Pragmatic Lessons from χPT on low-energies weak current in nuclei

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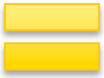
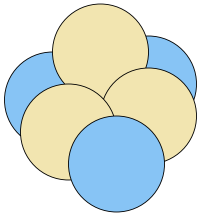
Introduction

- ***On the verge*** of a precision era in few-body nuclear physics:
  - Available methods for solving exactly the Schrödinger equation for few body systems, from their nucleonic degrees of freedom:
    - No core shell model.
    - Expansions in Hyperspherical Harmonics.
  - High precision nuclear interaction, phenomenological or χPT based:
    - Spectra of light nuclei.
    - Transitions and cross-sections.
- Will allow ***parameter free*** calculations of nuclear wave functions and low-energy reaction rates, with sub-percentage accuracy.
- How can we use this to gain understanding on interesting problems?



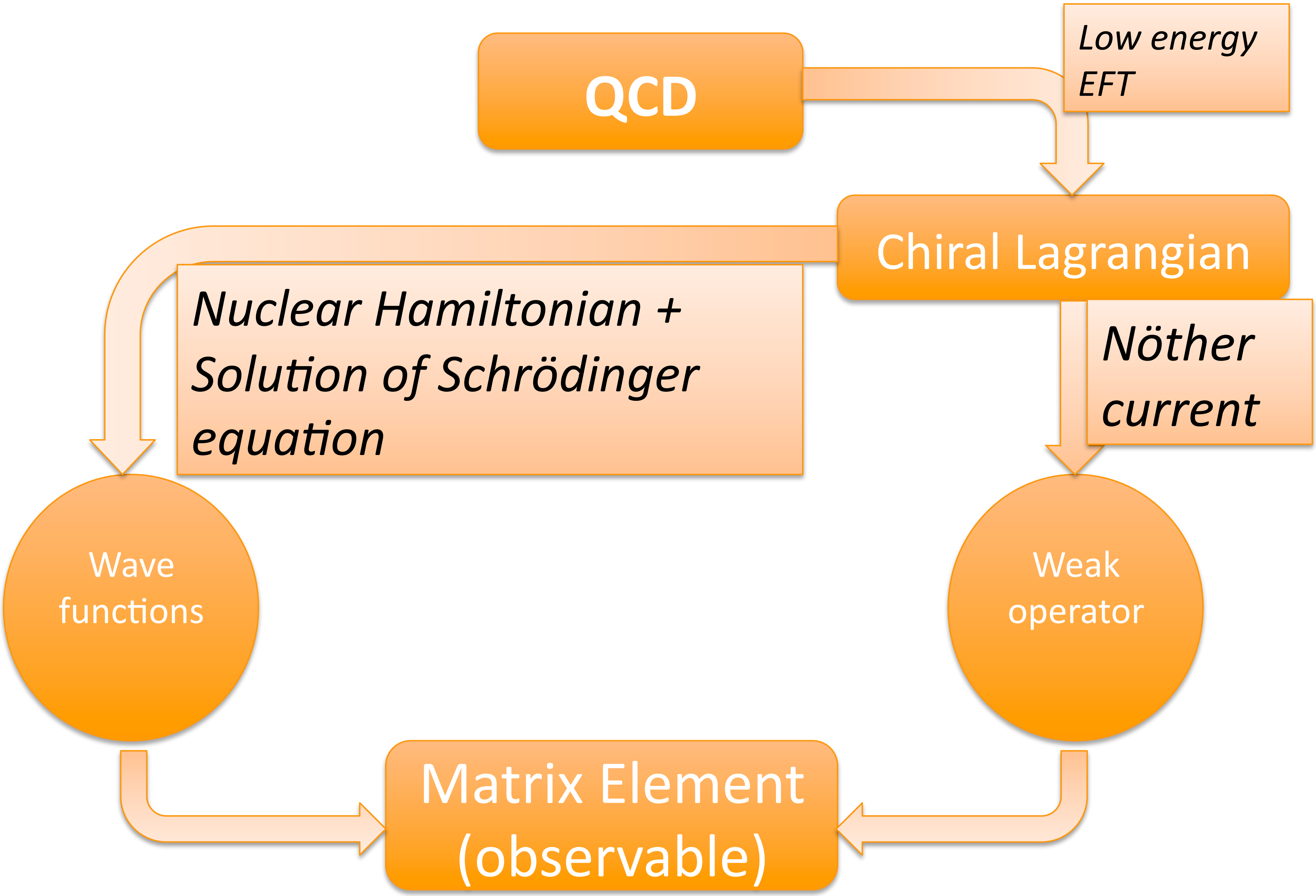


Light Nuclei

Outline

- Using χPT for calculating low-energy weak reactions.
- Applications:
  - *Constraining the nuclear force using triton β-decay and an inside look into correlations in the nucleus*.
  - *β-decay of ^{6}He – a difference between standard nuclear physics approach and* χPT approach.
  - *"Unexpected" success at higher energies: weak structure of the nucleon from μ-capture on ^{3}He*.
  - *Predictive force: Neutrino reactions with light nuclei in Supernovae*.
- Weak interaction in Holographic QCD: *easy access to the size of low-energy constants.*

χPT approach for low-energy EW nuclear reactions:



Effective Field Theory for low energy QCD

- We are aiming at energies which are relevant for nuclear phenomena – well below QCD breaking scale ~ 1 GeV.
- The constituent quarks are the up and down quarks.
- Their masses are small with respect to the QCD scale.

$$m_u = 2 \pm 1\,\mathrm{MeV} \text{ and } m_d = 5 \pm 2\,\mathrm{MeV}$$

- QCD Lagrangian with only the up and down quarks of vanishing mass: $\mathcal{L}^q_{QCD} = i\bar{q}\gamma^\mu \mathcal{D}_\mu q = i\bar{q}_R\gamma^\mu \mathcal{D}_\mu q_R + i\bar{q}_L\gamma^\mu \mathcal{D}_\mu q_L$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}\ ;\ q_{L,R} \equiv \frac{1}{2}(1 \pm \gamma_5)q$$

Effective Field Theory for low energy QCD

- Clearly, the QCD Lagrangian is invariant under:

$$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \mapsto \exp\left(-i\vec{\Theta}^R \cdot \frac{\vec{\tau}}{2}\right) \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \mapsto \exp\left(-i\vec{\Theta}^L \cdot \frac{\vec{\tau}}{2}\right) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

Or

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \mapsto \exp\left[-i\vec{\Theta}^V \cdot \frac{\vec{\tau}}{2}\right] \begin{pmatrix} u \\ d \end{pmatrix}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \mapsto \exp\left[-i\gamma_5\vec{\Theta}^A \cdot \frac{\vec{\tau}}{2}\right] \begin{pmatrix} u \\ d \end{pmatrix}$$

$$SU(2)_R \times SU(2)_L \cong SU(2)_V \times SU(2)_A$$

- This is an approximate symmetry of the Lagrangian due to the mass term:

$$M_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} = \underbrace{\frac{1}{2}(m_u + m_d)I + \underbrace{\frac{1}{2}(m_u - m_d)\tau_3}_{breaks\ SU(2)_V}}_{breaks\ SU(2)_A}$$

  - This creates deviations of the order $\dfrac{m_u \pm m_d}{M_N}$

Effective Field Theory for low energy QCD

- If this was a symmetry of the vacuum, there were approximate parity doublets in the QCD spectrum. However,
  - Nucleons of positive parity: p($\frac{1}{2}^+$,938.3), n($\frac{1}{2}^+$,939.6), I= ½
  - Nucleons of negative parity N($\frac{1}{2}^-$,1535), I= ½.
  - Mesons of Isospin 1: $\rho(1^-,770)$ and $a_1(1^+,1260)$
- Masses are very different → No parity doublets in the spectrum.

Conclusion: The chiral symmetry is ***spontaneously*** broken

- The Goldstone-Nambu spin zero bosons are the pions:
  - They are not massless due to the explicit symmetry breaking, though -

$$\xi = \exp\left(i\frac{\vec{\pi}\cdot\vec{\tau}}{2f_\pi}\right)$$

$$\left(\frac{m_\pi}{m_N}\right)^2 \approx 0.02.$$

Transformation rules:

- Goldstone's theorem states that $U \equiv \xi^2$, belongs to the $(\overline{2},2)$ representation of $SU(2)_L x SU(2)_R$: $U \to LUR^\dagger$
- A nonlinear realization of the symmetry (as $U^\dagger U = 1,\ |U| = 1$).
- The resulting transformation rules: $h \in SU(2)_V$

$$N \mapsto hN$$

$$\xi \mapsto L\xi h^\dagger = h\xi R^\dagger$$

- Invariant terms:

$$\overline{N}N,\ \overline{N}\gamma^\mu D_\mu N,\ \overline{N}\gamma^\mu \gamma_5 a_\mu N$$

$$with:$$

$$D_\mu = \partial_\mu + iv_\mu$$

$$v_\mu = -\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right)\ ;\quad a_\mu = -\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right)$$

Effective field theory (EFT) for nuclear physics: **Chiral perturbation theory (χPT)**

- **Symmetries are important *NOT* degrees of freedom:**
  - **In QCD – an approximate chiral symmetry:**

$$SU(2)_L \times SU(2)_R \cong SU(2)_V \times SU(2)_A \rightarrow SU(2)_V$$

  - **Pions – Goldstone bosons of the broken symmetry.**
- **Choose Λ – the cutoff of the theory.** (400-800 MeV)
- **Identify Q – the energy scale of the process.** (around 100 MeV)
- **In view of Q and Λ -Identify the relevant degrees of freedom.** (pions and nucleons).
- **Write all the possible operators which agree with the symmetries of the underlying theory (INFINITE)**
- **Calculate Feynman diagrams (INFINTE)**
- **Find a systematic way to organize diagrams according to their contribution**

Weinberg's Power Counting Scheme

- Each Feynman diagram can be characterized by: $\left(\frac{Q}{\Lambda_\chi}\right)^\nu$
- Q~100 MeV is the relevant momentum of the process or pion masses in the diagram.
- $\Lambda_\chi \sim 1\ \text{GeV}$ is the chiral symmetry breaking scale.
- Weinberg showed: $\nu \geq 0$

↓

Chiral Perturbation Theory

- In addition, expand in the nucleon's mass (take Λ_χ~M_N) ➔Heavy Baryon χPT.

The power counting

$$\text{Power} = -2 + 2A - 2C + 2L + \sum_{\text{all vertices}} \Delta_i$$

with

$A =$ number of nucleons;

$C =$ number of seperately connected pieces;

$L =$ number of loops;

$$\Delta_i = d_i + \frac{n_i}{2} - 2,$$

where

$d_i =$ number of derivatives,

$n_i =$ number of nucleon operators.

The Lagrangian we use

$$\mathcal{L} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$$

- Pion Lagarngian: $\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\,\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger + m_\pi^2\left(U + U^\dagger\right)\right]$

- Nucleon-Pion Lagrangian: $\mathcal{L}_{\pi N}^{(2)} = \bar{N}\left\{i\gamma_\mu \mathcal{D}^\mu + g_A \gamma^\mu \gamma_5 a_\mu - M_0\right\} N$

$$\delta_1 \mathcal{L}_{\pi N}^{(3)} = \frac{2\hat{c}_3}{M_N} \bar{N} N \mathrm{Tr}\left(\mathrm{a}_\mu \mathrm{a}^\mu\right)$$

$$\delta_2 \mathcal{L}_{\pi N}^{(3)} = i\frac{\hat{c}_4}{M_N} \bar{N}[a_\mu, a_\nu]\sigma^{\mu\nu} N,$$

- Nucleon-Nucleon contact terms.
  - Allowed, and also needed to remove divergences.
  - Represent short range correlations.

$$\mathcal{L}_4 = -2D_1\left(\bar{N}\gamma^\mu\gamma_5 a_\mu N\right)\left(\bar{N}N\right)$$

ab initio methods to solve the Schrödinger equation

- Expanding the wave functions in a known basis to get an exact solution to the equation.
- Using effective interaction approach to accelerate the convergence (mainly for A>3).

