National Nuclear Physics Summer School 2026

Seattle, WA

June 29 - July 11, 2026

Quantum Entanglement in Nuclear and High Energy Physics

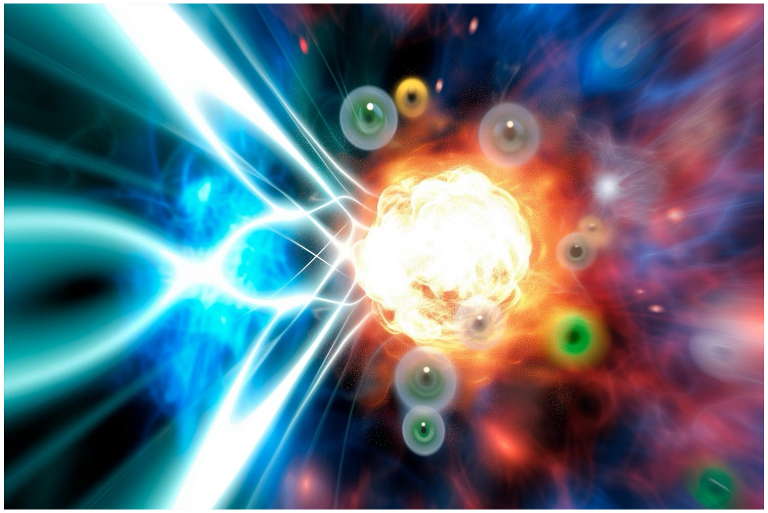


Image: SciTechDaily "Maximal entanglement inside the proton"

Dmitri Kharzeev

Center for Nuclear Theory
Stony Brook University

C2QA Co-design Center for Quantum Advantage

U.S. Department of Energy Office of Science

Brookhaven National Laboratory

Outline:

Lecture 1

1. Classical statistical physics from entanglement

2. Maximal entanglement from geometry of Hilbert space

3. Quantum simulations of jet production, and approach to maximal entanglement

4. Parton model from high-energy decoherence

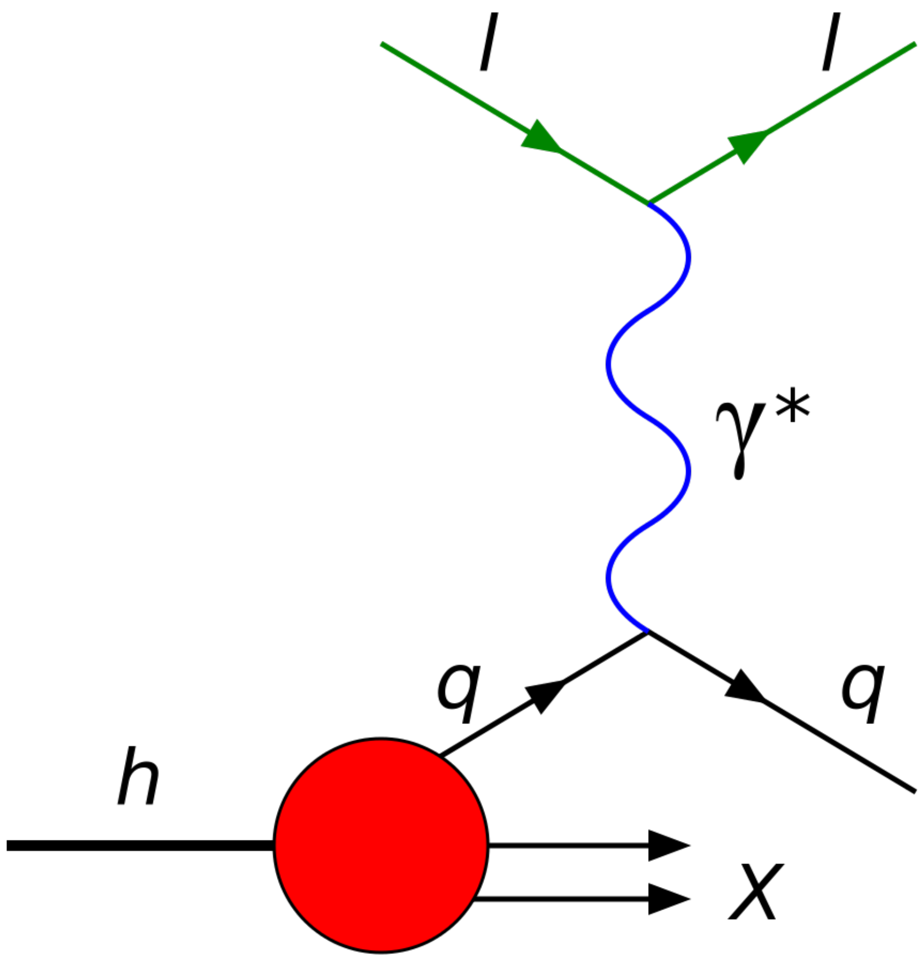
5. Measures of entanglement in jet fragmentation

6. (Some of) the things I did not cover: a guide

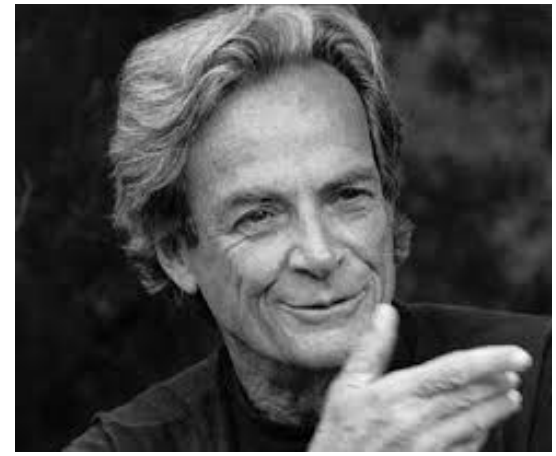
Lecture 2

Review: DK, quant-ph/2601.00405 (Acta Phys. Polon. B57, 4)

The parton model: 50 years of success



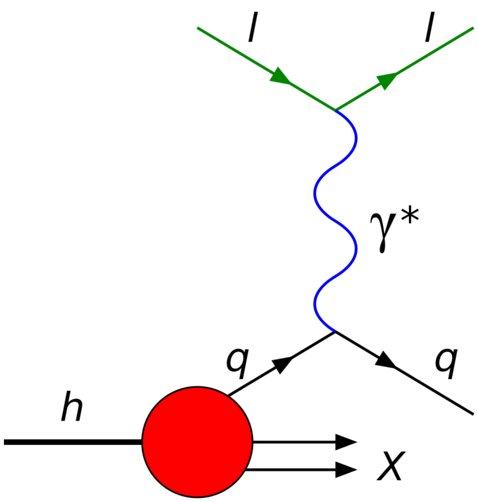

J. Bjorken


R. Feynman


V. Gribov

In fifty years that have ensued after the birth of the parton model, it has become an indispensable building block of high energy physics – so we have to understand it

The puzzle of the parton model



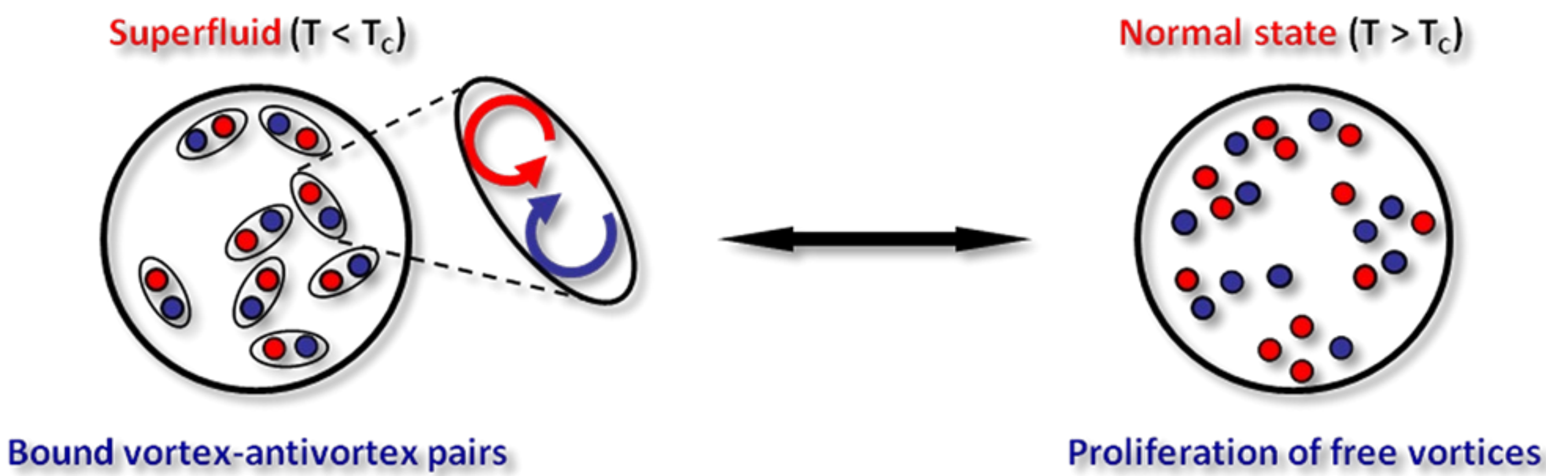
In parton model, the proton is pictured as a collection of point-like quasi-free partons that are frozen in the infinite momentum frame due to Lorentz dilation.

The DIS cross section is given by the incoherent sum of cross sections of scattering off individual partons.

How to reconcile this with quantum mechanics?

The puzzle of the parton model

In quantum mechanics, the proton is a <u>pure state</u> with <u>zero entropy</u>. Yet, a collection of free partons does possess entropy… Boosting to the infinite momentum frame does not help, as a Lorentz boost cannot transform a pure state into a mixed one.



The crucial importance of entropy in (2+1)D systems: BKT phase transition (Nobel prize 2016)

PHYSICAL REVIEW D **95**, 114008 (2017)

Deep inelastic scattering as a probe of entanglement

Dmitri E. Kharzeev[1,2,*] and Eugene M. Levin[3,4,†]

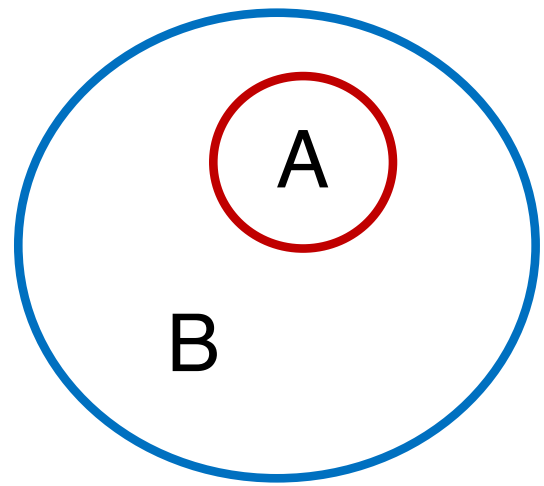
Our proposal: the key to solving this apparent paradox is entanglement.

DIS probes only a part of the proton's wave function (region A). We sum over unobserved region B; in quantum mechanics, this corresponds to accessing the density matrix of a mixed state

$$\hat{\rho}_A = \mathrm{tr}_{\mathrm{B}} \hat{\rho}$$

with a non-zero entanglement entropy

$$S_A = -\mathrm{tr}\left[\hat{\rho}_{\mathrm{A}} \ln \hat{\rho}_{\mathrm{A}}\right]$$



The quantum mechanics of partons and entanglement

What is "region B" in DIS? It may be the phase!

