Quantum state detection of Ba-138 ion Qubit

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Barium ion is an actively studied qubit candidate for quantum computers, hence it is important to study how to make accurate and efficient measurements of Barium ions. One problem that arises in measuring the state of a Barium ion, which utilizes state dependent fluorescence, is that it enters a dark state where the photons cannot interact with the ion and fluoresce. In this paper, we simulate a Barium ion under state dependent fluorescence to investigate how short alternating pulses of laser beams can break the ion out of a dark state and potentially improve its measurement process in a novel way.

I. INTRODUCTION

Since the creation of the quadruple ion trap by Wolfgang Paul, researchers have been studying how to use trapped ions to build a viable quantum computer. One important criterion for constructing a quantum computer is a qubit-specific measurement capability [1]. Measurement is a fundamentally important process in quantum computation and thus, it is crucial to make the measurement process as accurate and efficient as possible. In the case of trapped ion computing, the quantum state is measured by state dependent fluorescence. Achieving high fidelity with the state dependent fluorescence measurement method is thus of big interest in the field of trapped ion quantum computation.

Of the many candidate ions that can be used as a qubit, our study focuses on using a singly ionized Barium 138 ion. There are many advantages to the Ba-138 qubit such as the existence of a metastable excited state and the fact that visible wavelength lasers drive electric-dipole transitions. When measuring the quantum state of Ba-138 ion, typically two lasers illuminate the ion and the number of scattered photons of one specific wavelength is measured. One problem in this measurement process is that when two lasers that each drive separate transitions are shone on the ion, the ion can get trapped into a so-called 'coherent dark state' [2]. In this state, the ion is virtually transparent to the lasers, not absorbing or emitting any photons. State dependent fluorescence leverages the fact that, on average, the number of photons scattered depends on the ion's initial state. Hence, if an ion whose initial state is supposed to fluoresce a number of photons gets stuck in a coherent dark state during the measurement process, the measurement becomes very inefficient.

In this study, we investigate a method to enhance the quantum state detection process for Barium-138 ions by simulating their quantum systems under state dependent fluorescence. Specifically, we investigate how alternating laser pulses can be a novel method to break the ion out of a coherent dark state and increase the ion's average probability of being in the $6P_{1/2}$ state through a non-steady state evolution. An increased probability of Ba-138 ion being in the $6P_{1/2}$ state directly increases the number of 493 nm photons scattered and is thus expected to improve the measurement fidelity for Barium ion qubits.

II. BACKGROUND INFORMATION

A. Structure of the Barium ion

Orbitals in an atom has a specific energy and angular momentum associated with it. A singly ionized Barium 138 ion has one valence electron in the $6S_{1/2}$ [1] orbital. The electron in the $6S_{1/2}$ ground state can then be excited into either the $5D_{5/2}$ orbital or the $6P_{1/2}$ orbital. In each of these excited states, the electron can spontaneously decay back into the ground state. From the $6P_{1/2}$ state, the electron decays back to the ground state in an average lifetime of approximately 7 ns [3] whereas in the $5D_{5/2}$ state, the electron has a lifetime of 35 s [4]. This is significantly longer than the time scales for trapped ion quantum gate operations. For this reason the $6S_{1/2}$ and $5D_{5/2}$ orbitals are used as the coherent stable states for which we apply quantum operations. In other words, they act as the 0-bit and 1-bit of the qubit. After performing computations via quantum gates, measurement of the Barium qubit is performed by state dependent fluorescence.

B. Evolution of Ba ion under external field

To understand state dependent fluorescence measurements, it is crucial to understand how radiation affects the quantum system of an ion. When an ion is exposed to an oscillating external electric field, i.e. radiation, with angular frequency ω, there is an energy associated with the alignment of the ion's electric dipole moment with respect to the field. If the angular frequency ω is

[1] Ion state notation: 1st number is the energy level n; 2nd letter is the orbital angular momentum l; subscripted number is the total angular momentum j.

