National Nuclear Physics Summer School 2026

Seattle, WA

June 29 - July 11, 2026

Quantum Entanglement in Nuclear and High Energy Physics

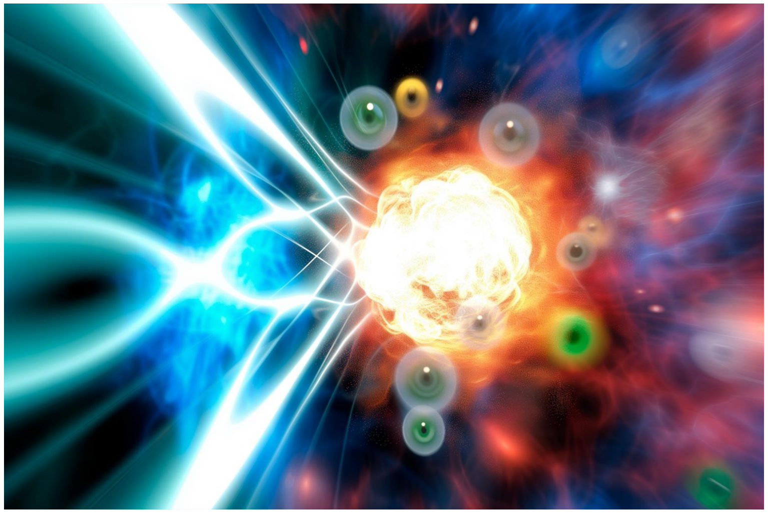


Image: SciTechDaily "Maximal entanglement inside the proton"

Dmitri Kharzeev

Center for Nuclear Theory
Stony Brook University

C2QA Co-design Center for Quantum Advantage

U.S. Department of Energy Office of Science

Brookhaven National Laboratory

Outline:

Lecture 1

1. Classical statistical physics from entanglement

2. Maximal entanglement from geometry of Hilbert space

3. Quantum simulations of jet production, and approach to maximal entanglement

4. Parton model from high-energy decoherence

5. Measures of entanglement in jet fragmentation

6. (Some of) the things I did not cover: a guide

Lecture 2

Review: DK, quant-ph/2601.00405 (Acta Phys. Polon. B57, 4)

Questions:

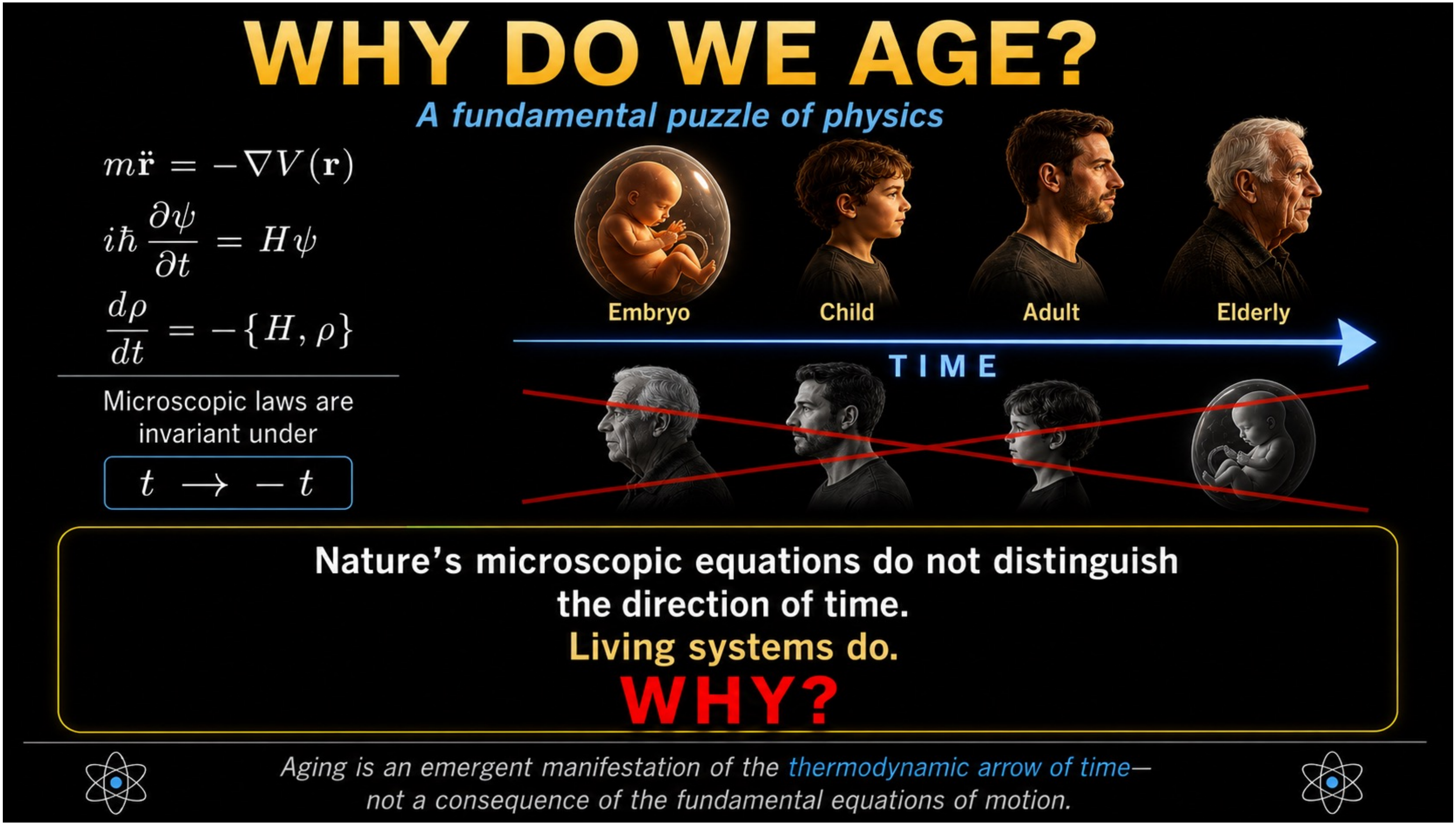
1. Why do we age?

2. Why is parton model probabilistic?

3. How is “fast thermalization” achieved in high-energy collisions?

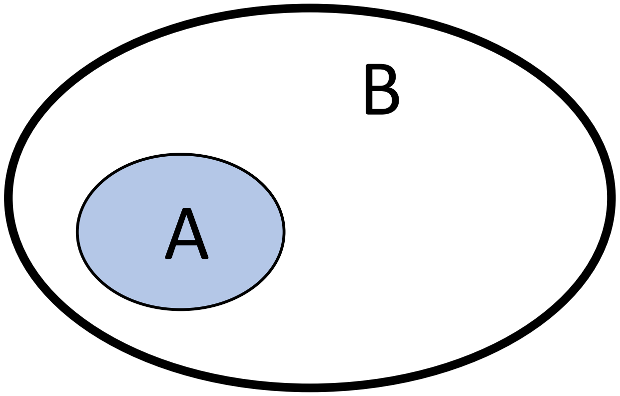
4. What is the role of entanglement inside hadrons at small x, and in the fragmentation of jets?

5. How to observe entanglement in hadron collisions?


WHY DO WE AGE?
A fundamental puzzle of physics
$m\ddot{\mathbf{r}} = -\nabla V(\mathbf{r})$
$i\hbar \frac{\partial \psi}{\partial t} = H\psi$
$\frac{d\rho}{dt} = -\{H, \rho\}$
Microscopic laws are invariant under
$t \longrightarrow -t$
Embryo
Child
Adult
Elderly
TIME
Nature's microscopic equations do not distinguish the direction of time.
Living systems do.
WHY?
Aging is an emergent manifestation of the thermodynamic arrow of time—
not a consequence of the fundamental equations of motion.

Two ideas:

1. Small subsystems of pure "typical" quantum states evolve towards maximally entangled states, if the underlying theory is chaotic (not integrable).



$$\rho_A = \mathrm{Tr}_B \rho$$

2. Maximal entanglement + Energy conservation =

Thermalization (not T-invariant!)

Statistics of classical states

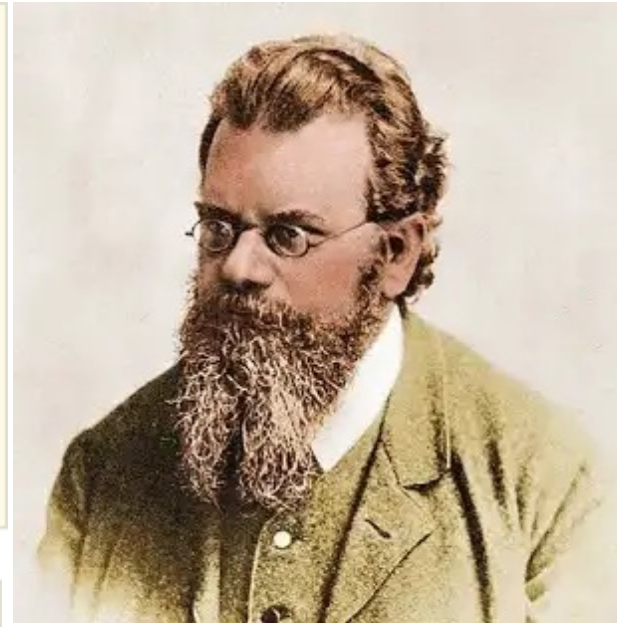
Über die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung, respective den Sätzen über das Wärmegleichgewicht.

Von dem c. M. **Ludwig Boltzmann** in Graz.

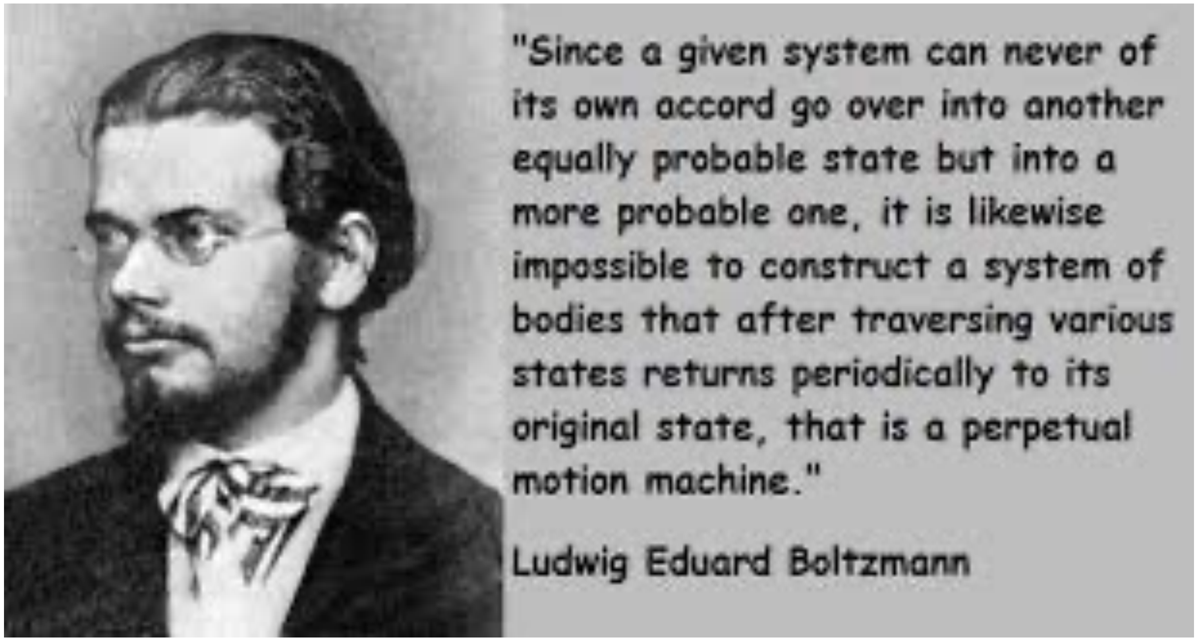
Eine Beziehung des zweiten Hauptsatzes zur Wahrscheinlichkeitsrechnung zeigte sich zuerst, als ich nachwies, dass ein analytischer Beweis desselben auf keiner anderen Grundlage

