Consistent Calculations of Electro-weak Reactions on Light Nuclei: Correlations, Currents and Applications

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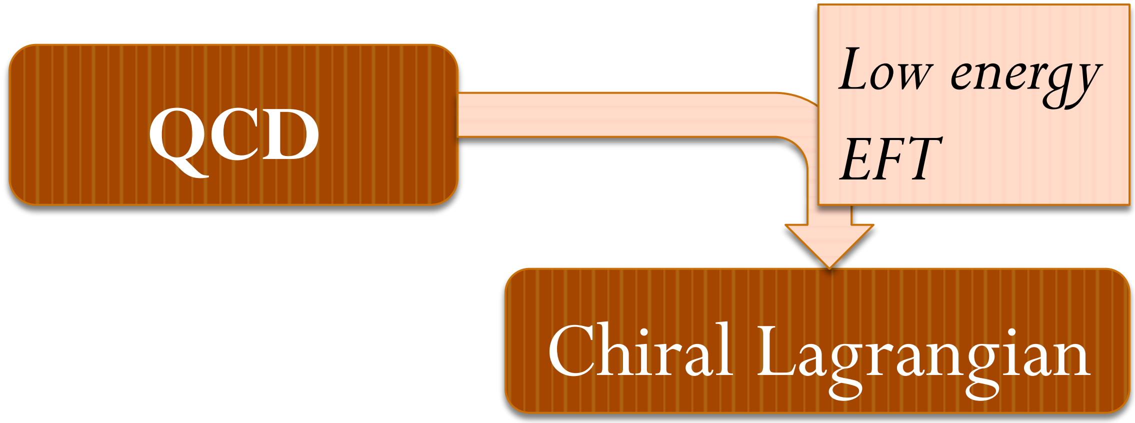
Introduction and outline

- The modern approach to nuclear physics relies on effective field theory for QCD at low-energies: chiral perturbation theory (χPT). Today, nuclear forces AND meson exchange currents were derived from χPT.
- When fully developed, χPT will allow consistent and parameter free checks of QCD within nuclear physics.
- Here, I will present a new approach to calibrate the chiral Lagrangian, using weak observable, namely the triton β-decay rate.
- Some consequences and resulting applications will be presented.

DG, Quaglioni, Navratil, in preparation.

χPT approach for low-energy EW nuclear reactions:



Effective Field Theories

- An effective theory is supposed to be valid up to some energy scale Λ.
- The most important thing in a theory are its symmetries.
  - The most important thing in an effective theory is that it will agree with the symmetries of the fundamental theory.
- In view of these – identify the relevant degrees of freedom.
- Write the most general Lagrangian consistent with the assumed symmetries. $\hat{\mathcal{L}}_{eff} = \sum_A c_A \hat{\mathcal{L}}_A$
  - c_A should be calibrated from experiments.
- Develop a perturbation scheme.

Effective field theory (EFT) for nuclear physics: Chiral perturbation theory (χPT)

- Symmetries are important *NOT* degrees of freedom:
  - In QCD – an approximate chiral symmetry:
    $$SU(2)_L \times SU(2)_R \cong SU(2)_V \times SU(2)_A \rightarrow SU(2)_V$$
  - Pions – Goldstone bosons of the broken symmetry.
- Choose Λ – the theory cutoff. (400-800 MeV)
- Identify Q – the energy scale of the process. (around 100 MeV)
- In view of Q and Λ -Identify the relevant degrees of freedom. (pions and nucleons).
- Write all the possible operators which agree with the symmetries of the underlying theory (INFINITE)
- Calculate Feynman diagrams (INFINTE)
- Find a systematic way to organize diagrams according to their contribution

Effective Field Theory for low energy QCD

- We are aiming at energies which are relevant for nuclear phenomena – well below QCD breaking scale ~ 1 GeV.
- The constituent quarks are the up and down quarks.
- Their masses are small with respect to the QCD scale.

$$m_u = 2 \pm 1\,\mathrm{MeV} \text{ and } m_d = 5 \pm 2\,\mathrm{MeV}$$

- QCD Lagrangian with only the up and down quarks of vanishing mass:

$$\mathcal{L}^q_{QCD} = i\bar{q}\gamma^\mu \mathcal{D}_\mu q = i\bar{q}_R\gamma^\mu \mathcal{D}_\mu q_R + i\bar{q}_L\gamma^\mu \mathcal{D}_\mu q_L$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \; ; \; q_{L,R} \equiv \frac{1}{2}(1 \pm \gamma_5) q$$

Effective Field Theory for low energy QCD

- Clearly, the QCD Lagrangian is invariant under:

$$\underbrace{\begin{aligned} q_R &= \begin{pmatrix} u_R \\ d_R \end{pmatrix} \mapsto \exp\left(-i\vec{\Theta}^R \cdot \frac{\vec{\tau}}{2}\right)\begin{pmatrix} u_R \\ d_R \end{pmatrix} \\ q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} \mapsto \exp\left(-i\vec{\Theta}^L \cdot \frac{\vec{\tau}}{2}\right)\begin{pmatrix} u_L \\ d_L \end{pmatrix} \end{aligned}}_{SU(2)_R \times SU(2)_L}$$

Or

$$\underbrace{\begin{aligned} q &= \begin{pmatrix} u \\ d \end{pmatrix} \mapsto \exp\left[-i\vec{\Theta}^V \cdot \frac{\vec{\tau}}{2}\right]\begin{pmatrix} u \\ d \end{pmatrix} \\ q &= \begin{pmatrix} u \\ d \end{pmatrix} \mapsto \exp\left[-i\gamma_5\vec{\Theta}^A \cdot \frac{\vec{\tau}}{2}\right]\begin{pmatrix} u \\ d \end{pmatrix} \end{aligned}}_{SU(2)_V \times SU(2)_A}$$

$$SU(2)_R \times SU(2)_L \cong SU(2)_V \times SU(2)_A$$

- This is an approximate symmetry of the Lagrangian due to the mass term:

$$M_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} = \underbrace{\frac{1}{2}(m_u + m_d)I + \underbrace{\frac{1}{2}(m_u - m_d)\tau_3}_{breaks\ \ SU(2)_V}}_{breaks\ \ SU(2)_A}$$

- This creates deviations of the order $\dfrac{m_u \pm m_d}{M_N}$

Effective Field Theory for low energy QCD

- If this was correct, even approximately, there were parity doublets in the QCD spectrum. However,
  - Nucleons of positive parity: p(½$^+$,938.3), n(½$^+$,939.6), I= ½
  - Nucleons of negative parity N(½$^-$,1535), I= ½.
  - Mesons of Isospin 1: ρ(1^-,770) and a_1(1^+,1260)
- Masses are very different ➔ No parity doublets in the spectrum.

Conclusion: The chiral symmetry is ***spontaneously*** broken

- The Goldstone-Nambu spin zero bosons are the pions:
  - They are not massless due to the explicit symmetry breaking, though -

$$\xi = \exp\left(i\frac{\vec{\pi}\cdot\vec{\tau}}{2f_\pi}\right)$$

$$\left(\frac{m_\pi}{m_N}\right)^2 \approx 0.02.$$

Transformation rules:

- Goldstone's theorem states that $U \equiv \xi^2$, belongs to the $(\bar{2},2)$ representation of SU(2)$_L$xSU(2)$_R$: $U \to LUR^\dagger$.
- A nonlinear realization of the symmetry (as $U^\dagger U = 1,\ |U| = 1$).
- The resulting transformation rules: $h \in SU(2)_V$

$$N \mapsto hN$$

$$\xi \mapsto L\xi h^\dagger = h\xi R^\dagger$$

- Invariant terms: $\bar{N}N,\ \bar{N}\gamma^\mu D_\mu N,\ \bar{N}\gamma^\mu \gamma_5 a_\mu N$

$$with:$$

$$D_\mu = \partial_\mu + i v_\mu$$

$$v_\mu = -\frac{i}{2}\left(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi\right)\ ;\quad a_\mu = -\frac{i}{2}\left(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi\right)$$

Weinberg's Power Counting Scheme

- Each Feynman diagram can be characterized by: $\left(\frac{Q}{\Lambda_\chi}\right)^\nu$
- Q~100 MeV is the relevant momentum of the process or pion masses in the diagram.
- $\Lambda_\chi \sim 1\ \text{GeV}$ is the chiral symmetry breaking scale.
- Weinberg showed: $\nu \geq 0$

$\Downarrow$

Chiral Perturbation Theory

- In addition, expand in the nucleon's mass (take Λ_χ~M_N) ➔Heavy Baryon χPT.

The power counting

$$\text{Power} = -2 + 2A - 2C + 2L + \sum_{\text{all vertices}} \Delta_i$$

with

$A =$ number of nucleons;

$C =$ number of seperately connected pieces;

$L =$ number of loops;

$$\Delta_i = d_i + \frac{n_i}{2} - 2,$$

where

$d_i =$ number of derivatives,

$n_i =$ number of nucleon operators.

