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Single Photon Blockade in Nanocavities with Weak Kerr Nonlinearity (Application)

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ABSTRACT

Achieving single-photon Fock states have many applications in quantum information processing [4]. To produce single-photon Fock states, we require the use of nonlinear optical processes [1]. In this work, we describe the methodology implemented by [3] to achieve single-photon Fock states by using a weak Kerr nonlinearity. We then apply [3] methods for a Silicon cavity by running QuTIP simulations to investigate the behavior of the cavity. We find that we are able to produce the Fock states for a Silicon-based cavity.

1. INTRODUCTION

Achieving single-photon Fock states have many applications in quantum information processing [4]. To produce single-photon Fock states, we require the use of nonlinear processes [1]. We will now describe these nonlinear optical processes by taking a detour to electromagnetism.

Elegantly, we can describe the propagation of light in matter using Maxwell's equations.

$$\nabla . D = 0 \qquad\qquad \nabla . B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad\qquad \nabla \times H = J + \frac{\partial D}{\partial t}$$

These equations describe the propagation of light through linear materials. Linear materials interact proportionally to the applied electric field E. For instance, if a light lay goes from air to water, we expect the ray to bend according to Snell's Law, where the frequency of the light is conserved, but the wavelength shifts. The displacement field D is given equation 1, and accounts for the electromagnetic effects of polarization P and an applied electric field E. The polarization P is given by equation 2, and is linear to the applied electric field E, and describes light propagating through a linear media. χ is known as the electric susceptibility, which indicates the degree of polarization of a dielectric material in response to an applied electric field. The greater the electric susceptibility, the greater the ability of a material to polarize (separate into dipole moments) in response to E.

$$D = \epsilon_0 E + P \tag{1}$$

$$P = \epsilon_0 \chi E \tag{2}$$

We can generalize the polarization (equation 2) to equation 3. As a reminder the $\epsilon_0 \chi E$ is the linear term, and describes light propagation through linear media. The $\chi^{(2)}$ and $\chi^{(3)}$ terms (and terms thereafter) are the nonlinear terms, and describe light propagation through nonlinear materials. The properties of the incident light are changed due to the material's response. Unlike light propagating through linear media, the light's frequency can change. Two common examples of nonlinear optical processes are second and third harmonic generation, which is described in detail in Figures 1 and 2, respectively. In our work, we will focus on the $\chi^{(3)}$ nonlinearity, which is known as a Kerr nonlinearity. A common material with a Kerr nonlinearity is Silicon, which has the most well developed fabrication process. Our work consists of analyzing a cavity made of Silicon. A cavity (refer to Figure 3 for visualization) can be

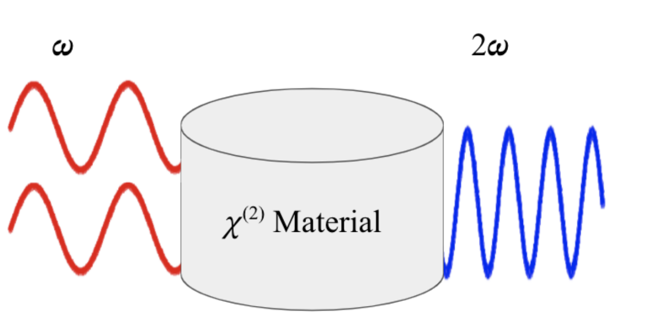

Figure 1. A schematic depicting second harmonic generation. Two photons with frequency ω enter a $\chi^{(2)}$ nonlinear material, and are annihilated inside. One photon is created with frequency 2ω satisfying conservation of energy.