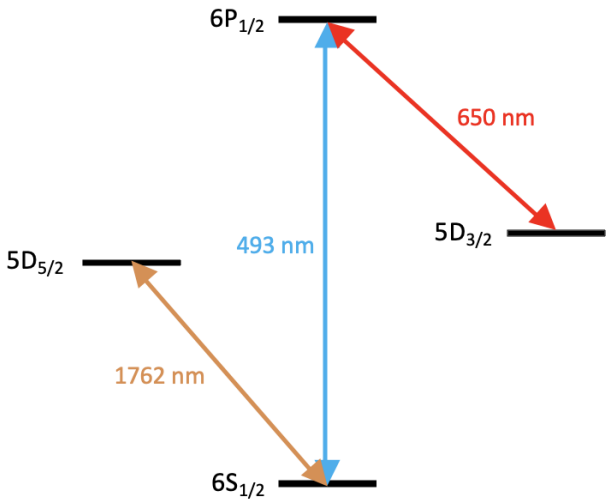

FIG. 1: Structure of the Barium-138 ion

near the resonant frequency[2], the ion can undergo Rabi oscillations via an electric-dipole transition. During a Rabi oscillation, the ion changes its state periodically between the two stationary states related to the resonant frequency in a sinusoidal manner [5]. The hamiltonian represented below is responsible for the time evolution of a Rabi oscillation [6].

$$\hat{H}_I(t) = \vec{r}\cdot\vec{E}_0\cos(\omega t) = \frac{\hbar}{2}\Omega\left(e^{i\omega t}\left|0\right\rangle\left\langle 1\right| + e^{-i\omega t}\left|1\right\rangle\left\langle 0\right|\right) \tag{1}$$

The Rabi frequency, $\Omega = \frac{eE_0}{\hbar}\left\langle 0\right|\vec{r}\cdot\vec{e}_{\text{rad}}\left|1\right\rangle$, is a value related to the coupling between the two states, in this example, $|0\rangle$ and $|1\rangle$. It also describes how quickly the ion will transition between the states. In other words, it is the frequency of the ion's periodic oscillation between the two states.

C. State Dependent Fluorescence

Typically, to measure the state of a barium ion, a 493 nm laser whose frequency corresponds to the energy difference between the $6S_{1/2}$ and $6P_{1/2}$ states illuminates the ion to initiate a Rabi oscillation between the two states. While the ion goes through a Rabi oscillation between the two states, it spontaneously emits a 493 nm photon in a random direction. If the ion was prepared in the $5D_{5/2}$ state, it will not be excited by the 493 nm laser nor will it scatter any photons. By measuring the number of scattered 493 nm photons, it can be determined whether the ion was in the $6S_{1/2}$ ground state (i.e. the 0-bit) or the $5D_{5/2}$ excited state (i.e. the 1-bit). One complication, though, is that from the $6P_{1/2}$ state the electron can also spontaneously decay into the $5D_{3/2}$ state. The rate of each decay processes are at a comparable scale having a branching ratio of approximately 3:1. To prevent from the electrons being optically pumped into the $5D_{3/2}$ state, a 650 nm laser illuminates the ion to initiate a Rabi oscillation between the $5D_{3/2}$ and $6P_{1/2}$ state.

Theoretically, if the ion was initially prepared in the 1-bit $5D_{5/2}$ state, no scattered 493 nm photons must be measured. However, due to noises and other imperfections in the measurement process, some scattered photons are detected. Because the spontaneous decay process is totally random, the number of scattered 493 nm photons may be different even for two same ions exactly prepared in the same state. Hence, the ion state measurement process is repeated a number of times for initially known states of ions to populate a histogram. The histogram consists of two distributions overlapped, each corresponding to the photon count of different initial states. This histogram is used as a guideline for determining which number of scattered photons observed corresponds to which initial state of the ion. The more separated these distributions are, the more certainty it gives to the state dependent fluorescence measurement. To improve this certainty, it is thus important to make the ion scatter as many 493 nm photons as possible when initially prepared in the 0-bit $6S_{1/2}$ state. This means our goal is to maximize the ion's average probability of being in the $6P_{1/2}$ state while it goes through a series of transitions between the $6S_{1/2}$, $6P_{1/2}$, and $5D_{3/2}$ states.