The Lagrangian to fourth order

$$\mathcal{L} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$$

- Pion Lagarngian: $\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger + m_\pi^2 \left(U + U^\dagger\right)\right]$
- Nucleon-Pion Lagrangian: $\mathcal{L}^{(2)}_{\pi N} = \bar{N}\left\{i\gamma_\mu \mathcal{D}^\mu + g_A \gamma^\mu \gamma_5 a_\mu - M_0\right\} N$

$$\delta_1 \mathcal{L}^{(3)}_{\pi N} = \frac{2\hat{c}_3}{M_N} \bar{N} N \mathrm{Tr}\left(\mathrm{a}_\mu \mathrm{a}^\mu\right)$$

$$\delta_2 \mathcal{L}^{(3)}_{\pi N} = i\frac{\hat{c}_4}{M_N} \bar{N}[a_\mu, a_\nu]\sigma^{\mu\nu} N,$$

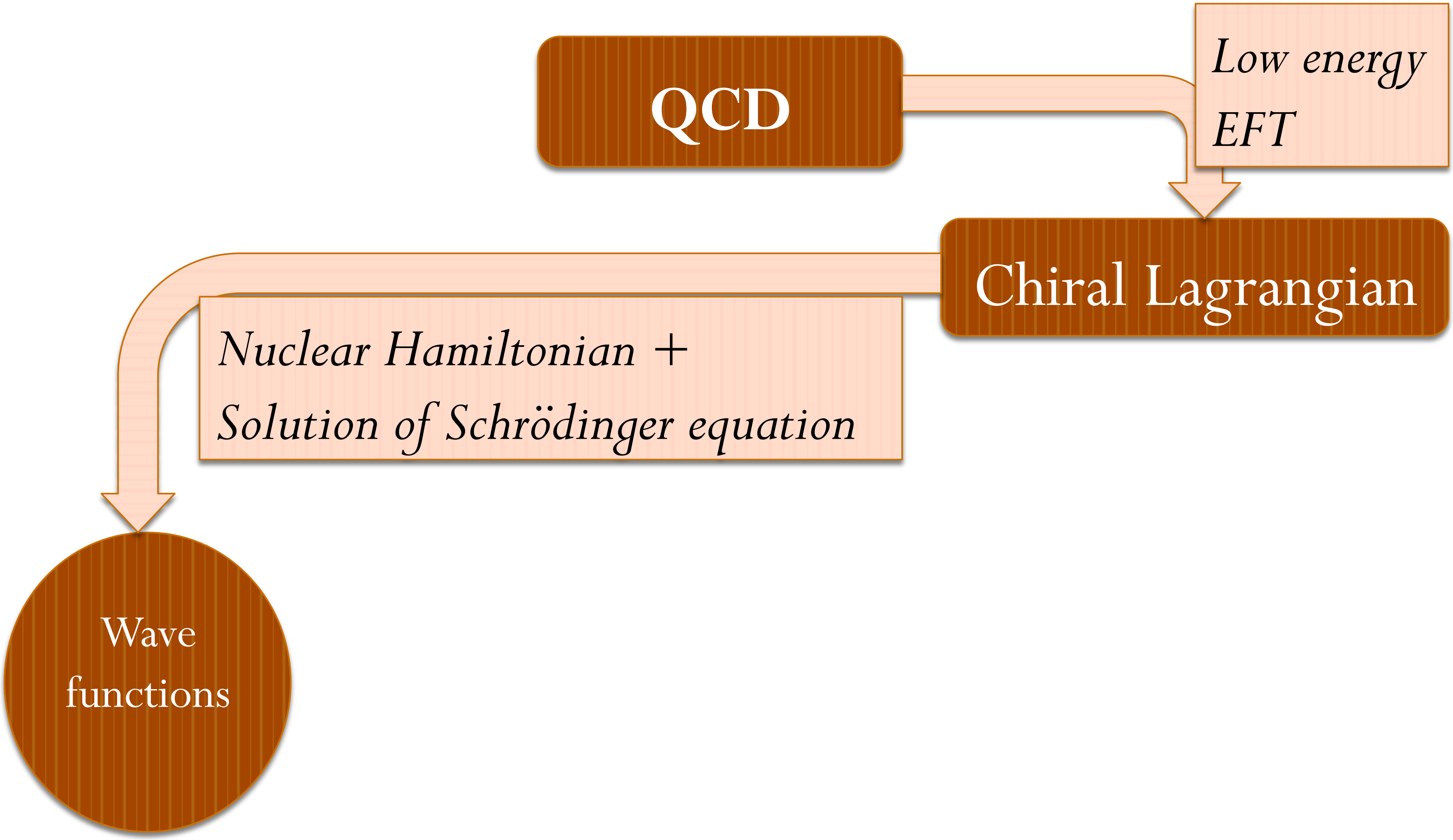
- Nucleon-Nucleon contact terms.
  - Allowed, and also needed to remove divergences.
  - Represent short range correlations.

$$\mathcal{L}_4 = -2D_1\left(\bar{N}\gamma^\mu\gamma_5 a_\mu N\right)\left(\bar{N}N\right)$$

The big deal in χPT

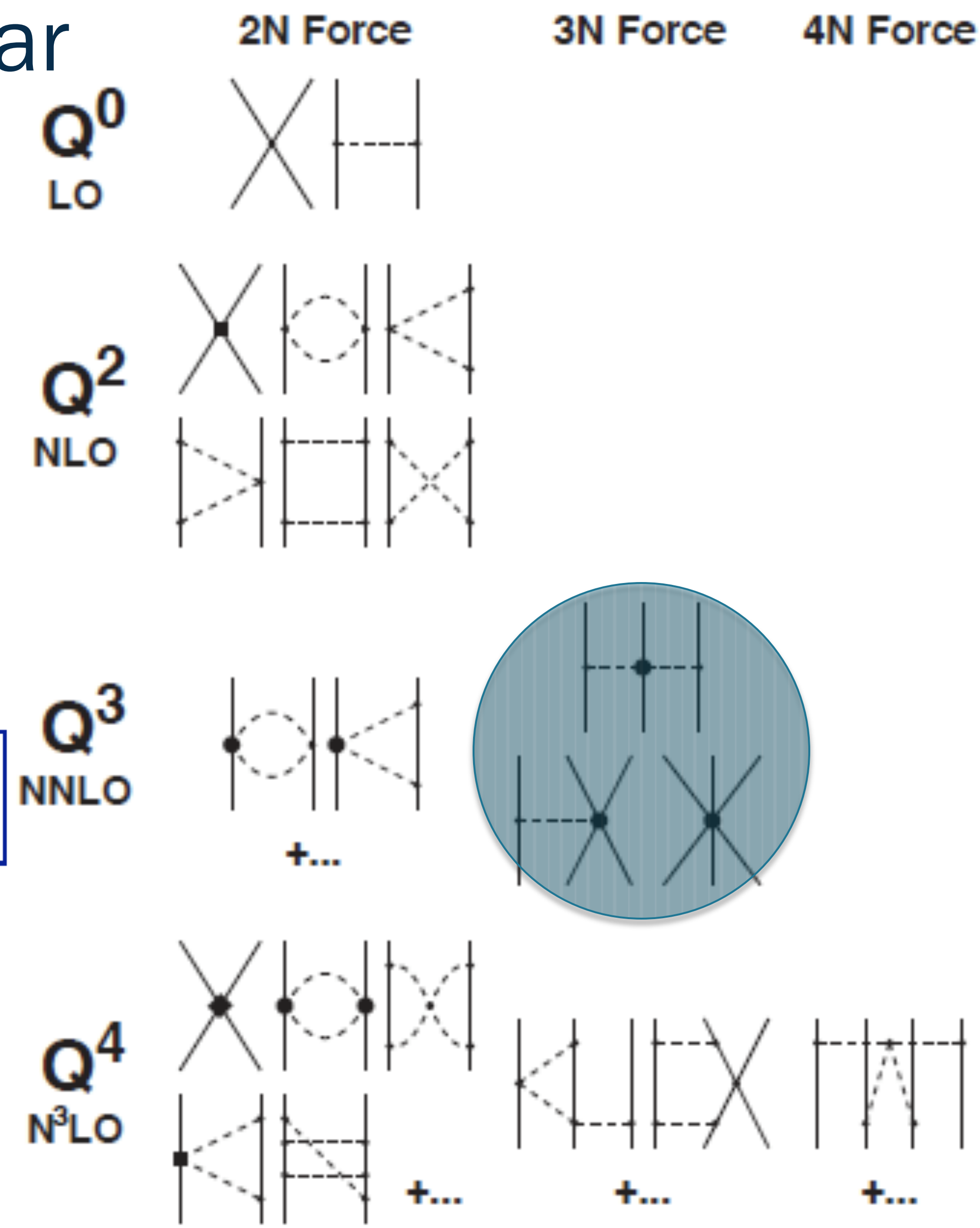
- A perturbation theory/expansion in small parameter of the observable, gives control over the accuracy of the calculation.
- Varying the cutoff gives estimate of the theoretical error-bar.
- Allows connection between *a-priori* not related operators.
- In particular the nuclear force and the electro-weak currents in the nucleus (that the SU(2)xSU(2) structure is a gauging of).
- The calculations are predictions of QCD.

χPT approach for low-energy EW nuclear reactions:



Hierarchy of Nuclear Forces in χPT

- Only contact terms cannot be calibrated in the pion or pion/nucleon system.
- The 2N terms are calibrated to reproduce phase shifts.

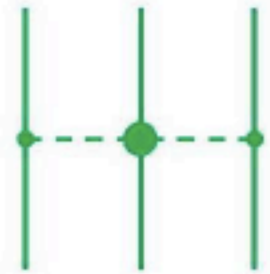


χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

3 nucleon forces at N^{2}LO

TPE-3NF



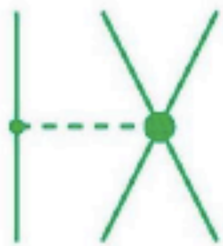
$$V^{3NF}_{TPE} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i \neq j \neq k} \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} F^{\alpha\beta}_{ijk} \tau_i^\alpha \tau_j^\beta$$

with $\vec{q}_i \equiv \vec{p}_i{}' - \vec{p}_i$, where $\vec{p}_i$ and $\vec{p}_i{}'$ are the initial and final momenta of nucleon i, respectively, and

$$F^{\alpha\beta}_{ijk} = \delta^{\alpha\beta}\left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\, \vec{q}_i \cdot \vec{q}_j\right] + \frac{c_4}{f_\pi^2} \sum_\gamma \epsilon^{\alpha\beta\gamma}\, \tau_k^\gamma\, \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$

No new parameters!