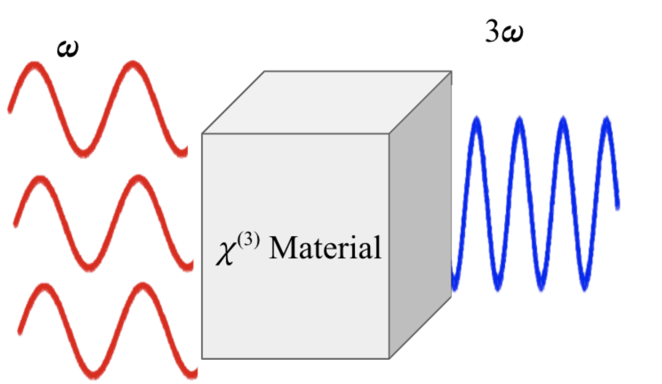

Figure 2. A schematic depicting third harmonic generation. Three photons with frequency ω enter a $\chi^{(3)}$ nonlinear material, and are annihilated inside. One photon is created with frequency 3ω satisfying conservation of energy.

thought of as simply a resonator for electromagnetic radiation. Cavities are not perfect, and can let the fields escape from the inside (or absorb them), which leads to damping.

$$P = \epsilon_0[\chi E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + ...] \tag{3}$$

We will leave the explanation behind the theory of generating single photon Fock states in a cavity made of Si to my collaborator, Matthew D. Stearns. In this work, we will describe the results we obtained from the QuTIP simulations.

2. APPLICATION/RESULTS

2.1. *QuTIP Simulations for Silicon Cavity*

To simulate the effectiveness of generating single photon Fock states for a Silicon cavity, we used an open-source software: Quantum Toolbox in Python (QuTIP). QuTIP simulates the dynamics of open quantum systems [6]. We use the QuTIP Lindblad Master Equation Solver to solve the master equation 4, given by [3]. In essence, QuTIP is solving for the number of photons present in the cavity and the $g^2(0,t)$ correlation function. The simulation includes the the nonlinear coupling strength U, which we calculated in the next section for a Silicon cavity. For the cavity loss rate κ, we chose a value of 1 MHz. Additionally, we must specify in the simulation the number of basis Fock state N, which we chose to be 60 states for our example. However, we note that N should be larger to obtain better results, but this increases computation time. We now present our graphical results from the QuTIP simulations as Figures 4, 5, 6, and 7.

$$\frac{d}{dt}\hat{\rho} = -i\left[\hat{H}_{\text{target}}, \hat{\rho}\right] + \kappa\mathcal{D}[\hat{a}]\hat{\rho} \tag{4}$$

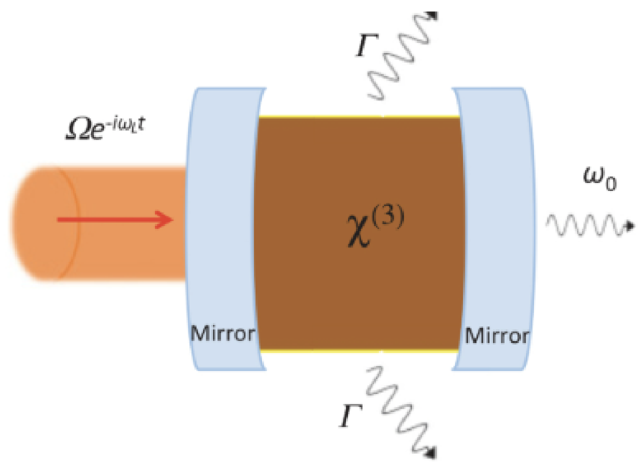

Figure 3. A schematic diagram of a cavity from [2]. On the left-hand side, we have a pumping laser (source), and in the middle is the $\chi^{(3)}$ material, which could be made of Silicon. The Γ term can be thought as the cavity loss rate, and ω_0 is the light created in the cavity.