Hyperspherical Harmonics (EIHH)

- Correct long range behavior.
- Difficult to antisymmetrize.

A_{max}~7, reactions,

No Core Shell Model (NCSM)

- Incorrect long range behavior.
- Antisymmetrization – easier.
- Rather indifferent to local/ nonlocal forces.

A_{max}~15, spectra

*Barnea, Leidemann, Orlandini, **Phys. Rev. C, 63** 057002 (2001).*

Navratil, Vary, Barrett, ***Phys. Rev. Lett., 84*** 5728 (2000).

Benchmark calculation of a four body bound state with a realistic NN potential AV8'

Method	E_b [MeV]	Matter radius [fm]
FY	25.94(5)	1.485(3)
CRCGV	25.90	1.482
SVM	25.92	1.486
HH	25.90(1)	1.483
GFMC	25.93(3)	1.490(5)
NCSM	25.80(20)	1.485
EIHH	25.944(10)	1.486

Kamada *et al*, Phys. Rev. C **64**, 044001 (2001)

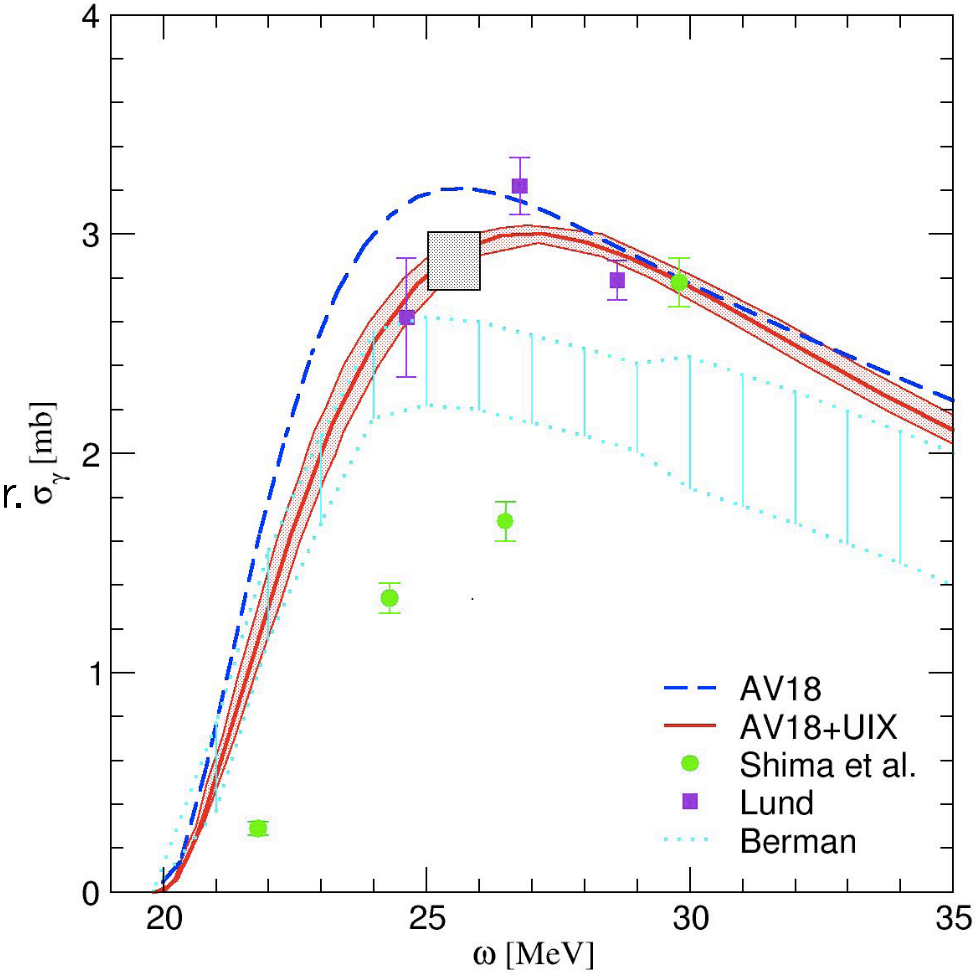
Calculation of ^{4}He bound state with *state of the art* NN+NNN potentials AV18+UIX

E_{exp}=28.296 MeV

	E_b [MeV]	Matter radius [fm]
EIHH [DG et. al]	28.418	1.432
FY [Nogga et. al]	28.50	
HH [Viviani et. al]	28.46	1.428
GFMC [Wiringa et. al]	28.34	1.43

Photoabsorption on ^{4}He

- At low-energy:
  - Scattering constrained by current conservation (Siegert theorem).
  - Governed by the dipole operator.



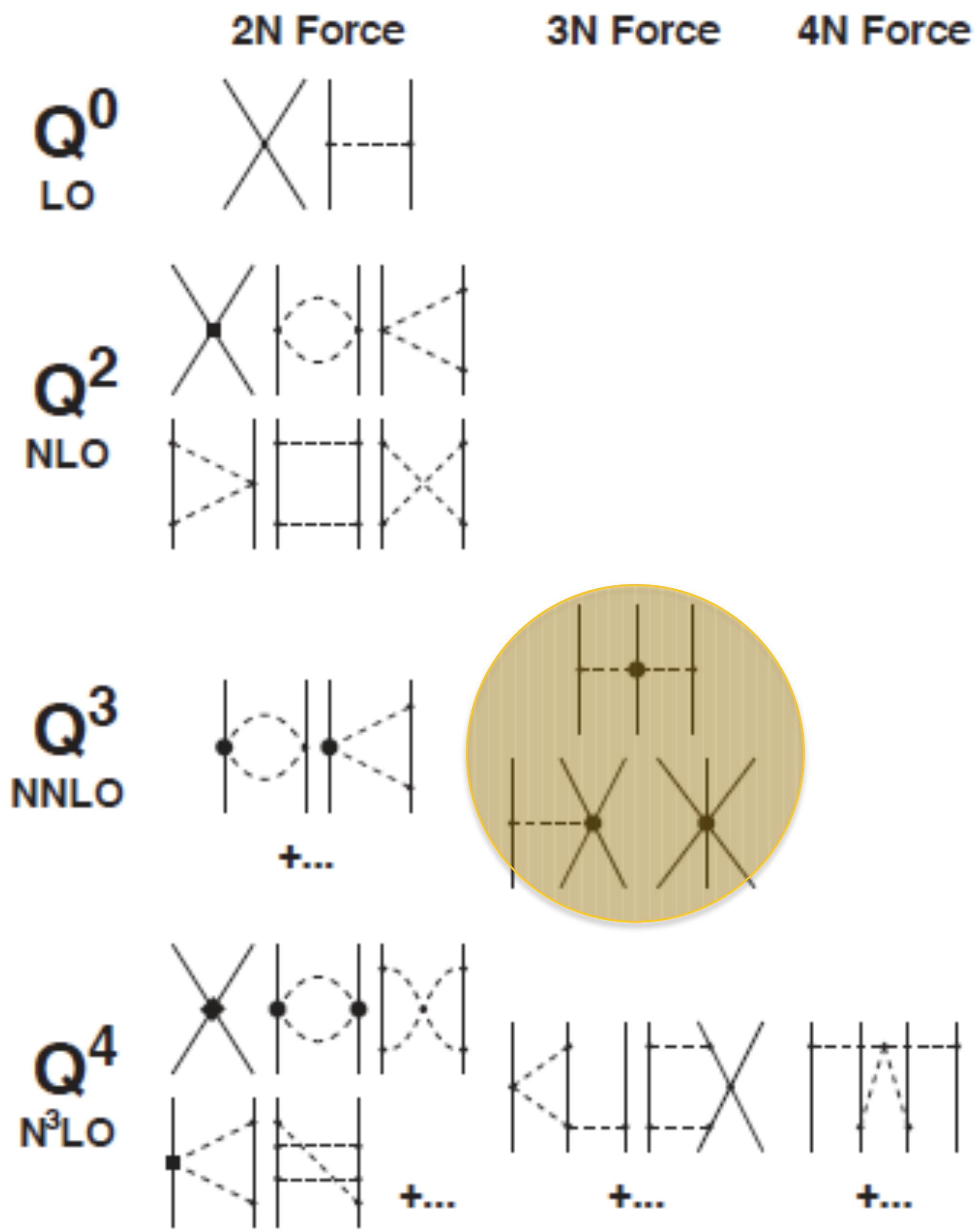
DG, Bacca, Branea, Leidemann, Orlandini, **Phys. Rev. Lett. 96**, 112301 (2007)

Hierarchy of Nuclear Forces in χPT

- Only contact terms cannot be calibrated in the pion or pion/ nucleon system.
- The 2N terms are calibrated to reproduce phase shifts.

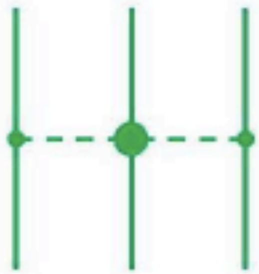
χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^{3}LO	NNLO	NLO	AV18
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04



3 nucleon forces at N^2LO

TPE-3NF



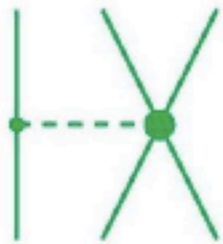
$$V^{3NF}_{TPE} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i\neq j\neq k} \frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(q_i^2+m_\pi^2)(q_j^2+m_\pi^2)} F^{\alpha\beta}_{ijk}\, \tau_i^\alpha \tau_j^\beta$$

with $\vec{q}_i \equiv \vec{p}_i\,' - \vec{p}_i$, where $\vec{p}_i$ and $\vec{p}_i\,'$ are the initial and final momenta of nucleon i, respectively, and

$$F^{\alpha\beta}_{ijk} = \delta^{\alpha\beta}\left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\,\vec{q}_i\cdot\vec{q}_j\right] + \frac{c_4}{f_\pi^2}\sum_\gamma \epsilon^{\alpha\beta\gamma}\,\tau_k^\gamma\,\vec{\sigma}_k\cdot[\vec{q}_i\times\vec{q}_j]$$

No new parameters!

