

Low-energy Weak reactions: *From QCD to Astrophysics*

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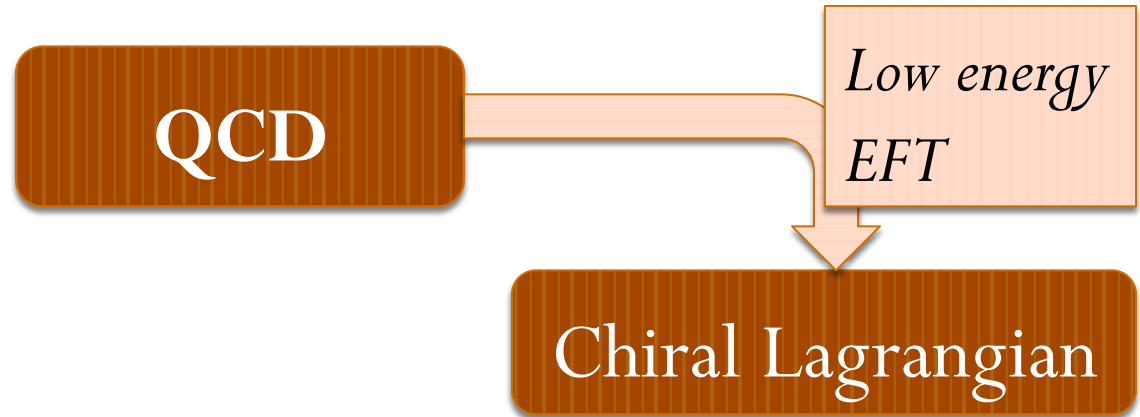
Introduction

- Precision era in few-body nuclear physics:
 - Available methods for solving exactly the Schrödinger equation for few body systems, from their nucleonic degrees of freedom:
 - Green's Function Monte Carlo.
 - No core shell model.
 - Expansions in Hyperspherical Harmonics.
 - High precision nuclear interaction, phenomenological or χ PT based:
 - Spectra of light nuclei.
 - Transitions and cross-sections.
- Allows parameter free calculations of nuclear wave functions and low-energy reaction rates, with sub-percentage accuracy.
- How can we use this to gain understanding on interesting problems?

Outline

- Using χ PT for calculating low-energy electro-weak reactions.
- Applications:
 - Nuclear Physics: *constraining the nuclear force using triton β -decay and an inside look into correlations in the nucleus.*
 - Nucleon: *Weak structure of the nucleon from μ -capture on ${}^3\text{He}$.*
 - Astrophysics: *Neutrino reactions with light nuclei in Supernovae.*
 - QCD: *no signature for g_A suppression in nuclear matter from β -decay of ${}^6\text{He}$?*
- Weak interaction in Holographic QCD: *easy access to the size of low-energy constants.*

Modern approach for low-energy EW nuclear reactions:



Effective field theory (EFT) for nuclear physics: Chiral perturbation theory (χ PT)

- Symmetries are important **NOT** degrees of freedom:
 - In QCD – an approximate chiral symmetry:
$$SU(2)_L \times SU(2)_R \cong SU(2)_V \times SU(2)_A \rightarrow SU(2)_V$$
 - Pions – Goldstone bosons of the broken symmetry.
- Choose **Λ** – the theory cutoff. (400-800 MeV)
- Identify **Q** – the energy scale of the process. (around 100 MeV)
- In view of **Q and Λ** -Identify the relevant degrees of freedom. (pions and nucleons).
- Write all the possible operators which agree with the symmetries of the underlying theory (INFINITE)
- Calculate Feynman diagrams (INFINITE)
- Find a systematic way to organize diagrams according to their contribution

Weinberg's Power Counting Scheme

- Each Feynman diagram can be characterized by: $\left(\frac{Q}{\Lambda_\chi}\right)^v$
- $Q \sim 140$ MeV is the relevant momentum of the process or pion masses in the diagram.
- $\Lambda_\chi \sim 1$ GeV is the chiral symmetry breaking scale.
- Weinberg showed: $v \geq 0$



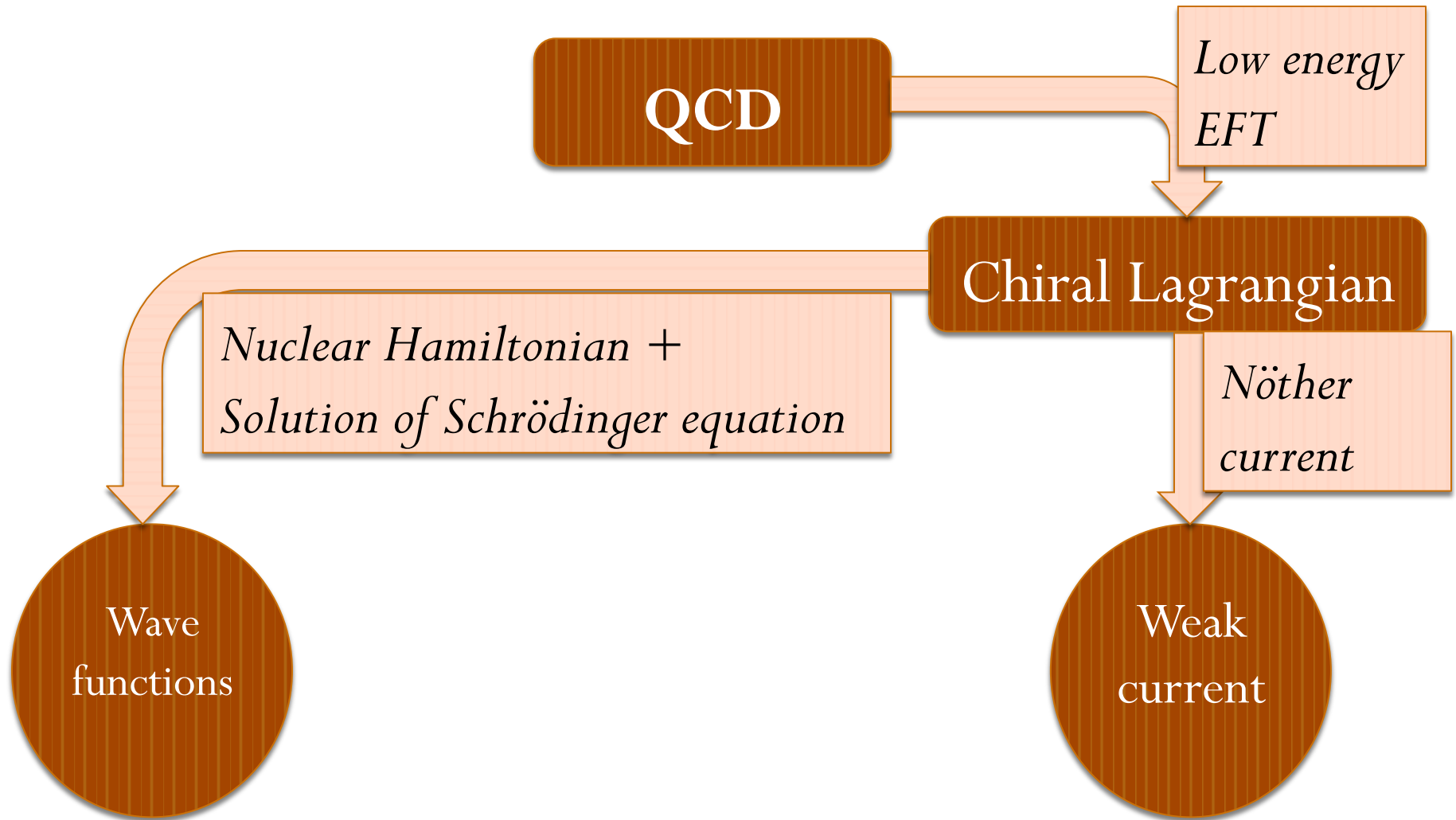
Chiral Perturbation Theory

- In addition, expand in the inverse of the nucleon's mass (take $\Lambda_\chi \sim M_N$) $\rightarrow$ Heavy Baryon χ PT.

The big deal in χ PT

- A perturbation theory/ expansion in small parameter of the observable, gives control over the accuracy of the calculation.
- Varying the cutoff gives estimate of the theoretical error-bar.
- Allows connection between *a-priori* not related operators.
- In particular the nuclear force and the electro-weak currents in the nucleus (that the $SU(2) \times SU(2)$ structure is a gauging of).
- When the low-energy constants are known: the calculations are predictions of QCD.

χ PT approach for low-energy EW nuclear reactions:



Weak interaction with the nucleus

$$\hat{H}_W = -\frac{G_F V_{ud}}{\sqrt{2}} \int d^3\vec{x} \hat{j}_\mu^+(\vec{x}) \hat{J}^\mu_-(\vec{x})$$

