STRANGE STARS

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Natural Units

$$\boxed{\hbar = c = 1}$$

$$\text{Mass, Energy, Momentum} \Rightarrow MeV$$

$$\text{Force} \Rightarrow MeV^2$$

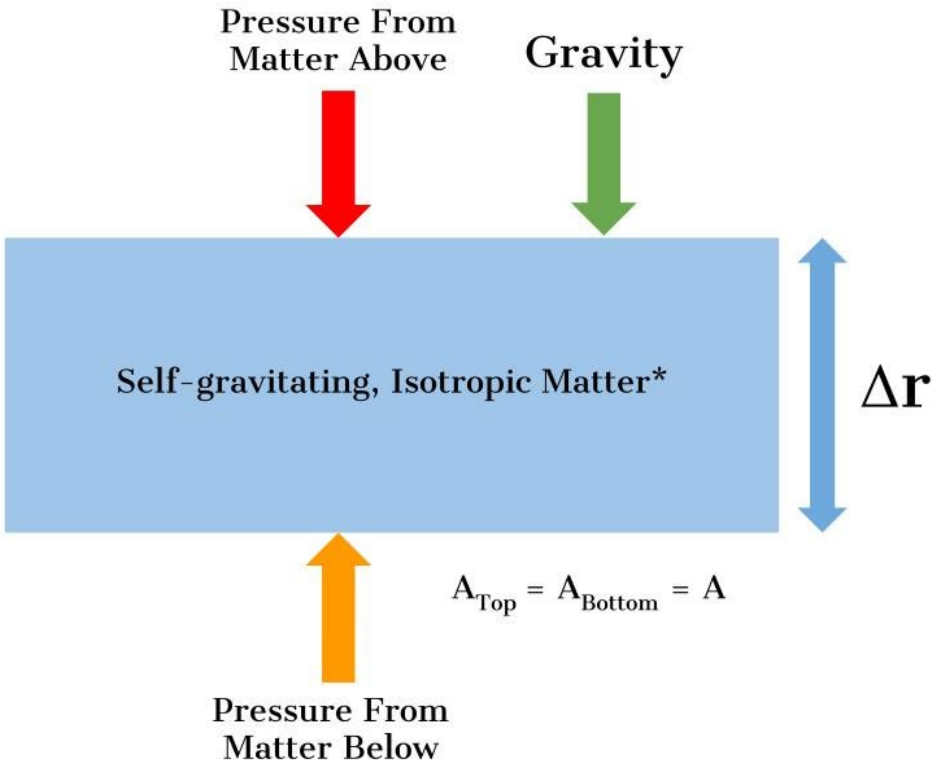
$$\text{Charge Density} \Rightarrow MeV^3$$

$$\text{Pressure, Energy Density} \Rightarrow MeV^4 \text{ or } \frac{MeV}{fm^3}$$

$$\text{Length, Time} \Rightarrow MeV^{-1}$$

$$\rho(r) = \frac{\epsilon(r)}{c^2} = \epsilon(r)$$

Star Structure



*Uniform Sphere in Equilibrium [1]

$$\sum F = 0$$

$$\boxed{P(r)A}\boxed{-P(r+\Delta r)A}\boxed{-\frac{GM(r)\rho(r)A\Delta r}{r^2}}=0$$

$$\Delta P = P(r+\Delta r) - P(r) = -\frac{GM(r)\rho(r)}{r^2}\Delta r$$

$$\boxed{\begin{aligned} \frac{dP}{dr} &= -\frac{GM(r)\epsilon(r)}{r^2} \\ \frac{dM}{dr} &= 4\pi r^2 \epsilon(r) \end{aligned}}$$

[1] Silbar, Reddy, Neutron Stars for Undergraduates, *Am.J.Phys. 72 (2004) 892-905*

Tolman-Oppenheimer-Volkov Equations

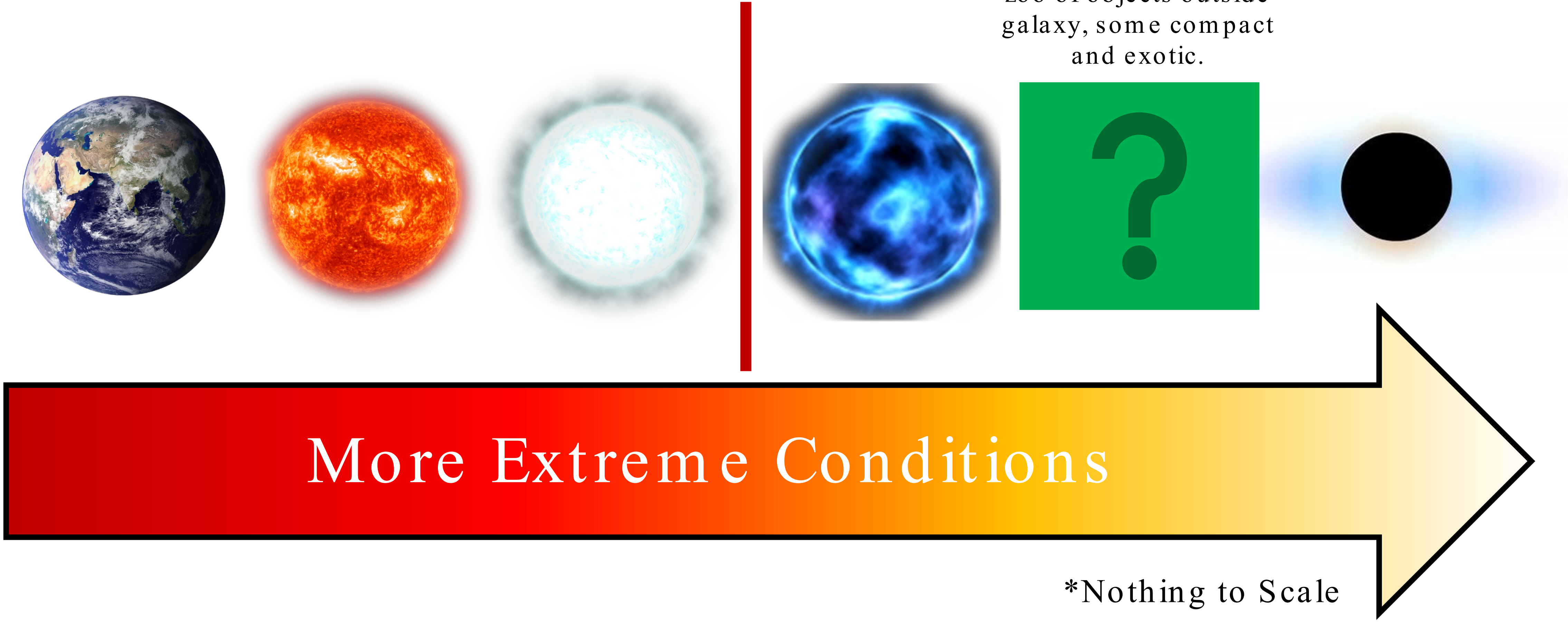
$$\frac{dP}{dr} = -\frac{G\epsilon(r)M(r)}{r^2}\left[1+\frac{P(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r)$$