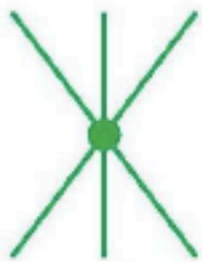
OPE-3NF



$$V^{3NF}_{OPE} = c_D \sum_{i\neq j\neq k} \frac{\vec{\sigma}_j\cdot\vec{q}_j}{q_j^2+m_\pi^2}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\vec{\sigma}_i\cdot\vec{q}_j)$$

New parameter $\mathcal{L}_4 = -2D_1\,(\bar{N}\gamma^\mu\gamma_5 a_\mu N)\,(\bar{N}N)$

Contact-3NF



$$V^{3NF}_{ct} = c_E \sum_{j\neq k} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\,.$$

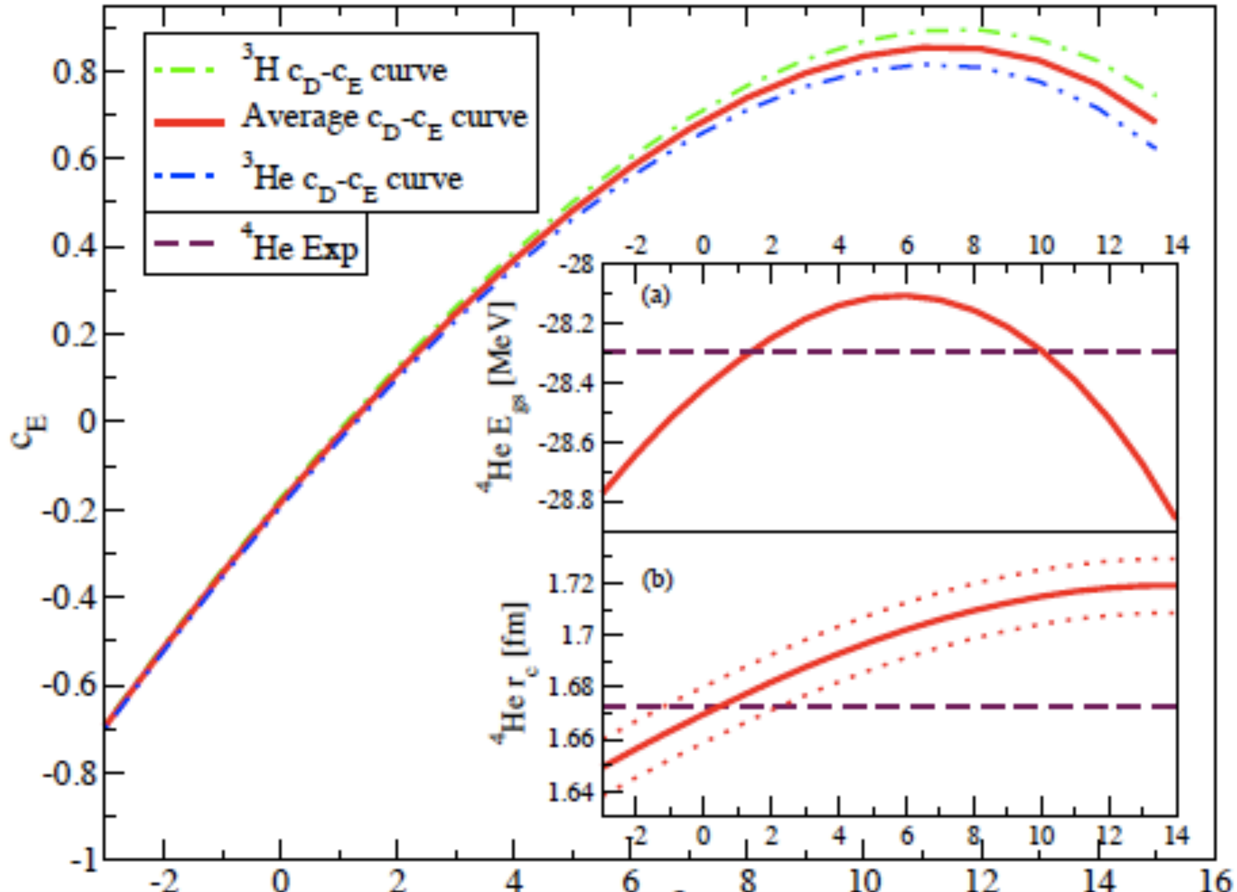
New parameter

Strategy : calibrate c_D and c_E from nuclear matter observables,
and then predict other observables.

Attempts to calibrate the contact parameters

Other attempts were to use 3 nucleon scattering lengths as a second observable.

The problem is the cross-correlation of the different observables.



Calibrating the contact parameters using weak observables?(!)

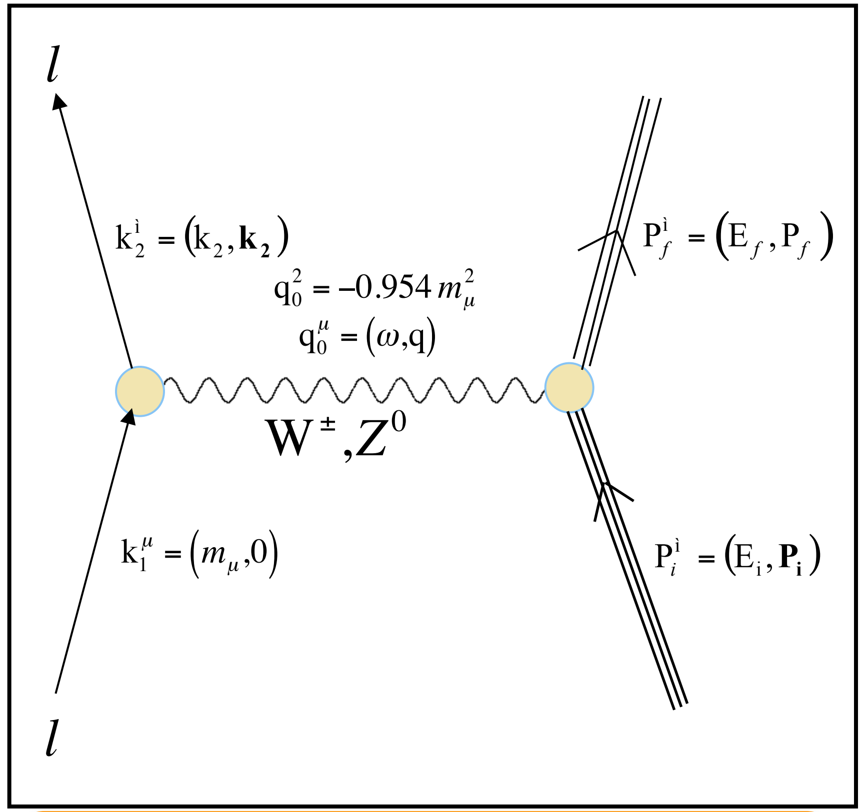
Navratil *et al*, **Phys. Rev. Lett. 99**, 042501 (2007).

Weak interaction with the nucleus

$$\hat{H}_W = -\frac{G_F V_{ud}}{\sqrt{2}} \int d^3\vec{x}\, \hat{j}_\mu{}^+(\vec{x})\, \hat{J}^{\mu\,-}(\vec{x})$$

Lepton current

Nuclear current



Scattering operator

⇕

SU(2) Currents in the nucleus

$$W,Z \;\; propagator = \frac{g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 + M_W^2} \xrightarrow[q \ll M_W]{} \frac{g_{\mu\nu}}{M_W^2}$$

Weak currents in the nucleus

- The standard model dictates only the structure of the currets:
  - Charged current $\mathcal{J}_\mu^{(\pm)} = \frac{\tau_\pm}{2}(J_\mu^V + J_\mu^A)$
  - Neutral current: $\mathcal{J}_\mu^{(0)} = (1 - 2\cdot\sin^2\theta_W)\frac{\tau_0}{2}J_\mu^V + \frac{\tau_0}{2}J_\mu^A - 2\cdot\sin^2\theta_W\frac{1}{2}J_\mu^V$
- The current of polar (axial) vector symmetry is the Noether current of the QCD Lagrangian, with respect to $SU(2)_V$ [$SU(2)_A$] symmetry.
- Includes:
  - single nucleon currents.
  - Meson exchange currents.

Weak Currents in the Nucleus from χPT

- $SU(2)_L x SU(2)_R$ is the gauging of the weak force.
- Weak currents are thus the Nother current of this symmetry.

$$\mathcal{J}^{a\mu} \equiv -\frac{\partial \mathcal{L}}{\partial(\partial_\mu \epsilon^a(x))}$$

- In χPT:
  - Single nucleon currents come at leading order (and receive momentum dependent corrections at higher orders).
  - Meson exchange currents start at N^2LO.

T.-S. Park *et al*, **Phys. Rev. C 67**, 055206 (2003); DG PhD thesis **arXiv: 0807.0216**

Single Nucleon Currents

$$\hat{J}^{\mu\mathbf{V}} = \bar{u}(p')\left[F_V(q^2)\gamma^\mu + \frac{i}{2M_N}F_M(q^2)\sigma^{\mu\nu}q_\nu + \frac{g_S}{m_\mu}q^\mu\right]u(p)$$

Vector

Magnetic

Second class currents

$$\hat{J}^{\mu\mathbf{A}} = -\bar{u}(p')\left[G_A(q^2)\gamma^\mu\gamma_5 + \frac{g_P(q^2)}{m_\mu}\gamma_5 q^\mu + \frac{ig_t}{2M_N}\sigma^{\mu\nu}\gamma_5 q_\nu\right]u(p)$$

Axial

Induced
Pseudo-Scalar

- q dependence is due to pion loops.
- Second class currents vanish to this order!

Weinberg **Phys. Rev.**, 112, 1375 (1958)

Meson Exchange currents

- Vector currents, protected by charge conservation (or CVC), do not include contact parameters, up to fourth order.
- Axial currents are more complicated, in configuration space:

$$\hat{\mathcal{A}}^{i,a}_{12}(\vec{r_{ij}}) = \frac{g_A}{2Mf_\pi^2}\hat{d}_r\mathcal{O}^{i,a}_{\ominus}\delta^{(3)}_\Lambda(\vec{r}_{ij}) + \frac{g_A m_\pi^2}{2Mf_\pi^2}\mathcal{O}^{i,a}_P y^\pi_{1\Lambda}(r_{12}) -$$

$$-\frac{g_A m_\pi^2}{2Mf_\pi^2}\left[\frac{\hat{c}_3}{3}(\mathcal{O}^{i,a}_{\oplus}+\mathcal{O}^{i,a}_{\ominus}) + \frac{2}{3}(\hat{c}_4+\frac{1}{4})\mathcal{O}^{i,a}_{\otimes}\right] y^\pi_{0\Lambda}(r_{ij})$$

$$-\frac{g_A m_\pi^2}{2Mf_\pi^2}\left[\hat{c}_3(\mathcal{T}^{i,a}_{\oplus}+\mathcal{T}^{i,a}_{\ominus}) - (\hat{c}_4+\frac{1}{4})\mathcal{T}^{i,a}_{\otimes}\right] y^\pi_{2\Lambda}(r_{ij})$$

$$\delta^{(3)}_\Lambda(\vec{r}) \equiv \int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\vec{r}}S^2_\Lambda(\vec{k}^2),$$

$$y^\pi_{\Lambda 0}(r) \equiv \int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\vec{r}}S^2_\Lambda(\vec{k}^2)\frac{1}{\vec{k}^2+m_\pi^2}$$

$$y^\pi_{\Lambda 1}(r) \equiv -\frac{\partial}{\partial r}y^\pi_{\Lambda 0}(r),$$

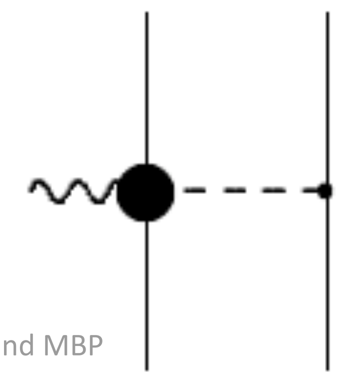
$$y^\pi_{\Lambda 2}(r) \equiv \frac{1}{m_\pi^2}r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}y^\pi_{\Lambda 0}(r)$$

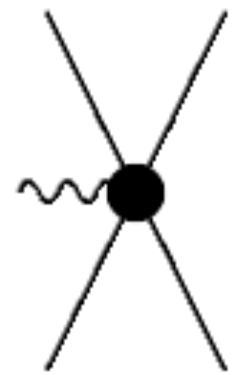
$$\vec{\mathcal{O}}^a_P = -\frac{m_\pi}{\ldots}(\vec{\tau}^{(1)}\times\vec{\tau}^{(2)})^a(\vec{P}_1\vec{\sigma}^{(2)}\cdot\hat{r}_{12}+\vec{P}_2\vec{\sigma}^{(1)}\cdot\hat{r}_{12})$$

$$\hat{d}_R \equiv \frac{M_N}{\Lambda_\chi g_A}c_D + \frac{1}{3}M_N(c_3+2c_4)+\frac{1}{6}$$

1 pion exchange

Contact term





CONSTRAINING THE NUCLEAR FORCE USING ^3H BETA-DECAY

DG, S. Quaglioni, P. Navratil, arxiv: 0812.4444.

