Neutrino Oscillations in Dense Media

Ellen Gates –Department of Physics and Astronomy, University of Missouri
Advisor: Vincenzo Cirigliano –Institute for Nuclear Theory, University of Washington

Outline

i. Introduction

ii. Quantum Kinetic Equations

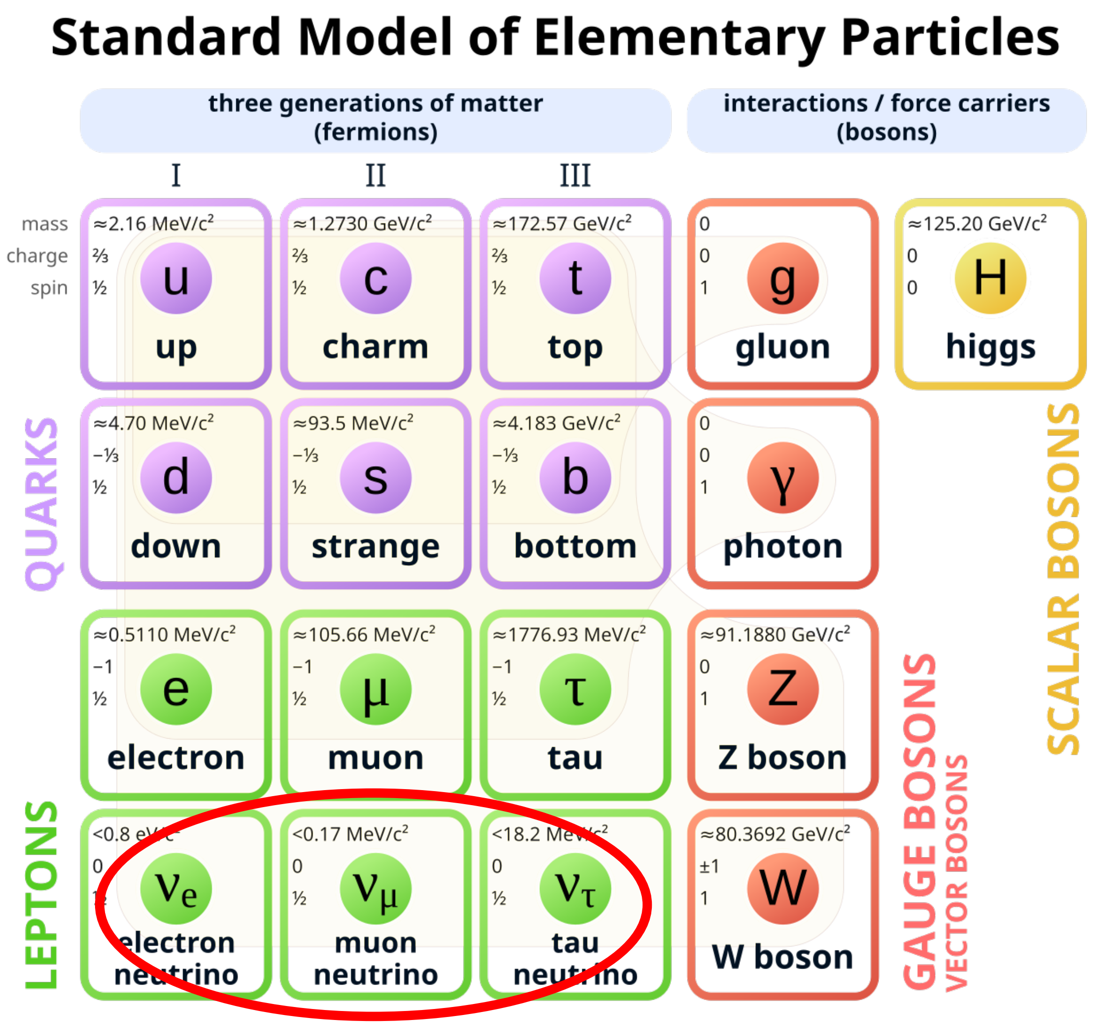
iii. Results

iv. Future Work

Outline

Neutrinos

- Neutral leptons
- Weakly interacting
- Three flavors



Importance

- Most abundant particle in the universe
- Present in many astronomical settings
    - Early Universe/nucleosynthesis
    - Stars
    - Supernova Explosions
    - Black holes and neutron stars

$$p + e^- \rightarrow n + \nu_e$$

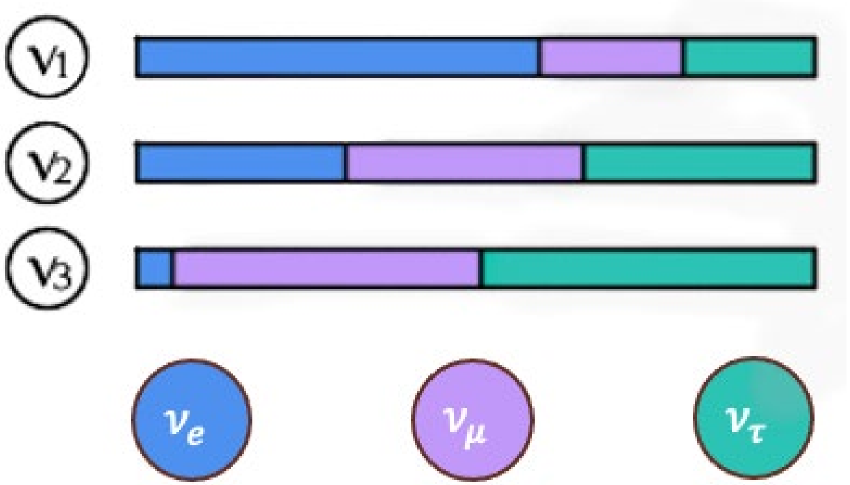
Neutrino Oscillations

- Massless? no
- Flavors composed of mass eigenstates
- If they have different masses, they can change flavor