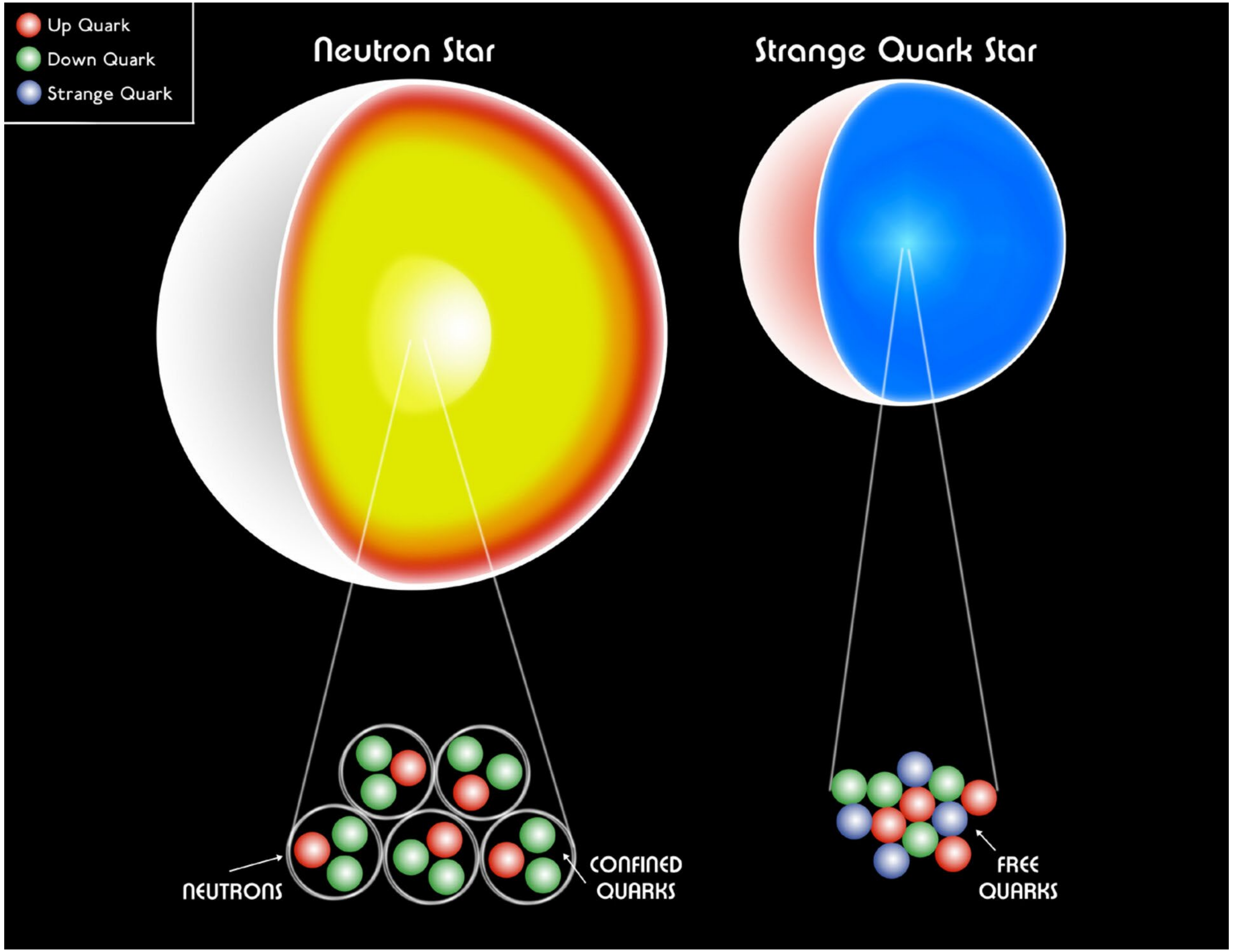
- To solve these equations, find P(ϵ).
- P(ϵ) depends on composition.

A Tour of Compact Objects

Gravitational waves will allow future study of a zoo of objects outside galaxy, some compact and exotic.