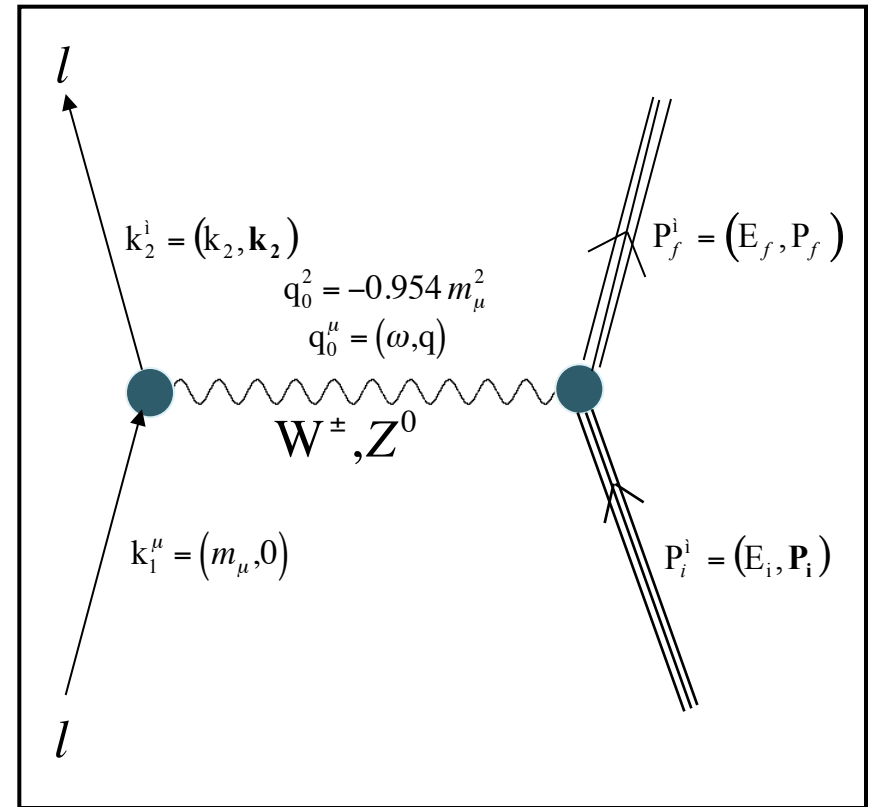
Lepton
current

Nuclear
current

Scattering operator

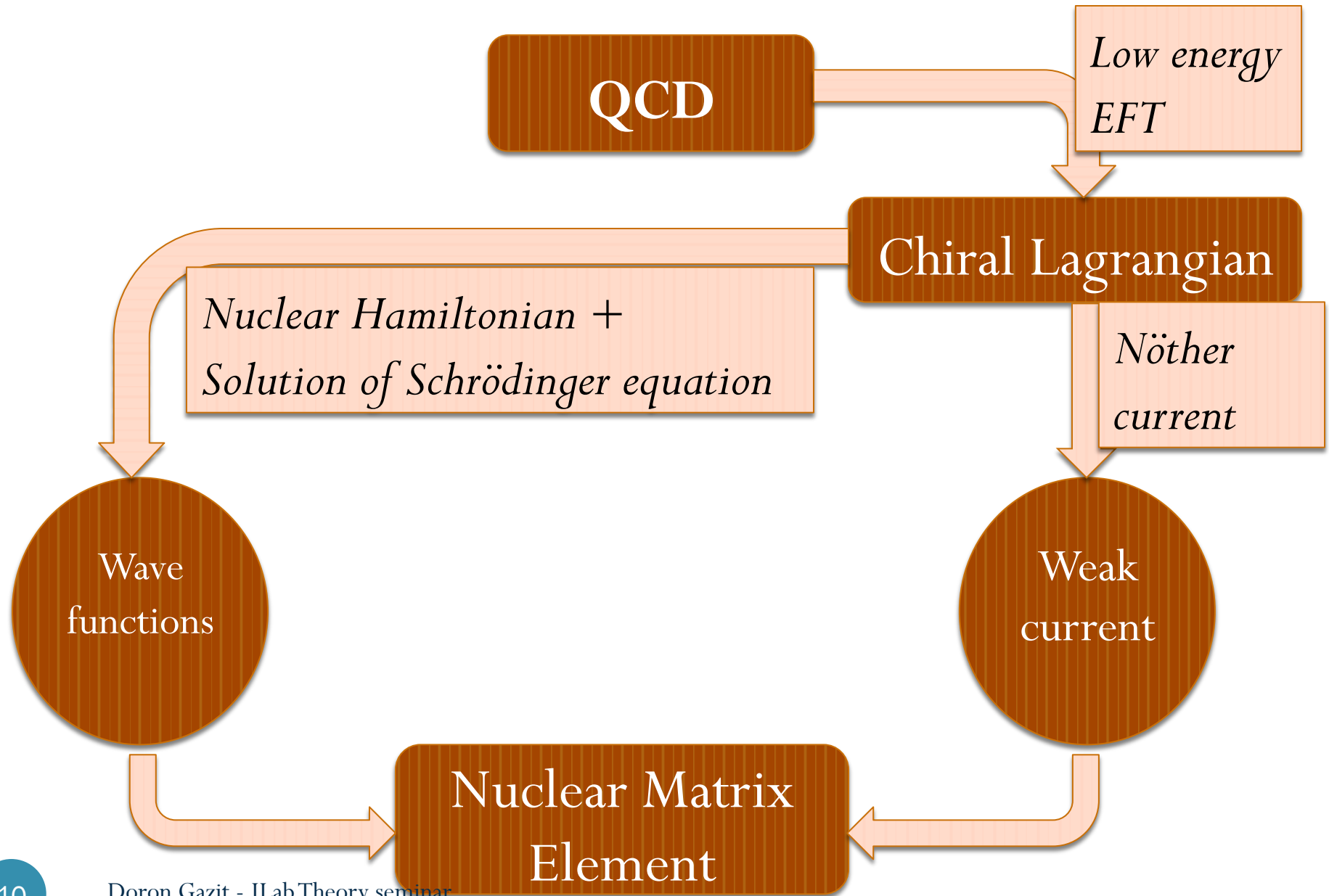


Currents in the nucleus



$$W, Z \text{ propagator} = \frac{g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 + M_W^2} \xrightarrow{q \ll M_W} \frac{g_{\mu\nu}}{M_W^2}$$

χ PT approach for low-energy EW nuclear reactions:

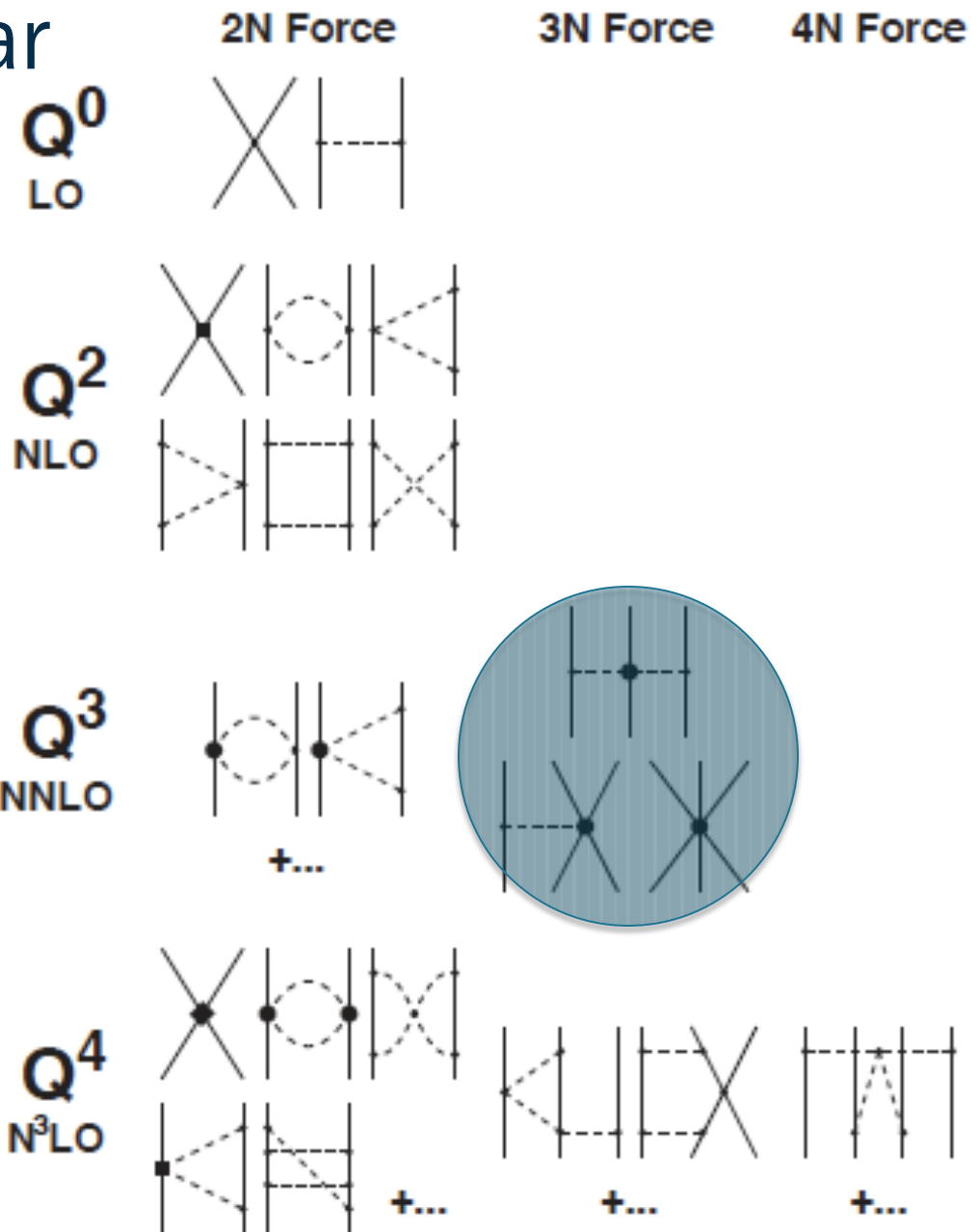


Hierarchy of Nuclear Forces in χ PT

- Only contact terms cannot be calibrated in the pion or pion/nucleon system.
- The 2N terms are calibrated to reproduce phase shifts.

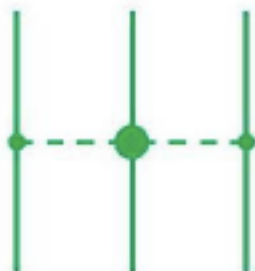
χ^2/datum for the reproduction of the
1999 *np* database

Bin (MeV)	# of data	N ³ LO	NNLO	NLO	AV18
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04



3 nucleon forces at N²LO

TPE-3NF



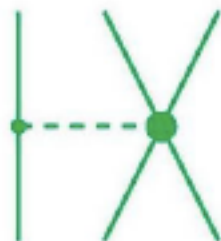
$$V_{\text{TPE}}^{3\text{NF}} = \left(\frac{g_A}{2f_\pi} \right)^2 \frac{1}{2} \sum_{i \neq j \neq k} \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

with $\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$, where $\vec{p}_i$ and $\vec{p}_i'$ are the initial and final momenta of nucleon i , respectively, and

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \vec{q}_i \cdot \vec{q}_j \right] + \frac{c_4}{f_\pi^2} \sum_\gamma \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$

No new parameters!

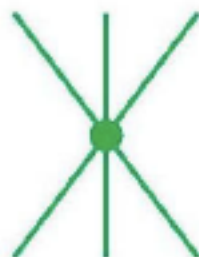
OPE-3NF



$$V_{\text{OPE}}^{3\text{NF}} = C_D \sum_{i \neq j \neq k} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{q_j^2 + m_\pi^2} (\tau_i \cdot \tau_j) (\vec{\sigma}_i \cdot \vec{q}_j)$$

One New param. $\mathcal{L}_4 = -2D_1 (\bar{N} \gamma^\mu \gamma_5 a_\mu N) (\bar{N} N)$

