# Fundamental Symmetries

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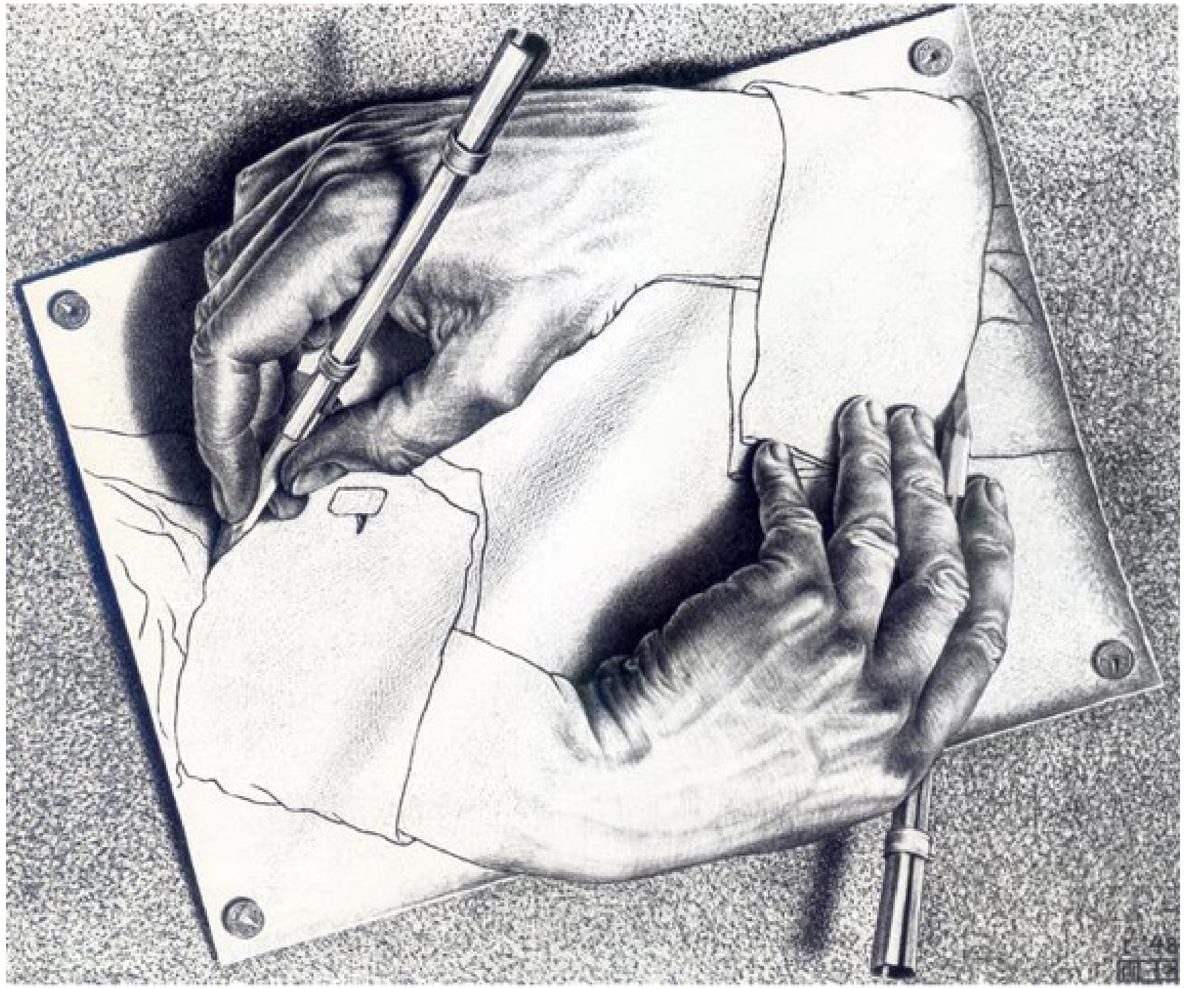


- Standard Model : Inadequacies
- Experimental Tests of Standard Model and Symmetries
  - Electric Dipole Moment Searches : $e,\ \mu,\ n,\ p$, nuclei
  - Parity Violation : MOLLER at JLab
  - Charged Lepton Flavor Violation : $\mu N \to eN$
  - Baryon Number Violation : Proton Decay
  - Precision Test of the Standard Model : Muon g-2
- Summary and Outlook

- My experience : experimentalist, worked on polarized deep-inelastic scattering, muonium hyperfine structure (test of bound state QED, muon mass), BNL muon g-2, electron EDM searches in polar diatomic molecules, polarized proton-proton scattering with PHENIX collaboration at RHIC - to measure $\Delta g$ and $\Delta \bar{u}$ and $\Delta \bar{d}$, FNAL muon g-2, CeNTREX TIF Schiff Moment search, Mu2e

- What is origin of the observed matter-antimatter asymmetry?
  - SM prediction off by $>$6 orders of magnitude
- SM doesn't explain $1/3$ relation between quark and lepton charges
- What is the origin of neutrino mass?
- What is dark matter? What is dark energy?
- What about axions and magnetic monopoles?
- Can we explain the extreme hierarchy of masses and strengths of forces?
- Why are there 3 families? Can the electroweak and strong forces be unified?

$\Rightarrow$ What about gravity ???

- Standard Model low-energy limit of a more fundamental theory - but what is it ???

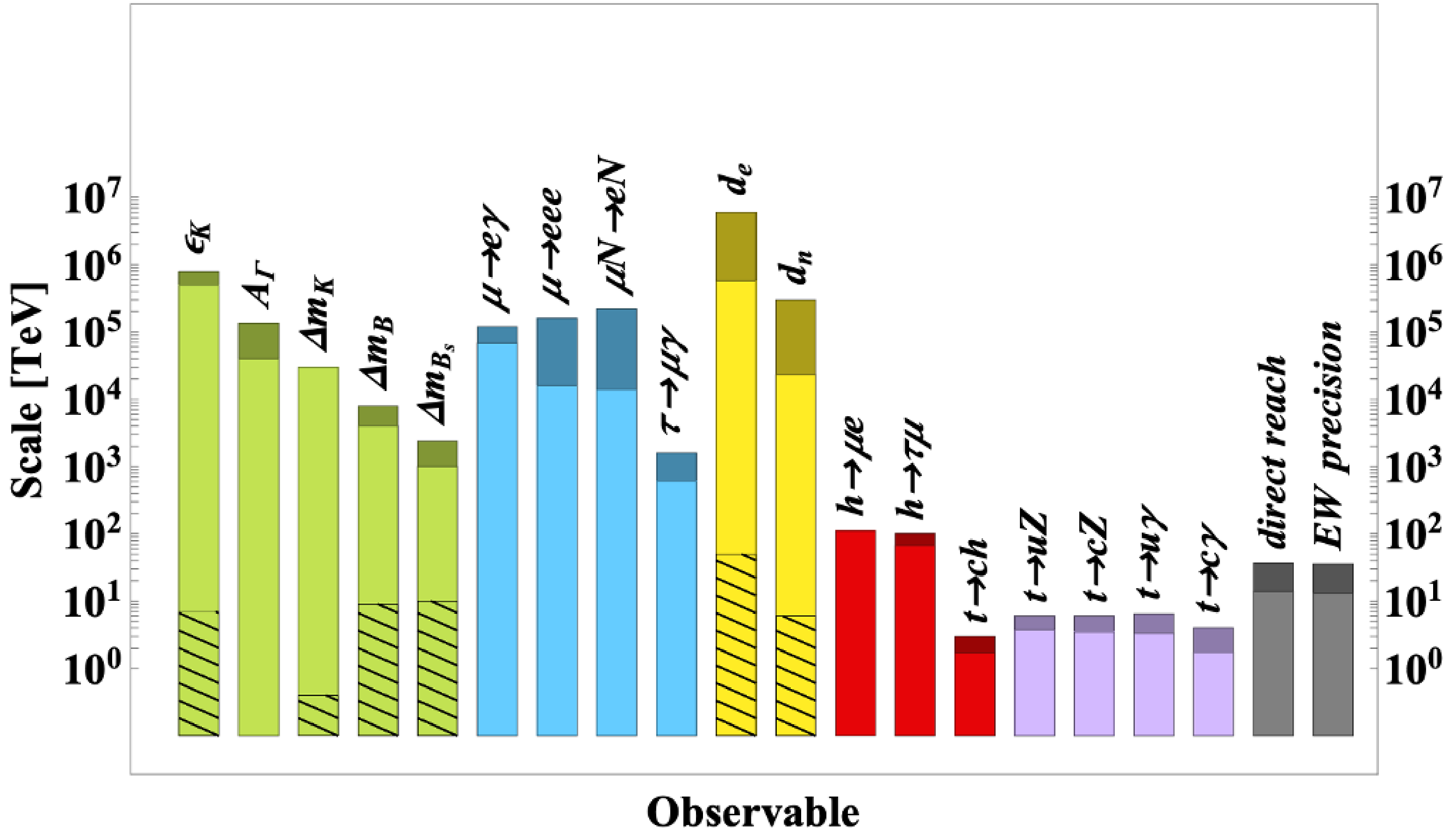



Fig. 5.1: Reach in new physics scale of present and future facilities, from generic dimension six operators. Colour coding of observables is: green for mesons, blue for leptons, yellow for EDMs, red for Higgs flavoured couplings and purple for the top quark. The grey columns illustrate the reach of direct flavour-blind searches and EW precision measurements. The operator coefficients are taken to be either $\sim 1$ (plain coloured columns) or suppressed by MFV factors (hatch filled surfaces). Light (dark) colours correspond to present data (mid-term prospects, including HL-LHC, Belle II, MEG II, Mu3e, Mu2e, COMET, ACME, PIK and SNS).

arXiv:1910.11975v2 [hep-ex]

# What is a Permanent Electric Dipole Moment (EDM) ?

- Non-relativistic Hamiltonians of bare spin 1/2 particle with magnetic moment $\vec{\mu}$ and EDM $\vec{d}$

$$H_{\text{Magnetic Dipole}} = -\vec{\mu} \cdot \vec{B} \; = \; -\mu\vec{\sigma} \cdot \vec{B}$$

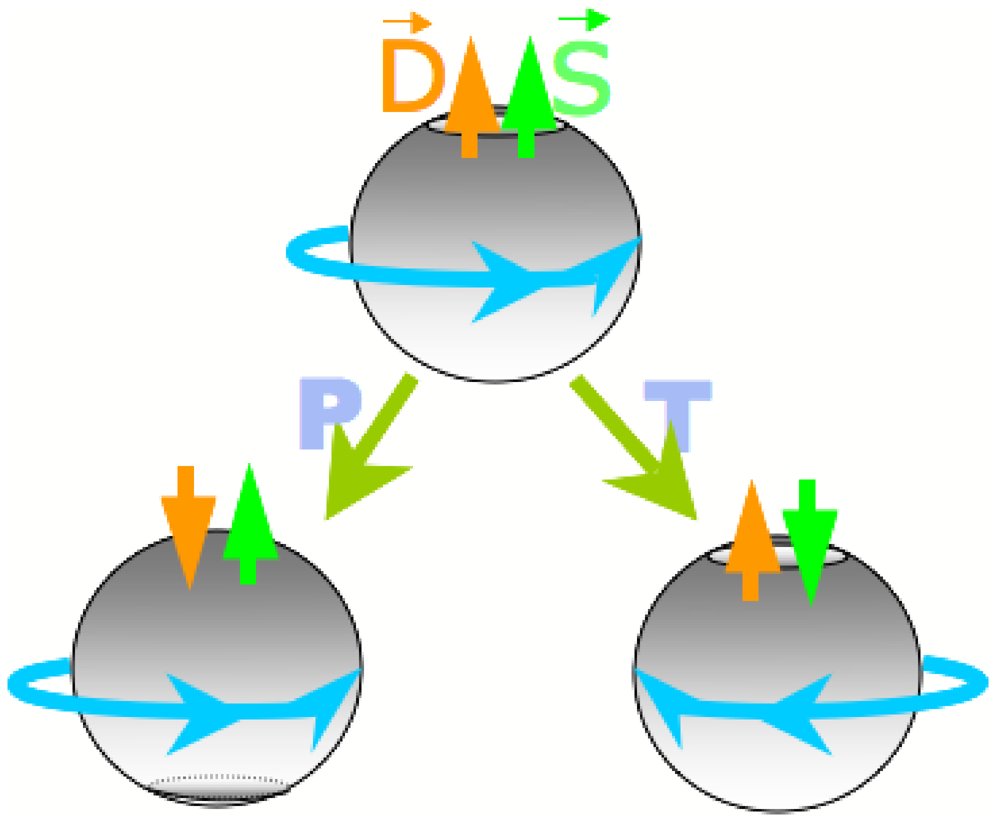
$$H_{\text{Electric Dipole}} = -\vec{d} \cdot \vec{E} \; = \; -d\vec{\sigma} \cdot \vec{E}$$

- EDM is analog of magnetic dipole moment
- Manifests itself as a linear Stark effect

Behavior of Moments under Parity and Time Reversal

| | $\vec{\sigma} \sim \vec{r} \times \vec{p}$ | $\vec{B} \sim \vec{j} \times \vec{r}/\lvert\vec{r}\rvert^3$ | $\vec{E} \sim -\vec{\nabla} V$ |
|---|---|---|---|
| P | even | even | odd |
| T | odd | odd | even |

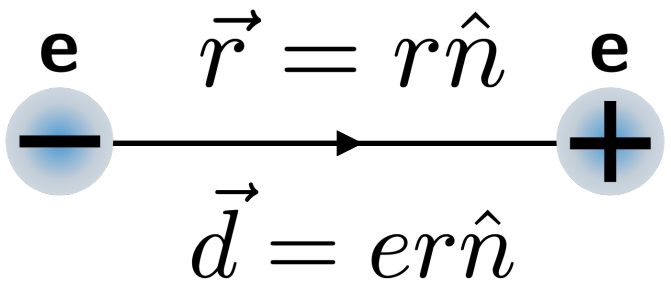
- $H_{\text{Magnetic Dipole}}$ is P-even and T-even
- $H_{\text{Electric Dipole}}$ is P-odd and T-odd !!!