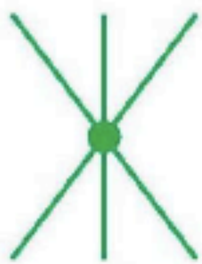
OPE-3NF



$$V^{3NF}_{OPE} = c_D \sum_{i \neq j \neq k} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{q_j^2 + m_\pi^2} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)(\vec{\sigma}_i \cdot \vec{q}_j)$$

One New param. $\mathcal{L}_4 = -2D_1\, (\bar{N}\gamma^\mu\gamma_5 a_\mu N)\, (\bar{N}N)$

Contact-3NF



$$V^{3NF}_{ct} = c_E \sum_{j \neq k} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\ .$$

One new parameter,

Strategy : calibrate c_D and c_E from nuclear matter observables, and then predict other observables.

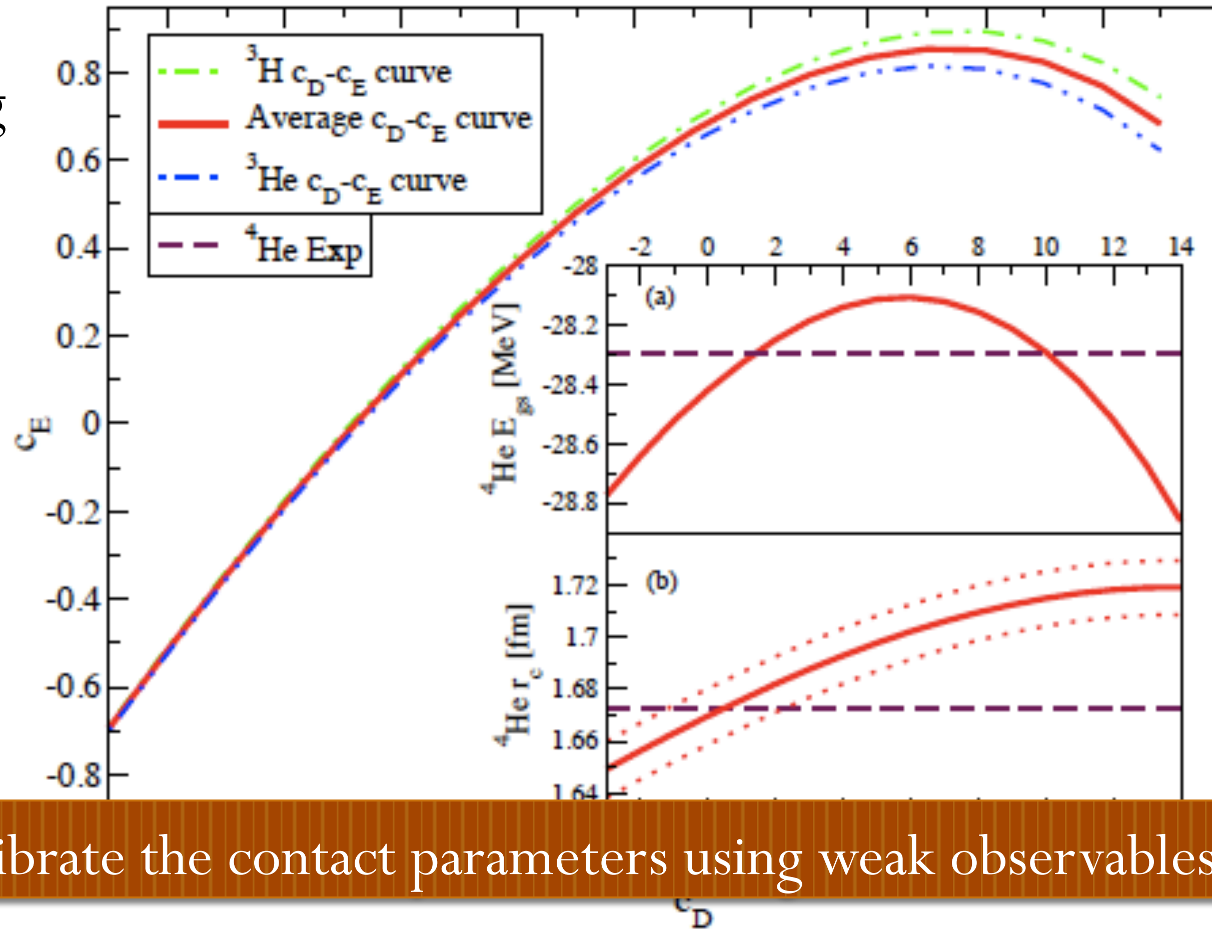
Some remarks

- For now, only N2LO 3NF exist.
- Only for cutoff of 600 MeV.
- Clearly – more work is called for!
- A good way to treat the cutoff dependence is to use the $V_{\text{low k}}$ approach (Bacca, Nogga, Schwenk, DG, in preparation).

Attempts to calibrate the contact parameters

Other attempts were to use 3 nucleon scattering lengths as a second observable.

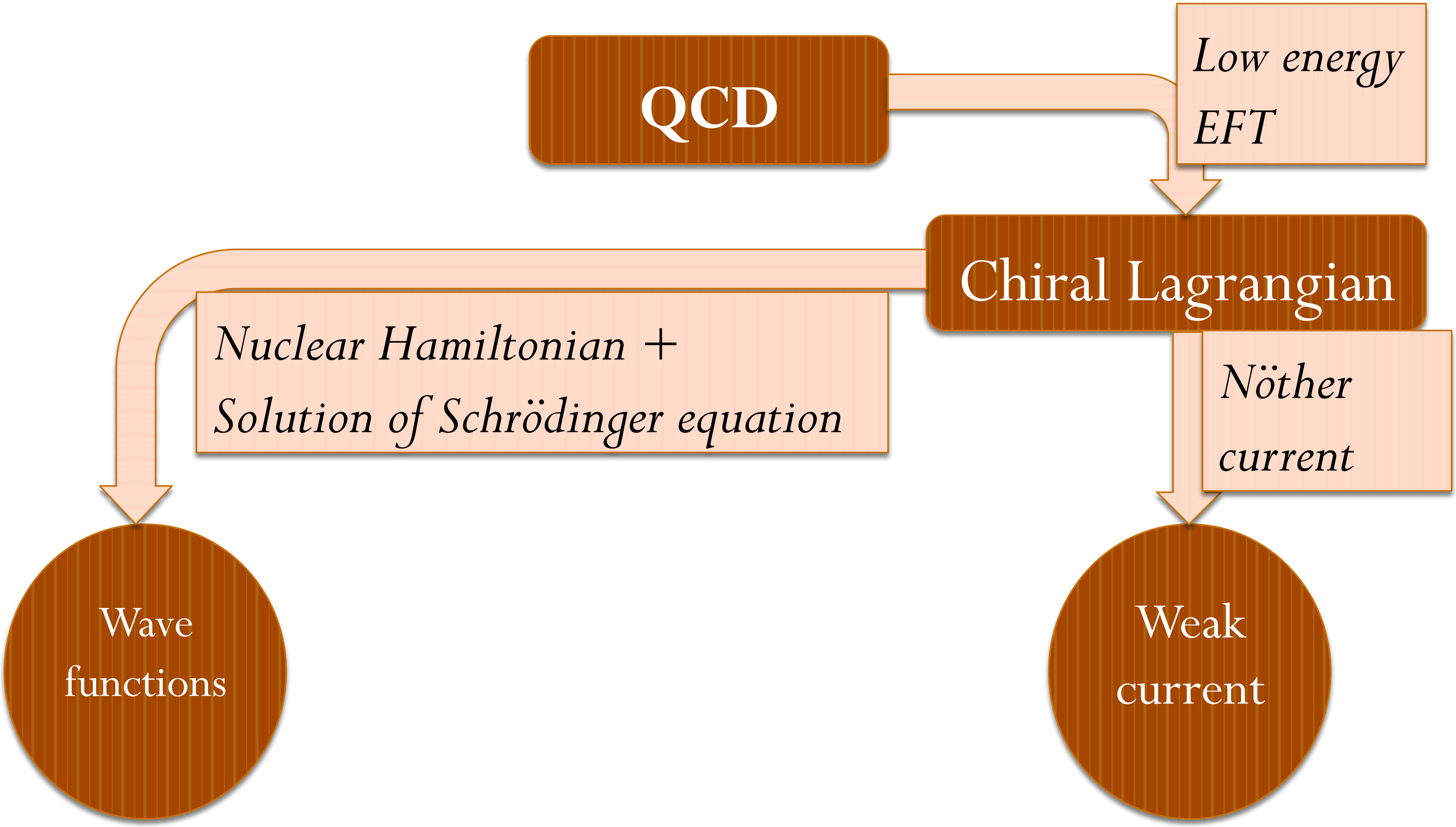
The problem is the cross-correlation of the different observables.



Here, we choose to calibrate the contact parameters using weak observables

Navratil *et al*, **Phys. Rev. Lett. 99**, 042501 (2007).