Contact-3NF



$$V_{\text{ct}}^{3\text{NF}} = C_E \sum_{j \neq k} \tau_j \cdot \tau_k \cdot$$

One new parameter,

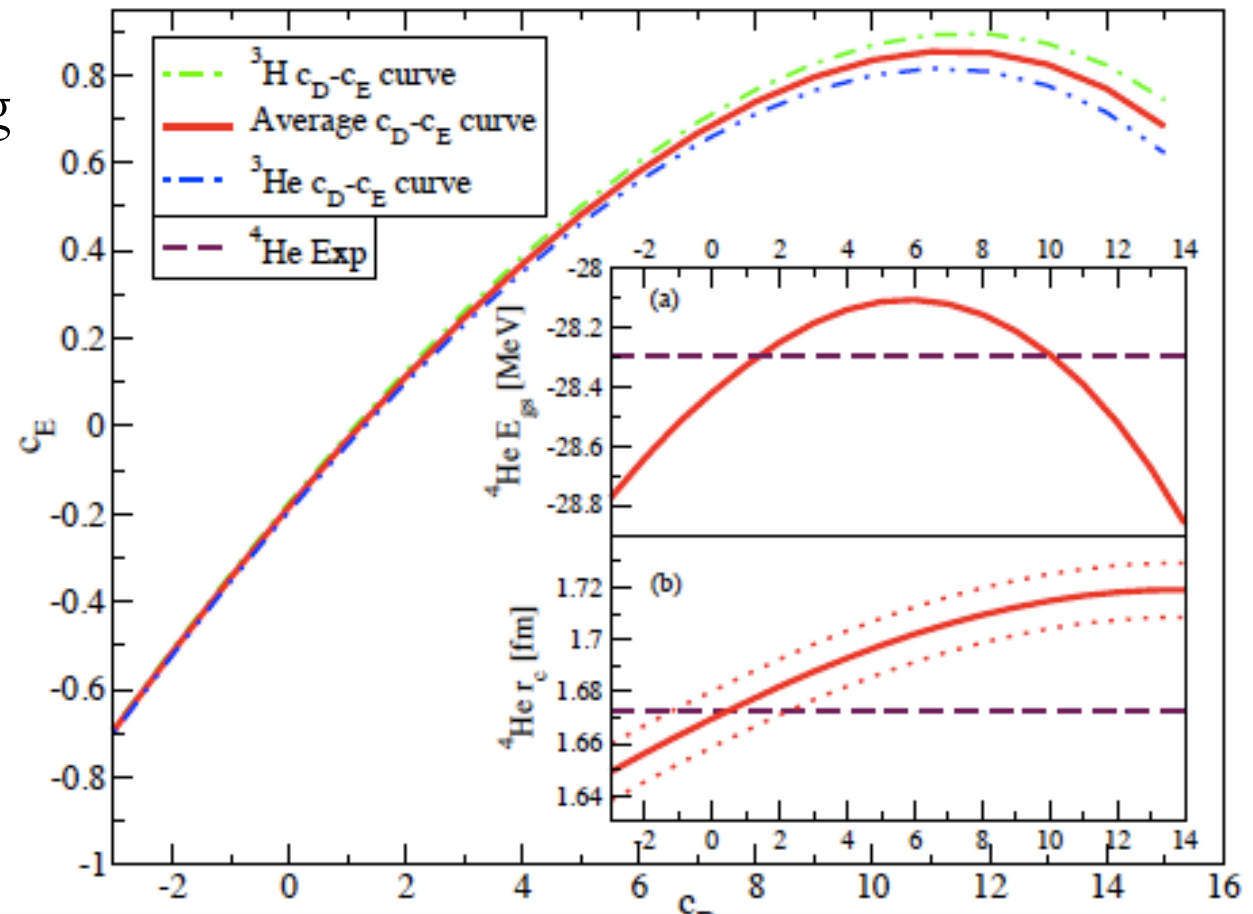
Strategy : calibrate c_D and c_E from nuclear matter observables, and then predict other observables.

Attempts to calibrate the contact parameters

Other attempts were to use 3 nucleon scattering lengths as a second observable.

The problem is the cross-correlation of the different observables.

This leads to an error bar of about $\Delta c_D \sim \pm 1$



Here, we choose to calibrate the contact parameters using weak observables

Some remarks

- For now, only $N^2\text{LO}$ 3NF exist.
- However, they include all LEC to $N^3\text{LO}$.
- Clearly – more work is called for!

...However, one can achieve very interesting results and conclusions, regarding the nature of correlations inside the nucleus...

Weak currents in the nucleus

- The standard model dictates only the structure of the currents:

- Charged current $\mathcal{J}_\mu^{(\pm)} = \frac{\tau_\pm}{2} (J_\mu^V + J_\mu^A)$

- Neutral current: $\mathcal{J}_\mu^{(0)} = (1 - 2 \cdot \sin^2 \theta_W) \frac{\tau_0}{2} J_\mu^V + \frac{\tau_0}{2} J_\mu^A - 2 \cdot \sin^2 \theta_W \frac{1}{2} J_\mu^V$

- The current of polar (axial) vector symmetry is the Noether current of the QCD Lagrangian, with respect to $SU(2)_V$ [$SU(2)_A$] symmetry.
- Includes:
 - single nucleon current (leading order)
 - Meson exchange currents. (start at N³LO)

Single Nucleon Currents

$$\hat{J}^{\mu V} = \bar{u}(p') \left[\boxed{F_V(q^2)} \gamma^\mu + \frac{i}{2M_N} \boxed{F_M(q^2)} \sigma^{\mu\nu} q_\nu + \boxed{\frac{g_S}{m_\mu}} q^\mu \right] u(p)$$

Vector

Magnetic

Second class currents

$$\hat{J}^{\mu A} = -\bar{u}(p') \left[\boxed{G_A(q^2)} \gamma^\mu \gamma_5 + \boxed{\frac{g_P(q^2)}{m_\mu}} \gamma_5 q^\mu + \boxed{\frac{ig_t}{2M_N}} \sigma^{\mu\nu} \gamma_5 q_\nu \right] u(p)$$

Axial

Induced

Pseudo-Scalar



- q dependence is due to pion loops.
- Second class currents vanish to this order!

Weinberg **Phys. Rev.**, 112, 1375 (1958)

Meson Exchange currents

- Vector currents, protected by charge conservation (or CVC), do not include contact parameters, up to fourth order.
- Axial currents are more complicated, in configuration space:

$$\hat{A}_{12}^{i,a}(\vec{r}_{ij}) = \frac{g_A}{2Mf_\pi^2} \hat{d}_r \mathcal{O}_\ominus^{i,a} \delta_\Lambda^{(3)}(\vec{r}_{ij}) - \frac{g_A m_\pi^2}{2Mf_\pi^2} \mathcal{O}_P^{i,a} y_{1\Lambda}^\pi(r_{12}) - \frac{g_A m_\pi^2}{2Mf_\pi^2} \left[\frac{\hat{c}_3}{3} (\mathcal{O}_\oplus^{i,a} + \mathcal{O}_\ominus^{i,a}) + \frac{2}{3} (\hat{c}_4 + \frac{1}{4}) \mathcal{O}_\otimes^{i,a} \right] y_{0\Lambda}^\pi(r_{ij}) - \frac{g_A m_\pi^2}{2Mf_\pi^2} \left[\hat{c}_3 (\mathcal{T}_\oplus^{i,a} + \mathcal{T}_\ominus^{i,a}) - (\hat{c}_4 + \frac{1}{4}) \mathcal{T}_\otimes^{i,a} \right] y_{2\Lambda}^\pi(r_{ij})$$

$$\delta_\Lambda^{(3)}(\vec{r}) \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} S_\Lambda^2(\vec{k}^2),$$

$$y_{\Lambda 0}^\pi(r) \equiv \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} S_\Lambda^2(\vec{k}^2) \frac{1}{k^2 + m_\pi^2}$$

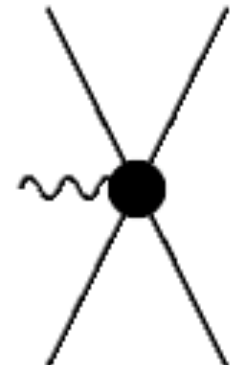
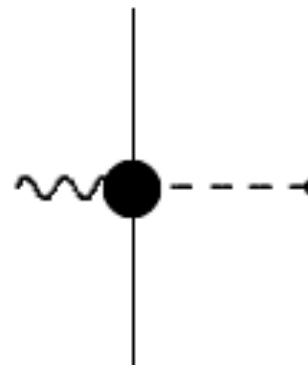
$$y_{\Lambda 1}^\pi(r) \equiv -\frac{\partial}{\partial r} y_{\Lambda 0}^\pi(r),$$

$$y_{\Lambda 2}^\pi(r) \equiv \frac{1}{m_\pi^2} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} y_{\Lambda 0}^\pi(r)$$

$$\hat{d}_R \equiv \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

1 pion exchange

Contact term



Some remarks

- Meson exchange current involves only TWO nucleons.
- Thus, in principle c_D can be calibrated using two-body weak processes.
- So – three nucleon force constrained at the two nucleon level!
- There are no low-energy weak observables that are measured accurately enough.
- However, many 3 nucleon processes are measured very well.

Constraining the Nuclear force using ${}^3\text{H}$ β -decay

DG, S. Quaglioni, P. Navratil, arxiv: 0812.4444.

Nuclear Matrix Elements

- A multipole decomposition of the currents is very helpful:

$$\hat{C}_{JM}(q) = \int d\vec{x} j_J(qx) Y_{JM}(\hat{x}) \hat{\mathcal{J}}_0(\vec{x})$$

$$\hat{E}_{JM}(q) = \frac{1}{q} \int d\vec{x} \vec{\nabla} \times [j_J(qx) \vec{Y}_{J JM}(\hat{x})] \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

$$\hat{M}_{JM}(q) = \int d\vec{x} j_J(qx) \vec{Y}_{J JM}(\hat{x}) \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

$$\hat{L}_{JM}(q) = \frac{i}{q} \int d\vec{x} \vec{\nabla} [j_J(qx) Y_{JM}(\hat{x})] \cdot \hat{\vec{\mathcal{J}}}(\vec{x})$$

- Usually, the low energy and selection rules mean that only a small number of multipoles contribute.

β decay rate for $q \rightarrow 0$

$$(fT_{1/2})_t = \frac{K/G_V^2}{(1 - \delta_c) + 3\pi \frac{f_A}{f_V} \langle E_1^A \rangle^2}$$

$$\langle E_1^A \rangle = |\langle {}^3\text{He} || \bar{E}_1^A || {}^3\text{H} \rangle|$$

- At the leading order:

$$E_1^A|_{\text{LO}} = i g_A (6\pi)^{-1/2} \overbrace{\sum_{i=1}^A \sigma_i \tau_i^+}^{\text{GT}}$$

- This is the reason for the common name: experimental Gamow-Teller.
- For the triton β -decay:

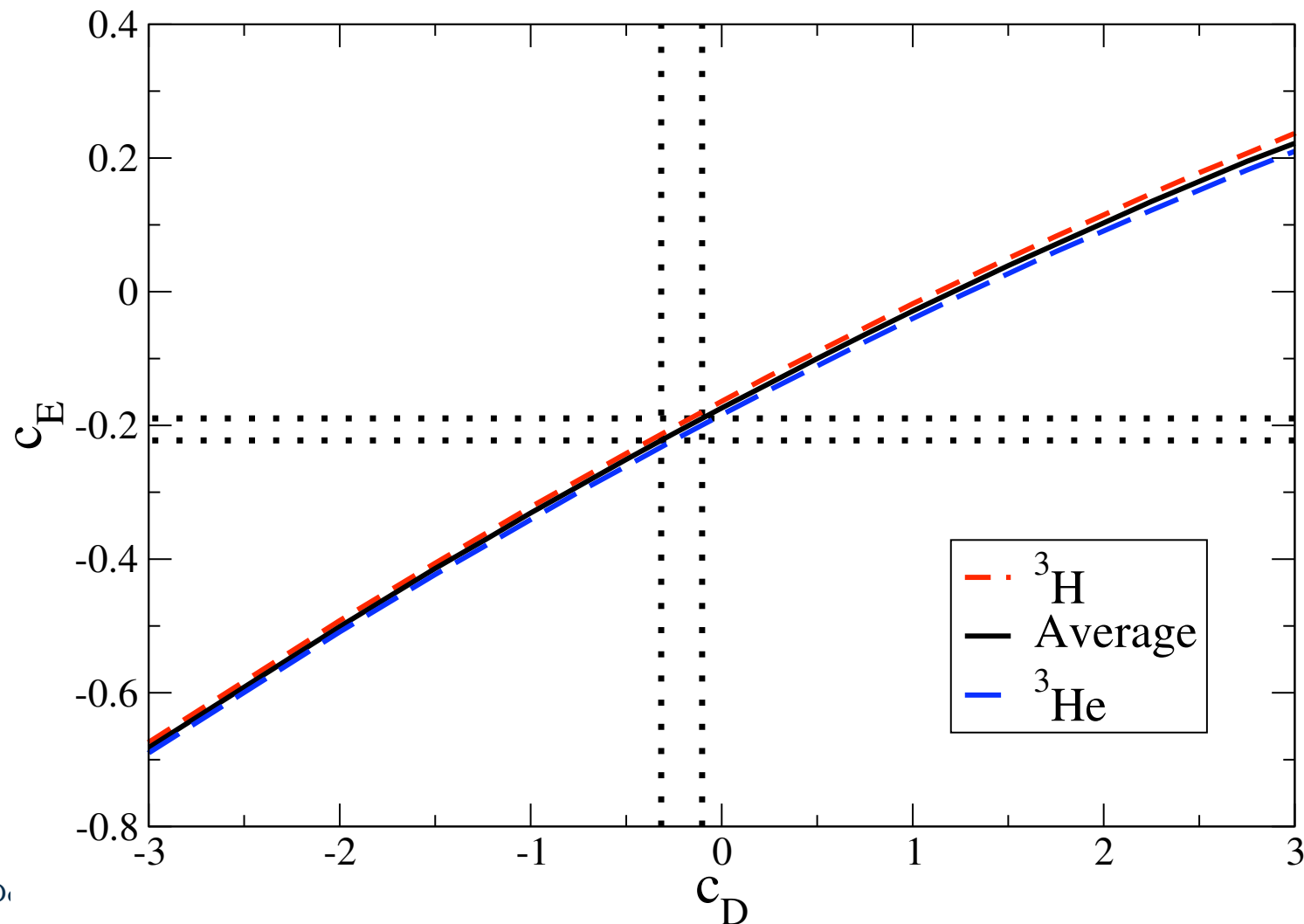
$$\langle E_1^A \rangle|_{\text{expt}} = 0.6848 \pm 0.0011$$

$$-0.3 \leq c_D \leq -0.1$$

⇓

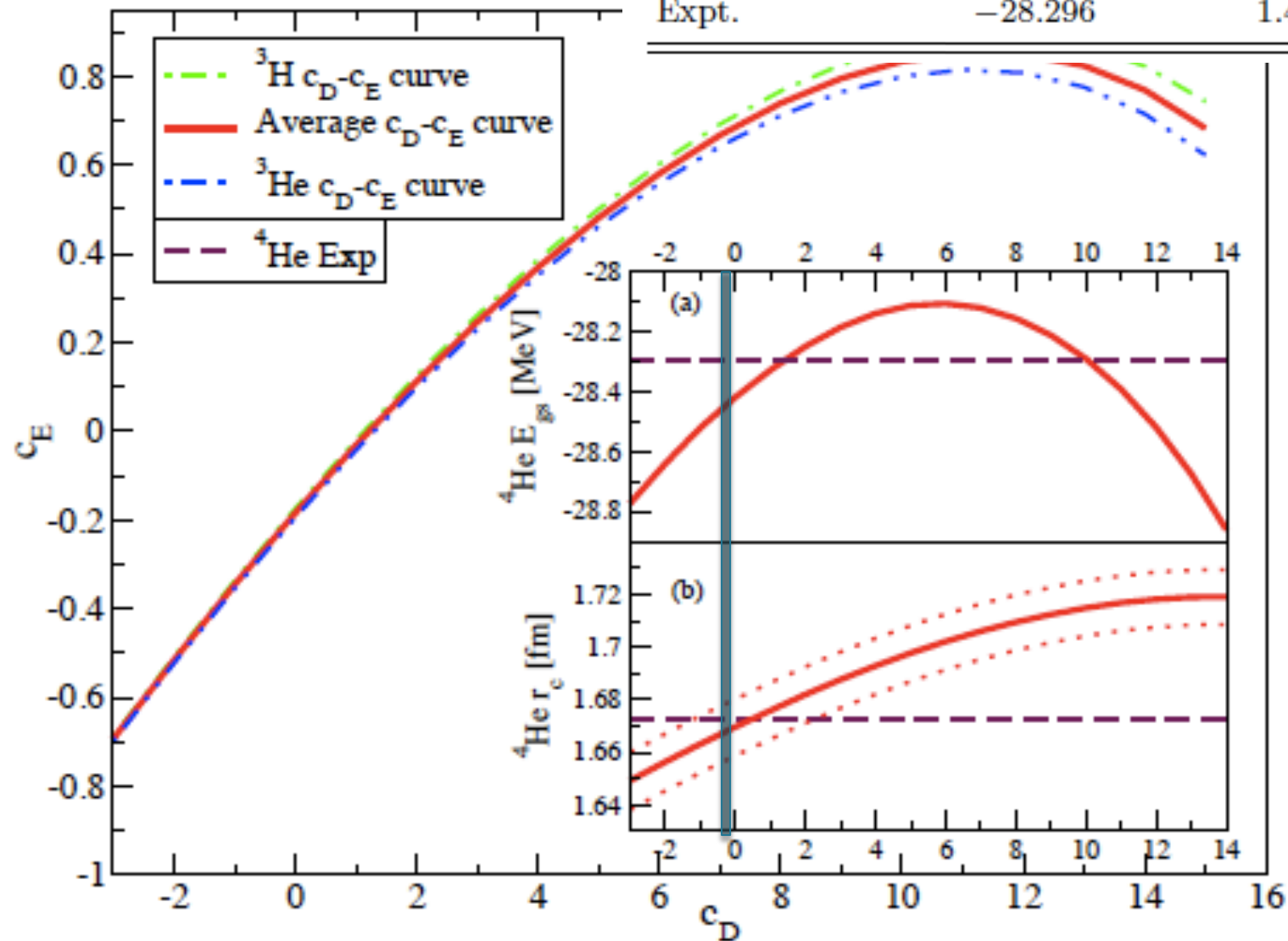
$$c_E \in [-0.220, -0.189]$$

Calibration result



A prediction of ^4He

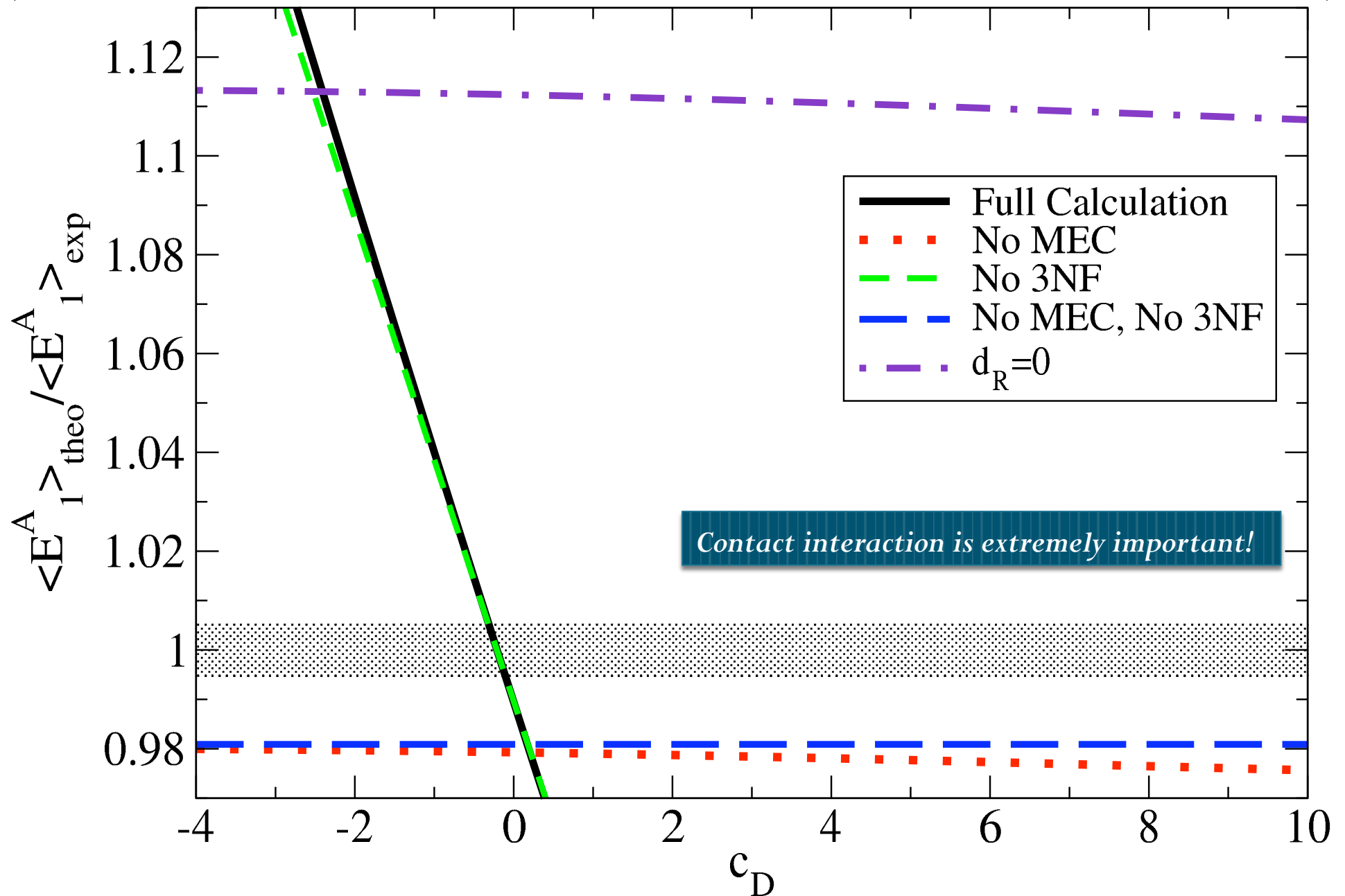
	^4He	
	$E_{\text{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$
NN	$-25.39(1)$	$1.515(2)$
$NN+NNN$	$-28.50(2)$	$1.461(2)$
Expt.	-28.296	$1.467(13)$ [24]



Not all is good yet...

- What is the effect of the missing 3NF diagrams?
- p-shell nuclei seem to suggest $c_D \sim -1$.
 - Renormalizing effect of the missing 3NF?
 - Numerical problems when calculating p-shell nuclei?
- There is still uncertainty, due to poorly known LEC - c_4 :
 - Still has to be checked consistently.
- Checked only with a specific χ PT Force:
 - No cutoff dependence.