$\Rightarrow$ For fundamental particle to have an EDM, P and T must be violated

# EDMs and Parity

- Permanent EDM behaves like separation of positive and negative charge along particle spin angular momentum
- Such a moment breaks time-reversal ($\mathbf{T}$) and parity symmetry ($\mathbf{P}$) simultaneously
- What are some implications for the detection of an EDM?
- Consider a molecular electric dipole moment $\vec{d} = er\hat{n}$




$$
\begin{aligned}
\hat{d}_z &\equiv er\hat{z}\cdot \\
\hat{d}_z\,|\hat{n}\rangle &= er\hat{z}\cdot\hat{n}\,|\hat{n}\rangle \\
\hat{\Pi}^{-1}\hat{d}_z\hat{\Pi}\,|\hat{n}\rangle &= \hat{\Pi}^{-1}\hat{d}_z\,|{-\hat{n}}\rangle \\
&= \hat{\Pi}^{-1}(er\hat{z}\cdot -\hat{n})\,|{-\hat{n}}\rangle \\
&= (-er\hat{z}\cdot\hat{n})\hat{\Pi}^{-1}\,|{-\hat{n}}\rangle \\
&= (-er\hat{z}\cdot\hat{n})\,|\hat{n}\rangle \\
&= -\hat{d}_z\,|\hat{n}\rangle
\end{aligned}
$$

$$
\Rightarrow \langle\Psi|\,\hat{\boldsymbol{d}}\,|\Psi\rangle = \langle\Psi|\,\binom{+}{-}\hat{\boldsymbol{d}}\binom{+}{-}\,|\psi\rangle = \langle\Psi|\,\Pi^{-1}\hat{\boldsymbol{d}}\Pi\,|\Psi\rangle = -\langle\Psi|\,\hat{\boldsymbol{d}}\,|\Psi\rangle = 0
$$

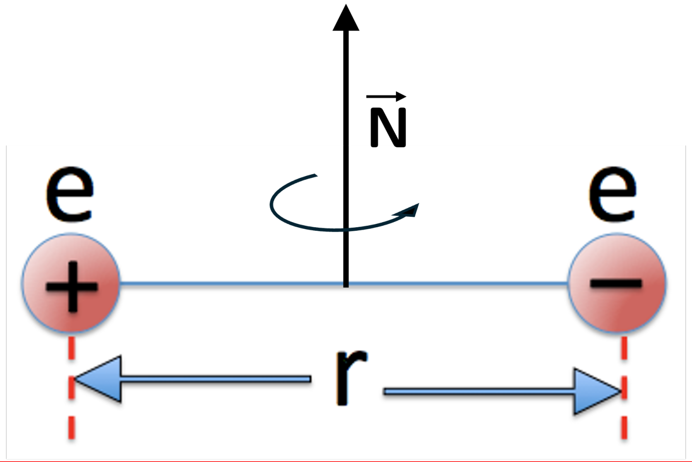
$\Rightarrow$ $\langle\Psi|\,\hat{\boldsymbol{d}}\,|\Psi\rangle = 0$ if $|\Psi\rangle$ has definite parity

$\Rightarrow$ **Atom in state of definite parity has no first order Stark shift, no observable EDM**

# Don't Polar Molecules have Electric Dipole Moments ?

Dipole moment of a polar molecule :

$$\vec{d} = \sum e_i \vec{r}_i \; = \; e\, r\, \hat{z} \; \simeq \; \mathrm{e\ a_0} \; \simeq \; 5 \times 10^{-9}\ \mathrm{e \cdot cm}$$




Reconsider the EDM of a polar molecule :

- Dipole moment parallel to internuclear axis ⇒ averaged out by rotation
- Do polar molecules really exhibit a linear Stark shift under $H_{\rm EDM} = -\vec{d} \cdot \vec{E}_{\rm ext}$ ?
- Energy eigenstates $\Psi_i$ are eigenstates of parity but $H_{\rm EDM} = -\vec{d} \cdot \vec{E}_{\rm ext}$ is P-odd

$$E_i' = E_i + \langle \Psi_i | H_{\rm EDM} | \Psi_i \rangle + \sum \frac{|\langle \Psi_j | H_{\rm EDM} | \Psi_i \rangle|^2}{E_i - E_j} \simeq E_i + \frac{(\vec{d} \cdot \vec{E}_{\rm ext})^2}{E_i - E_j}$$

$$|\Psi_i'\rangle \approx |\Psi_i\rangle + |\Psi_j\rangle \frac{\langle \Psi_j | H_{\rm EDM} | \Psi_i \rangle}{E_i - E_j}$$

- $\vec{E}_{\rm ext}$ field mixes opposite parity states - *induces* dipole, E shift *quadratic* in $\vec{E}_{\rm ext}$
- No linear Stark shift !

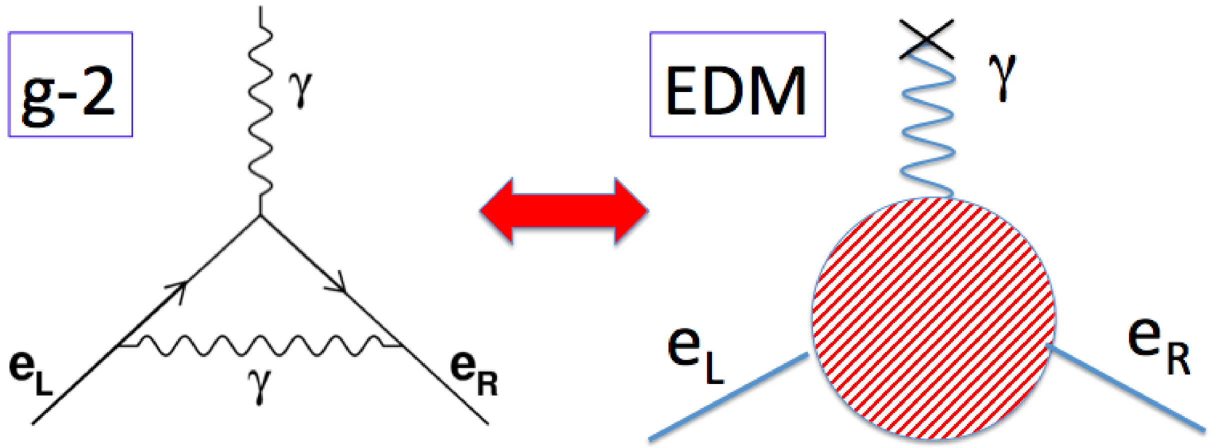
- Only permanent EDM makes mixed parity ground state and *linear* Stark effect

## Why do we expect the electron, proton, neutron, nucleus ... EDMs $d \neq 0$ ?

- EDMs violate P, T : through CPT theorem T-violation $\Leftrightarrow$ CP-violation
- P-violation observed, CP-violation observed in K, B, and D mesons, and bottom lambda baryon $(\Lambda_b)$
- Can generate EDM using Standard Model physics through radiative corrections

$\Rightarrow$ In same way radiative corrections make $g_e \neq 2.0000$, RC can make $d_e \neq 0$

$\Rightarrow$ Construct diagram with enough loops to incorporate P and CP-violating processes




- In SM need at least 4 loops - predicts $|d_e| \leq 1 \times 10^{-38}$ e·cm
- 8 orders of magnitude below current limit $|d_e| < 4.1 \times 10^{-30}$ e·cm !

JILA $HfF^+$,T. Roussy *et al.*, Science **381** (6653) 46-50, 2023.

- Reference scale "dipole moment" of a molecule $\approx e \times a_0 \approx 5 \times 10^{-9}$ e·cm

## Current limits on electron, proton, neutron, nuclear EDMs

| Particle / System | Upper Limit ($e \cdot \mathrm{cm}$) | C.L. | Type | Reference |
|---|---|---|---|---|
| Electron ($e^-$) | $\lvert d_e \rvert < 4.1 \times 10^{-30}$ | 90% | Direct ($HfF^+$, JILA) | Roussyet *al.* [1] |
| Neutron ($n$) | $\lvert d_n \rvert < 1.8 \times 10^{-26}$ | 90% | Direct (UCN, PSI) | Abel *et al.* [2] |
| Mercury ($^{199}$Hg) | $\lvert d_{\mathrm{Hg}} \rvert < 7.4 \times 10^{-30}$ | 95% | Direct (Atomic) | Graner *et al.* [3] |
| Proton ($p$) | $\lvert d_p \rvert < 2.0 \times 10^{-25}$ | 95% | Indirect (from $^{199}$Hg) | Graner *et al.* [3] |
| Deuteron ($d$) | $\lvert d_d \rvert < 2.5 \times 10^{-17}$ | 95% | Direct (Storage Ring, COSY) | Andres *et al.* [4] |

⇒ $d_d \approx -0.7 \times 10^{-34}\, e\,\mathrm{cm}$, $d_u \approx -0.15 \times 10^{-34}\, e\,\mathrm{cm}$, $d_n^{\mathrm{CKM}} = \frac{4}{3} d_d - \frac{1}{3} d_u \approx -0.9 \times 10^{-34}\, e\mathrm{cm}$

⇒ No evidence for a non-zero electric dipole moment of a fundamental particle, despite searching since the 1950s !

⇒ Should we give up?


[1 ] T. Roussy *et al.* (JILA $HfF^+$), "An improved bound on the electron's electric dipole moment," *Science* **381** (6653), 46-50 (2023).

[2 ] C. Abel *et al.* (nEDM Collaboration), "Measurement of the Permanent Electric Dipole Moment of the Neutron," *Phys. Rev. Lett.* **124**, 081803 (2020).

[3 ] B. Graner, Y. Chen, E. G. Lindahl, and B. R. Heckel, "Reduced Limit on the Permanent Electric Dipole Moment of $^{199}$Hg," *Phys. Rev. Lett.* **116**, 161601 (2016).

[4 ] A. Andres *et al.* (JEDI Collaboration), "First Experimental Limit on the Permanent Electric Dipole Moment of the Deuteron," *Phys. Rev. Lett.* **136**, 241801 (2026).

# Partial list of current and future EDMs

⇒ There is no evidence for a non-zero permanent electric dipole moment of a fundamental particle ⇒ but there may be soon !

| Collaboration | Species | Method | Sensitivity ($10^{-29}$ $e$ cm) | Status | Duration (years) |
|---|---|---|---|---|---|
| PanEDM I | n | UCN (ILL) | 380 | Commissioning | 5 |
| PanEDM II | n | UCN (ILL) | 79 | Commissioning | 8 |
| Beam EDM | n | beam (ILL) | 500 | proof-of-principle | ? |
| n2EDM | n | UCN (PSI) | 110 | Start data-taking | 4 |
| n2EDMagic | n | UCN (PSI) | 50 | Construction | 5 |
| nEDMsf | n | UCN (ESS) | 20 | Development | 7 |
| TUCAN | n | UCN (TRIUMF) | 100 | Commissioning | |
| nEDM | n | UCN (LANL) | 200 | Construction | 5 |
| ACME III | e ThO ($^3\Delta_1$) | Beam | 0.1 | Commissioning | |
| JILA III | e $ThF^+$ ($^3\Delta_1$) | Ion trap | | Commissioning | |
| Imperial II | e YbF ($^2\Sigma$) | $\mu$K beam | 0.1 | Commissioning | 3 |
| Imperial III | e YbF ($^2\Sigma$) | Lattice | 0.01 | Construction | 6 |
| NL-eEDM I | e BaF ($^2\Sigma$) | Slow beam | 0.5 | Commissioning | 3 |
| NL-eEDM II | e BaOH ($^2\Sigma$) | Lattice | 0.1 | Construction | 6 |
| PolyEDM | e SrOH ($^2\Sigma$) | Lattice | | Construction | |
| $EDM^3$ | e BaF ($^2\Sigma$) | Matrix | | Construction | |
| DOCET | e BaF ($^2\Sigma$) | Matrix | | Construction | 3 |
| CeNTREX | p $^{205}$TlF | Beam | | Commissioning | |
| HeXe | $^{129}$Xe | $^3$He-comagnetometer | 10 | Construction | 4 |
| RaX | e, Ra, RaF,RaOH | cold beam, trap | | Development | |
| quMercury | $^{199}$Hg | ultracold atoms | 1 | Construction | 5 |
| ALADDIN | $\Lambda_c^+, \Xi_c^+$ | collider | $1 \times 10^{13}$ | Development | ?+2 |
| muEDM I | $\mu$ | frozen-spin | $4 \times 10^8$ | Commissioning | 3 |
| muEDM II | $\mu$ | frozen-spin | $6 \times 10^6$ | Conception | 10 |
| pEDM I | p | frozen-spin | 1 | Development | 5 |
| pEDM II | p | frozen-spin | 0.01 | Conception | 5 |

Adapted from M. Athanasakis-Kaklamanakis *et al.*, Community input to the European Strategy on particle physics: Searches for Permanent Electric Dipole Moments, arXiv:2505.22281v1 [hep-ex]

## Why do we expect the electron, proton, neutron, ... EDMs $d \neq 0$ ?

- SM prediction is so small ⇒ any observation $d_{n,p,e} \neq 0$ definitive evidence of new physics

Reasons to expect there is new physics leading to $d_{n,p,d,e}$ large enough to detect :

- Sakharov showed CP-violation required to generate matter-antimatter asymmetry in universe
  - CP-violation in SM $> 10^5$ too small to account for observations
  - Expect new sources of CP-violation
  - EDMs could be dramatically enhanced
- Most SM extensions predict many new particles and CP-violating phases
  - Predict dramatically enhanced EDMs : $|d_e| \approx 10^{-26} - 10^{-31}$ e·cm !
    $|d_{n,p,d}| \approx 10^{-25} - 10^{-31}$ e·cm !

