Effects of Neutrino-Matter Interactions on Neutrino Quantum Dynamics in Astrophysical Environments

Student: Leo Ahlburg[1]
Mentors: Vincenzo Cirigliano and Yukari Yamauchi[1]
[1] Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA

September 25, 2025

Abstract

Neutrino flavor evolution in dense astrophysical media, such as binary neutron star mergers and core-collapse supernovae, is sensitive to neutrino-matter interactions. Dense media are theoretically predicted to exhibit appreciable neutrino-neutrino interaction effects due to the high number density of neutrinos. In this project, we derive matter-neutrino interaction Hamiltonian terms from the Standard Model effective Lagrangian, and discuss plans to develop a numerical simulation that includes these matter terms. Until recently, simulations of neutrino many-body systems have exclusively used a truncated Hamiltonian that neglects terms that mediate momentum changing scattering processes. We combine these self interaction Hamiltonian terms with the terms that mediate scattering off electrons and nucleons, treating these matter particles as a classical background field. With this Hamiltonian, we study neutrino many-body evolution in a finite, isotropic, inhomogeneous medium in equilibrium. This framework aims to capture effects such as the Mikheyev-Smirnov-Wolfenstein (MSW) resonance in a many-body context, opening the door to future investigations of flavor evolution in extreme astrophysical conditions.

Contents

1 Introduction and Background

1.1 Introduction

The neutrino is a nearly massless particle that experiences a phenomenon that no other Standard Model particle does: flavor oscillation. In the last decades the study of neutrino flavor evolution has been ongoing, though primarily in a one-body context. Neutrino flavor evolution in hot and dense astrophysical media is of particular interest as it probes fundamental questions in our understanding of nuclear synthesis, in both the early universe and from core collapse supernovae or neutron star mergers. Typically, astrophysical neutrinos are modeled using the Quantum Kinetic Equations (QKEs), but recent publications question the validity of the QKEs since they are derived using a one body approach and thus may leave out vital many-body effects [1].

Until recently, all approaches to neutrino many-body systems have used the truncated Hamiltonian, $H_{\nu\nu}^{(T)}$, which only mediates forward momentum changing processes (processes which preserve or swap the momentum of two neutrinos). In recent years it has been argued that use of the truncated Hamiltonian may not be valid, and as a response a first-principles treatment of many-body neutrino systems with the full Hamiltonian including non-forward scattering terms, $H_{\nu\nu}^{(F)}$, has been developed in reference [2]. In this project we closely follow the framework and formalism developed in [2], and extend the treatment to account for neutrino-matter interactions, which are prevalent in astrophysical media of interest that are dense in matter. By doing this we aim to contribute to the broader development of a more realistic approach to quantum many-body neutrino systems in an astrophysical context.

1.2 Background on Matter Effects

Exploring how neutrinos interact with matter as they propagate is key to expanding our understanding of many-body neutrino systems, and thus nuclear synthesis in hot and dense astrophysical media. Two classic examples which are of vital importance to neutrino experiments today are solar neutrinos created via nuclear processes in the Sun, who traverse a significant amount of solar matter before reaching detectors on earth, and atmospheric neutrinos, produced by cosmic rays, that could pass through the entire earth before reaching a detector on the other side. Although neutrino-matter interactions are extremely weak, there is good reason to believe that their cumulative effects can be quite significant, particularly in dense media [3]. To good approximation, one can consider matter as a sea of electrons and nucleons, as quark degrees of freedom are negligible in the neutrino energy scales we consider ($\ll$ 100 GeV) [4].

The allowed forward elastic scattering processes that a neutrino can undergo in a sea of matter are depicted in Fig. 1. In media we are interested in, the muon and tau number density is essentially zero, so electron-flavor neutrinos are the only flavor which undergoes the charged-current W-exchange interaction depicted in Fig. (5) (a). Additionally, all neutrinos regardless of flavor can undergo the neutral-current Z-exchange shown in Fig. (5) (b). These give rise to an interaction potential energy, which is flavor dependent because process (a) only affects electron-flavor neutrinos. If we consider solar neutrinos that are created via the process $p+p \rightarrow {}^2H+e^{+}+\nu_e$, at the point of creation the neutrino flux would be composed primarily of electron-flavor neutrinos.