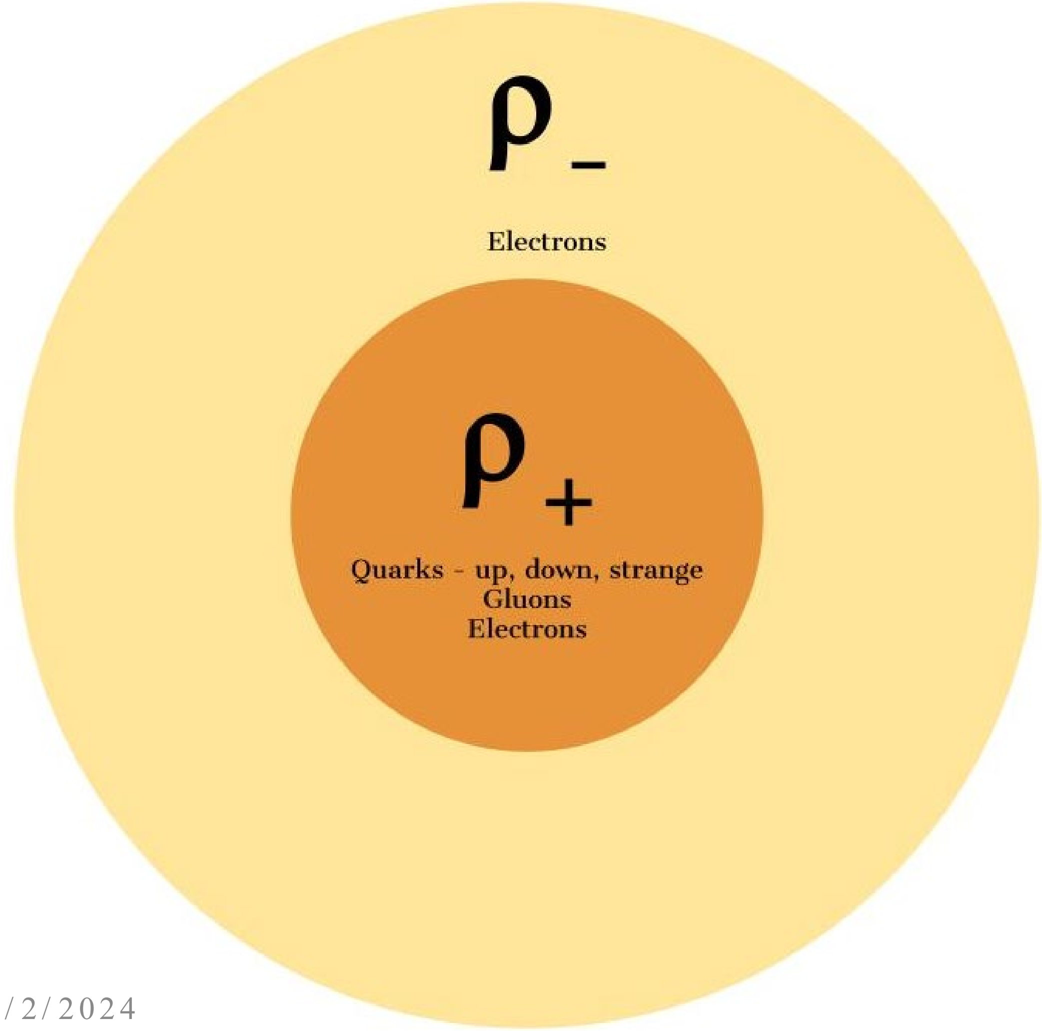


An Option: Mixed-Phase Strange Stars



[2] Image Credit Swinburne University

Matter in Mixed-Phase Equilibrium



$$d \leftrightarrows u + e^-$$

(Neutrinos excluded)

Strange quark decay and formation

To-Do List

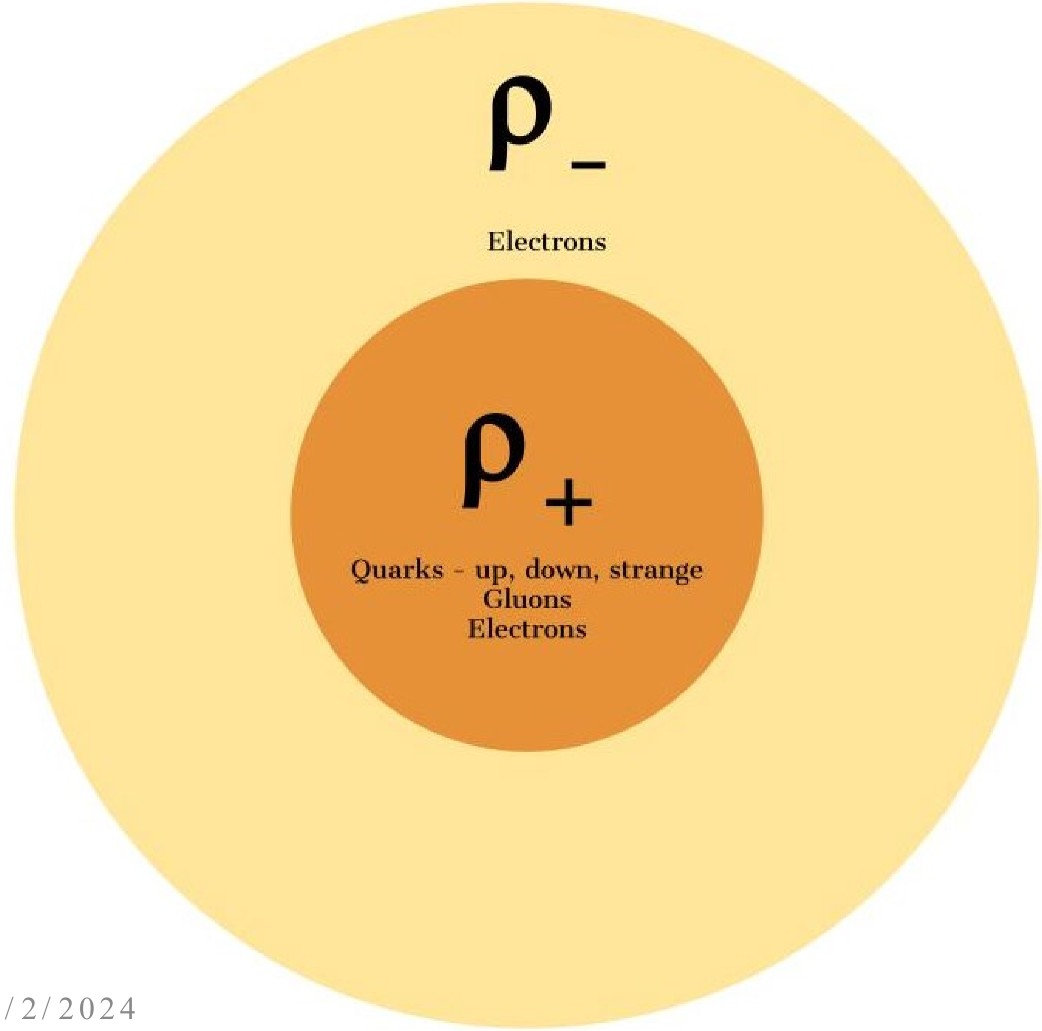
NOTE:

P – pressure
p - momentum
ρ – charge density

- ❑ Find pressures
- ❑ Find associated energy densities
- ❑ Find $P(\epsilon)$
- ❑ Solve TOV Equations
- ❑ Predict compact object characteristics

Where to start?

Step 1: Identify a Parameter



What is conserved?

- Baryon Number (B_i)
- Charge (Q_i)

Use chemical potentials (μ_B, μ_e) as parameters.

