Finding the most compact configuration of neutron stars

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Background / Purpose

- **Neutron stars (NS's)** are dense remnants of collapsed stars, in hydrostatic equilibrium between outward pressure and inward gravity. They are expected to have a **maximum mass**; if exceeded, gravitational collapse into a black hole starts.
- The neutron star **equation of state (EoS)** governs interior matter interactions and possible masses and radii.
- The **mass-radius curve** and EoS are poorly constrained by observation and theory, but their **one-to-one correspondence** means that information about one lends information about the other. By solving the **Tolman-Oppenheimer-Volkoff (TOV) equations** for a given EoS, one obtains the corresponding mass-radius curve.[1–2]
- The **slope** of the EoS is related to the **sound speed** through the material by $\frac{dp}{d\epsilon} = c_s^2$.
- In 1996, Koranda, Stergioulas, and Friedman introduced a **compactness conjecture** that the radius of a neutron star of any particular mass is minimized by a **piecewise** EoS with a **phase transition** from a lower density sound speed $c_{s,\,\text{lower}} = 0$ to a higher density sound speed $c_{s,\,\text{upper}} = 1$, the **causal limit.**[3]
- Since the mass estimation of PSR J1614-2230 in 2010, it has been apparent that the EoS must accommodate a maximum neutron star mass of **at least 2 solar masses.**[4] In this project, we test the validity of the compactness conjecture for NS's of $2\,\mathrm{M}_\odot$, and study how imposing the constraint $M_{\max} \geq 2\,\mathrm{M}_\odot$ affects radius and EoS constraints.
- **Figs. 1 and 2** demonstrate the correspondence between a piecewise EoS and the resulting mass-radius curve (also discussed in Alford & Han (2016)[5]. The maximum mass is ~2 solar masses.

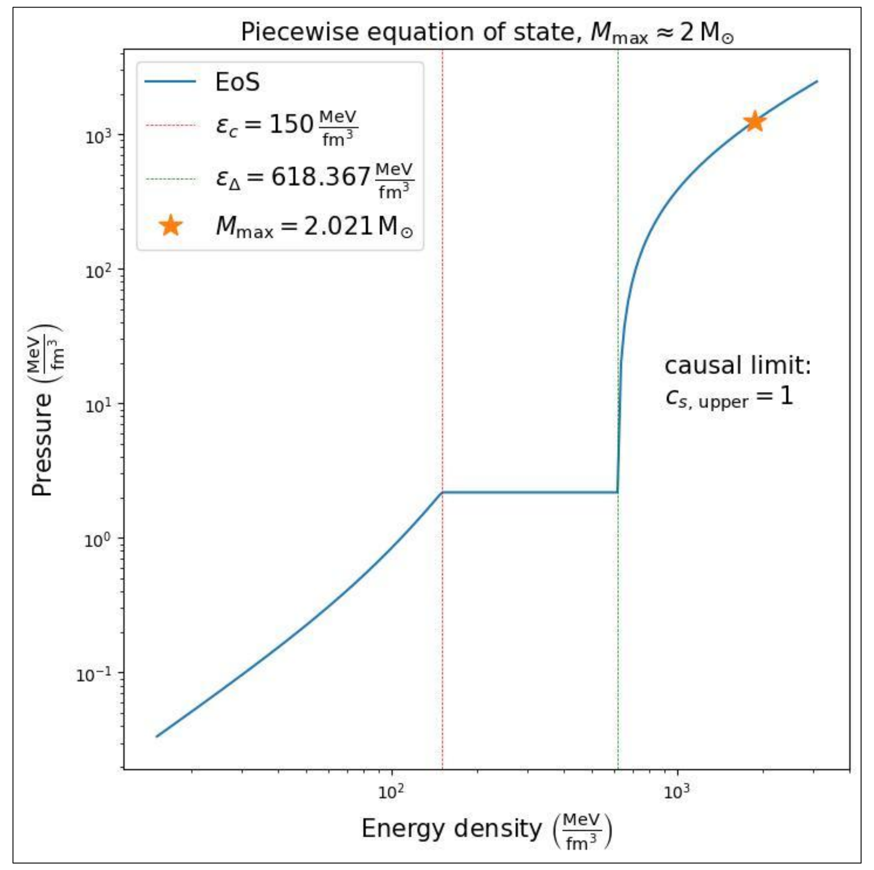

Fig. 1: Piecewise equation of state that yields a maximum mass of ~2 solar masses