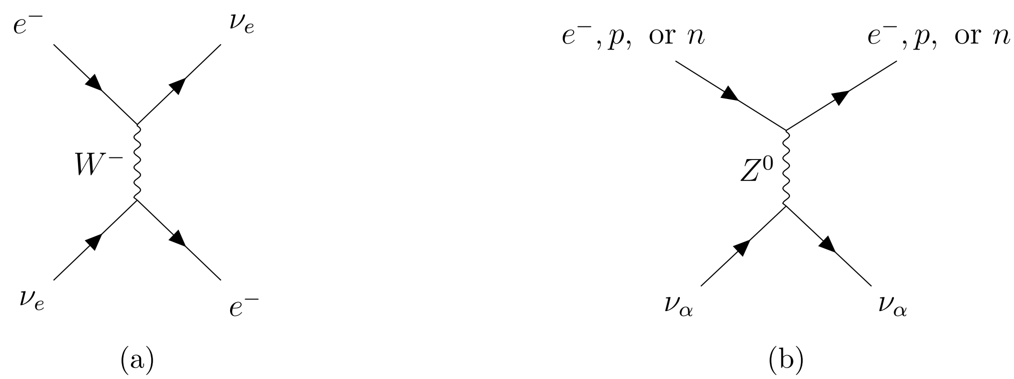

Figure 1: Possible neutrino forward scattering interactions with matter (a) Charged-current via W^- exchange. Only occurs for electron flavor neutrinos. (b) Neutral-current via Z^0 exchange. Occurs for any flavor.

Models have been constructed to explore the effects of the interactions in Fig. 1 on the flavor evolution of single body solar neutrino systems as they propagate from the center of the Sun to its edge [3]. Notably, the Sun Hamiltonian takes on a form identical to the vacuum Hamiltonian, but with different parameters which depend on the number density of matter at a given point. This model produces the so-called Mikheyev-Smirnov-Wolfenstein (MSW) effect in which electron-flavor neutrinos created via nuclear processes within the Sun are almost totally converted to muon-flavor neutrinos [5, 6]. The varying number density of matter across the width of the Sun gives rise to a resonance radius at which flavor mixing is maximal, whereas the rest of the time the mixing is minimal [3]. This is graphically depicted in Fig. 2. In this project, we aim to create a framework in which we can reproduce and observe how matter effects such as MSW manifest themselves in a many-body context.

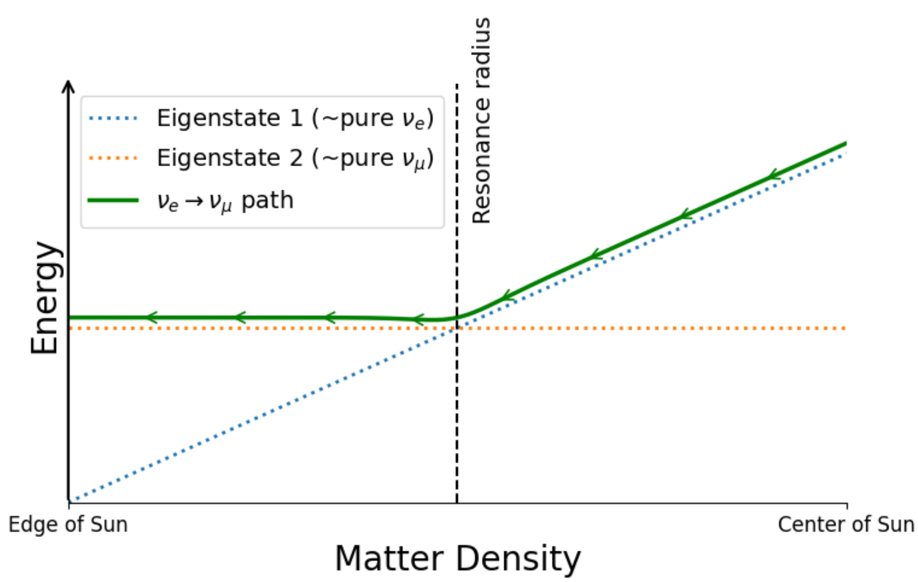

Figure 2: Efficient conversion of neutrinos via MSW effect. ν_e are born near the center of the Sun (top right) where mixing is minimal until a resonance radius is reached, where nearly all ν_e are converted to ν_μ. Then mixing becomes minimal once again as the neutrino travels away from the critical resonance radius and makes its way to the outer edge of the Sun. Adapted from [3] Fig. 6