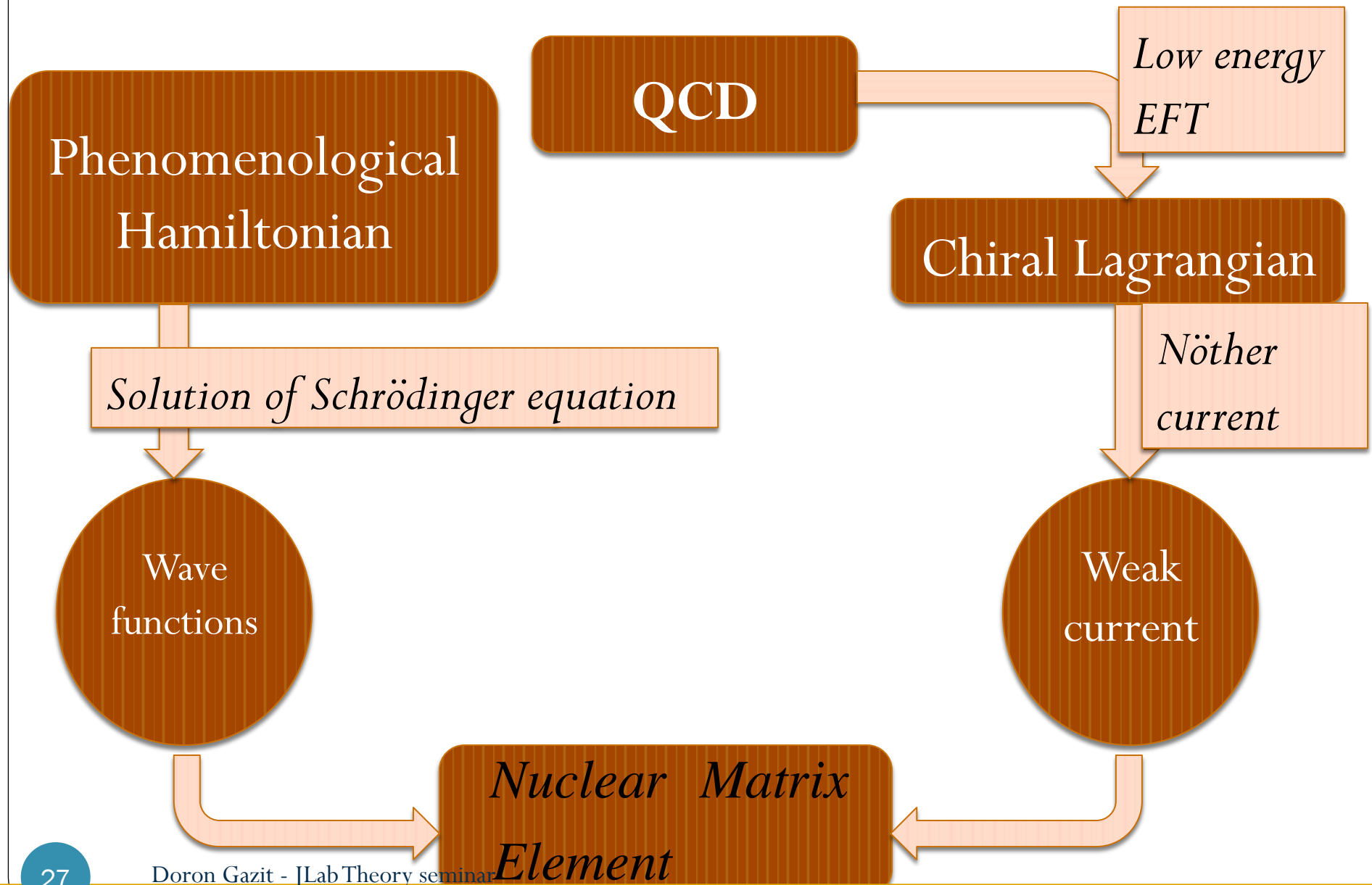
What can we learn about correlations in the wave function?



The apparent conclusion

- For GT type of operators, the short range correlations in the wave functions are not important for the observable.
- The long tail behavior of the potential is more essential.
- Is this the origin of the success of EFT*: hybrid calculations of weak reactions, using phenomenological forces in combination with χ PT based currents?
 - One unknown parameter in MEC (d_R) calibrated using the triton half-life.

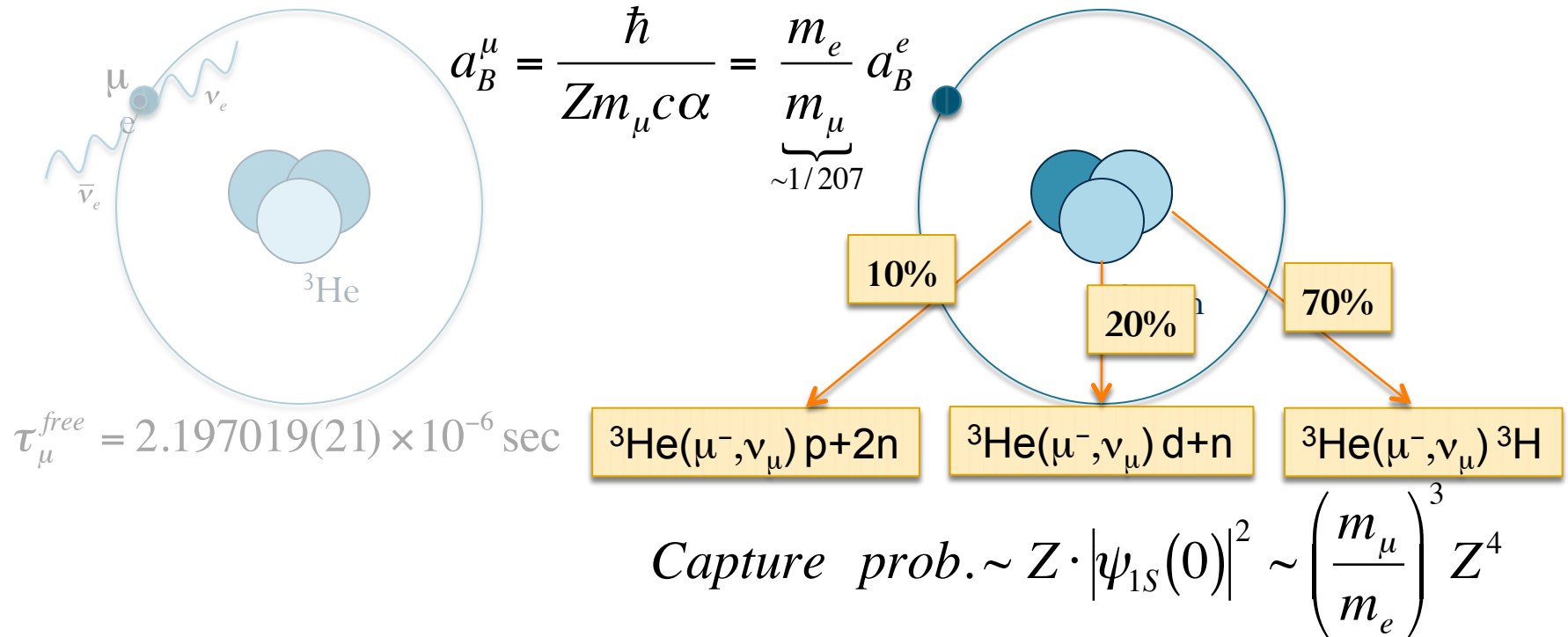
EFT approach for low-energy nuclear reactions:*



Extracting the weak structure of the nucleon from μ -capture on ^3He

DG, Phys. Lett. B **666**, 471 (2008).

The decay of a muonic ${}^3\text{He}$: *competition*



- The rates become comparable for $Z \sim 10$.
- The Z^4 law has deviations – mainly due to nuclear effects.
- In order to probe the weak structure of the nucleon, one has to keep the nuclear effects under control.

Why don't we stay in the single nucleon level?

The MuCap collaboration (PSI) measuring:

$$\Gamma(\mu^- p \rightarrow \nu_\mu n)_{1S}^{\text{singlet}} = 725.0 \pm 13.7_{\text{stat}} \pm 10.7_{\text{syst}} \text{ Hz}$$

Expecting to achieve 1% accuracy.

For the (exclusive) process ${}^3\text{He}(\mu^-, \nu_\mu) {}^3\text{H}$
an incredible measurement ($\pm 0.3\%$):

$$\Gamma(\mu^- + {}^3\text{He} \rightarrow \nu_\mu + t)_{\text{stat}} = 1496 \pm 4 \text{ Hz}$$

A parameter free, percentage level accuracy calculation of the process is a great challenge to nuclear physics – which is now possible!!

MuCap, **Phys. Rev. Lett.** **99**, 032002 (2007).

Ackerbauer *et al*, **Phys. Lett.** **B417**, 224 (1998).

Calculation:

- We take the phenomenological AV18 (NN) and UIX (NNN) nuclear forces.

Method	Binding Energy [MeV]	
	${}^3\text{H}$	${}^3\text{He}$
EIHH	8.471(2)	7.738(2)
CHH	8.474	7.742
FY	8.470	7.738
Experimental	8.482	7.718

$$\Gamma = \left\{ \frac{2G^2 |V_{ud}|^2 E_\nu^2}{2J_{{}^3\text{He}} + 1} \left(1 - \frac{E_\nu}{M_{{}^3\text{H}}} \right) |\psi_{1s}^{av}|^2 \Gamma_N \right\} (1 + RC)$$

$$\Gamma = 1499(2)_\Lambda (3)_{NM} (5)_t (6)_{RC} = 1499 \pm 16 \text{ Hz}$$

$$\Gamma_{EXP} = 1496 \pm 4 \text{ Hz}$$

Constraints on the weak structure of the nucleon from muon capture on ^3He

$$\hat{J}^{\mu V} = \bar{u}(p') \left[\boxed{F_V(q^2)} \gamma^\mu + \frac{i}{2M_N} \boxed{F_M(q^2)} \sigma^{\mu\nu} q_\nu + \boxed{\frac{g_S}{m_\mu}} q^\mu \right] u(p)$$

Vector

Magnetic

Second class currents

$$\hat{J}^{\mu A} = -\bar{u}(p') \left[\boxed{G_A(q^2)} \gamma^\mu \gamma_5 + \boxed{\frac{g_P(q^2)}{m_\mu}} \gamma_5 q^\mu + \boxed{\frac{ig_t}{2M_N}} \sigma^{\mu\nu} \gamma_5 q_\nu \right] u(p)$$

Axial

Induced
Pseudo-Scalar

Induced pseudo-scalar:

- From χ PT [Bernard, Kaiser, Meissner, PRD 50, 6899 (1994); Kaiser PRC 67, 027002 (2003)]:

$$g_P(-0.954m_\mu^2) = 7.99(0.20)$$

- From muon capture on proton [Czarnecki, Marciano, Sirlin, PRL 99, 032003 (2007); V. A. Andreev et. al., PRL 99, 032004(2007)]:

$$g_P(-0.88m_\mu^2) = 7.3(1.2)$$

- This work: $g_P(-0.954m_\mu^2) = 8.13(0.6)$

$$g_P(q^2) = \frac{2m_\mu g_{\pi pn} f_\pi}{m_\pi^2 - q_\mu^2} - \frac{1}{3} g_A m_\mu M_N \langle r_A^2 \rangle = 7.99(20)$$

Induced Tensor:

- From QCD sum rules: $\frac{g_t}{g_A} = -0.0152(53)$
- Experimentally [Wilkinson, Nucl. Instr. Phys. Res. A 455, 656 (2000)]:

$$\left| \frac{g_t}{g_A} \right| < 0.36 \text{ at } 90\%$$

- This work: $\frac{g_t}{g_A} = -0.1(0.68)$

$$\delta J^{\mu A} = \frac{ig_t}{2M_N} \sigma^{\mu\nu} \gamma_5 q_\nu$$

Induced scalar (limits CVC):

- “Experimentally” [Severijns et. al., RMP 78, 991 (2006)]:
$$g_s = 0.01 \pm 0.27$$

- This work:
$$g_s = -0.005 \pm 0.04$$

$$\delta J^{\mu V} = \frac{g_s}{m_\mu} q^\mu$$

Neutrino interaction with $A=3,4$ nuclei in Supernovae

Too small to measure...