χPT approach for low-energy EW nuclear reactions:



Weak interaction with the nucleus

$$\hat{H}_W = -\frac{G_F V_{ud}}{\sqrt{2}} \int d^3\vec{x}\, \hat{j}_\mu^{\,+}(\vec{x})\, \hat{J}^{\mu\,-}(\vec{x})$$

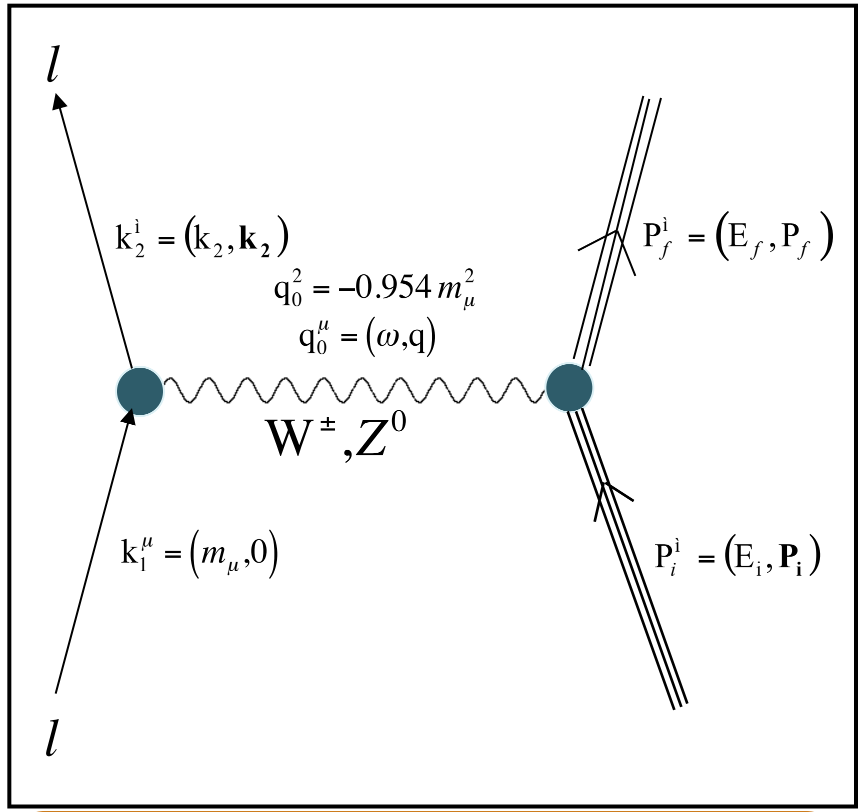
Lepton current

Nuclear current

Scattering operator

⇕

Currents in the nucleus



$$W \quad propagator = \frac{g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 + M_W^2} \xrightarrow[q \ll M_W]{} \frac{g_{\mu\nu}}{M_W^2}$$

Weak currents in the nucleus

- The standard model dictates only the structure of the currets:
  - Charged current $\mathcal{J}_\mu^{(\pm)} = \frac{\tau_\pm}{2}\left(J_\mu^V + J_\mu^A\right)$
  - Neutral current: $\mathcal{J}_\mu^{(0)} = (1 - 2\cdot\sin^2\theta_W)\frac{\tau_0}{2}J_\mu^V + \frac{\tau_0}{2}J_\mu^A - 2\cdot\sin^2\theta_W\frac{1}{2}J_\mu^V$
- The current of polar (axial) vector symmetry is the Noether current of the QCD Lagrangian, with respect to $SU(2)_V$ [$SU(2)_A$] symmetry.
- Includes:
  - single nucleon current (leading order)
  - Meson exchange currents.

Weak Currents in the Nucleus from χPT

- $SU(2)_L x SU(2)_R$ is the gauging of the electro-weak force.
- Weak currents are thus the Nother current of this symmetry.

$$\mathcal{J}^{a\mu} \equiv -\frac{\partial \mathcal{L}}{\partial(\partial_\mu \epsilon^a(x))}$$

- In χPT:
  - Single nucleon currents come at leading order (and receive momentum dependent corrections at higher orders).
  - Meson exchange currents start at N^2LO.

T.-S. Park *et al*, **Phys. Rev. C 67**, 055206 (2003); DG PhD thesis **arXiv: 0807.0216**

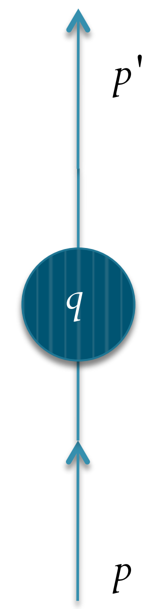
Single Nucleon Currents

$$\hat{J}^{\mu V} = \bar{u}(p')\left[F_V(q^2)\gamma^\mu + \frac{i}{2M_N}F_M(q^2)\sigma^{\mu\nu}q_\nu + \frac{g_S}{m_\mu}q^\mu\right]u(p)$$

Vector Magnetic Second class currents

$$\hat{J}^{\mu A} = -\bar{u}(p')\left[G_A(q^2)\gamma^\mu\gamma_5 + \frac{g_P(q^2)}{m_\mu}\gamma_5 q^\mu + \frac{ig_t}{2M_N}\sigma^{\mu\nu}\gamma_5 q_\nu\right]u(p)$$

Axial Induced Pseudo-Scalar



- q dependence is due to pion loops.
- Second class currents vanish to this order!

Weinberg **Phys. Rev.**, 112, 1375 (1958)

Meson Exchange currents

- Vector currents, protected by charge conservation (or CVC), do not include contact parameters.
- Axial currents are more complicated, in configuration space:

$$\hat{A}^{i,a}_{12}(\vec{r_{ij}}) = \boxed{\frac{g_A}{2Mf_\pi^2}\hat{d}_r \mathcal{O}^{i,a}_{\ominus}\delta^{(3)}_\Lambda(\vec{r}_{ij})} + \frac{g_A m_\pi^2}{2Mf_\pi^2}\mathcal{O}^{i,a}_P y^\pi_{1\Lambda}(r_{12}) -$$

$$-\frac{g_A m_\pi^2}{2Mf_\pi^2}\left[\frac{\hat{c}_3}{3}(\mathcal{O}^{i,a}_\oplus + \mathcal{O}^{i,a}_\ominus) + \frac{2}{3}(\hat{c}_4 + \frac{1}{4})\mathcal{O}^{i,a}_\otimes\right] y^\pi_{0\Lambda}(r_{ij}$$

$$-\frac{g_A m_\pi^2}{2Mf_\pi^2}\left[\hat{c}_3(\mathcal{T}^{i,a}_\oplus + \mathcal{T}^{i,a}_\ominus) - (\hat{c}_4 + \frac{1}{4})\mathcal{T}^{i,a}_\otimes\right] y^\pi_{2\Lambda}(r_{ij})$$

$$\delta^{(3)}_\Lambda(\vec{r}) \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} S^2_\Lambda(\vec{k}^2),$$

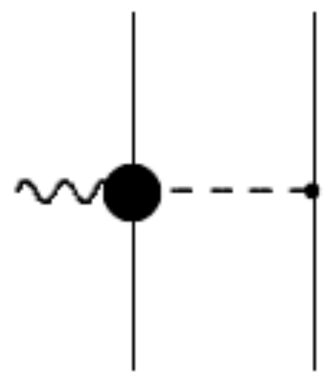
$$y^\pi_{\Lambda 0}(r) \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} S^2_\Lambda(\vec{k}^2)\frac{1}{\vec{k}^2 + m_\pi^2}$$

$$y^\pi_{\Lambda 1}(r) \equiv -\frac{\partial}{\partial r} y^\pi_{\Lambda 0}(r),$$

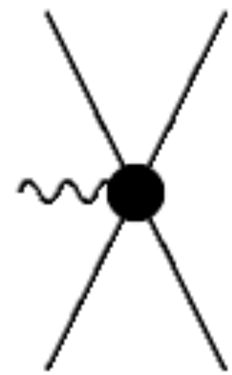
$$y^\pi_{\Lambda 2}(r) \equiv \frac{1}{m_\pi^2} r \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} y^\pi_{\Lambda 0}(r)$$

$$\boxed{\hat{d}_R \equiv \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N(c_3 + 2c_4) + \frac{1}{6}}$$

1 pion exchange



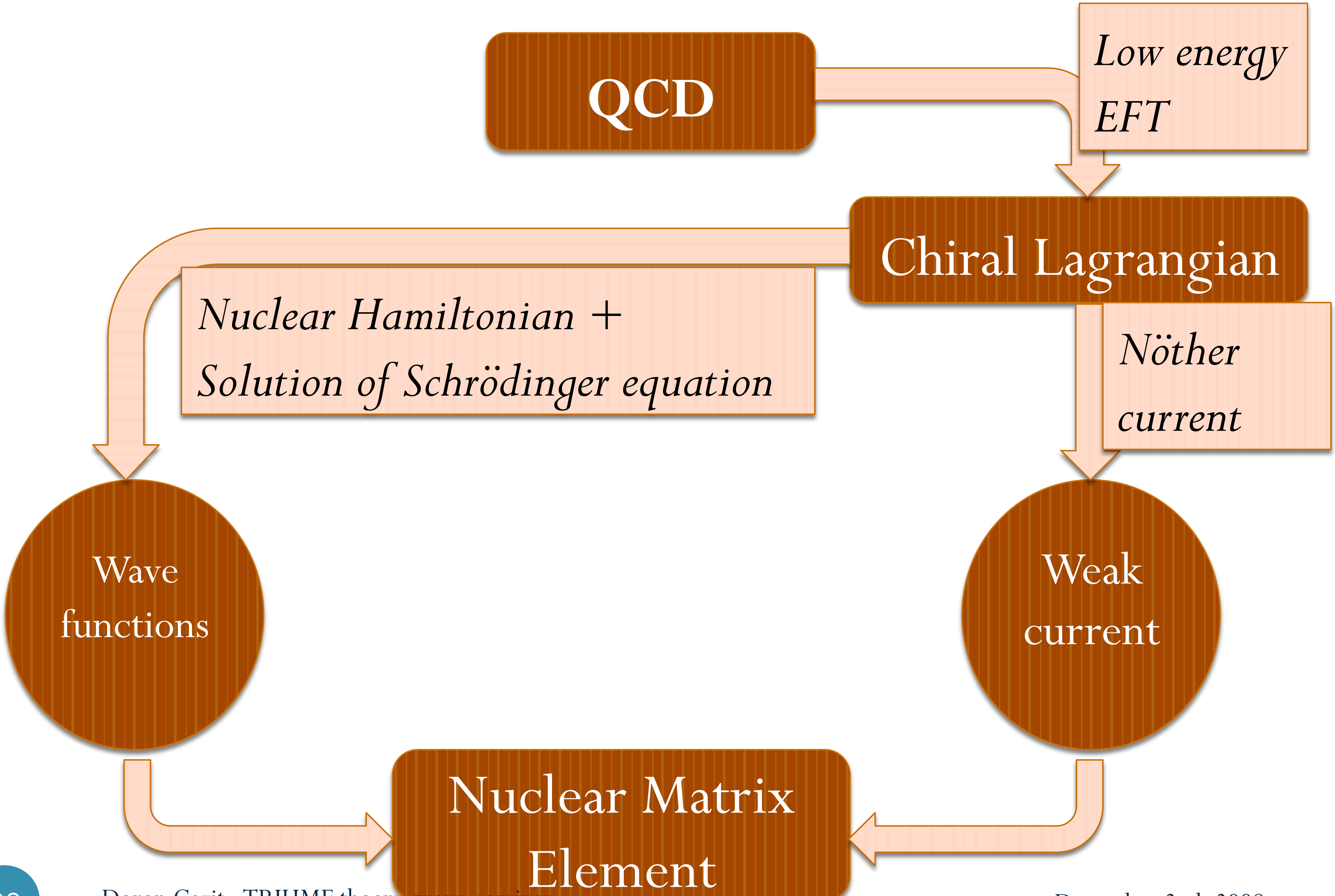
Contact term



Some remarks

- Meson exchange current involves only TWO nucleons.
- Thus, in principle c_D can be calibrated using two-body weak processes.
- So – three nucleon force constrained in the two nucleon level!
- The most attractive process –
  - Muon capture on deuteron – an experiment at PSI "MuD (MuSun)" aims to measure this process to 1%.

