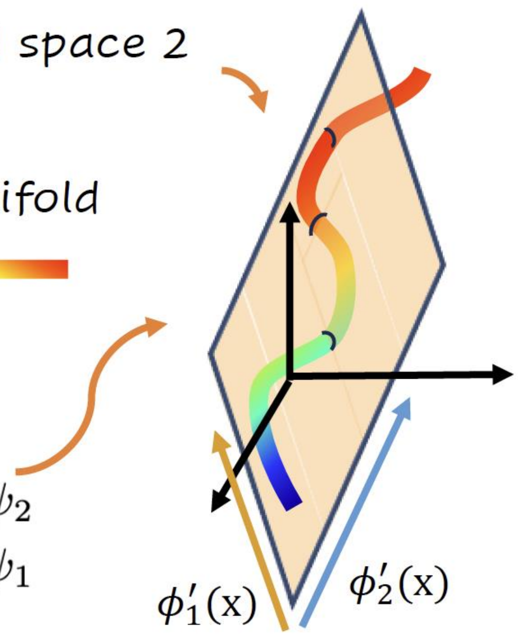


New software ROSE is 3 orders of magnitude faster than standard finite differences integration methods

Odel et al., Phys. Rev. C **109**, 044612 (2024)

RBM Emulator for Coupled-Channels





$$[T_1 + V_1 - E_1]\,\psi_1 = -U_{12}\psi_2$$
$$[T_2 + V_2 - E_2]\,\psi_2 = -U_{21}\psi_1$$

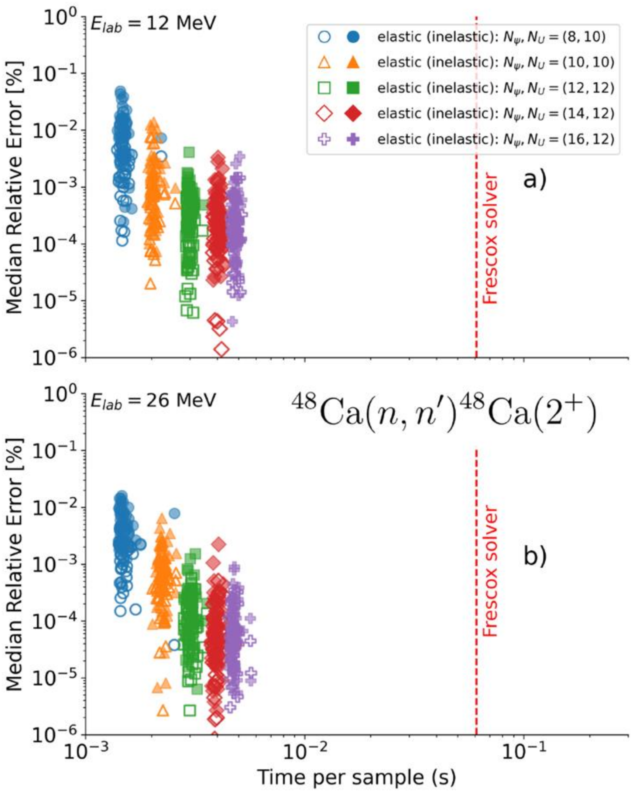
Variational Principle

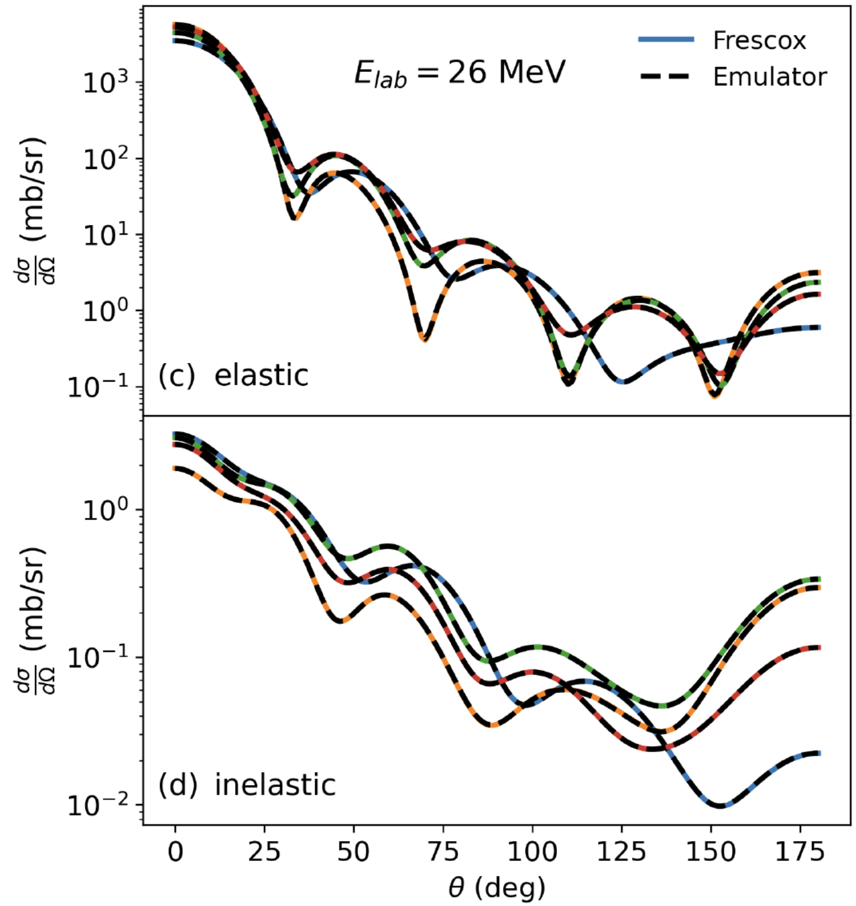
$$\sum_{i}^{N_{\text{basis}}} \beta_j \langle \psi_i | H - E | \psi_j \rangle = 0 \quad \text{for } i = 1, 2, .., N_{\text{basis}}$$

****Petrov-Galerkin: Solve coupled-system simultaneously.**

Catacora-Rios et al., PRC 113 (2026) 044623

Physics Driven Emulator
CC-RBM: Coupled-Channel Emulator

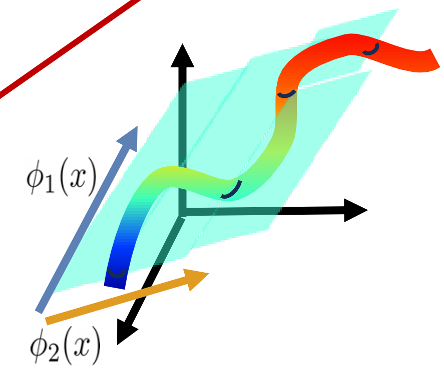




New software CC-RBM has been extended for coupled channel scattering problems including collective excitations of the target

Catacora-Rios et al., PRC 113 (2026) 044623

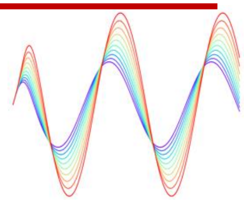
Emulators: learning the equations from data



(One way to do it: RBM)

1) Find good reduced coordinates

$$\hat{\phi}(x) = \phi_0 + \sum_k^n a_k \phi_k(x)$$



Principal Component Analysis

(One way to do it: Galerkin Projection)

2) Find their dynamics

$$\Big(H - E\Big)|\phi\rangle = 0$$

$$\langle \phi_j | \Big(H - E\Big) | \hat{\phi} \rangle = 0$$

Galerkin Projection

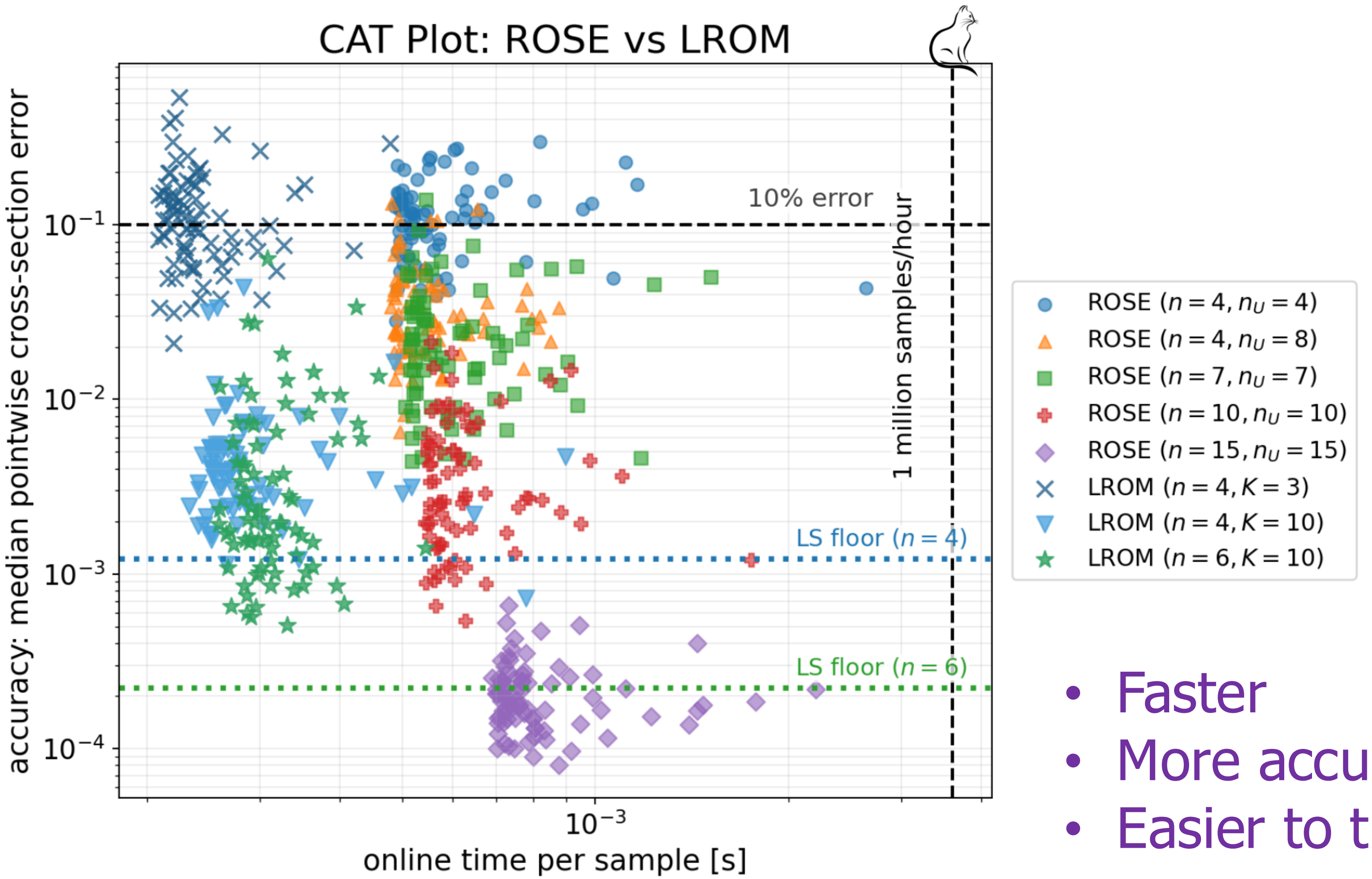
Instead: **learn** (implicit) equations from data:

$$\begin{bmatrix} \mathbf{M}(\alpha) \end{bmatrix}_{n \times n} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix}_n = \begin{bmatrix} c_1(\alpha) \\ c_2(\alpha) \\ \vdots \end{bmatrix}_n$$

Fit from data

$$\mathbf{M}(\alpha) = \mathbf{M}_0 + f_1(\alpha)\mathbf{M}_1 + \ldots$$

Performance of LROM versus ROSE



Pablo Giuliani

Daniel Shiu

- Faster
- More accurate
- Easier to train

Cross section explorer with LROM emulator

https://lrom.kyle.ee/

$^{40}Ca(n,n)$ at 14.1 MeV

1) take only the real interaction and explore the role of V, R, a (depth, radius and difuseness)
2) What happens when you add the imaginary parts
3) In what way does W_d and W_s impact the angular distribution differently?
4) And what happens to the total cross section?

$^{40}Ca(n,n)$ at 100 MeV

What are the differences when you explore the same reaction at much higher energies?

Pablo Giuliani

Kyle Godbey

Cross section explorer with LROM emulator

lrom.kyle.ee

Neutron scattering on Sn isotopes

1) How does the cross section change with beam energy?
2) How does the cross section change with mass?
3) How well does theory agree with the data?

Pablo Giuliani

Kyle Godbey

Questions?

Optical potential

○ Where does the optical potential come from?