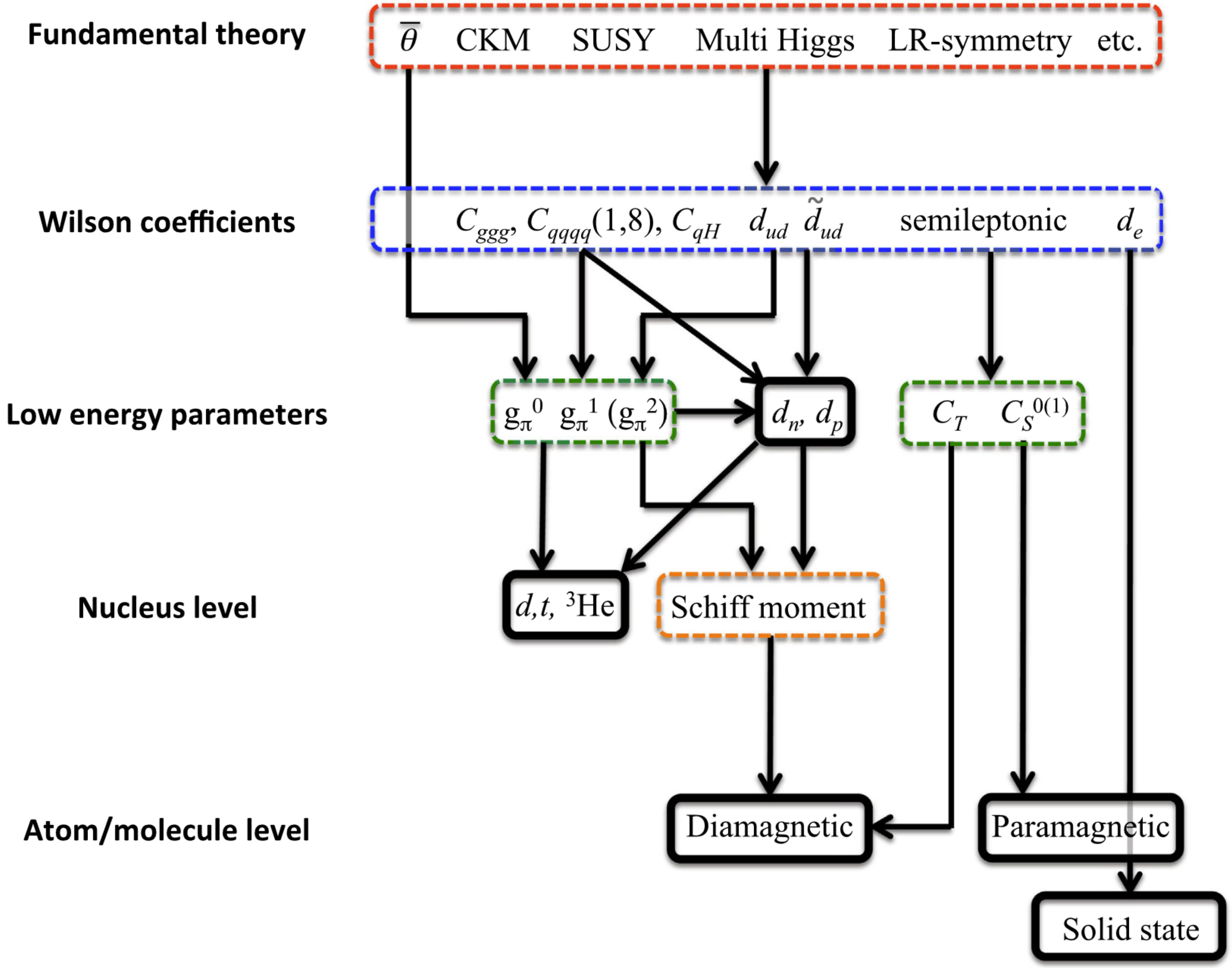
⇒ Observed matter-antimatter asymmetry and theoretical prejudice suggest significant sources of T-violation beyond SM

⇒ $d_{n,p,d,e} \neq 0$ definitive evidence of new physics

⇒ Predicted $d_{n,p,d,e}$ within range accessible to new experiments

⇒ Good time to look for EDMs ! Must-do physics !

# Connection between high energy fundamental theory and low energy observables




T.E. Chupp, P. Fierlinger, M.J. Ramsey-Musolf, and J.T. Singh, Rev. Mod. Phys., Vol. **91**, 015001 (2019)

# Effective Low Energy MSSM $CP$-violating Lagrangian

(From D. Demir *et al.*, Nucl. Phys. B **680**, 339 (2004))

$$\mathcal{L}_{\rm eff} = \frac{g_s^2}{32\pi^2}\bar{\Theta}G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} + \frac{1}{3}wf^{abc}G^a_{\mu\nu}\tilde{G}^{\nu\beta,b}G^{\mu,c}_{\beta} - \frac{i}{2}\sum_{i=e,u,d,s} d_i\bar{\Psi}_i\gamma_5\sigma^{\mu\nu}\Psi_i F_{\mu\nu} - \frac{i}{2}\sum_{i=e,u,d,s} d^c_i\bar{\Psi}_i g_s\gamma_5\sigma^{\mu\nu}\lambda^a\Psi_i G^a_{\mu\nu}$$

- Contributions : $\bar{\Theta}$, Weinberg 3-gluon, EDMs of $e$ and quarks $d_i$, chromo-edms of quarks $d^c_i$
- $|d_n|$ limits $\rightarrow$ $\bar{\Theta} < 1 \times 10^{-10}$, *a priori* $\bar{\Theta} \approx 0 - 2\pi$
  - If Peccei-Quinn axions exist $\bar{\Theta} \rightarrow 0$
  - Radiative corrections to $\bar{\Theta}$ may induce non-negligible EDM
- The CP-odd term cubic in $G^a_{\mu\nu}$ seldom dominates the EDM of a nucleon
- For given manner of SUSY breaking $w,\ d_i,\ d^c_i$ can be calculated
  - From quark level to nucleon level involves nuclear models : $w,\ d_{u,d,s},\ d^c_{u,d,s} \Rightarrow d_n,\ d_p$
  - $d_n = -d_p \approx 3\times 10^{-16}\,\bar{\theta}$ e·cm if $CP$-violation due to $\bar{\theta}_{\rm QCD}$
  - $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u + 0.83e(d^c_u + d^c_d) - 0.27e(d^c_u - d^c_d)$
  - $d_p = \frac{4}{3}d_u - \frac{1}{3}d_d + 0.83e(d^c_u + d^c_d) + 0.27e(d^c_u - d^c_d)$
  - $d_n = \eta\,(\Delta_d d_d + \Delta_u d_u + \Delta_s d_s)$, ...
  - $d_p \approx d_n$ if dominated by heavy quarks, $d_d$ from other combinations of terms

$\Rightarrow$ Need measurements in many systems $d_p,\ d_n,\ d_d$, ... to extract parameters of CP violation

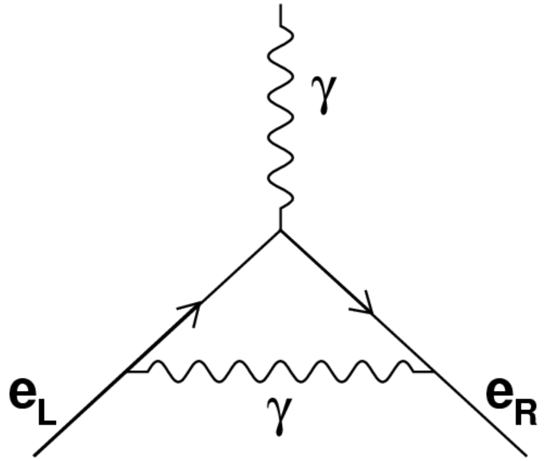
  - $d_e$ "easily" extracted from EDM, $d_A$, observed in atom or molecule

**TABLE III.**

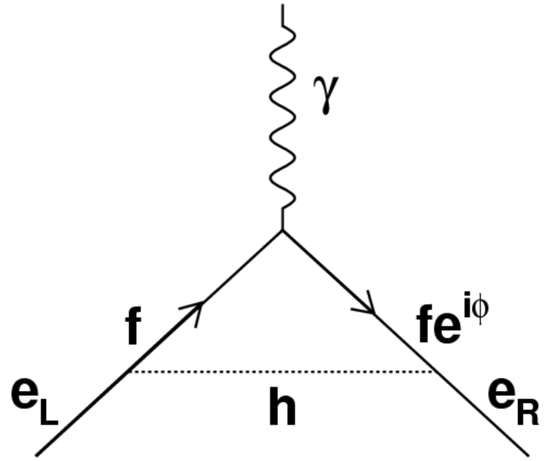
Limits on $CP$-violating observables from the $^{199}$Hg EDM limit. Each limit is based on the assumption that it is the sole contribution to the atomic EDM. In principle, the result for $\mathbf{d}_n$ supercedes [11] as the best neutron EDM limit.

| Quantity | Expression | Limit | Ref. |
|---|---|---|---|
| $\mathbf{d}_n$ | $\mathbf{S}_{\rm Hg}/(1.9\ {\rm fm}^2)$ | $1.6\times10^{-26}\ e\ {\rm cm}$ | [21] |
| $\mathbf{d}_p$ | $1.3\times\mathbf{S}_{\rm Hg}/(0.2\ {\rm fm}^2)$ | $2.0\times10^{-25}\ e\ {\rm cm}$ | [21] |
| $\bar{g}_0$ | $\mathbf{S}_{\rm Hg}/(0.135\ e\,{\rm fm}^3)$ | $2.3\times10^{-12}$ | [5] |
| $\bar{g}_1$ | $\mathbf{S}_{\rm Hg}/(0.27\ e\,{\rm fm}^3)$ | $1.1\times10^{-12}$ | [5] |
| $\bar{g}_2$ | $\mathbf{S}_{\rm Hg}/(0.27\ e\,{\rm fm}^3)$ | $1.1\times10^{-12}$ | [5] |
| $\bar{\theta}_{QCD}$ | $\bar{g}_0/0.0155$ | $1.5\times10^{-10}$ | [22,23] |
| $(\tilde{d}_u-\tilde{d}_d)$ | $\bar{g}_1/(2\times10^{14}\ {\rm cm}^{-1})$ | $5.7\times10^{-27}\ {\rm cm}$ | [25] |
| $C_S$ | $\mathbf{d}_{\rm Hg}/(5.9\times10^{-22}\ e\ {\rm cm})$ | $1.3\times10^{-8}$ | [15] |
| $C_P$ | $\mathbf{d}_{\rm Hg}/(6.0\times10^{-23}\ e\ {\rm cm})$ | $1.2\times10^{-7}$ | [15] |
| $C_T$ | $\mathbf{d}_{\rm Hg}/(4.89\times10^{-20}\ e\ {\rm cm})$ | $1.5\times10^{-10}$ | see text |

Our result can also be used to place limits on $P$, $T$-odd scalar, pseudoscalar, and tensor electron-nucleon interactions (described by $C_S$, $C_P$, and $C_T$) which may induce an atomic EDM independent of the Schiff moment. In $^{199}$Hg the tensor interaction is expected to dominate. Many recent calculations of the tensor coefficient $C_T$ have been performed; the




$\Longleftrightarrow$




⇒ Energy shift from anomalous mag. moment

$$\Delta E \approx (g-2)\ \mu_B\ |\mathbf{B}|/2 \approx \frac{\alpha}{2\pi}\ \frac{e\hbar}{2m_e c}\ |\mathbf{B}|$$

⇒ Energy shift from an electric dipole moment

$$\Delta E \approx d_e \cdot \mathbf{E} \approx \frac{\alpha}{2\pi}\ \frac{e\hbar}{2m_e c}\ |\mathbf{E}| \times \left(\frac{f}{e}\right)^2 \sin(\phi) \left(\frac{m_e}{m_h}\right)^2$$

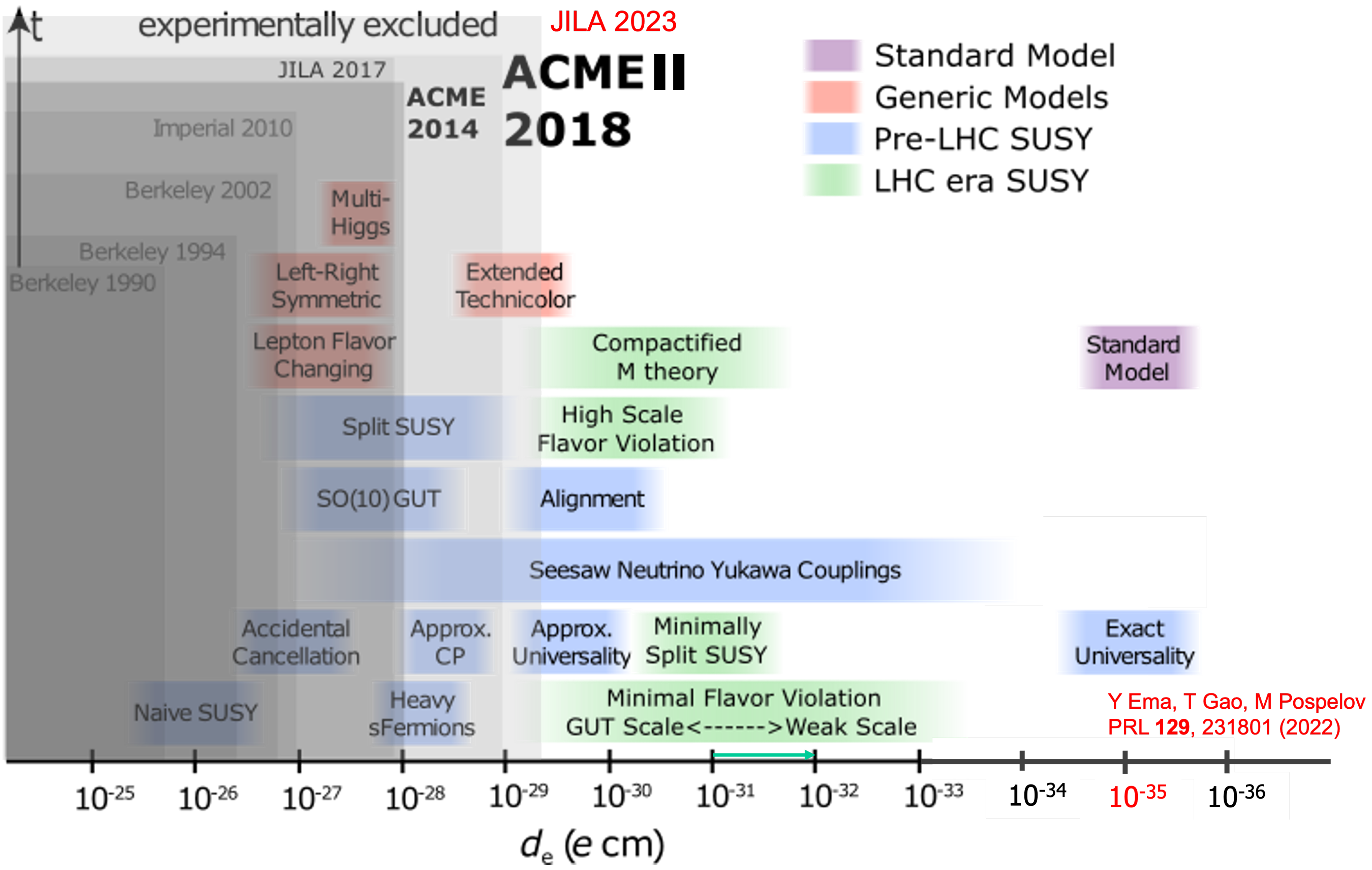
$$d_e \approx e\,\frac{\alpha}{4\pi}\ \sin(\phi)\ \frac{m_e}{m_h^2}, \quad \sin(\phi) \approx 1$$

$$\Rightarrow d_e \approx \frac{1}{137 \cdot 4\pi}\ \frac{1.05 \times 10^{-27}}{2 \cdot 9.1 \times 10^{-28} \cdot 3 \times 10^{10}} \left(0.5 \times 10^{-6}\right)^2 \left(\frac{1\ \mathrm{TeV}}{m_h}\right)^2\ e \cdot \mathrm{cm}$$

$$\approx 5 \times 10^{-27} \left(\frac{1\ \mathrm{TeV}}{m_h}\right)^2\ e \cdot \mathrm{cm}; \quad \text{for quarks } d_f \text{ almost 10 times larger}$$

⇒ Current limit $|d_e| < 4.1 \times 10^{-30}\,e$cm probes 30 TeV mass scale, future experiments even more !