χPT approach for low-energy EW nuclear reactions:



Nuclear Matrix Elements

- A multipole decomposition of the currents is very helpful:

$$\hat{C}_{JM}(q) = \int d\vec{x} j_J(qx) Y_{JM}(\hat{x}) \hat{\mathcal{J}}_0(\vec{x})$$

$$\hat{E}_{JM}(q) = \frac{1}{q} \int d\vec{x} \vec{\nabla} \times [j_J(qx) \vec{Y}_{JJM}(\hat{x})] \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

$$\hat{M}_{JM}(q) = \int d\vec{x} j_J(qx) \vec{Y}_{JJM}(\hat{x}) \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

$$\hat{L}_{JM}(q) = \frac{i}{q} \int d\vec{x} \vec{\nabla} [j_J(qx) Y_{JM}(\hat{x})] \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

- Usually, the low energy and selection rules mean that only a small number of multipoles contribute.

β decay rate for $q \rightarrow 0$

$$(fT_{1/2})_t = \frac{K/G_V^2}{(1-\delta_c) + 3\pi \frac{f_A}{f_V} \langle E_1^A \rangle^2}$$

$$\langle E_1^A \rangle = |\langle {}^3\mathrm{He} \| \hat{E}_1^A \| {}^3\mathrm{H} \rangle|$$

- At the leading order:

$$E_1^A|_{\mathrm{LO}} = i\, g_A (3\pi)^{-1/2} \overbrace{\sum_{i=1}^{A} \sigma_i \tau_i^+}^{\mathrm{GT}}$$

- This is the reason for the common name: experimental Gamow-Teller.
- For the triton β-decay:

$$\langle E_1^A \rangle|_{expt} = 0.6848 \pm 0.0011$$

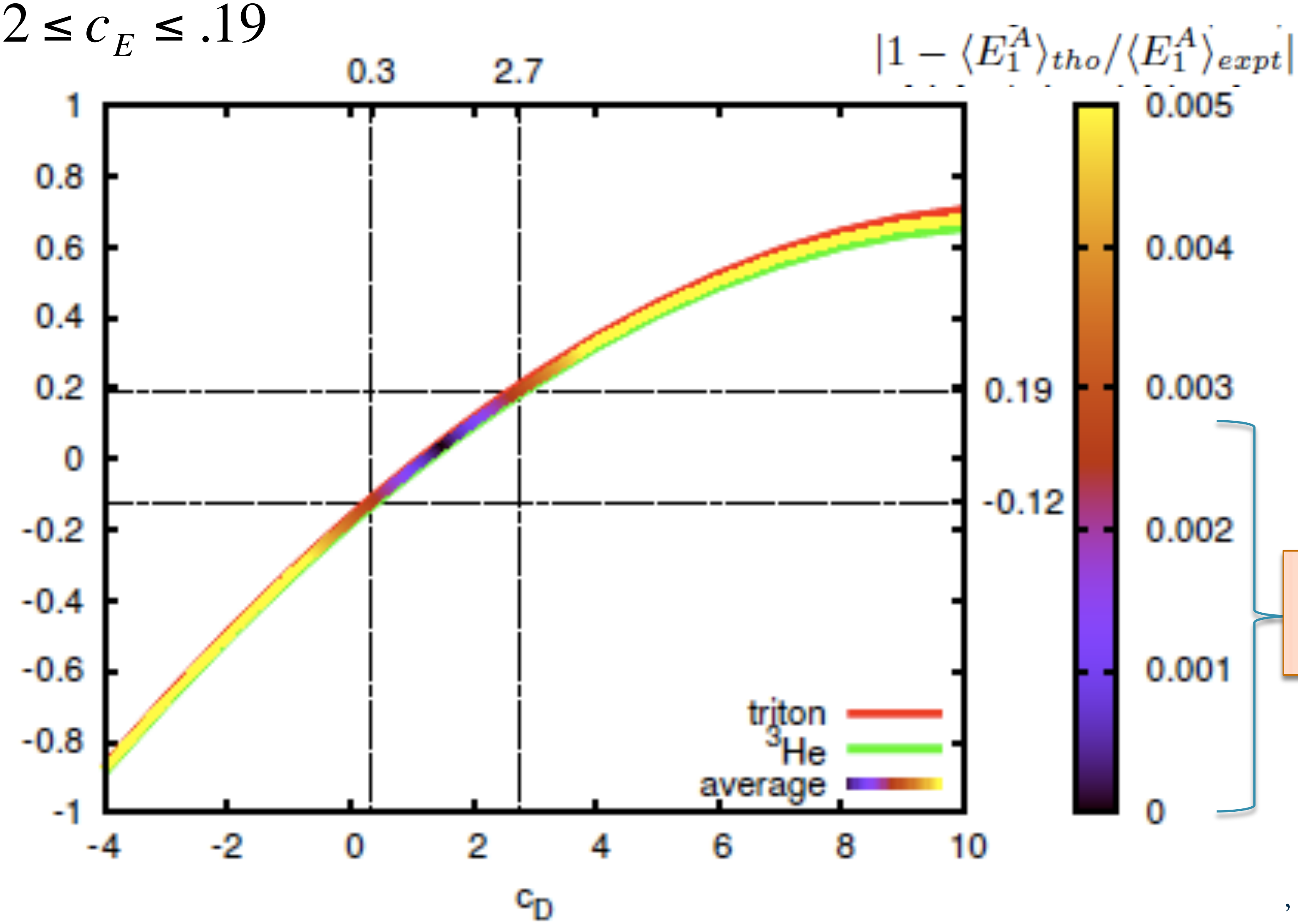
Akulov, Mamyrin, **Phys. Lett. B 610**, 45 (2005)
Simpson, **Phys. Rev. C 35**, 752 (1987)
Schiavilla, **Phys. Rev. C 58**, 1263 (1998)

$$0.3 \le c_D \le 2.7$$

$$\Downarrow$$

$$-.12 \le c_E \le .19$$

Calibration result

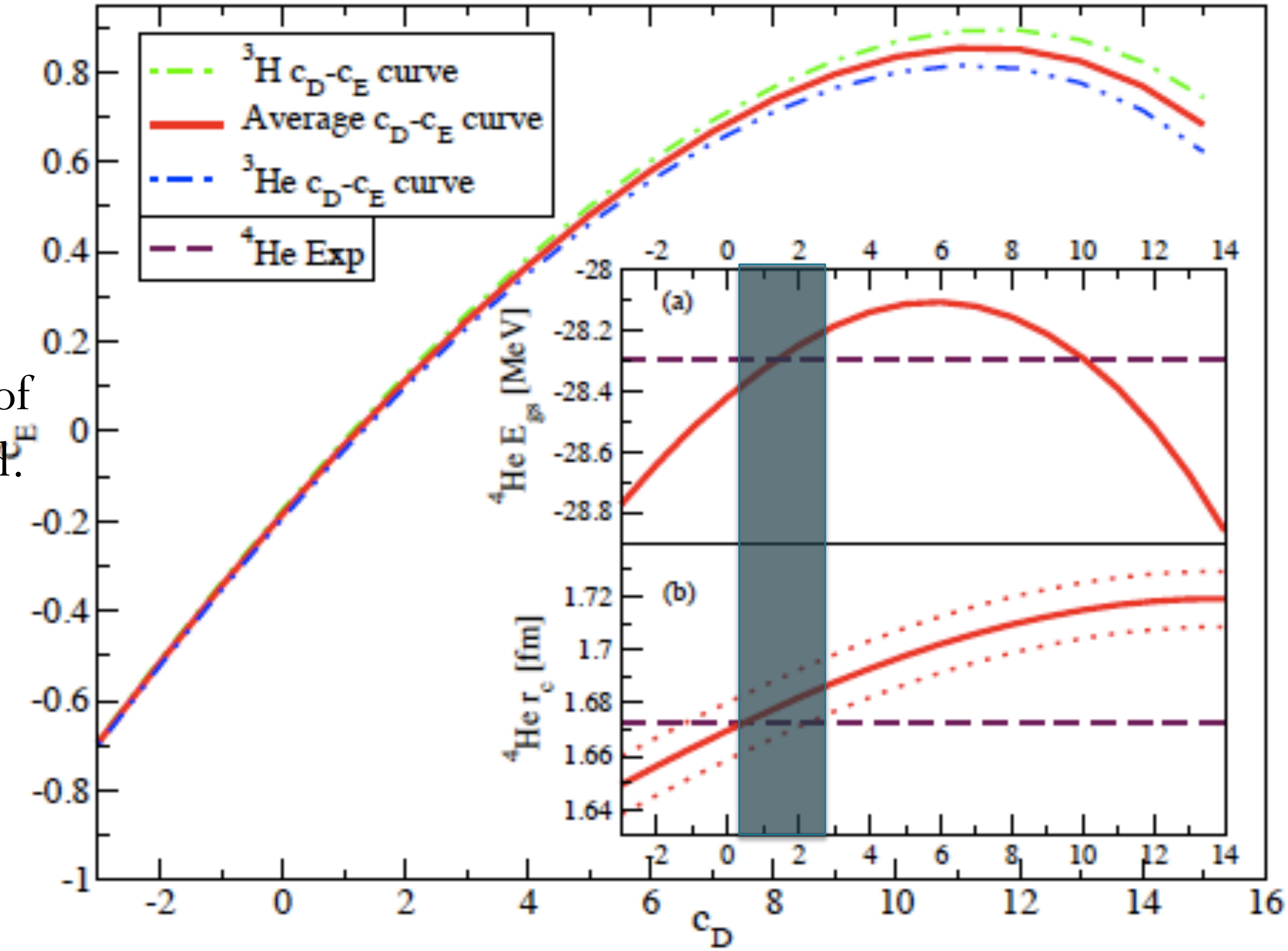


exp. error bar

, 2008

A prediction of ^{4}He

According to the charge radius – it looks like lower values of c_D are favored.