2 Results

2.1 Formalism

In what follows we adhere closely to the formalism developed in [2]. In systems where the typical energy of a neutrino is much less than the electroweak scale, the Hamiltonian takes the form

$$H = H_{kin} + H_{\nu\nu} + H_{\nu-m} \tag{1}$$

From now on we will limit our discussion to $H_{\nu-m}$, and use results for H_{kin} and $H_{\nu\nu}$ from [2]. We now introduce some notation to facilitate the derivation in the following section. We expand the neutrino fields in terms of helicity 4-spinors and creation/annihilation operators in the mass basis

$$\nu_i(x) = \sum_{h=\pm} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(u(\mathbf{p}, h) a_i(\mathbf{p}, h) e^{-ipx} + v(\mathbf{p}, h) b_i^\dagger(\mathbf{p}, h) e^{ipx} \right) \tag{2}$$

where $a_i(\mathbf{p}, h)$ and $b_i(\mathbf{p}, h)$ are creation/annihilation operators and $u(\mathbf{p}, h)$ and $v(\mathbf{p}, h)$ are helicity 4-spinors that correspond to neutrinos and antineutrinos respectively. The label $h \in \{+, -\}$ refers to helicity and $i \in 1, 2$ refers to the mass eigenstate. We normalize the creation and annihilation operators to have mass dimension -3/2 and satisfy the commutation relation:

$$\{a_\alpha(\mathbf{p}, h), a_\beta^\dagger(\mathbf{p}', h')\} = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{hh'} \delta_{\alpha\beta} \tag{3}$$

Additionally, the spinors are dimensionless and normalized:

$$u^\dagger(\mathbf{p}, h) u(\mathbf{p}, h') = v^\dagger(\mathbf{p}, h) v(\mathbf{p}, h') = \delta_{hh'} \tag{4}$$

The helicity 4-spinors read as follows

$$\begin{aligned} u(\mathbf{p}, +) &= \sqrt{\frac{E + |\mathbf{p}|}{2E}} \begin{pmatrix} r(\mathbf{p}) \xi_+(\hat{\mathbf{p}}) \\ \xi_+(\hat{\mathbf{p}}) \end{pmatrix}, \quad & u(\mathbf{p}, -) &= \sqrt{\frac{E + |\mathbf{p}|}{2E}} \begin{pmatrix} \xi_-(\hat{\mathbf{p}}) \\ r(\mathbf{p}) \xi_-(\hat{\mathbf{p}}) \end{pmatrix} \\ v(\mathbf{p}, +) &= \sqrt{\frac{E + |\mathbf{p}|}{2E}} \begin{pmatrix} \xi_-(\hat{\mathbf{p}}) \\ -r(\mathbf{p}) \xi_-(\hat{\mathbf{p}}) \end{pmatrix}, & v(\mathbf{p}, -) &= \sqrt{\frac{E + |\mathbf{p}|}{2E}} \begin{pmatrix} -r(\mathbf{p}) \xi_+(\hat{\mathbf{p}}) \\ \xi_+(\hat{\mathbf{p}}) \end{pmatrix} \end{aligned} \tag{5}$$

where $E = \sqrt{\mathbf{p}^2 + m^2}$ and $r(\mathbf{p}) = m/(E + |\mathbf{p}|)$.