Nuclear Matrix Elements

- A multipole decomposition of the currents is very helpful:

$$\hat{C}_{JM}(q) = \int d\vec{x} j_J(qx) Y_{JM}(\hat{x}) \hat{\mathcal{J}}_0(\vec{x})$$

$$\hat{E}_{JM}(q) = \frac{1}{q} \int d\vec{x} \vec{\nabla} \times [j_J(qx) \vec{Y}_{JJM}(\hat{x})] \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

$$\hat{M}_{JM}(q) = \int d\vec{x} j_J(qx) \vec{Y}_{JJM}(\hat{x}) \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

$$\hat{L}_{JM}(q) = \frac{i}{q} \int d\vec{x} \vec{\nabla} [j_J(qx) Y_{JM}(\hat{x})] \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

- Usually, the low energy and selection rules mean that only a small number of multipoles contribute.

β decay rate for $q \rightarrow 0$

$$(fT_{1/2})_t = \frac{K/(G^2|V_{ud}|^2)}{|\mathrm{F}|^2 + \frac{f_A}{f_V} g_A^2 |\mathrm{GT}|^2} .$$

$$\mathrm{F} \equiv \sqrt{\frac{4\pi}{2\mathrm{J_i}+1}} \langle \mathrm{C_0^V} \rangle$$

$$\mathrm{GT} \equiv \sqrt{\frac{6\pi}{2\mathrm{J_i}+1}} \frac{\langle \mathrm{E_1^A} \rangle}{\mathrm{g_A}}$$

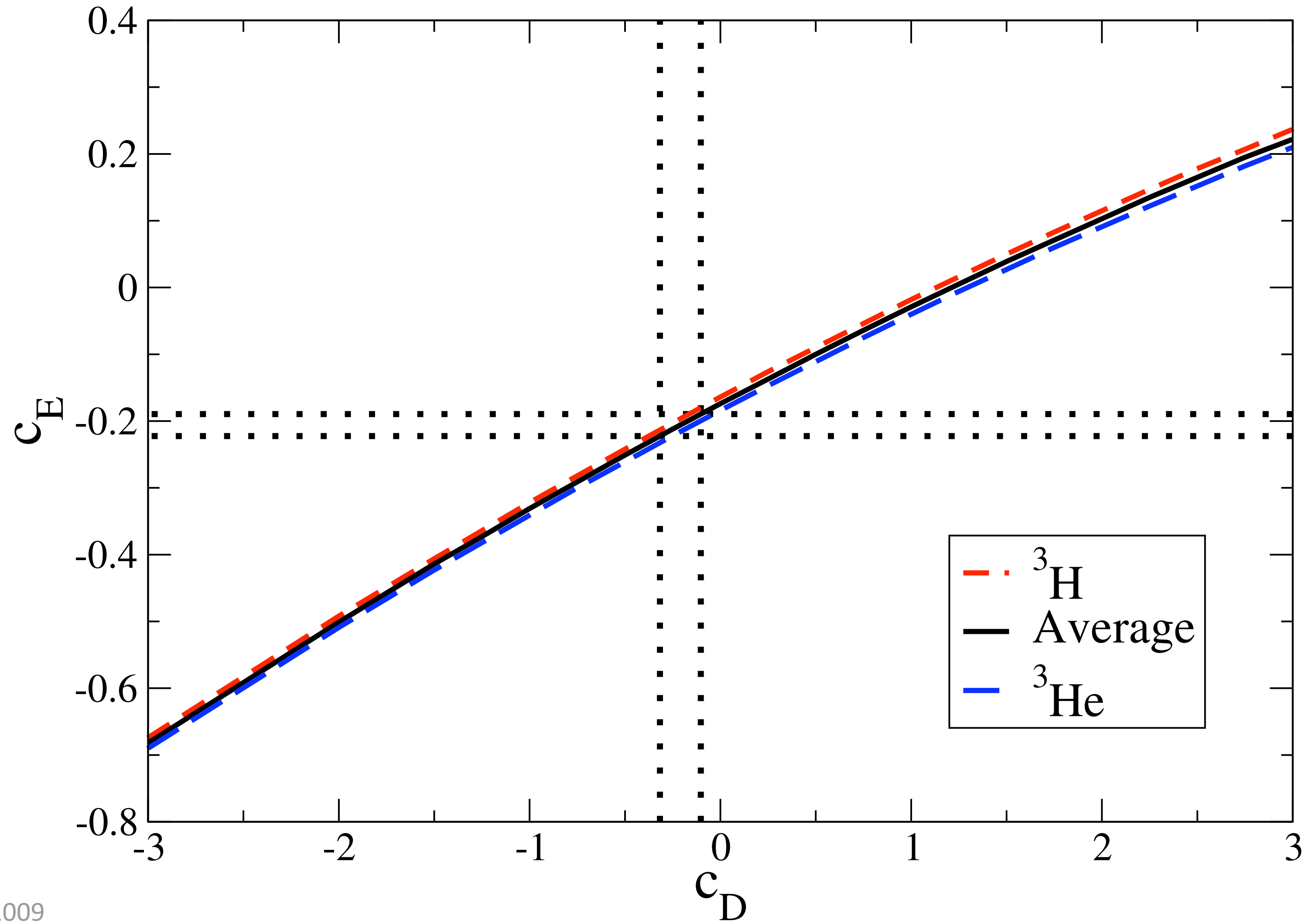
- At the leading order: $\mathrm{GT}\big|_{LO} = \sum \tau_i^+ \vec{\sigma}_i$
- This is the origin of the commonly used name: experimental (empirical) Gamow-Teller.
- For the triton β-decay:

$$\left\langle E_1^A \right\rangle\Big|_{emp} = 0.6848 \pm_{exp} 0.0007 \left(\pm_{g_A} 0.0007 \right)$$

Akulov, Mamyrin, **Phys. Lett. B 610**, 45 (2005)
Simpson, **Phys. Rev. C 35**, 752 (1987)
Schiavilla, **Phys. Rev. C 58**, 1263 (1998)

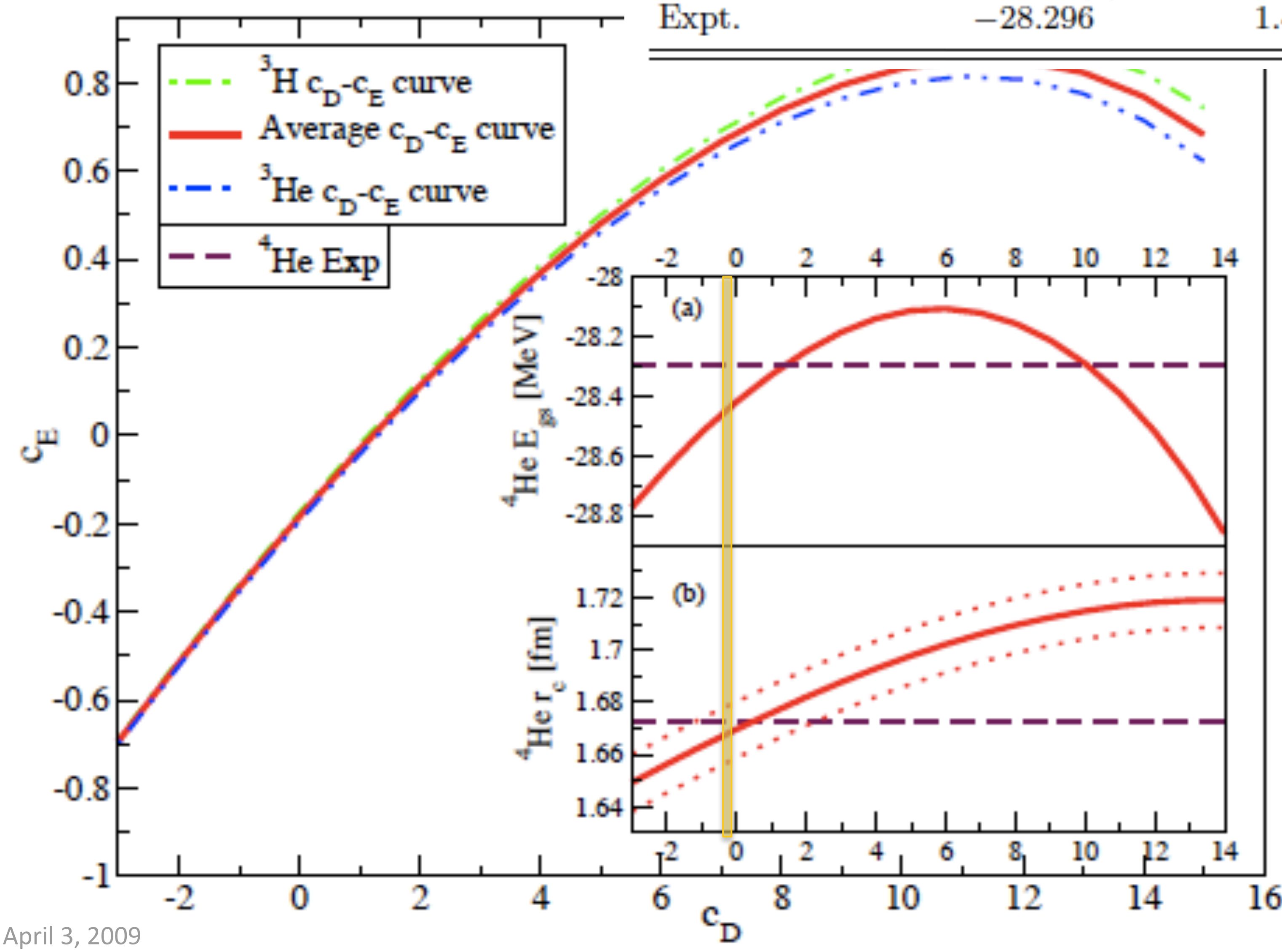
$$-0.3 \leq c_D \leq -0.1$$
$$\Downarrow$$
$$c_E \in [-0.220, -0.189]$$