D. Zeeman Splitting

In the presence of a magnetic field, an anomalous Zeeman effect splits the Ba-138 ion's energy levels according to the projection of the total angular momentum of each orbital onto the quantization axis[3]. Due to Zeeman splitting, the Ba-138 ion becomes a 8-level quantum system as seen in FIG. 2.

In this fine structure of Ba-138 ion, the allowed transitions are governed by the selection rule, which can be understood by conservation of angular momentum. A photon has one quantum of angular momentum, so while interacting with a photon, an ion can either gain or lose one quantum of angular momentum. An ion's total angular momentum is a sum of its orbital angular momentum and electron spin angular momentum. Since an electron's spin value does not change, an ion interacting with a

[2] Frequency corresponding to the energy difference between two levels within the atom.

[3] Axis along which the magnetic field is aligned. Typically chosen to be the z-axis

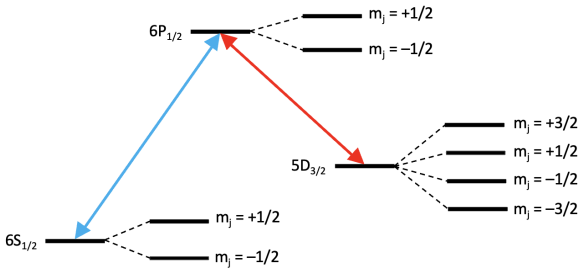

FIG. 2: Structure of the 8-level Barium-138 ion

photon must gain or lose one quantum of orbital angular momentum ($\Delta j = \pm 1$). The Photon's projection of angular momentum onto the quantization axis m_j is either +1, 0, or -1, and this quantity is also conserved. The m_j value of the photon depends on its polarization. A right-handed circularly polarized light propagating along the quantization axis drives a $\sigma+$ transition, where $\Delta m_j = +1$. A left-handed circularly polarized light propagating along the quantization axis drives a $\sigma-$ transition, where $\Delta m_j = -1$. Lastly, a light polarized along the quantization axis drives a π transition where $\Delta m_j = 0$.

III. RESULTS

To investigate the state dependent fluorescence process of a Barium ion qubit, its quantum system is simulated using Qutip in python. Since the ion is interacting with an external radiation and vacuum, which causes spontaneous emission, it is an open quantum system whose dynamics are described by the Lindblad Master equation [7]. To solve the Ba ion state evolving over time, we use the qutip.mesolve() method.

For a quick survey of the effectiveness of alternating laser pulses in improving the measurement, we start off by simulating a simpler 3-level quantum system of Ba-138 ion consisting of the $6S_{1/2}$, $6P_{1/2}$, and $5D_{3/2}$ states without Zeeman splitting (subsection III A). Then, we move on to simulating the full 8-level system of Ba-138 ion with Zeeman splitting (subsection III B).

The following values were used for the simulation. The decay rates were calculated by taking the inverse of the lifetime whose values were taken from the National Institute of Standards and Technology [8]. The Rabi frequencies were approximated by calculating the intensity of the laser at its beam waist.

A. 3-Level system of Ba ion

For the 3-level Ba ion qubit, the ion is initialized in the $6S_{1/2}$, 0-bit, state. As seen in FIG. 3, when both lasers are turned on the ion gradually enters into a steady state where the probability of being in the $6P_{1/2}$ state approaches zero, confirming the presence of a coherent dark state for a 3-level Ba ion system.

TABLE I: Decay rates and Rabi frequency values used in the Simulation

Decay Rate (μs^{-1})	
$6P_{1/2} \rightarrow 6S_{1/2}$	95.3
$6P_{1/2} \rightarrow 5D_{3/2}$	31.0
Rabi Frequency (μs^{-1})	
493 nm laser	258
650 nm laser	148

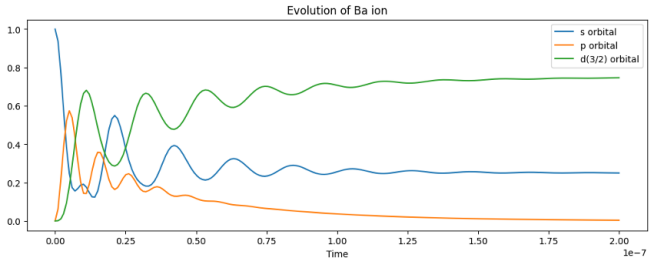

FIG. 3: Simulated time evolution of the 3-level quantum system of Ba-138 ion over a 200 ns time period with both 493 nm and 650 nm resonant lasers on. $6S_{1/2}$, $6P_{1/2}$, and $5D_{3/2}$ states are abbreviated as s orbital, p orbital d(3/2) orbitals, respectively. The vertical axis shows the probability of the ion occupying each orbital.