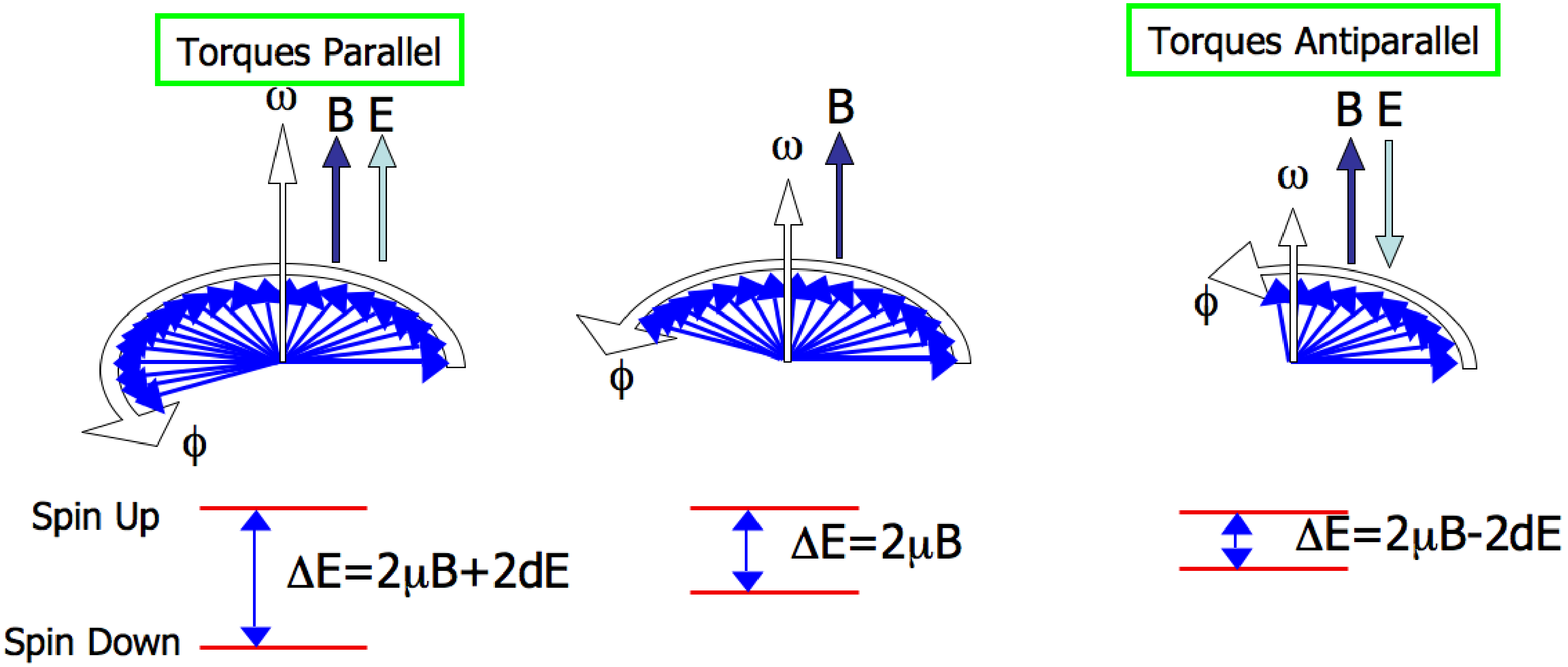
# Electron EDMs : what can we learn?




- $d_e$ is powerful probe of new physics, probing scales of 10s of TeV $\Rightarrow$ even a null result is interesting !
- Many possible sources of CP violation; need EDM searches in $\mu$, $\tau$, n, p, d, $^3$He, ... (thanks to Dave DeMille for plot)

# Algorithm for finding an EDM

- Put system with unpaired spins in parallel $E$ and $B$ fields
- Spin polarize system perpendicular to fields (superposition of spin up and down)
- Torques from $E$ and $B$ fields lead to precession through angle $\phi$ in coherence time $\tau$
- Flip $E$ wrt $B$, look for change in $\phi$ ($i.e.$ look for energy shift).




- Look for precession frequency shift $\Delta\nu = 4dE/h$
- For $E = 100$ kV/cm, $d_e = 1 \times 10^{-27}$ e·cm $\Rightarrow$ $\Delta\nu \approx 20$ nHz $\Leftrightarrow$ $B \approx$ few $\times 10^{-14}$ G
- Lessons : Maximize $E$ and precession angle $\phi$ $\Leftrightarrow$ maximize observation time $\tau$, and counting statistics

## Importance of closely spaced states of opposite parity and large electric fields

- Toy model of hydrogen atom with permanent electron electric dipole moment $d_e$
- A permanent EDM would make atomic ground state a superposition of states of opposite parity
- Atom polarized by electric field of EDM into a superposition of $|s\rangle$ and $|p\rangle$ states

$$|\Psi\rangle = |s\rangle - \left[\frac{\langle p|\,\boldsymbol{d}_e \cdot \boldsymbol{E}_{int}\,|s\rangle}{E_s - E_p}\right]|p\rangle$$

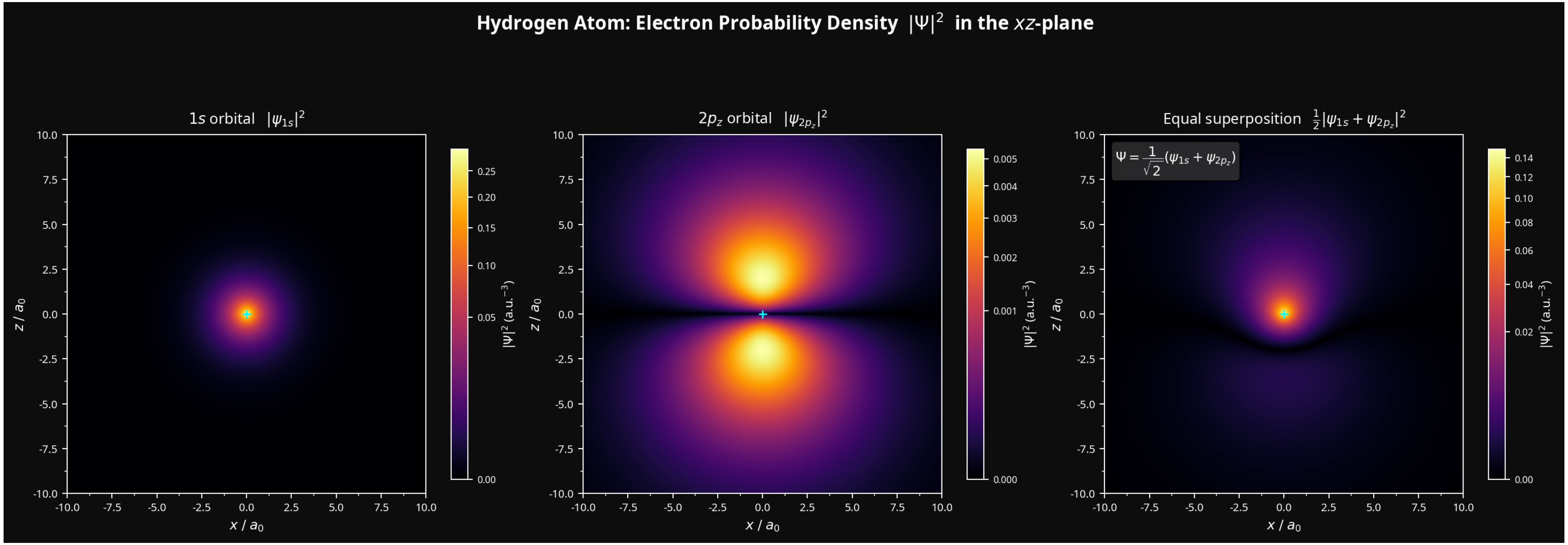
- Admixture of opposite parity state depends on $d_e$ and wavefunctions
- Now apply external field $\boldsymbol{E}_0$, with Stark interaction $H' = -e\boldsymbol{r} \cdot \boldsymbol{E}_0$
- Can interpret first order energy shift from EDM as $\Delta E_{edm} = -\boldsymbol{d}_a \cdot \boldsymbol{E}_0 = -\boldsymbol{d}_e \cdot \boldsymbol{E}_{\text{eff}}$

$$\begin{aligned}
\Delta E_{edm} &= \langle\Psi|\,H'\,|\Psi\rangle \\
&= \left[\langle s| + \langle p|\left[\frac{\langle p|\,\boldsymbol{d}_e \cdot \boldsymbol{E}_{int}\,|s\rangle}{E_s - E_p}\right]\right]\left[-e\boldsymbol{r} \cdot \boldsymbol{E}_0\right]\left[\frac{\langle p|\,\boldsymbol{d}_e \cdot \boldsymbol{E}_{int}\,|s\rangle}{E_s - E_p}\,|p\rangle + |s\rangle\right] \\
&\approx \langle p|\,\boldsymbol{d}_e \cdot \boldsymbol{E}_{int}\,|s\rangle\,\frac{\langle p|\,e\boldsymbol{r} \cdot \boldsymbol{E}_0\,|s\rangle}{E_s - E_p} \\
&\approx d_e E_{int}\,\frac{\langle p|\,e\boldsymbol{r} \cdot \boldsymbol{E}_0\,|s\rangle}{E_s - E_p} \;\approx\; d_e E_{\text{eff}}
\end{aligned}$$

$\Rightarrow$ EDM signal much larger when opposite parity states closely spaced in energy and in large electric fields

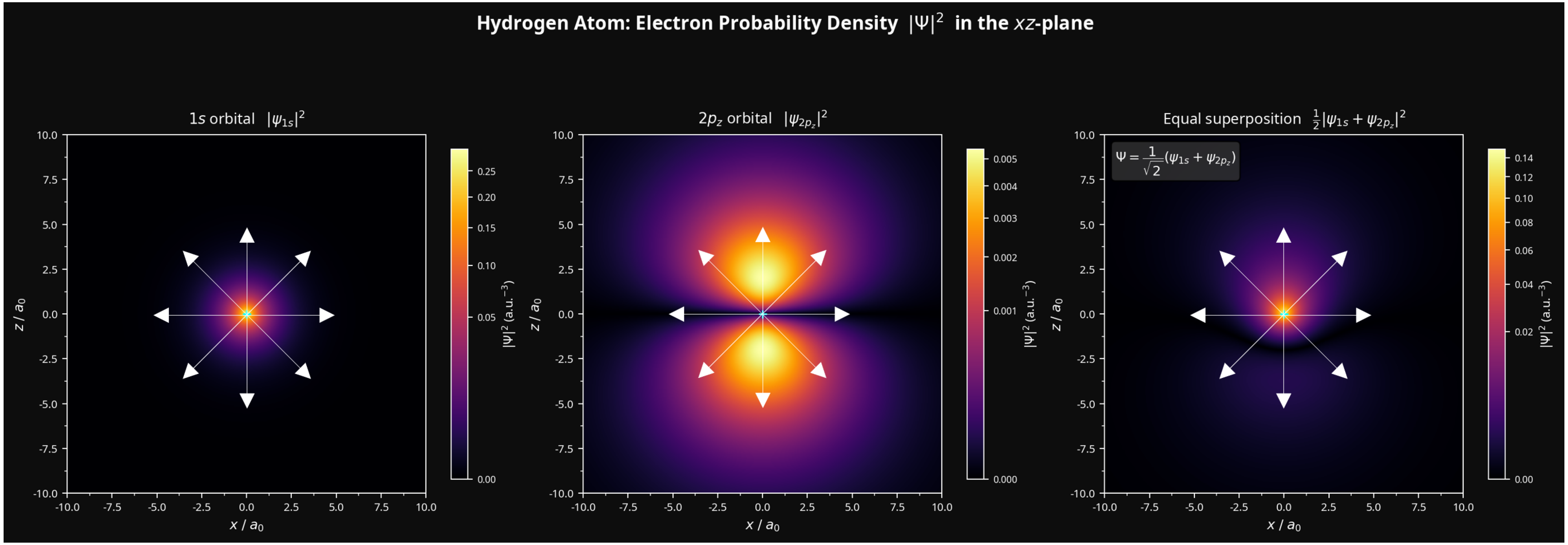
# Toy model of hydrogen atom

- Consider fully polarized state $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\Psi_{1s}\rangle + |\Psi_{2p_z}\rangle\right)$
- $\Psi_{1s} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \quad \Psi_{2p_z} = \frac{1}{4\sqrt{2\pi a_0^3}} e^{-r/2a_0} \cos\theta$
- Total Charge density: $\rho(\vec{r}) = |\Psi(\vec{r})|^2 = \frac{1}{2}\left(\Psi_{1s}^2(\vec{r}) + \Psi_{2p_z}^2(\vec{r}) + 2\Psi_{1s}(\vec{r})\Psi_{2p_z}(\vec{r})\right)$



# Toy model of hydrogen atom with electric field of nucleus

- Consider fully polarized state $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\Psi_{1s}\rangle + |\Psi_{2p_z}\rangle\right)$
- $\Psi_{1s} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \quad \Psi_{2p_z} = \frac{1}{4\sqrt{2\pi a_0^3}} e^{-r/2a_0} \cos\theta$
- Total Charge density: $\rho(\vec{r}) = |\Psi(\vec{r})|^2 = \frac{1}{2}\left(\Psi_{1s}^2(\vec{r}) + \Psi_{2p_z}^2(\vec{r}) + 2\Psi_{1s}(\vec{r})\Psi_{2p_z}(\vec{r})\right)$




