Global Analysis of New Physics Interactions

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Abstract

Recent discussion of violation of unitarity of the Cabibbo-Kobayashi-Maskawa matrix illuminates a need for physics beyond the Standard Model (SM). In this paper, we report efforts of constructing a method of probing the SM as an effective field theory (SMEFT). Through this method, we hope to demonstrate statistically significant evidence that the SMEFT aligns more accurately with experimental measurements than its core SM constituent. Of particular interest is the reproduction of analysis of V_{ud} and V_{us} constraints, as well as a demonstration of right-handed interactions with the weak force. In addition, we highlight the inclusion of new Parity Violation observables in our code in the search for a potential impact on determination of Wilson Coefficients.

1 Background Information

The Standard Model is an attempt to explain matter at its most irreducible level, with a generalization to include how matter interacts with other matter. The natural language of the Standard Model is the Lagrangian. A lagrangian that is invariant under a certain transformation of its field argument obeys certain symmetry laws, which in turn correspond to conserved quantities by Noether's Theorem. Hence the Standard Model can be defined by its particle content and its symmetries.

The action is defined as the integral of the lagrangian density in Minkowski Space,

$$S = \int \mathscr{L}(\phi, \partial\phi) d^4x, \tag{1}$$

where the principle of least action,

$$\delta S = 0, \tag{2}$$

gives rise to the Euler Lagrange Equations. The action has a more direct application to particle physics, in terms of the path integral formulation where the probability amplitude for a system to transform from one state to another is given by the sum over all possible paths, weighted by the action-

$$\int \mathscr{D}[\phi] e^{iS/\hbar}. \tag{3}$$

The path integral formulation can be pictorially represented in a Feynman Diagram, in which the mathematical details of perturbation theory are encoded in certain diagrammatic rules. A simple example of a Feynman diagram is the typical beta decay, in which a down quark in the neutron converts to an up quark in the resulting proton, along with an electron and an electron antineutrino:

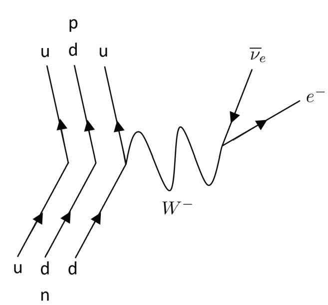

Figure 1: Beta Decay Feynman Diagram

The presence of the W^- boson, along with the flavor change of the up to down quark, indicates that this is a weak process. The number of vertices in the diagram correspond to interaction strength (perturbation order). In this case, the interaction strength is given by $g^2 V_{ud}$, where g is related to the fermi constant, G_F, and the boson mass, m_W, by $G_F = g^2/(4\sqrt{2}m_W^2)$. V_{ud} is a constant specific to the flavor change of the up to down quark.

These diagrams for weak processes in which quark flavor changes extend beyond just beta decay. For example, Kaon decays contain flavor changes of up to strange quarks, which gives rise to the V_{us} term. All possible flavor changes can be encoded in a matrix which relates the down quark weak eigenstates to the mass eigenstates, known as the Cabibbo–Kobayashi–Maskawa (CKM) Matrix:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} \tag{4}$$

The symmetries of the SM lead to a CKM matrix that is unitary, which implies

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1 \tag{5}$$

Experimentally, the term V_{ij} is directly related to the decay rate of a process. The decay rate Γ is proportional to $\mid V_{ij} \mid^2$, allowing for the comparison of experiment against theory as a test of validity of the standard model.

Experimental measurements of decay rates of these CKM matrix elements [2] demonstrate a tension in unitary of 2.2 standard deviations:

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9985(5) \tag{6}$$

Such a tension implies the opportunity to explore beyond the standard model physics.

Treating the Standard Model as an effective field theory (SMEFT) allows for the introduction of small perturbations to the SM Lagrangian [3]:

$$\mathscr{L} = \mathscr{L}_{SM} + \sum_k C_k Q_k \tag{7}$$

Where k runs over all possible beyond the standard model (BSM) interactions, C denotes BSM Wilson Coefficients which arise from renormalization, and Q denotes operators specific to the new interactions. For each k there exist new C_k and corresponding Q_k that denote a specific interaction. The C_k coefficients determine the strengths of the new interactions induced by BSM physics. These couplings can affect the theoretical predictions of observables, which can be compared to experimental measurements. This ultimately gives a way of quantifying how well the new theory holds in a chi-squared goodness of fit test, comparing theory against experiment.