We then introduce the helicity Pauli spinors which obey the identity $(\overrightarrow{\sigma} \cdot \hat{\mathbf{p}}) \xi_\pm(\hat{\mathbf{p}}) = \pm \xi_\pm(\hat{\mathbf{p}})$

$$\xi_+(\hat{\mathbf{p}}) = \begin{pmatrix} \cos\frac{\theta_\mathbf{p}}{2} \\ e^{i\phi_\mathbf{p}} \sin\frac{\theta_\mathbf{p}}{2} \end{pmatrix}, \qquad \xi_-(\hat{\mathbf{p}}) = \begin{pmatrix} -e^{-i\phi_\mathbf{p}} \sin\frac{\theta_\mathbf{p}}{2} \\ \cos\frac{\theta_\mathbf{p}}{2} \end{pmatrix} \tag{6}$$

where $\theta_\mathbf{p}$ and $\phi_\mathbf{p}$ are the polar and azimuthal angles of $\hat{\mathbf{p}} = \frac{\mathbf{p}}{|\mathbf{p}|}$.

Since we ignore antineutrinos, we include only the left-helicity creation and annihilation operators. So $a_i(\mathbf{p}, -)$ becomes $a_i(\mathbf{p})$.

Additionally, we write down the basis change relation between mass and flavor bases, as it is convenient to express the Hamiltonian in the flavor basis

$$\begin{aligned} a_e(\mathbf{p}) &= \cos\theta\, a_1(\mathbf{p}) + \sin\theta\, a_2(\mathbf{p}) \\ a_\mu(\mathbf{p}) &= -\sin\theta\, a_1(\mathbf{p}) + \cos\theta\, a_2(\mathbf{p}) \end{aligned} \tag{7}$$

2.2 Matter Hamiltonian

In the case of neutrinos in astrophysical environments of interest, we replace quark degrees of freedom with nucleon degrees of freedom since the energy of an average neutrino is well below the electroweak scale. Thus, the Standard Model effective Lagrangian terms that mediate neutrino-neutrino and neutrino-matter interactions expressed in terms of 4-spinors read [4] (repeated indices are summed over):

$$\mathcal{L}_{\nu\nu} = -\frac{G_F}{\sqrt{2}}\bar{\nu}_\alpha\gamma_\mu P_L\nu_\alpha\bar{\nu}_\beta\gamma^\mu P_L\nu_\beta, \tag{8}$$

$$\mathcal{L}_{\nu e} = -2\sqrt{2}G_F(\bar{\nu}_\alpha\gamma_\mu P_L Y_L\nu_\alpha\bar{e}\gamma^\mu P_L e + \bar{\nu}_\alpha\gamma_\mu P_L Y_R\nu_\alpha\bar{e}\gamma^\mu P_R e), \tag{9}$$

$$\mathcal{L}_{\nu N} = -\sqrt{2}G_F\sum_{N=p,n}\bar{\nu}_\alpha\gamma_\mu P_L\nu_\alpha\bar{N}\gamma^\mu(C_V^{(N)} - C_A^{(N)}\gamma_5)N, \tag{10}$$

where the gamma matrices are given by

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^0 \\ \sigma^0 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{11}$$

the projection operators $P_{L,R} = (\mathbb{1} \mp \gamma_5)/2$ and

$$Y_L = \begin{pmatrix} \frac{1}{2} + \sin^2\theta_W & 0 & 0 \\ 0 & -\frac{1}{2} + \sin^2\theta_W & 0 \\ 0 & 0 & -\frac{1}{2} + \sin^2\theta_W \end{pmatrix}, \quad Y_R = \sin^2\theta_W \times \mathbb{1} \tag{12}$$

With $g_A \cong 1.27$, the nucleon couplings are given by

$$\begin{aligned} C_V^{(p)} &= \frac{1}{2} - 2\sin^2(\theta_W), & C_V^{(n)} &= -\frac{1}{2} \\ C_A^{(p)} &= \frac{g_A}{2}, & C_A^{(n)} &= -\frac{g_A}{2} \end{aligned} \tag{13}$$

We first consider $H_{\nu e}$. Converting the Lagrangian density into its corresponding Hamiltonian, we get

$$H_{\nu e} = 2\sqrt{2}G_F\int dx^3(\bar{\nu}_\alpha\gamma_\mu P_L Y_L\nu_\alpha\bar{e}\gamma^\mu P_L e + \bar{\nu}_\alpha\gamma_\mu P_L Y_R\nu_\alpha\bar{e}\gamma^\mu P_R e) \tag{14}$$