- Electron only sees net field from nucleus when electon cloud is polarized

# Toy model of hydrogen atom with electric field of nucleus and electron EDM

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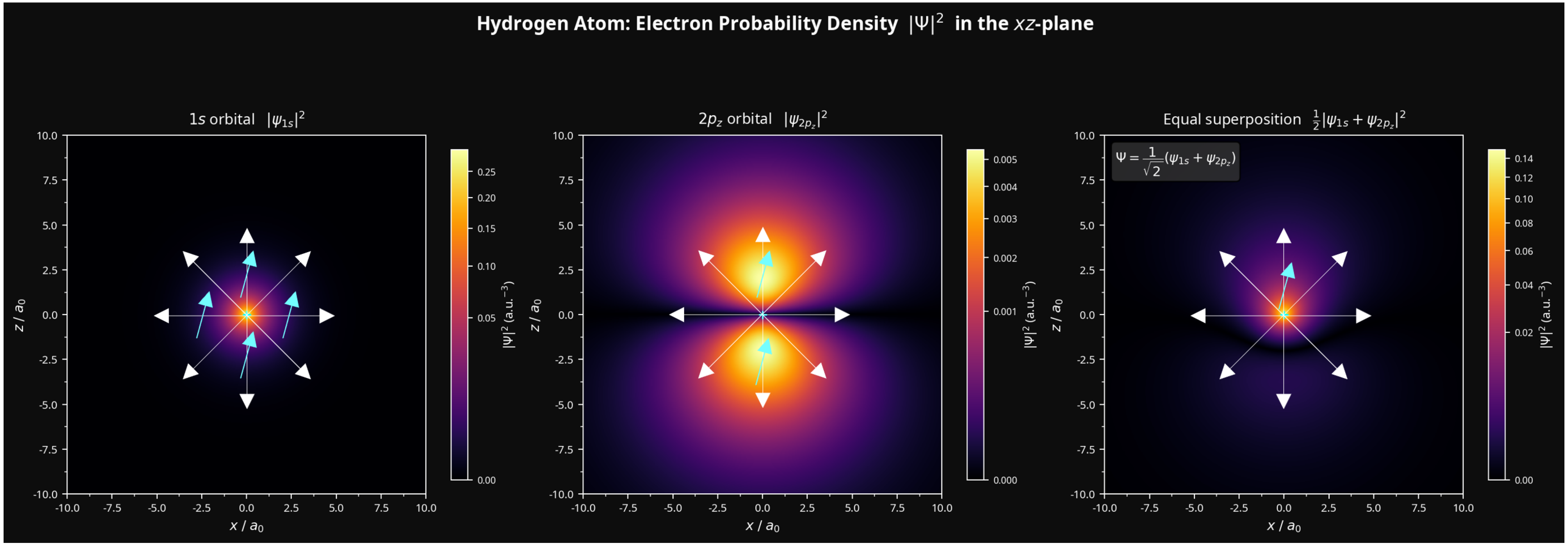
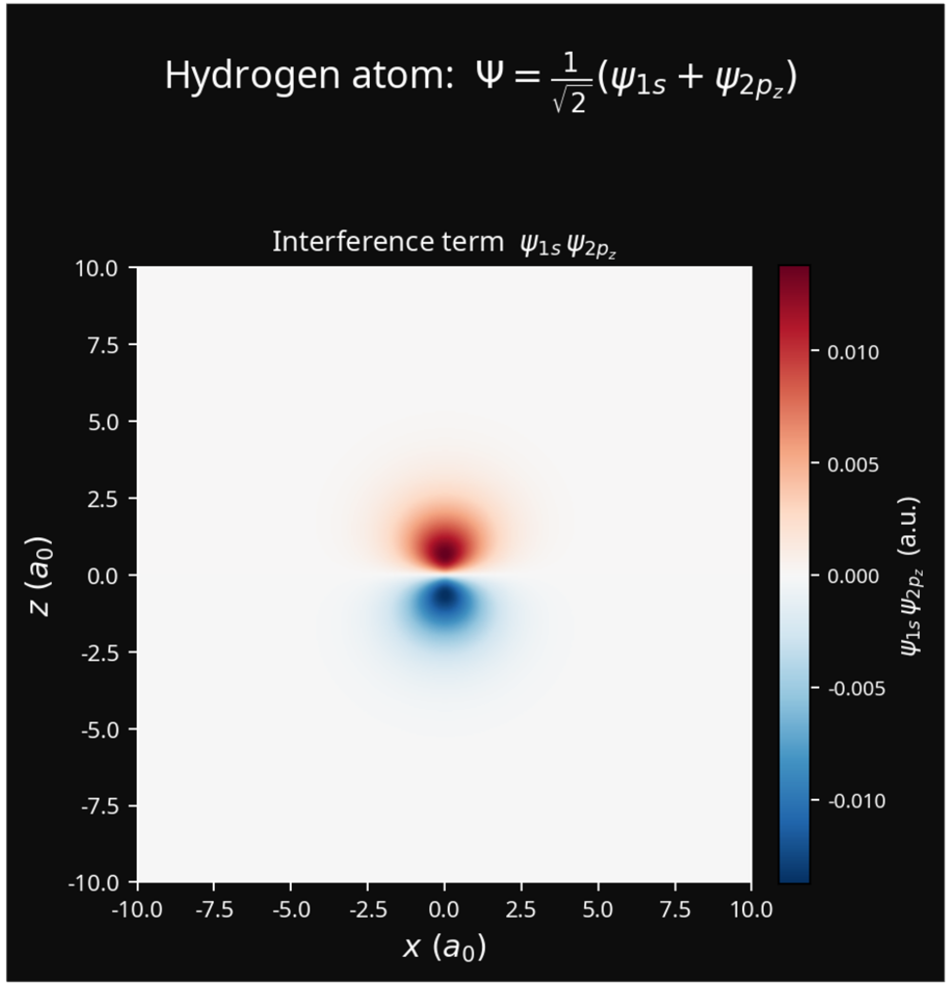
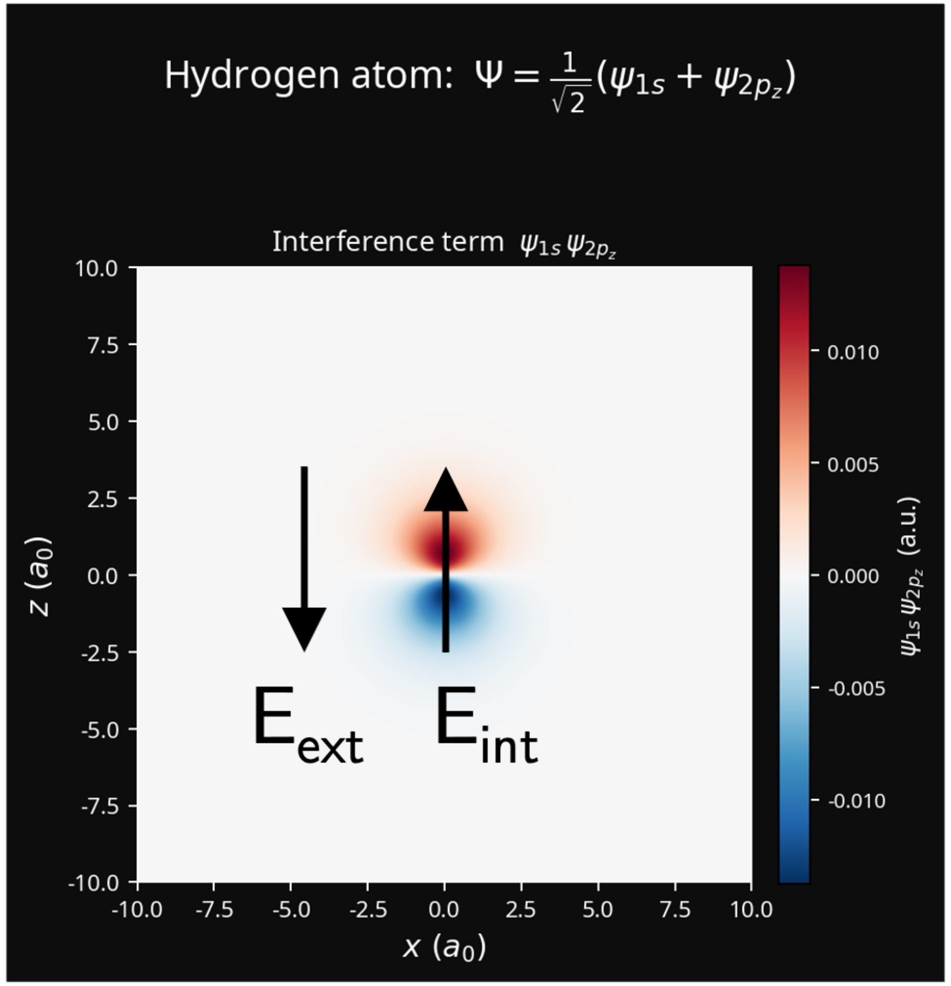



- Electron EDM only sees net field from nucleus when electon cloud is polarized

# Schiff's Theorem: L.I. Schiff, Phys. Rev. **132**, 2194 (1963)




- **Schiff's theorem**: A neutral atom or molecules with non-relativistic pointlike constituents will be in equilibrium under electrostatic forces and exhibit no energy shift due to EDM of electron or nucleus
- A neutral atom/molecule in a static external $\boldsymbol{E}_{ext}$ field will remain at rest, average electric field $\langle \vec{E}_{ext} + \vec{E}_{int} \rangle = 0$
- Atom/molecule polarized such that external $\boldsymbol{E}_{ext}$ is cancelled by redistribution of electrons
- Net electric field on electron or nuclear EDM will be zero! No EDM detectable!

# Evading Schiff's Theorem: L.I. Schiff, Phys. Rev. **132**, 2194 (1963)

- **Schiff's theorem**: A neutral atom or molecules with non-relativistic pointlike constituents will be in equilibrium under electrostatic forces and exhibit no energy shift due to EDM of electron or nucleus
- A neutral atom/molecule in static external $\boldsymbol{E}_{ext}$ field will remain at rest, average electric field on constituents must be $\varnothing$
- Atom/molecule polarized such that external $\boldsymbol{E}_{ext}$ is cancelled by redistribution of electrons

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- Loopholes (see D. Budker, D. Kimball, D. DeMille, *Atomic Physics*, Oxford Univ. Press, 2008)
  - When electron near nucleus, (1) moving relativistically, (2) see significant magnetic fields, $< \boldsymbol{E} > \neq 0$
  - (3) Nucleus is not pointlike, could lead to observable nuclear EDM

---

$$\begin{aligned} H_{edm} &= -\boldsymbol{d}_a \cdot \boldsymbol{E}_{ext} \\ &= -\boldsymbol{d}_e \cdot \boldsymbol{E}_{int} \end{aligned}$$

$\boldsymbol{d}_a$ = net atomic/molecular EDM
$\boldsymbol{E}_{ext}$ = external electric field
$\boldsymbol{d}_e$ = electron EDM
$\boldsymbol{E}_{int}$ = internal electric field seen by electron

Average force on electron must be zero: $< \boldsymbol{F} >=< \boldsymbol{F}_{elec} + \boldsymbol{F}_{mag} >= 0, \quad \boldsymbol{F}_{mag} = \nabla \boldsymbol{\mu}_e \cdot \boldsymbol{B}_{nuc}$

$$\boldsymbol{B}_{nuc} = \boldsymbol{E}_{nuc} \times \frac{\boldsymbol{v}}{c} \Rightarrow \left(\frac{Ze}{r^2}\right) \cdot \left(\frac{Z\alpha c}{c}\right) \Rightarrow \nabla B_{nuc} \approx \left(\frac{2Ze}{r^3}\right) \cdot (Z\alpha) = \frac{2Z^4 e}{a_0^3} \cdot Z\alpha, \text{ where } r \approx a_0/Z$$

Average force on electron must be zero: $< \boldsymbol{F} > = < \boldsymbol{F}_{elec} + \boldsymbol{F}_{mag} > = 0, \quad \boldsymbol{F}_{mag} = \nabla \boldsymbol{\mu}_e \cdot \boldsymbol{B}_{nuc}$

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$$\mu_e = g\frac{e}{2mc}\frac{\hbar}{2} \approx \frac{2}{mc}\frac{\hbar}{2}, \quad \text{substitute } \alpha = \frac{e^2}{\hbar c},\ a_0 = \frac{\hbar^2}{me^2}$$
$$\approx \frac{\alpha}{2} e\, a_0$$

$$\boldsymbol{E}_{\text{eff}} \approx \frac{\boldsymbol{F}_{mag}}{e} \approx \frac{1}{e}\nabla \boldsymbol{\mu}_e \cdot \boldsymbol{B}_{nuc} \approx \frac{1}{e}\left(\frac{\alpha}{2} e\, a_0\right)\left(\frac{2Z^5 e\alpha}{a_0^3}\right) \approx Z^5\alpha^2 \frac{e}{a_0^2} \approx Z^3\alpha^2 \left(\frac{e}{(a_0/Z)^2}\right)$$

$$\text{Using } V \approx \frac{4\pi}{3}\left(\frac{a_0}{Z}\right)^3, \text{ and } |\Psi_s(0)|^2 \approx \frac{Z}{\pi a_0^3}$$

$$< \boldsymbol{d}_a \cdot \boldsymbol{E}_{ext} > \approx \boldsymbol{d}_e \cdot \boldsymbol{E}_{\text{eff}} \approx d_e\, |\Psi_s(0)|^2\, V\, Z^3\alpha^2 \left(\frac{e}{(a_0/Z)^2}\right)$$

$$\Rightarrow d_a \approx Z^3\alpha^2\, d_e$$

- For heavy atoms with large $Z$, $d_a >> d_e$, enhanced by factors of several hundred!