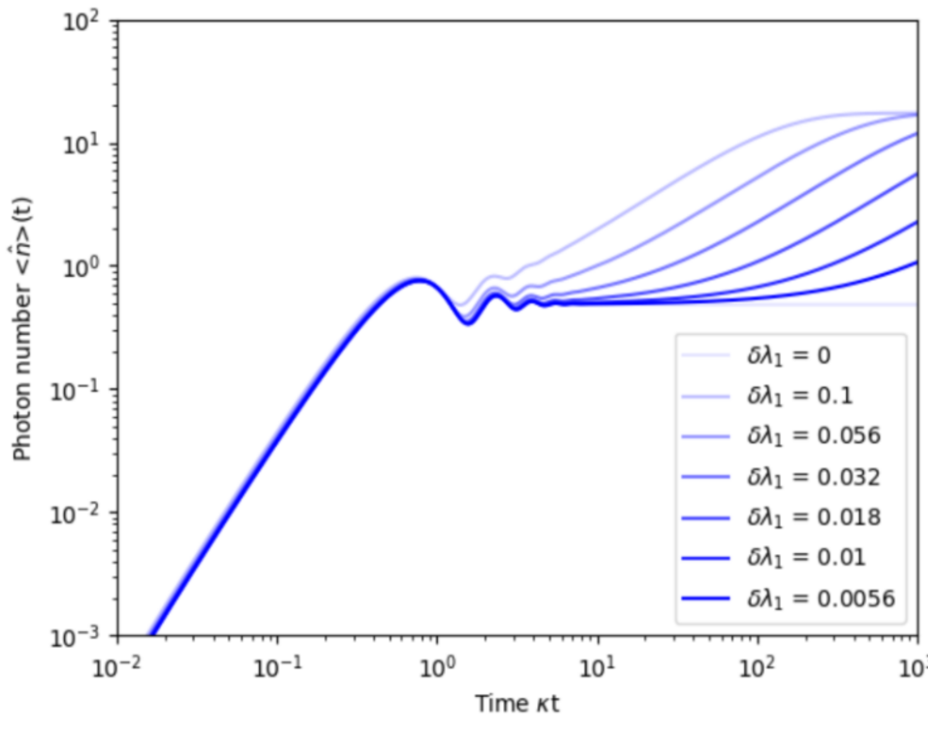

Figure 4. Plot showing the photon number (in log-scale) vs. time (in log-scale) for seven different drive mismatch amplitudes ($\delta\lambda_1$). We remind the reader that the nonlinear drive amplitude is given by $\tilde{\Lambda_1} = -\tilde{\Lambda_3}(1+\delta\lambda_1) = -\tilde{\Lambda_3}r$.. We note that an ideal single photon Fock state is generated when $\delta\lambda_1 = 0$, where we notice that the photon number approaches 1 indicating a single photon Fock state.

2.2. *Calculating the Nonlinear Coupling Strength U for a Silicon cavity*

To calculate the nonlinear coupling strength U for a Silicon cavity, we used the equation derived in [1], which is given by equation 5.

