Effects of Landau Quantization on Neutrino Emission and Absorption

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Introduction

Young neutron stars cool primarily via weak interactions that emit neutrinos. One of the most efficient pathways, the Direct Urca process, may help explain how some neutron stars cool so rapidly.

$$n \to p + e + \bar{\nu}$$

$$p + e \to n + \nu$$

Fig. 1: The two reactions in the Direct Urca process.

This process is normally suppressed below very high densities due to the requirement that momentum be conserved. However, in the presence of a strong magnetic field ($B \gtrsim 10^{16}$ G), energy quantization for protons and electrons enables the process to take place at lower densities. Our work investigates the consequences of this behavior, both for the cooling of individual magnetars and for the radiation environment in neutron star mergers.

Quasiclassical Approximation

The standard method of calculating the Direct Urca emissivity is the quasiclassical (QC) approximation, in which we treat the discrete energy levels of the protons and electrons as a continuum. Using this approximation the emissivity is easily computed as a prefactor times a simple numerical integral.

Naïvely, the QC approximation breaks down when the spacing between energy levels is not small compared to the Fermi energy (~100 MeV); this usually happens around a magnetic field strength $B \approx 10^{16}$ G.

Our work showed that the QC approximation remains valid above this field strength. At $B \approx 5 \times 10^{16}$ G, the QC result agrees with the full calculation at threshold; when averaged across the entire star, the QC emissivity is within half an order of magnitude of the correct result up to 10^{17} G.

Our Improvements

Our calculation incorporates a number of improvements over the standard approximation. Most importantly, we take a discrete sum over energy levels instead of assuming that they form a continuum. This introduces singularities in the density of states when the *z*-component of a particle's momentum is exactly zero, which happens upon the population of a new energy level. Fig. 2 shows the resulting emissivity enhancements.

The full calculation involves a computationally intensive phase space integral. We thus developed a *semi-analytic approximation* that maintains the accuracy of the full calculation while remaining nearly as fast as the quasiclassical method.

Other additions include:

- relativistic effects in the matrix element
- mean-field nuclear interactions
- thermal population of energy levels
- energy level splitting due to the proton's spin

The enhancements and suppressions generally offset to within half an order of magnitude, but the resonances themselves remain visible at low temperatures (Fig. 2).

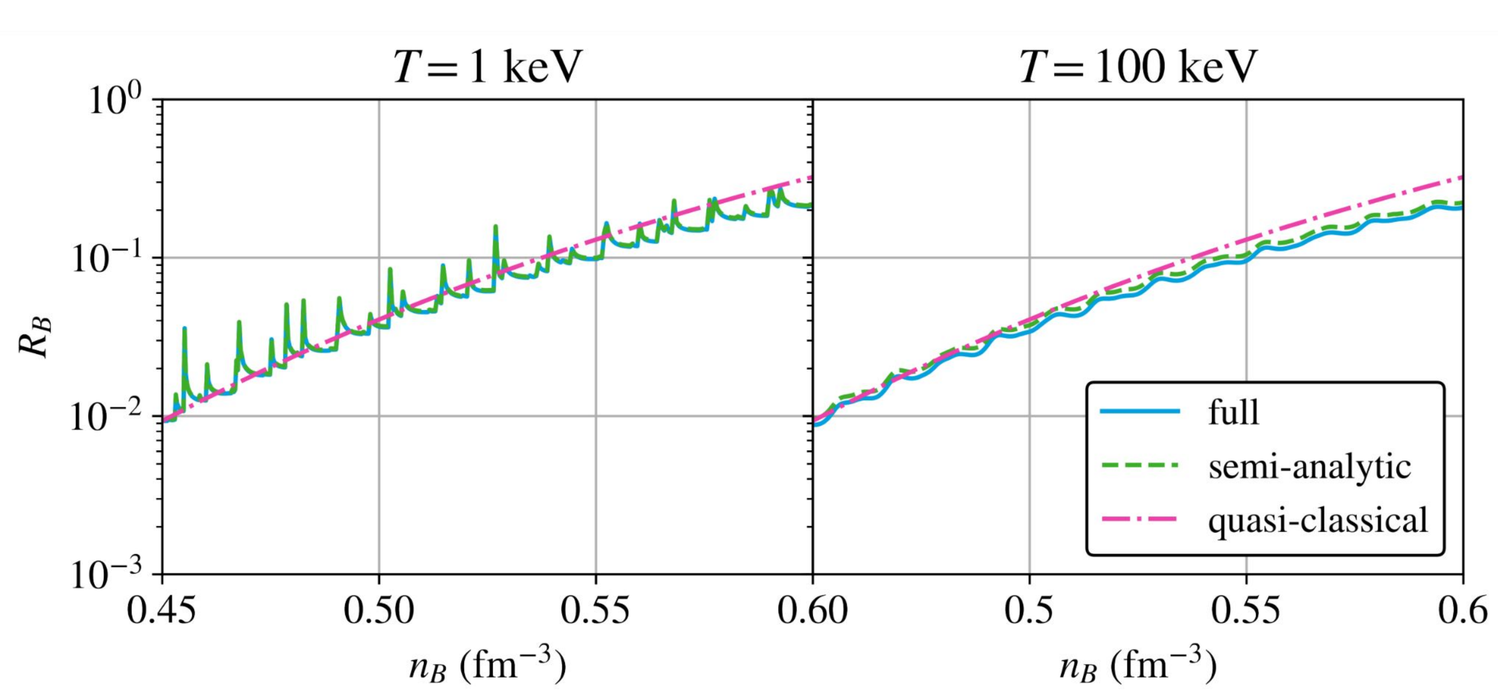

Fig. 2: Comparison of emissivity results at the Direct Urca threshold, with $B = 5 \times 10^{16}$ G. In the left panel, the semi-analytic approximation is indistinguishable from the exact result. Compare the spikes in the left plot to the smeared-out enhancement in the right plot.