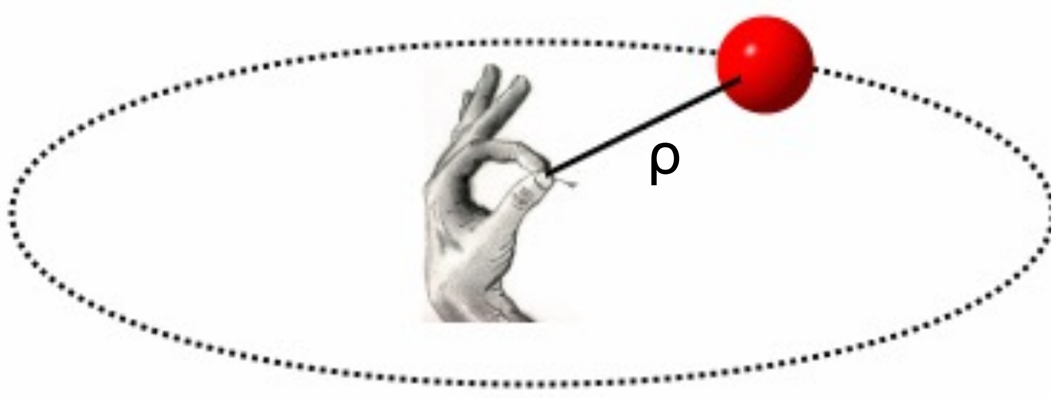
DK, Phil. Trans. Royal Soc (2022); arXiv:2108.08792

DIS takes an instant snapshot of the proton's wave function. This snapshot cannot measure the phase of the wave function.

Classical analogy:

$$z = \rho \, \exp(i\omega t)$$

Instant snapshot can measure the amplitude ρ, but not the angular velocity ω !



The quantum mechanics of partons and entanglement

A simple quantum mechanical model (proton rest frame):

DK, Phil. Trans. Royal Soc (2022); arXiv:2108.08792

Expand the proton wave function in oscillator Fock states:

$$|n\rangle = \frac{1}{\sqrt{n!}} \prod_i^n a_i^\dagger |0\rangle,$$

$$|\Psi\rangle = \sum_n \alpha_n |n\rangle,$$

The density matrix:

$$\hat{\rho} = |\Psi\rangle\langle\Psi\rangle = \sum_{n,n'} \alpha_n \, \alpha_{n'}^* \, |n\rangle\langle n'|,$$

depends on time:

$$\hat{\rho}(t) = \sum_{n,n'} e^{i(n'-n)\omega t} \, \hat{\rho}(t=0).$$

But this time dependence cannot be measured by a light front – it crosses the hadron too fast, at time $t_{light} = R,$

Decoherence in high energy interactions

DK, Phil. Trans. Royal Soc (2022)

Therefore, the observed density matrix is a trace over an unobserved phase:

$$\hat{\rho}_{parton} = \mathrm{Tr}_{\varphi}\hat{\rho} = \sum_{n,n'} \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{i(n'-n)\varphi}\ \alpha_n\ \alpha^*_{n'} |n\rangle\langle n'| = \sum_n |\alpha_n|^2\ |n\rangle\langle n|.$$

U(1) Haar measure

"Haar scrambling" = decoherence
Y.Sekino, L.Susskind '08

After "Haar scrambling", the density matrix becomes diagonal in parton basis (Schmidt basis) –

Probabilistic parton model!

This is a density matrix of a mixed state, with non-zero entanglement entropy!

The quantum mechanics of partons and entanglement

The parton model density matrix:

$$\hat{\rho}_{parton} = \sum_n p_n \, |n\rangle\langle n|$$

is mixed, with purity

$$\gamma_{parton} = \mathrm{Tr}(\rho^2_{parton}) = \sum_n p_n^2 < 1.$$

entanglement entropy

$$S_E = -\sum_n p_n \ln p_n$$

Parton model expressions for expectation values of operators:

$$\langle\hat{\mathcal{O}}\rangle = \mathrm{Tr}(\hat{\mathcal{O}}\hat{\rho}_{parton}) = \sum_n p_n \langle n|\hat{\mathcal{O}}|n\rangle;$$

The quantum mechanics of partons and entanglement on the light cone

The density matrix on the light cone:

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \sum_{n,n'}^{\infty} \int d\Gamma_n \, d\Gamma_{n'} \, \Psi^*_{n'}(x_{i'}, \vec{k}_{\perp i'}) \Psi_n(x_i, \vec{k}_{\perp i}) |n\rangle\langle n'|.$$

Haar scrambling: on the light cone, $t_i - z_i = x_i^- = 0,$
but t, z and $x^+ = z + t$ cannot be independently determined:

$$\int \frac{dx^+}{2\pi} e^{i(P_n^- - P_{n'}^-)x^+} = \delta(P_n^- - P_{n'}^-),$$

⬇

$$\hat{\rho}_{parton} = \mathrm{Tr}_{x^+} |\Psi\rangle\langle\Psi| = \sum_{n}^{\infty} \int d\Gamma_n \, |\Psi_n(x_i, \vec{k}_{\perp i})|^2 |n\rangle\langle n|,$$

Phase-occupation number uncertainty relation and parton model

$$\Delta\phi \Delta n \geq \frac{1}{2} \left| \langle \Psi | [\hat{\phi}, \hat{n}] | \Psi \rangle \right|$$

High energies – phase cannot be measured, number is fixed:

parton model applies

Low energies - phase shifts can be measured, number is uncertain:

parton model does not apply

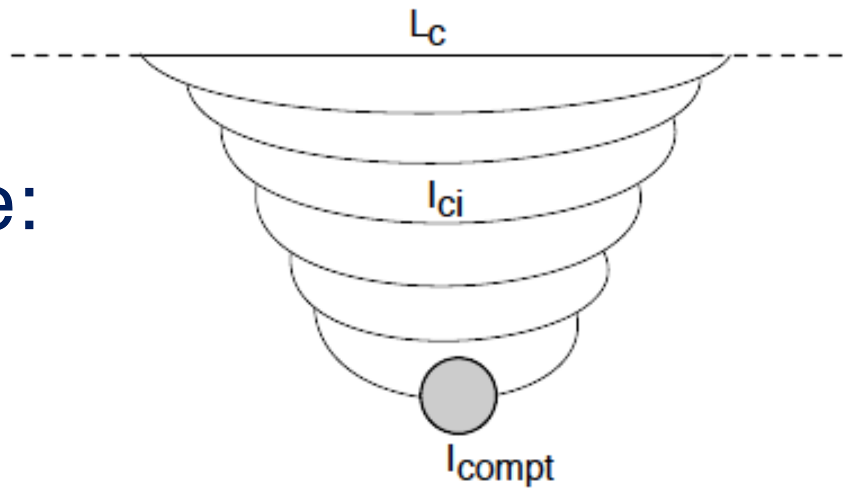
PHYSICAL REVIEW A VOLUME 48, NUMBER 4 OCTOBER 1993

Measurement of number-phase uncertainty relations of optical fields

D. T. Smithey, M. Beck, J. Cooper,* and M. G. Raymer

The entanglement entropy from QCD evolution

Space-time picture
in the proton's rest frame:



The evolution equation:

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

The entanglement entropy from QCD evolution

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

Solve by using the generating function method
(A.H. Mueller ‘94; E. Levin, M. Lublinsky ‘04):

$$Z(Y,u) = \sum_n P_n(Y) u^n.$$

Solution:

$$P_n(Y) = e^{-\Delta Y}(1 - e^{-\Delta Y})^{n-1}.$$

The resulting von Neumann entropy is

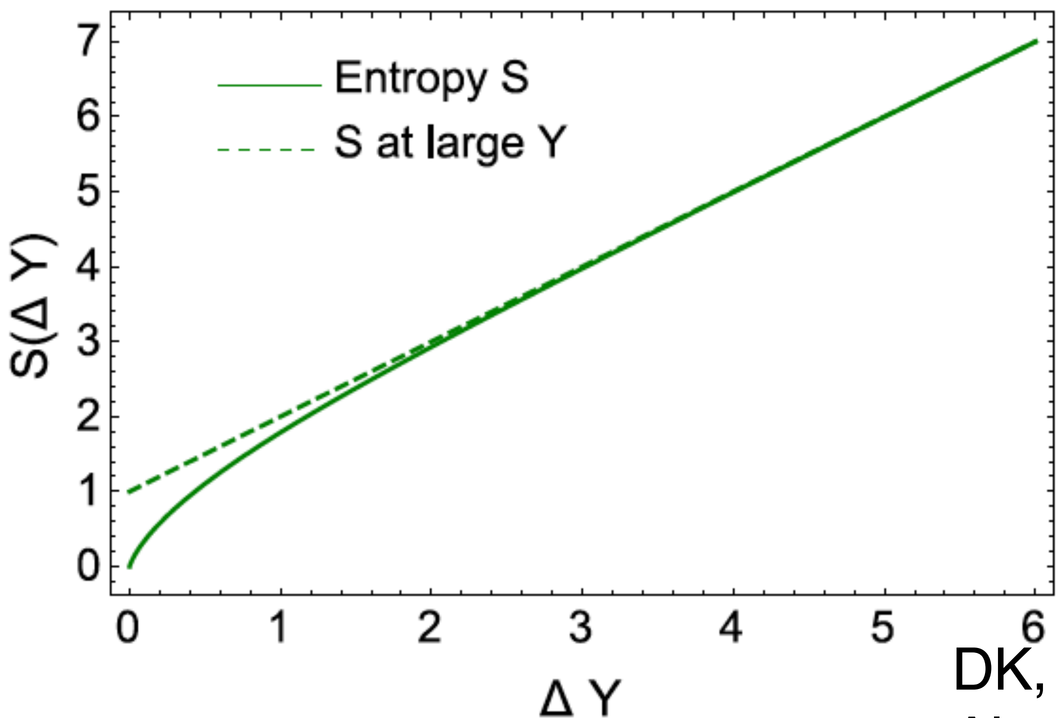
$$S(Y) = \ln(e^{\Delta Y} - 1) + e^{\Delta Y} \ln\left(\frac{1}{1 - e^{-\Delta Y}}\right)$$

DK, E. Levin, arXiv:1702.03489; PRD

The entanglement entropy from QCD evolution

At large ΔY , the entropy becomes

$$S(Y) \to \Delta Y$$



This "asymptotic" regime starts rather early, at

$$\Delta Y \simeq 2$$

DK, E. Levin, arXiv:1702.03489; PRD
Also: K. Kutak, arXiv:1103.3654

Linear dependence on rapidity is a consequence of (approximate) conformal invariance:

PHYSICAL REVIEW D **110,** 074008 (2024)

Universal rapidity scaling of entanglement entropy inside hadrons from conformal invariance

Umut Gürsoy,[1] Dmitri E. Kharzeev,[2,3] and Juan F. Pedraza[4]

[1]*Institute for Theoretical Physics and Center for Extreme Matter and Emergent Phenomena, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands*
[2]*Center for Nuclear Theory, Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA*
[3]*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*
[4]*Instituto de Física Teórica UAM/CSIC, Calle Nicolás Cabrera 13-15, Madrid 28049, Spain*

description. In this paper, we use an effective conformal field theoretic description of hadrons on the light cone to show that the linear dependence of the entanglement entropy on rapidity found in parton description is a general consequence of approximate conformal invariance and does not depend on the assumption of weak coupling. Our result also provides further evidence for a duality between the parton and string descriptions of hadrons.

$$S_A = \frac{c}{6}\Delta\eta + \ldots,$$

The entanglement entropy from QCD evolution

At large ΔY (x ~ 10^{-3}) the relation between the entanglement entropy and the structure function

$$xG(x) = \langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\Delta}$$

becomes very simple:

$$S = \ln[xG(x)]$$

DK, E. Levin, arXiv:1702.03489; PRD 95 (2017)

The entanglement entropy from QCD evolution

What is the physics behind this relation?

$$S = \ln[xG(x)]$$

It signals that all $\exp(\Delta Y)$ partonic states have about equal probabilities $\exp(-\Delta Y)$ – in this case the **entanglement entropy is maximal**, and the proton is a **maximally entangled state** (a new look at the parton saturation and CGC?)

DK, E. Levin, arXiv:1702.03489; PRD 95 (2017)

Experimental tests

What is the relation between the parton and hadron multiplicity distributions?