Particle Parameterization

$$\mu_i = B_i \mu_B + Q_i \mu_e$$

u

$$\mu_{up} = \frac{1}{3}\mu_B + \frac{2}{3}\mu_e$$

d

$$\mu_{down} = \frac{1}{3}\mu_B - \frac{1}{3}\mu_e$$

s

$$\mu_{strange} = \frac{1}{3}\mu_B - \frac{1}{3}\mu_e$$

e⁻

$$\mu_{e^-} = -\mu_e$$

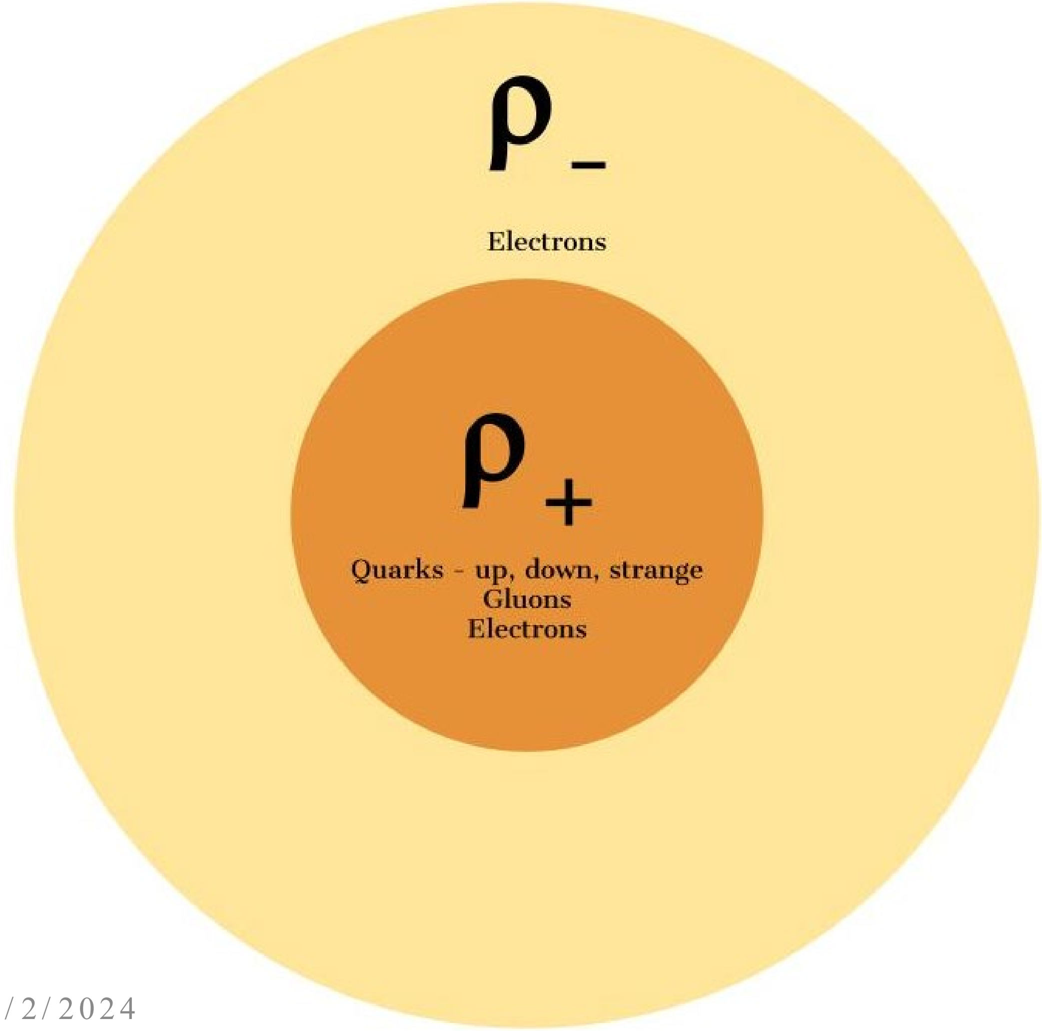
GOAL:

Two parameters ($\boldsymbol{\mu}_{\mathrm{B}}$, $\boldsymbol{\mu}_{\mathrm{e}}$)

↓

One Parameter ($\boldsymbol{\mu}_{\mathrm{B}}$)

Mixed Phase Gibbs Equilibrium(cont.)



- Equal pressure in each phase.

$$P_{e^-} + P_{quarks} = P_{e^-}$$

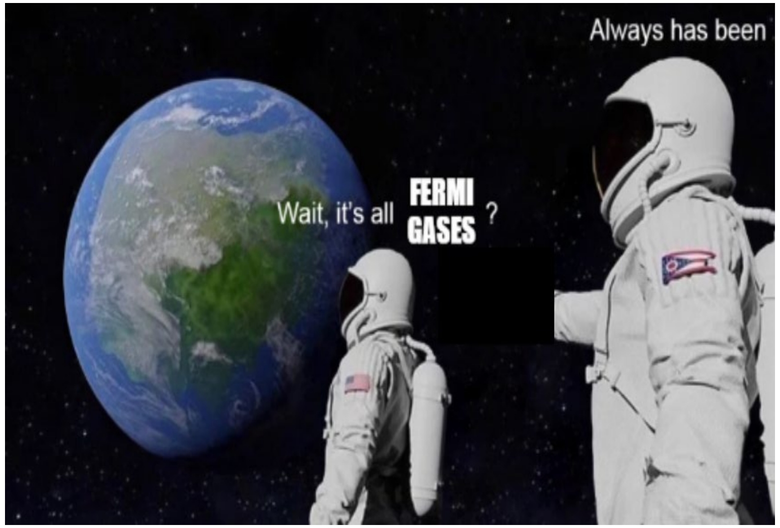
$$P_{quarks}(\mu_B, \mu_e) = \left(\sum_{i=u,d,s} P_{F_i} \right) - B = 0$$

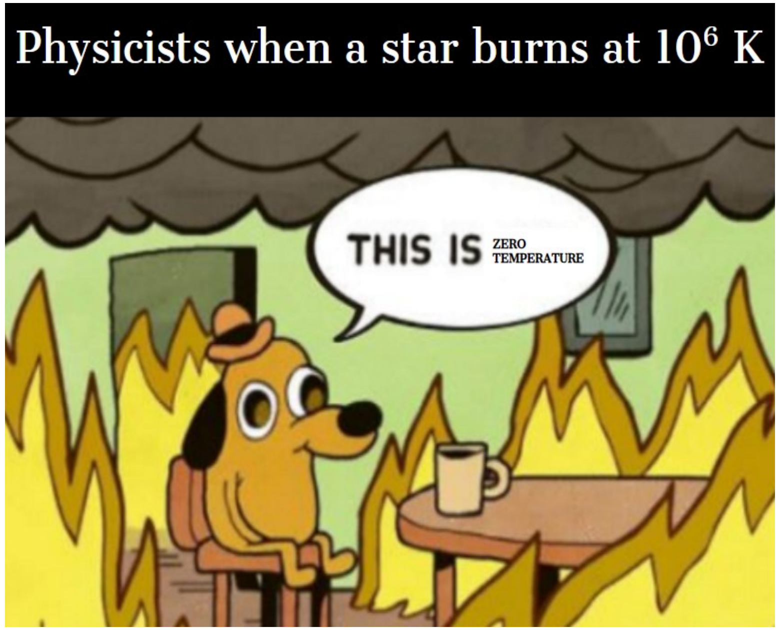
Modeling Strange Star Matter

For each constituent, $P_{thermal} << P_{degeneracy}$

$$\Downarrow$$

Zero-Temperature,
Relativistic, Non-Interacting,
Fermi Gas





Zero-Temperature, Relativistic, Non-Interacting Fermi Gas

$$\boxed{\mu_i = \sqrt{p^2 + m_i^2}}$$

$$\boxed{x_i = \frac{p_{F_i}}{m_i} = \frac{\sqrt{(B_i\mu_B + Q_i\mu_e)^2 - m_i^2}}{m_i}}$$

$$\boxed{P_{F_i} = \frac{g_i}{6\pi^2}\int_0^{p_{F_i}} \frac{p^4}{\sqrt{p^2 + m_i^2}}dp}$$