Consider the original many-body problem nucleon-nucleus N+A

$$H(\mathbf{r}_0;\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)\Psi(\mathbf{r}_0;\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A) = E\Psi(\mathbf{r}_0;\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$

Split the Hamiltonian into:

- ○ kinetic energy of the projectile
- ○ the interaction of the projectile with all nucleons of the target
- ○ internal Hamiltonian of the target

$$H(\mathbf{r}_0;\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A) = T_0 + \sum_{i=1}^{A} V(\mathbf{r}_{0i}) + H_A(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$

The solutions for the target Hamiltonian form a complete set:

$$H_A(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)\Phi_i(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A) = \epsilon_i\Phi_i(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$

The general solution for N+A can be written in terms of the complete set above:

$$\Psi(\mathbf{r}_0;\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A) = \sum_{ij} \chi_i(\mathbf{r}_0)\Phi_j(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$

Optical potential

○ Feshbach projection

Since at this point we still assume in our reaction model that the target stays in the ground state, we need to project the problem into the target ground state.

P is the projection operator: $$P = |\Phi_0\rangle\langle\Phi_0|$$

It picks up the elastic component: $$P\Psi = \chi_0\Phi_0$$

Properties of projection operators

$$P^2\Psi = P\Psi$$

$$Q^2\Psi = Q\Psi$$

$$Q = 1 - P$$

$$PQ\Psi = QP\Psi = 0$$

Now apply it to the full equation: $$(E - H)(P + Q)\Psi = 0$$

Wong, Introduction to Nuclear Physics, Wiley

Optical potential

- After some algebra:

$$\left\{E - T_0 - \langle\Phi_0|V|\Phi_0\rangle - \langle\Phi_0|VQ\frac{1}{E-QHQ}QV|\Phi_0\rangle\right\}\chi_0 = 0$$

$$V \equiv \sum_{i=1}^{A} V(r_{0i})$$

Potential acting between projectile and target nucleons

$$H_A\Phi_0 = \epsilon_0\Phi_0 = 0$$

Interpretation for the formal propagator: multiple scattering in Q-space

$$\frac{1}{E-QHQ} = \frac{1}{E}\left\{1 + \frac{1}{E}QHQ + \frac{1}{E}QHQ\frac{1}{E}QHQ + \cdots\right\}$$

- The scattering equation can be rewritten: $(E - T_0 - \mathcal{V}(\boldsymbol{r}_0))\chi_0 = 0$

  with the effective potential: $\mathcal{V}(\boldsymbol{r}_0) = \langle\Phi_0|V|\Phi_0\rangle + \langle\Phi_0|VQ\frac{1}{E-QHQ}QV|\Phi_0\rangle$

Optical potential

- The scattering equation can be rewritte $(E - T_0 - \mathcal{V}(\boldsymbol{r}_0))\chi_0 = 0$
  with the effective potential: $\mathcal{V}(\boldsymbol{r}_0) = \langle \Phi_0 | V | \Phi_0 \rangle + \langle \Phi_0 | VQ \frac{1}{E - QHQ} QV | \Phi_0 \rangle$

- This potential is generally non-local which gives rise to some complications:

$$(E - T_0)\chi_0(\boldsymbol{r}_0) = \mathcal{V}(\boldsymbol{r}_0)\chi_0(\boldsymbol{r}_0) + \int f(\boldsymbol{r}_0, \boldsymbol{r}'_0)\chi_0(\boldsymbol{r}'_0)d\boldsymbol{r}'_0$$

Often this is approximated to a local version.

Optical model: replace this microscopic potential by a model potential obtained phenomenologically:

$$(E - T_0 - U_{\text{opt}})\chi_0 = 0$$

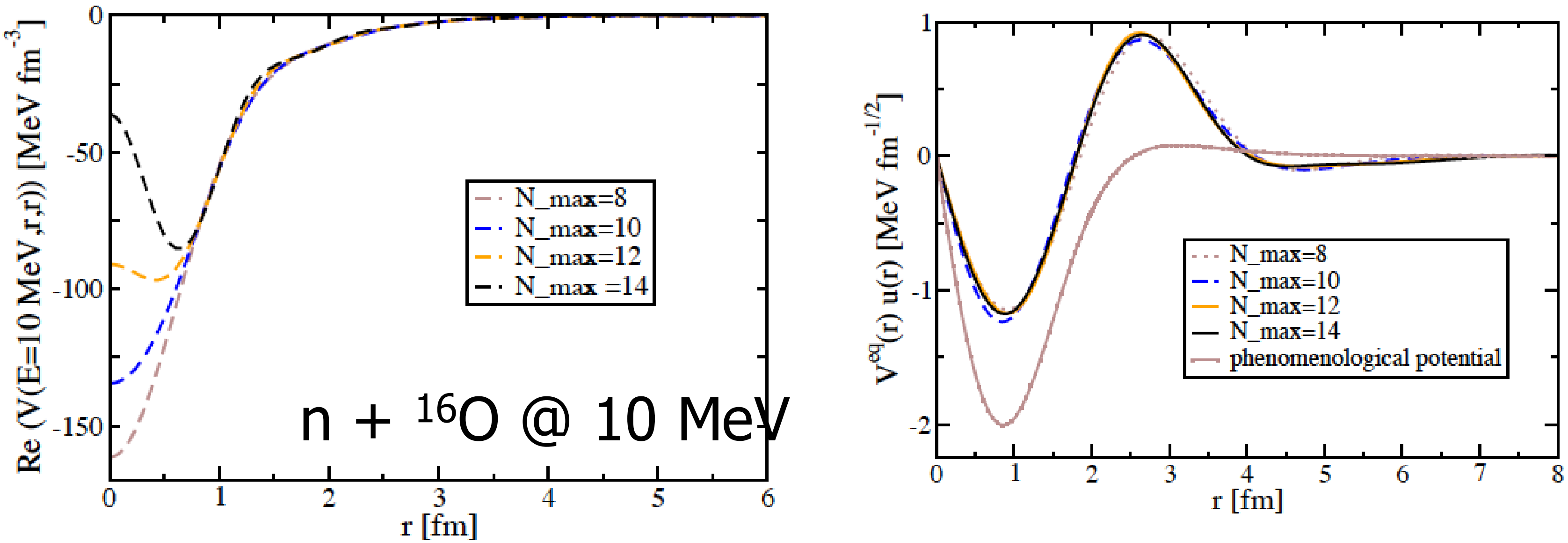
Scattering into Q-space may never return to elastic – loss of flux
Optical potential needs to have an imaginary term!

Microscopic N-A optical potential

- V_{eff} is self energy extracted from coupled-cluster CCSD

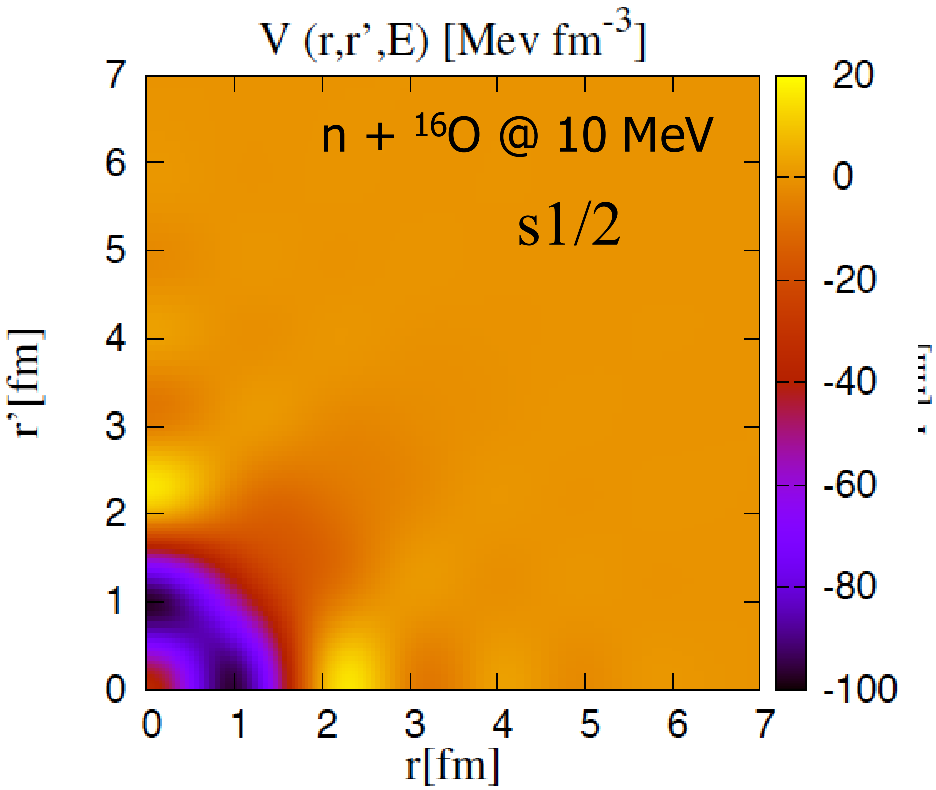
$$G(\alpha,\beta,E) = G^{(0)}(\alpha,\beta,E) + \sum_{\gamma,\delta} G^{(0)}(\alpha,\gamma,E)\Sigma^{*}(\gamma,\delta,E)G(\delta,\beta,E).$$

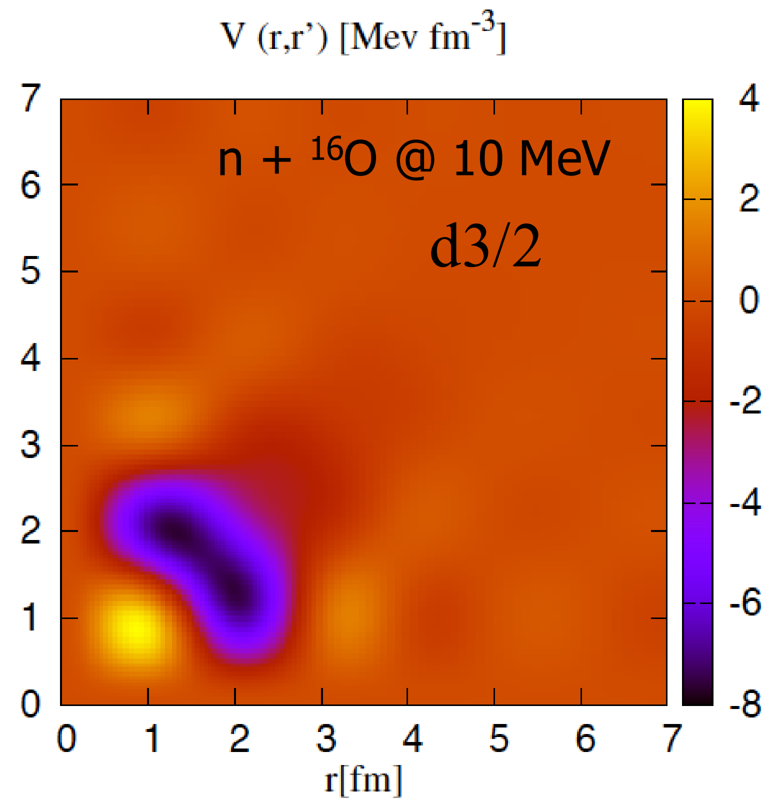
Ab-initio Hamiltonian: NN_{opt}
Basis: HO and Breggren
Extend for convergence of potential.



Rotureau et al., PRC95, 024315 (2017)

Microscopic N-A optical potential





The effective interaction is non-local!

Rotureau et al., PRC95, 024315 (2017)

Microscopic N-A optical

- There remains an energy dependence!
- Absorption is small from E=0-10 MeV.

n + ^{16}O

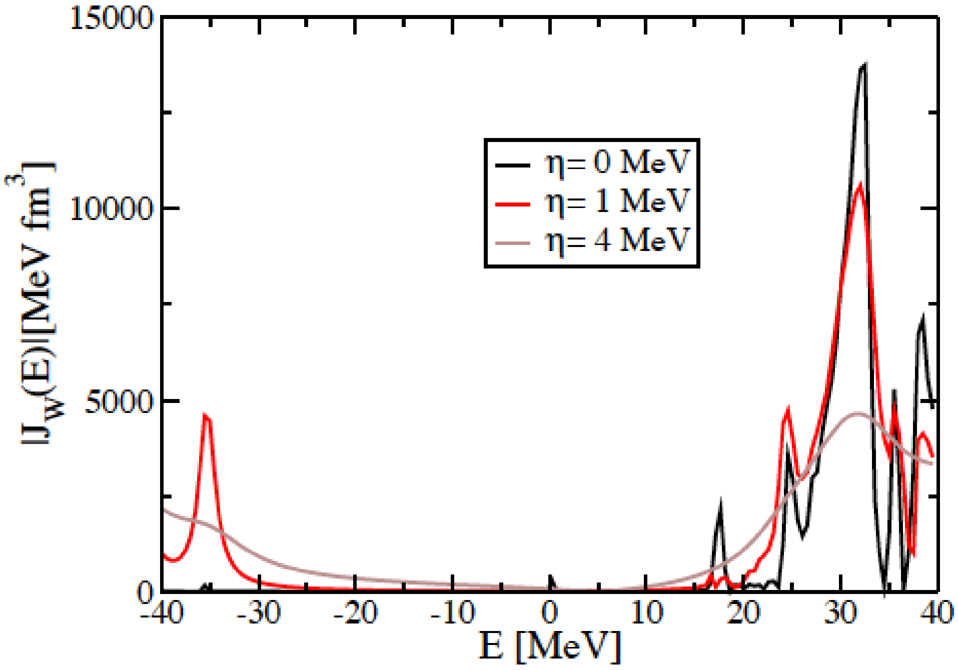


FIG. 11. Neutron s-wave imaginary volume integral $J_W(E)$ for several values of η. Calculations were performed at $N_{\max} = 14$ with 50 discretized $s_{1/2}$ shells.

