Thermal Evolution of Dark Matter-Admixed Neutron Stars

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Abstract

This paper investigates the potential identification of dark matter through heating signatures observed in neutron stars. Neutron stars, the dense remnants of massive stars that undergo supernova explosions, provide a unique environment for dark matter detection due to their extreme density and strong gravitational fields. Using a Python-based model, the structure and thermal behavior of neutron stars are analyzed by solving the Tolman-Oppenheimer-Volkoff (TOV) equations and the heat equation with dark matter-induced heating terms. The model is used to predict the mass-radius relationship and cooling curves for neutron stars, taking into account various equations of state (EOS) and dark matter fractions. By comparing the theoretical cooling curves with observational data, the study aims to detect deviations that could indicate the presence and influence of dark matter within neutron stars. The results could provide insights into the nature of dark matter and its interactions with baryonic matter in extreme astrophysical environments.

1 Neutron Stars

1.1 Neutron Star Formation

Neutron stars are the dense remnants of massive stars that explode as supernovae. This process begins with the rapid collapse of the star's core, occurring within 0.5 to 1 second, which creates a shock wave that triggers the supernova [3]. The remaining core contracts and forms a proto-neutron star (PNS), marking the start of a critical phase in the star's evolution [5].

During this Kelvin-Helmholtz phase, the PNS undergoes deleptonization, a process where the star loses electrons primarily through electron capture and the emission of neutrinos. As the PNS cools, it emits neutrinos, which carry away most of the energy—about 10^{53} ergs [3]. These neutrinos are vital to the dynamics of the supernova, as they can reignite the shock wave, ensuring the explosion's success [5].

As the neutron star continues to cool, neutrino emission and surface radiation play key roles, with the cooling rate influenced by factors like the star's mass, magnetic field, and insulating layers [6]. Neutrino emission is the more efficient cooling process when the star is hotter, but after about 10^6 years, surface radiation of photons becomes the dominant cooling mechanism. The Kelvin-Helmholtz phase is particularly significant for detecting neutrinos on Earth, as the sustained emission during this period can be observed using various detectors [3].

1.2 Identifying Dark Matter

Neutron stars, with their extreme density and strong gravitational fields, are also excellent candidates for identifying dark matter. They are highly efficient at capturing dark matter particles, which transfer kinetic energy to the star's particles as they fall into the star at high speeds, causing the star to overheat—a process known as kinetic heating [2]. Additionally, when a neutron within the star fills a lower energy level, it leaves a gap at its original higher level, triggering a chain reaction that releases energy as radiation and kinetic energy. This process also contributes to the heating of the neutron star [2].

The effectiveness of dark matter capture by neutron stars depends on the mass of the dark matter particles. For particles with masses less than that of a neutron, only a small fraction of neutrons near their Fermi surface can gain enough momentum to scatter to a higher state. This phenomenon, known as 'Pauli-blocking,' results in a capture cross section that is inversely proportional to the square root of the dark matter's energy and mass [2].

Dark matter capture can occur in various layers of the neutron star, including the inner crust, where scattering with superfluid neutrons—primarily through the excitation of phonons—can occur for dark matter particles with masses below about 10 MeV [2]. The high speeds of dark matter particles captured by neutron stars help overcome the challenges posed by the lower speeds of dark matter on Earth, potentially leading to significant improvements in Earth-bound dark matter searches, particularly for interactions that depend on spin or speed [2].

In comparing the measured surface temperature of neutron stars with theoretical models, this project aims to determine what, if any, signatures of dark matter can be found from neutron star cooling measurements. These signatures would result from specific dark matter-induced heating mechanisms under consideration. While the dark matter fraction within the star certainly affects the heating process, it cannot be directly observed. Instead, any deviations from expected cooling results in the absence of dark matter could indirectly suggest the presence and influence of dark matter

2 Modeling A Star

2.1 Initial Stellar Structure

To model the initial stellar structure, a code was built utilizing Python. The code is designed to understand the structure of neutron stars. It uses a set of equations, known as the Tolman-Oppenheimer-Volkoff (TOV) equations, to describe the balance between the star's gravity and its internal pressure, which determines the star's structure.

The TOV equations describe the structure of a spherically symmetric star in general relativity. They are given by:

$$\frac{dP(r)}{dr} = -\frac{G(\rho(r) + P(r)/c^2)\left(M(r) + 4\pi r^3 P(r)/c^2\right)}{r\,(r - 2GM(r)/c^2)} \quad (1)$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \quad (2)$$

where:

- $P(r)$ is the pressure at radius r,
- $\rho(r)$ is the energy density at radius r,
- and $M(r)$ is the mass enclosed within radius r.

To start, the code gathers data about the neutron star's crust and core from the equation of state. The equation of state used comes from Baym-Pethick-Sutherland and gives data on pressure, energy density, and baryon number [1].

For the core of the neutron star, the code calculates various properties at different points, like the mass density and energy density. It creates a function to predict how these properties change from the crust to the core. The code then combines the data for both the crust and the core into a single model that describes the entire neutron star. It defines mathematical equations that show how pressure and mass change as you move from the center of the star outward.