## What about the Weak Interaction?

- Can write total Hamiltonian of an atom as: $H = H_{\text{EM}} + H_{\text{weak}}$
- $H_{\text{EM}}$: Electromagnetic and kinetic part, comprising Coulomb interaction, spin-orbit coupling, etc. Commutes with parity operator: $[H_{\text{EM}}, \hat{\Pi}] = 0$
- $H_{\text{weak}}$: Weak interaction between electrons and quarks via $Z_0$ exchange fundamentally violates parity, so $[H_{\text{weak}}, \hat{\Pi}] \neq 0$

⇒ Total atomic Hamiltonian doesn't commute with parity $[H, \hat{\Pi}] \neq 0$

### Consequence: Atomic/molecular states are not pure parity eigenstates

- Since $[H, \hat{\Pi}] \neq 0$, energy eigenstates are not parity eigenstates
- Weak interaction creates superposition of states of opposite parity
- Nominal $S_{1/2}$ states contains tiny admixture of $P_{1/2}$

$$|\tilde{S}_{1/2}\rangle = |S_{1/2}\rangle + \sum_n \frac{\langle nP_{1/2}| H_{\text{weak}} |S_{1/2}\rangle}{E_S - E_{nP}} |nP_{1/2}\rangle$$

- Has same structure as parity mixing from external electric field, but due to internal weak force
- Weak interaction is weak and very short range, parity admixture is tiny, order $10^{-11}$

- Weak interaction in atoms is contact interaction of electrons and nucleus
- Electrons move relativistically near heavy nucleus, proper treatment requires Dirac equation
- The nuclear-spin independent (NSI) parity non-conserving (PNC) Dirac Hamiltonian is:

$$H_{\text{weak}} = \frac{G_F}{2\sqrt{2}} Q_W \rho(\boldsymbol{r}) \gamma_5$$

- $G_F$ is Fermi coupling constant
- $\rho(\boldsymbol{r})$ is nucleon density distribution
- $\gamma_5$ Dirac matrix that mixes upper and lower components of electron spinor. These components have opposite parity, so $\gamma_5$ is the operator that explicitly breaks parity
- $Q_W = -N + Z(1 - 4\sin^2\theta_W)$ is weak charge of nucleus
- $N$=number of neutrons, $Z$=number of protons, $\sin^2\theta_W \approx 0.23$ so $(1 - 4\sin^2\theta_W) \approx 0.08)$ is small
- Effect scales like $Z^3$, with one factor from $Q_W \propto N \propto Z$, and $Z^2$ from electron density near nucleus

# What about the Weak Interaction?

- Non-relativisitic limit from Foldy-Wouthuysen transformation
- $\gamma_5$ reduces to term proportional to electron spin dotted with momentum

$$H_{\text{weak}}^{NR,NSI} = \frac{G_F}{4\sqrt{2}m_e c} Q_W \left( \boldsymbol{\sigma} \cdot \boldsymbol{p}\, \delta^3(\boldsymbol{r}) + \delta^3(\boldsymbol{r})\, \boldsymbol{\sigma} \cdot \boldsymbol{p} \right)$$

- Parity violation: from scalar product $\boldsymbol{\sigma} \cdot \boldsymbol{p}$, since under parity $\boldsymbol{\sigma} \to \boldsymbol{\sigma}$ (axial vector) and $\boldsymbol{p} \to -\boldsymbol{p}$ (polar vector) so under parity $H_{\text{weak}} \to -H_{\text{weak}}$
- Time-Reversal conservation: under time-reversal, $\boldsymbol{\sigma} \to -\boldsymbol{\sigma}$ and $\boldsymbol{p} \to -\boldsymbol{p}$ so $H_{\text{weak}} \to H_{\text{weak}}$
- Imaginary Phase i: since $\boldsymbol{p} = -i\hbar\nabla$, $H_{\text{weak}}$ has purely imaginary coupling between states of opposite parity - doesn't lead to linear Stark shift

# Why doesn't Weak Interaction lead to EDM and linear Stark shift?

- Weak interaction mixes states of opposite parity - resulting atomic energy eigenstates are states of mixed parity
- Why doesn't this lead to a permanent atomic EDM and linear Stark shift?
- Recall matrix element mixing $S$ and $P$ states $\langle P|\, H_{\text{weak}}\, |S\rangle$ is pure imaginary
- Parity-mixed atomic energy eigenstate has imaginary, tiny, mixing coefficient $\eta$:

$$|\tilde{S}\rangle = |S\rangle + i\eta\,|P\rangle$$

Now calculate expectation value of permanent EDM $\boldsymbol{d}$ from this mixed state:

$$\begin{aligned}\langle\tilde{S}|\,\hat{\boldsymbol{d}}\,|\tilde{S}\rangle &= [\langle S| - i\eta\,\langle P|]\,\hat{\boldsymbol{d}}\,[|S\rangle + i\eta\,|P\rangle] \\ &= \langle S|\,\hat{\boldsymbol{d}}\,|S\rangle + i\eta\,\langle S|\,\hat{\boldsymbol{d}}\,|P\rangle - i\eta\,\langle P|\,\hat{\boldsymbol{d}}\,|S\rangle + \langle P|\,\hat{\boldsymbol{d}}\,|P\rangle \\ &= 0\end{aligned}$$

- True because $\langle S|\,\hat{\boldsymbol{d}}\,|S\rangle = \langle P|\,\hat{\boldsymbol{d}}\,|P\rangle = 0$ and $\langle S|\,\hat{\boldsymbol{d}}\,|P\rangle = \langle P|\,\hat{\boldsymbol{d}}\,|S\rangle$ since dipole operator $\hat{\boldsymbol{d}}$ is a real observable operator
- Weak interaction doesn't violate time-reversal symmetry, leads to imaginary parity mixing coefficient
- Weak interaction imaginary mixing coefficient prevents atom from acquiring permanent EDM
- Atom/molecule will not exhibit linear Stark shift because of weak interaction

$\Rightarrow$ Permanent EDMs require both $T$- and $P$-violation, uniquely characterized by linear Stark shift

# The Quantum Two-Level System: A Universal Quantum Prototype

- Hamiltonian formulation
- Rotating-wave approximation
- Rabi oscillations
- Transition probability

Consider a quantum system with exactly two energy eigenstates: $|g\rangle$ (ground), $|e\rangle$ (excited)

Energies: $E_g = -\hbar\omega_0/2$ and $E_e = +\hbar\omega_0/2$, so the transition frequency is $\omega_0$.

The **bare Hamiltonian** is:

$$\hat{H}_0 = \frac{\hbar\omega_0}{2}\hat{\sigma}_z = \frac{\hbar\omega_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

The general state at time $t$ is:

$$|\Psi(t)\rangle = c_e(t)\,|e\rangle + c_g(t)\,|g\rangle$$

with $|c_e|^2 + |c_g|^2 = 1$.

## Physical realizations

- Atomic hyperfine levels (e.g. $^{133}$Cs, $^{87}$Rb)
- Nuclear spin-$\frac{1}{2}$ in NMR
- Optical transitions in trapped ions
- Superconducting qubits
- Electron spin in quantum dots

**Why two levels?** Many real systems have a pair of states that are nearly isolated from all others, making the two-level approximation both tractable and highly accurate.

Apply a monochromatic oscillating field of frequency $\omega$ and amplitude $V_0$. The coupling to the atom is off-diagonal (it drives transitions between $|g\rangle$ and $|e\rangle$):

$$\hat{H}_{\text{int}}(t) = \hbar\Omega\cos(\omega t)\,\hat{\sigma}_x = \frac{\hbar\Omega}{2}\begin{pmatrix} 0 & e^{i\omega t}+e^{-i\omega t} \\ e^{i\omega t}+e^{-i\omega t} & 0 \end{pmatrix}$$

where $\Omega = V_0/\hbar$ is the **Rabi frequency** (units of rad/s).

The total Hamiltonian is $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}(t)$. The Schrödinger equation $i\hbar\,\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$ gives coupled equations for the amplitudes:

$$i\dot{c}_e = \frac{\omega_0}{2}c_e + \Omega\cos(\omega t)\,c_g \tag{1}$$

$$i\dot{c}_g = -\frac{\omega_0}{2}c_g + \Omega\cos(\omega t)\,c_e \tag{2}$$

Note: these equations contain both *slowly* and *rapidly* oscillating terms.
Go to the **rotating frame** to isolate the slow dynamics.

## (1) Rotating frame substitution

Go into a frame rotating at $\omega$. Define slowly-varying amplitudes:

$$\tilde{c}_e(t) = c_e(t)\, e^{+i\omega t/2}, \qquad \tilde{c}_g(t) = c_g(t)\, e^{-i\omega t/2}$$

Substituting and expanding $\cos(\omega t) = \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$ yields terms oscillating at $(\omega_0 - \omega)$ and at $(\omega_0 + \omega)$.

## (2) Rotating-Wave Approximation (RWA)

Near resonance, $\omega \approx \omega_0$, so $(\omega_0 - \omega) \ll (\omega_0 + \omega)$. The rapidly oscillating terms at $(\omega_0 + \omega)$ average to zero on any timescale of interest and are *dropped*.

## (3) Rotating-frame Hamiltonian

After the RWA the Schrödinger equation becomes:

$$i\hbar \frac{d}{dt} \begin{pmatrix} \tilde{c}_e \\ \tilde{c}_g \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} -\delta & \Omega \\ \Omega & \delta \end{pmatrix} \begin{pmatrix} \tilde{c}_e \\ \tilde{c}_g \end{pmatrix}$$

where $\delta = \omega - \omega_0$ is the **detuning**.

This is a *time-independent* $2 \times 2$ eigenvalue problem — exactly solvable.

## Eigenvalues of the rotating-frame Hamiltonian:

The matrix $\frac{\hbar}{2}\begin{pmatrix} -\delta & \Omega \\ \Omega & \delta \end{pmatrix}$ has eigenvalues $\pm\frac{\hbar}{2}\Omega_R$, where:

$$\Omega_R \equiv \sqrt{\Omega^2 + \delta^2}$$

is the **generalised Rabi frequency**.

## Exact solution:

Starting in the ground state $|\Psi(0)\rangle = |g\rangle$ (i.e. $\tilde{c}_e(0) = 0$, $\tilde{c}_g(0) = 1$), the time evolution gives:

$$\tilde{c}_e(t) = -\frac{i\Omega}{\Omega_R} \sin\left(\frac{\Omega_R t}{2}\right)$$

## The Rabi Transition formula:

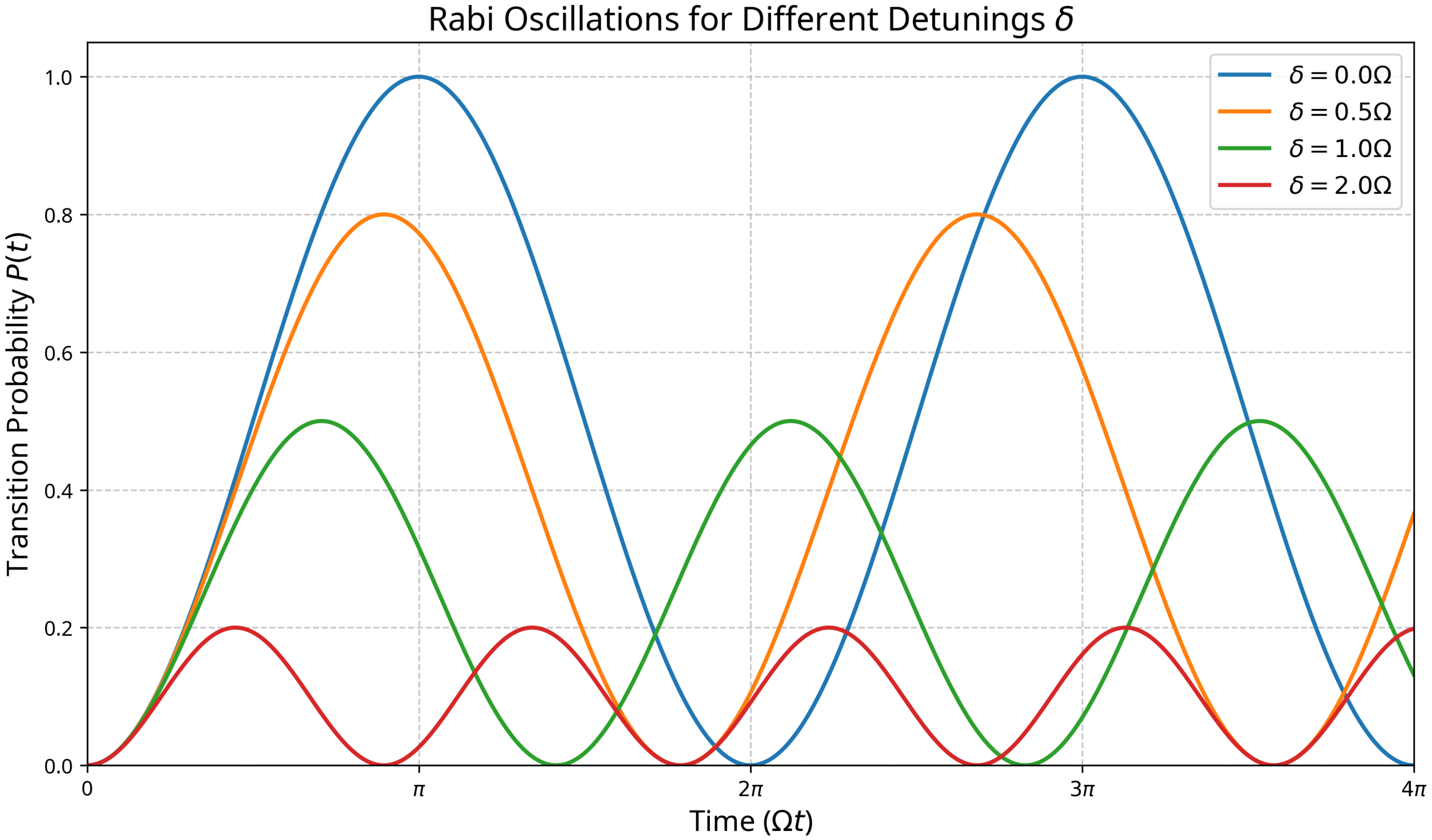
The probability of finding the system in $|e\rangle$ at time $t$ is:

$$\boxed{P_{g\to e}(t) = |\tilde{c}_e(t)|^2 = \frac{\Omega^2}{\Omega_R^2} \sin^2\left(\frac{\Omega_R\, t}{2}\right) = \frac{\Omega^2}{\Omega^2 + \delta^2} \sin^2\left(\frac{\sqrt{\Omega^2 + \delta^2}\; t}{2}\right)}$$

On Resonance ($\delta = 0$)
$P(t) = \sin^2\left(\frac{\Omega t}{2}\right)$
Complete population inversion at $t = \pi/\Omega$ ($\pi$-**pulse**).