Rotureau et al., PRC95, 024315 (2017)

Some additional reading

Theory road map:
http://fribusers.org/8_THEORY/3_DOCUMENTS/Blue_Book_FINAL.pdf

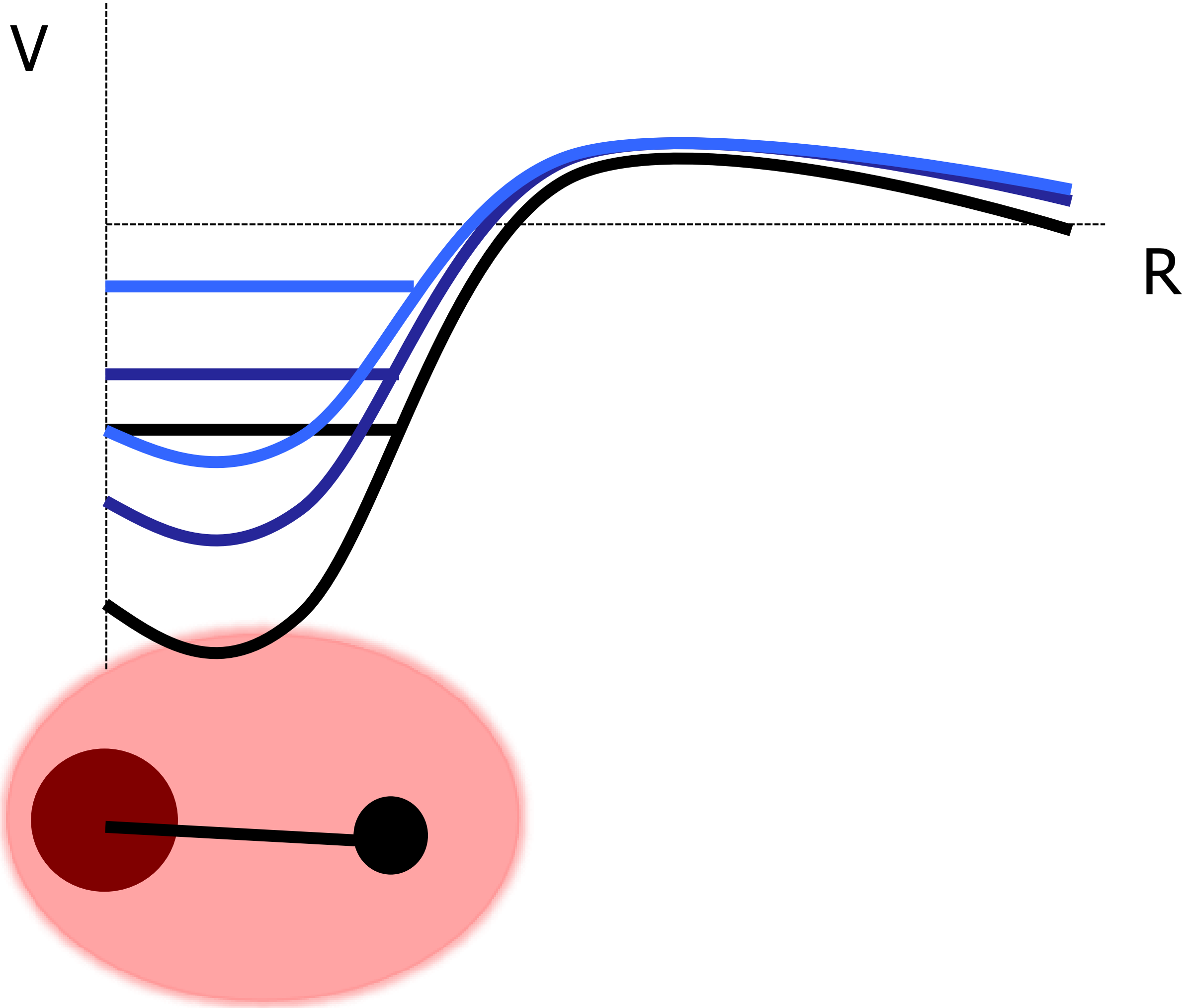
Research opportunities with rare isotopes
http://books.nap.edu/openbook.php?record_id=11796&page=1

Nuclear force and Effective field theories
Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. **65** (2010)

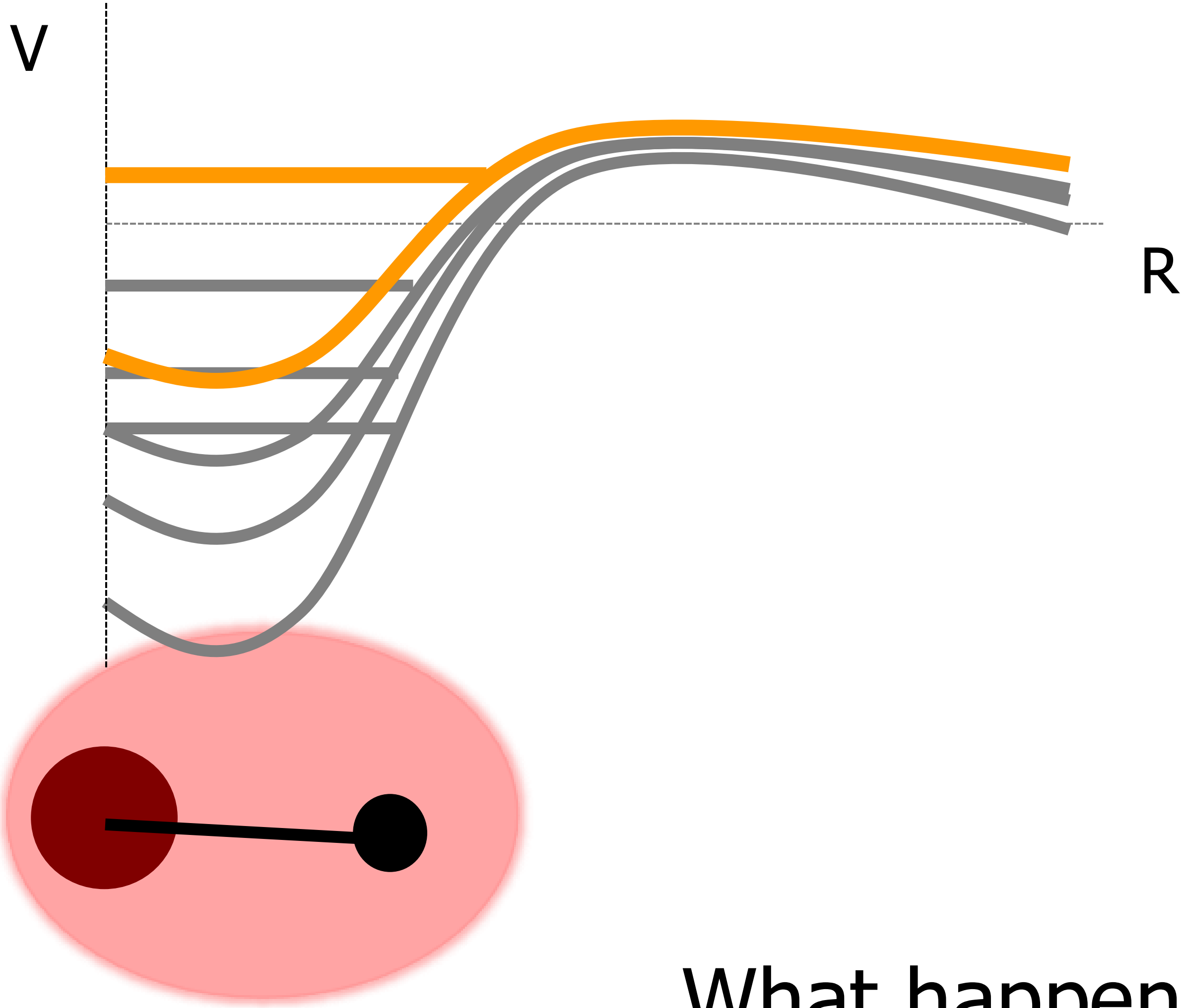
Nuclear reactions for nuclear astrophysics:
Thompson and Nunes, Cambridge University Press

Joint institute for nuclear astrophysics:
http://www.jinaweb.org

Real interactions and bound states



Real interactions and bound states

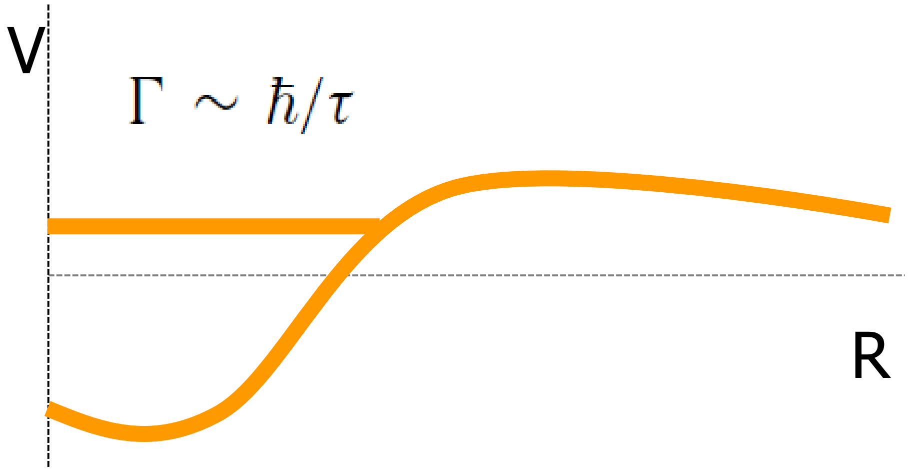


What happens with E>0?

Resonances and phase shifts

- particles trapped inside a barrier

$\Gamma \sim \hbar/\tau$



- Resonance characterized by J, E, $\Gamma > 0$

$$\Delta t \sim \hbar \mathrm{d}\delta(E)/\mathrm{d}E$$

- will show rapid rise of phase shift

For the cross section to have a Breit-Wigner form

$$\delta_{\mathrm{res}}(E) = \arctan\left(\frac{\Gamma/2}{E_{\mathrm{r}} - E}\right) + n(E)\pi$$

the resonance energy corresponds to $\delta = \pi/2$

There is usually a background in addition to the resonance part:

$$\delta(E) = \delta_{\mathrm{bg}}(E) + \delta_{\mathrm{res}}(E)$$

Resonances: the alpha-n example

$$\chi_L^{\text{ext}}(R) \to e^{i\delta_L}[\cos\delta_L \sin(kR - L\pi/2) + \sin\delta_L \cos(kR - L\pi/2)]$$
$$= e^{i\delta_L}\sin(kR + \delta_L - L\pi/2).$$

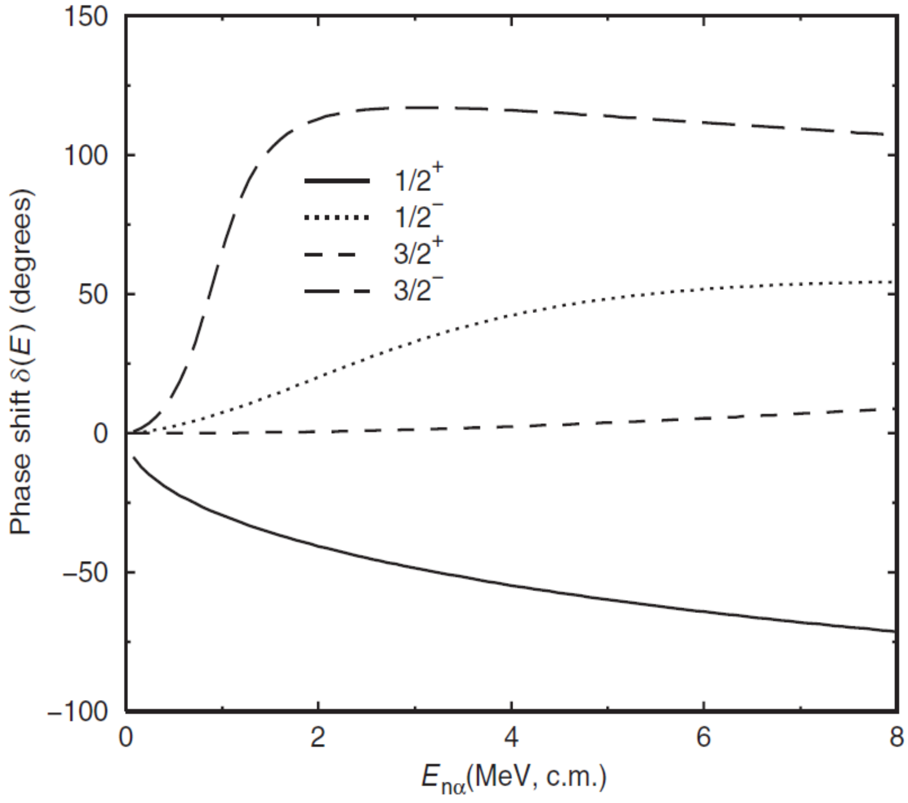


Fig. 3.2. Examples of resonant phase shifts for the $J^\pi = 3/2^-$ channel in low-energy n–α scattering, with a pole at $E = 0.96 - i0.92/2$ MeV. There is only a hint of a resonance in the phase shifts for the $J^\pi = 1/2^-$ channel, but it does have a wide resonant pole at $1.9 - i6.1/2$ MeV.