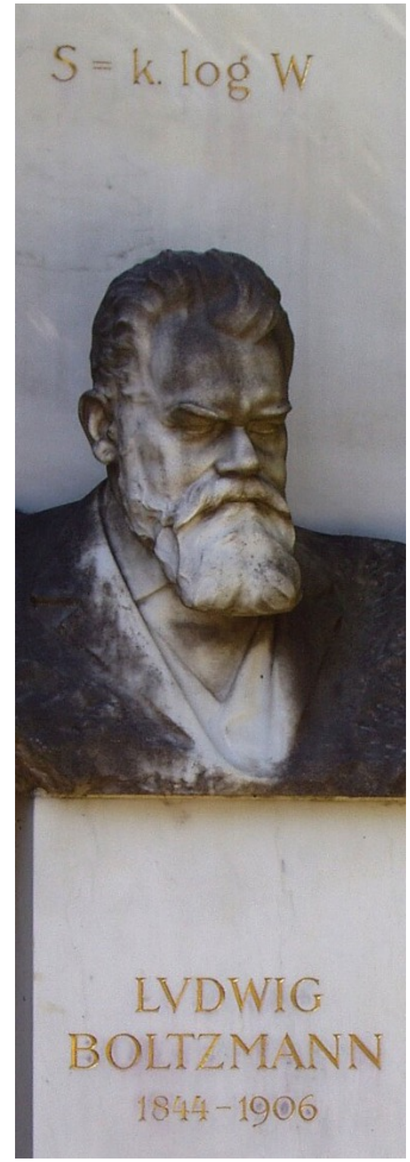
mit folgenden Worten: „Es ist klar, dass jede einzelne gleichförmige Zustandsvertheilung, welche bei einem bestimmten

Anfangszustande nach Verlauf einer bestimmten Zeit entsteht, ebenso unwahrscheinlich ist, wie eine einzelne noch so ungleichförmige Zustandsvertheilung, gerade so wie im Lottospiele jede einzelne Quinterne ebenso unwahrscheinlich ist, wie die Quinterne



“It is clear that in thermal equilibrium all possible states of the system … are equally probable”





In classical statistical mechanics,
the system is driven to the most probable state
with the largest entropy – equilibrium.

What if the system is quantum? Do quantum **sub**systems evolve to the state with the largest entanglement entropy (maximally entangled states)?

Is this the fundamental origin of thermalization?

Quantum entanglement

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

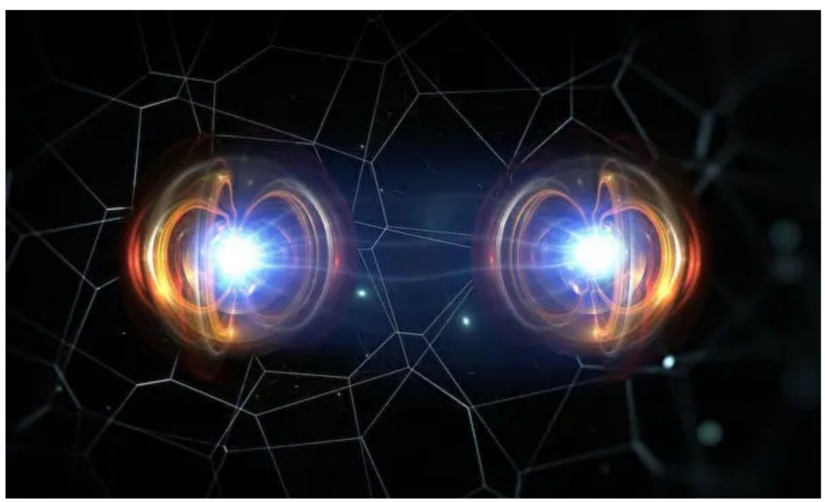
A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Consider two separated quantum particles that had previously interacted

Measurement of momentum of one particle



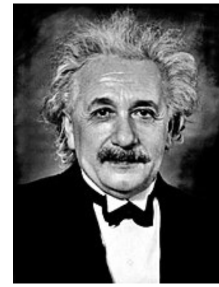
prevents the knowledge of another particle's coordinate!

``I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from ***spooky action at a distance****”*

A. Einstein, letter to

M. Born, 1947

Quantum entanglement



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

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OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

Describing entanglement: the density matrix

EPR state (2 qubits):

$$\frac{|0\rangle_A|0\rangle_B \pm |1\rangle_A|1\rangle_B}{\sqrt{2}}$$

The corresponding density matrix:

$$\rho_{\rm AB}\left(\frac{|00\rangle \pm |11\rangle}{\sqrt{2}}\right) = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}\frac{\langle 00| \pm \langle 11|}{\sqrt{2}} = \frac{|00\rangle\langle 00| \pm |00\rangle\langle 11| \pm |11\rangle\langle 00| + |11\rangle\langle 11|}{2}$$

If the state of B is unknown, A is described by the reduced density matrix:

$$\rho_A = \mathrm{tr}_B(\rho_{AB}) = \frac{|0\rangle\langle 0|\ \langle 0|0\rangle \pm |0\rangle\langle 1|\ \langle 0|1\rangle \pm |1\rangle\langle 0|\ \langle 1|0\rangle + |1\rangle\langle 1|\ \langle 1|1\rangle}{2}$$

$$= \frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\mathbf{I}}{2}.$$

Mixed state! $\rho_A \neq \rho_A^2$

Das Dämpfungsproblem in der Wellenmechanik.

Von **L. Landau** in Leningrad.

(Eingegangen am 27. Juli 1927.)

Es wird eine Formel für die wellenmechanische Behandlung der Dämpfung aufgestellt. Mit ihrer Hilfe werden einige diesbezügliche Fragen untersucht; auch die Kohärenzerscheinungen finden ihre Aufklärung. Ein Ausdruck für spontane Emission wird ermittelt, und die Intensitätsfrage der Spektrallinien auf diese Weise gelöst.

§ 1. Gekoppelte Systeme in der Wellenmechanik. In der Wellenmechanik kann ein System nicht eindeutig definiert werden; wir haben es immer mit einer Wahrscheinlichkeitsgesamtheit zu tun (statistische Auffassung)[1]. Ist das System mit einem anderen gekoppelt, so tritt in seinem Verhalten eine doppelte Unbestimmtheit auf.

Der Zustand des ersten Systems sei charakterisiert durch die Größen a_n in

$$\psi = \sum a_n \psi_n; \tag{1}$$

für das zweite gelte

$$\psi' = \sum b_r \psi'_r. \tag{2}$$

Die Schrödingersche Funktion für beide Systeme zusammen ist dann:

$$\Psi = \psi \psi' = \sum_n \sum_r a_n b_r \psi_n \psi'_r = \sum_n \sum_r c_{nr} \psi_n \psi'_r, \tag{3a}$$

wo

$$c_{nr} = a_n b_r. \tag{3b}$$

Tritt eine Kopplung auf, so wird c_{nr} Funktion der Zeit und kann nicht mehr der Gleichung (3b) gemäß zerlegt werden. Es können also a_n und b_r hier nicht mehr einzeln angewandt werden.

Quantifying entanglement: von Neumann entropy

$$\rho = \sum_n p_n |n\rangle\langle n|$$

Entanglement entropy:

$$S = -\mathrm{tr}\rho \ln \rho = -\mathrm{p_n} \ln \mathrm{p_n}$$

Pure states:

$$S = 0$$

e.g.

$$p_0 = 1,\ p_{n\neq 0} = 0$$

Mixed states:

$$S \neq 0$$

e.g. for EPR $\rho_A = \frac{\mathbf{I}}{2}$

$$p_0 = p_1 = \frac{1}{2} \rightarrow S = \ln 2$$

Maximally entangled states

Consider the entanglement entropy

$$S = -\mathrm{tr}\rho \ln \rho = -\sum_n p_n \ln p_n$$

for the case of N states with equal probabilities

$$p_n = 1/N$$

Then $$S = -N \frac{1}{N} \ln(1/N) = \ln N$$

This looks like the Boltzmann formula!

Von Neumann on entropy and ergodicity in quantum mechanics

Beweis des Ergodensatzes und des *H*-Theorems in der neuen Mechanik.

Von **J. v. Neumann** in Berlin.

(Eingegangen am 10. Mai 1929.)

systems that are correct most of the time. In particular, first, how the peculiar, seemingly irreversible behavior of entropy arises, and second, why the statistical properties of the (fictitious) micro-canonical ensemble can be attributed to the incompletely known (real) system[1]. And these questions shall be attacked with the means of quantum mechanics.

Eur. Phys. J. H **35**, 201–237 (2010)
DOI: 10.1140/epjh/e2010-00008-5

Proof of the ergodic theorem and the H-theorem in quantum mechanics*

Historical document

Beweis des Ergodensatzes und des *H*-Theorems in der neuen Mechanik.

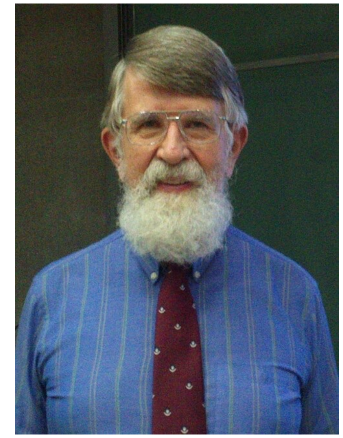
Von **J. v. Neumann** in Berlin.

(Eingegangen am 10. Mai 1929.)

To sum up, in quantum mechanics one can prove the ergodic theorem and the *H*-theorem in full rigor and without disorder assumptions; thus, the applicability of the statistical-mechanical methods to thermodynamics is guaranteed without relying on any further hypotheses[20]. Of course, this is compatible with the fact that also the time-dependent Schrödinger equation, on which quantum mechanics is grounded, has reversibility and recurrence properties just like the differential equations of classical mechanics [17], and therefore cannot alone explain irreversible phenomena[21].

Maximal entanglement and the Page theorem

For a random pure state in the Hilbert space of dimension mxn, the average entanglement entropy of a subsystem of dimension m<n is



$$\boxed{\langle S(\rho_A)\rangle = \ln m - \frac{m}{2n} + \mathcal{O}\left(\frac{1}{n^2}\right)}.$$

VOLUME 71 30 AUGUST 1993 NUMBER 9

Average Entropy of a Subsystem

Don N. Page*
CIAR Cosmology Program, Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2J1
(Received 7 May 1993)

If a quantum system of Hilbert space dimension mn is in a random pure state, the average entropy of a subsystem of dimension $m \leq n$ is conjectured to be $S_{m,n} = \sum_{k=n+1}^{mn} \frac{1}{k} - \frac{m-1}{2n}$ and is shown to be $\simeq \ln m - \frac{m}{2n}$ for $1 \ll m \leq n$. Thus there is less than one-half unit of information, on average, in the smaller subsystem of a total system in a random pure state.