Let us assume they are the same ("EbyE parton-hadron duality"); then the hadron multiplicity distribution should be given by

$$P_n(Y) \;=\; e^{-\Delta Y}(1-e^{-\Delta Y})^{n-1}.$$

Consider moments

$$C_q = < n^q > / < n >^q$$

Fluctuations in hadron multiplicity

The moments can be easily computed by using the generating function

$$C_q = \left(u \frac{d}{du} \right)^q Z(Y, u) \Bigg|_{u=1}$$

We get

$$C_2 = 2 - 1/\bar{n}; \quad C_3 = \frac{6(\bar{n}-1)\bar{n}+1}{\bar{n}^2};$$

$$C_4 = \frac{(12\bar{n}(\bar{n}-1)+1)(2\bar{n}-1)}{\bar{n}^3}; \quad C_5 = \frac{(\bar{n}-1)(120\bar{n}^2(\bar{n}-1)+30\bar{n})+1}{\bar{n}^4}.$$

Fluctuations in hadron multiplicity

Numerically, for $\bar{n} = 5.8 \pm 0.1$ at $|\eta|<0.5$, E_{cm}=7 TeV we get:

theory	exp (CMS)	theory, high energy limit
C_2 = 1.83	C_2 = 2.0+-0.05	C_2 = 2.0
C_3 = 5.0	C_3 = 5.9+-0.6	C_3 = 6.0
C_4 = 18.2	C_4 = 21+-2	C_4 = 24.0
C_5 = 83	C_5 = 90+-19	C_5 = 120

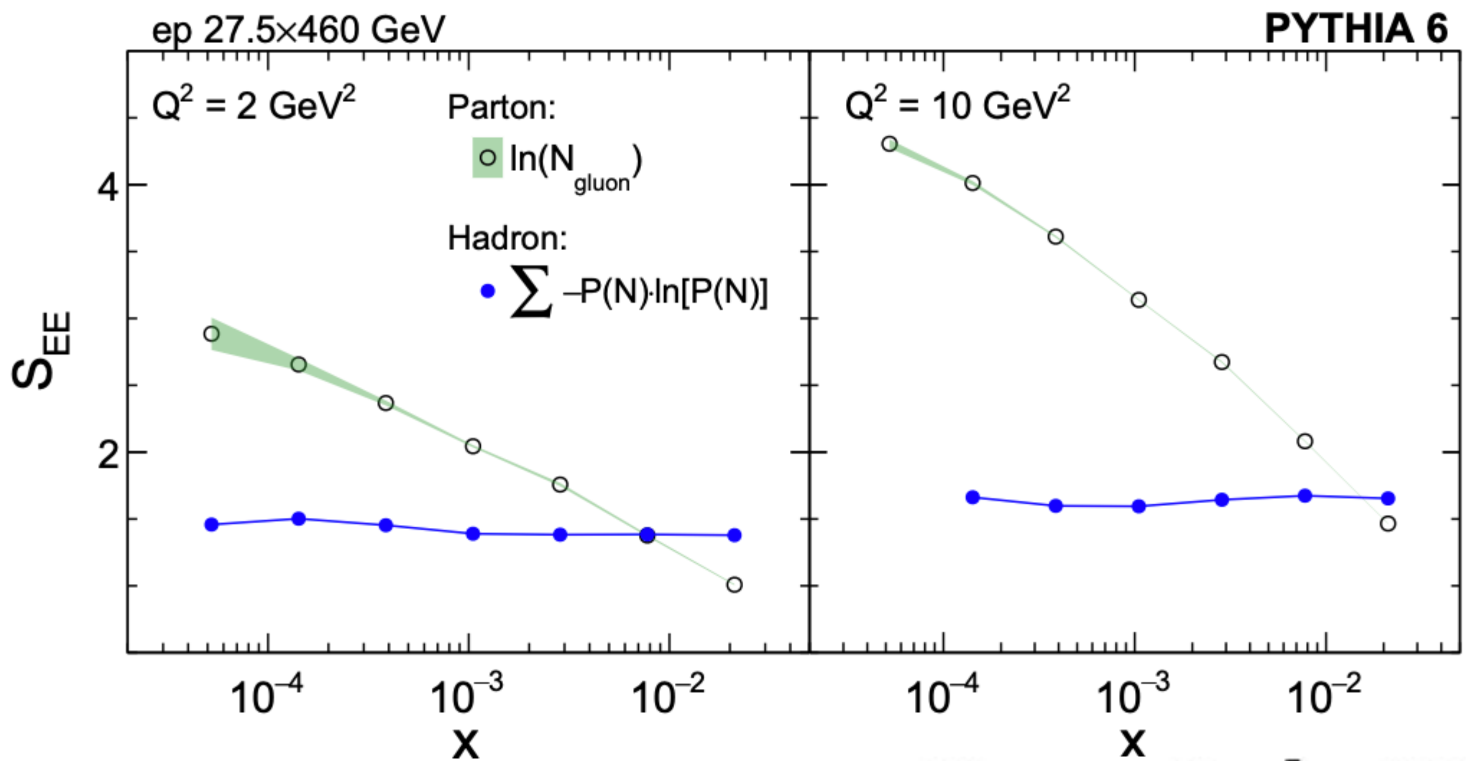
It appears that the multiplicity distributions of final state hadrons are very similar to the parton multiplicity distributions – this suggests that the entropy is close to the entanglement entropy

Test of the entanglement at the LHC

PHYSICAL REVIEW LETTERS **124**, 062001 (2020)

Einstein-Podolsky-Rosen Paradox and Quantum Entanglement at Subnucleonic Scales

Zhoudunming Tu[1,*] Dmitri E. Kharzeev,[2,3] and Thomas Ullrich[1,4]



MC generator PYTHIA: $S = \ln[xG(x)]$

is not satisfied at small x (no entanglement)

Test of the entanglement at the LHC

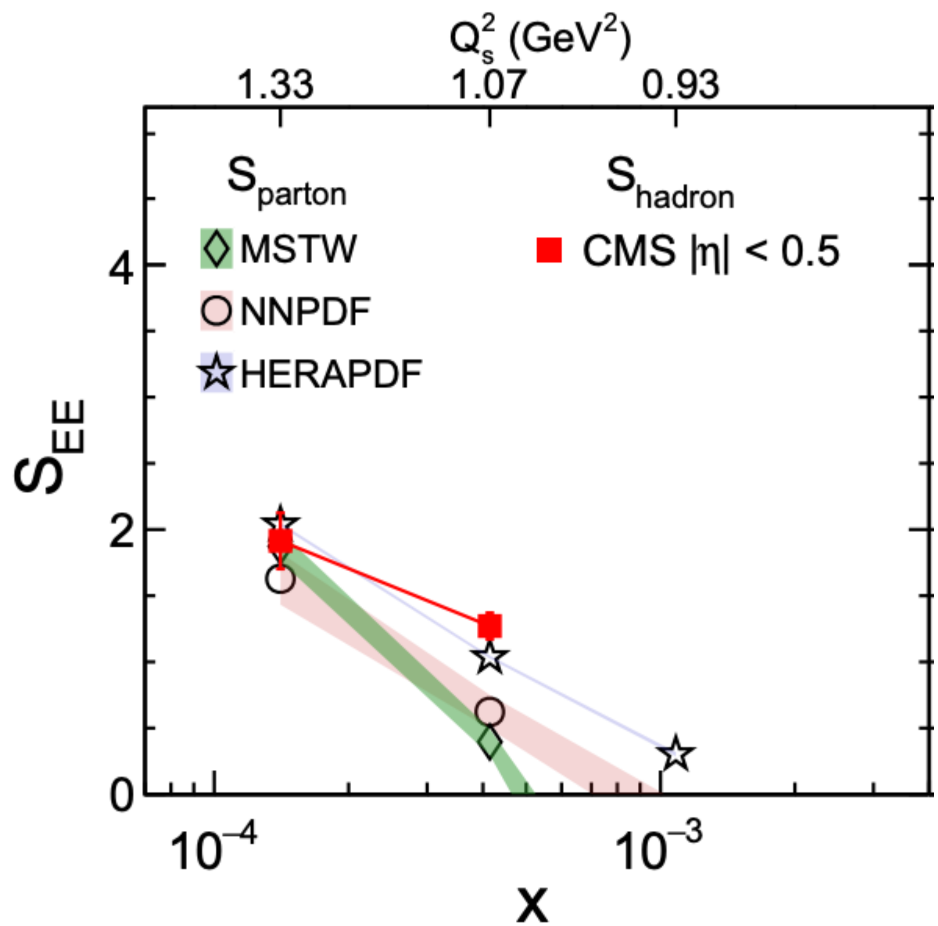
LHC data:

arXiv:1904.11974

$$S = \ln[xG(x)]$$

is satisfied at small x (**entanglement?!**)

K. Tu, DK, T. Ullrich, arXiv:1904.11974; PRL (2020)



Evidence for the maximally entangled low x proton in Deep Inelastic Scattering from H1 data

Martin Hentschinski[1] and Krzysztof Kutak[2]

[1]Departamento de Actuaria, Física y Matemáticas, Universidad de las Americas Puebla, San Andrés Cholula, 72820 Puebla, Mexico
[2] Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342, Kraków, Poland

December 14, 2021

Abstract

We investigate the proposal by Kharzeev and Levin of a maximally entangled proton wave function in Deep Inelastic Scattering at low x and the proposed relation between parton number and final state hadron multiplicity. Contrary to the original formulation we determine partonic entropy from the sum of gluon and quark distribution functions at low x, which we obtain from an unintegrated gluon distribution subject to next-to-leading order Balitsky-Fadin-Kuraev-Lipatov evolution. We find for this framework very good agreement with H1 data. We furthermore provide a comparison based on NNPDF parton distribution functions at both next-to-next-to-leading order and next-to-next-to-leading with small x resummation, where the latter provides an acceptable description of data.

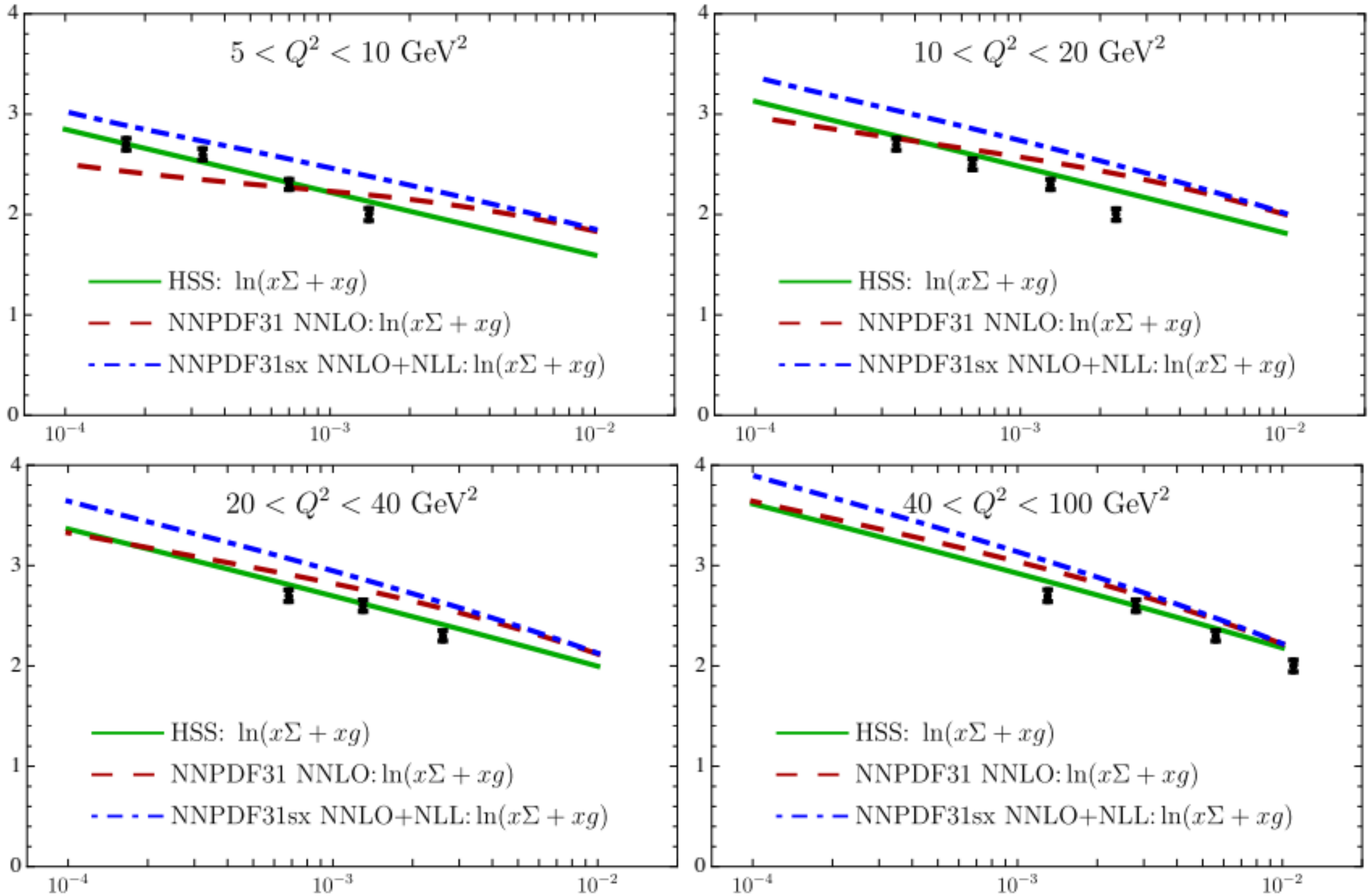


Figure 1: *Partonic entropy versus Bjorken x, as given by Eq. (1) and Eq. (2). We furter show results based on the gluon distribution only as well as a comparison to NNPDFs. Results are compared to the final state hadron entropy derived from the multiplicity distributions measured at H1 [19]*