Calibration result



A prediction of ^4He

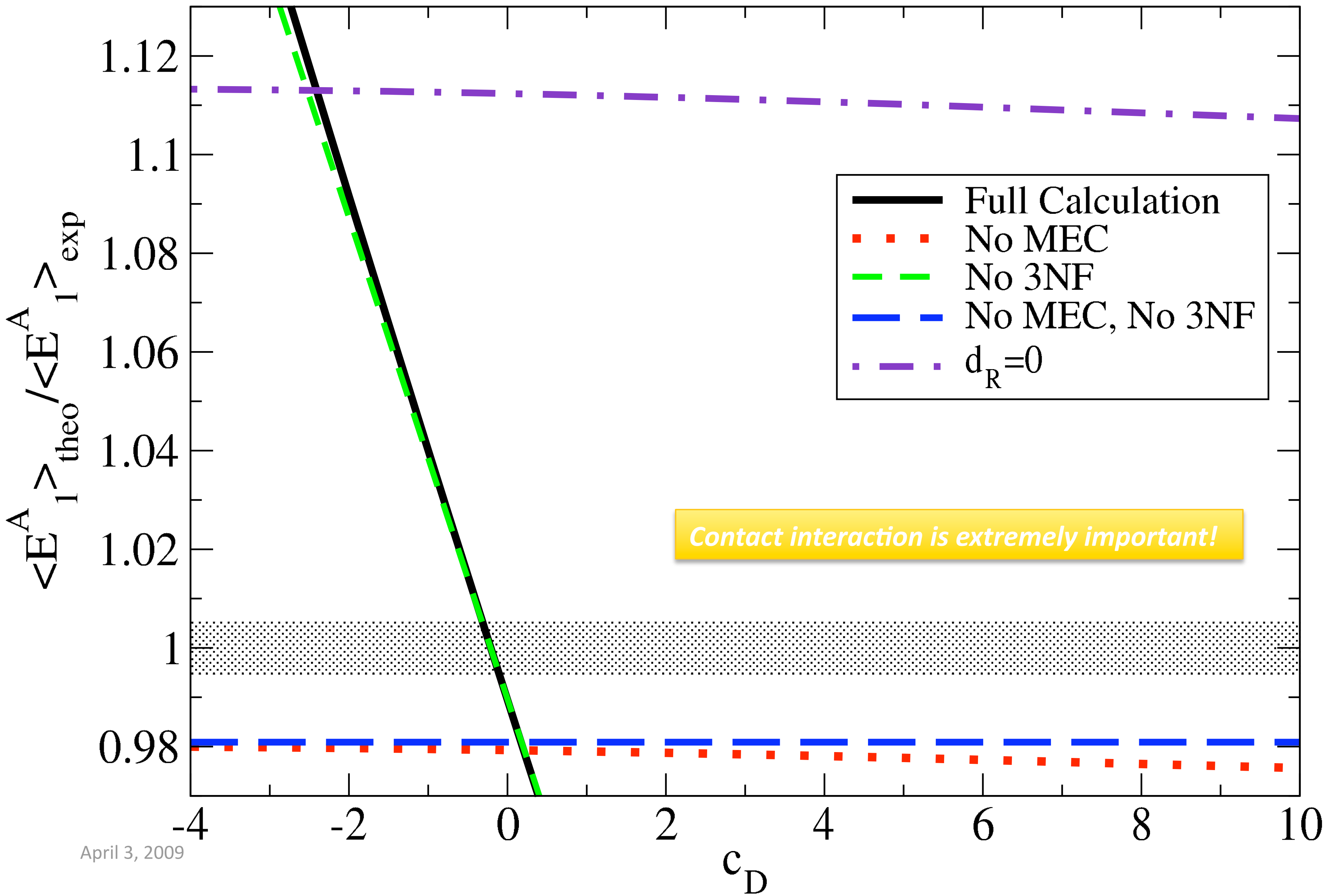
	^4He	
	$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$
NN	$-25.39(1)$	$1.515(2)$
$NN+NNN$	$-28.50(2)$	$1.461(2)$
Expt.	-28.296	$1.467(13)$ [24]



Not all is good yet…

- ***What is the correct way to do a consistent calculation?***
- Checked only with a specific χPT Force:
  - No cutoff dependence.
  - Well, we just talked about that for 2 weeks…
- What is the effect of the missing 3NF diagrams?
- p-shell nuclei seem to suggest c_D~-1.
  - Renormalizing effect of the missing 3NF?
  - Numerical problems when calculating p-shell nuclei?
- There is still uncertainty, due to poorly known LECs (c_4):
  - Still has to be checked consistently.
- …
- …

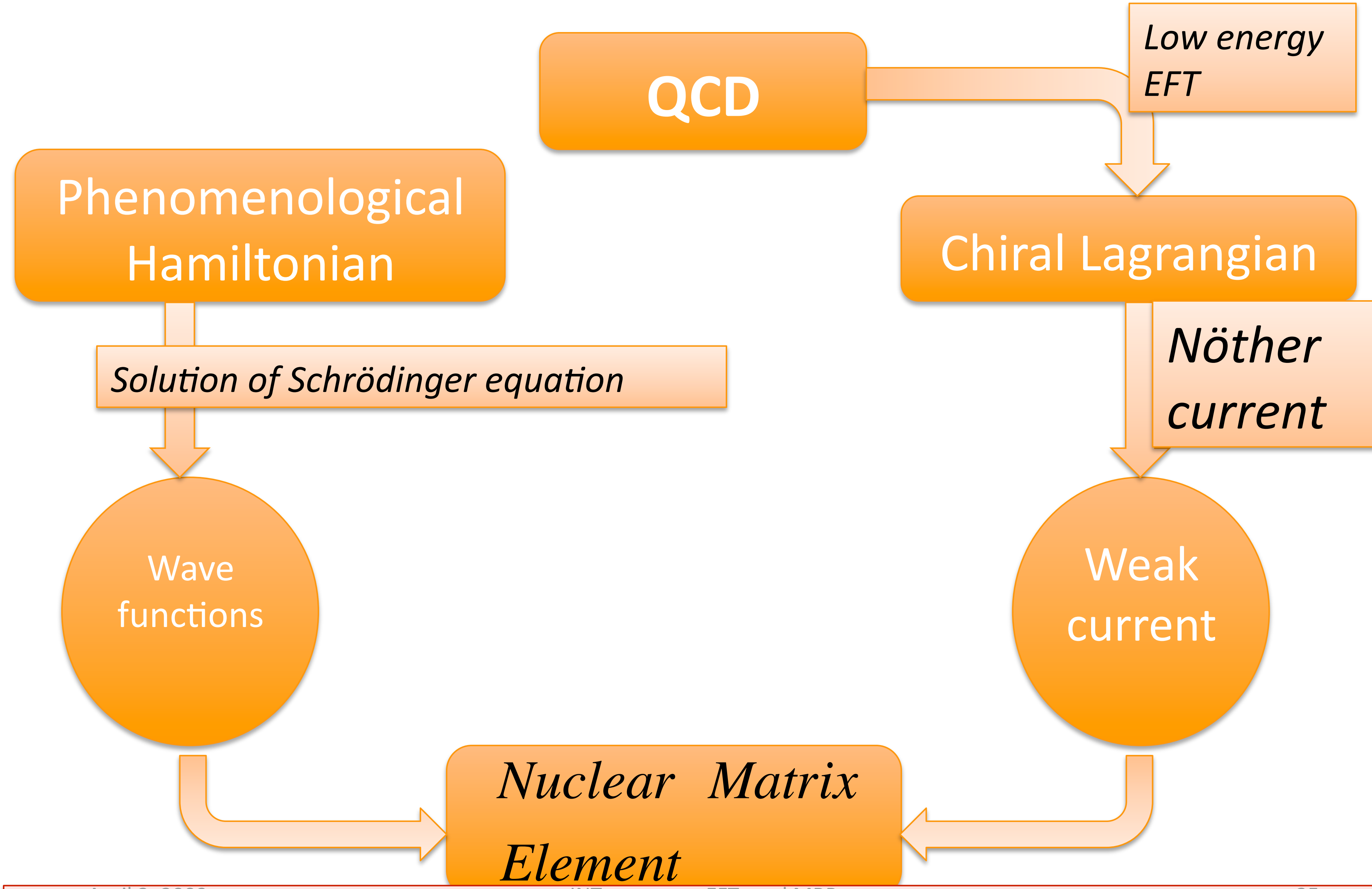
What can we learn about correlations in the wave function?



The apparent conclusion

- For GT type of operators, the short range correlations in the wave functions are not important for the observable.
- Is this the origin of the success of EFT*: hybrid calculations of weak reactions, using phenomenological forces in combination with χPT based currents?
  - One unknown parameter in MEC (d_R) calibrated using the triton half-life.

EFT approach for low-energy nuclear reactions:*



T.-S. Park *et al*, **Phys. Rev. C 67**, 055206 (2003), M. Rho **arXiv:** nucl-th/061003.

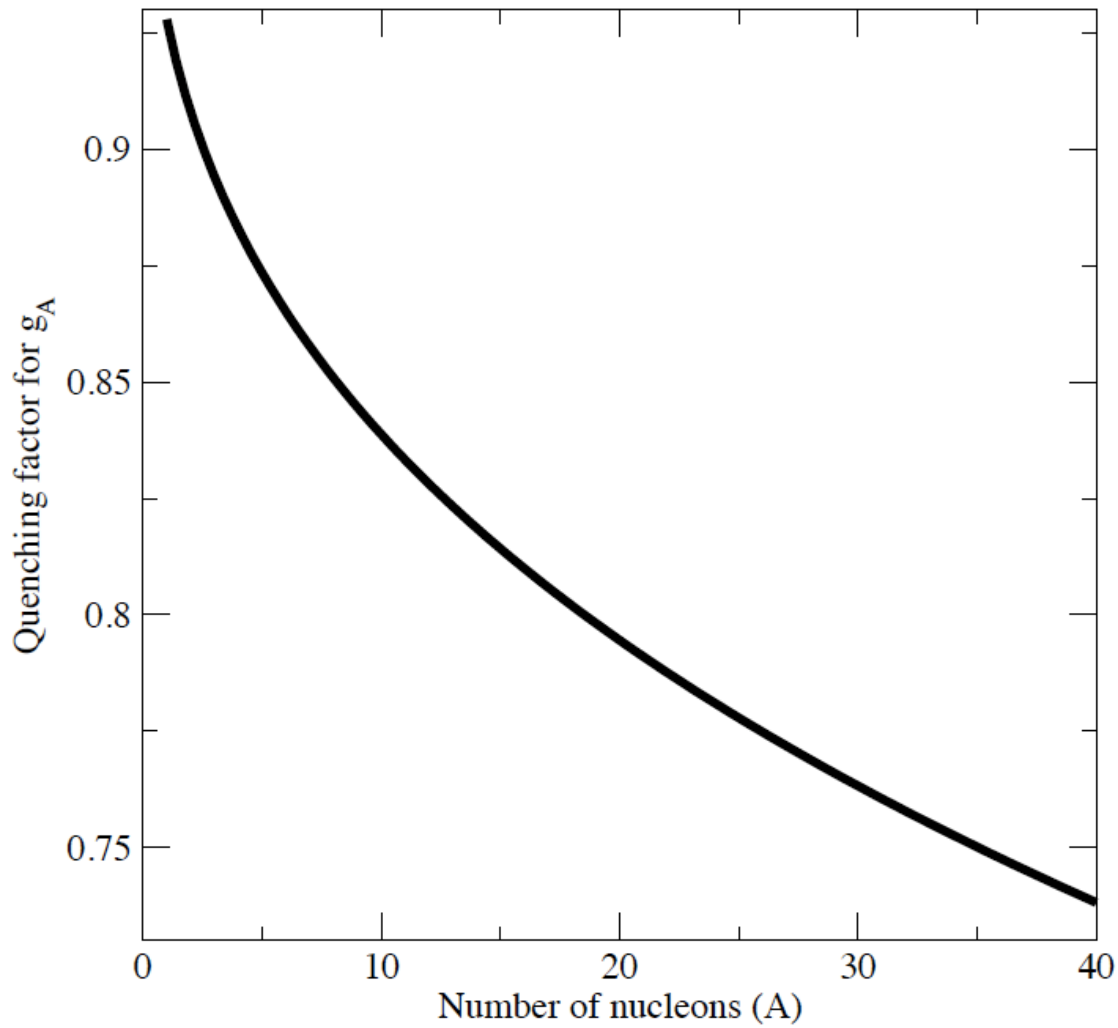
WHAT CAN WE LEARN FROM ^{6}HE BETA-DECAY ABOUT THE SUPPRESSION OF G_A IN NUCLEAR MATTER? SNPA VS. EFT BASED MEC?