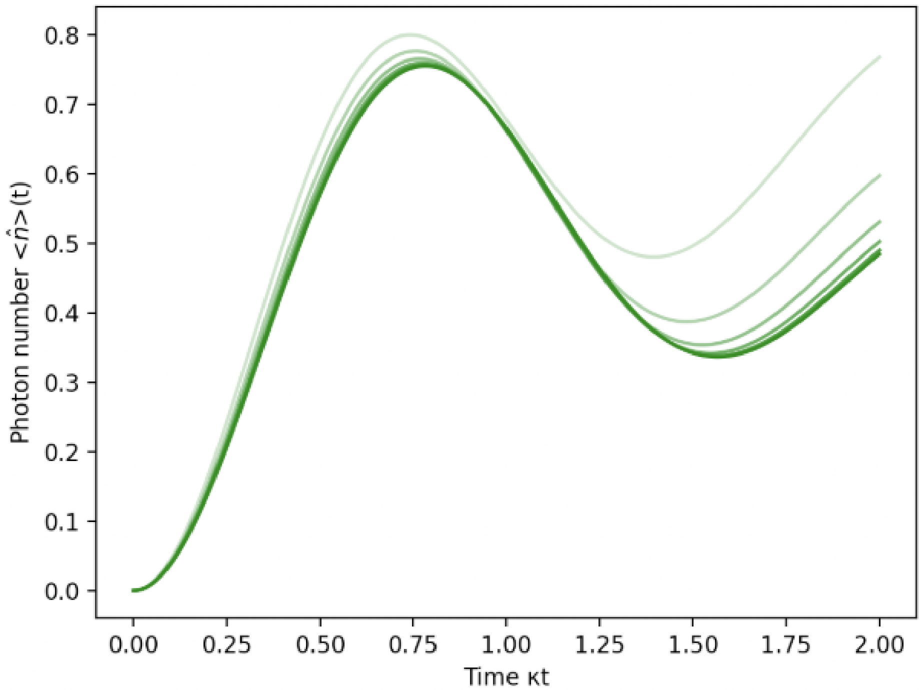

Figure 5. Plot showing the photon number (in linear-scale) vs. time (in linear-scale) for seven different drive mismatch amplitudes. ($\delta\lambda_1$).

$$U = \frac{3(\hbar\omega)^2}{4\epsilon_0 V_e} \frac{\chi^{(3)}}{\epsilon_r^2} \tag{5}$$

For our analysis, we used the values provided in [2], which are highlighted in Table 2.2.

$\chi^{(3)}$ ($\mathrm{m^2/V^2}$)	n	λ (nm)	V_e ($\mathrm{m^3}$)
0.45×10^{-18}	3.4	1550	10^{-20}

Table 1. Parameters taken from [2] to calculate U for a Silicon cavity. $\chi^{(3)}$ is the third-order nonlinear susceptibility, n is the index of refraction, λ is the wavelength of light, and V_e is the effective cavity mode volume. The index of refraction n is related to the relative dielectric constant ϵ_r by $\epsilon_r^2 = n^4$.

Since we know the wavelength of light λ, we can figure out $\hbar\omega$ by using the following relation.

$$\hbar\omega = \frac{\hbar c}{\lambda} = \frac{197.3 \text{ nm ev}}{1550 \text{ nm}} \frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ ev}} = 2.04 \times 10^{-20} \text{ J} \tag{6}$$

Once we know $\hbar\omega$, we will now plug in all the parameters to obtain U. We note that ϵ_0 is the permittivity of free space.

$$U = \frac{3(2.04 \times 10^{-20})^2}{4(8.85 \times 10^{-12})(10^{-20})} \frac{0.45 \times 10^{-18}}{(3.4)^4} = 1.19 \times 10^{-29} \text{ J} \tag{7}$$

We then divide our U value by Planck's constant to obtain the nonlinear coupling strength in units of MHz. Thus, we take U to be 0.018 MHz, which will we use for our QuTIP simulations.