Maximal entanglement: experimental tests at HERA and EIC

Probing the Onset of Maximal Entanglement inside the Proton in Diffractive Deep Inelastic Scattering

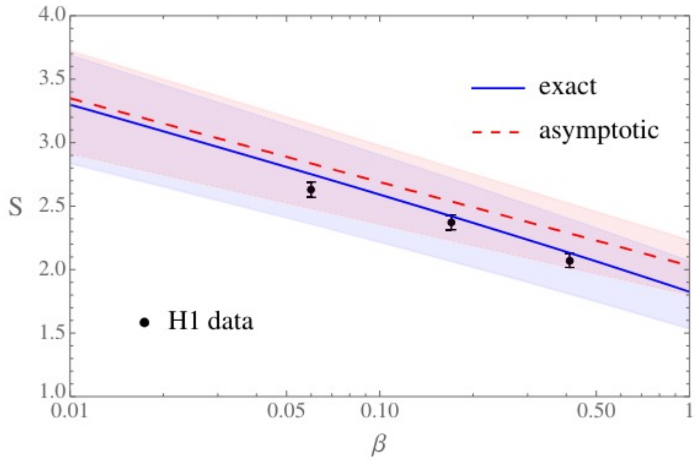
Martin Hentschinski, Dmitri E. Kharzeev, Krzysztof Kutak, and Zhoudunming Tu

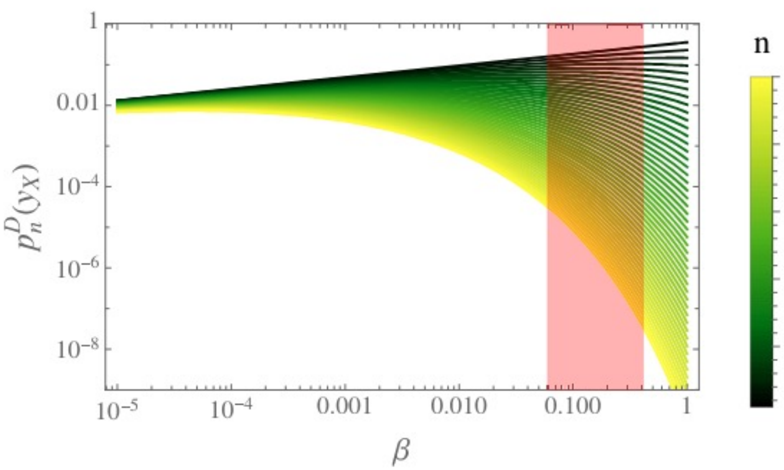
Phys. Rev. Lett. **131**, 241901 – Published 13 December 2023

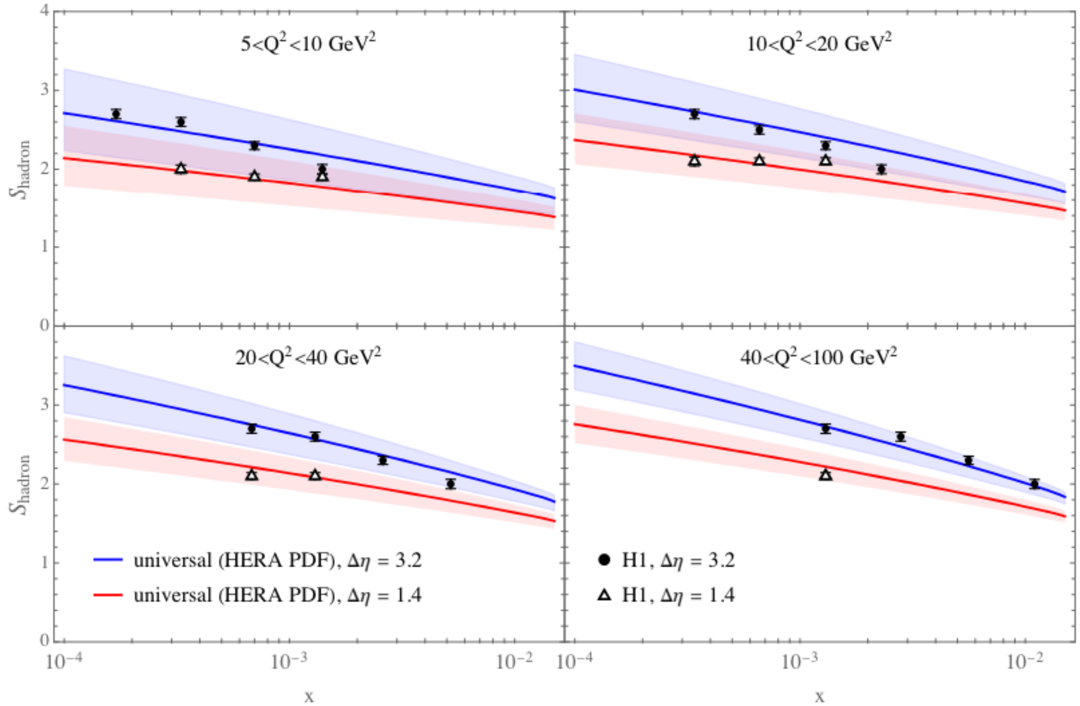
QCD evolution of entanglement entropy

Martin Hentschinski,[1,*] Dmitri E. Kharzeev,[2,3,†] Krzysztof Kutak,[4,‡] and Zhoudunming Tu[3,§]

arXiv:2408.01259, Rep.Prog.Phys.(2025)



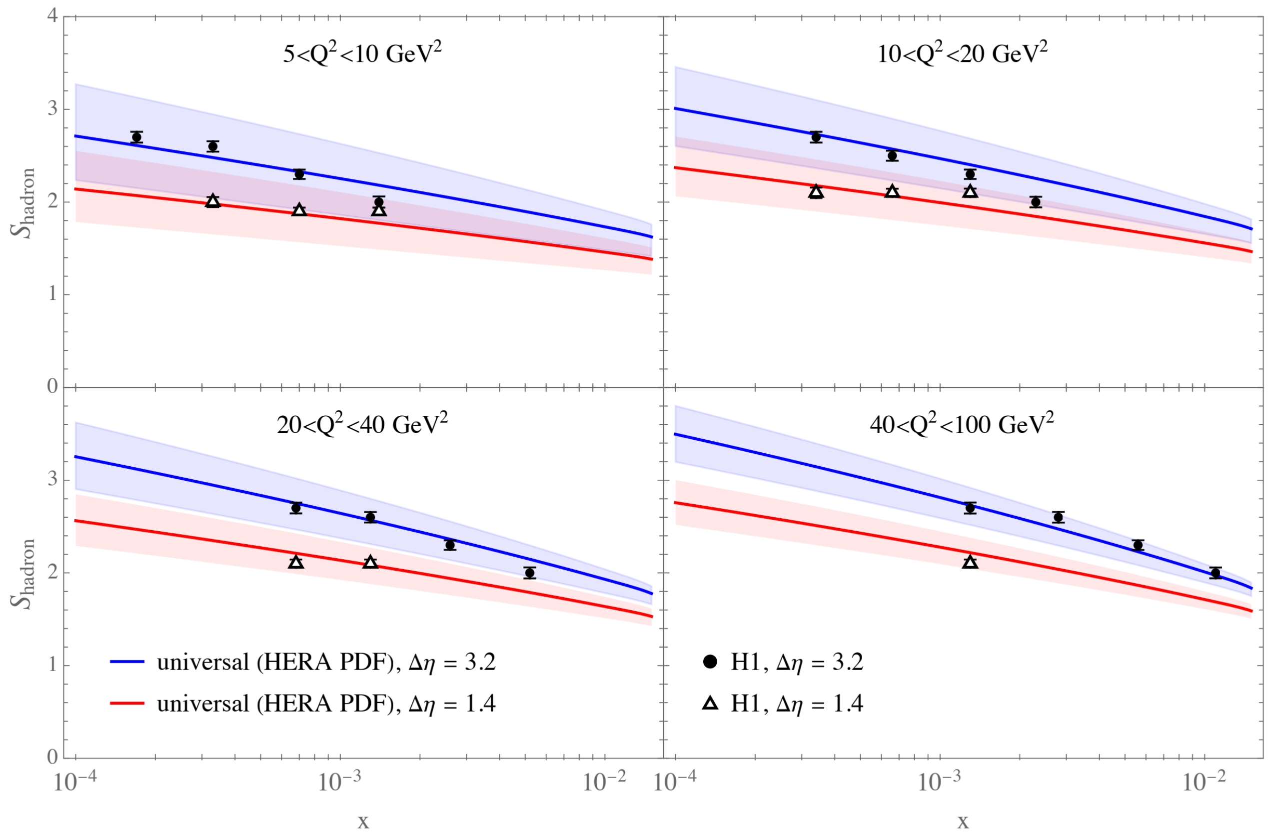




QCD evolution of entanglement entropy

Martin Hentschinski[1], Dmitri E Kharzeev[2,3], Krzysztof Kutak[4] and Zhoudunming Tu[3,*]

arXiv: 2408.01259; Reports on Progress in Physics, 2024

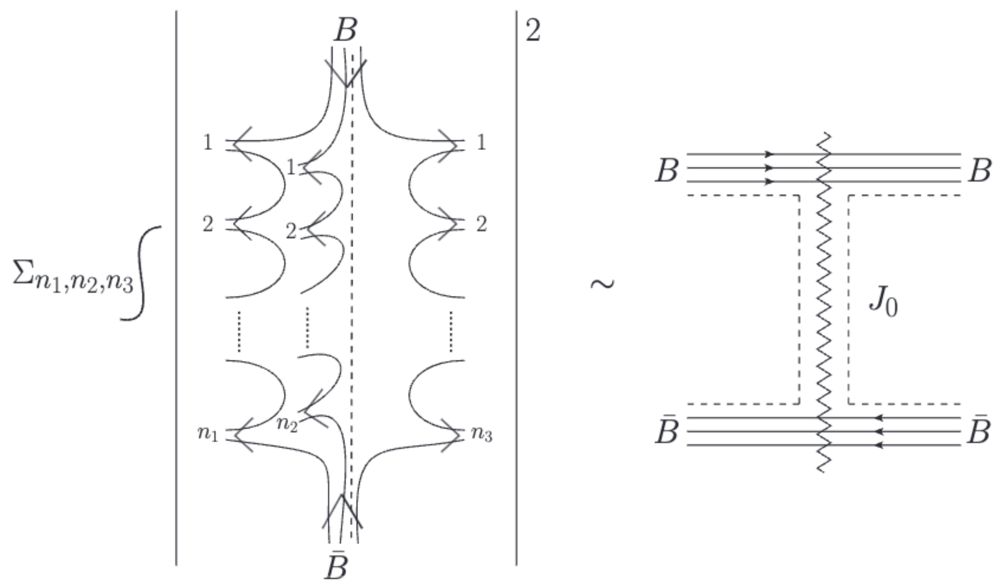


Maximal entanglement agrees with H1 measurements in different rapidity windows

Entanglement of quarks inside the proton, baryon number transport, and the Feynman-Wilson gas