To solve these equations, the code uses a numerical method called the Runge–Kutta method. This method helps in predicting the structure of the neutron star step by step by solving the TOV equations.

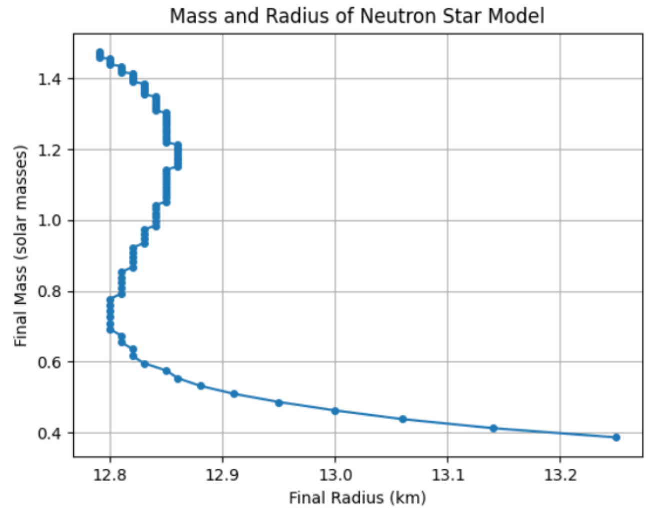

Figure 1: Theoretical star models with central pressures ranging from 10^{33} Pa to 10^{34} Pa

The code's input is a range of central pressures, it calculates how each pressure affects the star's mass and size. It collects this information and creates a graph that shows how the neutron star's mass and radius vary with different central pressures. This helps in visualizing and understanding the relationship between the pressure at the center of the star and its overall size and mass.

Figure 1 shows the mass-radius relationship for neutron star models obtained from the theoretical model described. The observed patterns consistent with results reported in the current literature on neutron star structure.

This behavior reflects the equation of state (EOS) used in the models, where stiffer EOSs predict larger radii for a given mass. The mass-radius relationship is significant in understanding the internal structure of neutron stars, particularly the density profile and the possible presence of dark matter, in their cores.

2.2 Evolving Structure With Time

To evolve the structure with time another code was constructed to solve the heat equation, which is a modified version of the diffusion equation.

$$\frac{c_V}{n_b}\frac{\partial(\mathrm{e}^{\nu/2}T)}{\partial t} = \frac{\partial}{\partial N_b}\left((4\pi r^2)^2 n_b \kappa e^{\nu/2}\frac{\partial}{\partial N_b}\left(e^{\nu/2}T\right)\right) + \frac{e^{\nu}}{n_b}\left(H - Q\right). \quad (3)$$

where:

- c_V is the constant volume heat capacity,
- n_b is the baryon number fraction
- $e^{\nu/2}T$ is the redshifted temperature
- N_b is the baryon number
- κ is the thermal conductivity
- Q is the cooling due to neutrino emission,
- and H is the dark matter induced heating term

The code sets up characteristic parameters for neutron stars, including scales for temperature, density, and thermal conductivity. It then uses data on baryon number and radii to create a model of the neutron star's internal structure.

The star is split into shells of fixed enclosed baryon number. As the star evolves, the total baryon number doesn't change because dark matter particles are assumed to have a baryon number of 1. However, the radius can change since the structure changes as the dark matter accumulates [4].

The program uses interpolation to estimate properties based on existing data, and it applies various formulas to account for different physical processes, such as neutrino emission from plasmons and bremsstrahlung (radiation caused by the acceleration of charged particles) as well as thermal conductivity from electron-ion and electron-electron interactions [6]. Overall, the code aims to provide a comprehensive analysis of the thermal behavior of neutron stars by calculating key thermal properties based on their internal structure and the conditions within different regions.

Another function computes several core physical properties given input parameters such as temperature, baryon number density, proton fraction, and effective neutron and proton masses. These properties include neutrino emissivity, thermal conductivity, and specific heat capacity. The function utilizes established models and constants from relevant literature, to calculate these properties in erg/cm^3/s for emissivity, erg/cm/K/s for thermal conductivity, and erg/K/cm^3 for specific heat capacity [6].

The code further interpolates various core properties—such as baryon number density and radial coordinates—based on given data to ensure accurate simulations. The simulation discretizes the baryon number N_b and time steps, applying the Crank–Nicolson method to solve the thermal evolution over time. This includes adapting thermal conductivity coefficients, updating physical properties with new temperature values, and handling boundary conditions and time-stepping to model the cooling of the neutron star effectively.

Figure 2 displays the theoretical cooling curve of a neutron star, showing the surface temperature as a function of time. This curve deviates from other current models in which cooling starts rapidly and slows along the stars lifetime.