Treating the electrons as an isotropic classical mean field background, we replace $\bar{e}\gamma^\mu P_L e$ and $\bar{e}\gamma^\mu P_R e$ with their expectation values $\langle\bar{e}\gamma^\mu P_L e\rangle$ and $\langle\bar{e}\gamma^\mu P_R e\rangle$. In unpolarized matter left and right handed electrons populate the medium in equal numbers, so we have

$$\langle\bar{e}\gamma^\mu P_L e\rangle = \langle\bar{e}\gamma^\mu P_R e\rangle = \frac{1}{2}\langle\bar{e}\gamma^\mu e\rangle \tag{15}$$

Under our assumption of isotropy, only the $\mu = 0$ density component survives

$$\langle\bar{e}\gamma^\mu e\rangle = \langle\bar{e}\gamma^0 e\rangle\delta^{\mu 0} = \langle\hat{n}_e(x)\rangle\delta^{\mu 0} = n_e(x)\delta^{\mu 0} \tag{16}$$

where $\hat{n}(x)$ is the number density operator. So our Hamiltonian becomes

$$H_{\nu e} = \sqrt{2} G_F \int dx^3 (\bar{\nu}_\alpha \gamma_0 P_L Y_L \nu_\alpha + \bar{\nu}_\alpha \gamma_0 P_L Y_R \nu_\alpha) n(x) \tag{17}$$

We now pause to consider $H_{\nu N}$. Rewriting the Lagrangian density in terms of its corresponding Hamiltonian, we get

$$H_{\nu N} = \sqrt{2} G_F \int dx^3 \sum_{N=p,n} \bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha \bar{N} \gamma^\mu (C_V^{(N)} - C_A^{(N)} \gamma_5) N, \tag{18}$$

By similarly treating nucleons as an isotropic classical mean field background, we take the expectation value of the term bilinear in the nucleon fields. With the assumption of isotropy the axial term $C_A^{(N)} \gamma_5$ dies, and the Hamiltonian becomes

$$H_{\nu N} = \sqrt{2} G_F \int dx^3 \sum_{N=p,n} \bar{\nu}_\alpha \gamma_0 P_L \nu_\alpha C_V^{(N)} n_N(x), \tag{19}$$

We now expand the electron and nucleon Hamiltonians in terms of creation and annihilation operatros in the flavor basis using Eqs. (2, 5, 6, 11, 12). Imposing the ultra-relitivistic limit where $m_i/|\mathbf{p}| \ll 1$, and considering the two flavor case of elecron and muon flavor neutrinos, $H_{\nu e}$ and $H_{\nu N}$ become

$$\begin{aligned} H_{\nu e} &= \sqrt{2} G_F \int d^3x \, \frac{d\mathbf{p}'}{(2\pi)^3} \frac{d\mathbf{p}}{(2\pi)^3} \, n_e(x) e^{ix(p-p')} h(\mathbf{p}', \mathbf{p}) \\ &\times \left[\left(2\sin^2\theta_W + \tfrac{1}{2} \right) a_e^\dagger(\mathbf{p}') a_e(\mathbf{p}) + \left(2\sin^2\theta_W - \tfrac{1}{2} \right) a_\mu^\dagger(\mathbf{p}') a_\mu(\mathbf{p}) \right] \end{aligned} \tag{20}$$

$$\begin{aligned} H_{\nu N} &= \sqrt{2} G_F \int d^3x \, \frac{d\mathbf{p}'}{(2\pi)^3} \frac{d\mathbf{p}}{(2\pi)^3} \, e^{ix(p-p')} h(\mathbf{p}', \mathbf{p}) \\ &\times \left[\left(\tfrac{1}{2} - 2\sin^2\theta_W \right) n_p(x) - \tfrac{1}{2} n_n(x) \right] a_\alpha^\dagger(\mathbf{p}') a_\alpha(\mathbf{p}) \end{aligned} \tag{21}$$

where the helicity 4-spinor overlap is

$$h(\mathbf{p}', \mathbf{p}) = e^{i(\phi_{\mathbf{p}'} - \phi_{\mathbf{p}})} \sin(\frac{\theta_{\mathbf{p}'}}{2}) \sin(\frac{\theta_{\mathbf{p}}}{2}) + \cos(\frac{\theta_{\mathbf{p}'}}{2}) \cos(\frac{\theta_{\mathbf{p}}}{2}) \tag{22}$$

Immediately apparent is the fact that our $H_{\nu-m}$ terms are quadratic in our creation and annihilation operators, which is in stark contrast to $H_{\nu\nu}$, which is quartic in the operators [2], although they are both interaction potential terms.