D. Frenklakh, DK, G. Rossi, G. Veneziano, arXiv:2405.04569

JHEP07(2024)262



Feynman-Wilson gas

(1+1)d gas	High energy interactions
Volume	Total rapidity
Particle’s coordinate	Rapidity of the produced hadron
Short-range particle correlations	Short-range rapidity correlations
Partition function	Generating function for cross sections
Fugacity	Parameter of the generating function

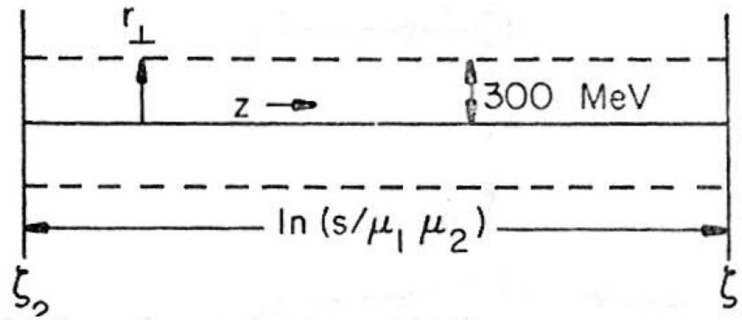
Feynman-Wilson gas

Some Experiments on Multiple Production*

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University,

Ithaca, New York 14850



scaling laws.[10] The best way to introduce these scaling laws is, I think, to use an analogy invented by Feynman.[11] This analogy links multiparticle production cross sections to the multiparticle distribution functions of a classical gas, with the total cross section becoming the partition function of a gas. This analogy is very much on Feynman's mind when he discusses his parton model of high energy collisions, although it is not discussed in his papers.

11. R. P. Feynman, private communication.

Short-range rapidity correlations – “multiperipheral” mechanism of high energy interactions

Theory of High-Energy Scattering and Multiple Production.

D. AMATI and A. STANGHELLINI

CERN - Geneva

S. FUBINI

Istituto di Fisica dell’Università - Torino
CERN - Geneva

(ricevuto il 4 Luglio 1962)

Summary. — In this paper we propose a theoretical model for high-energy interaction, the basic idea of which is that the high-energy processes are reducible to the low-energy ones, through a peripheral mechanism. The asymptotic properties of this model are studied by means

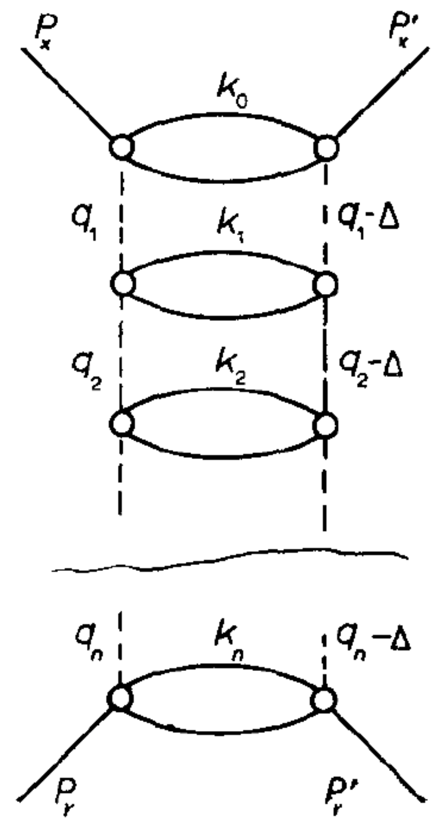


Fig. 3.

Feynman-Wilson gas

Generating functional of exclusive cross sections is given by:

$$\Sigma[z(x)] = \sum_n \int \prod_{j=1}^{n} (dx^j z(x^j)) \frac{1}{\sigma_t} \frac{d\sigma(a+b \to x^1, x^2 \ldots x^n)}{dx^1 dx^2 \ldots dx^n}$$

x_j are "coordinates of the final-state particles (rapidity, p_t, …)

$$\Sigma[z(x) = 1] = 1.$$

Exclusive differential cross sections can be obtained by taking partial functional derivatives at z=0.

Feynman-Wilson gas

m-particle inclusive cross sections

$$\rho_m(x^1, x^2 \ldots x^m) = \frac{1}{\sigma_t} \sum_X \frac{d\sigma(a + b \to x^1, x^2 \ldots x^m + X)}{dx^1 dx^2 \ldots dx^m}$$

are obtained by m-order partial derivatives at $z(x^j)=1$.

Now we can use the cluster decomposition to express these inclusive cross sections in terms of connected correlators defined through (at z(x)=1):

$$\log \Sigma[z(x)] = \sum_m \frac{1}{m!} \int \prod_{j=1}^{m} [dx^j (z(x^j) - 1)] c_m(x^1, x^2 \ldots x^m) \equiv p[z(x)] V$$

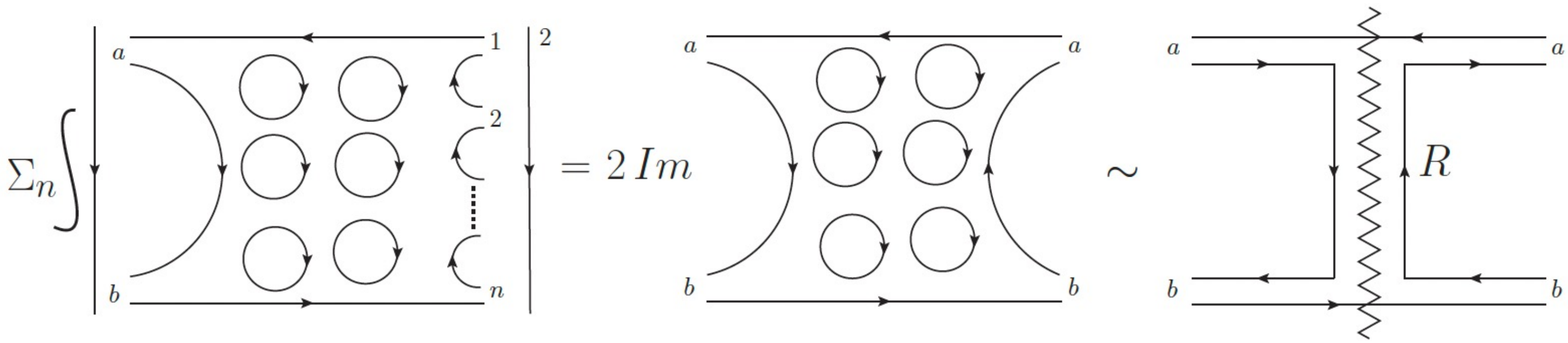
⇧ "grand canonical partition function"

⇧ "pressure"

$$\log \Sigma = \frac{PV}{k_B T}$$

Feynman-Wilson gas: planar bootstrap and Regge theory

At high energies, hadron cross sections are described by Regge theory:



The energy dependence of cross sections is determined by the Reggeon intercept

Total

$$\sigma_t^{pl} \sim s^{\alpha_{\mathbb{R}}-1}$$

Exclusive 2->2

$$\sigma_{excl}^{pl} \sim s^{2\alpha_{\mathbb{R}}-2}$$

Feynman-Wilson gas: planar bootstrap and Regge theory

Therefore, the pressure of the planar Feynman-Wilson gas yields the leading Reggeon intercept:

$$\sigma_t^{pl} \sim s^{\alpha_{\mathbb{R}}-1} \qquad \sigma_{excl}^{pl} \sim s^{2\alpha_{\mathbb{R}}-2}$$

$$\log \Sigma = \frac{PV}{k_B T}$$

$$\Sigma_{pl}(z) \to \frac{\sigma_{excl}^{pl}}{\sigma_t^{pl}} \sim s^{\alpha_{\mathbb{R}}-1} \Rightarrow -p(0) = 1 - \alpha_{\mathbb{R}}(0)$$

Feynman-Wilson gas: planar bootstrap and Regge theory

Use the Feynman-Wilson gas analogy to compute the Reggeon intercept using the cluster decomposition!

$$\Sigma_{pl}(z) \equiv \exp\left(Y p(z)\right) = \exp\left(Y \sum_{m\geq 1} c_m \frac{(z-1)^m}{m!}\right) \; ;$$

$$p(1) = 0 \; , \; p'(1)Y = c_1 Y = \langle n \rangle \; , \; p''(1)Y = c_2 Y = \langle n(n-1)\rangle - \langle n \rangle^2 \; ,$$

$$p'''(1)Y = c_3 Y = \langle n(n-1)(n-2)\rangle - \langle n \rangle^3 - 3c_1c_2Y^2 \; ,$$

⬇

$$-p(0) = 1 - \alpha_{\mathbb{R}}(0) = \sum_m c_m \frac{(-1)^{m+1}}{m!} = \frac{\langle n \rangle}{Y} - \frac{c_2}{2} + \dots$$

Reggeon intercept ⟷ Multi-hadron correlations

Feynman-Wilson gas: planar bootstrap and Regge theory

For vanishing correlations (Poisson distributions), we get

$$-p(0) = 1 - \alpha_{\mathbb{R}}(0) = \sum_m c_m \frac{(-1)^{m+1}}{m!} = \frac{\langle n \rangle}{Y} - \frac{c_2}{2} + \dots$$

⬇

$$\alpha_{\mathbb{R}}(0) = 1 - \frac{\langle n \rangle}{Y}$$

This result agrees with

Multiperipheral Bootstrap Model*

G. F. Chew and A. Pignotti

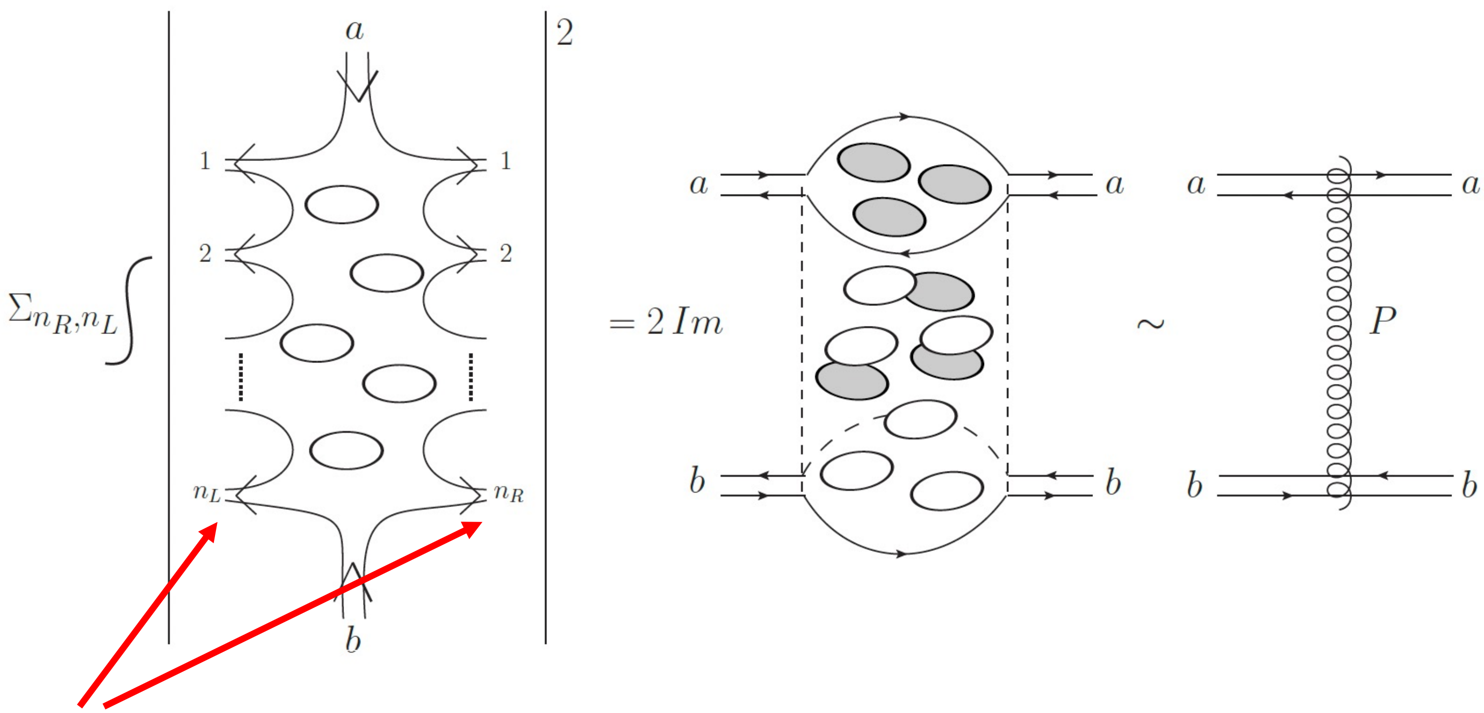
Lawrence Radiation Laboratory, University of California, Berkeley, California 94704

(Received 3 July 1968)

What about the Pomeron?