Next, we incorporated an alternating sequence of 493 nm and 650 nm laser pulses into the simulation. In all cases, the alternating sequence starts with the 492 nm laser pulse first and the two laser pulses have no overlap between each other. FIG. 4 shows how different intervals for each laser pulse affect the average probability of the ion being in the $6P_{1/2}$ state. Three different optimization algorithms were employed for comparison. The most optimal point was determined to be a 43.74 ns pulse of 493 nm laser alternating with a 26.64 ns pulse of 650 nm laser resulting in an average probability of 0.254 in the $6P_{1/2}$ state. FIG. 5 shows the non-steady state evolution of the 3-level Ba ion system with the optimal alternating laser pulses identified from FIG. 4, demonstrating how alternating laser pulses can break the ion from its coherent dark state.

B. 8-Level system of Ba ion

For the 8-level Ba ion qubit, the ion is initialized in the $6S_{1/2}, m_j = -\frac{1}{2}$ state. The hamiltonian evolving the 8-level Ba ion qubit system is shown in appendix A1. While the magnetic field is along the z-axis, both lasers propagate in the y-direction, being linearly polarized in

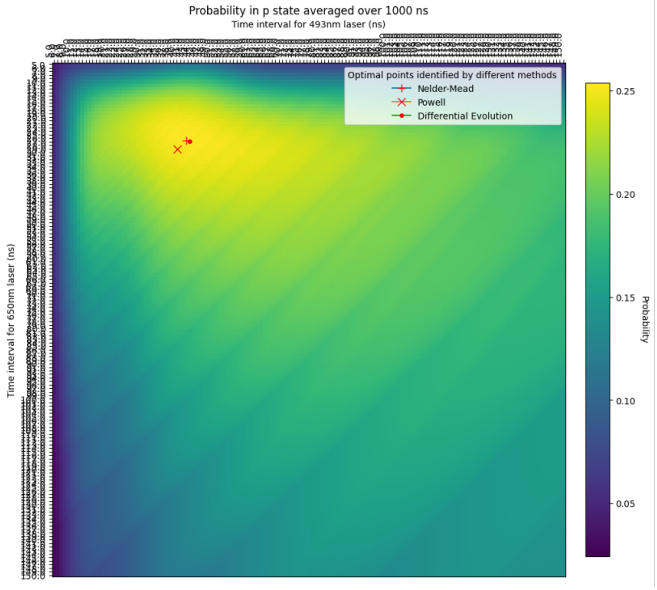

FIG. 4: Heat map showing the average probability of the 3-level Ba ion being in the $6P_{1/2}$ state while varying the interval lengths of the two laser pulses from 5 to 150 ns. Optimal points identified by 3 different optimization algorithms from the numpy module are each marked with distinct red symbols.

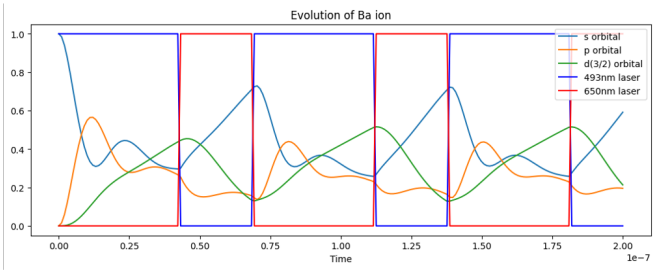

FIG. 5: Simulated time evolution of the 3-level Ba-138 ion over a 200 ns time period with alternating 493 nm and 650 nm laser pulses at identified optimal intervals–43.74 ns of 493 nm laser pulse followed by 26.64 ns of 650 nm laser pulse.