- Surveys of "empirical Gamow-Teller" show that $g_A \rightarrow 1$, as A grows.
- This has been related to:
  - Restoration of axial symmetry.
  - Lack of correlations in the calculation.
  - Loop corrections from nucleonic excitations.
  - Something beyond the standard model?
- Schiavilla and Wiringa showed that for ^{6}He, the suppression is about 4%. The MEC actually increased the suppression!!
  - A real effect?
  - ***Problems in the weak current?***

DG, S. Vaintraub, N. Barnea, arXiv:0903.1048 (2009).

What does it mean $g_A \rightarrow 1$?

- Take the experimental value of the half life.
- Extract the empirical GT.
- Calculate GT via shell model (assumes LO, sometimes RC are added).
- The ratio between GT(shell model) and GT(emp) is g_A.
- Plot g_A as a function of the nuclear mass A.



Calculation Approach (1)

- I apologize, but we have to use a hybrid approach:
  - JISP16 NN potential is used to calculate the ground states WF of ^{6}He, ^{6}Li, ^{3}H, ^{3}He.
  - EFT based MEC.
- Calculate the ^{3}H decay rate, as a function of d_R for various cutoff values.
- Calibrate $d_R(\Lambda)$ by fitting the half life of ^{3}H to the experimental.

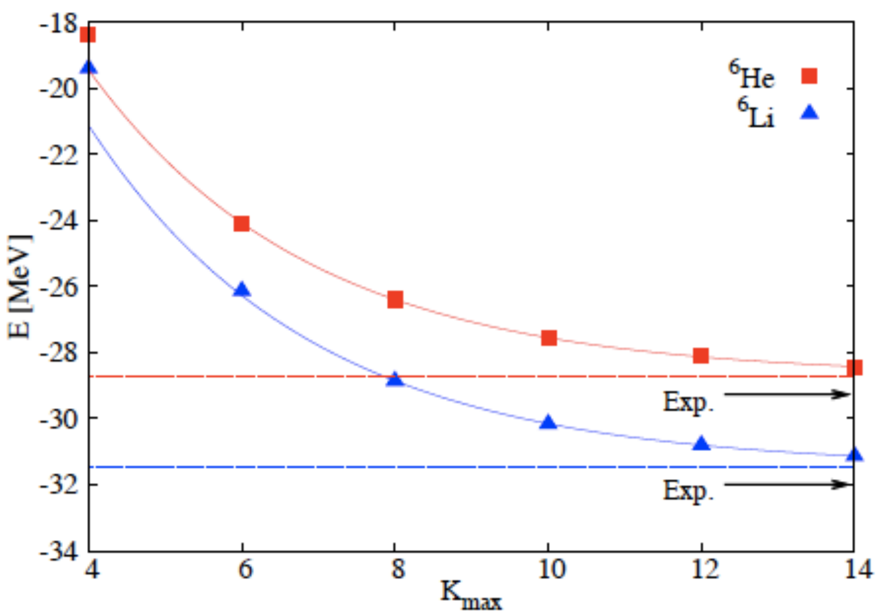
	^{3}H		^{3}He		
K_{max}	B.E.	radius	B.E.	radius	GT$\|_{LO}$
4	8.094	1.632	7.364	1.653	1.6656
6	8.233	1.656	7.512	1.680	1.6620
8	8.319	1.677	7.604	1.704	1.6575
10	8.351	1.691	7.641	1.720	1.6547
12	8.360	1.697	7.651	1.727	1.6538
14	8.365	1.701	7.657	1.733	1.6530
16	8.367	1.704	7.660	1.736	1.6526
18	8.367	1.705	7.661	1.738	1.6524
$[20]_V$	8.354		7.648		
$[20]_E$	8.496(20)		7.797(17)		
Exp.	8.482		7.718		

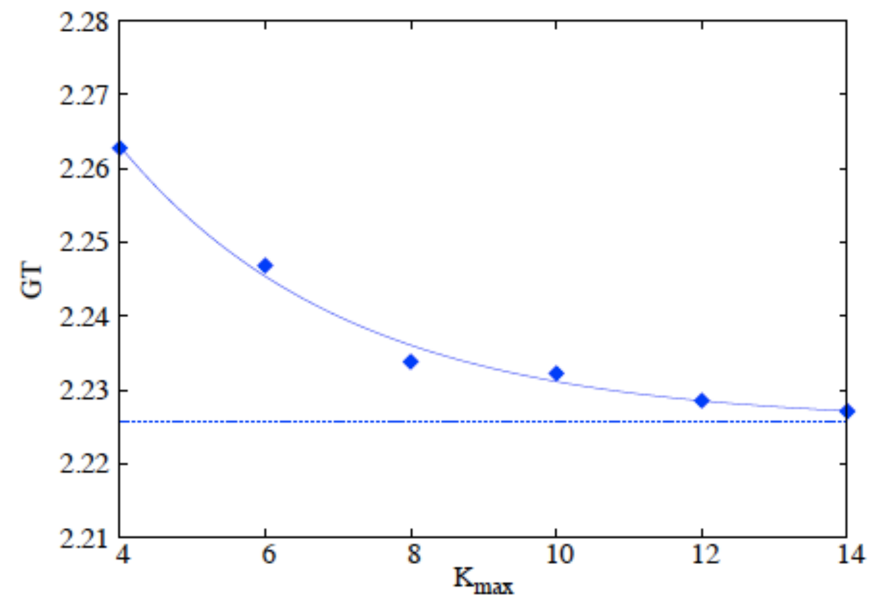
Potential model	GT$\|_{LO}$
AV18+3NF [32]	1.598(2)
Bonn+3NF [33]	1.621(2)
Nijm+3NF [34]	1.605(2)
JISP16 [This work]	1.6524(2)
Expt.	1.656(3)

$$\hat{d}_r(\Lambda_\chi = 500\,\mathrm{MeV}) = 0.583(27)_t(38)_{g_A}$$
$$\hat{d}_r(\Lambda_\chi = 600\,\mathrm{MeV}) = 0.625(25)_t(35)_{g_A}$$
$$\hat{d}_r(\Lambda_\chi = 800\,\mathrm{MeV}) = 0.673(23)_t(33)_{g_A}.$$

Calculation Approach (2)

- Calculate 6-body WF and GT.





Calculation Approach (3)

- Add MEC at various cutoffs, and predict:

$$|\mathrm{GT}(^6\mathrm{He})|_{\mathrm{theo}} = 2.198(1)_\Lambda(2)_\mathrm{N}(4)_\mathrm{t}(5)_{\mathrm{g_A}} = 2.198 \pm 0.007$$

- Compare to experiment:

Potential	1-Body	Full
AV18/UIX – VMC	2.250(7)	2.281(7)
JISP16	2.225(2)	2.198(7)
Experiment		2.161(4)

- Remark on the origin of difference.
- Hope for the best ;)

Things to resolve

- Is there a qualitative difference between the SNPA based MEC and the EFT based MEC?
- Is this difference a result of the use of a too simplistic NN potential (JISP16)?
- In any case, with this calculation the experimental 6-body half life is reproduced.

$$\frac{g_A\left({}^6\mathrm{He}\right)}{g_A\left(n\right)} = 0.983 \pm 0.01$$

SNPA vs. χPT based MEC…

- SNPA based MEC have the following form:

$$A^a = A^a_{\mathrm{I}} + A^a_{\mathrm{II}}$$

$$\equiv [A^a(\Delta\pi) + A^a(\pi\rho) + A^a(\pi S)]$$

$$+ [A^a(\Delta\rho) + A^a(\rho S)],$$

$$A^a_{\mathrm{I}} = \frac{g_A}{2m_N f_\pi^2}\left\{ -\frac{4}{25} g_A^2 I_1 \frac{m_N}{m_\Delta - m_N} \mathcal{R}_\pi^2(k_2) \right.$$

$$\times [4\tau_2^a k_2 - (\vec{\tau}_1 \times \vec{\tau}_2)^a \boldsymbol{\sigma}_1 \times k_2]$$

$$-\frac{I_2}{4} \mathcal{R}_\rho(k_1) \mathcal{R}_\pi(k_2) \frac{m_\rho^2}{m_\rho^2 + k_1^2}$$

$$\times (\vec{\tau}_1 \times \vec{\tau}_2)^a [(1+\kappa)\boldsymbol{\sigma}_1 \times k_1 - 2i\bar{p}_1]$$

$$+\frac{I_1}{4} g_A^2 \mathcal{R}_\pi^2(k_2)[(\vec{\tau}_1 \times \vec{\tau}_2)^a \boldsymbol{\sigma}_1 \times k_2$$

$$\left. -\tau_2^a(-q + 2i\boldsymbol{\sigma}_1 \times \bar{p}_1)] \right\} \frac{\boldsymbol{\sigma}_2 \cdot k_2}{m_\pi^2 + k_2^2} + (1 \leftrightarrow 2)$$

$$1 \leftrightarrow I_2 \mathcal{R}_\rho(k_1) \mathcal{R}_\pi(k_2) \frac{m_\rho^2}{m_\rho^2 + k_1^2}, \tag{A24}$$

$$\hat{c}_3 \leftrightarrow -\frac{8}{25} g_A^2 I_1 \frac{m_N}{m_\Delta - m_N} \mathcal{R}_\pi^2(k_j), \tag{A25}$$

$$\hat{c}_4 + \frac{1}{4} \leftrightarrow \frac{4}{25} g_A^2 I_1 \frac{m_N}{m_\Delta - m_N} \mathcal{R}_\pi^2(k_j)$$

$$+ I_2 \mathcal{R}_\rho(k_1) \mathcal{R}_\pi(k_2) \frac{m_\rho^2}{m_\rho^2 + k_1^2} \frac{1+\kappa}{4}, \tag{A26}$$