Need parameter free calculations...

DG, Barnea **Phys. Rev. C** **70**, 048801 (2004); **Phys. Rev. Lett.** **75**, 192501 (2007); **Nucl. Phys. A** **790**, 356 (2007); **Few Body Syst.** (2008).

O'Connor, DG, Horowitz, Schwenk, Barnea, **Phys. Rev. C** **75**, 055803 (2007).

DG, PhD. thesis, **arXiv: 0807.0216** (2007).

Type II supernovae

- Core collapse supernovae are giant explosions of massive stars.
- 99% of the energy released is carried away by neutrinos, thus the phenomena inside are sensitive to neutrino interactions with matter.
- SNe are the probable site for r-process nucleosynthesis.

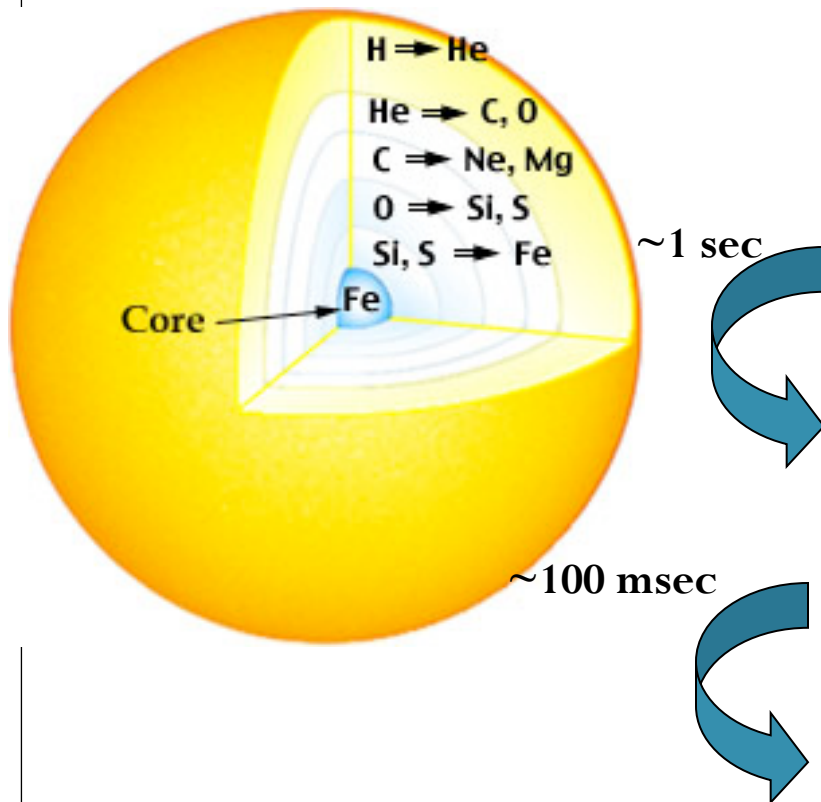


Supernova 1987A

The death of a massive star

“the nuclear physicist paradigm”

After millions of years of evolving...



- Iron peak nuclei don't burn to heavier nuclei
➔ no support to the core mass.

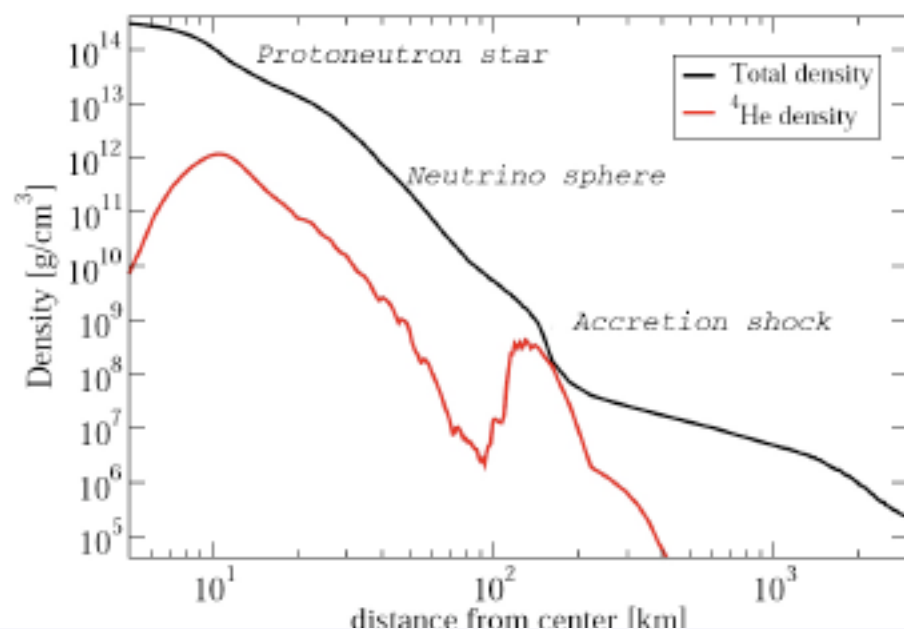
- The core becomes gravitationally unstable ➔ collapses.

- Nuclear forces halt the collapse, and drive an outgoing shock.

- The shock loses energy due to dissociation, neutrino radiation.

- The shock stalls...

Density profile



Nuclear saturation density

$$n_0 \approx 0.16 \text{ fm}^{-3} \\ \sim 10^{14} \text{ g/cm}^3$$

Matter temperature

$$T \approx 0.5 - 5 \text{ MeV}$$

Typical energies for the neutrino flux

$$\langle E_{\nu_e} \rangle \approx 12 \text{ MeV}$$

$$\langle E_{\bar{\nu}_e} \rangle \approx 15 \text{ MeV}$$

$$\langle E_{\nu_\mu} \rangle = \langle E_{\nu_\tau} \rangle \approx 24 \text{ MeV}$$

Post-shock microscopic input to SNe modeling

- EOS – determines the equilibrium composition:
 - Lately it was shown that due to the higher density at the neutrinosphere, this area is dominated by: free nucleons, deuteron, ^3H , ^3He , and ^4He .

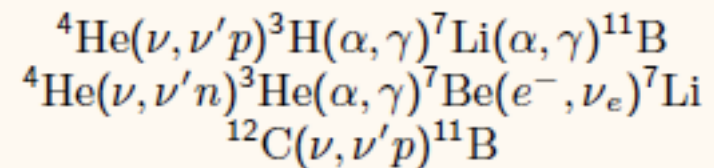
O'Connor, DG, Horowitz, Schwenk, Barnea, **Phys. Rev. C** **75**, 055803 (2007).

Arcones, Martinez-Pinedo, O'Connor, Schwenk, Janka, Horowitz, Langanke.

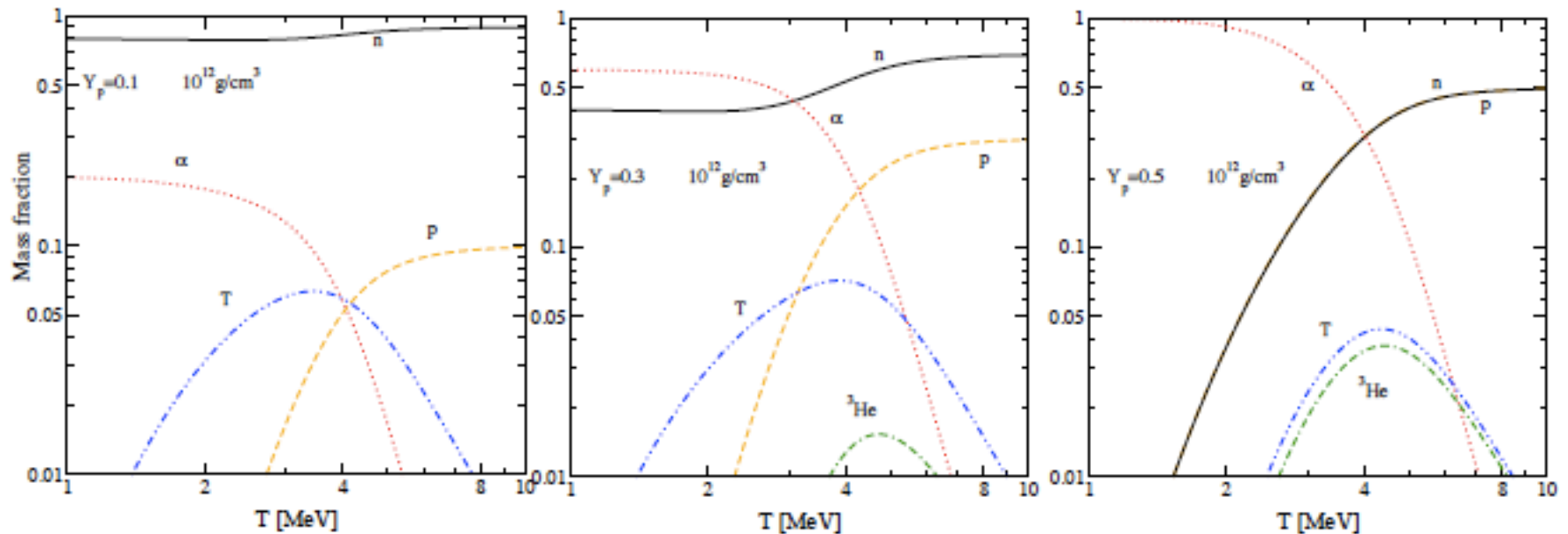
arXiv:0805.3752 (2008).

- Cross-sections for neutrino scattering, especially inelastic scattering:
 - Deposits energy – changes composition and temperature.
 - Change composition – ν nucleosynthesis.

Haxton, **Phys. Rev. Lett** **69**, 1999 (1988), Woosley *et al.*, **Astrophys. J.** **356**, 272 (1990).