1.1 Methods

The χ^2 goodness of fit test between the experimental and theoretical determination of an observable is given by the equation

$$\chi^2 = (O_{theory} - O_{expt})^2/\sigma^2. \tag{8}$$

In general, however, observables are correlated to each other. Thus it makes sense to use the chi-squared metric in terms of a covariance matrix,

$$\chi^2 = (O_{theory} - O_{expt})^T.Cov^{-1}.(O_{theory} - O_{expt}), \tag{9}$$

Where the observables are now vectors.

The covariance matrix also yields a computationally efficient way of calculating χ^2 values for many observables. The following procedure outlines the steps we take in a Mathematica function to determine the Wilson Coefficients and correlation matrix (which comes from normalizing the covariance matrix):

1. Define the χ^2 in terms of (undetermined) observables to be marginalized over using (9),

2. Construct the inverse covariance matrix by [1]

$$Cov_{ij}^{-1} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial C_i \partial C_j} \tag{10}$$

3. Calculate the inverse of the result above, and then calculate the correlation matrix using

$$Corr_{ij} = \sigma_i^{-1}.Cov_{ij}.\sigma_j^{-1} \tag{11}$$

4. Isolate the submatrix corresponding to the desired observables from the correlation matrix.

5. Invert step three to obtain a new covariance matrix containing only the desired observables

$$Cov_{nm} = \sigma_n.Corr_{nm}.\sigma_m \tag{12}$$

6. Finally, calculate the new χ^2 according to (10).

2 Results

The scope of this project includes reproducing figure 5 of [1], reproducing figure 1 of [2] setting theory parameters to their central values as well as assuming theory parameters are variables with uncertainties, and extending the existing code to include parity violation observables.

2.1 Right Handed Interactions

The Standard Model Lagrangian only allows left-handed fermions to interact with the weak force:

$$L = -\frac{g}{\sqrt{2}} W_\mu \overline{\psi}_L \gamma^\mu \psi_L. \tag{13}$$

However, the Standard Model Effective Theory allows the possibility to explore right-handed currents, which could offer an explanation for unitarity violation of the CKM matrix.

Two SMEFT Wilson Coefficients which explore right-handed interactions are the C_{Hud}^{11} and C_{Hud}^{12}. Collecting the χ^2 according to (9), we plot the contours of σ, 2σ, and 3σ in the C_{Hud}^{12} - C_{Hud}^{11} plane.

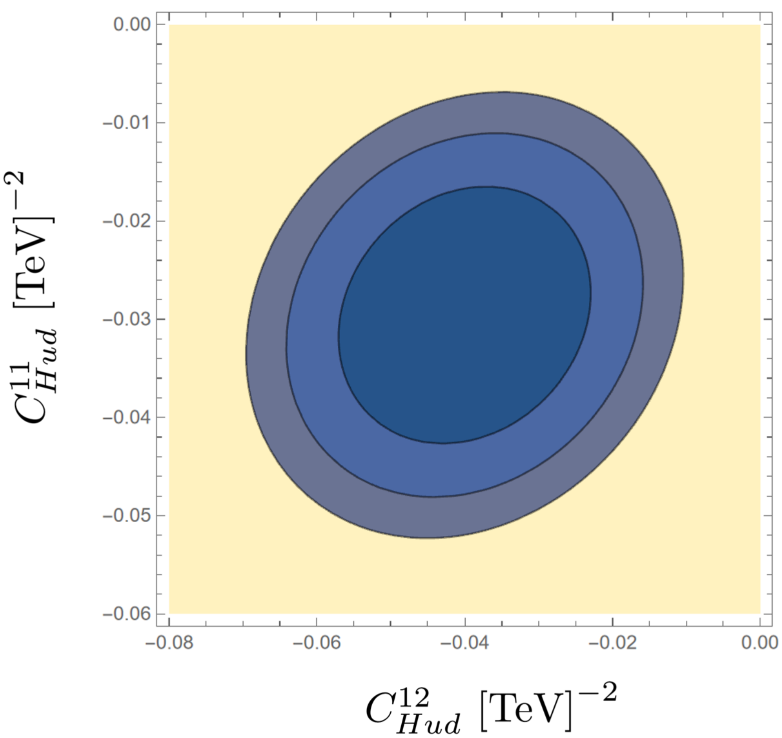

Figure 2: Contour plot displaying σ, 2σ, and 3σ standard deviations in the right-handed wilson coefficient plane.