Maximal entanglement and geometry of Hilbert space

Consider bipartite Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
with

$$\dim \mathcal{H}_A = m, \qquad \dim \mathcal{H}_B = n, \qquad m \leq n.$$

and a random pure state drawn from it.

The reduced density matrix of A is then given by a normalized Wishart matrix (X is a rectangular matrix whose entries are independent complex random numbers):

$$\rho_A = \frac{W}{\mathrm{Tr}\, W}, \qquad W = XX^\dagger$$

The eigenvalue distribution is given by

$$P(\{\lambda_i\}) \propto \delta\left(1 - \sum_{i=1}^{m} \lambda_i\right) \prod_{i=1}^{m} \lambda_i^{n-m} \prod_{i<j} (\lambda_i - \lambda_j)^2$$

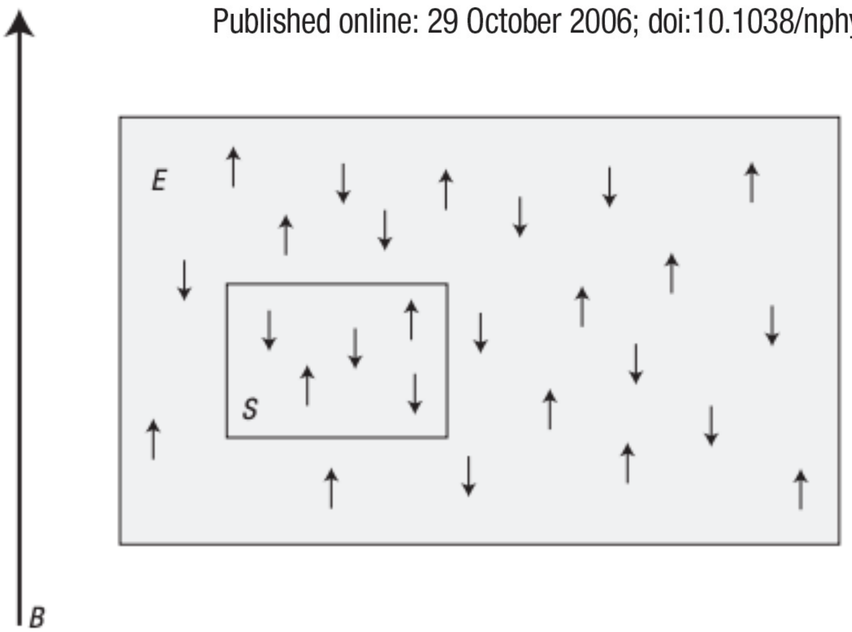
Maximal entanglement and thermalization

Entanglement and the foundations of statistical mechanics

SANDU POPESCU[1,2], ANTHONY J. SHORT[1]* AND ANDREAS WINTER[3]
[1] H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, UK
[2] Hewlett-Packard Laboratories, Stoke Gifford, Bristol BS12 6QZ, UK
[3] Department of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, UK
*e-mail: tony.short@bristol.ac.uk

nature physics
Published online: 29 October 2006; doi:10.1038/nphys444

Statistical mechanics is one of the most successful areas of physics. Yet, almost 150 years since its inception, its foundations and basic postulates are still the subject of debate. Here we suggest that the main postulate of statistical mechanics, the equal *a priori* probability postulate, should be abandoned as misleading and unnecessary. We argue that it should be replaced by a general canonical principle, whose physical content is fundamentally different from the postulate it replaces: it refers to individual states, rather than to ensemble or time averages. Furthermore, whereas the original postulate is an unprovable assumption, the principle we propose is mathematically proven. The key element in this proof is the quantum entanglement between the system and its environment. Our approach separates the issue of



In future work, we hope to go beyond the kinematic viewpoint presented here to address the dynamics of thermalization. In

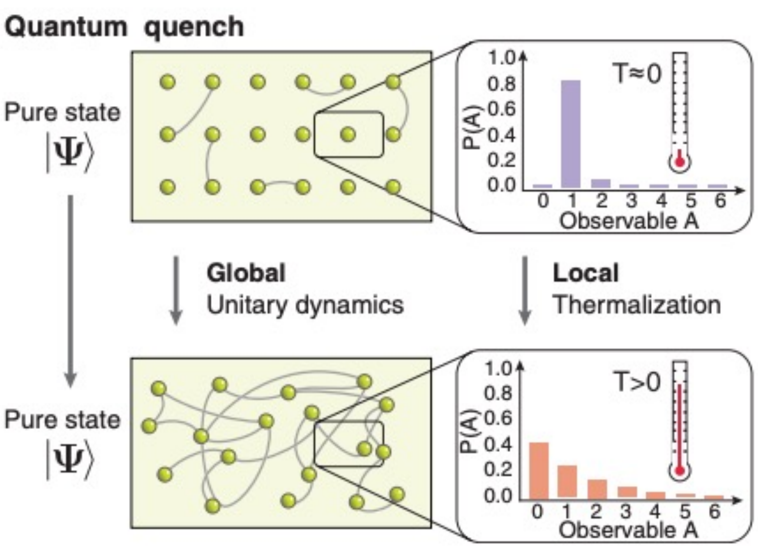
Maximal entanglement and thermalization

STATISTICAL PHYSICS

Quantum thermalization through entanglement in an isolated many-body system

Adam M. Kaufman, M. Eric Tai, Alexander Lukin, Matthew Rispoli, Robert Schittko, Philipp M. Preiss, Markus Greiner*

Statistical mechanics relies on the maximization of entropy in a system at thermal equilibrium. However, an isolated quantum many-body system initialized in a pure state remains pure during Schrödinger evolution, and in this sense it has static, zero entropy. We experimentally studied the emergence of statistical mechanics in a quantum state and observed the fundamental role of quantum entanglement in facilitating this emergence.



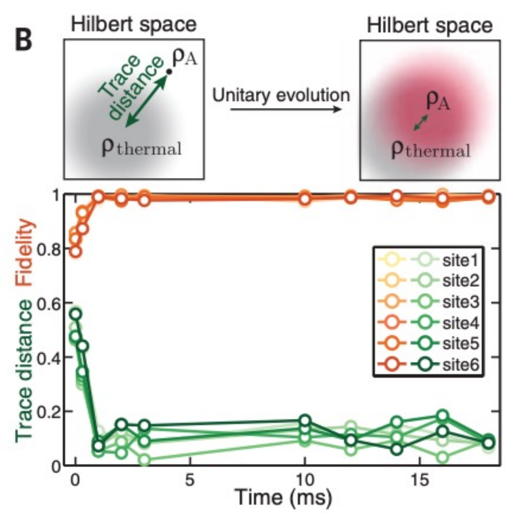
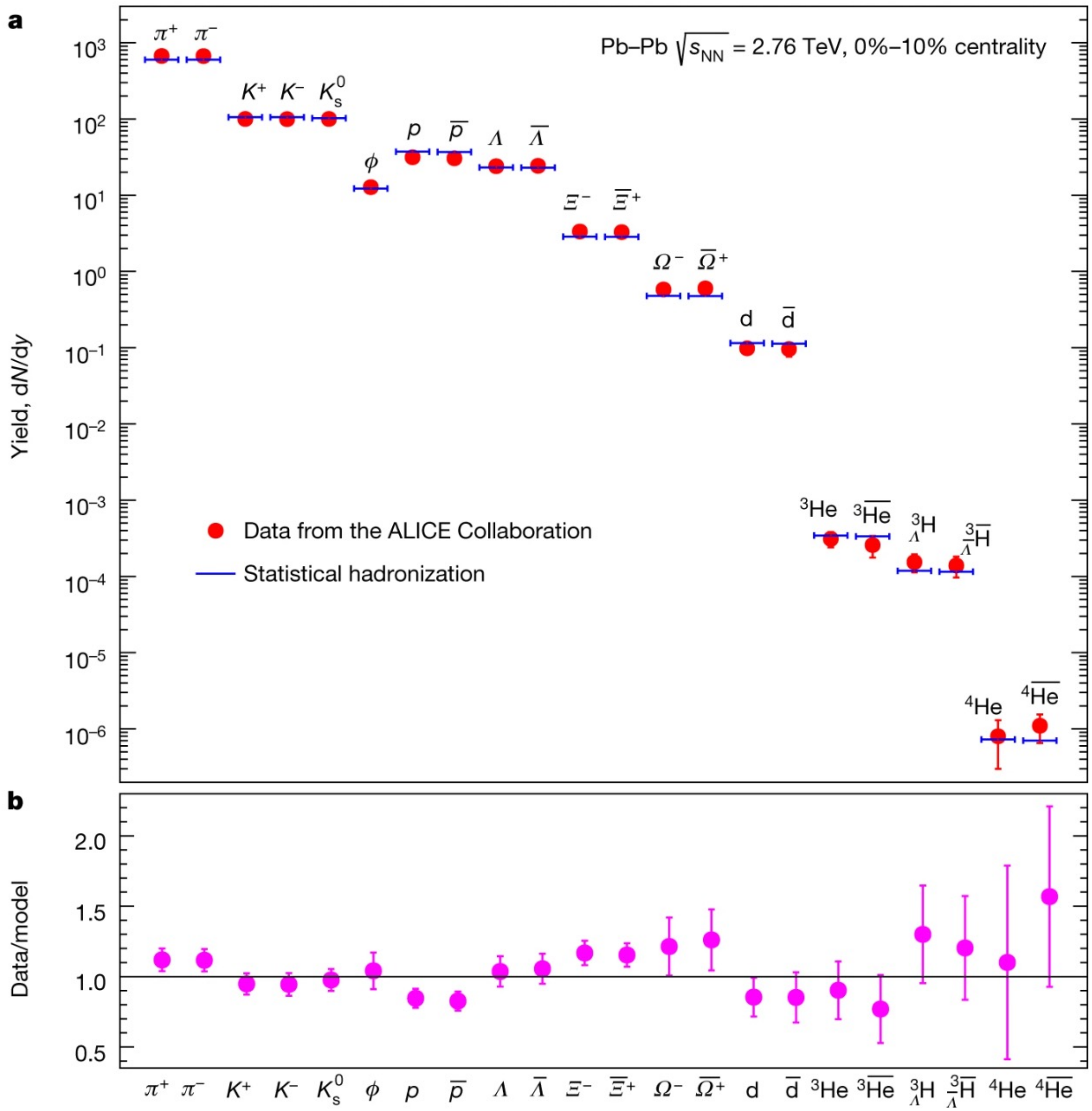


Fig. 5. Observation of local thermalization.