Results

The full calculation introduces order–1 corrections that may be important locally (e.g. for transport coefficients). When averaged across the star, however, the correction to the quasiclassical result is generally small (Fig. 3). This validates the quasiclassical approximation beyond its naïve bounds.

The same tools are applicable to calculating the cross-section for neutrino capture in a merger environment. We find an enhancement to capture processes of an order of magnitude or more (Fig. 4).

B (10^{16} G)	$\tilde{T}$ (keV)	Full Direct Urca	QC Direct Urca
2	1	2.83×10^{24}	2.74×10^{24}
2	100	1.65×10^{36}	2.74×10^{36}
5	1	1.67×10^{29}	1.05×10^{29}
5	100	1.35×10^{41}	1.05×10^{41}
10	1	1.48×10^{31}	6.89×10^{30}
10	100	1.33×10^{43}	6.89×10^{42}

Fig. 3: Total emissivity (erg/s) of a 1.4-solar-mass neutron star for Direct Urca calculated in full and in the QC approximation.

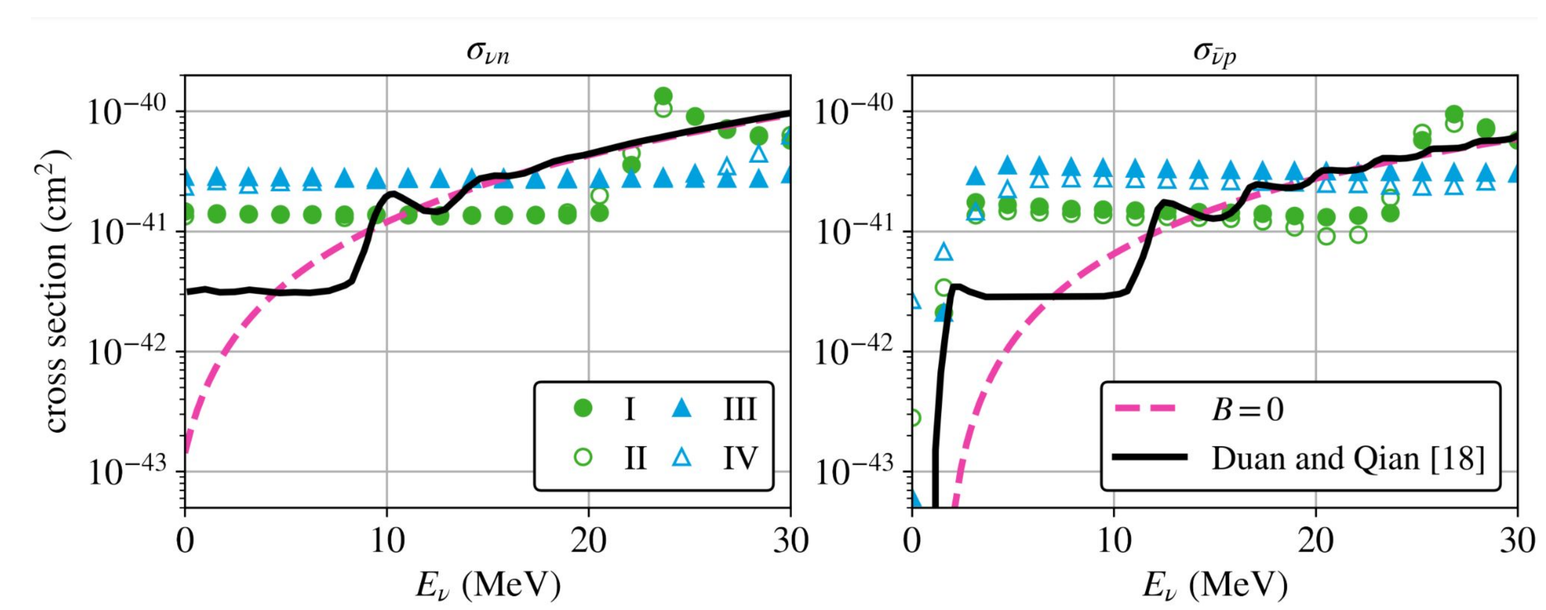

Fig. 4: Cross sections for neutrino capture on nucleons. Green circles correspond to $B = 5 \times 10^{16}$ G, while blue triangles are $B = 10^{17}$ G. Filled marks have T = 1 MeV while unfilled have T = 3 MeV and 8 MeV respectively. The black curve was calculated at $B = 10^{17}$ G, T = 2 MeV.

For references and acknowledgements, please see the full paper. Scan the QR code to go to arxiv.org/abs/2412.02925.