Resonances and cross sections

$$\delta_{\rm res}(E) = \arctan\left(\frac{\Gamma/2}{E_{\rm r} - E}\right) + n(E)\pi$$

- Breit-Wigner form

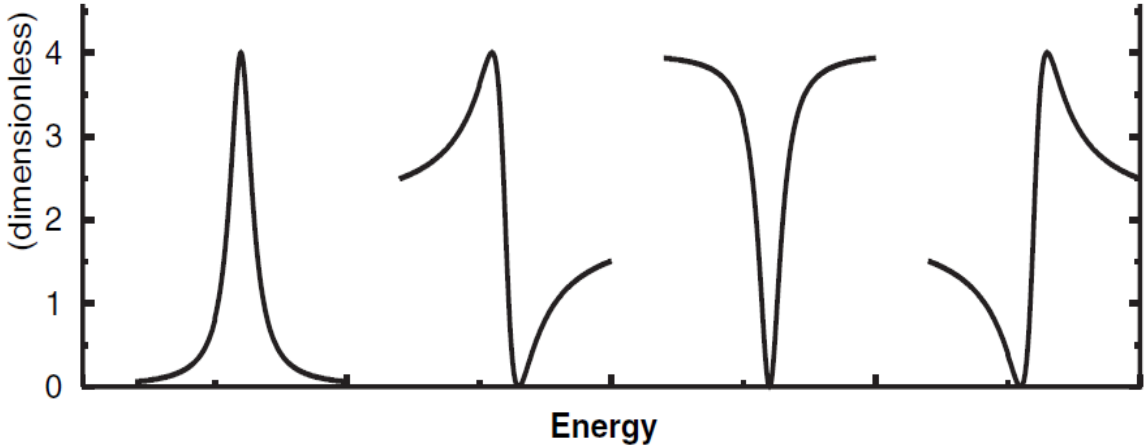
$$\sigma_{\rm el}^{\rm res}(E) \simeq \frac{4\pi}{k^2}(2L+1)\sin^2\delta_{\rm res}(E)$$

$$= \frac{4\pi}{k^2}(2L+1)\frac{\Gamma^2/4}{(E - E_{\rm r})^2 + \Gamma^2/4}$$

QUIZ TIME

Which of these cross sections corresponds to a resonance?

(a) b) c) d)



Resonance and backgrounds

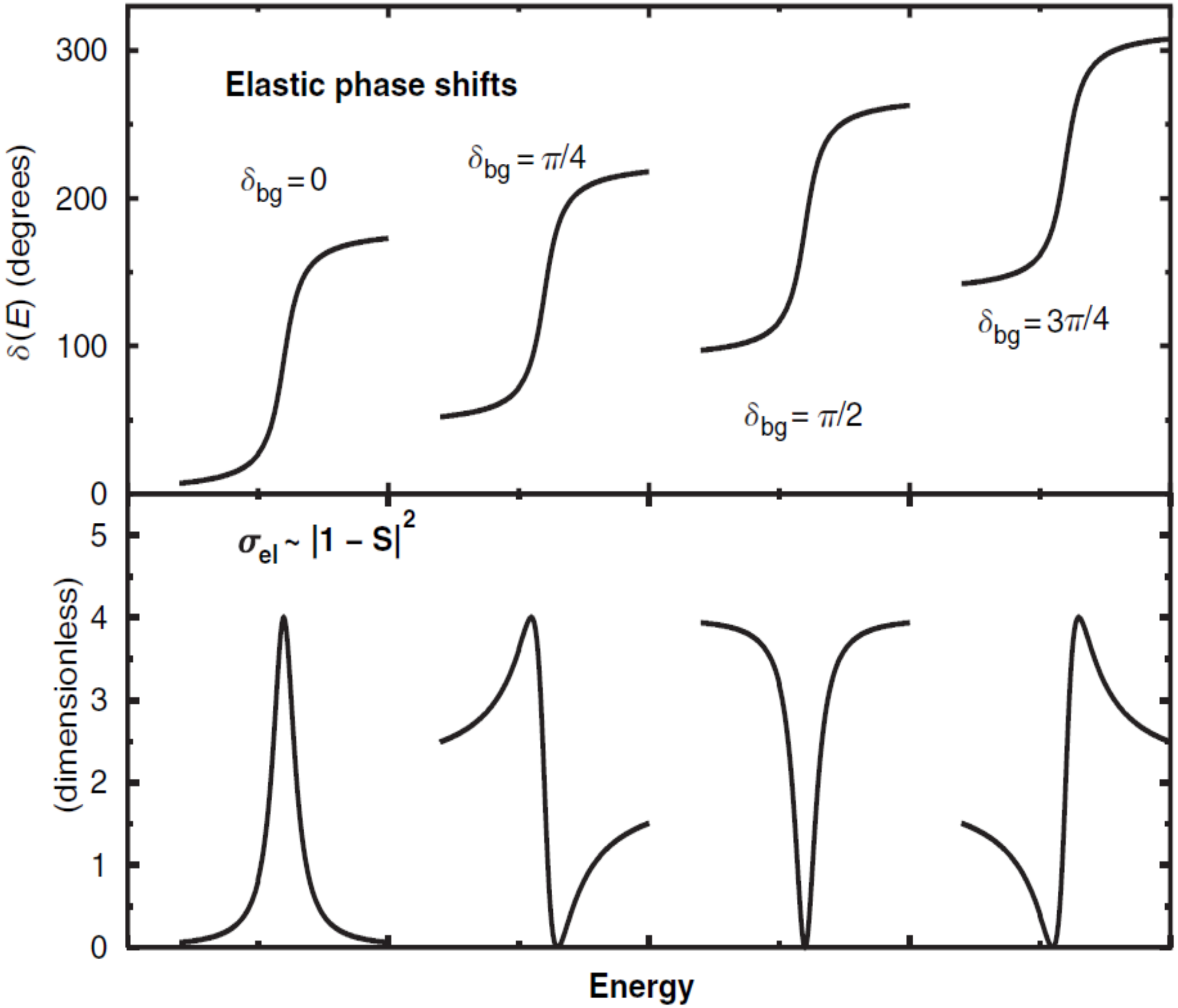


Fig. 3.3. Possible Breit-Wigner resonances. The upper panel shows resonant phase shifts with several background phase shifts $\delta_{bg} = 0, \pi/4, \pi/2$ and $3\pi/4$ in the same partial wave. The lower panel gives the corresponding contributions to the total elastic scattering cross section from that partial wave.

why do reactions? elastic

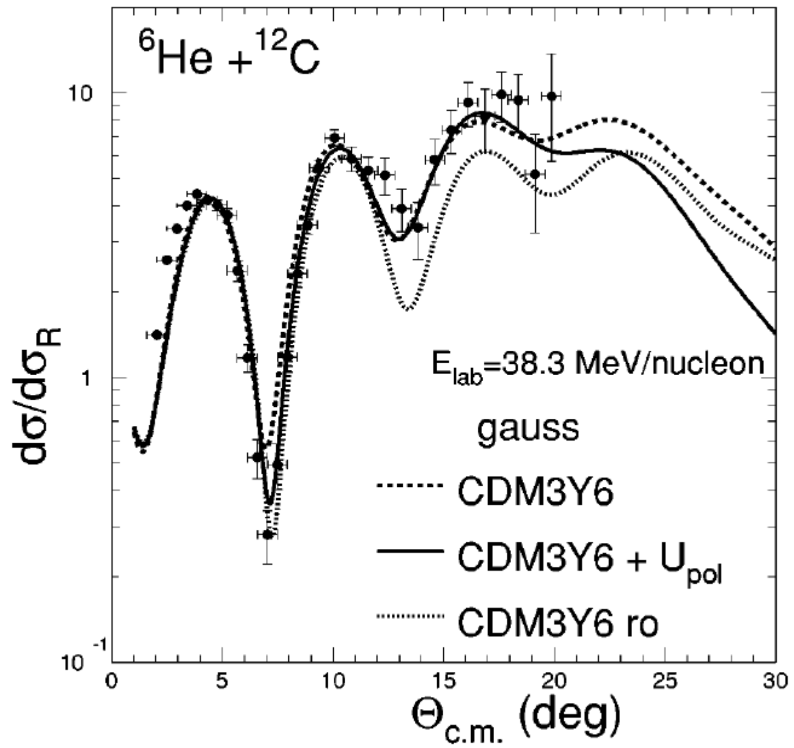
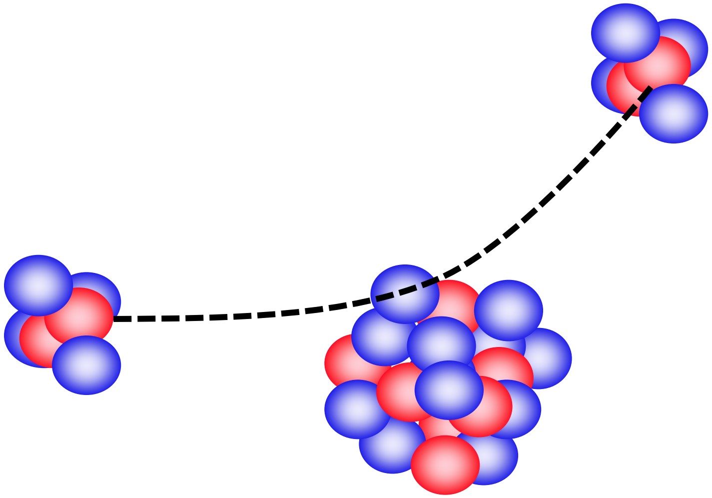


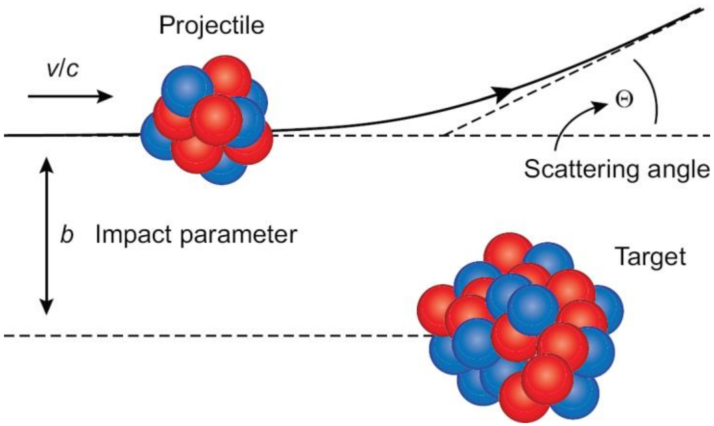
FIG. 10. Elastic scattering for ^{6}He+ ^{12}C at 38.3 MeV/nucleon in comparison with the OM results given by the real folded potential (obtained with the CDM3Y6 interaction and the Gaussian *ga* density for ^{6}He). The dashed curve is obtained with the unrenormalized folded potential only. The solid curve is obtained by adding a complex surface polarization potential to the real folded potential. Its parameters, and those of the imaginary part, are explained in the text. The dotted line is obtained by folding the CDM3Y6 interaction with the compact Gaussian density *ro*.