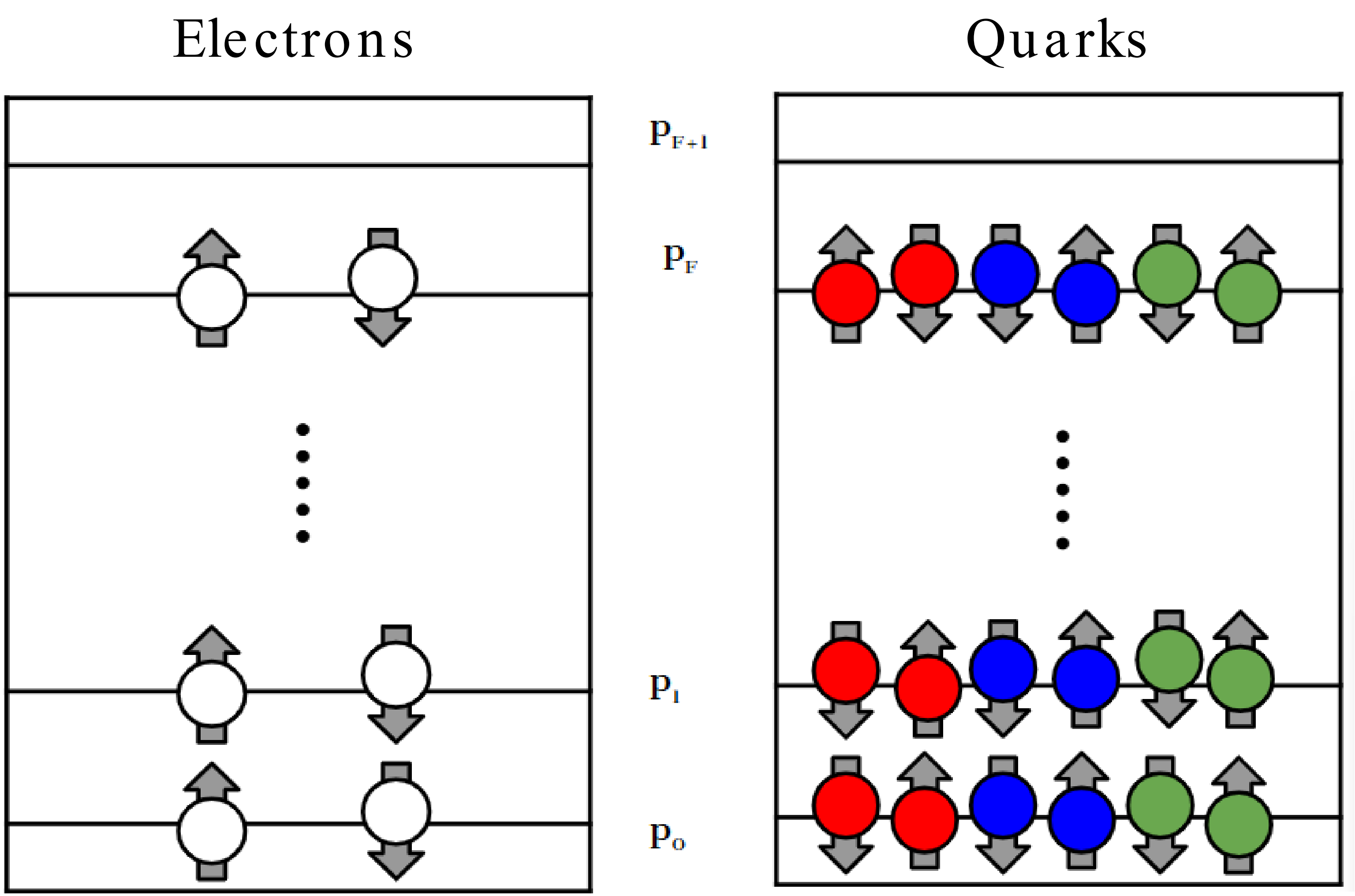
$$m_{up} = 2.3\ MeV$$

$$m_{down} = 4.8\ MeV$$

$$m_{strange} = 95\ MeV$$

$$m_{e^-} = 0.511\ MeV$$

Fermi Gas Degeneracy @ T = 0



$$g_i = \begin{cases} 2, i = e^- \\ 6, i = u, d, s \end{cases}$$

[3] Chakrabarty, Equation of State of Strange Quark Matter and Strange Star, *APS Phys. Rev. D* **43**, *62715 (1991)*

Two Parameters → One Parameter

$$P_{F_i} = \frac{g_i}{6\pi^2}\int_0^{p_{F_i}} \frac{p^4}{\sqrt{p^2+m_i^2}}dp$$

$$x_i = \frac{p_{F_i}}{m_i} = \frac{\sqrt{(B_i\mu_B + Q_i\mu_e)^2 - m_i^2}}{m_i}$$