Here, we have to reconcile the Feynman-Wilson gas with the topological expansion (TE) of QCD.

In TE, the Pomeron has topology of a cylinder:



Two copies of Feynman-Wilson gas!

Cylinder topology of the perturbative BFKL Pomeron

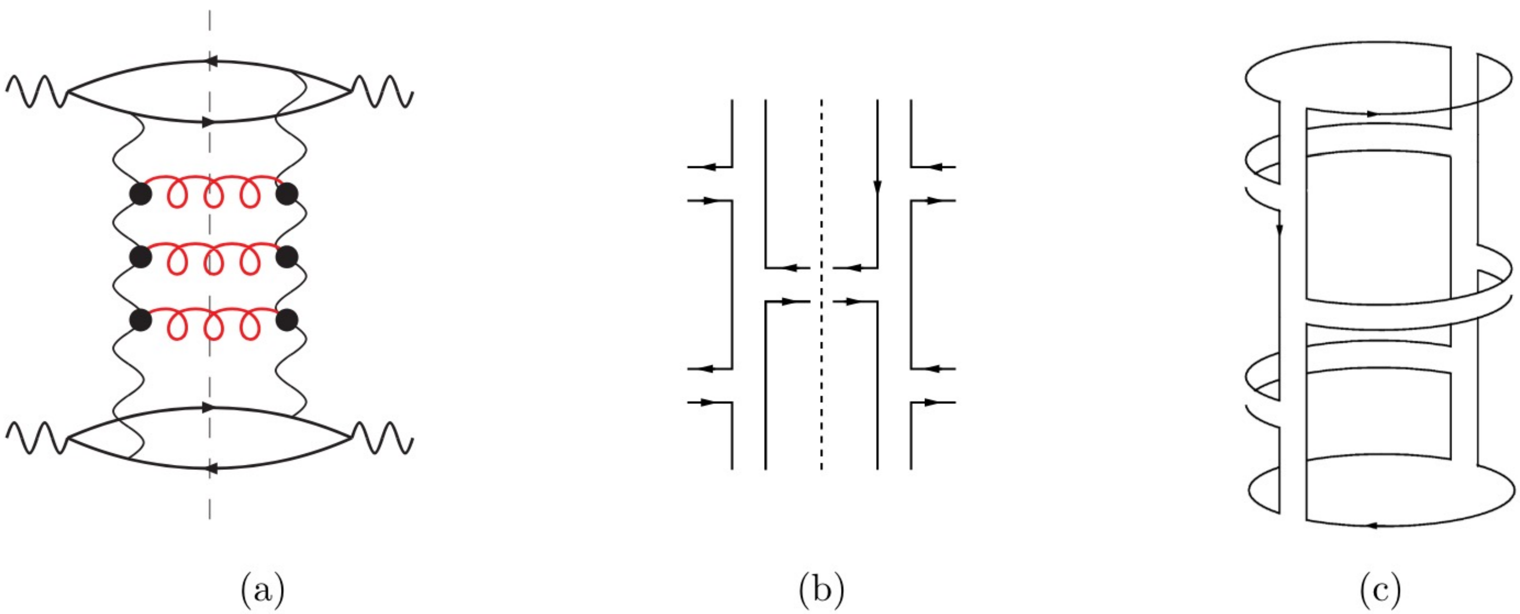


Figure 8: Multi-gluon emission within the MRK. (a) The cut Feynman diagram (b) Combination of relevant color diagrams of the production vertex. (c) The combination of (b) on the cylinder.

From: J.Bartels and M.Hentschinski, JHEP 08 (2009) 103

The Pomeron

The partition function of two-species Feynman-Wilson gas:

$$\Sigma_{cyl}(z_R, z_L) = \frac{1}{\sigma_t^{cyl}} \sum_{n_R+n_L\geq 2} z_R^{n_R} z_L^{n_L} \sigma^{cyl}(n_R, n_L) \Rightarrow \Sigma_{cyl}(1,1) = 1$$

The cluster expansion

$$\Sigma_{cyl}(z_R, z_L) \equiv \exp\left(Y p(z_R, z_L)\right) = \exp\left(Y \sum_{m_R+m_L\geq 1} c(m_R, m_L) \frac{(z_R-1)^{m_R}(z_L-1)^{m_L}}{m_R! m_L!}\right)$$

$$c(1,0)Y = \langle n_R \rangle \ , \ c(0,1)Y = \langle n_L \rangle \ , \ c(1,1)Y = \langle n_R n_L \rangle - \langle n_R \rangle \langle n_L \rangle \ , \ \ldots$$

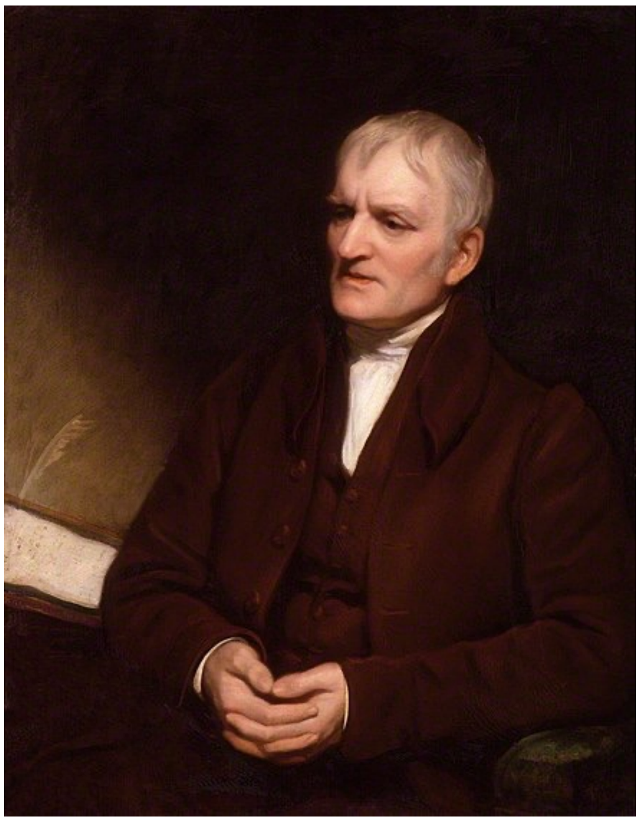
yields the Pomeron intercept:

$$\alpha_{\mathbb{P}} = 1 + \sum_{m_R, m_L \geq 1} \frac{c(m_R, m_L)}{m_R! m_L!}(-1)^{m_R+m_L} = 1 + \frac{\langle n_R n_L \rangle - \langle n_R \rangle \langle n_L \rangle}{Y} + \ldots$$

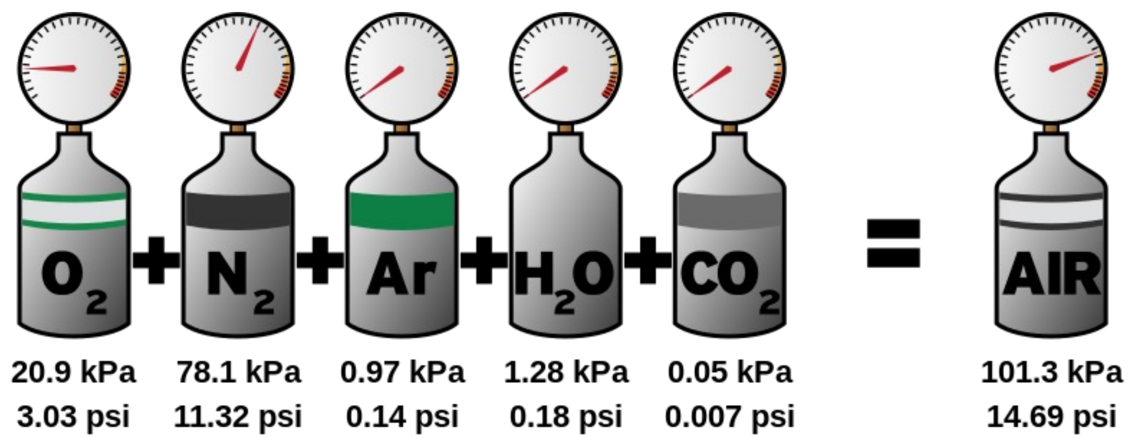
If correlations are neglected, the intercept is 1

The Pomeron

The absence of correlations (leading to intercept=1) is the analog of Dalton's law in Feynman-Wilson gas.



John Dalton 1766-1844



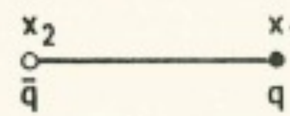
Also: Daltonism – color blindness

How to construct a gauge-invariant wave function of a baryon?

G. Rossi, G. Veneziano 1977

Table IIa

Simplest mesons and baryons : colour structure and string picture

HADRON	GAUGE INVARIANT OPERATOR	STRING PICTURE
$M_2 = q\bar{q}$ meson	$\bar{q}^{j_2}(x_2)\left[P\exp\left(ig\int_{x_1}^{x_2} A_\mu\, dx^\mu\right)\right]_{j_2}^{j_1} q_{j_1}(x_1)$	
M_o = quarkless meson	$Tr\left[P\exp\left(ig\oint A_\mu\, dx^\mu\right)\right]$	
$B_3 = qqq$ baryon	$\epsilon^{j_1 j_2 j_3}\left[P\exp\left(ig\int_{x_1}^{x} A_\mu\, dx^\mu\right) q(x_1)\right]_{j_1}$ $\left[P\exp\left(ig\int_{x_2}^{x} A_\mu\, dx^\mu\right) q(x_2)\right]_{j_2}$ $\left[P\exp\left(ig\int_{x_3}^{x} A_\mu\, dx^\mu\right) q(x_3)\right]_{j_3}$	

Baryon-number – flavor separation in high energy collisions

Physics Letters B 378 (1996) 238–246

Can gluons trace baryon number?