Off resonance ($\delta \neq 0$)
$P_{\max} = \Omega^2/(\Omega^2 + \delta^2) < 1$.
Oscillation frequency increases; peak probability decreases.

Rabi Oscillations for Different Detunings




**Important features:**

- **On resonance** ($\delta = 0$, blue): Complete inversion, $P_{\max} = 1$, Period $T = 2\pi/\Omega$
- **Small detuning** ($\delta = 0.5\Omega$, orange): $P_{\max} = 0.80$, faster oscillation.
- **Moderate detuning** ($\delta = \Omega$, green): $P_{\max} = 0.50$, $\Omega_R = \sqrt{2}\,\Omega$.
- **Large detuning** ($\delta = 2\Omega$, red): $P_{\max} = 0.20$, rapid oscillation.

**Generally:**

As detuning $|\delta|$ increases, the peak probability $\Omega^2/(\Omega^2 + \delta^2)$ decreases and the oscillation frequency $\Omega_R$ increases.
Important for state preparation and detection in EDM searches

## Summary of two-level system discussion

- The two-level Hamiltonian with an oscillatory drive.
- The rotating-frame transformation and the RWA.
- The exact Rabi formula:

$$P(t) = \frac{\Omega^2}{\Omega^2 + \delta^2} \sin^2\!\left( \frac{\sqrt{\Omega^2 + \delta^2}\, t}{2} \right)$$

- Resonance condition and the $\pi$-pulse.

### Important parameters:

| | |
|---|---|
| $\omega_0$ | bare transition frequency |
| $\omega$ | drive frequency |
| $\delta = \omega - \omega_0$ | detuning |
| $\Omega$ | Rabi frequency |
| $\Omega_R = \sqrt{\Omega^2 + \delta^2}$ | generalised Rabi freq. |

## Extensions

- **Bloch equations:** include spontaneous emission and de-phasing via relaxation rates $T_1$, $T_2$.
- **Adiabatic passage:** sweep $\delta(t)$ slowly to achieve robust population transfer (STIRAP, rapid adiabatic passage).
- **Pulse sequences:** $\pi/2$–$\pi$–$\pi/2$ (spin echo), composite pulses for error correction.
- **Ramsey spectroscopy:** two $\pi/2$ pulses separated by free evolution — the basis of atomic clocks, EDM experiments
- **Floquet theory:** exact treatment beyond the RWA.

# Superposition of Two-level System as Rotation of Spin Projection

- Consider a two-level quantum system $|b\rangle$ and $|a\rangle$ with energy splitting $\hbar\omega_0$
- Splitting could be due to Zeeman interaction $H_Z = -\boldsymbol{\mu} \cdot \boldsymbol{B}$ where spin up and spin down levels shift in opposite directions
- Make equal superposition of the two states at $t = 0$, $|\Psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|b\rangle + |a\rangle)$
- Expect $|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\omega_0 t/2}|b\rangle + e^{+i\omega_0 t/2}|a\rangle\right)$
- Determine spin projections in different directions as function of time

$$\begin{aligned}
< S_z(t) > &= \langle\Psi(t)|\, S_z \,|\Psi(t)\rangle \\
&= 0 \\
< S_x(t) > &= \langle\Psi(t)|\, S_x \,|\Psi(t)\rangle \\
&= \cos(\omega_0 t) \\
< S_y(t) > &= \langle\Psi(t)|\, S_y \,|\Psi(t)\rangle \\
&= \sin(\omega_0 t)
\end{aligned}$$

- How do we measure an EDM? We place the particles in parallel (or anti-parallel) electric and magnetic fields and measure their Larmor precession frequency.
- The energy of the states depends on the relative orientation of the spin and the fields:

$$h\nu = 2\mu B \pm 2dE$$

## The Experimental Protocol:

1. Measure the precession frequency $\nu_{\uparrow\uparrow}$ with $\vec{E}$ parallel to $\vec{B}$.
2. Reverse the electric field direction so $\vec{E}$ is anti-parallel to $\vec{B}$.
3. Measure the new precession frequency $\nu_{\uparrow\downarrow}$.

The difference in frequency isolates the EDM contribution:

$$\Delta\nu = \nu_{\uparrow\uparrow} - \nu_{\uparrow\downarrow} = \frac{4dE}{h} \quad \Longrightarrow \quad \boxed{d = \frac{h\Delta\nu}{4E}}$$

**Challenge:** We need to measure $\Delta\nu$ with extraordinary precision.

Before Ramsey, the standard technique was I.I. Rabi's method (1938).

- Atoms pass through a *single* continuous oscillating field region of length $L$.
- The interaction time is $\tau = L/v$.
- The transition probability is given by the Rabi formula:

$$P(\delta) = \frac{\Omega^2}{\Omega^2 + \delta^2} \sin^2\left(\frac{\sqrt{\Omega^2 + \delta^2}\,\tau}{2}\right)$$

- The linewidth (FWHM) of the central resonance is $\Delta\nu \approx \Omega/\pi \approx 1/\tau$.

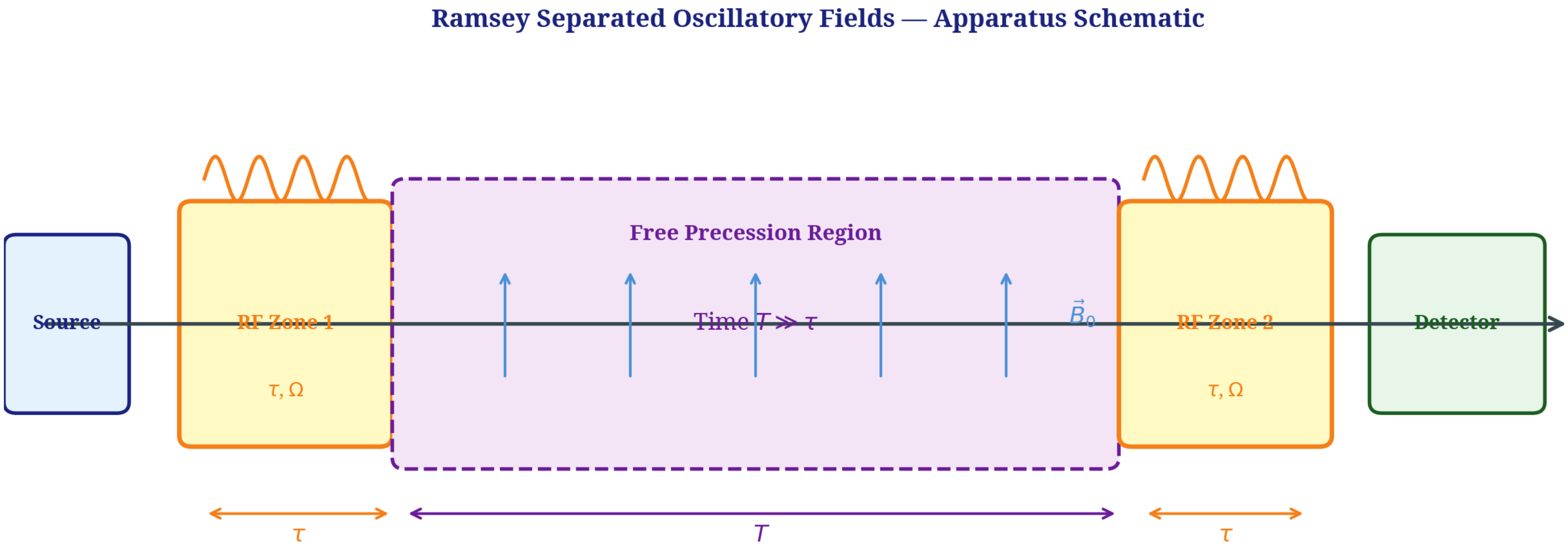
Limitations of the Rabi Method

- Linewidth is $\delta \approx \pi/\tau \approx \pi v/L$
- Narrower linewidth requires long interaction time $\tau$ and/or $L$
- This requires making the RF cavity very long.
- Difficult to maintain a perfectly uniform $B_0$ field over a large spatial volume. Field inhomogeneities broaden and wash out the resonance line.
- Tradeoff between resolution and systematic errors

## Ramsey's Insight (1949/1950)

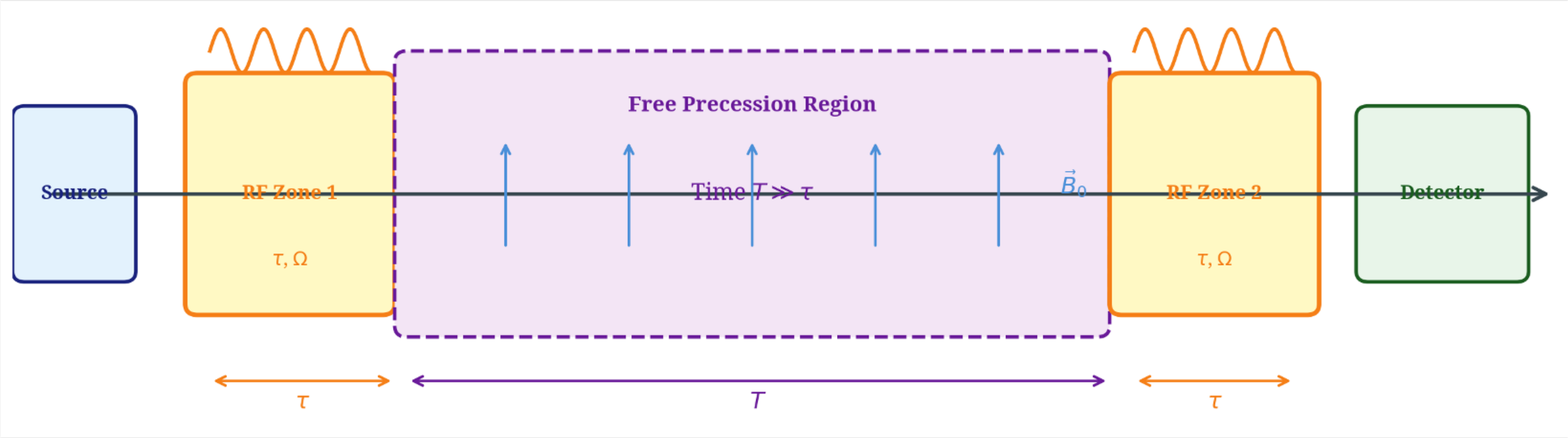
- Replace single long zone with two short zones separated by a long field-free region. Precision is set by drift time T.

Norman Ramsey (1949) solved this by separating the oscillatory field into two short regions, separated by a long field-free region.



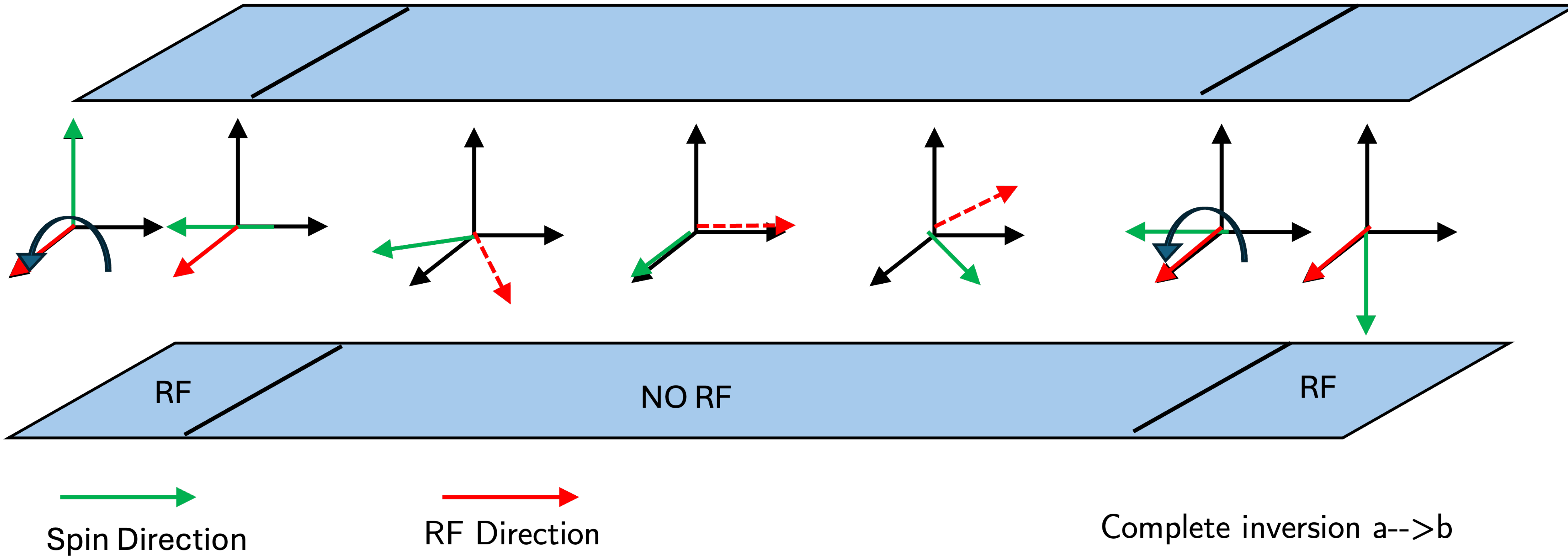

- **Zone 1:** First RF pulse of duration $\tau$ creates a coherent superposition (a $\pi/2$ pulse).
- **Free Precession:** Spins precess in $B_0$ (and $E$) for a long time $T \gg \tau$.
- **Zone 2:** Second RF pulse of duration $\tau$ recombines the states (interferometry).

# Ramsey's Method of Separated Oscillatory Fields: Physical Setup




| Region | Duration | Field Applied |
|---|---|---|
| First interaction zone | $\tau$ | Oscillating field $\mathbf{B}_\perp \cos(\omega t)$ |
| Free-precession (drift) | $T \gg \tau$ | Static field $\mathbf{B}_\parallel$ only |
| Second interaction zone | $\tau$ | Oscillating field (phase-coherent) |

- Key requirement is **phase coherence** between the two oscillating fields — both driven by the same oscillator.
- In the free-precession zone, the magnetic moment precesses at Larmor frequency $\omega_0 = \mu B_\parallel/\hbar$. Accumulated phase difference is $\phi = (\omega_0 - \omega)T = \delta T$.
- **Physical picture:** The first pulse creates a superposition state ($\pi/2$ rotation). The atom freely accumulates phase for time $T$. The second pulse reads out this phase.