As in [2], we now adapt our system to a box of finite volume V. Our continuous 3-momenta now become discrete triplets of integers according to the relation $(\mathbf{p})_{x,y,z} = [(2\pi)/L](\mathbf{z_p})_{x,y,z}$, and the continuous integrals take on the form of discrete sums according to

$$\int \frac{d^3p}{(2\pi)^3} \to \frac{1}{V} \sum_{\mathbf{z_p}}. \tag{23}$$

.

2.3 Fock Space

Using a nearly identical setup to [2], we work in a second quantized Fock space. Given momentum modes $\mathbf{p}_i$ ($i \in 1, ..., k$) and the flavor label α ($\alpha \in e, \mu$), each state is specified by its occupied discretized three-momentum, and flavor. In the system there are $2k$ single-particle states, and each single particle state can be either occupied or unoccupied. Thus, the dimension of the Fock space is 2^{2k}. Since our Hamiltonian preserves the total number of neutrinos, we work with fixed N, where

$$N = \sum_{i=1}^{k} \sum_{\alpha=e,\mu} n_{i\alpha} \tag{24}$$

and the dimension of the space is

$$d_{N,k} = \binom{2k}{N}. \tag{25}$$

The $d_{N,k}$ basis vectors are labeled by

$$\mathbf{n} = \{n_{1e}, n_{1\mu}, ..., n_{ke}, n_{k\mu}\}, \tag{26}$$

We define $|\mathbf{n}\rangle$ as a Slater Determinant of N single-particle states that constitute our many-body system. For example, the normalized basis state with neutrinos (p_1, e), (p_1, μ), and (p_2, e) is

$$|\mathbf{n}\rangle = \frac{a_e^\dagger(p_1)}{\sqrt{V}} \frac{a_\mu^\dagger(p_1)}{\sqrt{V}} \frac{a_e^\dagger(p_2)}{\sqrt{V}} |\mathbf{0}\rangle. \tag{27}$$

The conjugate state is obtained by applying the annihilation operators in reverse order, e.g.

$$\langle\mathbf{n}| = \langle\mathbf{0}| \frac{a_e(p_2)}{\sqrt{V}} \frac{a_\mu(p_1)}{\sqrt{V}} \frac{a_e(p_1)}{\sqrt{V}}, \tag{28}$$

which ensures $\langle\mathbf{n}|\mathbf{m}\rangle = \delta_{n,m}$. We have thus defined an orthonormal basis.

Applying $a_\alpha(\mathbf{p}_i)$ to a basis vector $|\mathbf{n}\rangle$ results in

$$a_\alpha(\mathbf{p}_i)|\mathbf{n}\rangle = V^{1/2} f_{n,i,\alpha}\, \delta_{n_{i\alpha},1}\, |\mathbf{n}^{[i\alpha]}\rangle, \tag{29}$$

where

$$n^{[i\alpha]} = n \text{ with } n_{i\alpha} \to 0 \tag{30}$$

and

$$f_{n,i,\alpha} = (-1)^{\sum_{(j,\beta)<(i,\alpha)} n_{j\beta}}. \tag{31}$$

In Eq. 31, the f-factors account for the fermionic antisymmetry of the wave function; when $a^\dagger$ or a is applied to a state, we pick up a number of factors of -1 equal to the number of single-particle states with non-zero occupation numbers preceding $n_{i,\alpha}$ according to our ordering scheme (order of increasing momenta from left to right, where electron labels precede muon labels). The Kronecker deltas encode action of creation or annihilation. If the corresponding occupation number of the state was zero, the Kronecker delta evaluates to zero and the state vanishes. If it were non-zero, the Kronecker delta evaluates to one and the state survives. For a more detailed description of the Fock space see Ref. [2].