The region of 3σ deviates significantly from the origin, demonstrating a possibility for right-handed weak interactions to be physical. It is worth noting that the regions above are slightly different than figure 5 of [1]; this is due to the way the χ^2 was calculated, using equation (9). A more accurate calculation would also include the uncertainties of several theoretically determined parameters (using Eq. 10) that have been neglected here.

2.2 CKM Unitarity Violation

Because V_{ub} is small [2], unitarity of the CKM matrix can be tested in the V_{ud} - V_{us} plane against the so-called unitarity circle,

$$V_{ud}^2 + V_{us}^2 = 1. \tag{14}$$

Again following equation (9) we attempted to reconstruct Figure 1 of [2]. The experimental data used comes from processes involving flavor changes from a down to an up quark or a strange to an up quark. Thus the plot contains different "bands" that restrict regions in terms of σ corresponding to different interactions.

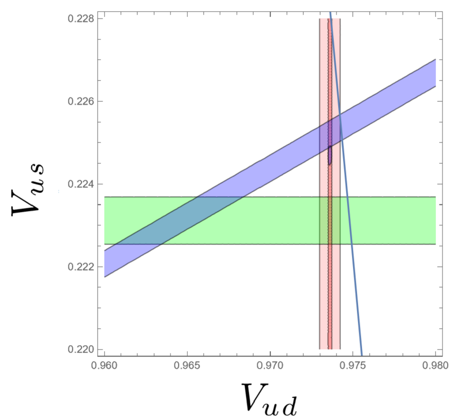

Figure 3: Contour plot of various χ^2 values corresponding to different processes, as a function of V_{ud} and V_{us}. The green band is derived from K_{l3} decays. The leftmost red band is derived from 0^+ decays, while the rightmost red band is derived from neutron decays. The diagonal band contains a certain ratio of Kaon to Pion decays (K_{l2}/π_{l2}). The small ellipse contains all of these fits combined, and the blue line is the unitarity circle.

Interestingly, the combined ellipse is much smaller than the results of [2]. This is due to the neglected uncertainties of the theory parameters and their correlations, which appear, for example, through the K_{l2}/π_{l2} decays that depend on several of them. This suggests there is significant correlation between the wilson coefficients comprising these decays. Here, K_{l2} corresponds to the process where a kaon decays into a charged lepton and a neutrino, and K_{l3} corresponds to the process where a kaon decays into a pion, charged lepton, and a neutrino.

To test this correlation, we recalculated the total χ^2 using the entire procedure outlined in the methods section. As a result, the ellipse size significantly increased, resembling that of [2]:

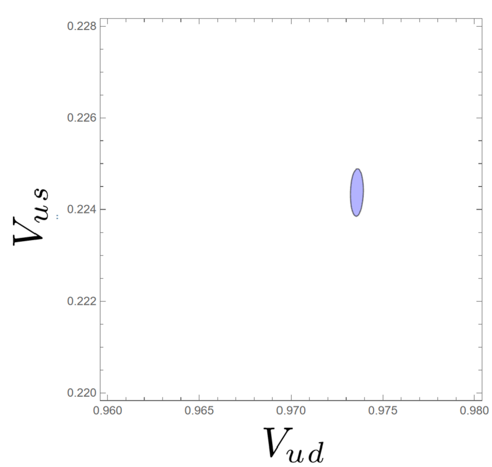

Figure 4: Total χ^2 ellipse with full list of theory parameters included.

2.3 Parity Violation Observables

So far when performing fits, the mathematica function uses data from Beta Decays, Electroweak Precision Observables, and LHC bounds. The function does not contain parameters related to Parity Violation or Coherent Elastic Neutrino-Nucleus Scattering. These processes are in principle sensitive to several of the BSM interactions that could explain the CKM unitarity discrepancy. Parity Violation can be observed in asymmetries of electron-deuterium scattering, in the weak charge of Cs 133, and in electron-proton scattering.