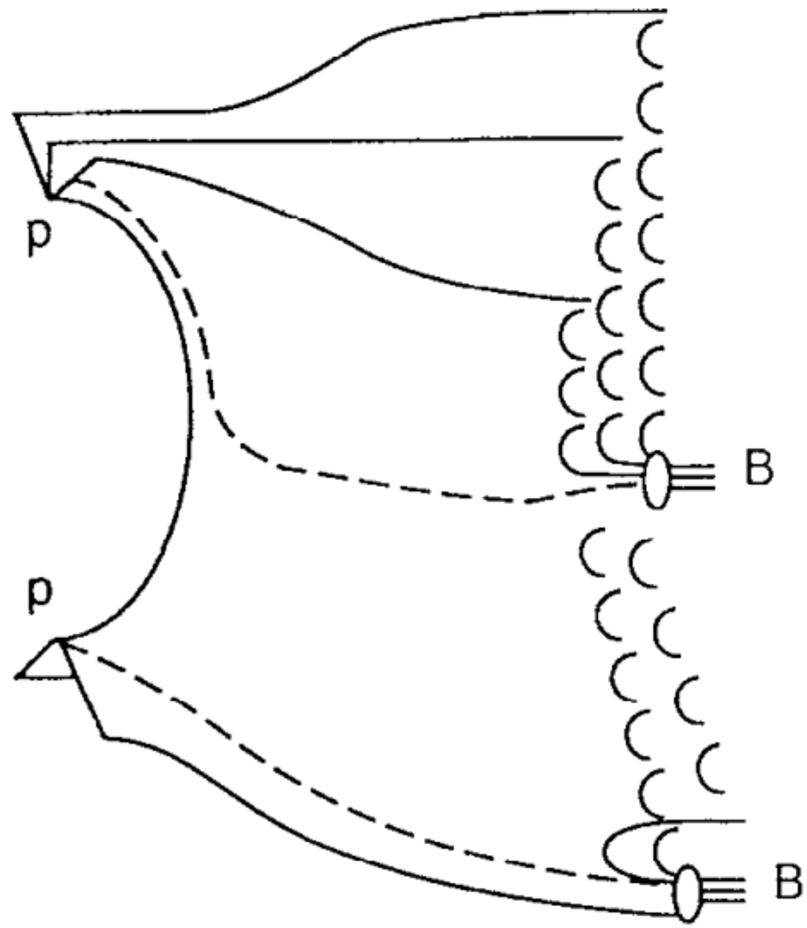
D. Kharzeev

Theory Division, CERN, CH-1211 Geneva, Switzerland
and Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany

Received 15 March 1996
Editor: R. Gatto

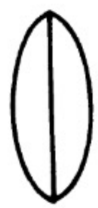
$$E_B \frac{d^3\sigma^{(1)}}{d^3p_B} = 8\pi G_p^M(0) G_p^P(0)\ f_B^{MP}(m_t^2)\ \left(\frac{\sqrt{s}\ m_t}{s_0}\right)^{\alpha_0^J(0)+\alpha_P(0)-2}$$

$$\times \left(exp[y^*(\alpha_P(0) - \alpha_0^J(0))] + exp[-y^*(\alpha_P(0) - \alpha_0^J(0))] \right).$$

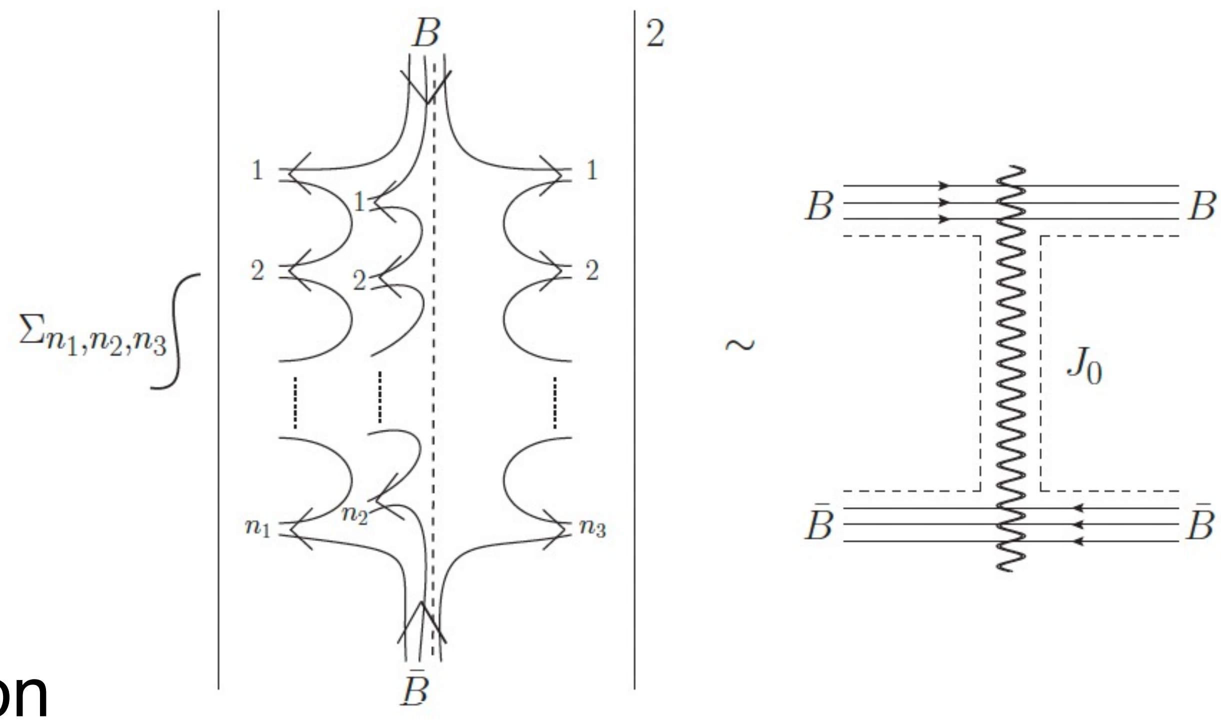


Feynman-Wilson gas with baryons

Consider baryon-antibaryon annihilation through the junction:



M_0^J



The cluster expansion

$$\Sigma_{ann}(z_1, z_2, z_3) \equiv e^{Y p(z_1,z_2,z_3)} =$$

$$= \exp\left(Y \sum_{m_1+m_2+m_3 \geq 1} c(m_1, m_2, m_3) \frac{(z_1-1)^{m_1}(z_2-1)^{m_2}(z_3-1)^{m_3}}{m_1! m_2! m_3!} \right)$$

Feynman-Wilson gas with baryons

The corresponding intercepts:

$$\alpha_{\mathbb{J}_4} = (2\alpha_{\mathbb{B}} - 1) + 2(1 - \alpha_{\mathbb{P}}) + (1 - \alpha_{\mathbb{R}}) - C_3 \sim -0.5 - 2C_{RL} - C_3$$
$$\alpha_{\mathbb{J}_2} = (2\alpha_{\mathbb{B}} - 1) + 3(1 - \alpha_{\mathbb{P}}) + 2(1 - \alpha_{\mathbb{R}}) - C_3 \sim 0 - 3C_{RL} - C_3$$
$$\alpha_{\mathbb{J}_0} = (2\alpha_{\mathbb{B}} - 1) + 3(1 - \alpha_{\mathbb{P}}) + 3(1 - \alpha_{\mathbb{R}}) - C_3 \sim 0.5 - 3C_{RL} - C_3$$

three-species correlations should be weaker and may be neglected;

C_{RL} is given by the Pomeron intercept.

All of the intercepts are thus fixed.

The junction-anti-junction intercept

$$\alpha_{\mathbb{P}} = 1 + C_{RL}$$

$$\alpha_{\mathbb{J}_0} = (2\alpha_{\mathbb{B}} - 1) + 3(1 - \alpha_{\mathbb{P}}) + 3(1 - \alpha_{\mathbb{R}}) - C_3 \sim 0.5 - 3C_{RL} - C_3$$

$$\Downarrow$$

$$\alpha_{\mathbb{J}_0} \simeq 0.26$$

$$\Downarrow$$

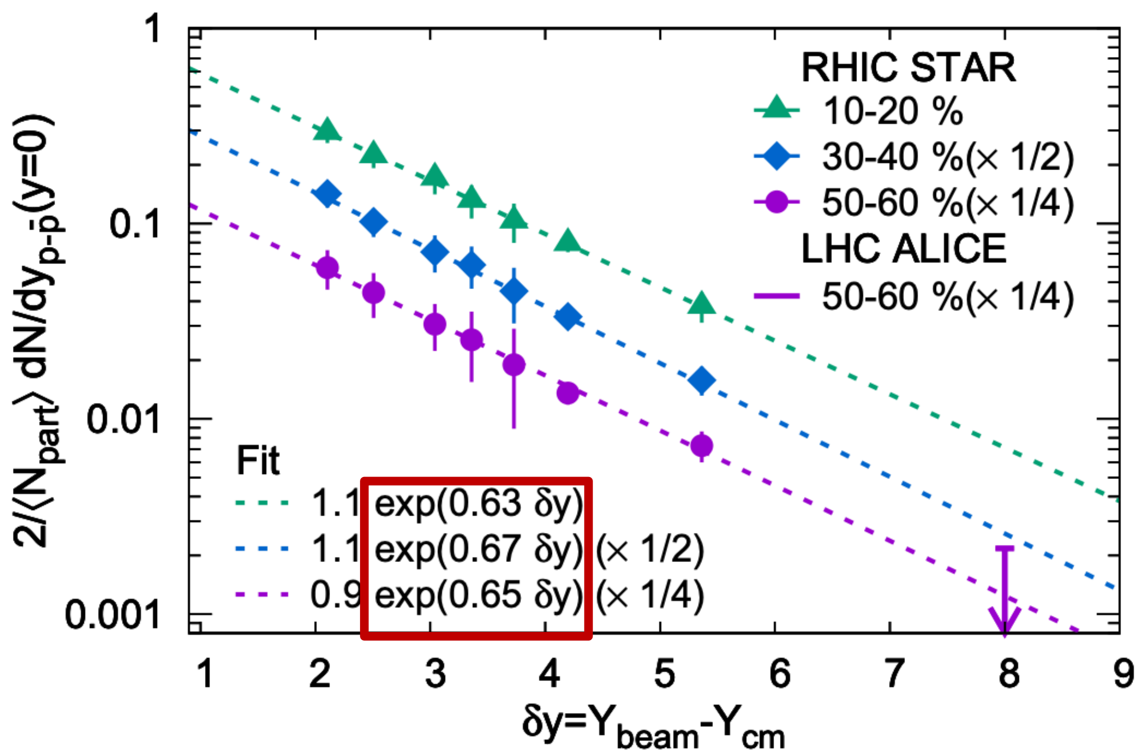
$$\text{beam rapidity dependence } e^{(\alpha_{\mathbb{J}_0} + \alpha_{\mathbb{P}} - 2)Y/2} = e^{-0.66\ Y/2}$$

D. Frenklakh, DK, G. Rossi, G. Veneziano, arXiv:2405.04569

Feynman-Wilson gas
+ topological expansion of QCD

beam rapidity dependence $e^{(\alpha_{\mathbb{J}_0}+\alpha_{\mathbb{P}}-2)Y/2} = e^{-0.66\ Y/2}$

D. Frenklakh, DK, G. Rossi, G. Veneziano, arXiv:2405.04569



Can the proton be “thermal”?

DK, K. Rajagopal, arXiv:2605.19058

At first glance, the proton is “obviously” a pure state with zero entropy, an eigenstate of the QCD Hamiltonian.

But: in QFT, one needs a UV cutoff – scale anomaly.
The modes above UV cutoff are unobservable at low resolution, and have to be traced over:

$$\mathcal{H} = \mathcal{H}_{\mathrm{IR}} \otimes \mathcal{H}_{\mathrm{UV}} \qquad \rho_{\mathrm{IR}} = \mathrm{Tr}_{\mathrm{UV}} \, |p\rangle\langle p|$$

The von Neumann entropy of the observable sector is non-zero

$$S_{\mathrm{IR}} = -\mathrm{Tr}\left(\rho_{\mathrm{IR}} \ln \rho_{\mathrm{IR}}\right)$$

and happens to be close to the thermal entropy of QGP droplet of proton size at hadronization transition.

Can the proton be "thermal"?