In nuclear and high energy physics, there is a long-standing puzzle of "early thermalization"

There is an ample evidence from experiments at RHIC, LHC and elsewhere that high energy heavy ion (and even pp and e^+e^- collisions) lead to some kind of fast thermalization:

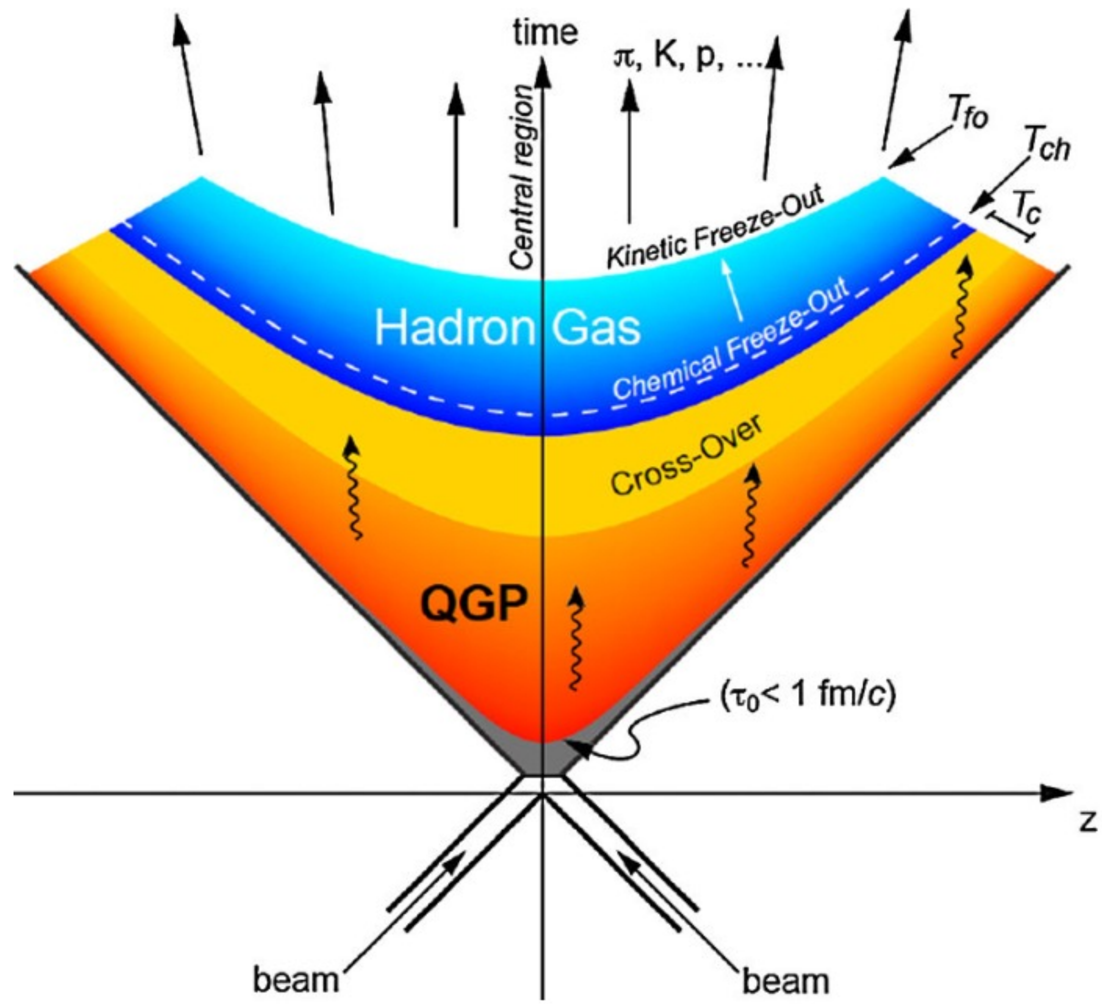
- Hadron abundances look thermal
- Hydrodynamics describes remarkably well the momentum spectra and azimuthal correlations of produced hadrons, assuming that the initial conditions are provided at a very early time $\tau \sim 0.5$ fm



A.Andronic, P.Braun-Munzinger, K.Redlich, J. Stachel, Nature 561 (2018) 321

What is the mechanism of this thermalization?

How can it happen so fast in a rapidly expanding system?



Is the mechanism of rapid thermalization linked to entanglement?

What is the real-time dynamics of thermalization?
Is it accompanied by entanglement entropy (EE) production?
Can it be slowed down or accelerated?

To answer these questions, we need a **real-time quantum simulation** in a field theory that is simple enough to solve numerically and still describes physical systems of interest.

Schwinger model is similar to QCD in a number of ways:
confinement, chiral condensate, anomaly, …

↓

Perform a real-time quantum simulation of e^+e^- annihilation in massive Schwinger model, with the goal of understanding the possible **link between entanglement and thermalization**

Center for Nuclear Theory

Stony Brook University

The team:

C2QA Co-design Center for Quantum Advantage

David Frenklakh (SBU->BNL)

Adrien Florio (SBU->Bielefeld)

Kazuki Ikeda (SBU->UMass)

Shuzhe Shi (SBU->Tsinghua)

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Andrea Palermo (SBU->Tours)

U.S. Department of ENERGY Office of Science

Science Highlights

Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

Adrien Florio[1,*], David Frenklakh[2,†], Kazuki Ikeda[2,3,‡], Dmitri Kharzeev[1,2,3,§], Vladimir Korepin[4,‖], Shuzhe Shi[5,2,¶], and Kwangmin Yu[6,**]

Phys. Rev. Lett. **131**, 021902 – **Published 13 July, 2023**

Quantum real-time evolution of entanglement and hadronization in jet production: Lessons from the massive Schwinger model

Adrien Florio[1,2,*], David Frenklakh[3,†], Kazuki Ikeda[2,3,‡], Dmitri Kharzeev[1,2,3,§], Vladimir Korepin[2,4,‖], Shuzhe Shi[5,3,¶], and Kwangmin Yu[6,**]

Phys. Rev. D **110**, 094029 – **Published 15 November, 2024**

Thermalization from quantum entanglement: Jet simulations in the massive Schwinger model

Adrien Florio[1,2,*], David Frenklakh[2,†], Sebastian Grieninger[3,‡], Dmitri E. Kharzeev[3,4,§], Andrea Palermo[3,‖], and Shuzhe Shi[5,6,¶]

Phys. Rev. D **112**, 094502 – **Published 7 November, 2025**

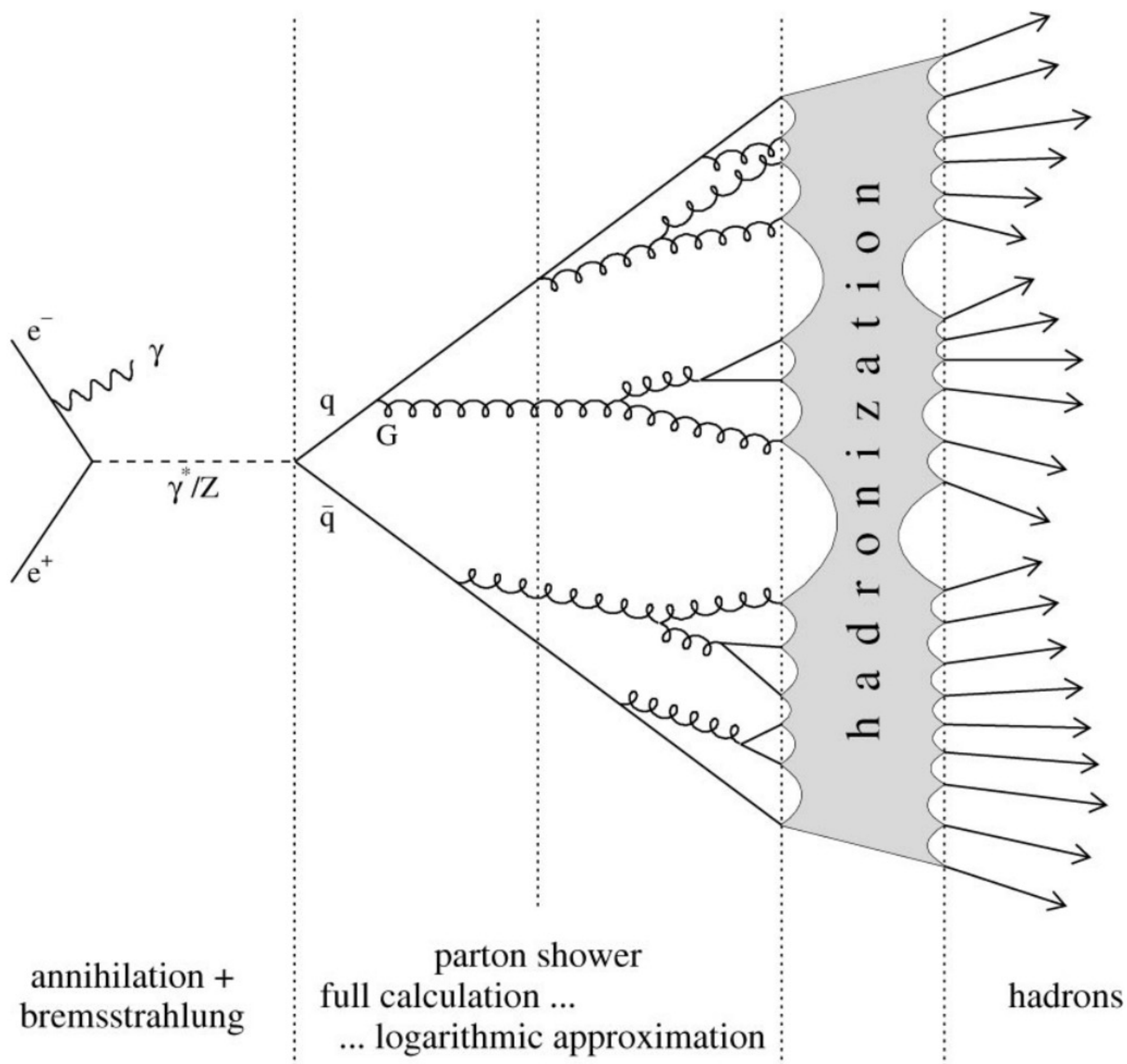
2510.23919
PRD113(2026)

Thermal nature of confining strings

Sebastian Grieninger,[1, 2, 3, 4, *] Dmitri E. Kharzeev,[3, 4, 5, †] and Eliana Marroquin[3, 4, ‡]

The setup

O. Biebel / Physics Reports 340 (2001) 165–289



Schwinger model: QED in (1+1) dimensions

$$S = \int \mathrm{d}^2x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g\theta}{4\pi} \epsilon^{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathrm{i}\gamma^\mu D_\mu - m)\psi \right]$$

J. Schwinger
1965 Nobel prize with R. Feynman and S. Tomonaga for QED

PHYSICAL REVIEW VOLUME 128, NUMBER 5 DECEMBER 1, 1962

Gauge Invariance and Mass. II*

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts

(Received July 2, 1962)

The possibility that a vector gauge field can imply a nonzero mass particle is illustrated by the exact solution of a one-dimensional model.

it is plausible that some other types of excitation will then be located at fairly small fractions of m_0. Thus, one could anticipate that the known spin-0 bosons, for example, are secondary dynamical manifestations of strongly coupled primary fermion fields and vector gauge fields. This line of thought emphasizes that the question "Which particles are fundamental?" is in- correctly formulated. One should ask "What are the fundamental fields?"

ACKNOWLEDGMENTS

I have had the benefit of conversations on this and related topics with Kenneth Johnson and Charles Sommerfield.