$$\mu_i = \sqrt{p^2+m_i^2}$$

$m_{up} = 2.3\ MeV$

$m_{down} = 4.8\ MeV$

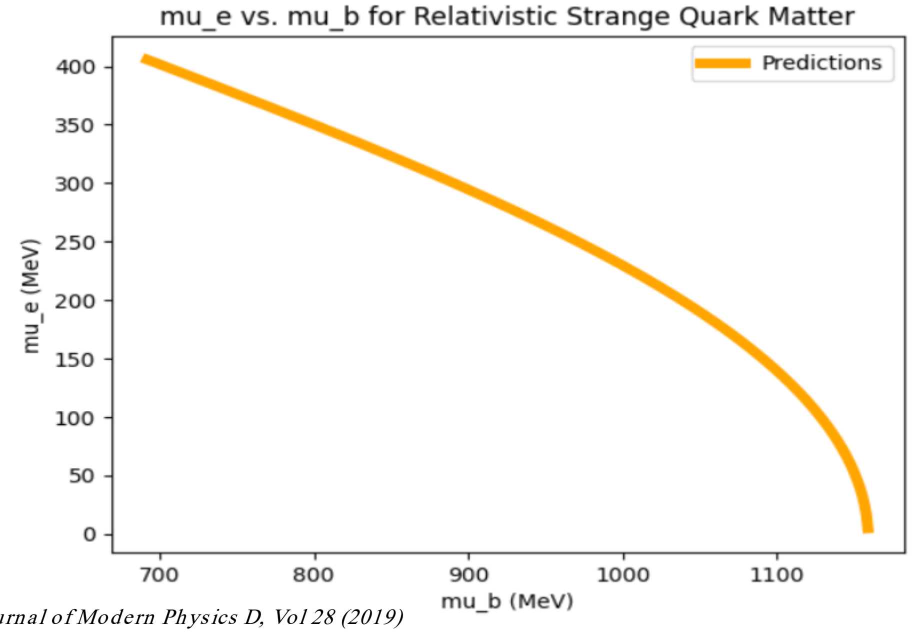
$m_{strange} = 95\ MeV$

$m_{e^-} = 0.511\ MeV$

$$P_{quarks}(\mu_B,\mu_e) = \left(\sum_{i=u,d,s} P_{F_i}\right) - B = 0$$

$$\frac{8\pi^2 B}{3} = \sum_{i=u,d,s} m_i^4\left[x_i(\frac{2}{3}x_i^2 - 1)\sqrt{1+x_i^2} + \operatorname{arcsinh}(x_i)\right]$$

Numerically solve $\rightarrow \mu_e(\mu_B)$



Now there is only one parameter (μ_B).

[4] Aziz et. Al, Constraining values of Bag Constant for strange star candidates, *International Journal of Modern Physics D, Vol 28 (2019)*

Calculating Pressures

$$\boxed{x_i = \frac{p_{F_i}}{m_i} = \frac{\sqrt{(B_i\mu_B + Q_i\mu_e)^2 - m_i^2}}{m_i}}$$

$$P = P_{F_{e^-}}(\mu_e)$$

$$= \frac{m_{e^-}^4}{8\pi^2}\left[x_{e^-}(\frac{2}{3}x_{e^-}^2 - 1)\sqrt{1 + x_{e^-}^2} + \mathrm{arcsinh}(x_{e^-})\right]$$

- For each μ_B, there is now an associated pressure of this strange object.

To-Do List

NOTE:

P – pressure
p - momentum
ρ – charge density

- ✅ Find pressures
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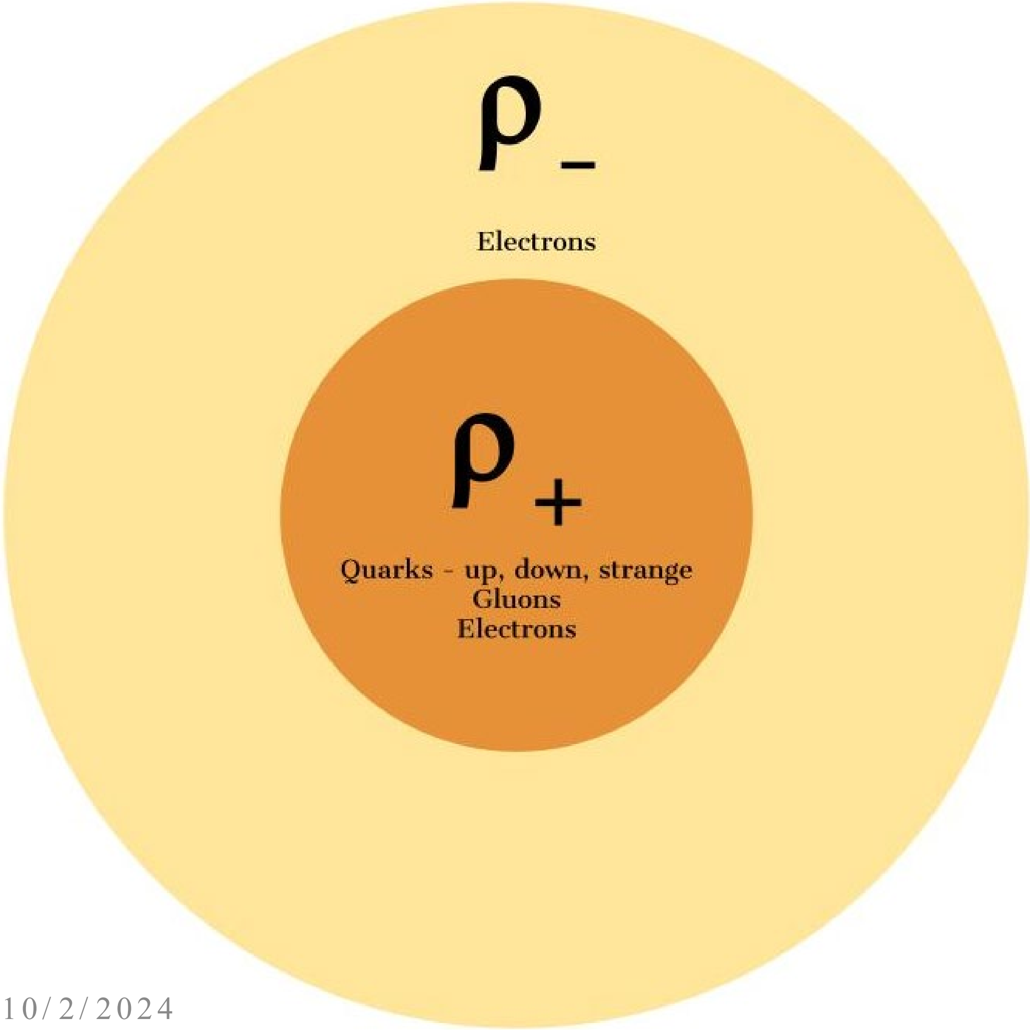
- ❑ Find P(ϵ)
- ❑ Solve TOV Equations
- ❑ Predict compact object characteristics