[Lapoux et al, PRC 66 (02) 034608]



traditionally used to extract optical potentials, rms radii, density distributions.

why do reactions? inelastic



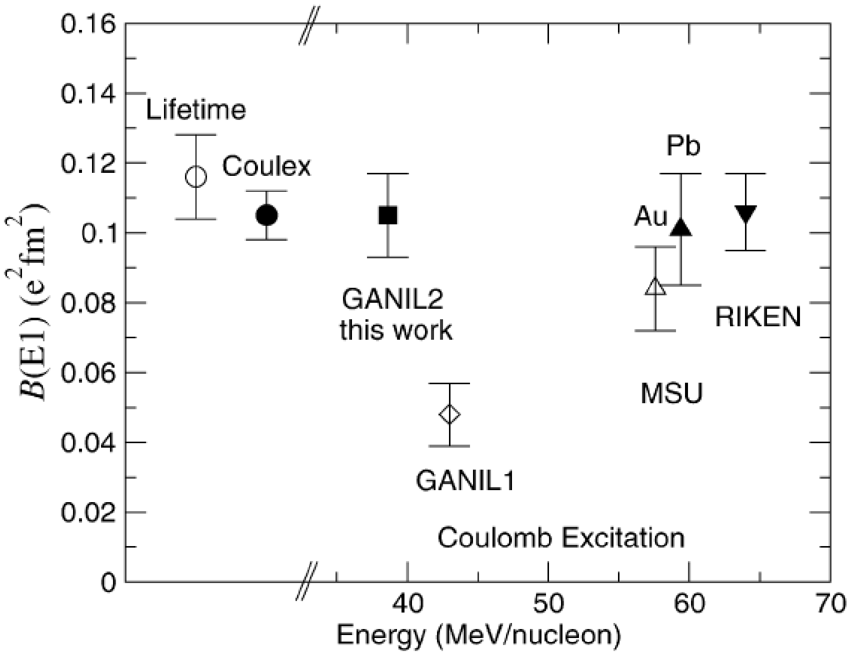
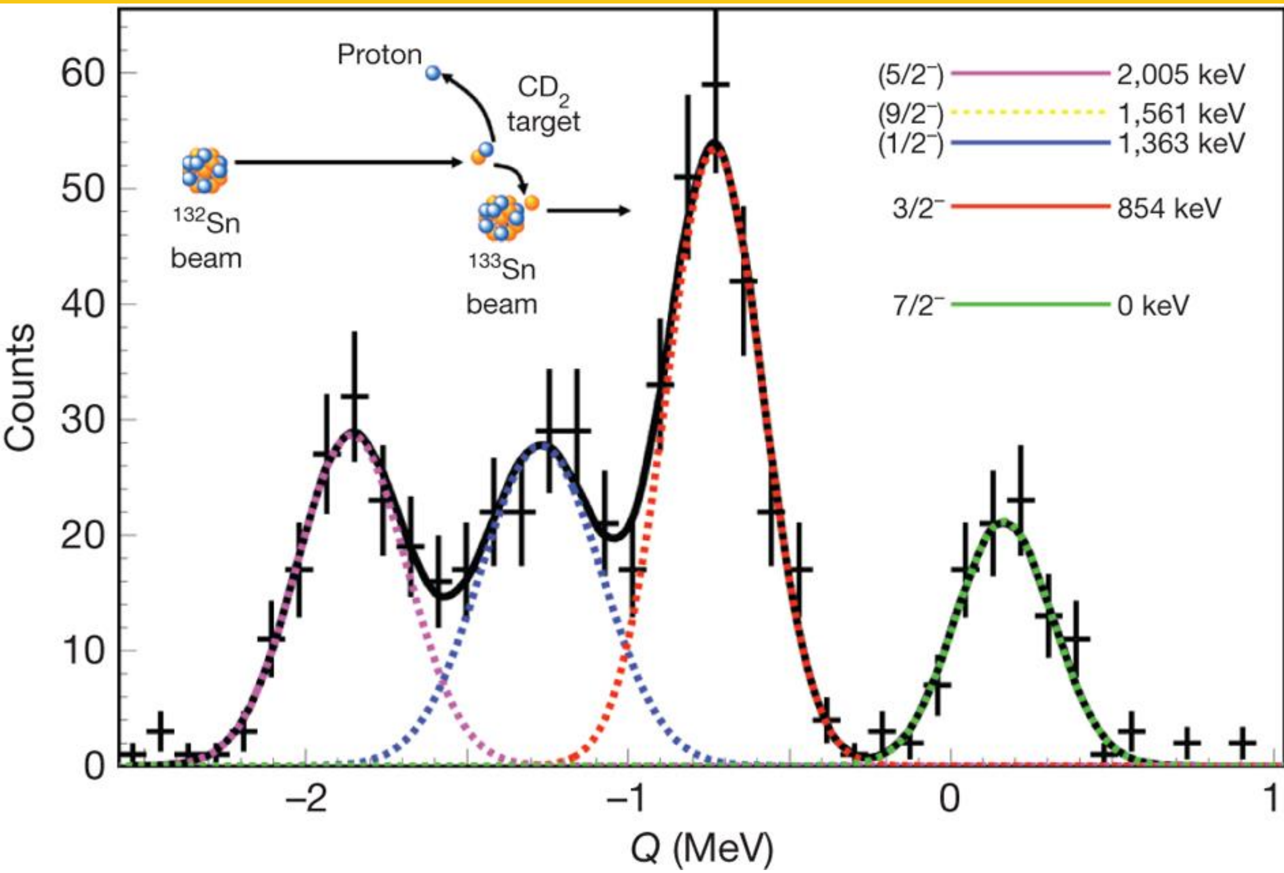


Fig. 2. Comparison of B(E1) values obtained from lifetime and Coulomb excitation measurements. The weighted average of lifetime measurements [3] (open circle) is plotted on the left along with the weighted average (solid circle) of three Coulomb excitation measurements (solid symbols). The individual Coulomb excitation measurements, GANIL (this work, square), MSU (up triangle) [6], RIKEN (down triangle) [7], and a previous GANIL experiment (diamond) [4], are plotted versus the beam energy.

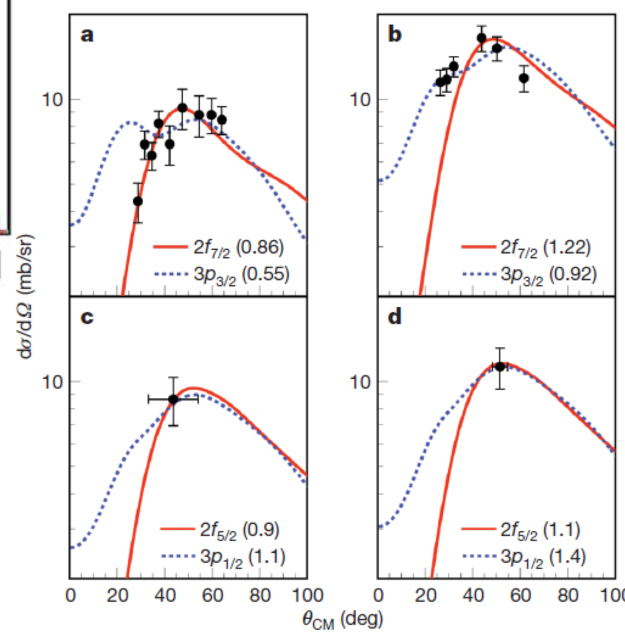
traditionally used to extract electromagnetic transitions or nuclear deformations

[Summers et al, PLB 650 (2007) 124]

why do reactions? transfer



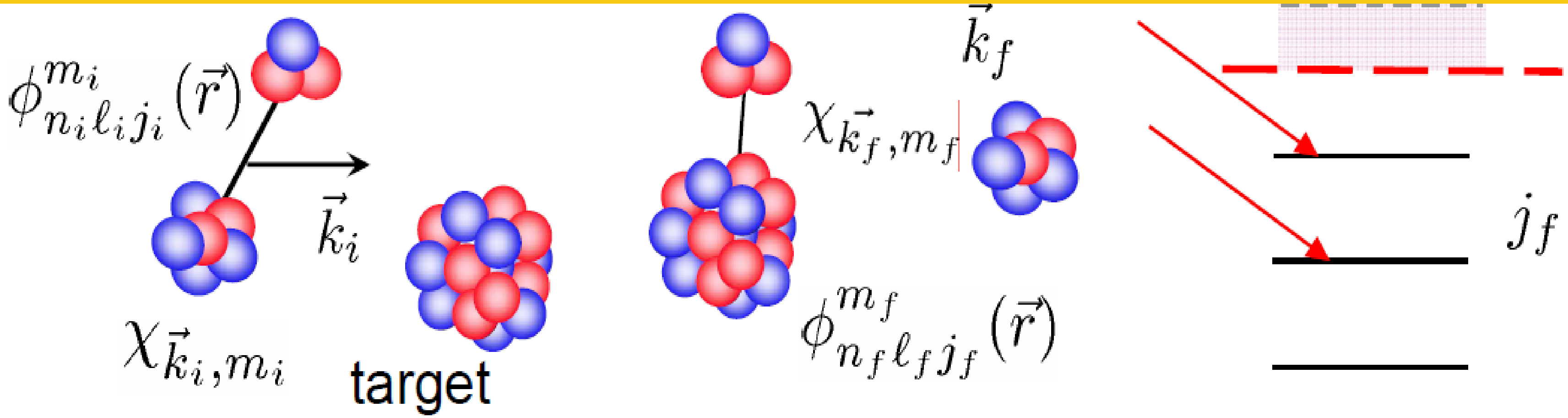
d(^{132}Sn,^{133}Sn)p@5 MeV/u



traditionally used to extract spin,parity and probabilities

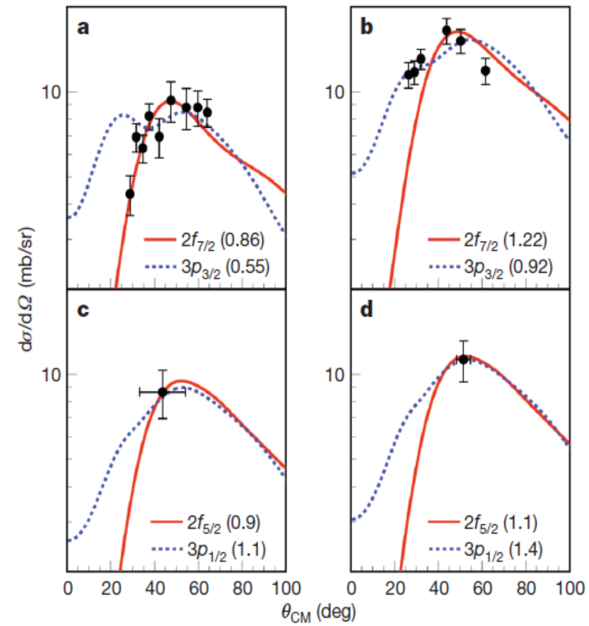
[K. Jones et al, Nature 465 (2010) 454]

why do reactions? transfer



traditionally used to extract spin,parity and orbital occupancy in valence shells

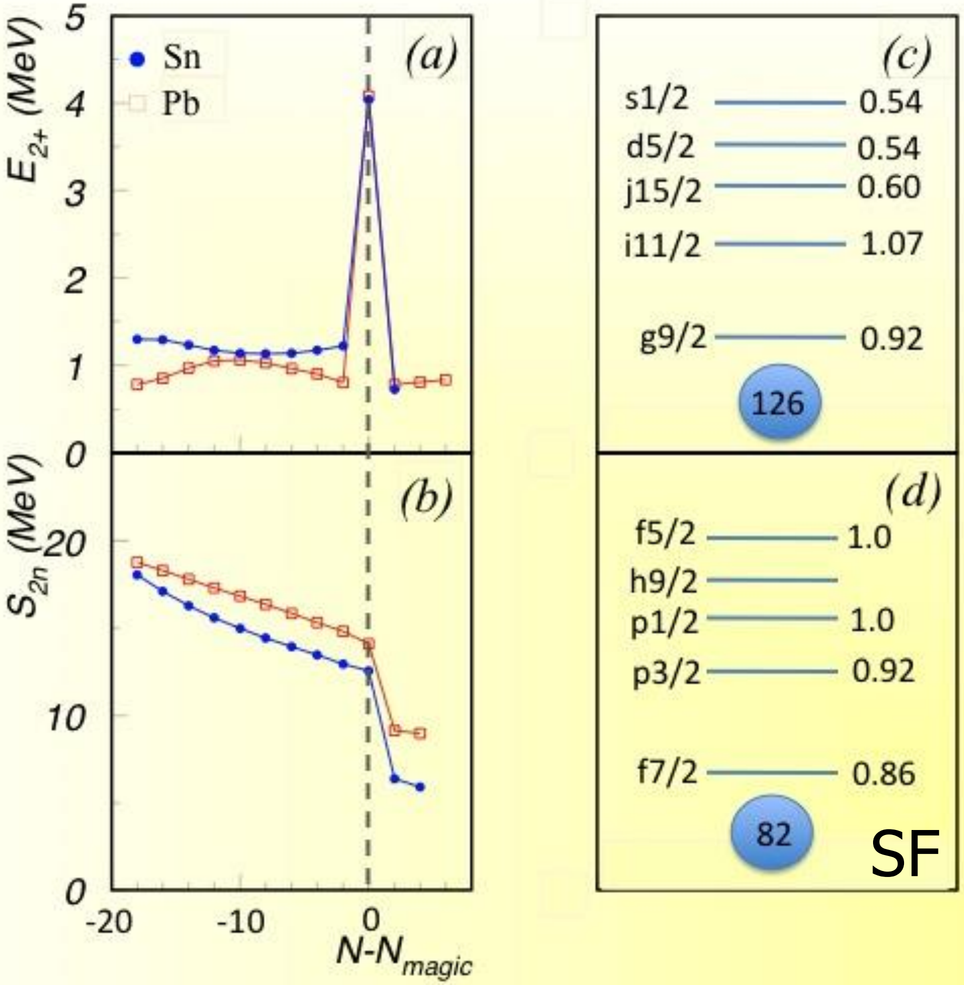
d(^{132}Sn,^{133}Sn)p@5 MeV/u



[K. Jones et al, Nature 465 (2010) 454]

