Homework: Nuclear Physics of Neutron Stars and Neutron Star Mergers

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I would be happy to help. Please let me know any questions.

I. MINIMUM DENSITY

Consider two stars of equal mass and equal radius in a circular orbit of frequency ν (angular frequency $\omega = 2\pi\nu$). Show that each star must have a density ρ greater than,

$$\rho > 12\pi \frac{\nu^2}{G}, \tag{1}$$

to avoid collision. Such a binary system generates quadrupole gravitational wave radiation at frequency $f_{GW} = 2\nu$. If the LIGO detectors are only sensitive to GW at frequencies $f_{GW} > 20$ Hz, what is the minimum density of any LIGO sources?

II. TOV MASS OF FREE FERMI GAS

A. Free Fermi gas equation of state

Show that the equation of state for a free relativistic Fermi gas of neutrons of mass m can be written,

$$p = \frac{5}{24\pi^2} k_f^3 E_f - \frac{1}{8\pi^2} k_f E_f^3 + \frac{m^4}{16\pi^2} \ln \frac{k_f + E_f}{E_f - k_f}, \tag{2}$$

$$\epsilon = \frac{1}{8\pi^2}(k_f^3 E_f + k_f E_f^3) - \frac{m^4}{16\pi^2} \ln \frac{k_f + E_f}{E_f - k_f}. \tag{3}$$

Here p is the pressure, ϵ the energy density, k_f the Fermi momentum and $E_f = (k_f^2 + m^2)^{1/2}$. What does $p = p(\epsilon)$ become in the high density limit $k_f \gg m$?

B. Interpolating the EOS

Write a table of k_f, p, and ϵ values and write a routine to interpolate your table and find $\epsilon = \epsilon(p)$. I suggest you use cgs units so p and ϵ have units of ergs/cm^3.

C. Numerically integrate the TOV equation

Numerically integrate the TOV equation,

$$\frac{dp}{dr} = -\frac{GM(r)\epsilon}{c^2 r^2}[1 + \frac{p}{\epsilon}][1 + \frac{4\pi r^3 p}{M(r)c^2}][1 - \frac{2GM(r)}{c^2 r}]^{-1} \tag{4}$$

and the enclosed mass,

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon / c^2. \tag{5}$$

Start from $r = 0$, $M(0) = 0$, and $p(0) = p_c$ where p_c is a constant. Use a step size of say $\delta r = 10$ cm. At each radius use your equation of state routine to find $\epsilon = \epsilon(p(r))$. Integrate until $p(R) = 0$. This gives the radius of the star R and the mass $M(R)$. Repeat for different values of p_c and show that the maximum mass of a neutron star with a free Fermi gas EOS is about $0.7 M_\odot$. This is the problem that Oppenheimer and Volkov solved in 1939. Hint, here are my plots of star mass versus p_c and mass versus radius.

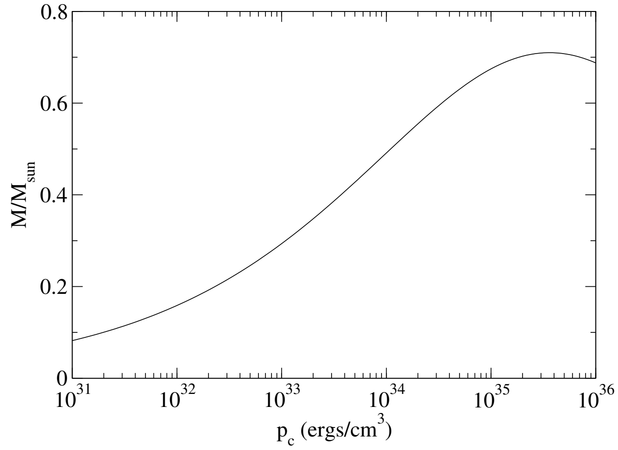

FIG. 1: Mass versus central pressure p_c for a free Fermi gas EOS.

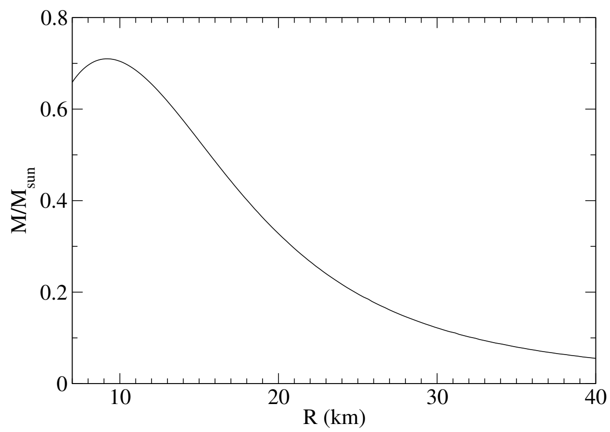

FIG. 2: Mass versus radius for a free Fermi gas EOS.