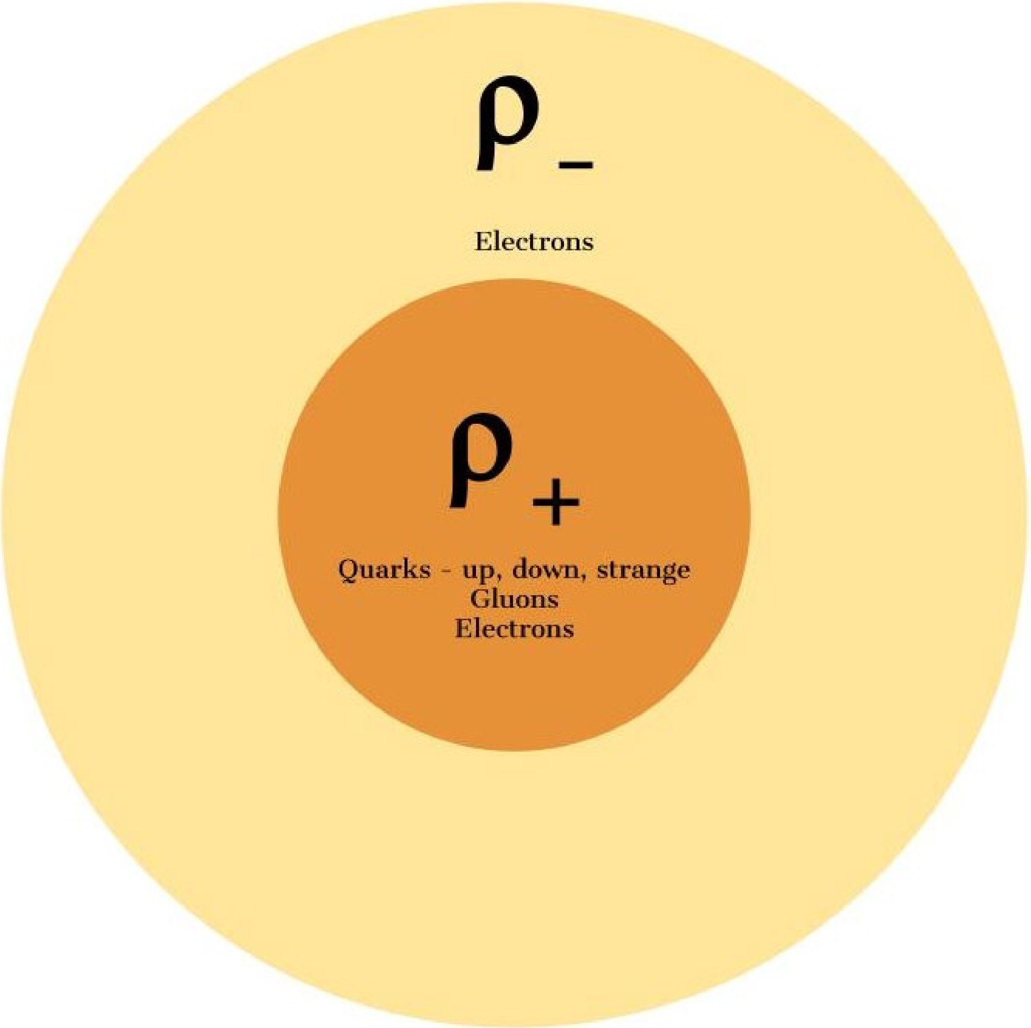
Calculating Energy Density



$$\epsilon_{total} = f\epsilon_{quarks} + \epsilon_{e^-}$$

$$f = \frac{\text{Volume of Quark-Electron matter}}{\text{Total Volume}}$$

Calculating Energy Density - Volume Fraction



$$f\rho_+ + \rho_- = 0$$

$$f = -\frac{\rho_-}{\rho_+}$$

$$\boxed{\rho = -\frac{\partial P}{\partial \mu_e}}$$

[5] Jaikumar et. al, The Strange Star Surface: A Crust with Nuggets, *Phys. Rev. Lett.* ***96****, 041101 (2006)*

Electron Phase Charge Density

$$\boxed{x_{e^-} = \frac{p_{F_{e^-}}}{m_{e^-}} = \frac{\sqrt{\mu_e^2 - m_{e^-}^2}}{m_{e^-}} \approx \frac{\mu_e}{m_{e^-}}}$$

$$P_{F_{e^-}} = \frac{m_{e^-}^4}{8\pi^2}\left[x_{e^-}\left(\frac{2}{3}x_{e^-}^2 \cancel{-1}\right)\sqrt{\cancel{1+}\, x_{e^-}^2} + \operatorname{arcsinh}(x_{e^-})\right]$$

$\boxed{\text{With electrons, } \mu_e >> m_{e^-}}$

$$\rho_- = \frac{\partial P_{F_{e^-}}}{\partial \mu_e} \approx \frac{m_{e^-}^4}{8\pi^2}\frac{\partial}{\partial \mu_e}\left[x_{e^-}\left(\frac{2}{3}x_{e^-}^2\right)(x_{e^-}) + \operatorname{arcsinh}(x_{e^-})\right]$$

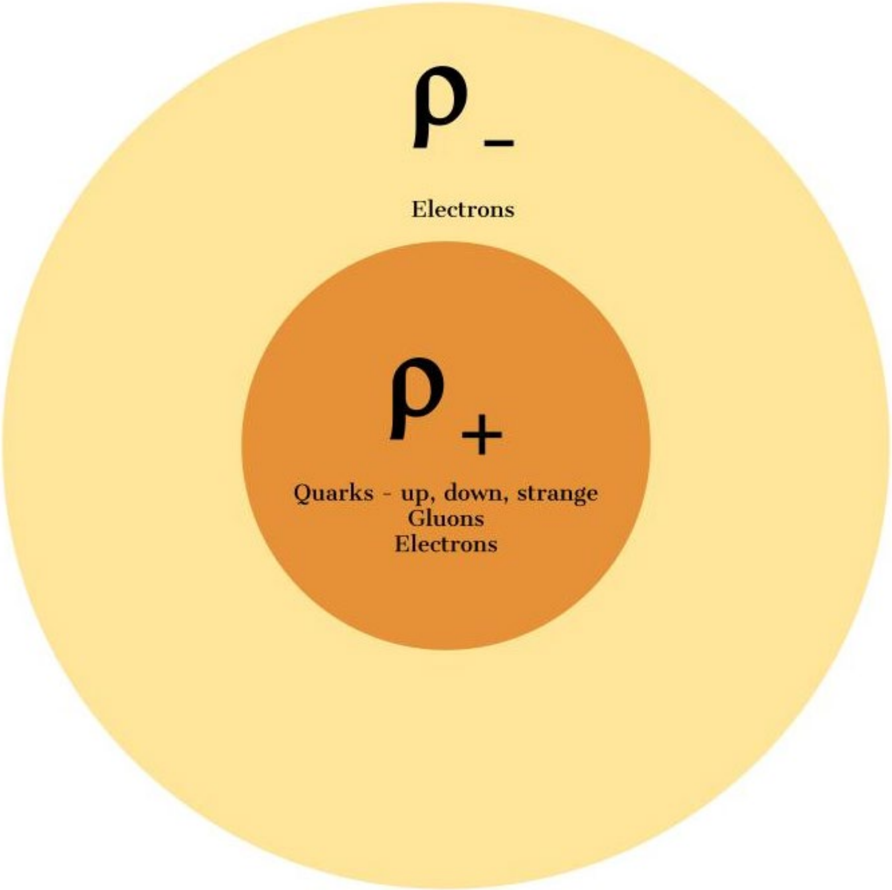
$$= \frac{m_{e^-}^4}{12\pi^2}\left[4\frac{\mu_e^3}{m_{e^-}^4} + \frac{3}{2}\cancel{\frac{1}{\sqrt{1+\frac{\mu_e^2}{m_{e^-}^2}}}}\right] \approx \frac{m_{e^-}^4}{12\pi^2}\left(4\frac{\mu_e^3}{m_{e^-}^4}\right) = \boxed{\frac{\mu_e^3}{3\pi^2}}$$