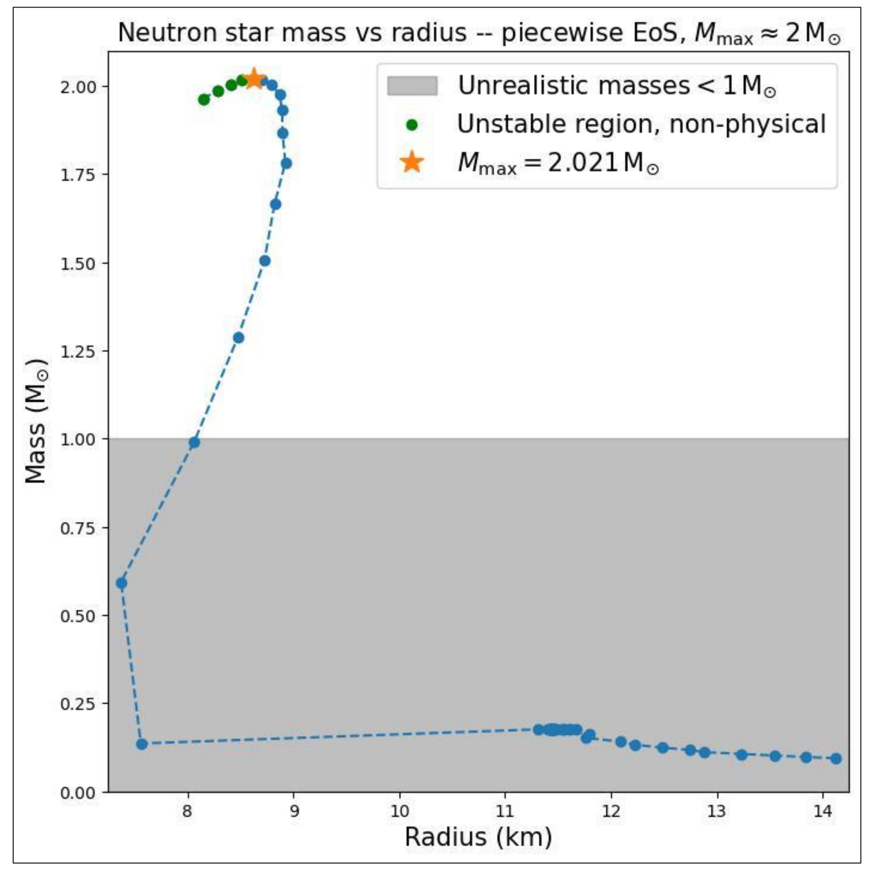

Fig. 2: Mass-radius curve with ~2 solar mass maximum yielded by piecewise equation of state

Methods

- We wrote original Python scripts to solve the TOV equations numerically, for any specified EoS.
- The script was tested iteratively by producing mass-radius curves for pure-neutron Fermi gas EoS's (the non-interacting and interacting cases), until the output values were consistent with results from the literature.[6]
- After validating the script's functionality, we constructed piecewise EoS's resembling Fig. 1. The lowest density region uses a neutron matter EoS from Gandolfi et al. (2014): Eq. (11) and $a = 13.0\,\mathrm{MeV}$, $\alpha = 0.49$, $b = 3.21\,\mathrm{MeV}$, $\beta = 2.47$ from Table 2.[7] The first phase transition occurs at **nuclear saturation density**, above which the EoS becomes flat until a second phase transition.
- To test the compactness conjecture, we vary the **density of the second phase transition**, ϵ_Δ, as well as the **high density sound speed** $c_{s,\,\text{upper}}$. For each EoS and corresponding mass-radius curve, we discard any unrealistic NS mass (less than 1 solar mass), and find both the **smallest remaining radius** and the **radius of a 2.00 solar mass NS**. The results of these searches are in **Figs. 3 and 4**. Finally, a cut of the mass-constrained grid search, for $c_{s,\,\text{upper}} = 1$, is in **Fig. 5**.