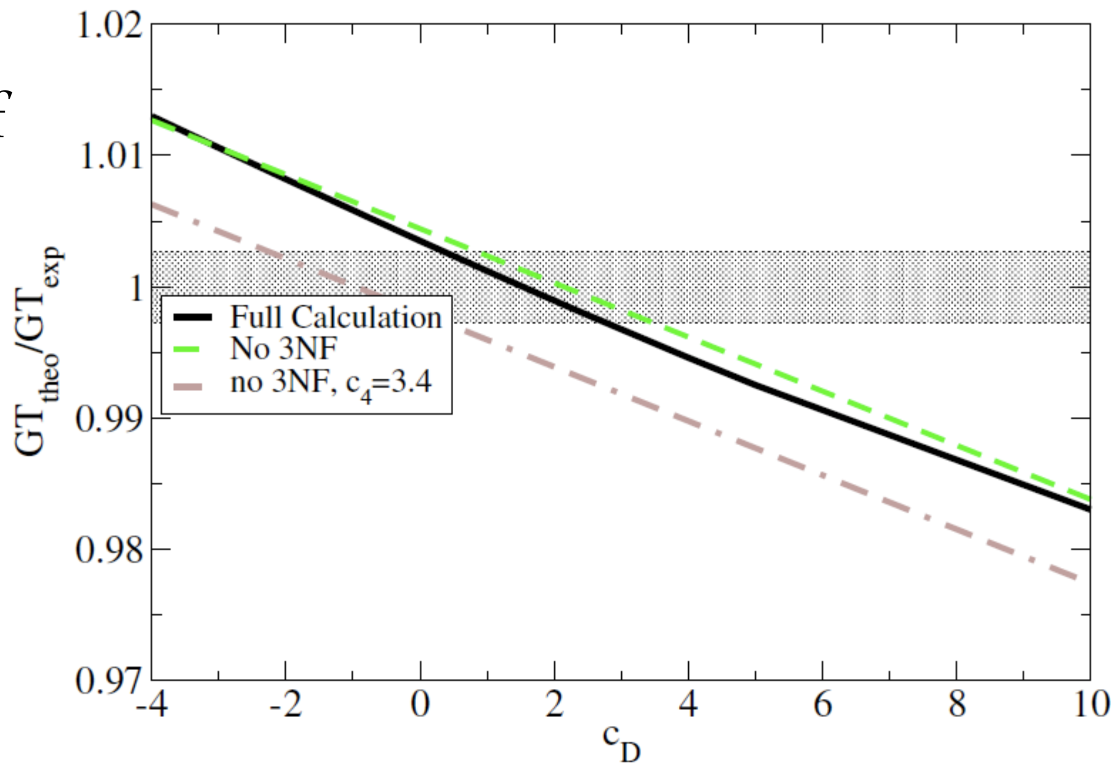
Some properties at c_D=1, c_E=-0.029

		^{3}H		nd		^{4}He		n^3H	
		$E_{\text{g.s.}}$	$\langle r_p^2\rangle^{1/2}$	^2a	^4a	$E_{\text{g.s.}}$	$\langle r_p^2\rangle^{1/2}$	^1a	^3a
NN	NCSM [16],[a]	−7.852(5)	1.650(5)	–	–	−25.39(1)	1.515(2)	–	–
NN	HH [20]	−7.854	1.655	1.100	6.342	−25.38	1.518	4.20	3.67
NN+NNN	NCSM [16],[a]	−8.473(5)	1.608(5)	–	–	−28.34(2)	1.475(2)	–	–
NN+NNN	HH [20]	−8.474	1.611	0.675	6.342	−28.36	1.476	3.99	3.54
Expt.		−8.482	1.60	–	–	−28.296	–	–	–
Expt. [21–23]		–	–	0.65(4)	6.35(2)	–	1.467(13)	4.98(29)	3.13(11)
Expt. [24, 25]		–	–	0.645(8)	–	–	–	4.45(10)	3.32(2)

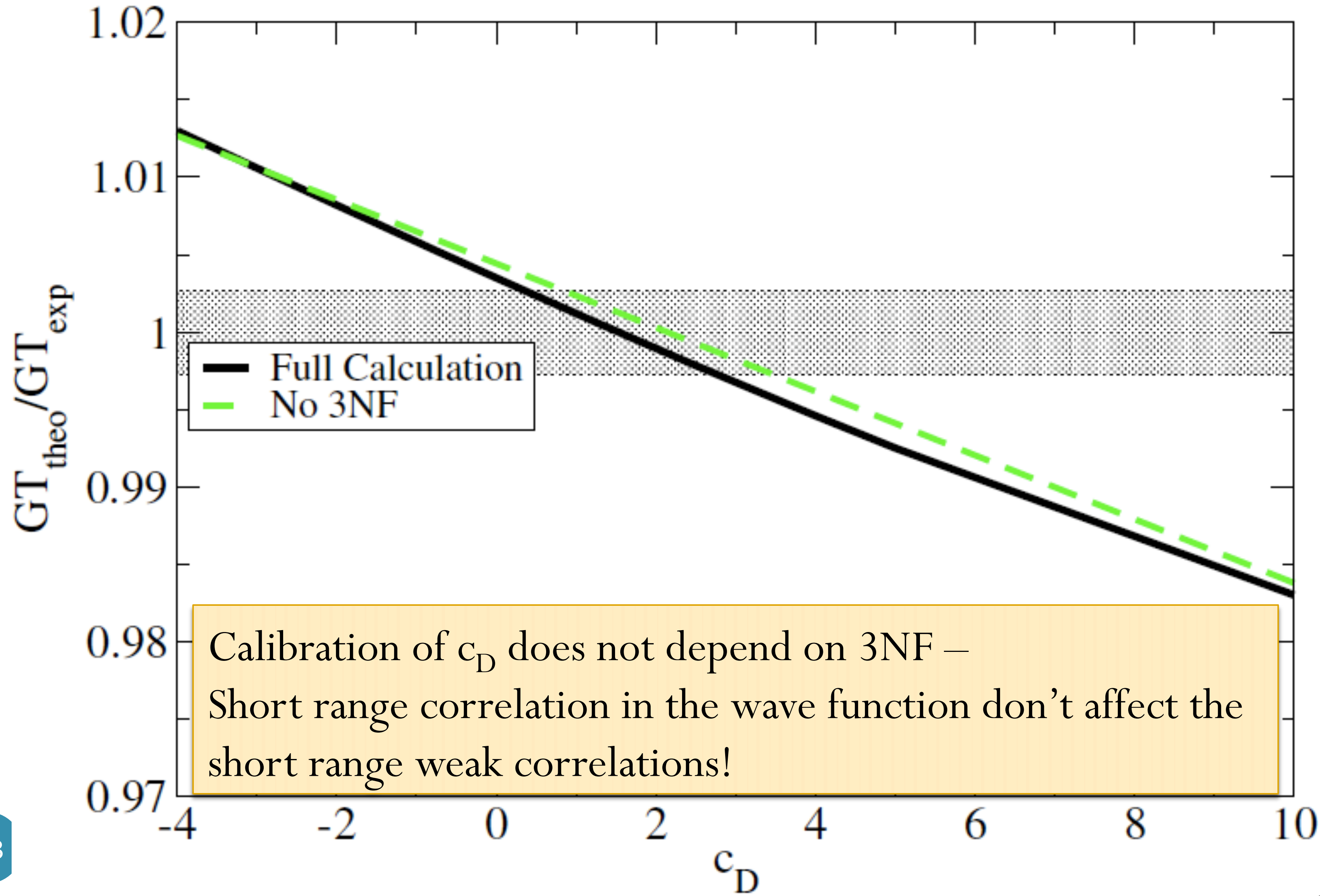
[20] A. Kievsky *et al.*, J. Phys. G **35**, 063101 (2008).
[21] W. Dilg *et al.*, Phys. Lett. B **36**, 208 (1971).
[22] E. Borie and G. A. Rinker, Phys. Rev. A **18**, 324 (1978); S. Kopecky *et al.*, Phys. Rev. Lett. **74**, 2447 (1995); P. Mohr and B. Taylor, Rev. Mod. Phys. A **596**, 367 (1996); I. Sick, Phys. Lett. B **576**, 62 (2003).
[23] H. Rauch *et al.*, Phys. Lett. B 165, 39 (1985).
[24] K. Schoen *et al.*, Phys. Rev. C **67**, 044005 (2003).
[25] G. M. Hale *et al.*, Phys. Rev. C **42**, 438 (1990).