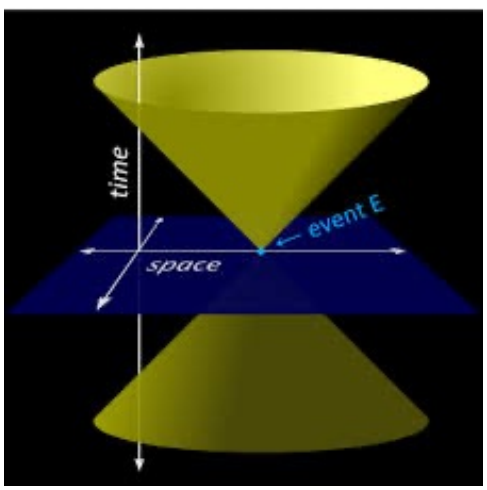
PHYSICAL REVIEW D VOLUME 10, NUMBER 2 15 JULY 1974

Vacuum polarization and the absence of free quarks

A. Casher,* J. Kogut,† and Leonard Susskind‡
Tel Aviv University, Ramat-Aviv, Tel Aviv, Israel
(Received 29 June 1973; revised manuscript received 4 October 1973)

This paper is addressed to the question of why isolated quark partons are not seen. It is argued that in vector gauge theories it is possible to have the short-distance and light-cone behavior of quark fields without real quark production in deep-inelastic reactions. The physical mechanism involved is the flow of vacuum-polarization currents which neutralize any outgoing quarks. Our ideas are inspired by arguments due to Schwinger and an intuitive picture of Bjorken. Two-dimensional (1 space, 1 time) vector gauge field theories provide exactly soluble examples of this phenomenon. The resulting picture of deep-inelastic final states predicts jets of hadrons and logarithmically rising multiplicities as conjectured by Bjorken and Feynman.

Massless Schwinger model coupled to external sources:



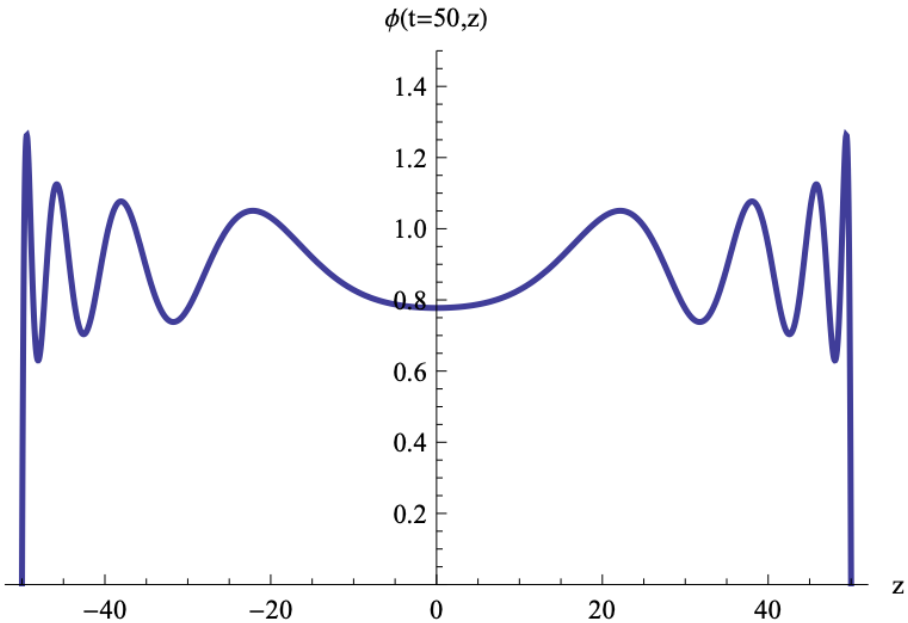
$$j_0^{\text{ext}} = g\delta(z-t), \quad j_1^{\text{ext}} = g\delta(z-t) \qquad \text{for } z>0,$$

$$j_0^{\text{ext}} = -g\delta(z+t), \quad j_1^{\text{ext}} = g\delta(z+t) \qquad \text{for } z<0,$$

In the massless case, can be solved exactly:

$$\phi(x) = \theta(t^2 - z^2)[1 - J_0(m\sqrt{t^2 - z^2})]$$

DK, F. Loshaj
Phys Rev D87 (2013) 7, 077501



String breaking due to production of quark-antiquark pairs;
the produced mesons form a rapidity plateau

To address thermalization, one needs to consider interacting mesons – this leads to the massive Schwinger model.

Non-integrable, no analytical solutions can be found – use digital quantum simulations!

Recent review: C. Robin, M. Savage, arXiv: 2604.26376

The Nobel Prize in Physics 2025

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2025 to

John Clarke
University of California, Berkeley, USA

Michel H. Devoret
Yale University, New Haven, CT and
University of California, Santa Barbara, USA

John M. Martinis
University of California, Santa Barbara
and Qolab, Los Angeles, CA, USA

"for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit"

Schwinger model: QED in (1+1) dimensions

The Hamiltonian:

$$H = \int \mathrm{d}x \left[\frac{1}{2}\left(\Pi + \frac{g\theta}{2\pi}\right)^2 + \bar{\psi}(\mathrm{i}\gamma_1 D_1 + m)\psi \right]$$

Gauge fixing, temporal gauge: $A_0 = 0.$

Generalized momentum:
(θ angle as background
electric field)

$$\Pi = \dot{A}_1 - \frac{g\theta}{2\pi}$$

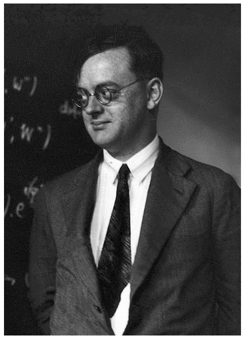
The staggered lattice Hamiltonian:

$$H_S^L = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\chi_n^\dagger \chi_{n+1} - \chi_{n+1}^\dagger \chi_n\right) + m\sum_{n=1}^{N}(-1)^n \chi_n^\dagger \chi_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}\left(L_{\mathrm{dyn},n} + L_{\mathrm{ext},n}(t)\right)^2$$

Schwinger model: QED in (1+1) dimensions

Jordan-Wigner transformation:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\prod_{i=0}^{n-1}(-\mathrm{i}Z_i), \qquad \chi_n^\dagger = \frac{X_n + \mathrm{i}Y_n}{2}\prod_{i=0}^{n-1}\mathrm{i}Z_i,$$



Dirac	staggered	spin
$\bar\psi\psi$	$\frac{(-1)^n}{a}\chi_n^\dagger\chi_n$	$\frac{(-1)^n}{2a}Z_n$
$\bar\psi\gamma_0\psi$	$\frac{1}{a}\chi_n^\dagger\chi_n$	$\frac{1}{2a}Z_n$
$\bar\psi\gamma_1\psi$	$\frac{1}{2a}[\chi_n^\dagger\chi_{n+1} + \chi_{n+1}^\dagger\chi_n]$	$\frac{1}{4a}[X_nY_{n+1} - X_nY_{n+1}]$
$\bar\psi\gamma_5\psi$	$\frac{(-1)^n}{2a}[\chi_n^\dagger\chi_{n+1} - \chi_{n+1}^\dagger\chi_n]$	$-\frac{\mathrm{i}(-1)^n}{4a}[X_nX_{n+1} + Y_nY_{n+1}]$
$\bar\psi\gamma_1\partial_1\psi$	$-\frac{1}{2a^2}[\chi_n^\dagger\chi_{n+1} - \chi_{n+1}^\dagger\chi_n]$	$-\frac{\mathrm{i}}{4a^2}[X_nX_{n+1} + Y_nY_{n+1}]$

From:
DK and Y. Kikuchi, Phys.Rev.Res. 2 (2020) 2, 023342

Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

Adrien Florio, David Frenklakh, Kazuki Ikeda, Dmitri Kharzeev, Vladimir Korepin, Shuzhe Shi, and Kwangmin Yu
Phys. Rev. Lett. **131**, 021902 – Published 13 July 2023

The form of Hamiltonian used in simulations:

$$H^L(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}[L_{\mathrm{dyn},n} + L_{\mathrm{ext},n}(t)]^2.$$

all-to-all qubit connectivity!

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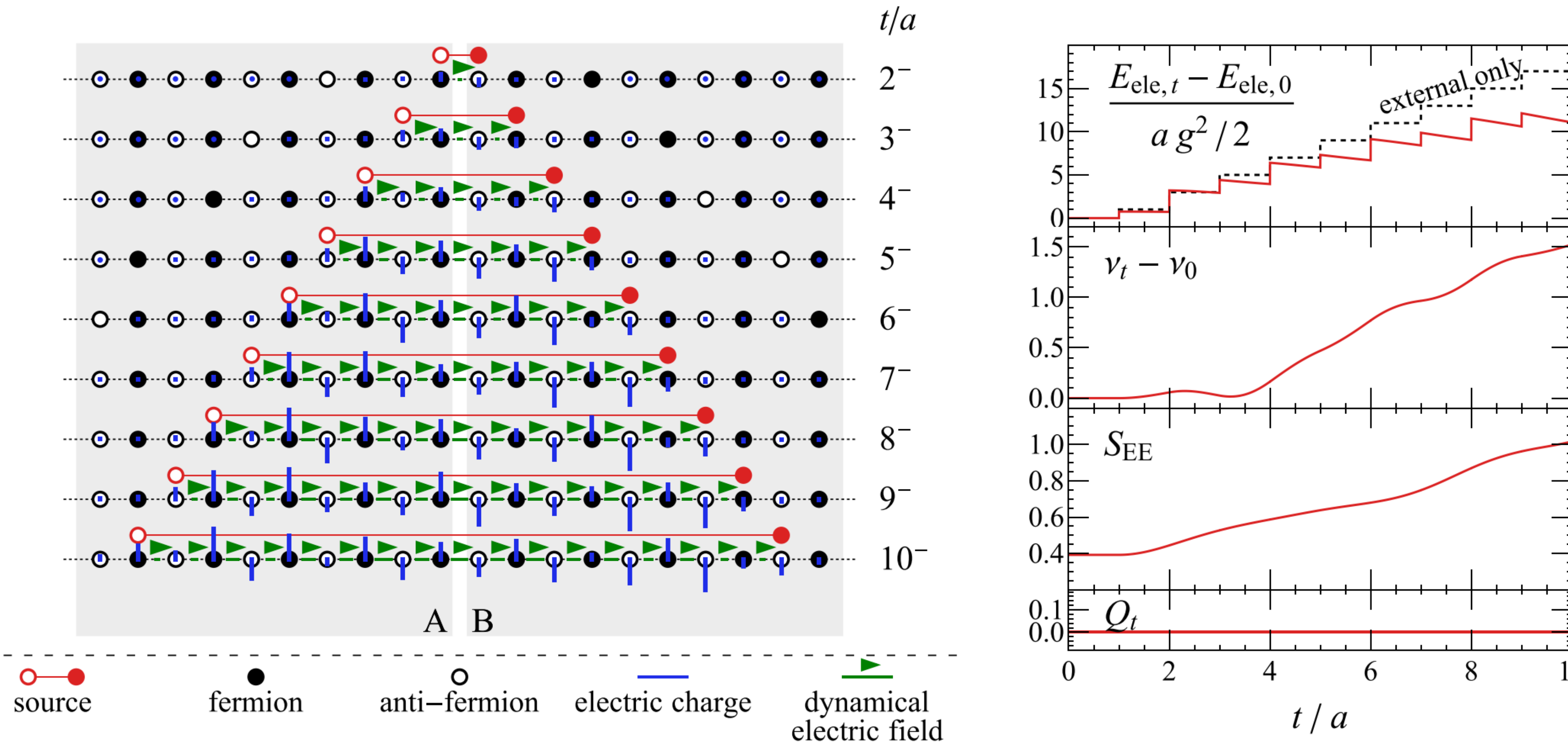
PHYSICAL REVIEW LETTERS **131,** 021902 (2023)