the xz-plane. The polarization of each laser is expressed with the following equations.

$$\vec{E}_{493nm} = \begin{bmatrix} \sin\alpha \\ 0 \\ \cos\alpha \end{bmatrix}, \vec{E}_{650nm} = \begin{bmatrix} \sin\beta \\ 0 \\ \cos\beta \end{bmatrix} \tag{2}$$

While keeping the same lengths of alternating laser pulses, we varied the polarization angles, α and β, from 0 to π to see the effects of polarization on the average probability in the p state. FIG. 6 shows that for all polarization angle of the lasers, the probability[4] in the $6P_{1/2}$ state approaches a value of 0.05437, differing only in the scale of 10^{-6}, which is appreciably close to zero.

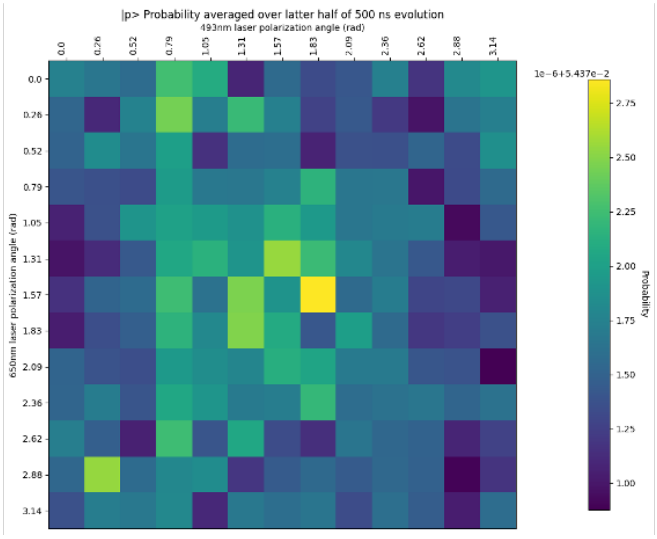

FIG. 6: Probability of 8-level Ba ion being in the $6P_{1/2}$ state over a 500 ns time evolution while varying the polarization angles of the laser from 0 to π. The horizontal axis is the polarization angle for the 493 nm laser and the vertical axis is the polarization angle for the 650 nm laser.

Next, with an arbitrary choice of $\alpha = \pi/2$ and $\beta = \pi/4$, we used the same optimization algorithm used from subsection III A to figure out if there is a new optimal set of alternating laser pulse intervals. Due to limited technological capacity, we were not able to generate a heat map, but the algorithm identified a 39.6 ns of 493 nm laser pulse alternating with a 81.7 ns of 650 nm laser pulse as an optimal sequence. This optimal sequence raised the probability in the $6P_{1/2}$ state to 0.0688 but was still a value close to zero. By plotting the actual evolution of the 8 level ion with the optimal sequence identified, shown in FIG. 7, we were able to infer that despite the 493 nm laser and 650 nm laser alternating, the 8-level Ba ion was entering another dark state where the average probability of the $6P_{1/2}$ state approaching zero.

Our hypothesis was that the different transitions occurring between the $6P_{1/2}$ state and the $5D_{3/2}$ state, namely the sigma transition and the pi transition, were interfering each other to create a coherent dark state optimally pumping the ions into the $5D_{3/2}$ state. To verify this hypothesis, we set up the 81.7 ns of 650 nm laser pulse to be divided into a pulse of x-polarized laser that runs a σ-transition (an equal linear combination of both

[4] Note that in this subsection, all the average probabilities are calculated by taking the average of the latter half of the P state values over total time evolution to put an emphasis on the end behavior of the evolution.

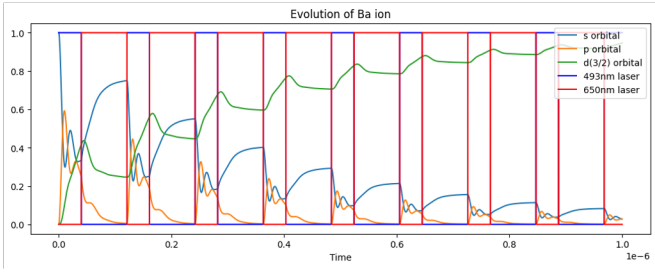

FIG. 7: Simulated time evolution of the 8-level quantum system of Ba-138 ions over a 1000 ns time period. The laser amplitude and probability in each state is shown in the y-axis with color lines indicated in the legend.