DK, K. Rajagopal, arXiv:2605.19058

The density matrix can be represented in terms of the modular Hamiltonian K

$$\rho_{\rm IR} = e^{-K}$$

For chaotic theories, and high-dimension of Hilbert space,

$$\rho_{\rm IR} \simeq \frac{1}{Z} \exp\left(-\frac{H_{\rm eff}}{T_{\rm eff}}\right)$$

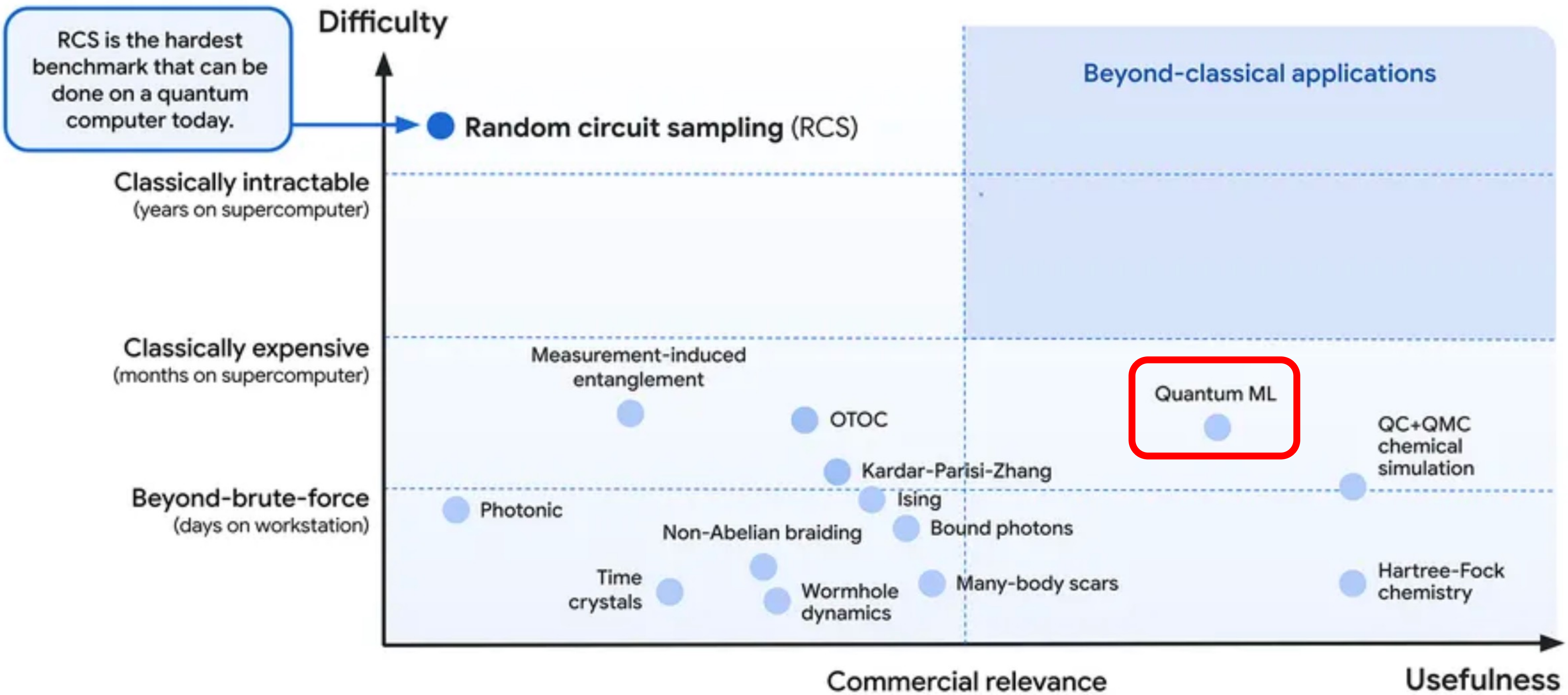
The proton mass can then be used to introduce an effective temperature:

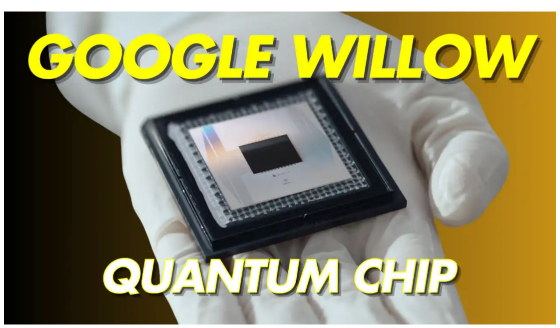
$$E_p = {\rm Tr}\, \rho_p H = -\frac{\partial}{\partial \beta} \ln Z(\beta).$$

$T_{eff} = 1/\beta \sim 160$ MeV

Argument based on discussion with Mark Srednicki

Quantum thermalization and AI/ML





Estimated time on Willow vs classical supercomputer	5 minutes vs. 10^{25} years

An obstacle to quantum deep neural networks:

PRX QUANTUM **2,** 040316 (2021)

Entanglement-Induced Barren Plateaus

Carlos Ortiz Marrero,[1,*] Mária Kieferová,[2,†] and Nathan Wiebe[3,4,‡]

[1] *Data Sciences and Analytics Group, Pacific Northwest National Laboratory, Richland, Washington 99354, USA*
[2] *Centre for Quantum Computation and Communication Technology, Centre for Quantum Software and Information, University of Technology Sydney, New South Wales 2007, Australia*
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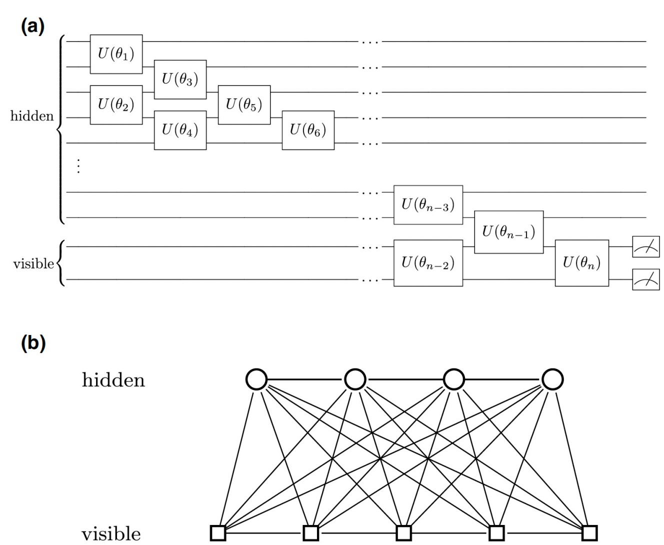
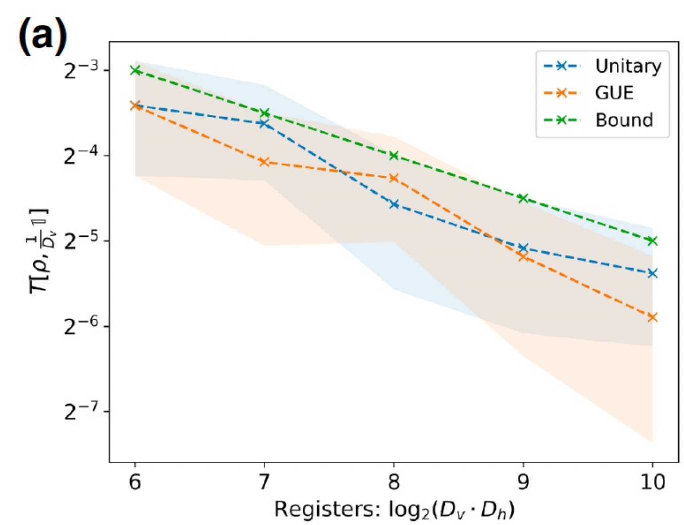
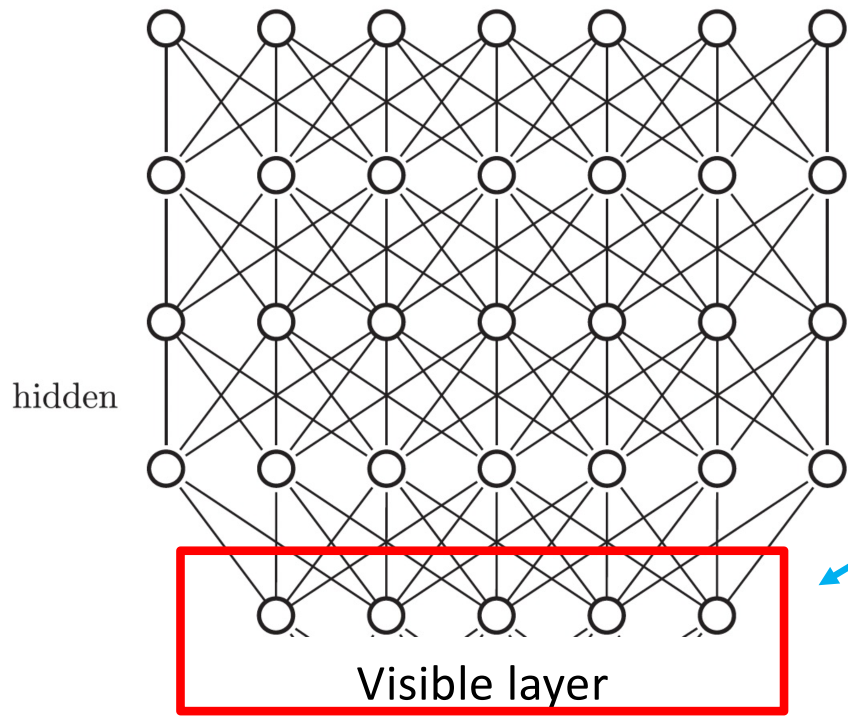
[4] *High Performance Computing Group, Pacific Northwest National Laboratory, Richland, Washington 99354, USA*





An obstacle to quantum deep neural networks:



Entangled pure state; entanglement entropy scales with the volume

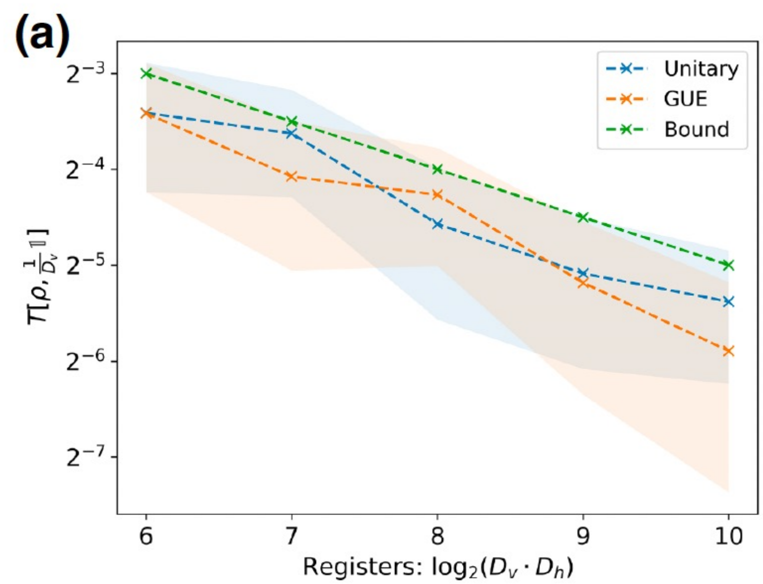
A small sub-system (the visible layer) is maximally entangled - **completely random output!**

Proposition 2. *Let $U \in \mathbb{C}^{D_v D_h \times D_v D_h}$ be drawn from a unitary 2-design and let $H = U^\dagger S U$ for some diagonal matrix $S \in \mathbb{C}^{D \times D}$, where $D = D_v D_h$. If either $\rho = U|0\rangle\langle 0|U^\dagger$ (unitary network) or $\rho = e^{-H}/Tr(e^{-H})$ (Boltzmann machine), then for any bounded operator $O_{obj} \in \mathbb{C}^{D_v \times D_v}$ acting on the visible subspace, we have that*

$$\left|Tr\left[(O_{obj} \otimes I_h)(\rho - I/D)\right]\right| \in O\left(\|O_{obj}\|_\infty \sqrt{\frac{D_v}{D_h}}\right),$$

Entanglement-Induced Barren Plateaus

Carlos Ortiz Marrero,[1,*] Mária Kieferová,[2,†] and Nathan Wiebe[3,4,‡]



Summary:

- Deep connection emerges between entanglement and thermalization in statistical physics – fundamentally interesting, crucially important for technology (qubit decoherence, quantum AI/ML,…)

- Studies of real-time behavior of quantum field theories relevant for nuclear/high energy/condensed matter physics are key to understanding thermalization, and ways to control it