Quark/Electron Phase Charge Density

$$\rho_+ = -\frac{\partial}{\partial \mu_e}\left[P_{F_{e^-}} + \sum_{i=u,d,s} P_{F_i}\right]$$

$$\boxed{\rho_+ = -\frac{\mu_e^3}{3\pi^2} - \sum_{n=0}^{3} c_n \mu_e^n}$$

Closing the Dolls



Charge Densities

↓

Volume Fraction of Quark Phase

↓

Energy Density

$$\epsilon_{e^-} \approx \frac{\mu_e^4}{4\pi^2 m_{e^-}} - \frac{\mu_e^3}{8\pi^2}\operatorname{arcsinh}\left(\frac{\mu_e}{m_{e^-}}\right)$$

$$\epsilon_{quarks} = \frac{3}{8\pi^2}\sum_{i=u,d,s} m_i^3\left[x_i(2x_i^2+1)\sqrt{1+x_i^2} - \operatorname{arcsinh}(x_i)\right]$$

$$\epsilon_{total} = f\epsilon_{quarks} + \epsilon_{e^-}$$

To-Do List

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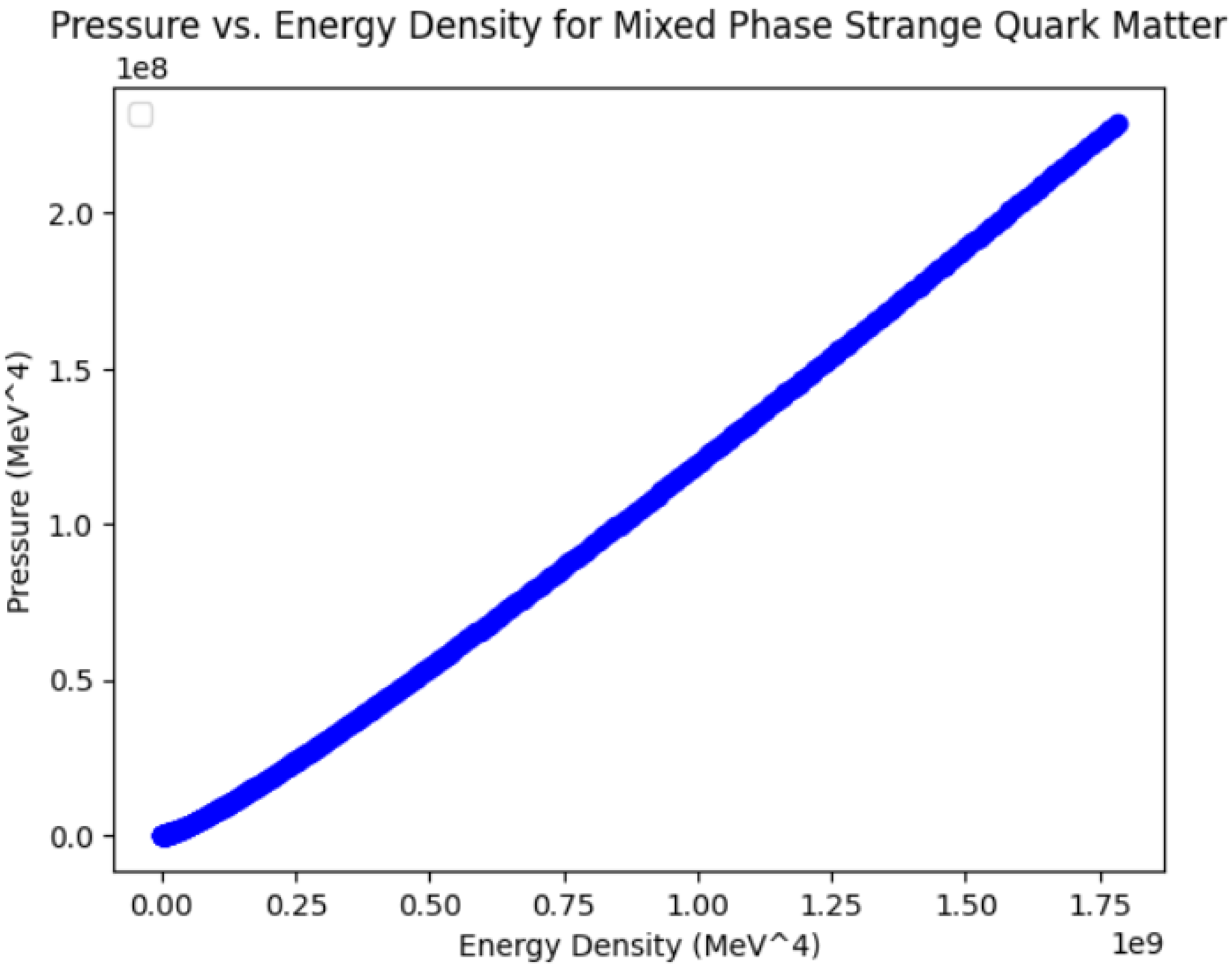

We can now…

- Change units
- Try different fits
- Solve the TOV equations
- Predict
- Repeat process with different matter

Exponential Fit: $P(\epsilon) \approx 0.00632\epsilon^{1.142}$

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- ✅ Find P(ϵ)
- ❑ Solve TOV Equations
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Acknowledgements

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Citations

[1] Silbar, Reddy, Neutron Stars for Undergraduates, *Am.J.Phys. 72 (2004) 892-905*

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[5] Jaikumar et. al, The Strange Star Surface: A Crust with Nuggets, *Phys. Rev. Lett.* ***96**, 041101 (2006)*

Fermi Gas Energy Formula

$$\epsilon_i = \frac{E_i}{V} = \frac{g_i}{2\pi^2} \int_0^{p_{F_i}} p^2 \sqrt{p^2 + m_i^2} \, dp$$