$$\hat{c}_3 = (-5.58 \pm 0.08,\ -5.49 \pm 0.01,\ -5.82 \pm 0.08)$$

$$\hat{c}_4 = (3.26 \pm 0.05,\ 3.29 \pm 0.01,\ 3.30 \pm 0.04).$$

$$\mathrm{N^3LO:} \quad \hat{c}_3 = -4.96 \pm 0.23,$$

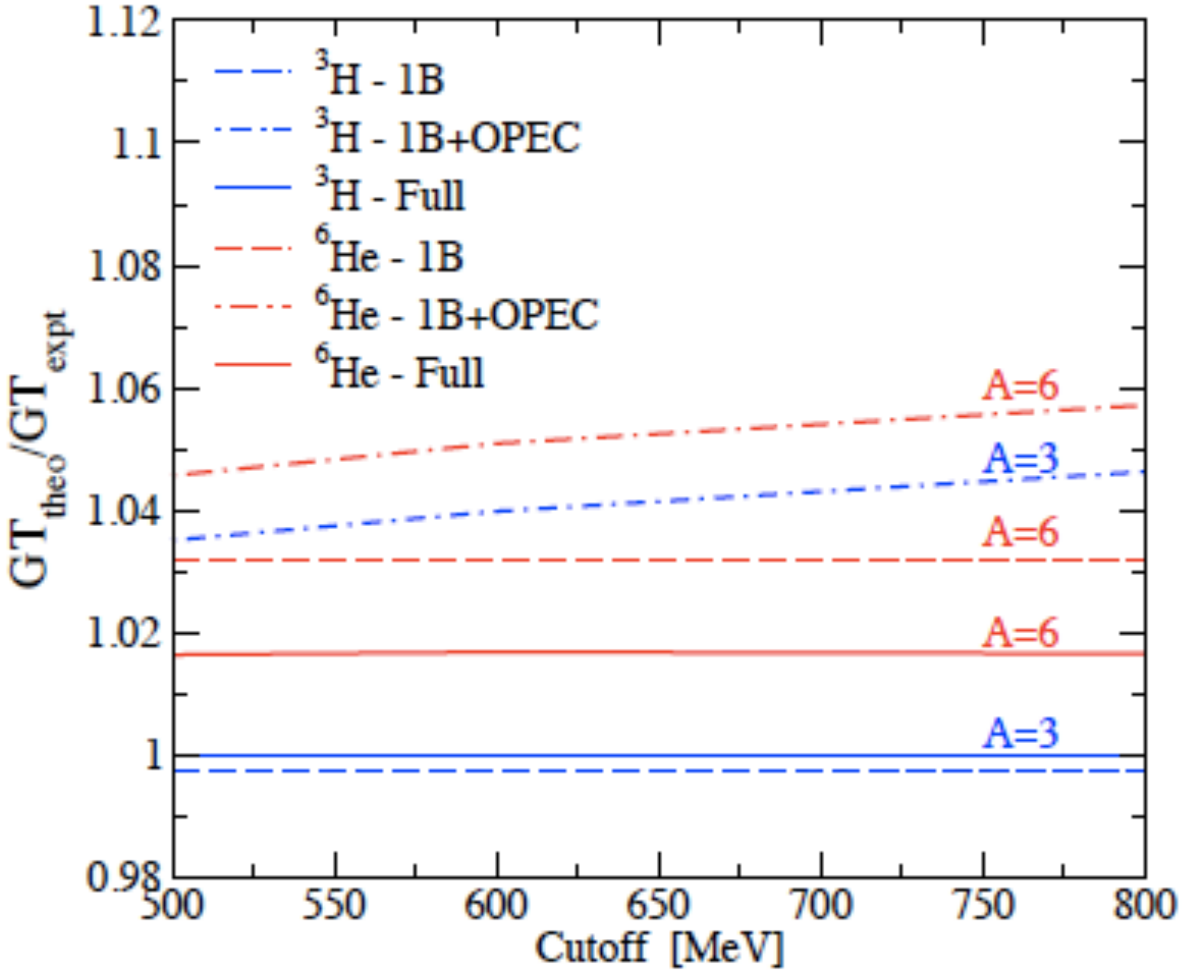
$$\hat{c}_4 = 3.40 \pm 0.09.$$

Difference arise in the contact interaction and calibration

$$A^a(\rho\Delta) = \frac{g_A}{2m_N f_\pi^2} I_2 \frac{(1+\kappa)^2}{50 m_N (m_\Delta - m_N)} \mathcal{R}_\rho^2(k_2) \frac{m_\rho^2}{m_\rho^2 + k_2^2}$$
$$\times [4\tau_2^a (\boldsymbol{\sigma}_2 \times \boldsymbol{k}_2) \times \boldsymbol{k}_2 - (\vec{\tau}_1 \times \vec{\tau}_2)^a \boldsymbol{\sigma}_1$$
$$\times [(\boldsymbol{\sigma}_2 \times \boldsymbol{k}_2) \times \boldsymbol{k}_2]] + (1 \leftrightarrow 2). \qquad \text{(A35)}$$

- This term is N^5LO.
- No operator in SNPA corresponds to the EFT contact interaction.
- Calibration of MEC is done in the 3-body level, by calibrating $g_{\pi N\Delta}$, which in EFT just contributes to c_3.

Different contributions to the decay



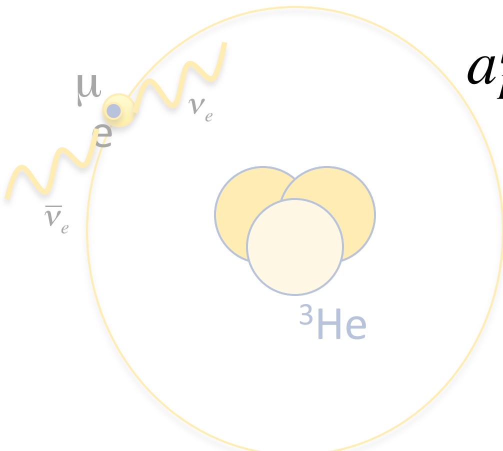
Conclusions

- The ^{6}He beta decay was used as a test case for the weak currents. The calculation is essentially "without free parameters".
- A reliable calculation of the WF, has been accomplished, using JISP16 potential.
- A qualitative difference was found between the MEC contribution in SNPA and in χPT, originating in the contact interaction.
  - Would be interesting to see what would be the effect on heavier nuclei.
- Good agreement with experiment was found (1.7% difference compared with 5.4% in SNPA).
- A consistent calculation within χPT is the next step.

EXTRACTING THE WEAK STRUCTURE OF THE NUCLEON FROM μ-CAPTURE ON ^{3}HE

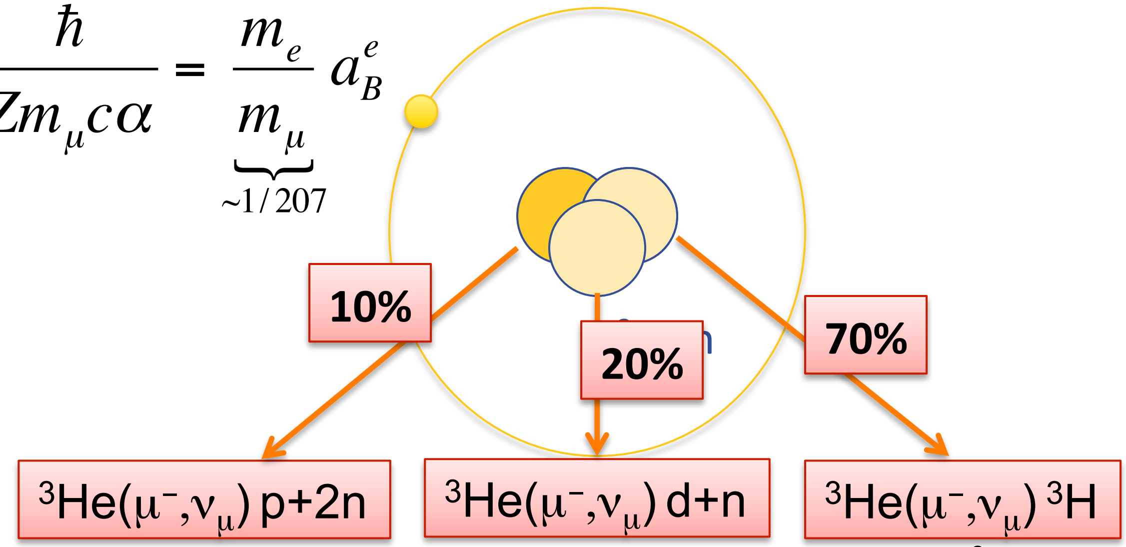
DG, Phys. Lett. B **666**, 471 (2008).

The decay of a muonic ^{3}He: *competition*



$$a_B^\mu = \frac{\hbar}{Z m_\mu c \alpha} = \underbrace{\frac{m_e}{m_\mu}}_{\sim 1/207} a_B^e$$

$$\tau_\mu^{free} = 2.197019(21) \times 10^{-6} \sec$$



$$Capture \quad prob. \sim Z \cdot |\psi_{1S}(0)|^2 \sim \left(\frac{m_\mu}{m_e}\right)^3 Z^4$$

- The rates become comparable for Z~10.
- The Z^4 law has deviations – mainly due to nuclear effects.
- In order to probe the weak structure of the nucleon, one has to keep the nuclear effects under control.

Why don't we stay in the single nucleon level?

The MuCap collaboration (PSI) measuring:

$$\Gamma\left(\mu^{-} p \rightarrow \nu_{\mu} n\right)_{1S}^{\text{singlet}} = 725.0 \pm 13.7_{stat} \pm 10.7_{syst}\,\text{Hz}$$

Expecting to achieve 1% accuracy.

For the (exclusive) process $^3\text{He}(\mu^-,\nu_\mu)\,^3\text{H}$

an incredible measurement (±0.3%):

$$\Gamma\left(\mu^{-} + {}^{3}\text{He} \rightarrow \nu_{\mu} + \text{t}\right)_{stat} = 1496 \pm 4\,\text{Hz}$$

A parameter free, percentage level accuracy calculation of the process is a great challenge to nuclear physics – which is now possible!!

MuCap, **Phys. Rev. Lett. 99**, 032002 (2007).

Ackerbauer *et al*, **Phys. Lett. B417**, 224 (1998).

Previous results

- Ab-initio calculations, based on phenomenological MEC or Δ:
  - Congleton and Truhlik [PRC, 53, 956 (1996)]: 1502±32 Hz.
  - Marcucci et. al. [PRC, 66, 054003(2002)]: 1484±4 Hz.

Radiative corrections to the process

- Muon capture has prominent radiative corrections.
- **Czarnecki, Marciano, Sirlin PRL 99, 032003 (2007)**, showed that radiative corrections increase the cross section by 3.0±0.4%.
- This ruins the good agreement of the old calculations.
- But…

Calculation:

- We take the phenomenological AV18 (NN) and UIX (NNN) nuclear forces.