Not all is good yet… two remarks:

- p-shell nuclei seem to suggest $c_D \sim -1$.
  - Renormalizing effect of the missing 3NF?
  - Numerical problems?
- There is still uncertainty, due to unknown value of c_4:
  - Still has to be checked consistently.



What are the important parts of the wave functions?

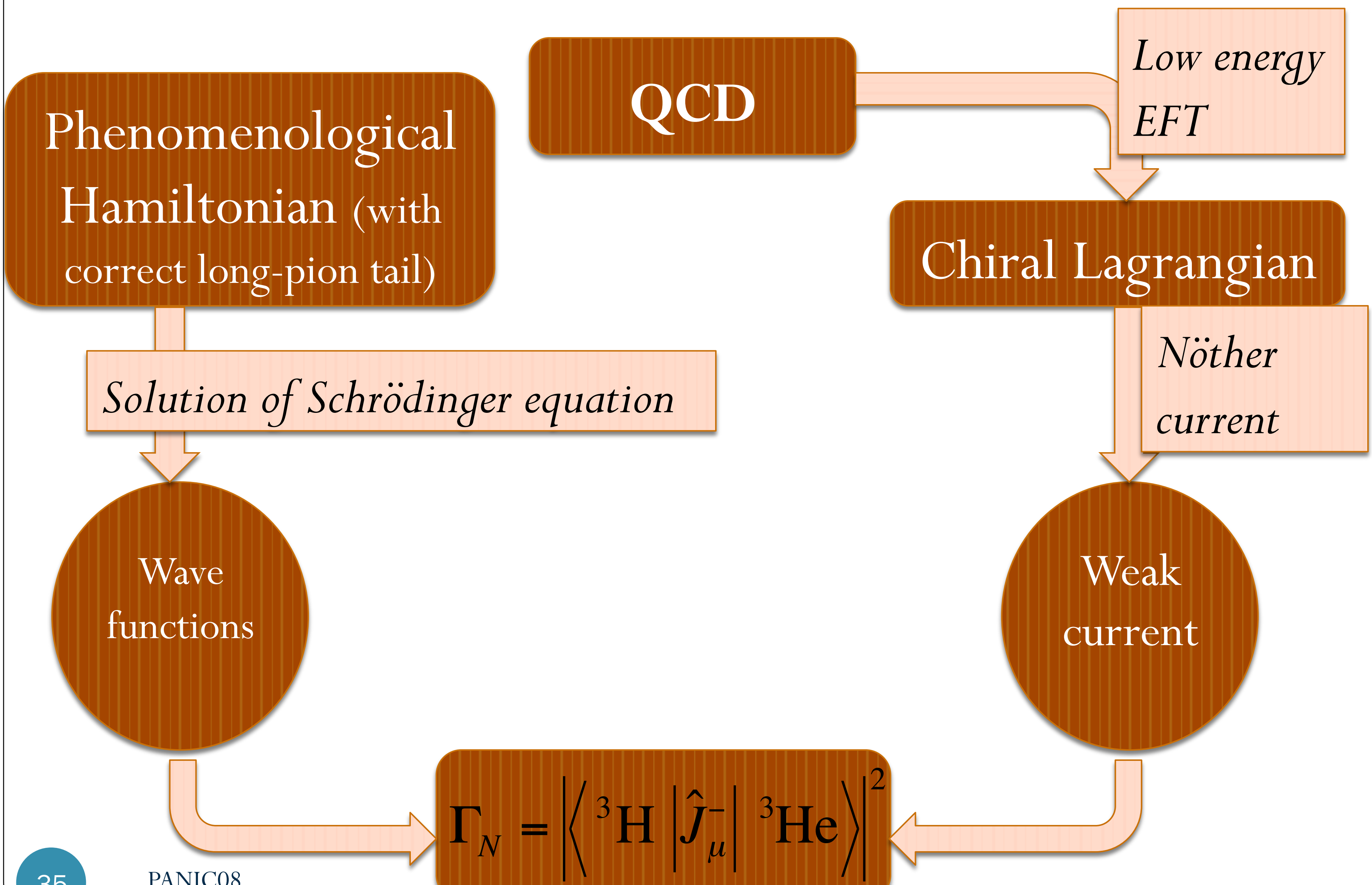


Calibration of c_D does not depend on 3NF –
Short range correlation in the wave function don't affect the short range weak correlations!

The apparent conclusion

- For GT type of operators, the short range correlations in the wave functions are not important for the observable.
- The long tail behavior of the potential is more essential.
- Is this the origin of the success of EFT*: hybrid calculations of weak reactions, using phenomenological forces in combination with χPT based currents?
  - One unknown parameter in MEC (d_R) calibrated using the triton half-life.

EFT approach for low-energy nuclear reactions:*



T.-S. Park *et al*, **Phys. Rev. C 67**, 055206 (2003), M. Rho **arXiv:** nucl-th/061003.

Uses of EFT*

- Many possible applications:
  - Extracting the weak structure of the nucleon from muon capture on ^{3}He.

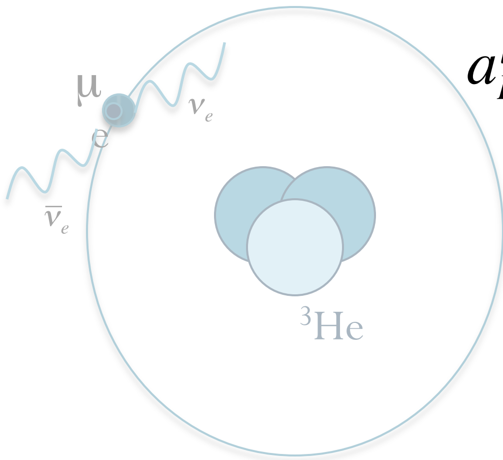
    DG, **Phys. Lett. B 666**, 472 (2008).
  - Predicting neutrino reactions on light nuclei, essential for supernova calculations.

    DG, Barnea, **Phys. Rev. Lett. 98**, 192501 (2007); DG et al. **Phys. Rev. C 75**, 055803 (2007); DG, PhD thesis.
  - Calculating weak reactions for the sun (*hep* and *pp*).

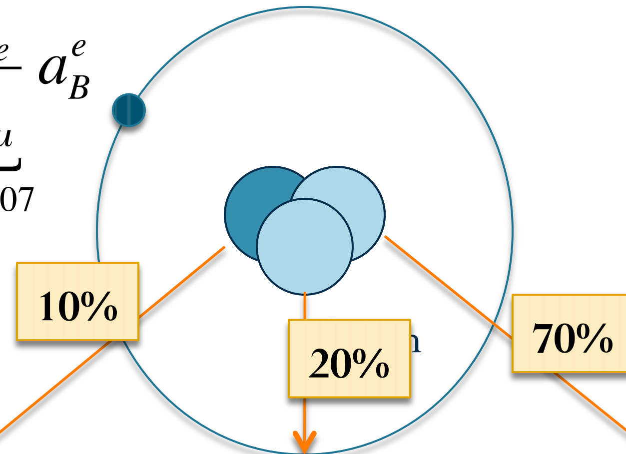
    Park et al, **Phys. Rev. C 67**, 055206 (2003).
  - Understanding the role of MEC in heavier nuclei.

    Vaintraub, DG, Barnea, in preparation.

The decay of a muonic ^{3}He: *competition*



$$a_B^\mu = \frac{\hbar}{Z m_\mu c\alpha} = \underbrace{\frac{m_e}{m_\mu}}_{\sim 1/207} a_B^e$$



$$\tau_\mu^{free} = 2.197019(21)\times 10^{-6}\ \text{sec}$$

^{3}He(μ^-,ν_μ) p+2n

^{3}He(μ^-,ν_μ) d+n

^{3}He(μ^-,ν_μ) ^{3}H

$$Capture\quad prob. \sim Z\cdot\left|\psi_{1S}(0)\right|^2 \sim \left(\frac{m_\mu}{m_e}\right)^3 Z^4$$

- The rates become comparable for Z~10.
- The Z^4 law has deviations – mainly due to nuclear effects.
- In order to probe the weak structure of the nucleon, one has to keep the nuclear effects under control.

In other words…

- Solve the nuclear problem from the nuc… freedom.
- However:
  - This is possible only…
  - The nuclear … are complicated – in particul… of the muon with meson cu…

A parameter free, percentage level accuracy calculation of the process is a great challenge to nuclear physics – which is now possible!!

The MuCap … g.

Γ(… 10.7_{syst} Hz

… % accuracy.

MuCap, **Phys. Rev. Lett. 99**, 032002 (2007).

For the (exclusive) process $^3\mathrm{He}(\mu^-,\nu_\mu)\,^3\mathrm{H}$ an incredible measurement (±0.3%):

$$\Gamma\left(\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + \mathrm{t}\right)_{stat} = 1496 \pm 4\,\mathrm{Hz}$$

Ackerbauer *et al*, **Phys. Lett. B417**, 224 (1998).