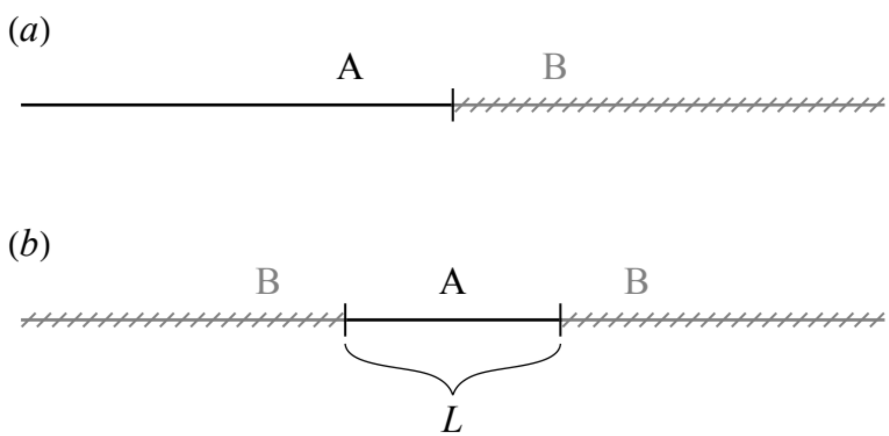
Screening of electric field, modification of the vacuum, growth of entanglement entropy!

What can we do to understand a possible approach to thermalization in our system?

Quantum simulation of entanglement and hadronization in jet production: lessons from the massive Schwinger model

Adrien Florio,[1,2,*] David Frenklakh,[3,†] Kazuki Ikeda,[2,3,‡] Dmitri Kharzeev,[1,2,3,§] Vladimir Korepin,[2,4,¶] Shuzhe Shi,[3,5,**] and Kwangmin Yu[6,††]

Let us start by examining the entanglement spectrum:



Entanglement among the quark and antiquark jet

Entanglement among the central region and the rest of the system

The entanglement spectrum

$$S_{EE}(t) = -\mathrm{Tr}_L[\rho_L(t) \ln \rho_L(t)] = -\sum_{i=1}^{2^{N/2}} \lambda_i \ln \lambda_i.$$

$$\rho_L(t) = \mathrm{Tr}_R \rho(t) = \sum_{i=1}^{2^{N/2}} \lambda_i(t) |\psi_i^L(t)\rangle\langle\psi_i^L(t)|,$$

At late times, a huge number of entanglement eigenstates start to contribute, with comparable eigenvalues – approach to the **maximal entanglement and thermalization**?

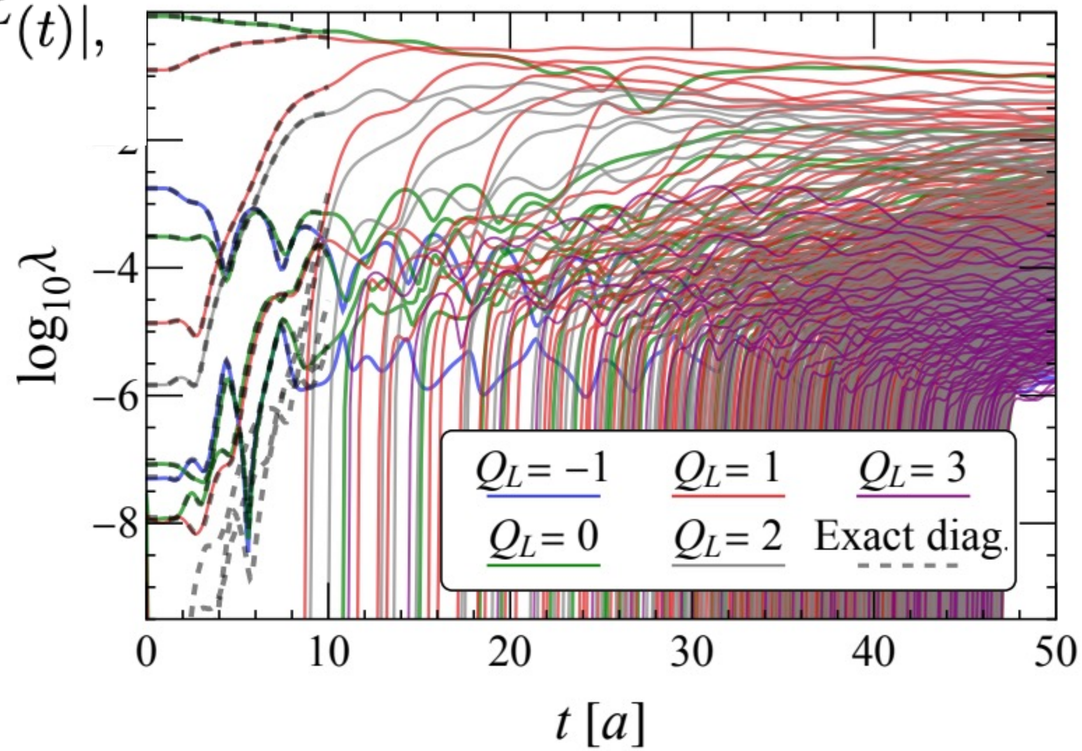


FIG. 2. Symmetry-resolved entanglement spectrum evolution for the lattice size $N = 100$, $m = 1/(4a), g = 1/(2a)$. For comparison the spectrum obtained with exact diagonalization for $N = 20$ at the same mass and coupling is shown as dashed curves.

Tests of maximal entanglement

Renyi entropy

$$S_\alpha(t) \equiv \frac{\ln \mathrm{Tr}_L(\rho_L(t)^\alpha)}{1-\alpha} = \frac{\ln \sum_{i=1}^{2^{N/2}} \lambda_i^\alpha}{1-\alpha}.$$

"Entangleness"

$$\mathcal{E} \equiv \frac{1-\mathrm{tr}\rho_L^2}{1-2^{-N/2}} = \frac{1-\sum_{i=1}^{2^{N/2}} \lambda^2}{1-2^{-N/2}}.$$

$$\mathcal{E}[\mathrm{MES}] = 1.$$

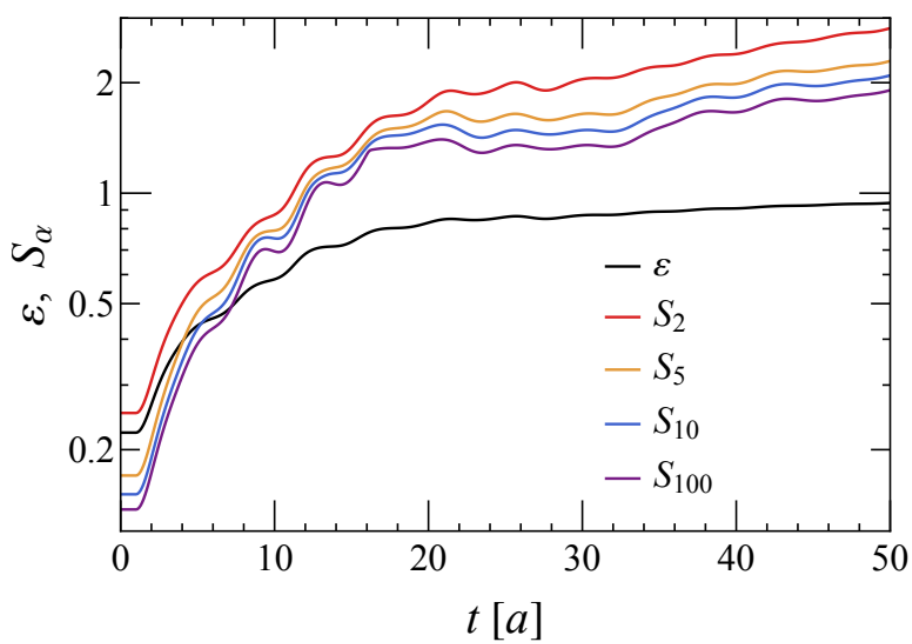
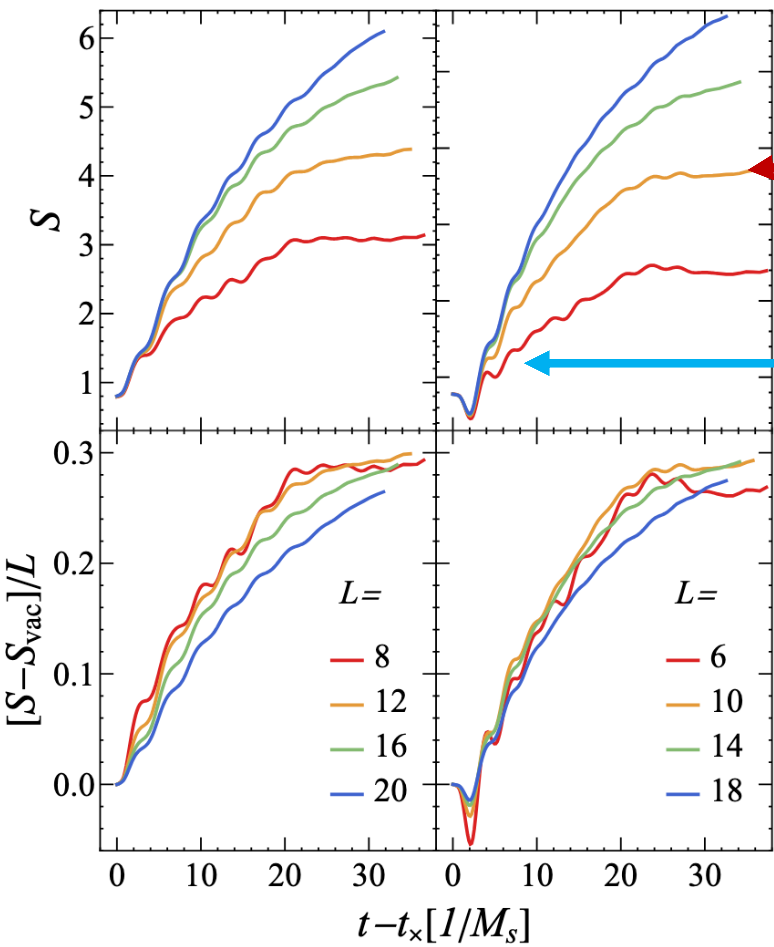


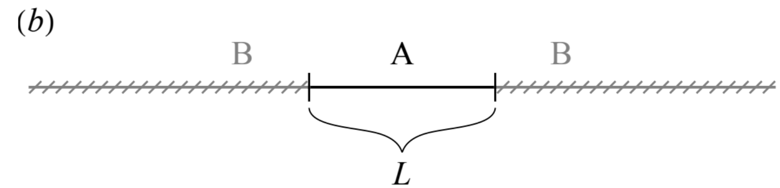
FIG. 3. Entangleness (black) and Rényi entropy with $\alpha = 2$ (red), 5 (gold), 10 (blue), and 100 (purple).