reactions probe magicity

Doubly magic nuclei

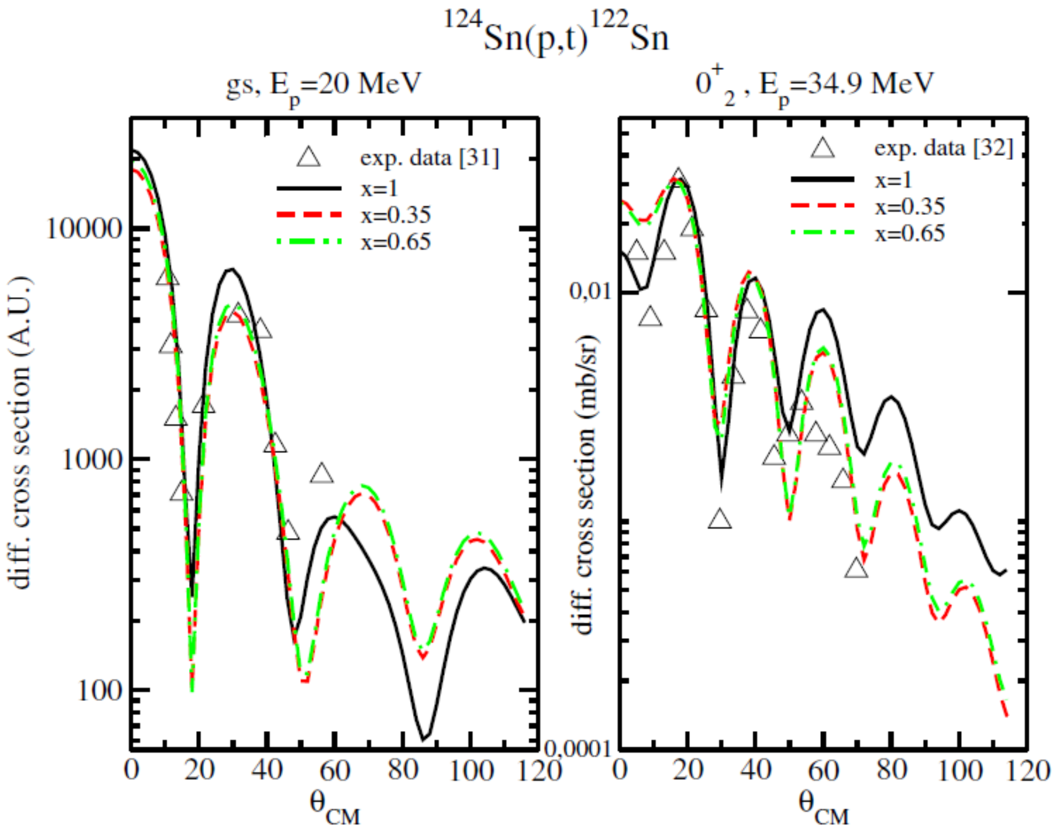


^{208}Pb(d,p)^{209}Pb

^{132}Sn(d,p)^{133}Sn

[K. Jones et al, Nature 465 (2010) 454]

why do reactions? transfer

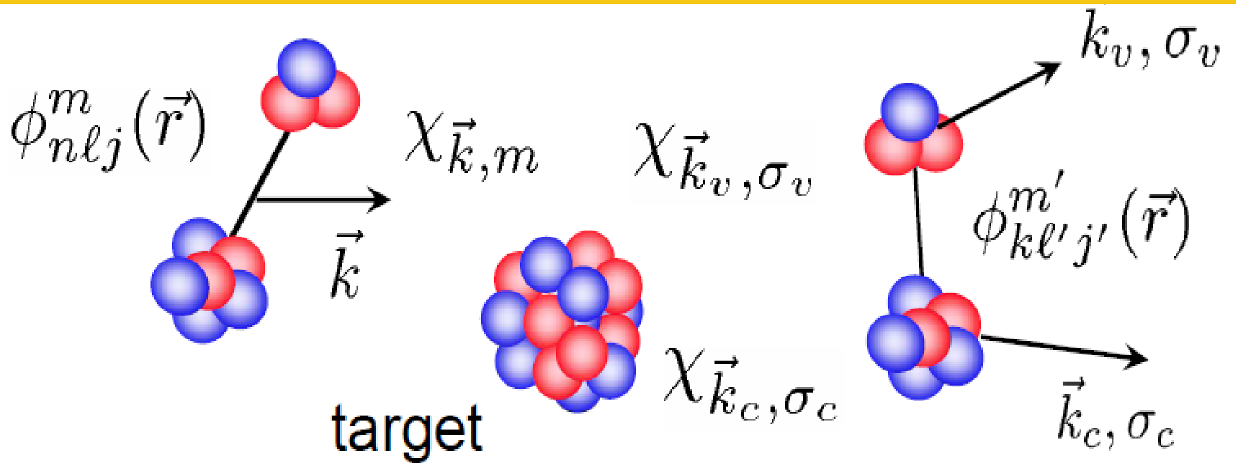


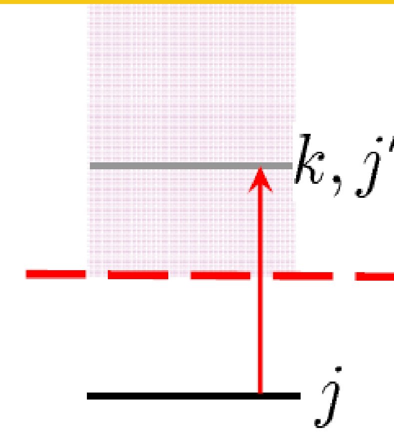
x=pairing parameter

[Pllumbi et al., Phys. Rev. C, 034613 (2011)

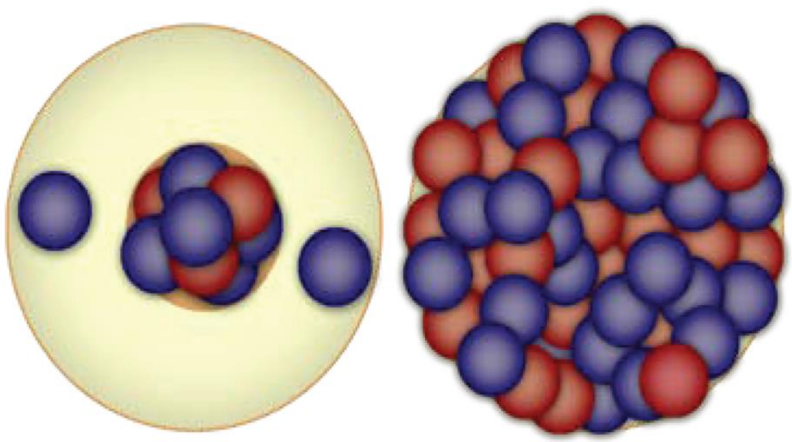
traditionally used to study two nucleon correlations and pairing

why do breakup?





traditionally used to extract properties of loosely bound states (halos)

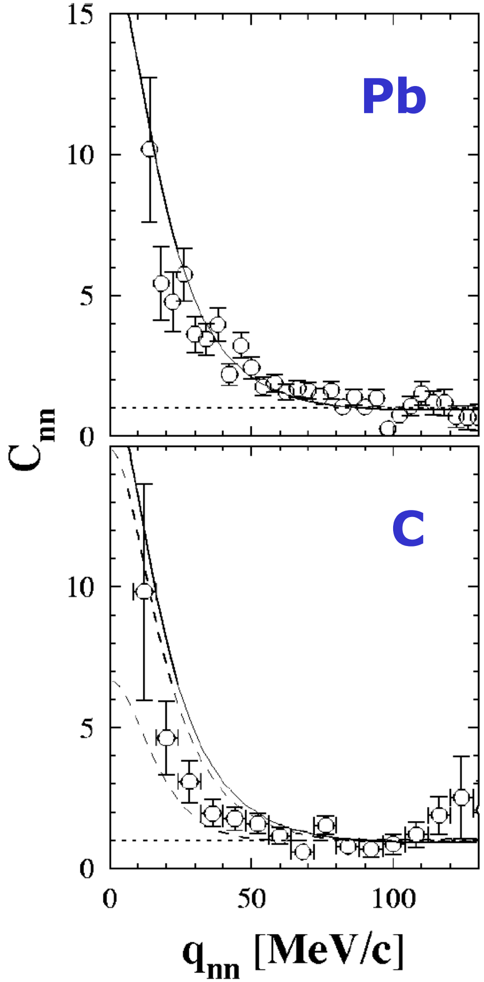


^{11}Li ^{208}Pb

why do reactions? breakup

two nucleon correlation function

$^{14}Be \rightarrow n+n+^{12}Be$



[Marques et al, PRC 64 (2001) 061301]

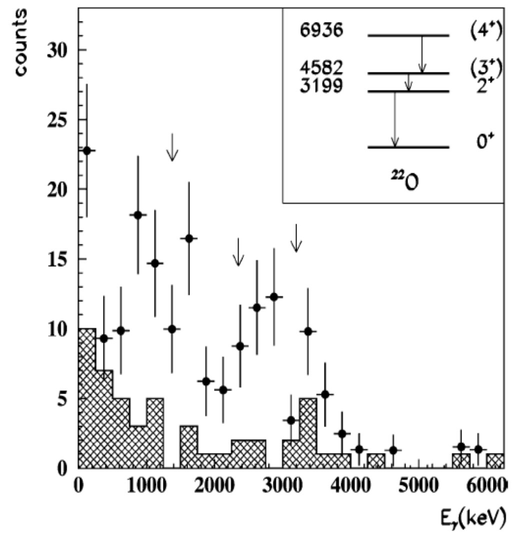
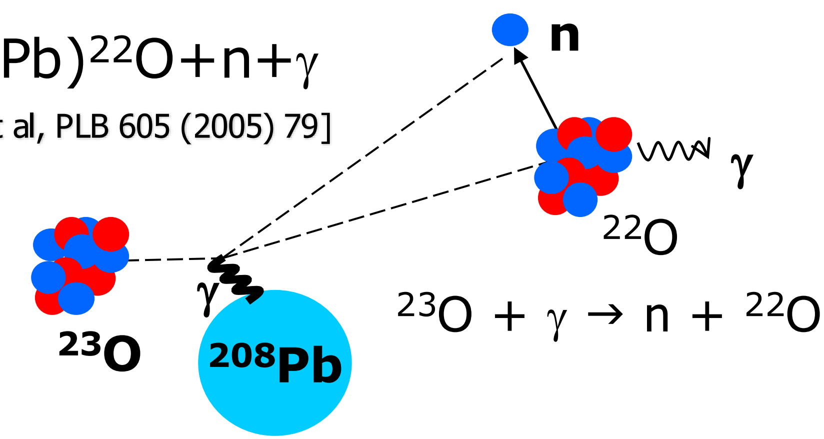


Fig. 1. Doppler corrected γ-ray spectra measured in coincidence with an ^{22}O fragment and one neutron for Pb (symbols) and C (shaded area) targets. Arrows indicate the strongest γ transitions as expected from the ^{22}O level scheme of Ref. [10] (partial level scheme shown as inset; level energies are in keV).

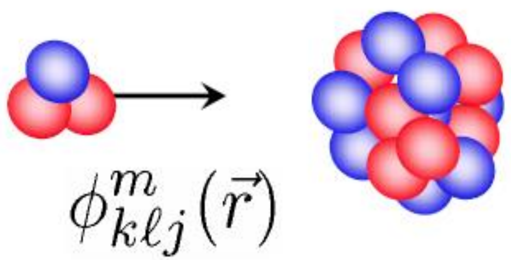
$^{23}O(Pb,Pb)^{22}O+n+\gamma$

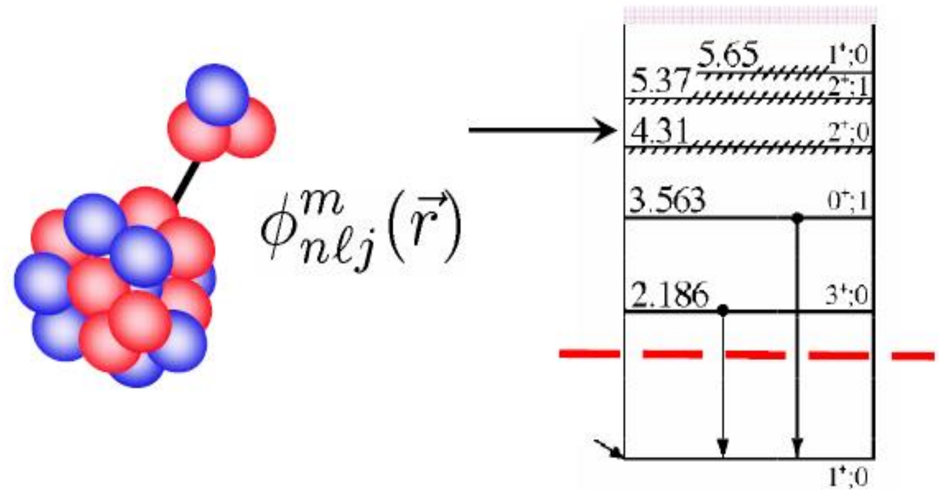
[Nociforo et al, PLB 605 (2005) 79]