According to (8), we can rewrite the SMEFT wilson coefficients in terms of the SM wilson coefficients. In our model independent study, these become the parameters we marginalize over. Quoting Ref. [3], we state the effective field theory coefficients:

$$C_{1u}^{e,SMEFT} = \frac{\sqrt{2}}{4G_F}(C_{lq}^{(3)} - C_{lq}^{(1)} + C_{eu} + C_{qe} - C_{lu} - |V_{ud}|^2(C_{\phi q}^{(3)} - C_{\phi q}^{(1)}) + C_{\phi u}), \tag{15}$$

$$C_{2u}^{e,SMEFT} = \frac{\sqrt{2}}{4G_F}(C_{lq}^{(3)} - C_{lq}^{(1)} + C_{eu} - C_{qe} + C_{lu} - (1-4s_w^2)(|V_{ud}|^2(C_{\phi q}^{(3)} - C_{\phi q}^{(1)}) + C_{\phi u})), \tag{16}$$

$$C_{1d}^{e,SMEFT} = \frac{\sqrt{2}}{4G_F}(-C_{lq}^{(3)} - C_{lq}^{(1)} + C_{ed} + C_{qe} - C_{ld} + C_{\phi q}^{(3)} + C_{\phi q}^{(1)} + C_{\phi d}), \tag{17}$$

$$C_{2d}^{e,SMEFT} = \frac{\sqrt{2}}{4G_F}(-C_{lq}^{(3)} - C_{lq}^{(1)} + C_{ed} - C_{qe} + C_{ld} - (1-4s_w^2)(C_{\phi q}^{(3)} - C_{\phi q}^{(1)}) + C_{\phi d})). \tag{18}$$

Here, s_W is the weak mixing angle, $\sin\theta_W$, and G_F is the Fermi constant. Each wilson coefficient describes a different coupling; for instance, $C_{\phi u}$ describes a coupling of an up quark to the Higgs field.

In an effort to see an impact on wilson coefficients, we implemented the following observables in the code:

1. Weak charge of ^{133}Cs, $Q_w(^{133}\text{Cs})$:

$$\begin{aligned} Q_w(^{133}\text{Cs}) = -73.24 - 2(Z(2(C_{1u} + .1888) + (C_{1d} - .3419)+ \\ n((C_{1u} + .1888) + 2(C_{1d} - .3419))), \end{aligned} \tag{19}$$

   Where Z is the atomic number and n is the number of neutrons. The experimentally measured value is -72.94, with an uncertainty of 0.43. From this we used (9) to construct the χ^2 in terms of the undetermined SMEFT wilson coefficients.

2. Parity Violation in electron-deuterium scattering asymmetry (for which there are two observables):

$$\begin{aligned} A_1 = -87.7 * 10^{-6} + 1.156 * 10^{-4}((2(C_{1u} + 0.1888) - (C_{1d} - 0.3419)) \\ +0.348(2(C_{2u} + 0.0352) - (C_{2d} - 0.0249)) \end{aligned} \tag{20}$$

$$\begin{aligned} A_2 = -158.9 * 10^{-6} + (2(C_{1u} + 0.1888) - (C_{1d} - 0.3419)+ \\ 0.594(2(C_{2u} + 0.0352) - (C_{2d} - 0.0249))) \end{aligned} \tag{21}$$

   A_1 is measured to be $-91.10 * 10^{-6}$ with an uncertainty of $3.11 * 10^{-6}$. A_2 is measured to be $-160.80 * 10^{-6}$, with an uncertainty of $6.39 * 10^{-6}$.

3. Parity Violation in Electron - Proton Scattering (approximating the proton's weak charge):

$$Q_p = 0.0710 - 2(2(C_{1u} + C_{1d})), \tag{22}$$

With an approximate value of 0.0704 and an uncertainty of 0.0047.

3 Future Work

Further analysis remains to be carried out for the Parity Violation Observables discussed above. The new total χ^2 for these PV Observables was incorporated in the fit function. Now we must compare the results (correlation matrix and determination of Wilson Coefficients) with and without the inclusion of PV χ^2 terms to determine if PV has any significant impact on BSM explanations of the CKM unitarity discrepancy or other inconsistencies of the Standard Model. Similar to the results of the V_{ud} - V_{us} plot, we would hope to see significant changes in the bands constraining the values of our parameters.

Another possibility would be to incorporate the Coherent Elastic Neutrino–Nucleus Scattering Observables in the fit as well in order to further expand the scope of the fit function. These Observables would require different SMEFT Wilson Coefficients as outlined by equation (3.23) of Ref. [3].

The final goal of this project is to make the mathematica notebook more user friendly and accessible to the public. This can be accomplished by streamlining the fitting process, making it easier for users to understand what is going on.

Acknowledgement

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