Approach to maximal entanglement!
(in a subspace of the full Hilbert space)

Transition from area to volume law scaling of entanglement

von Neumann entropy:





Volume law (~ L)
("thermal")

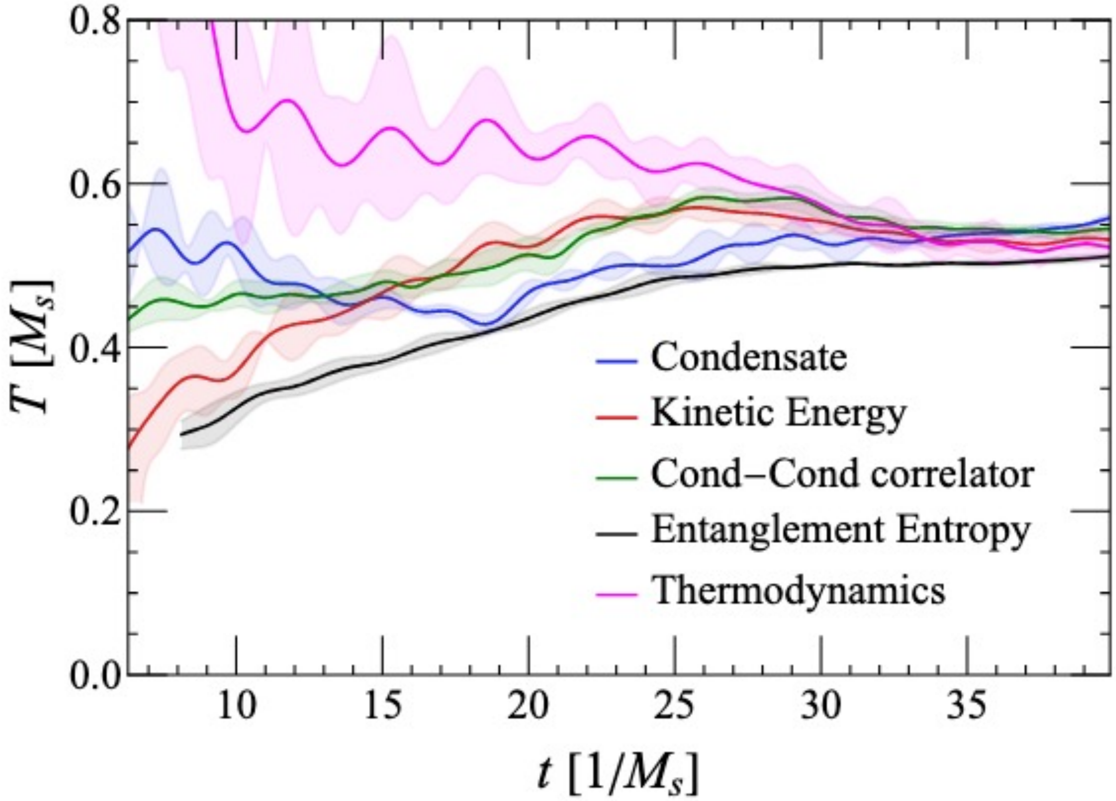
Area law
(L independent)

Thermalization from quantum entanglement: jet simulations in the massive Schwinger model

Adrien Florio,[1,2,*] David Frenklakh,[2,†] Sebastian Grieninger,[3,‡] Dmitri E. Kharzeev,[3,4,§] Andrea Palermo,[3,¶] and Shuzhe Shi[5,6,**]

arXiv:2506.14983, PRD '25

Expectation values of local operators approach the thermal ones

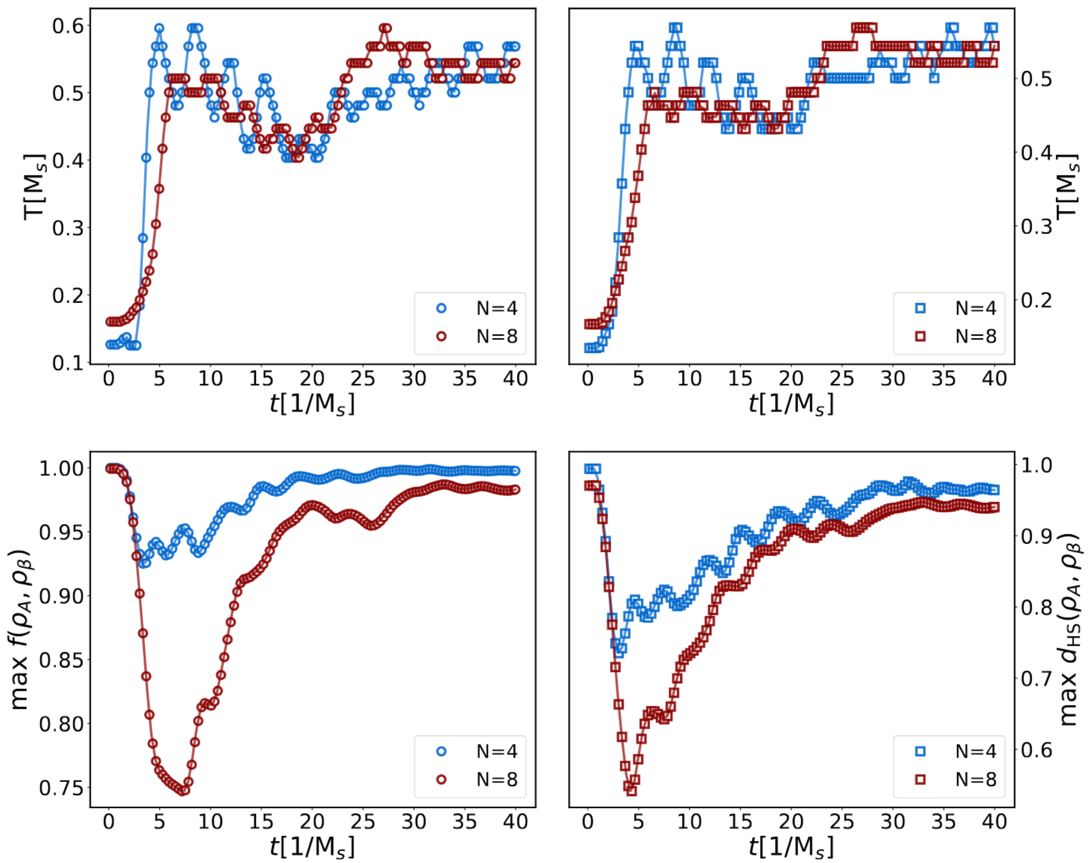


Thermalization from quantum entanglement: jet simulations in the massive Schwinger model

arXiv:2506.14983, PRD

Adrien Florio,[1,2,*] David Frenklakh,[2,†] Sebastian Grieninger,[3,‡]
Dmitri E. Kharzeev,[3,4,§] Andrea Palermo,[3,¶] and Shuzhe Shi[5,6,**]

Overlap with thermal density matrix approaches unity:



Overlap:

$$f(\rho_A, \rho_\beta) \equiv \frac{\mathrm{Tr}(\rho_A\, \rho_\beta)}{\sqrt{\mathrm{Tr}\, \rho_A^2\, \mathrm{Tr}\, \rho_\beta^2}}$$

Hilbert-Schmidt distance:

$$D_{\mathrm{HS}}(\rho_A, \rho_\beta) = \sqrt{\mathrm{Tr}[(\rho_A - \rho_\beta)^2]}.$$

$$d_{\mathrm{HS}}(\rho_A, \rho_\beta) = 1 - D_{\mathrm{HS}}(\rho_A, \rho_\beta)$$

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The physical meaning of Schmidt states

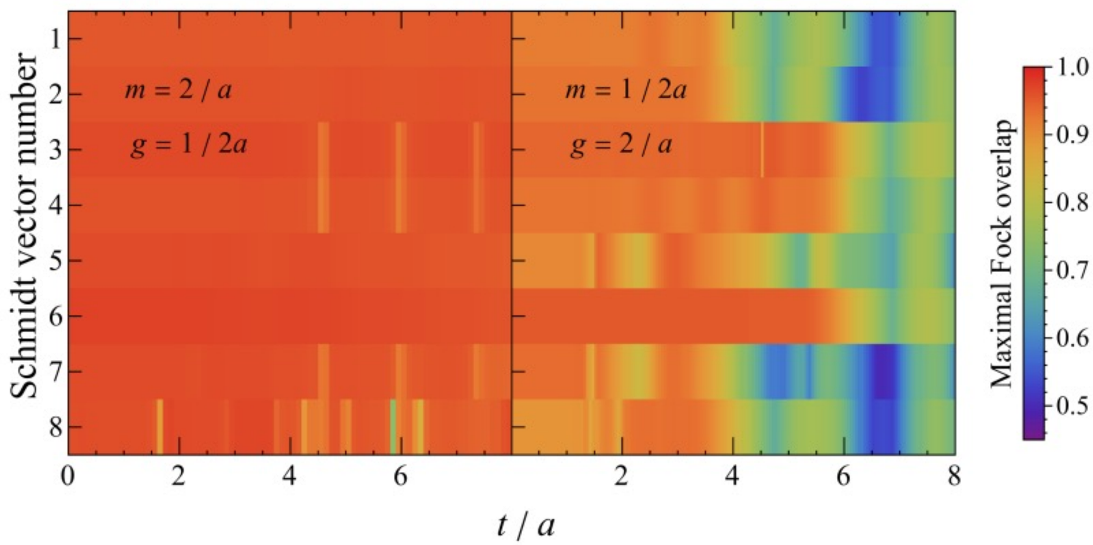
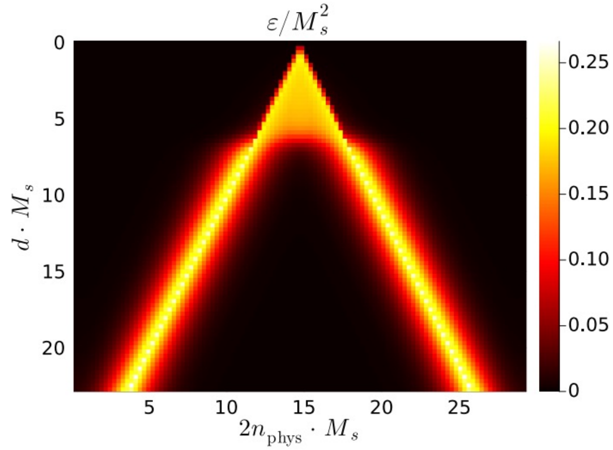


FIG. 5. Maximal overlap of each Schmidt vector with any Fock state. Comparison between $m = 2/a, g = 1/(2a)$ on the left panel and $m = 1/(2a), g = 2/a$ on the right panel is shown. In both cases, $N = 16$. To study continuous evolution, we choose to consider the 8 leading Schmidt vectors in the vacuum state at $t = 0$ and follow their evolution. Because of the level crossing in Schmidt spectrum, at later times these vectors are not necessarily the 8 leading Schmidt vectors.

Transition from “quark-antiquark” states at early times to “mesons” at late times –

Hadronization seen in real time!