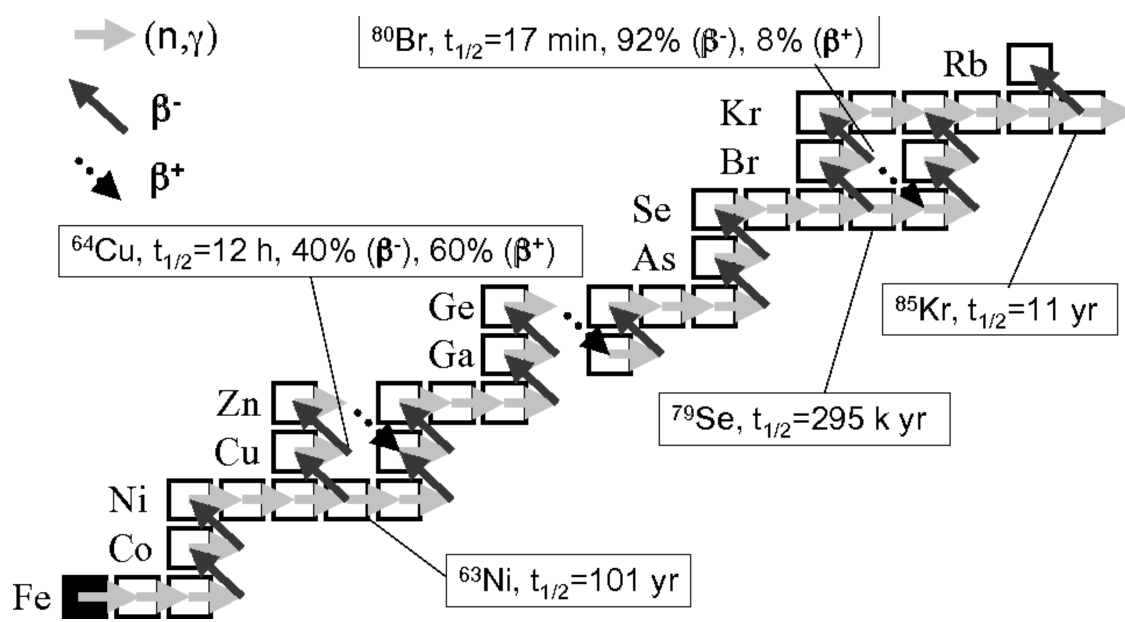
Why do capture?

<u>Capture</u>

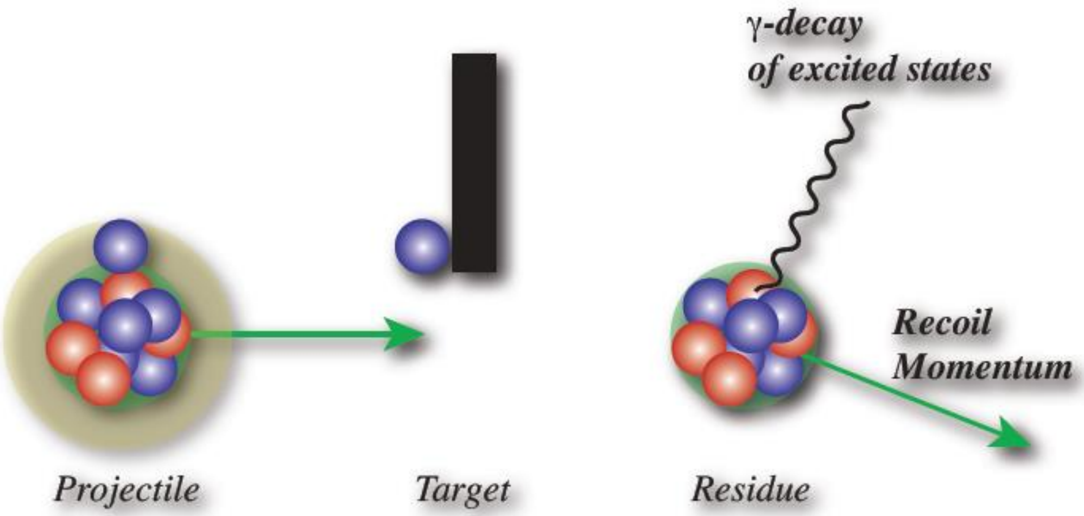




Traditionally needed for astrophysics: nucleosynthesis networks

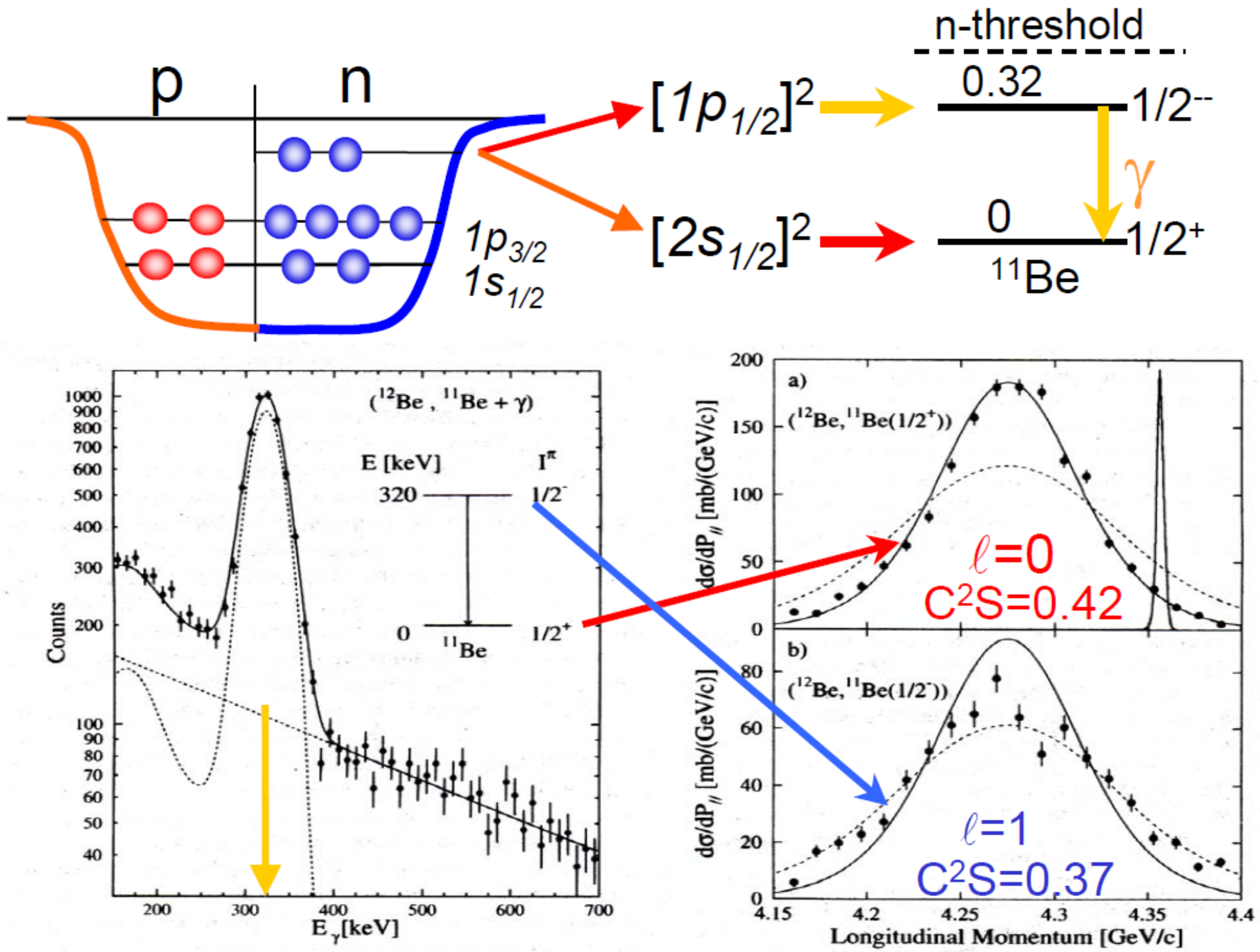


Why do reactions? knockout



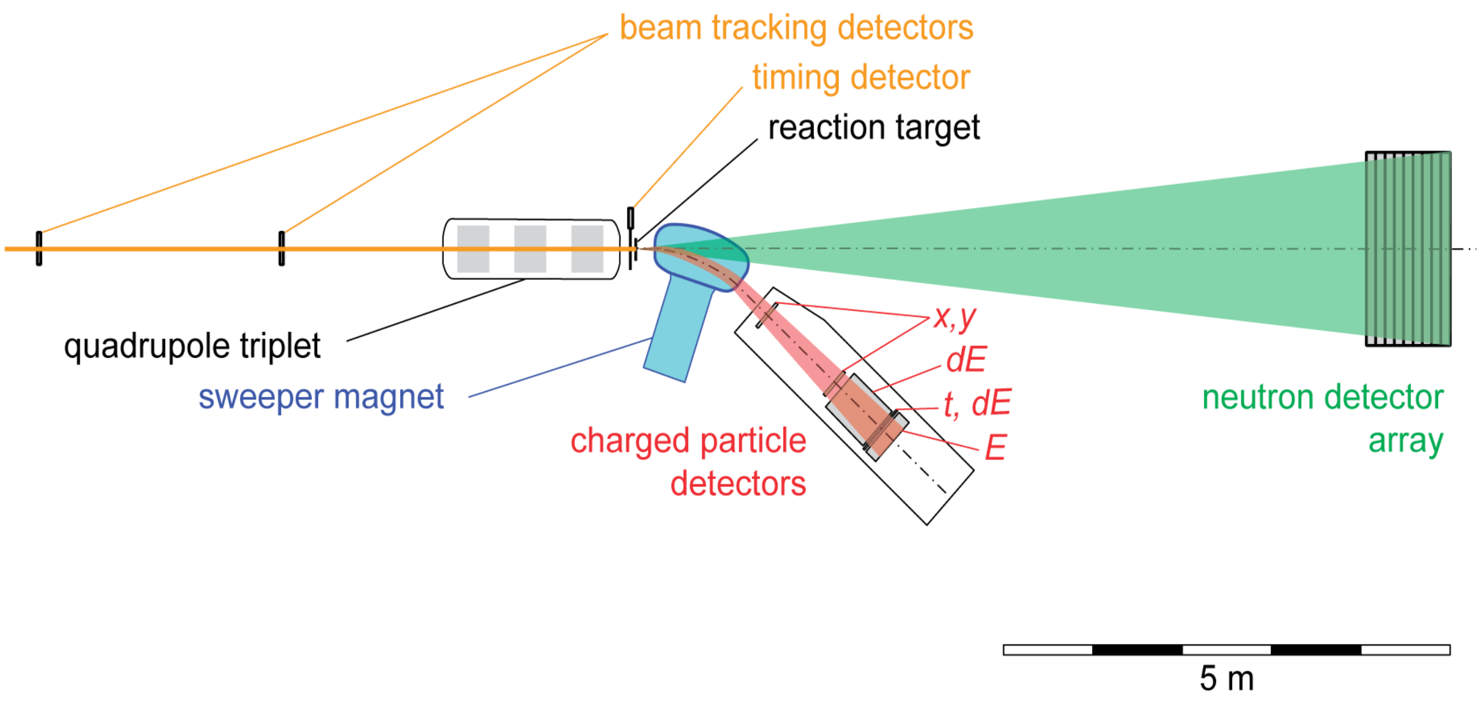
- Just like (e,e'p) but with a nuclear probe
- Includes elastic and inelastic breakup as well as transfer
- Needs less beam than transfer or breakup, integrated information