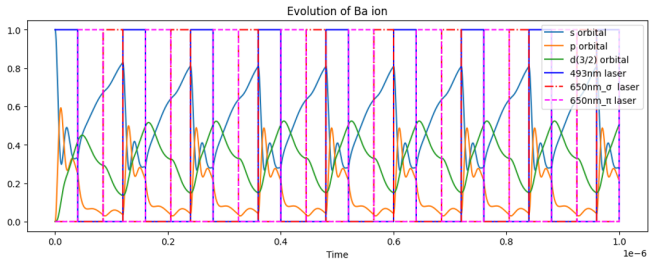

FIG. 9: Simulated time evolution of the 8-level Ba-138 ions over a 1000 ns time period. The amplitude of each laser pulses and the probability in each state is shown in the y-axis with color lines indicated in the legend.

$\sigma+$ and $\sigma-$ transitions) followed by a pulse of z-polarized laser that runs a π-transition. Then, we set up a optimization algorithm to find the most optimal length of the z-polarized laser pulse (FIG. 8). It was identified that a 35.90 ns of x-polarized laser pulse followed by a 44.10 ns of z-polarized laser pulse maximized the probability in the $6P_{1/2}$ state to be 0.162. FIG. 9 shows the simulated non steady state evolution of the 8-level Ba ion at the optimal laser sequences identified.

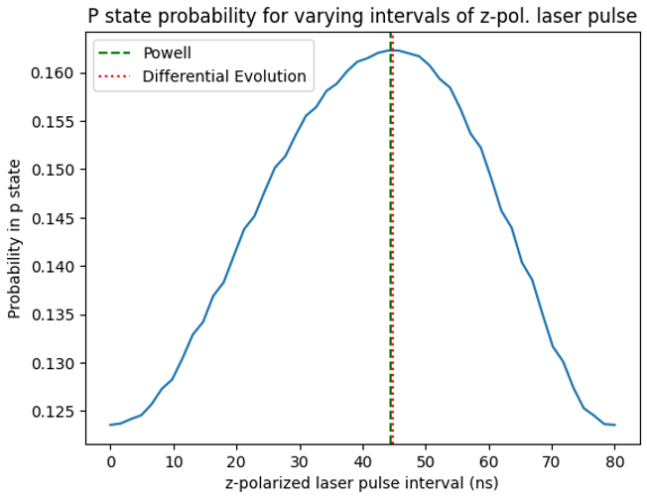

FIG. 8: Average probability of the 8-level Ba ion being in the $6P_{1/2}$ state while varying the intervals of the differently polarized 650 nm laser pulses. Horizontal axis shows the length of the z-polarized 650 nm laser pulse. The length after subtracting the z-polarized interval length from 80 ns is the length of the x-polarized laser pulse. The horizontal lines show the optimal interval lengths identified by different optimization methods.

IV. DISCUSSION

Our simulation results have shown that alternating laser pulses, instead of two laser beams that drive different transitions turned on at the same time, is able to prevent the Ba ion from entering into a coherent dark state. By finding an optimal sequence of alternating laser pulses, we were able to maximize the probability in the $6P_{1/2}$ state to 0.254 for 3-level Ba ions and 0.162 for 8-level Ba ions. If experimental results support these results, the alternating laser pulse technique is expected to enhance the measurement process of Ba ion qubits and could potentially improve general performance of other quantum computers made of ions that rely on state dependent fluorescence measurements. One challenge to verifying these results experimentally is creating a sequence of short alternating laser pulses of different polarizations.