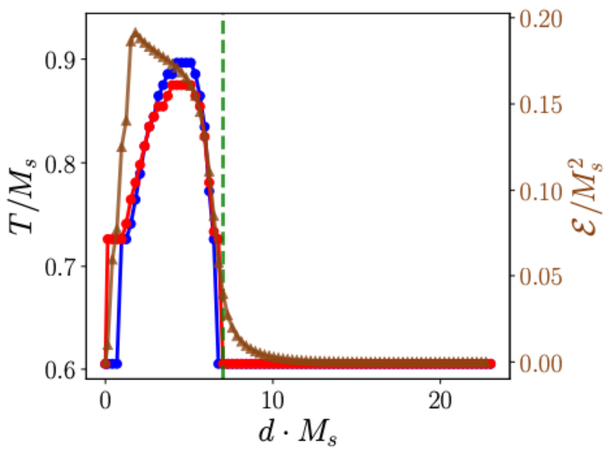
What happens when the sources are static?

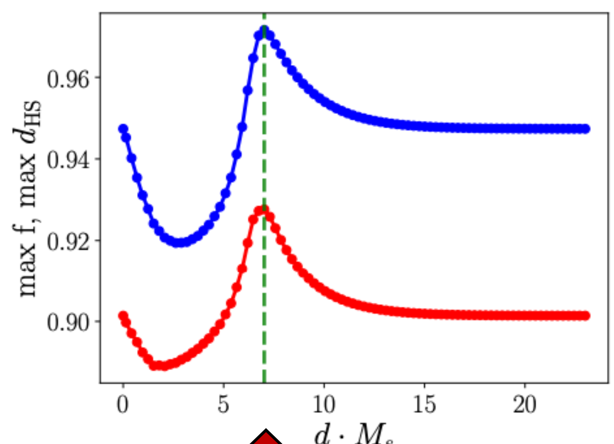


arXiv: 2510.23919,
PRD113(2026)3

Thermal nature of confining strings

Sebastian Grieninger,[1,2,3,4,*] Dmitri E. Kharzeev,[3,4,5,†] and Eliana Marroquin[3,4,‡]





overlap

Hilbert-Schmidt

At the critical break-up distance, the string is thermal!

The hadrons are produced thermally equilibrated

Maximal entanglement erases long-distance confinement

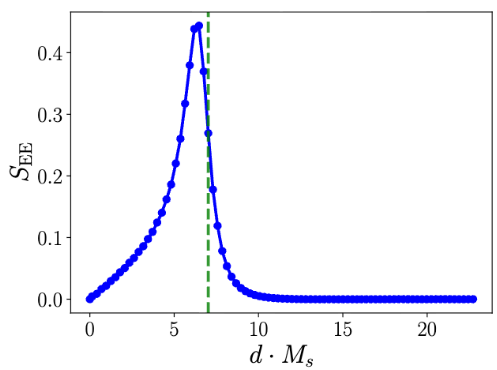
As the string stretches, the energy stored in the string grows:

$$E(L) \simeq \sigma L,$$

As a result, the dimensionality of the corresponding Hilbert space grows:

$$\dim \mathcal{H}(E) = \Omega(E) = e^{S(E)}.$$

Therefore, the conditions for Page theorem are satisfied, and the density matrix of a subsystem becomes maximally entangled



⇨ string breaking results!

arXiv: 2510.23919,
PRD113(2026)3

Thermal nature of confining strings

Sebastian Grieninger,[1,2,3,4,*] Dmitri E. Kharzeev,[3,4,5,†] and Eliana Marroquin[3,4,‡]

String breaking and complexity

IQuS@UW-21-119, NT@UW-26-1

The Quantum Complexity of String Breaking in the Schwinger Model

Sebastian Grieninger [1,*] Martin J. Savage [1,†] and Nikita A. Zemlevskiy [1,‡]
[1] *InQubator for Quantum Simulation (IQuS), Department of Physics, University of Washington, Seattle, WA 98195, USA*
(Dated: April 15, 2026)

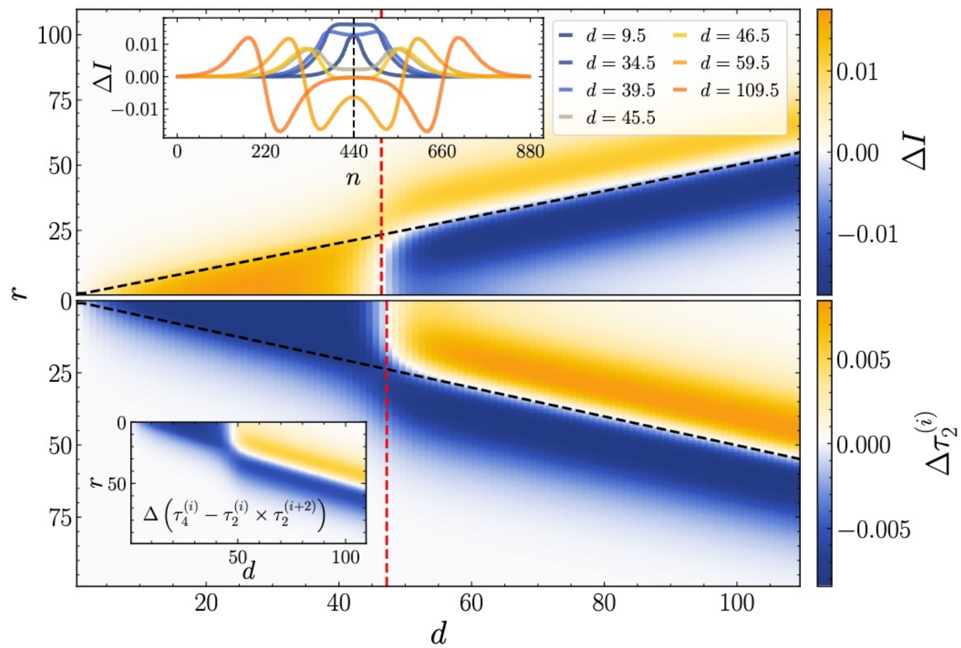


FIG. 3. *Local and multipartite entanglement in string breaking.* Top: the local vacuum-subtracted MI as a function of

Entanglement as a probe of hadronization

Jaydeep Datta,[1,*] Abhay Deshpande,[1,2,†] Dmitri E. Kharzeev,[3,4,‡] Charles Joseph Naïm,[1,§] and Zhoudunming Tu[5,¶]

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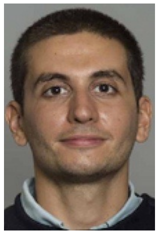
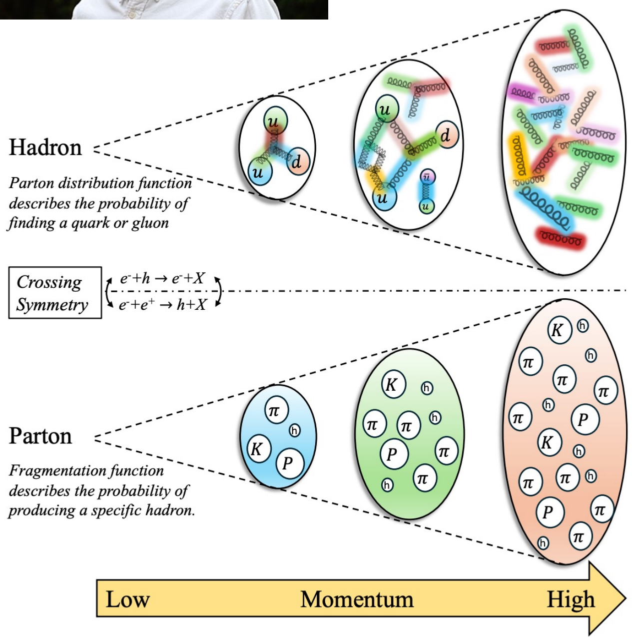
[5]*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA*

(Dated: October 30, 2024)


arXiv:2410.22331, Phys. Rev. Lett.(2025)



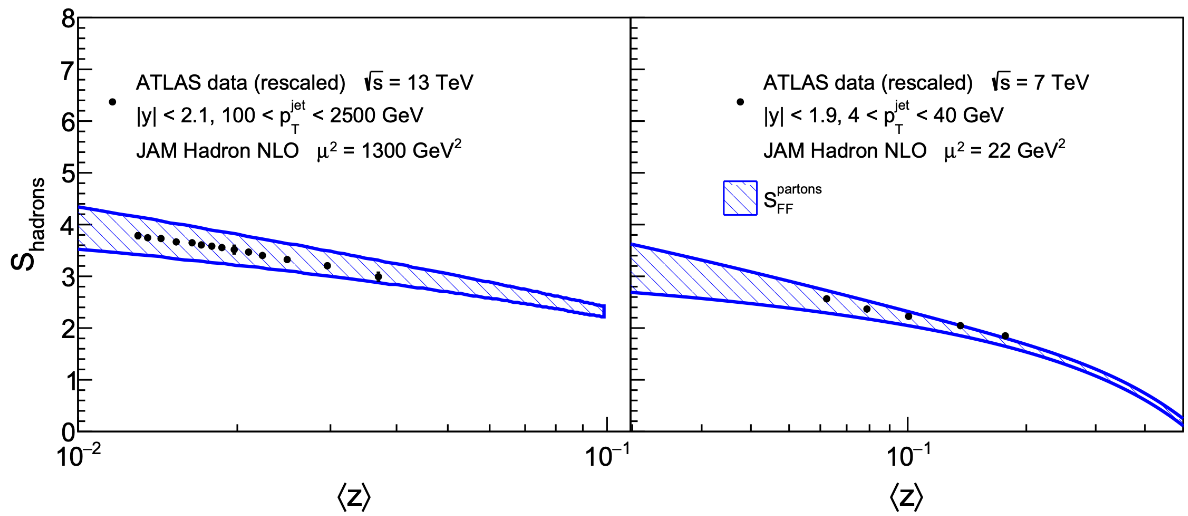


FIG. 3. The entropy S_{hadrons} as a function of $\langle z\rangle$ for $S_{\mathrm{FF}}^{\mathrm{partons}}$ — incorporating gluons, u-(anti)quarks, and d-(anti)quarks — is shown using JAM fragmentation functions at NLO for $\mu^2 = 1300$ GeV^2, compared with ATLAS data at $\sqrt{s} = 13$ TeV [45] (left). Additionally, the results at $\mu^2 = 22$ GeV^2 are compared with ATLAS data at $\sqrt{s} = 7$ TeV [43] (right). The uncertainties are calculated at the 1σ level. The total entropy $S_{\mathrm{FF}}^{\mathrm{partons}}$ is derived from the sum of the individual entropies of each parton, with each contribution normalized by the average fraction of jets produced by that parton from PYTHIA simulation.

Evidence for maximal entanglement from jet fragmentation

Detecting quantum correlations in inclusive jet fragmentation

Our quantum simulations suggest the presence of quantum entanglement in rapidity bins that are not too far from each other – say, below Δy ~ 2.

These correlations can be detected through joint multiplicity distribution in two distinct rapidity bins, related to the mutual information:

$$I(A:B) = S_A + S_B - S_{A\cup B},$$

$$I(A\!:\!B) \simeq \sum_{n_1,n_2} P(n_1,n_2)\, \ln\left[\frac{P(n_1,n_2)}{P_1(n_1)\, P_2(n_2)}\right] \;\geq\; 0.$$

DK, quant-ph/2601.00405 (Acta Phys. Polon. B57, 4)