Method	Binding Energy [MeV]	
	^{3}H	^{3}He
EIHH	8.471(2)	7.738(2)
CHH	8.474	7.742
FY	8.470	7.738
Experimental	8.482	7.718

$$\Gamma = \left\{ \frac{2G^2 |V_{ud}|^2 E_\nu^2}{2J_{^3\mathrm{He}}+1} \left(1-\frac{E_\nu}{M_{^3\mathrm{H}}}\right) |\psi_{1s}^{av}|^2 \Gamma_N \right\} (1+RC)$$

$$\Gamma = 1499(2)_\Lambda (3)_{NM} (5)_t (6)_{RC} = 1499 \pm 16 \ \mathrm{Hz}$$

$$\Gamma_{EXP} = 1496 \pm 4 \,\mathrm{Hz}$$

CONSTRAINTS ON THE WEAK STRUCTURE OF THE NUCLEON FROM MUON CAPTURE ON ^{3}HE

$$\hat{J}^{\mu\mathbf{V}} = \bar{u}(p')\left[F_V(q^2)\gamma^\mu + \frac{i}{2M_N}F_M(q^2)\sigma^{\mu\nu}q_\nu + \frac{g_S}{m_\mu}q^\mu\right]u(p)$$

Vector | Magnetic | Second class currents

$$\hat{J}^{\mu\mathbf{A}} = -\bar{u}(p')\left[G_A(q^2)\gamma^\mu\gamma_5 + \frac{g_P(q^2)}{m_\mu}\gamma_5 q^\mu + \frac{ig_t}{2M_N}\sigma^{\mu\nu}\gamma_5 q_\nu\right]u(p)$$

Axial | Induced Pseudo-Scalar

Induced pseudo-scalar:

- From χPT [Bernard, Kaiser, Meissner, PRD 50, 6899 (1994); Kaiser PRC 67, 027002 (2003)]:

$$g_P\left(-0.954 m_\mu^2\right) = 7.99(0.20)$$

- From muon capture on proton [Czarnecki, Marciano, Sirlin, PRL 99, 032003 (2007); V. A. Andreev et. al., PRL 99, 032004(2007)]:

$$g_P\left(-0.88 m_\mu^2\right) = 7.3(1.2)$$

- This work:

$$g_P\left(-0.954 m_\mu^2\right) = 8.13(0.6)$$

$$g_P\left(q^2\right) = \frac{2 m_\mu g_{\pi pn} f_\pi}{m_\pi^2 - q_\mu^2} - \frac{1}{3} g_A m_\mu M_N \left\langle r_A^2 \right\rangle = 7.99(20)$$

Induced Tensor:

- From QCD sum rules: $$\frac{g_t}{g_A} = -0.0152(53)$$
- Experimentally [Wilkinson, Nucl. Instr. Phys. Res. A 455, 656 (2000)]:

$$\left|\frac{g_t}{g_A}\right| < 0.36 \text{ at } 90\%$$

- This work: $$\frac{g_t}{g_A} = -0.1(0.68)$$

$$\delta J^{\mu A} = \frac{i g_t}{2M_N}\sigma^{\mu\nu}\gamma_5 q_\nu$$

Induced scalar (limits CVC):

- "Experimentally" [Severijns et. al., RMP 78, 991 (2006)]: $g_S = 0.01 \pm 0.27$

- This work: $g_S = -0.005 \pm 0.04$

$$\delta J^{\mu V} = \frac{g_S}{m_\mu} q^\mu$$

Using string theory to calculate and constrain low-energy weak reactions in the real world.

WEAK INTERACTING HOLOGRAPHIC QCD

DG, Ho-Ung Yee, Phys. Lett. B **670**, 154 (2008).

Large N QCD has a dual classical theory in 5-D?!

- Large N factorization of gauge invariant theories:

$$\langle O_1(x_1)O_2(x_2)\cdots O_n(x_n)\rangle = \langle O_1(x_1)\rangle\langle O_2(x_2)\rangle\cdots\langle O_n(x_n)\rangle + O\left(\frac{1}{N^2}\right)$$

  – Implies a classical theory for gauge invariant operators (AKA master fields).
- RG running survives the large N limit, thus the master field is a function of the energy scale:

$$\langle O(x)\rangle(\mu)$$

  – The RG equations constrain flow in this scale
- Holographic QCD is a gravitational theory of gauge invariant fields in 5 dimensions.
  – 5^{th} dimension corresponds roughly to the energy scale.

Things that we know

AdS/CFT Duality proposal

$\mathcal{N}$=4 Super Yang-Mills theory in (3+1)D for $\lambda = g_{YM}^2 N_C$
$N_c \to \infty$, $g_{YM} \to 0$ and fixed but large

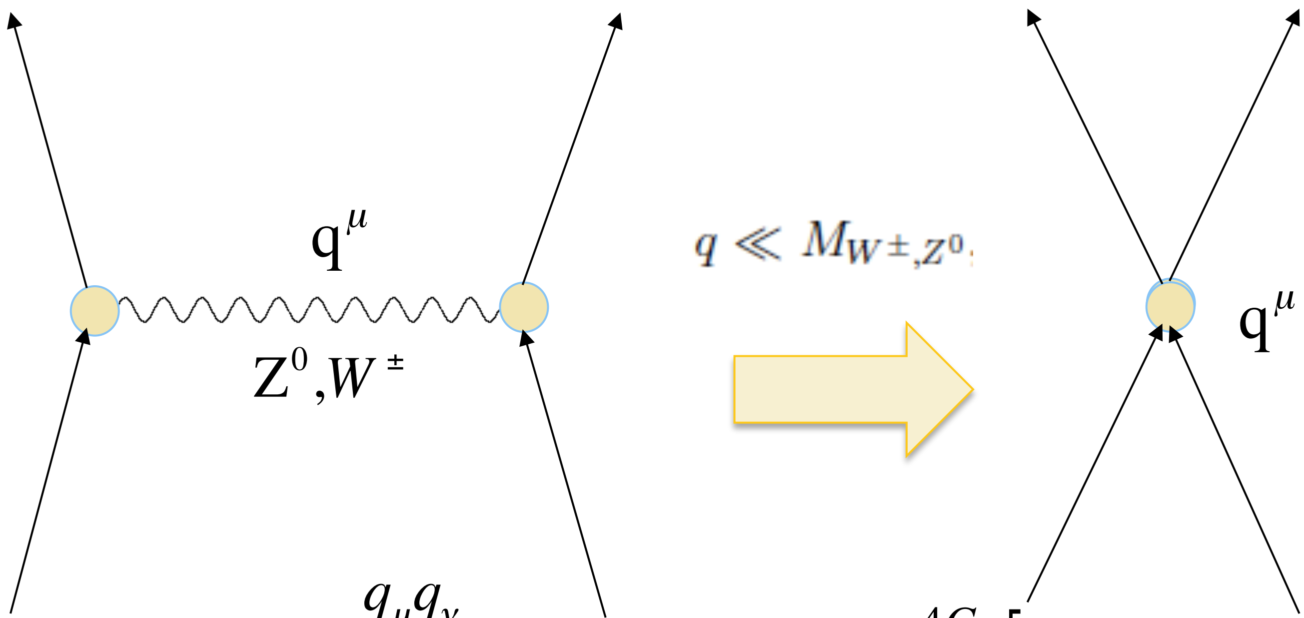
is equivalent to

Type IIB Supergravity in $AdS_5 \times S^5$ with size $\lambda^{1/4}$

We thus expect the dual theory of QCD…

- In the UV regime: highly nonlocal, corresponding to asymptotic freedom.
- In the IR regime: local, corresponding to the strongly correlated QCD.
- Thus, current models of Holographic QCD model the gravitational dual as a local theory.
- Properties of existing models of Holographic QCD:
  - Chiral symmetry.
  - Confinement.
  - Explain experimental observables to 20%.

Low-energy Weak interaction



$$T_{fi} \sim J^\mu \frac{g_{\mu\nu} + \frac{q_\mu q_\nu}{M_B^2}}{q^2 + M_B^2} J^\nu$$

$$\mathcal{H}_W = \frac{4G_F}{\sqrt{2}}\left[J_{W^+}J_{W^-} + \cos^2\theta_W \left(J_{Z^0}\right)^2\right] =$$

$$= \frac{4G_F}{\sqrt{2}}\left[\left(J_L^1\right)^2 + \left(J_L^2\right)^2 + \left(J_L^3 - \sin^2\theta_W J_Q\right)^2\right]$$

SU(2)$_L$ EM

$$J_L^a = \sum_f \overline{\psi}_f \sigma^a \psi_f$$

How to perturb the QCD Lagrangian?

Gauge

- Perturbation to the Lagrangian.
- Single trace operator $\mathcal{O}$.

- A Lagrangian pertutbation:

$$\Delta\mathcal{L} = \int d^4 x f(x) \mathcal{O}(x)$$

Gravity

- Deforming boundary conditions of field near UV boundary.

- A 5D field, such that:

$$\phi_{\mathcal{O}}\left(x^\mu, z\right) \underset{z\to\infty}{\sim} c_1\left(x^\mu\right) z^{-\Delta_-} + c_2\left(x^\mu\right) z^{-\Delta_+}$$

- Boundary conditions:

$$c_1(x) = f(x)$$
$$c_2(x) = \langle \mathcal{O}(x) \rangle$$

For a general functional perturbation of a single trace operator

$$\Delta\mathcal{L} = \int d^4 x F\left[\mathcal{O}(x)\right]$$

$$c_1(x) = \left.\frac{\delta F\left[\mathcal{O}\right]}{\delta\,\mathcal{O}}\right|_{\mathcal{O}\to c_2(x)}$$

$$c_2(x) = \left\langle \mathcal{O}(x) \right\rangle$$

The idea is general enough to implement in any Holographic Model.
We demonstrated on two models:

Top – Down Model: Sakai-Sugimoto Model

Bottom – Up Model: Hard/Soft Wall Model.

IMPLEMENTATION

How to calculate different reactions?

- Write equation of motion for the global gauge field (i.e. the $U(N_F)$ current).
- Solve it with the ***prescribed*** boundary conditions.
- If you'd like pions to be involved, do it by gauge fixing A_z.

$$A_z(+\infty) = \frac{1}{\sqrt{\pi\kappa}} \cdot \frac{\pi(x)}{1+z^2}$$

- For reactions that include nucleons, choose a model for baryons, and calculate baryon-pion coupling from the kinetic term, and from magnetic type of couplings:

$$S = i\int d^4x \int d\omega\left[\overline{\mathcal{B}}\gamma^M\left(\partial_M - iA_M\right)\mathcal{B} - m_{\mathcal{B}}(\omega) + C\overline{\mathcal{B}}\sigma^{MN}F_{MN}\mathcal{B} + \ldots\right]$$

Neutron b-decay

Sakai-Sugimoto

- With: $$\mathcal{L}_{\bar{n}pe^-\bar{\nu}_e} = \sqrt{2}G_F\Big[\bar{n}\gamma_\mu p + g_A\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\bar{n}\gamma^\nu\gamma_5 p - i(0.84)\bar{n}q^\nu\sigma_{\mu\nu}p\Big]\cdot\left(\bar{\nu}_L\gamma^\mu e_L\right)$$

$$g_A = 1.3$$

Hard/Soft wall model

- With: $$\mathcal{L}_{\bar{n}pe^-\bar{\nu}_e} = \sqrt{2}G_F\Big[\bar{n}\gamma_\mu p + g_A\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\bar{n}\gamma^\nu\gamma_5 p - i(0.48)D\bar{n}q^\nu\sigma_{\mu\nu}p\Big]\cdot\left(\bar{\nu}_L\gamma^\mu e_L\right)$$

$$g_A = 0.33 + 1.02D$$

$$g_A^{\exp} = 1.2695(29)$$

Parity non-conserving pion-nucleon coupling

- First example without an external source.
- We are interested in parity violating couplings of mesons to the nucleons.
- To this end, we consider only charged pion-nucleon coupling.
- In both models, the result in the zero *q* limit is identical to the current algebra result:

$$L_{N-\pi}^{weak} = -2G_F f_\pi \left(\bar{p}\gamma^\mu n\right)\left(\partial_\mu \pi^+\right)$$

- Still, a lot to be done!

Summary

- This is a prescription to include weak interactions in the framework of holographic QCD.
- Applicable up to energies of a few GeV, when strong coupling is still valid.
- We have shown its strength by using Sakai-Sugimoto and Hard/Soft wall models to calculate few exemplar reactions.
- The current approach, contrary to other approaches (such as χPT), gives not only the operator structure, but the numerical coefficients, to about 20%, and valid for energies above the chiral limit.

Final Remarks

- Weak reactions with light nuclei:
  - Can be used to study the basic symmetries of QCD.
  - Provide a hatch to the properties of heavier nuclei.
- Parameter free calculations, which will be done within χPT, would be able to constrain these observables.
  - For that, a microscopic calculation of LECs is needed.
- χPT, even at the current "phenomenological" level, can teach us a lot about the character of correlations in nuclei:
  - Different observables might depend differently on the short range physics.
  - Structure of weak MEC, from a more basic approach than a meson model.