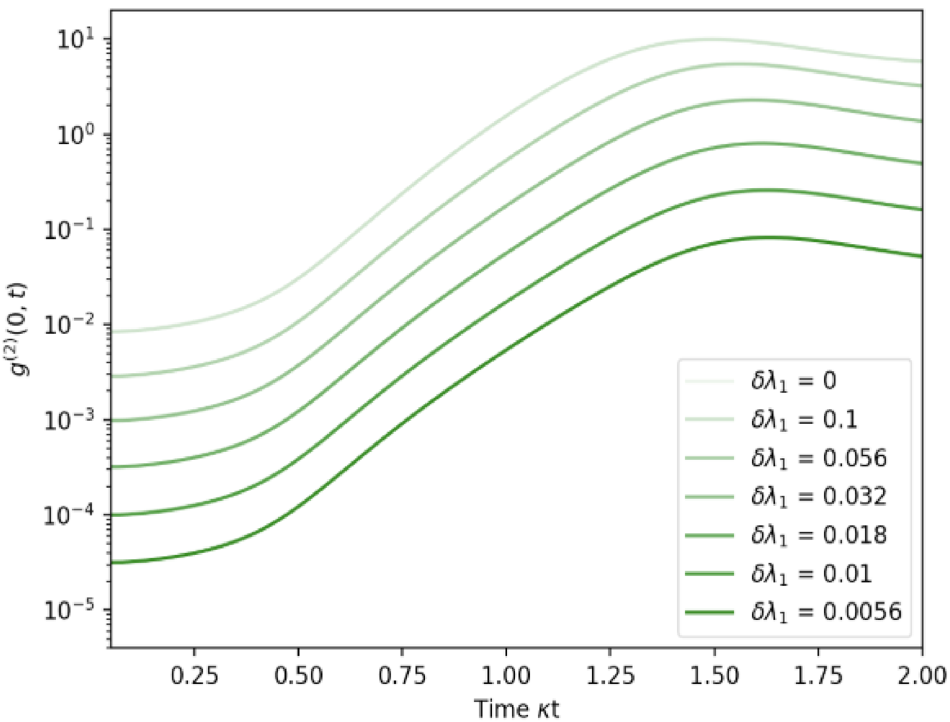

Figure 6. Plot slowing the $g^2(0,t)$ (in log-scale) correlation function as a function of time (in linear-scale) for seven different drive mismatch amplitudes. We note that the photons must behave quantum mechanically for the generation of single photon Fock states, which corresponds to a $g^2(0,t) < 1$.

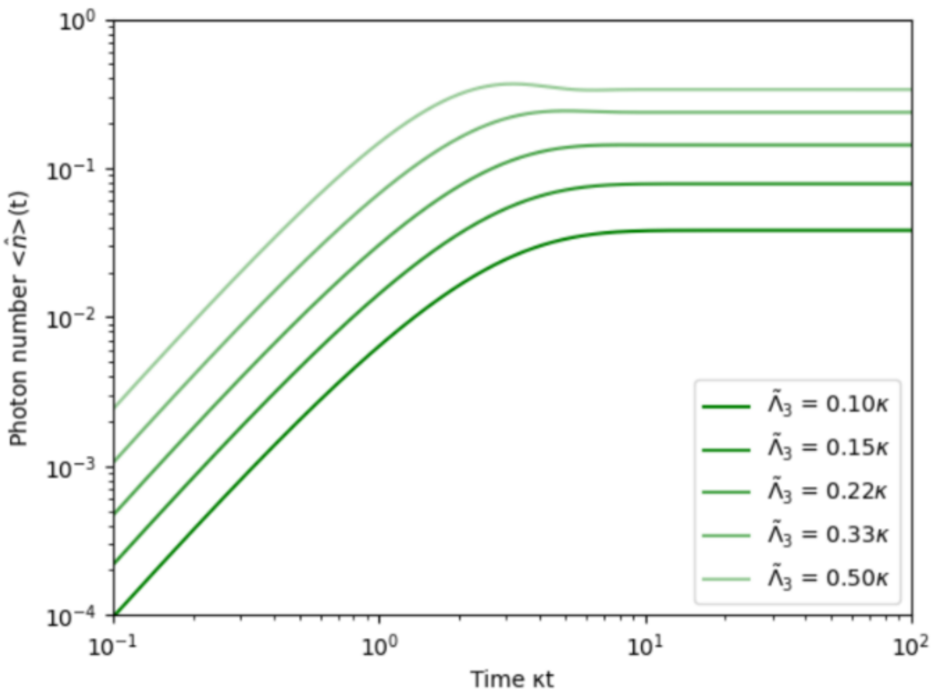

Figure 7. Plot showing the photon number (in log-scale) vs. time (in log-scale) for five different nonlinear drive amplitudes ($\tilde{\Lambda}_3$).

3. CONCLUSIONS/FUTURE WORK

In this work, we show our results from our QuTIP simulations to analyze the generation of single photon Fock states in a Silicon cavity. We find that we are able to produce such states in a Silicon cavity. Moving forward, we aim to implement such a system in the lab and discuss our results in a future paper.

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