Complete inversion a-->b

Consider a two-level atom $\{|a\rangle, |b\rangle\}$ with transition frequency $\omega_0$. In the rotating frame (RWA), the Hamiltonian is:

$$\hat{H} = \frac{\hbar}{2}\begin{pmatrix} \delta & \Omega \\ \Omega & -\delta \end{pmatrix}$$

**Solution to the Schrödinger equation** for initial conditions $a(t_0) = a_0$, $b(t_0) = b_0$:

$$a(t+t_0) = a_0\left[\cos\frac{\Omega_g t}{2} - \frac{i\delta}{\Omega_g}\sin\frac{\Omega_g t}{2}\right] - b_0\frac{i\Omega}{\Omega_g}\sin\frac{\Omega_g t}{2}$$
$$b(t+t_0) = -a_0\frac{i\Omega}{\Omega_g}\sin\frac{\Omega_g t}{2} + b_0\left[\cos\frac{\Omega_g t}{2} + \frac{i\delta}{\Omega_g}\sin\frac{\Omega_g t}{2}\right]$$

**In the free-precession zone** $(\Omega = 0)$: pure phase accumulation, $a \to a\, e^{-i\delta T/2}$, $b \to b\, e^{+i\delta T/2}$.

- **Note:** This uses oscillating field $\Omega$ along $\hat{x}$
- Using $\Omega e^{1\phi}$ changes this direction. Can use this freedom to make phase shifts between the two RF regions

# Transfer Matrices for Each Zone

- The full sequence is a product of three propagators. For the interaction zones:

$$U_{\mathsf{int}}(\tau) = \begin{pmatrix} \cos\frac{\Omega_g\tau}{2} - \frac{i\delta}{\Omega_g}\sin\frac{\Omega_g\tau}{2} & -\frac{i\Omega}{\Omega_g}\sin\frac{\Omega_g\tau}{2} \\ -\frac{i\Omega}{\Omega_g}\sin\frac{\Omega_g\tau}{2} & \cos\frac{\Omega_g\tau}{2} + \frac{i\delta}{\Omega_g}\sin\frac{\Omega_g\tau}{2} \end{pmatrix}$$

- For the free-precession zone ($\Omega = 0$):

$$U_{\mathsf{free}}(T) = \begin{pmatrix} e^{-i\delta T/2} & 0 \\ 0 & e^{+i\delta T/2} \end{pmatrix}$$

- The total evolution operator is:

$$U_{\mathsf{Ramsey}} = U_{\mathsf{int}}(\tau) \cdot U_{\mathsf{free}}(T) \cdot U_{\mathsf{int}}(\tau)$$

- Matrix multiplication produces cross terms — interference between transitions in the *first* vs. *second* zone.
- This is the origin of **Ramsey fringes**.

- Assuming the system starts in the ground state $|g\rangle = \binom{0}{1}$, we compute the final amplitude $c_e = (U_{\text{total}})_{12}$.
- After multiplying the matrices and taking the absolute square $P(\delta) = |c_e|^2$, we obtain the exact Ramsey transition probability $P = |b(2\tau + T)|^2$:

$$P_{\text{Ramsey}}(\delta, T, \tau) = \frac{4\Omega^2}{\Omega_g^2} \sin^2\left(\frac{\Omega_g \tau}{2}\right) \left[\cos\frac{\Omega_g \tau}{2} \cos\frac{\delta T}{2} - \frac{\delta}{\Omega_g} \sin\frac{\Omega_g \tau}{2} \sin\frac{\delta T}{2}\right]^2$$

## Physical Interpretation

- **Rabi envelope:** The prefactor is the transition probability for a single zone.
- **Interference factor:** The bracketed term contains the cosine of the accumulated phase $\delta T/2$.

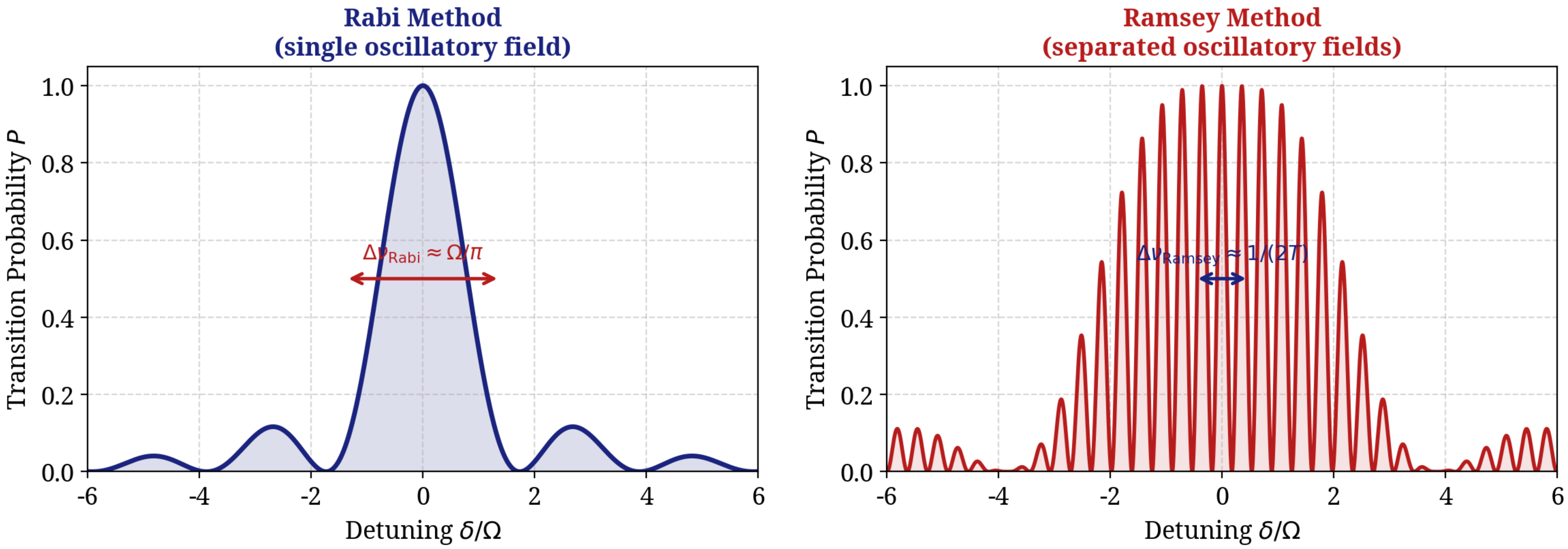
## The $\pi/2$-Pulse Limit

- For $\Omega_g \tau = \pi/2$ and $|\delta| \ll \Omega$:

$$P_{\text{Ramsey}} \approx \cos^2\left(\frac{\delta T}{2}\right)$$

- This is the **ideal Ramsey fringe** — a pure cosine-squared oscillation with period $\Delta\delta = 2\pi/T$.

- The state vector $\mathbf{r} = (\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle)$ traces a path on the unit sphere.
- **Step 1: First $\pi/2$ Pulse**
  Starting from south pole $|a\rangle$, rotate by $\pi/2$ about x-axis.
  State becomes equatorial superposition:
  $$\frac{1}{\sqrt{2}}(|a\rangle - i|b\rangle)$$
- **Step 2: Free Precession**
  Precess about z-axis at detuning rate $\delta$. Accumulated angle $\phi = \delta T$:
  - If $\delta = 0$: no precession.
  - If $\delta \neq 0$: rotates by $\delta T$.
- **Step 3: Second $\pi/2$ Pulse**
  Rotate by $\pi/2$ about x-axis. Final z-projection depends on $\delta T$:
  - If $\delta T = 0$: reaches $|b\rangle$, $P = 1$.
  - If $\delta T = \pi$: returns to $|a\rangle$, $P = 0$.
- **Oscillation between $P = 0$ and $P = 1$ produces the Ramsey fringes.**

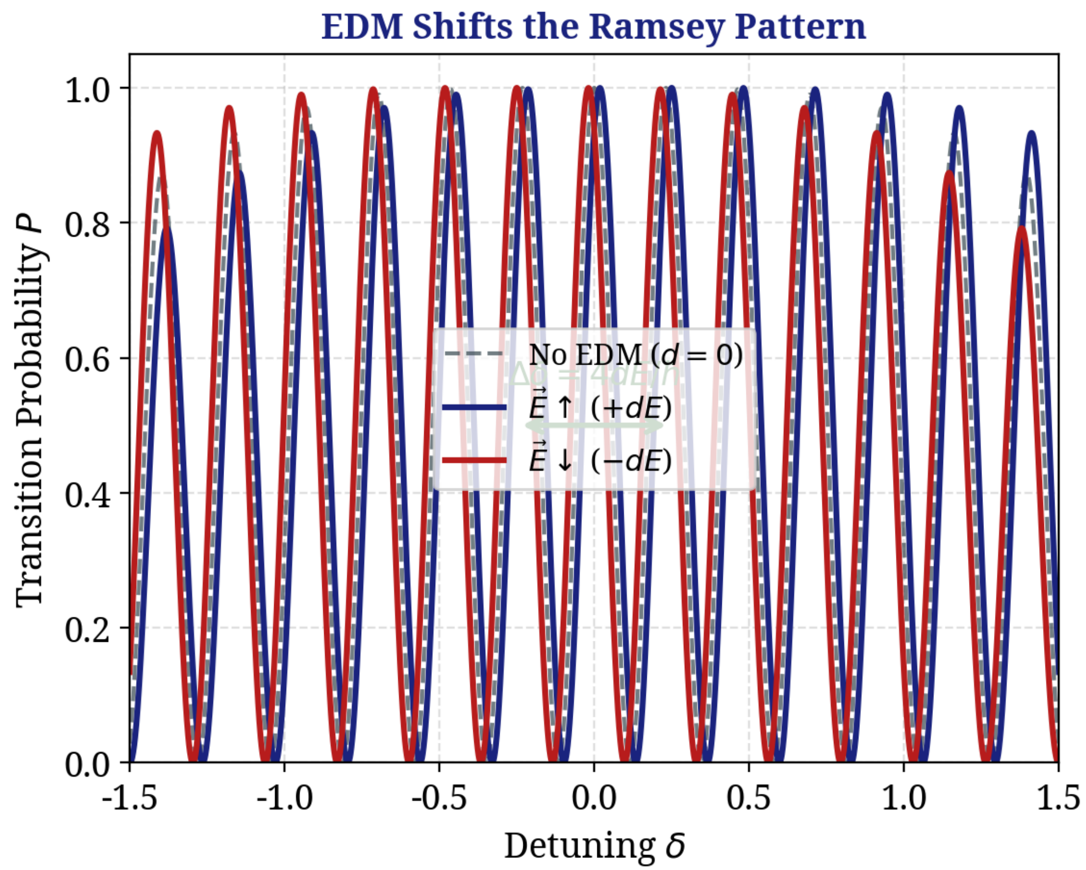


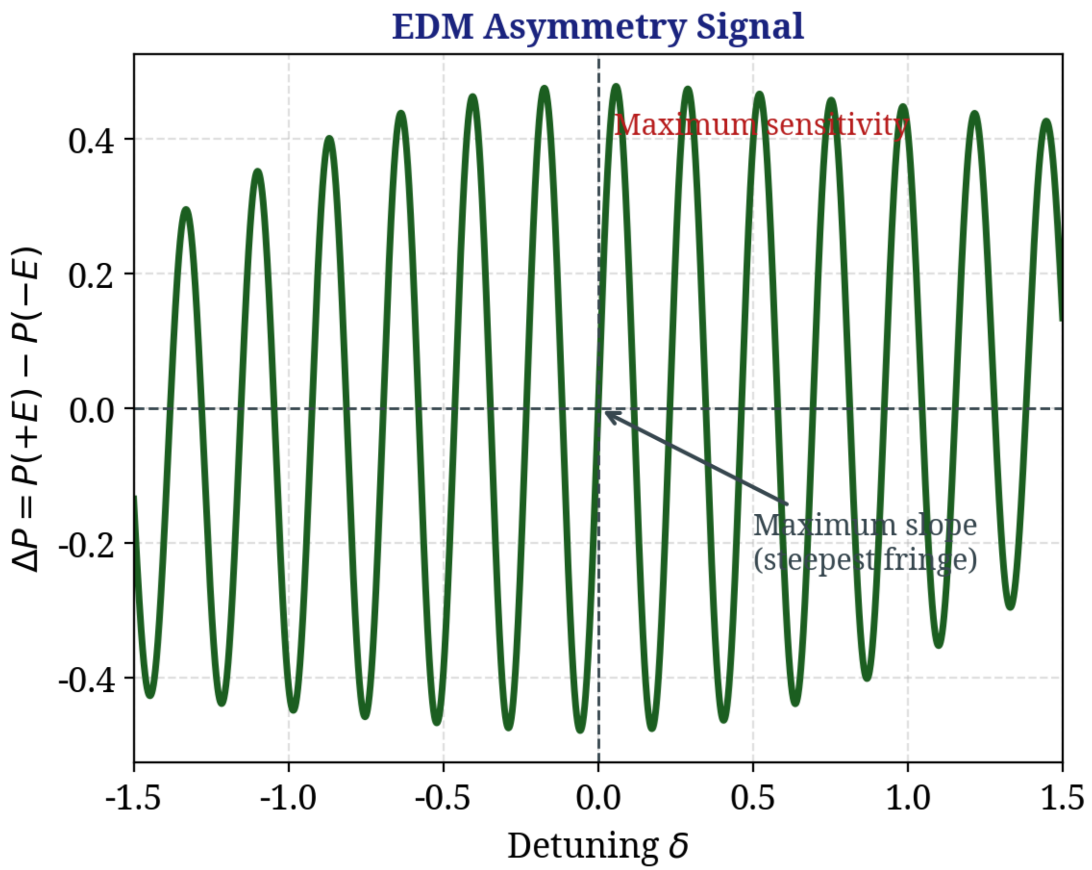

## Rabi Method

- Linewidth determined by total time in the RF field.
- Highly sensitive to $B$-field inhomogeneity over the entire RF cavity.

## Ramsey Method

- Linewidth determined by the *free precession* time $T$.
- RF zones can be very short; only the *average* $B$-field in the free region matters.
- Central fringe FWHM $\Delta\nu \approx 1/(2T)$




## The EDM Shift

- Reversing the E-field shifts the entire Ramsey fringe pattern by $\Delta\delta = 4dE/\hbar$
- Park the laser/RF frequency at the steepest part of the central fringe (maximum slope).
- A small horizontal shift $\Delta\delta$ translates into a large change in transition probability $\Delta P$.