Final result:

$$\Gamma = \left\{ \frac{2G^2 |V_{ud}|^2 E_\nu^2}{2J_{^3\mathrm{He}} + 1} \left(1 - \frac{E_\nu}{M_{^3\mathrm{H}}} \right) |\psi_{1s}^{av}|^2 \Gamma_N \right\} (1 + RC)$$

$$\Gamma = 1499(2)_\Lambda (3)_{NM} (5)_t (6)_{RC} = 1499 \pm 16 \ \ \mathrm{Hz}$$

$$\Gamma_{EXP} = 1496 \pm 4\,\mathrm{Hz}$$

Constraints on the weak structure of the nucleon from muon capture on ^3He

$$\hat{J}^{\mu V} = \bar{u}(p')\left[\boxed{F_V(q^2)}\gamma^\mu + \frac{i}{2M_N}\boxed{F_M(q^2)}\sigma^{\mu\nu}q_\nu + \boxed{\frac{g_S}{m_\mu}}q^\mu\right]u(p)$$

Vector

Magnetic

Second class currents

$$\hat{J}^{\mu A} = -\bar{u}(p')\left[\boxed{G_A(q^2)}\gamma^\mu\gamma_5 + \boxed{\frac{g_P(q^2)}{m_\mu}}\gamma_5 q^\mu + \boxed{\frac{ig_t}{2M_N}}\sigma^{\mu\nu}\gamma_5 q_\nu\right]u(p)$$

Axial

Induced
Pseudo-Scalar

Induced pseudo-scalar:

- From χPT [Bernard, Kaiser, Meissner, PRD 50, 6899 (1994); Kaiser PRC 67, 027002 (2003)]:

$$g_P\left(-0.954 m_\mu^2\right) = 7.99(0.20)$$

- From muon capture on proton [Czarnecki, Marciano, Sirlin, PRL 99, 032003 (2007); V. A. Andreev et. al., PRL 99, 032004(2007)]:

$$g_P\left(-0.88 m_\mu^2\right) = 7.3(1.2)$$

- This work: $g_P\left(-0.954 m_\mu^2\right) = 8.13(0.6)$

$$\boxed{g_P\left(q^2\right) = \frac{2 m_\mu g_{\pi pn} f_\pi}{m_\pi^2 - q_\mu^2} - \frac{1}{3} g_A m_\mu M_N \left\langle r_A^2 \right\rangle = 7.99(20)}$$

Induced Tensor:

- From QCD sum rules: $$\frac{g_t}{g_A} = -0.0152(53)$$
- Experimentally [Wilkinson, Nucl. Instr. Phys. Res. A 455, 656 (2000)]:

$$\left|\frac{g_t}{g_A}\right| < 0.36 \ \text{at} \ 90\%$$

- This work: $$\frac{g_t}{g_A} = -0.1(0.68)$$

$$\delta J^{\mu A} = \frac{i g_t}{2M_N} \sigma^{\mu\nu} \gamma_5 q_\nu$$

Induced scalar (limits CVC):

- "Experimentally" [Severijns et. al., RMP 78, 991 (2006)]: $$g_S = 0.01 \pm 0.27$$
- This work: $$g_S = -0.005 \pm 0.04$$

$$\delta J^{\mu V} = \frac{g_S}{m_\mu} q^\mu$$

β decay rates for heavier nuclei

- Experimental extraction of "Gamow-Teller" strengths from β-decay rates shows a needed suppression: $g_A \rightarrow 1$ starting from A=3 gradually to A=40, where it stays at $g_A=1$.

$$GT \sim g_A \sum \sigma_i \tau_{3,i}$$

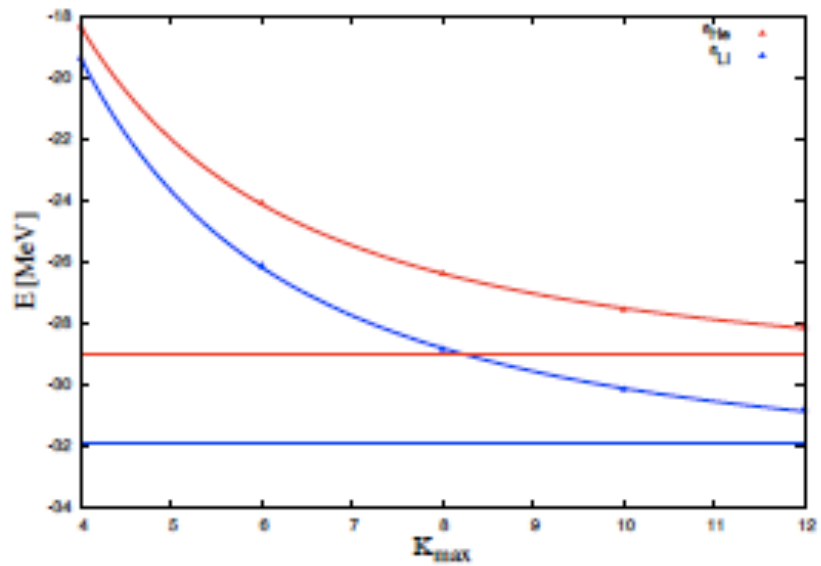
- Note that $g_A=1$ means a restoration of axial symmetry!
- Signatures of this restoration appear in hadron spectra at high energies.
- Is this a signature of the restoration of chiral symmetry in nuclear matter?
- But… this is only the leading order of the E_1^A operator.
- What is the effect of higher orders?
- After triton, the next β-decaying nucleus is ^{6}He, where consistent calculations are complicated.
- EFT* helps – allows the use of softer phenomenological potentials.

The JISP16 NN potential

The JISP16 reproduce the NN phase shifts in the range 0 – 300MeV.

Binding Energies

	AV18+UBIX	JISP16	Nature
D	2.24	2.24	2.24
^{3}H	8.48	8.35	8.48
^{3}He	7.74	7.65	7.72
^{4}He	28.5	28.3	28.3
^{6}He	-	29.0	29.29
^{6}Li	-	31.9	31.99



AV18+UBIX - Argonne V18 + Urbana IX
JISP16 - J-matrix Inverse Scattering Potential, Shirokov *et* al.

Single nucleon GT strengths

The β-decay half-life

$$\boxed{t_{1/2} = \frac{1}{f}\frac{\tau \log 2}{\langle F\rangle^2 + (F_A/F_V)^2 \langle GT\rangle^2}}$$

$$\langle GT\rangle \equiv \langle \Psi_f || \sum_j \boldsymbol{\sigma}_j \tau_j^+ || \Psi_i\rangle$$

$$\langle F\rangle \equiv \langle \Psi_f || \sum_j \tau_j^+ || \Psi_i\rangle$$

^{3}H - ^{3}He

Potential	$\langle GT\rangle$
AV18+3NF	$\sqrt{3}\cdot 0.923(1)$
Bonn+3NF	$\sqrt{3}\cdot 0.936(1)$
JISP16 [This work]	$\sqrt{3}\cdot 0.9544(4)$
Expt.	$\sqrt{3}\cdot 0.955(3)$

^{6}He - ^{6}Li

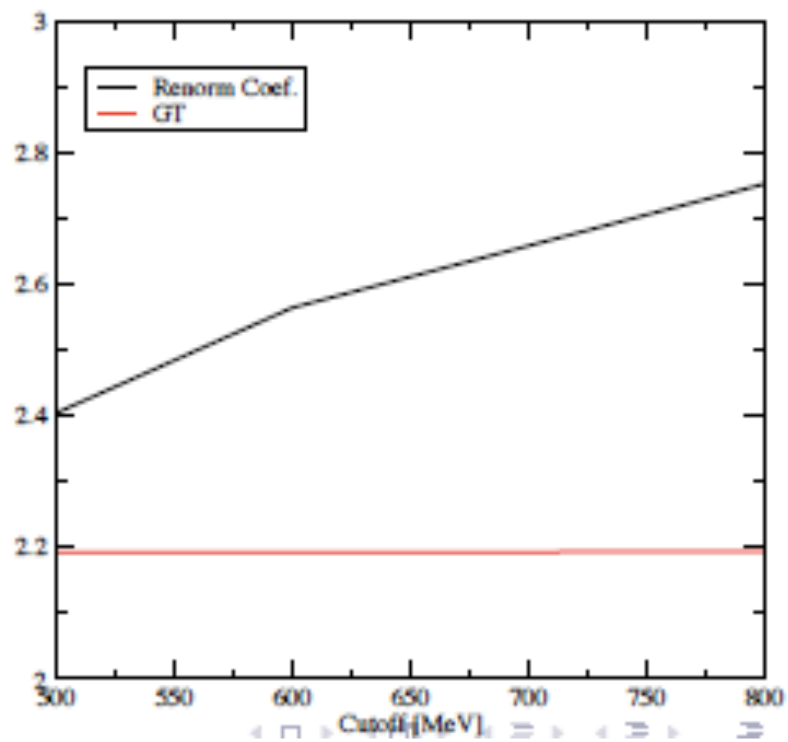
Potential	$\langle GT\rangle$
AV18/UIX - VMC	2.250(7)
AV18/IL2 - VMC	2.22(2)
AV18/IL2 - GFMC	2.182(25)
AV8/TM - NCSM	2.283(2)
JISP16 - [This work]	2.229(3)
Expt.	2.170(3)

Complete current

The Gamow-Teller ^{6}He - ^{6}Li matrix element

Potential	1-body	2-body
AV18/UIX - VMC	2.250(7)	2.281(7)
JISP16 [This work]	2.229(3)	2.193(2)
Expt.		2.170(3)

- The VMC calculation with MEC made things even worse for ^{6}He !
- HH calculations with EFT 2-body currents reconcile theory and experiment !
- The matrix-element is almost independent of the cutoff !



Conclusions from ^{6}He β decay

1. The 1-body current underpredict the ^{3}H β-decay $\langle GT \rangle$.
2. It overpredict the ^{6}He β-decay $\langle GT \rangle$.
3. 2-body currents derived from meson exchange model go in the wrong direction for ^{6}He !
4. In contrast, EFT 2-body currents lead to reconciliation between thoery and experiment.
5. The predicted ^{6}He β-decay half life is independent of the cutoff.

Final Remarks

- Weak reaction rates provide efficient observables to calibrate the three nucleon forces.
- In the (near) future, a combined calculation of muon capture rates on deuteron and ^{3}He will allow cross-checks of the calibration.
- Consistent calculation of ^{6}He decay rate, as well as electron capture on ^{7}Be, could light the way to heavier elements.
- In the meantime – EFT* has a new justification.
- These studies can:
  - Give parameter free QCD predictions for nuclear reactions.
  - Shed light on short range correlations in nuclei:
    - 3NF.
    - MEC.
- Solving QCD problems in the nuclear system: the $g_A \rightarrow 1$ problem.