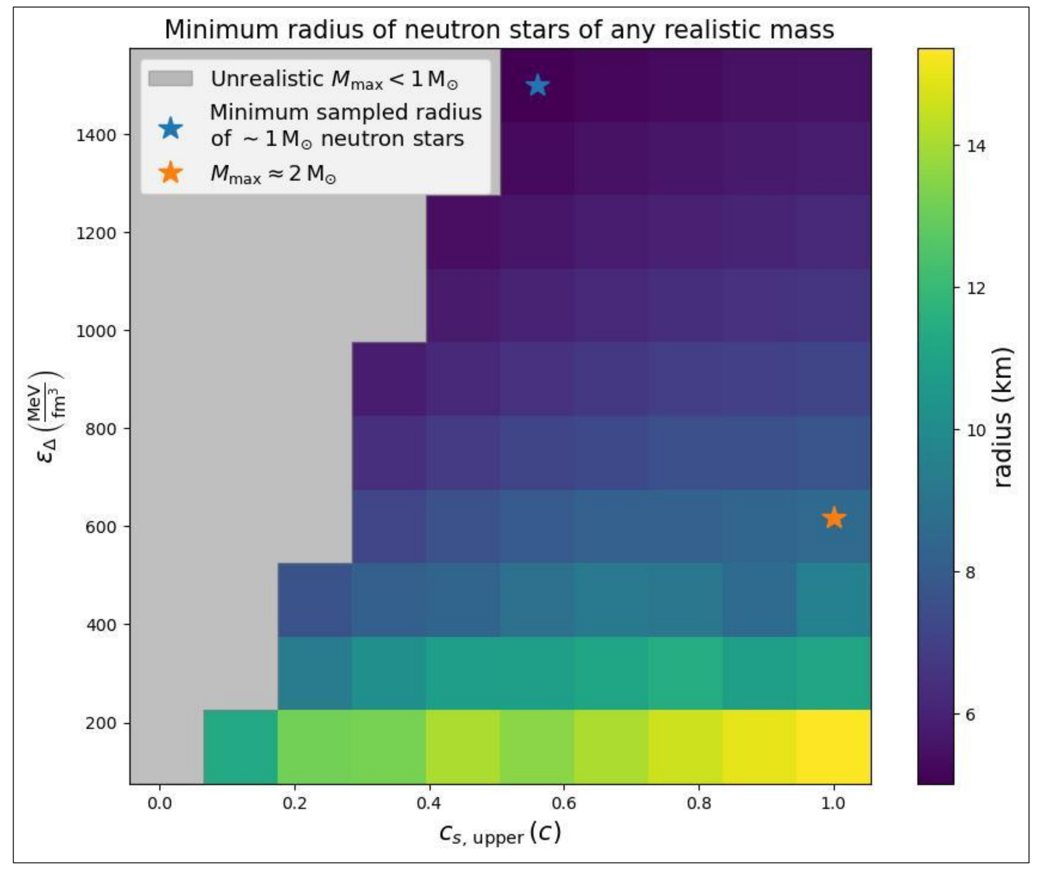

Fig. 3: Mass-agnostic grid search: smallest radius in mass-radius curve

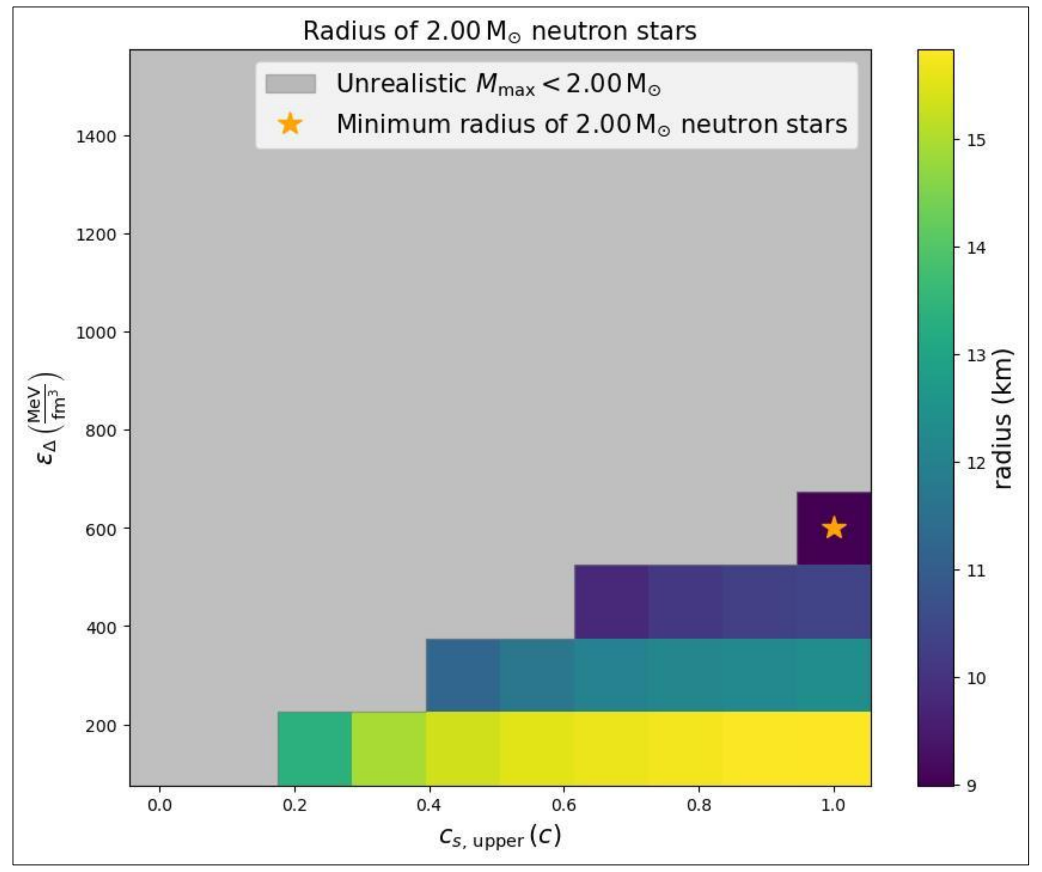

Fig. 4: Mass-constrained grid search: radius of 2.00 solar mass neutron stars

Results

- From the mass-agnostic grid search (**Fig. 3**), we see the general behavior of radius as a function of ϵ_Δ and $c_{s,\,\text{upper}}$: as ϵ_Δ increases, radius decreases, and for any fixed value of ϵ_Δ, as $c_{s,\,\text{upper}}$ increases, radius increases. The excluded region lies in the same part of parameter space where radius tends to decrease; this indicates that as radius decreases, mass also decreases.
- When the mass constraint (>1 solar mass) is imposed in **Fig. 4**, the excluded region shifts to larger masses to meet the constraint. This causes the minimum radius to be located at a smaller transition density. **Fig. 5** also demonstrates how the minimum radius configuration is directly adjacent to the excluded region.
- We see that the minimum radius of a 2.00 solar mass neutron star is yielded by the causal limit on high density sound speed, and that it corresponds to the mass-radius curve with a maximum mass of ~2 solar masses.