2.4 Hamiltonian Matrix Elements

We now write down the matrix elements of $H_{\nu e}$ and $H_{\nu N}$ in the occupation number basis. Using the notation developed in the previous section, the full matter Hamiltonian, including electron and nucleon terms, $H_{\nu-m}$ is as follows (we make the sums explicit for clarity)

$$\begin{aligned}\langle \mathbf{m}|H_{\nu-m}|\mathbf{n}\rangle = \sqrt{2}G_F\frac{1}{V}\sum_i\sum_j\sum_\alpha \int d^3x\, h(\mathbf{p}_i,\mathbf{p}_j)e^{i\mathbf{x}\cdot(\mathbf{p}_j-\mathbf{p}_i)} \\ \times\left[\kappa_\alpha n_e(x)+\left(\tfrac{1}{2}-2\sin^2\theta_W\right)n_p(x)-\tfrac{1}{2}n_n(x)\right] \\ \times\left(f_{m,i,\alpha}\,\delta_{m_{i,\alpha},1}\right)\left(f_{n,j,\alpha}\,\delta_{n_{j,\alpha},1}\right)\langle\mathbf{m}^{[i\alpha]}|\mathbf{n}^{[j\alpha]}\rangle\end{aligned} \tag{32}$$

where

$$\kappa_{e,\mu} = 2\sin^2\theta_W \pm \frac{1}{2}, \tag{33}$$

and

$$\mathbf{p}_i = \frac{2\pi}{L}\mathbf{z}_i. \tag{34}$$

3 Future Direction

3.1 Development of Numerical Simulation

Having derived from first principles the Hamiltonian for modeling many-body neutrino systems in matter, the next step is to include this matter term in numerical simulations. First we will write and adapt code from [2] to simulate astrophysical many-body quantum neutrino systems. We will then conduct a study in which we will vary parameters such as the number density of electrons and nucleons over a given volume. We can tune these parameters to model different astrophysical media, and we expect them to have an appreciable effect on the flavor evolution of neutrino systems. For instance, if the number density was too low at the center of the Sun and the change in number density of matter too little between the center and edge of the Sun, we predict that we would not observe the MSW effect [3]. The goal is to reproduce and quantify the effects of neutrino-matter interactions.

3.2 Varying Number of Neutrino Flavors

In the future we plan to collaborate with the project INTURN 25-4 to explore the effects of matter interactions in the context of neutrino systems with greater than two flavors. Combining the two frameworks, we hope to develop a more realistic and robust model for flavor evolution of many-body neutrino systems.

Acknowledgement

I would like to thank my mentors Vincenzo Cirigliano and Yukari Yamauchi for their time and knowledge they have shared with my during my journey so far. This research would not be possible without the support of the INT's U.S. Department of Energy grant No. DE-FG02-00ER41132. I would also like to acknowledge the support from the N3AS's National Science Foundation award No. 2020275.

References

[1] A. B. Balantekin et al. "Quantum information and quantum simulation of neutrino physics". In: *Eur. Phys. J. A* 59 (2023), p. 186. DOI: `10.1140/epja/s10050-023-01006-2`. arXiv: `2305.01150 [nucl-th]`.

[2] Vincenzo Cirigliano, Srimoyee Sen, and Yukari Yamauchi. *Neutrino many-body flavor evolution: the full Hamiltonian*. 2025. arXiv: `2404.16690 [hep-ph]`.

[3] Boris Kayser. *Neutrino Mass, Mixing, and Oscillation*. 2001. arXiv: `hep-ph/0104147 [hep-ph]`.

[4] Daniel N. Blaschke and Vincenzo Cirigliano. "Neutrino Quantum Kinetic Equations: The Collision Term". In: *Phys. Rev. D* 94 (2016), p. 033009. DOI: `10.1103/PhysRevD.94.033009`. arXiv: `1605.09383 [hep-ph]`.

[5] S. P. Mikheev and A. Yu. Smirnov. "Resonance Oscillations of Neutrinos in Matter". In: *Sov. Phys. Usp.* 30 (1987), pp. 759–790. DOI: `10.1070/PU1987v030n09ABEH002961`.

[6] L. Wolfenstein. "Neutrino Oscillations in Matter". In: *Phys. Rev. D* 17 (1978), pp. 2369–2374. DOI: `10.1103/PhysRevD.17.2369`.