Composition near neutrinosphere



Proton fraction

$$Y_p = \frac{n_p + 2n_\alpha + 2n_{^3\text{He}} + n_{^3\text{H}}}{n_b}$$

Mass fraction

$$X_i = \frac{A_i n_i}{n_b}$$

O'Connor, Gazit, Horowitz,
Schwenk, Barnea, PRC **75**,
055803 (2007)

Neutrino cross-sections

First *ab-initio* calculations:

- Within EFT*:
 - AV18+UIX NN+NNN interactions.
 - MEC from HB χ PT
- Theoretical accuracy of about 1%

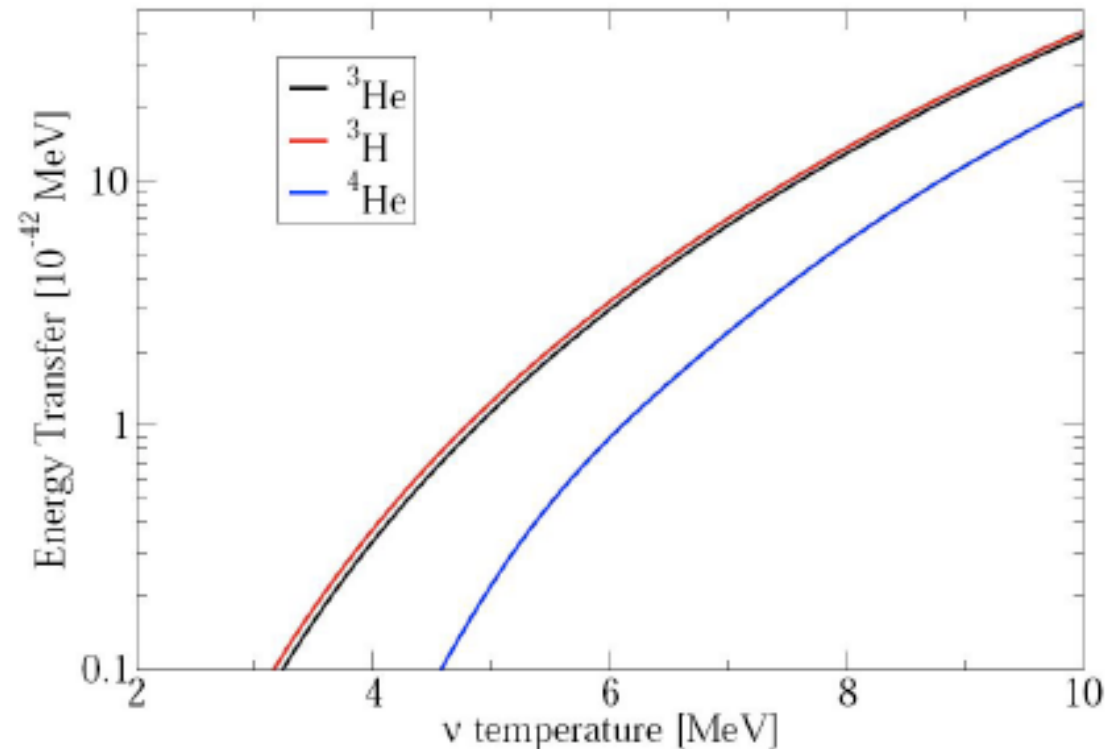


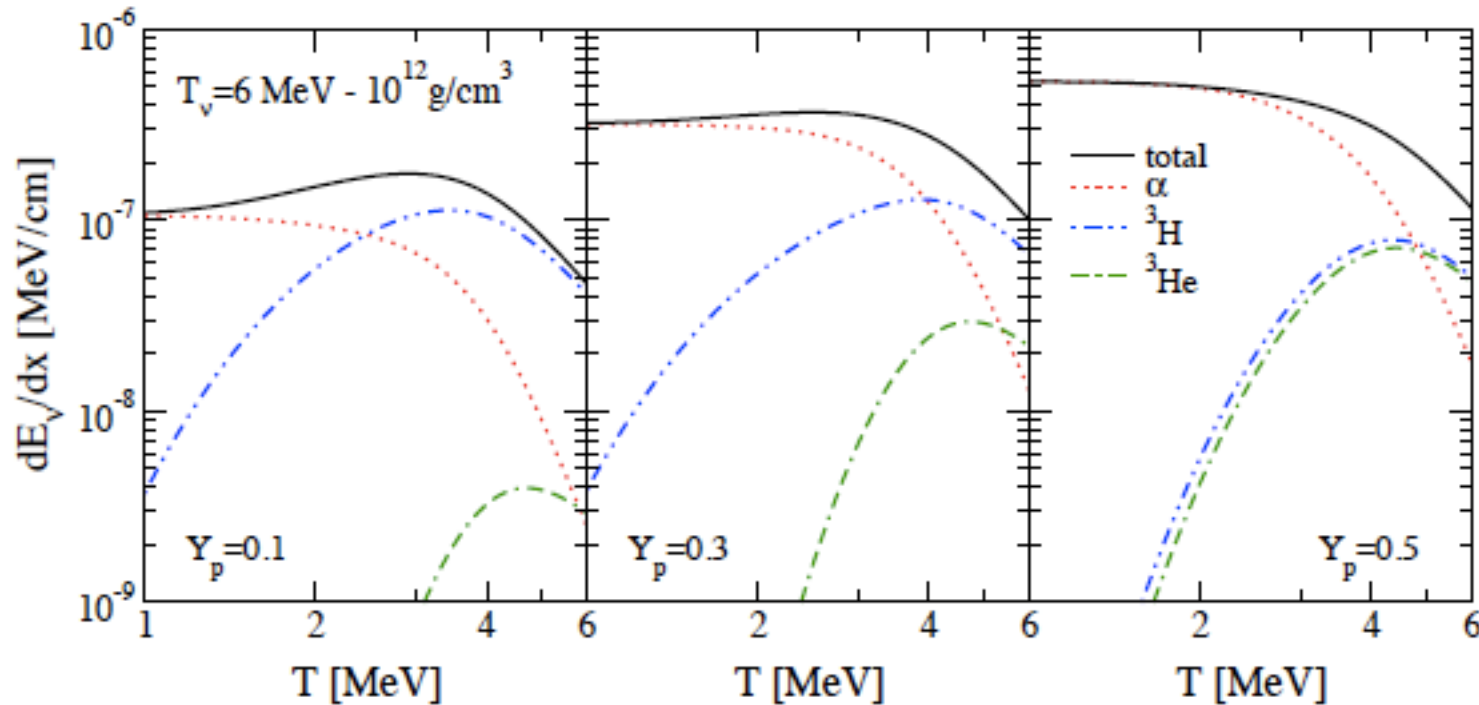
Figure: Temperature averaged neutral current energy transfer cross-section per nucleon

DG, Barnea **Phys. Rev. C** **70**, 048801 (2004); **Phys. Rev. Lett.** **75**, 192501 (2007); **Nucl. Phys. A** **790**, 356 (2007); **Few Body Syst.** (2008).

O'Connor, DG, Horowitz, Schwenk, Barnea, **Phys. Rev. C** **75**, 055803 (2007).

DG, PhD. thesis, **arXiv: 0807.0216** (2007).

Energy deposition near neutrinosphere



Neutrino energy loss dE_ν/dx due to inelastic reactions on ${}^3\text{He}$, ${}^4\text{He}$, T

$$\frac{dE_\nu}{dx} = n_b \sum_{i={}^3\text{H}, {}^3\text{He}, {}^4\text{He}} x_i \langle \omega \sigma \rangle_{i, T_\nu}$$

Summary

- $A=3$ nuclei can have a substantial influence on phenomena near the neutrinosphere.
 - Recent results [Arcones et. al] show a change in the electron antineutrino spectrum due to the $A=3$ nuclei.
- Effects of inelastic ν - ${}^4\text{He}$ reactions on the:
 - explosion mechanism.
 - ν -spectra.
 - ν -nucleosynthesisshould be checked in detailed simulations.

What can we learn from ${}^6\text{He}$ β -decay about the suppression of g_A in nuclear matter?

- Surveys of “experimental Gamow-Teller” shows that $g_A \rightarrow 1$, as A grows.
- This has been related to:
 - Restoration of axial symmetry.
 - Lack of correlations in the calculation.
 - Loop corrections from nucleonic excitations.
 - *Something beyond the standard model?*
- Schiavilla and Wiringa showed that for ${}^6\text{He}$, the suppression is about 4%.
The MEC actually increased the suppression!!
 - A real effect?
 - Problems in VMC?
 - **Problems in the weak current?**

DG, S. Vaintraub, N. Barnea, in preparation (2008).

The JISP16 NN potential

The **JISP16** reproduce the NN phase shifts in the range 0 – 300MeV.

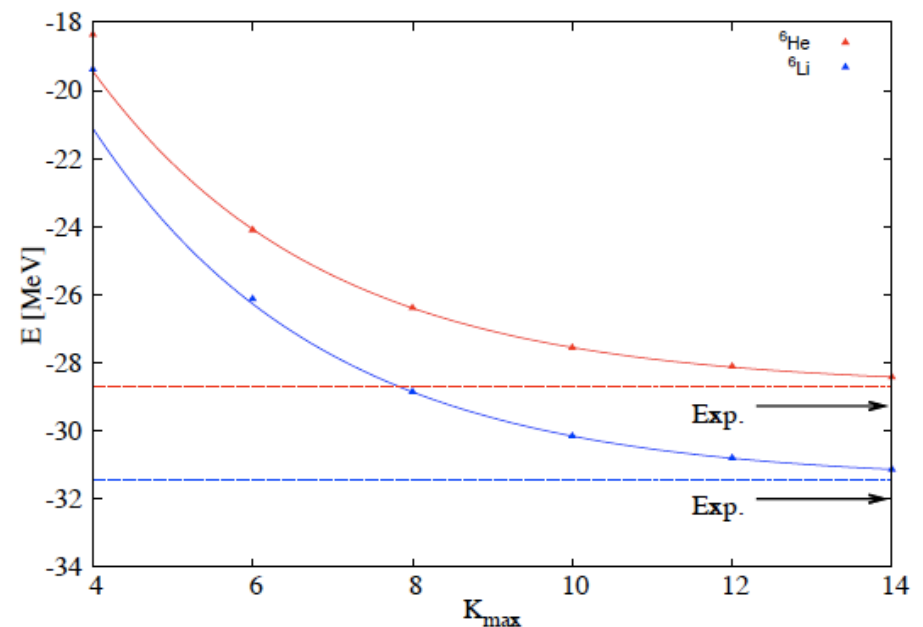
Binding Energies

	AV18+UBIX	JISP16	Nature
D	2.24	2.24	2.24
^3H	8.48	8.35	8.48
^3He	7.74	7.65	7.72
^4He	28.5	28.3	28.3
^6He	-	29.0	29.29
^6Li	-	31.9	31.99

AV18+UBIX - Argonne V18 + Urbana IX

JISP16 - J-matrix Inverse Scattering

Potential, Shirokov *et al.*

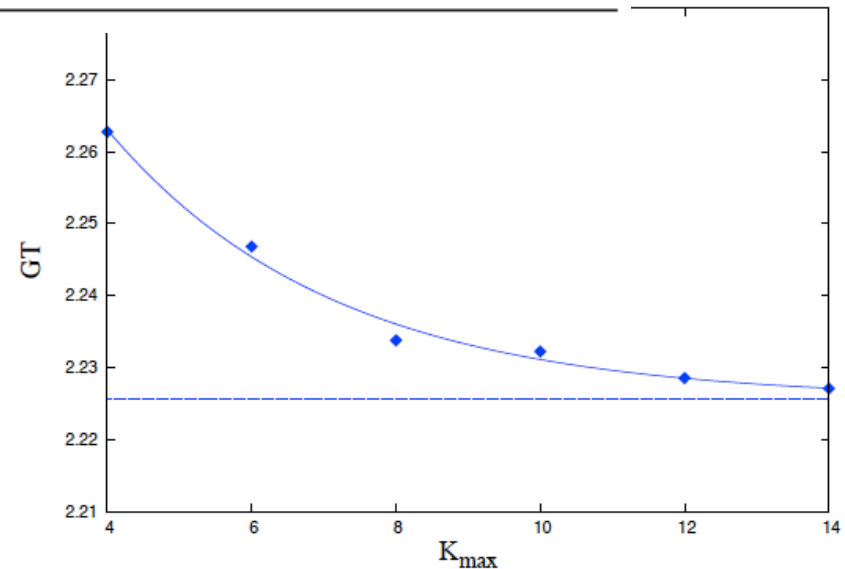


Single nucleon GT strengths

K_{max}	${}^6\text{He}$		${}^6\text{Li}$		$\langle GT \rangle$
	B.E.	radius	B.E.	radius	
4	18.367	1.840	19.392	1.859	2.263
6	24.103	1.902	26.124	1.909	2.247
8	26.392	1.979	28.854	1.984	2.234
10	27.560	2.051	30.156	2.051	2.232
12	28.112	2.112	30.797	2.110	2.229
14	28.424	2.165	31.132	2.160	2.227
∞	28.70(13)		31.46(5)		2.225(2)
[20]	28.32(28)		31.00(31)		
Exp.	29.269	2.18	31.995	2.09	2.170