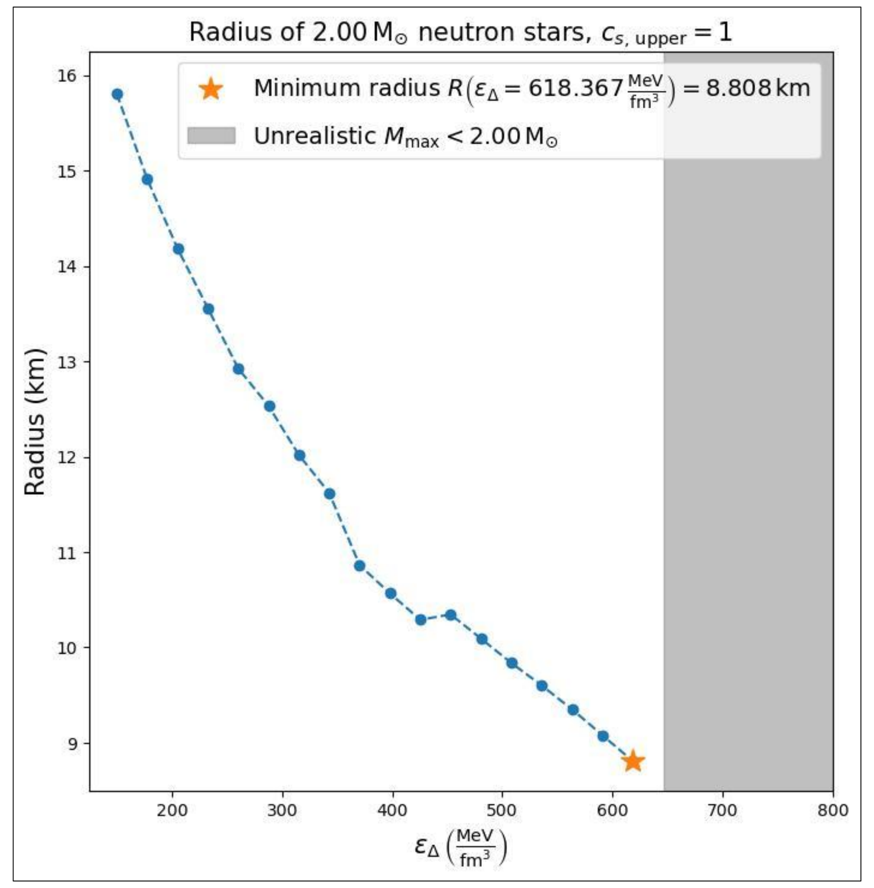

Fig. 5: Cut of mass-constrained grid search, for the causal limit of $c_{s,\,\text{upper}} = 1$

Conclusion

- We find that the compactness conjecture is supported by the fact that the minimum radius of 2.00 solar mass neutron stars corresponds to the causal limit of high density sound speed.
- We also find that the minimum radius configuration corresponds to the 2 solar mass maximum.
- Because we can impose the constraint $M_{\max} \geq 2\,\mathrm{M}_\odot$, this suggests that the most compact configuration of a 2 solar mass NS corresponds to a lower constraint on the EoS. The excluded region in Fig. 4 is an exclusion of any EoS that lies below the piecewise EoS in Fig. 1.
- The existence of an EoS lower bound helps our understanding of fundamental nuclear astrophysics.
- This warrants further study of what the EoS lower bound is. In future studies, the lowest density segment of the EoS can be revisited, and similar grid searches can be repeated with finer resolution.

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Acknowledgments

- This research was supported by the INT's U.S. Department of Energy grant No. DE-FG02-00ER41132 and the N3AS's National Science Foundation award No. 2020275.
- This poster presentation for the 2025 APS Global Physics Summit was supported by the APS's Division of Astrophysics Travel/Caregiver Grant.
- We are grateful to members of the INT Undergraduate Research Network and the Gravitational Wave Astronomy group at the University of Washington, Bothell for helpful discussions.