In the early stages of the neutron star's life, the cooling process is dominated by the extremely high initial temperatures resulting from the star's formation. During this phase, the cooling is primarily driven by the efficient neutrino emission. Neutrino emission mechanisms, such as bremsstrahlung and synchrotron radiation, are highly efficient at these high temperatures, leading to a rapid drop in the star's temperature, as reflected by the steep initial decline in the curve. The neutrino cooling is primarily driven by the

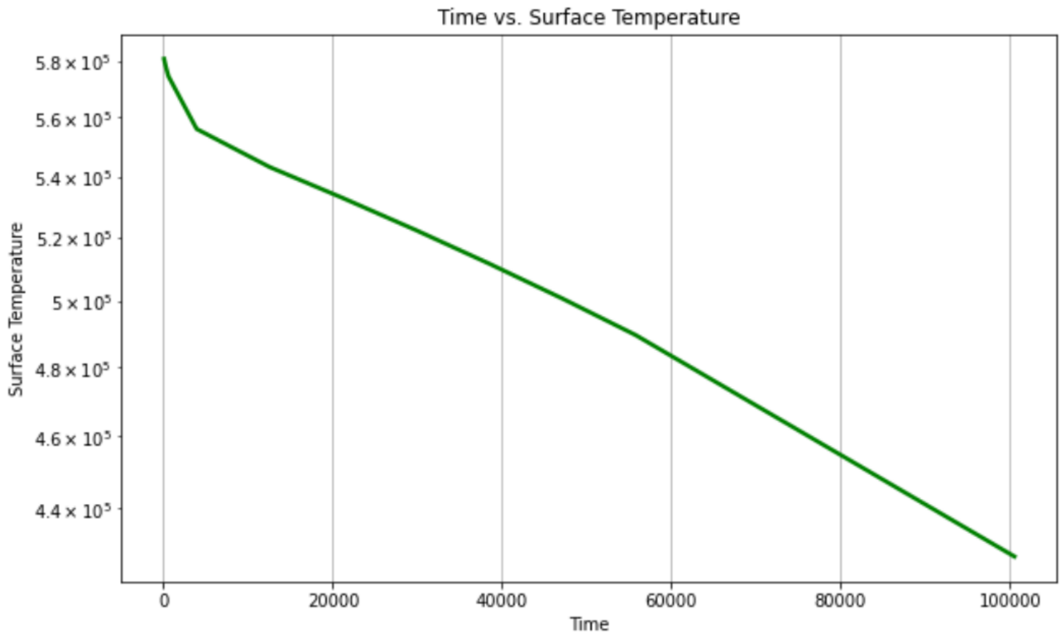

Figure 2: An example theoretical redshifted surface temperature curve of a neutron star with a central pressure of 5×10^{33} Pa.

modified Urca reaction in the core, which is more significant than bremsstrahlung and synchrotron radiation in the crust due to the core occupying a much larger volume of the star. Additionally, if direct Urca reactions are present, they will dominate over the modified Urca reactions. It is expected that as the temperature decreases, the rate of neutrino emission slows down, leading to a more gradual cooling process. This transition is represented in the curve, where the slope becomes less steep. The dominance of neutrino emission diminishes, and other cooling mechanisms begin to play a more significant role.

At later times, the cooling curve flattens out, indicating a transition to a slower cooling system. During this phase, the neutron star's temperature stabilizes, and the cooling rate is controlled by the less efficient emission of photons from the star's surface, as well as residual neutrino emission from the core. This phase is also influenced by the thermal relaxation between the core and the crust of the neutron star.

3 Dark Matter Fraction

The dark matter fraction is updated by integrating a specific equation over one time step. The change in the dark matter fraction of baryons, ΔY_d, is calculated using the formula

$$\Delta Y_d = e^{\nu/2} \frac{\Gamma}{n_b} \Delta t,$$

where Δt is the time step, and Γ is the volumetric creation rate of particles/number of particles created per unit volume and per unit time. This step is crucial for determining how much the dark matter fraction changes in each shell.

The star is adjusted to hydrostatic equilibrium through the Henyey method. This involves creating a series of finite-difference equations for each shell that depend on the pressure P, mass M, and radius r in that shell and its neighboring shells. These equations are then Taylor-expanded about their initial values, leading to the solution of a large matrix equation to find the corrections for ΔP, ΔM, and Δr.

Once these corrections are applied, updated values for P, r, and M are obtained. These values are then used, along with updated values of the metric potential ν, in a predictor-corrector method like Heun's method to refine the computations for the dark matter fraction Y_d and temperature T. This iterative process is repeated for each time step until the simulation covers the desired time span.

Finally, the Henyey method is extended to account for the presence of both ordinary matter and dark matter, requiring the solution of coupled equations for the pressures of these two components. These equations are more complex than those for a single fluid, but they provide a comprehensive description of the star's structure under the influence of dark matter.

This study highlights the potential of neutron stars as astrophysical laboratories for identifying dark matter through their thermal evolution. By modeling the structure and cooling behavior of neutron stars with the inclusion of dark matter-induced heating mechanisms, we demonstrate that deviations from standard cooling models could serve as indirect evidence of dark matter. Further work would involve modifying the dark matter induced heating term H and comparing with observational data to improve our understanding of dark matter's influence on neutron star dynamics.

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