TABLE I:

Potential model	$\langle GT \rangle$
AV18+3NF [28]	1.598(2)
Bonn+3NF [29]	1.621(2)
Nijm+3NF [30]	1.605(2)
JISP16 [This work]	1.6524(2)
Expt.	1.654(5)



Complete current

Potential	1-Body	Full
AV18/UIX – VMC	2.250(7)	2.281(7)
JISP16	2.225(3)	2.191(3)
Experiment		2.161(4)

$$GT|_{JISP16} = 2.190(4)_{\Lambda}(2)_N(4)_t(6)_{g_A} = 2.190 \pm 0.011$$

Things to think about from ${}^6\text{He}$ β decay

- Is there a qualitative difference between the SNPA based MEC and the EFT based MEC?
- Is this difference a result of the VMC wave functions?
- Is this difference a result of the use of a too simplistic NN potential (JISP16)?
- The current calculation implies no suppression of g_A :

$$\frac{g_A({}^6\text{He})}{g_A(n)} = 0.986 \pm 0.01$$

Weak Interacting Holographic QCD

Using string theory to calculate and constrain low-energy weak reactions in the real world.

DG, Ho-Ung Yee, Phys. Lett. B **670**, 154 (2008).

Large N QCD has a dual classical theory in 5-D?!

- Large N factorization of gauge invariant theories:

$$\langle O_1(x_1)O_2(x_2)\cdots O_n(x_n) \rangle = \langle O_1(x_1) \rangle \langle O_2(x_2) \rangle \cdots \langle O_n(x_n) \rangle + O\left(\frac{1}{N^2}\right)$$

- Implies a classical theory for gauge invariant operators (AKA master fields).
- RG running survives the large N limit, thus the master field is a function of the energy scale:

$$\langle O(x) \rangle(\mu)$$

- The RG equations constrain flow in this scale
- Holographic QCD is a gravitational theory of gauge invariant fields in 5 dimensions.
 - 5th dimension corresponds roughly to the energy scale.

Things that we know

AdS/CFT Duality proposal

$\mathcal{N}=4$ Super Yang-Mills theory in $(3+1)\text{D}$ for $N_c \rightarrow \infty$,
 $g_{\text{YM}} \rightarrow 0$ and fixed but large $\lambda = g_{\text{YM}}^2 N_c$

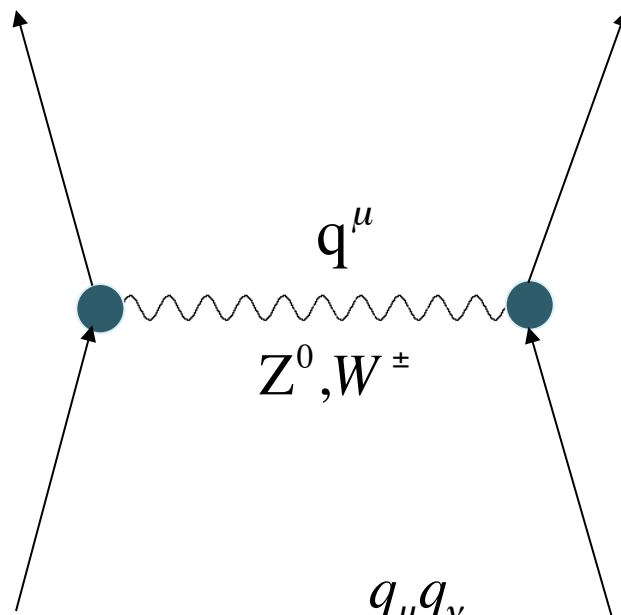
is equivalent to

Type IIB Supergravity in $\text{AdS}_5 \times S^5$ with size $\lambda^{1/4}$

We thus expect the dual theory of QCD...

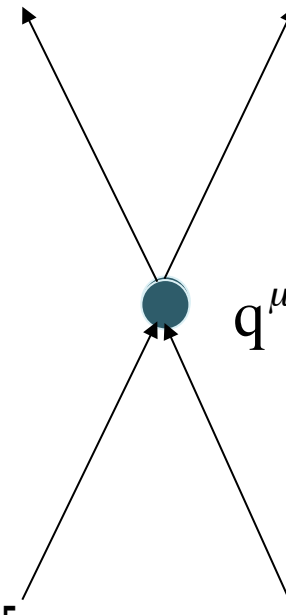
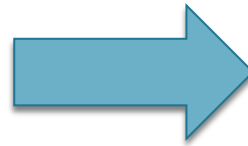
- In the UV regime: highly nonlocal, corresponding to asymptotic freedom.
- In the IR regime: local, corresponding to the strongly correlated QCD.
- Thus, current models of Holographic QCD model the gravitational dual as a local theory.
- Properties of existing models of Holographic QCD:
 - Chiral symmetry.
 - Confinement.
 - Explain experimental observables to 20%.

Low-energy Weak interaction



$$T_{fi} \sim J^\mu \frac{g_{\mu\nu} + \frac{q_\mu q_\nu}{M_B^2}}{q^2 + M_B^2} J^\nu$$

$q \ll M_{W^\pm, Z^0}$



$$\begin{aligned} \mathcal{H}_W &= \frac{4G_F}{\sqrt{2}} \left[J_{W^+} J_{W^-} + \cos^2 \theta_W (J_{Z^0})^2 \right] = \\ &= \frac{4G_F}{\sqrt{2}} \left[\underbrace{(J_L^1)^2 + (J_L^2)^2}_{\text{SU}(2)_L} + (J_L^3 - \sin^2 \theta_W J_Q)^2 \right] \end{aligned}$$

↓
EM

$$J_L^a = \sum_f \bar{\psi}_f \sigma^a \psi_f$$

How to perturb the QCD Lagrangian?

Gauge

- Perturbation to the Lagrangian.
- Single trace operator $\mathcal{O}$.

- A Lagrangian perturbation:

$$\Delta\mathcal{L} = \int d^4x f(x) \mathcal{O}(x)$$

Gravity

- Deforming boundary conditions of field near UV boundary.

- A 5D field, such that:

$$\phi_{\mathcal{O}}(x^\mu, z) \underset{z \rightarrow \infty}{\sim} c_1(x^\mu) z^{-\Delta_-} + c_2(x^\mu) z^{-\Delta_+}$$

- Boundary conditions:

$$c_1(x) = f(x)$$

$$c_2(x) = \langle \mathcal{O}(x) \rangle$$

For a general functional perturbation of
a single trace operator

$$\Delta\mathcal{L} = \int d^4x F[\mathcal{O}(x)]$$

$$c_1(x) = \left. \frac{\delta F[\mathcal{O}]}{\delta \mathcal{O}} \right|_{\mathcal{O} \rightarrow c_2(x)}$$

$$c_2(x) = \langle \mathcal{O}(x) \rangle$$

Implementation

The idea is general enough to implement in any Holographic Model.
We demonstrated on two models:

Top – Down Model: Sakai-Sugimoto Model

Bottom – Up Model: Hard/Soft Wall Model.

How to calculate different reactions?

- Write equation of motion for the global gauge field (i.e. the $U(N_F)$ current).

- Solve it with the *prescribed* boundary conditions.

- If you'd like pions to be involved, do it by gauge fixing A_z .

$$A_z(+\infty) = \frac{1}{\sqrt{\pi K}} \cdot \frac{\pi(x)}{1+z^2}$$

- For reactions that include nucleons, choose a model for baryons, and calculate baryon-pion coupling from the kinetic term, and from magnetic type of couplings:

$$S = i \int d^4x \int d\omega \left[\bar{B} \gamma^M (\partial_M - i A_M) B - m_B(\omega) + C \bar{B} \sigma^{MN} F_{MN} B + \dots \right]$$

Neutron b-decay

Sakai-Sugimoto

$$\mathcal{L}_{\bar{n}pe^{-}\bar{\nu}_e} = \sqrt{2}G_F \left[\bar{n}\gamma_\mu p + \right. \\ \left. + g_A \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \bar{n}\gamma^\nu \gamma_5 p - \right. \\ \left. - i(0.84) \bar{n}q^\nu \sigma_{\mu\nu} p \right] \cdot (\bar{\nu}_L \gamma^\mu e_L)$$

- With:

$$g_A = 1.3$$

Hard/Soft wall model

$$\mathcal{L}_{\bar{n}pe^{-}\bar{\nu}_e} = \sqrt{2}G_F \left[\bar{n}\gamma_\mu p + \right. \\ \left. + g_A \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \bar{n}\gamma^\nu \gamma_5 p - \right. \\ \left. - i(0.48) D \bar{n}q^\nu \sigma_{\mu\nu} p \right] \cdot (\bar{\nu}_L \gamma^\mu e_L)$$

- With:

$$g_A = 0.33 + 1.02D$$

$$g_A^{\text{exp}} = 1.2695(29)$$

Parity non-conserving pion-nucleon coupling

- First example without an external source.
- We are interested in parity violating couplings of mesons to the nucleons.
- To this end, we consider only charged pion-nucleon coupling.
- In both models, the result in the zero q limit is identical to the current algebra result:

$$L_{N-\pi}^{weak} = -2G_F f_\pi (\bar{p} \gamma^\mu n) (\partial_\mu \pi^+)$$

- Still, a lot to be done!

Summary

- This is a prescription to include weak interactions in the framework of holographic QCD.
- Applicable up to energies of a few GeV, when strong coupling is still valid.
- We have shown its strength by using Sakai-Sugimoto and Hard/Soft wall models to calculate few exemplar reactions.
- The current approach, contrary to other approaches (such as χ PT), gives not only the operator structure, but the numerical coefficients, to about 20%, and valid for energies above the chiral limit.

Final Remarks

- Electro-weak reactions with light nuclei:
 - Can be used to study the basic symmetries of QCD.
 - Are involved in stellar evolution, supernovae, and other astrophysical phenomena.
 - Provide a hatch to the properties of heavier nuclei.
- Parameter free calculations, which will be done within χ PT, would be able to constrain these observables.
 - For that, a microscopic calculation of LECs is needed.
- A lot to do...

Collaborators and Other helps

- Trento: G. Orlandini, W. Leidemann
- TRIUMF: S. Bacca, A. Schwenk
- ICTP: H.-U. Yee
- HUJI: E. Livne, N. Barnea, S. Vaintraub
- LLNL: S. Quaglioni, P. Navratil