Apart from future experimental work needed, some points of questions still remain about the results and simulations presented in this paper. Compared to the dark state created for the 3-level Ba ions when both 493 nm and 650 nm lasers are on at the same time, the dark state created for the 8-level Ba ions shown in FIG. 7 is less understood. If our hypothesis that the σ-transitions and π-transitions driven by different polarizations of the 650 nm laser creating a dark state was correct, we can expect a similar dark state being generated by different transitions driven by the 493 nm laser. However, for all linear polarization of 493 nm laser, no dark state was created where the ion was being pumped into the $6S_{1/2}$ state. Furthermore, by alternating the x-polarized and z-polarized laser beams of 650 nm laser, we prevented from σ-transitions and π-transitions occurring at the same time, but the x-polarized laser beam was already an even superposition of both $\sigma+$ and $\sigma-$ transitions. This leads us to question why the $\sigma+$ and $\sigma-$ transitions would not interfere to create a dark state. Overall, these questions raise our need to study and better understand the dark states created in the 8-level Ba ion system.

One way to further study this system would be mathematically analyzing the hamiltonian of the 8-level Bar-

ium ion for differently polarized lasers. Solving for the eigenstates of the total hamiltonian would reveal some insight into the dark states we observe in the simulations. Another way to study would be further improving the simulation so that the laser polarizations are specific enough to control each transitions ($\sigma+, \sigma-$, and π transitions) and see how the ion evolves differently under different laser combinations. Furthermore, in actual state dependent fluorescence measurement experiments, it is common to use elliptically polarized light. Some experiments that use such polarization have shown high fidelity in their measurements proving the absence of dark states. Hence, developing simulations with elliptical polarizations would also be a point of interest.

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Appendix A: 8-level Hamiltonian

$$H = \begin{bmatrix}
\Delta_{493} - u & 0 & \frac{\Omega_{493}}{\sqrt{3}}\cos\alpha & -\frac{\Omega_{493}}{\sqrt{3}}\sin\alpha & 0 & 0 & 0 & 0 \\
0 & \Delta_{493} + u & -\frac{\Omega_{493}}{\sqrt{3}}\sin\alpha & -\frac{\Omega_{493}}{\sqrt{3}}\cos\alpha & 0 & 0 & 0 & 0 \\
\frac{\Omega_{493}}{\sqrt{3}}\cos\alpha & -\frac{\Omega_{493}}{\sqrt{3}}\sin\alpha & -\frac{1}{3}u & 0 & -\frac{\Omega_{650}}{2}\sin\beta & -\frac{\Omega_{650}}{\sqrt{3}}\cos\beta & \frac{\Omega_{650}}{2\sqrt{3}}\sin\beta & 0 \\
-\frac{\Omega_{493}}{\sqrt{3}}\sin\alpha & -\frac{\Omega_{493}}{\sqrt{3}}\cos\alpha & 0 & \frac{1}{3}u & 0 & -\frac{\Omega_{650}}{2\sqrt{3}}\sin\beta & -\frac{\Omega_{650}}{\sqrt{3}}\cos\beta & \frac{\Omega_{650}}{2}\sin\beta \\
0 & 0 & -\frac{\Omega_{650}}{2}\sin\beta & 0 & \Delta_{650} - \frac{6}{5}u & 0 & 0 & 0 \\
0 & 0 & -\frac{\Omega_{650}}{\sqrt{3}}\cos\beta & -\frac{\Omega_{650}}{2\sqrt{3}}\sin\beta & 0 & \Delta_{650} - \frac{2}{5}u & 0 & 0 \\
0 & 0 & \frac{\Omega_{650}}{2\sqrt{3}}\sin\beta & -\frac{\Omega_{650}}{\sqrt{3}}\cos\beta & 0 & 0 & \Delta_{650} + \frac{2}{5}u & 0 \\
0 & 0 & 0 & \frac{\Omega_{650}}{2}\sin\beta & 0 & 0 & 0 & \Delta_{650} + \frac{6}{5}u
\end{bmatrix} \tag{A1}$$

The figure above is a matrix representation of the hamiltonian evolving the 8-level Barium ion used in the simulations for subsection III B. This hamiltonian was adapted from a dissertation by Sosnova [9]. Δ stands for the detuning of the laser from the resonant frequency. The detuning values were taken to be half the line-width which is the same as half the decay rate for each transition. Ω stand for the rabi frequency of the laser. u stands for the Bohr magneton multiplied by 1 gauss of magnetic field strength.

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