Knockout typical result: ^{12}Be



A. Navin *et al.*, Phys. Rev. Lett. **85**, 266 (2000)

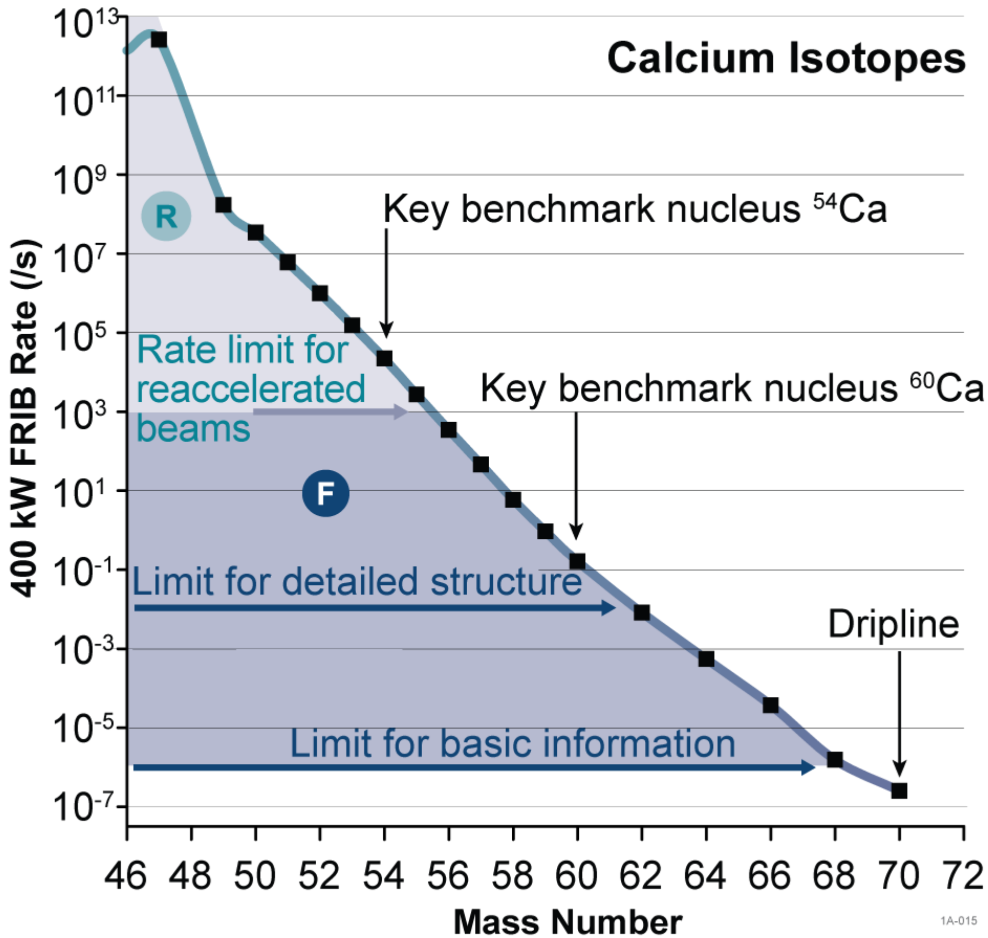
Reaction experiments



Typical setup for MONA experiments at NSCL

FRIB Features: Fast, Stopped, and Reaccelerated Beams

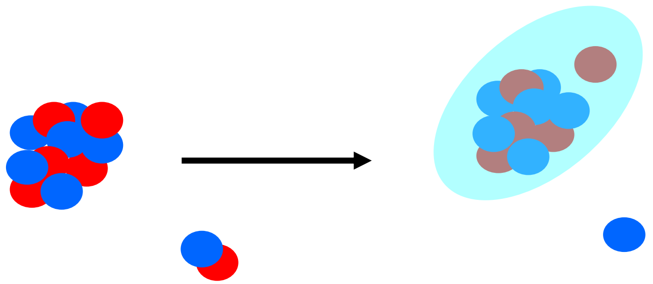
- Fast beams (>100 MeV/u)
  - Decay studies, knockout, Coulomb excitation, nuclear structure, limits of existence, EOS of asymmetric matter
- Stopped beams (0-100 keV)
  - Ion thermalization - fast, efficient
  - Precision experiments – masses, moments, atomic structure, symmetries
- Reaccelerated beams (0.2-20 MeV/u)
  - Ion thermalization and reacceleration
  - Detailed study of nucleus-nucleus collisions with exotic nuclei
  - Astrophysical reaction rates



why do reactions? astrophysics

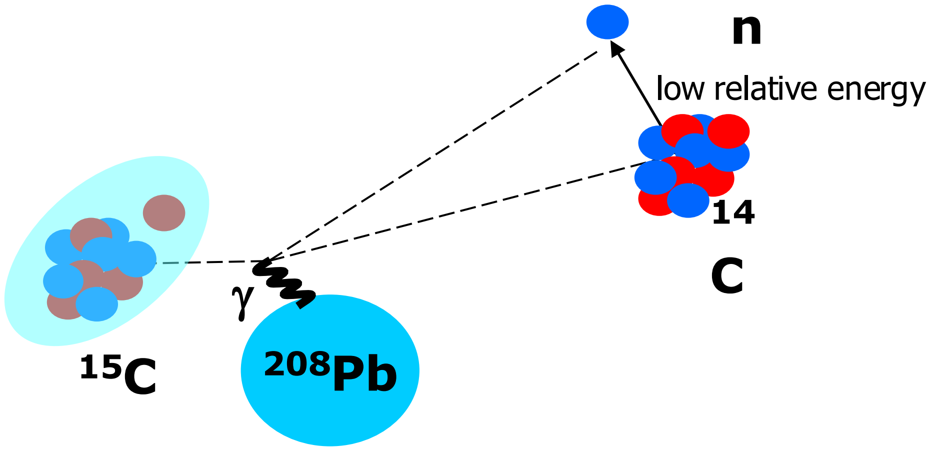
• direct measurement $^{14}C(n,\gamma)^{15}C$

• transfer reaction



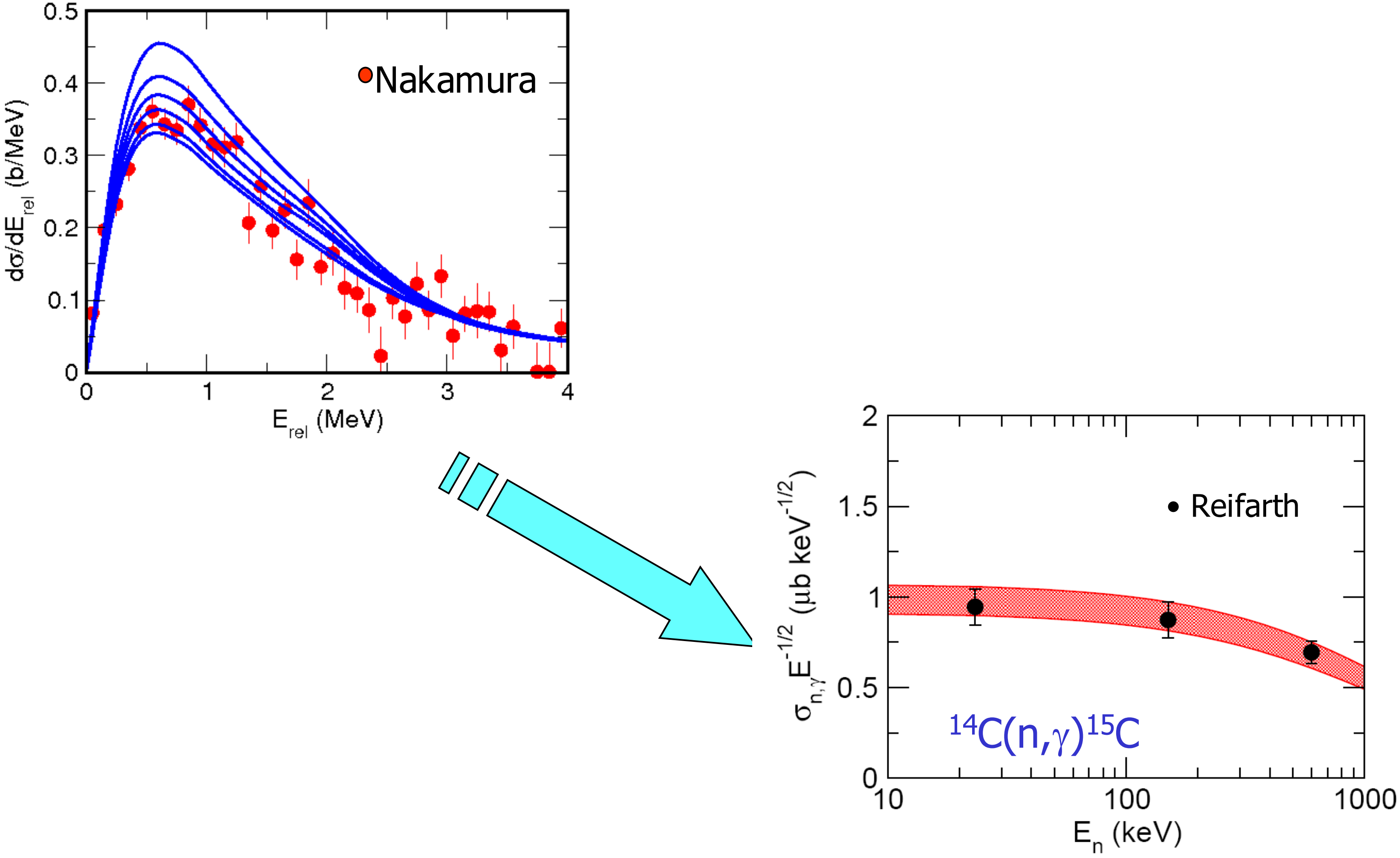
$^{14}C(d,p)^{15}C$

• Coulomb dissociation



breakup reactions and (n,γ)

$^{208}Pb(^{15}C,^{14}C+n)^{208}Pb$@68 MeV/u



Nakamura et al, NPA722(2003)301c
Reifarth et al, PRC77,015804 (2008)

Fusion of stable versus unstable nuclei

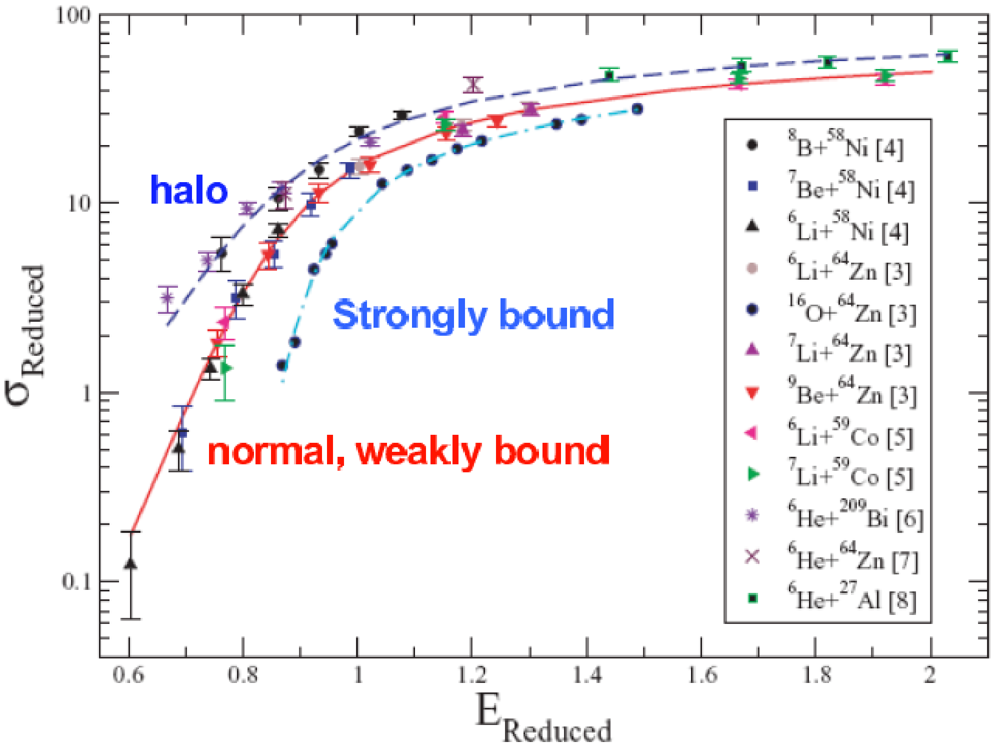
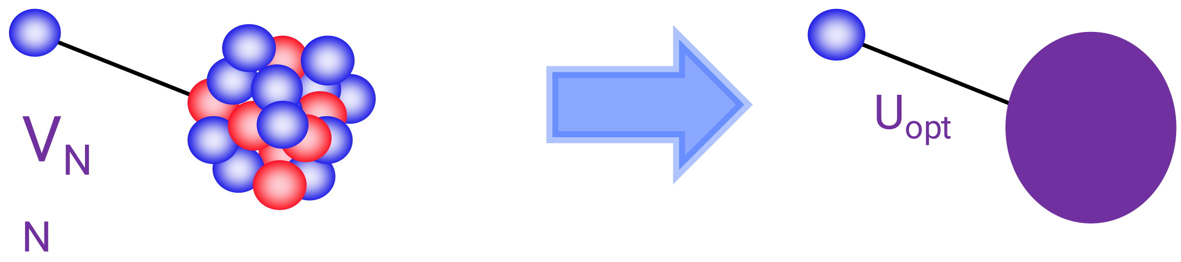


Fig. 8. Reduced cross sections for the fusion of halo, normal/weakly bound, and strongly bound nuclei. (Courtesy of J. Kolata).

After geometric effects are scaled out, fusion enhanced for halo nuclei!

The Optical Potential is an essential ingredient in reaction theory



It's the projection of the many-body scattering problem on the ground state: $$P\Psi(\vec{r},\vec{r}_1,\ldots,\vec{r}_A)=\phi_0(\vec{r})\Phi_0(\vec{r}_1,\ldots,\vec{r}_A)$$

End up with a single-channel scattering equation with potential:

$$V_{\rm opt}=\mathcal{V}_{00}+\sum_{j,k\neq 0}\mathcal{V}_{0j}\frac{1}{E-H_{jk}+i\eta}\mathcal{V}_{k0}$$

U_{opt}= V(R) + iW (R) can be obtained phenomenologically!

Hebborn, Nunes, et al., JPG 50, 060501 (2023)