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle$$

$$\alpha = e, \mu, \tau$$

$$i = 1, 2, 3$$



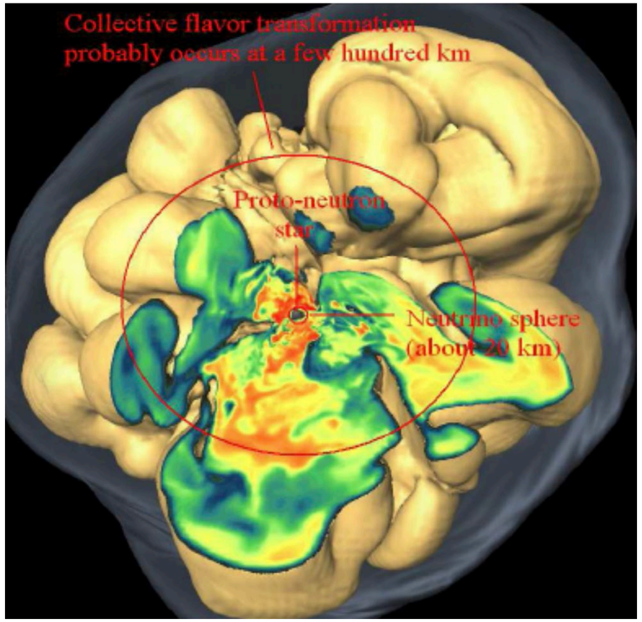
e.g. $$\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix}$$

$$|\nu_i(t)\rangle \approx e^{-m_i^2 t/2E_i}\,|\nu_i\rangle$$

Neutrinos in Dense Media

- Cannot neglect neutrino interactions in astrophysical setting – too dense
- Want to know the nature of oscillation over time

NASA, ESA, J. Hester and A. Loll (Arizona State University)



V. Cirigliano

Two Approaches

Quantum Many Body Approach

vs.

Quantum Kinetic Equations

Outline

i. Introduction

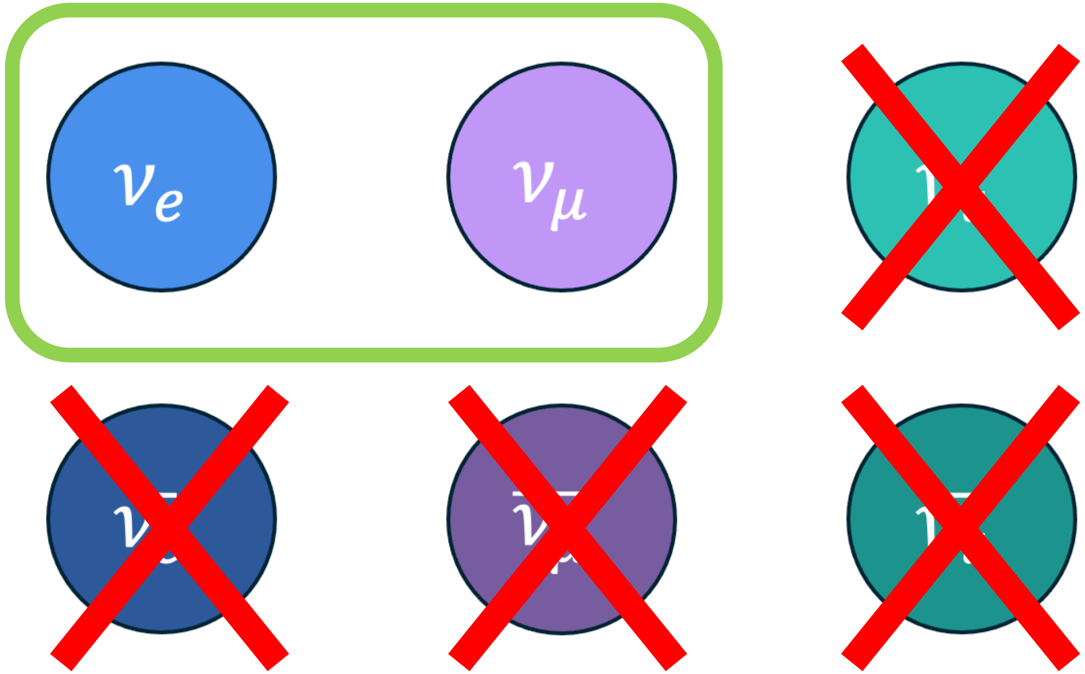
ii. Quantum Kinetic Equations

iii. Results

iv. Future Work

Simplifications

- Two flavor model
- No antineutrinos



Toy Model

- Supernova Environment
- 35 available momentum bins – discretized
- 10 neutrinos: 6 ν_e, 4 ν_μ

$$\bar{p} = (p_x, p_y, p_z)$$

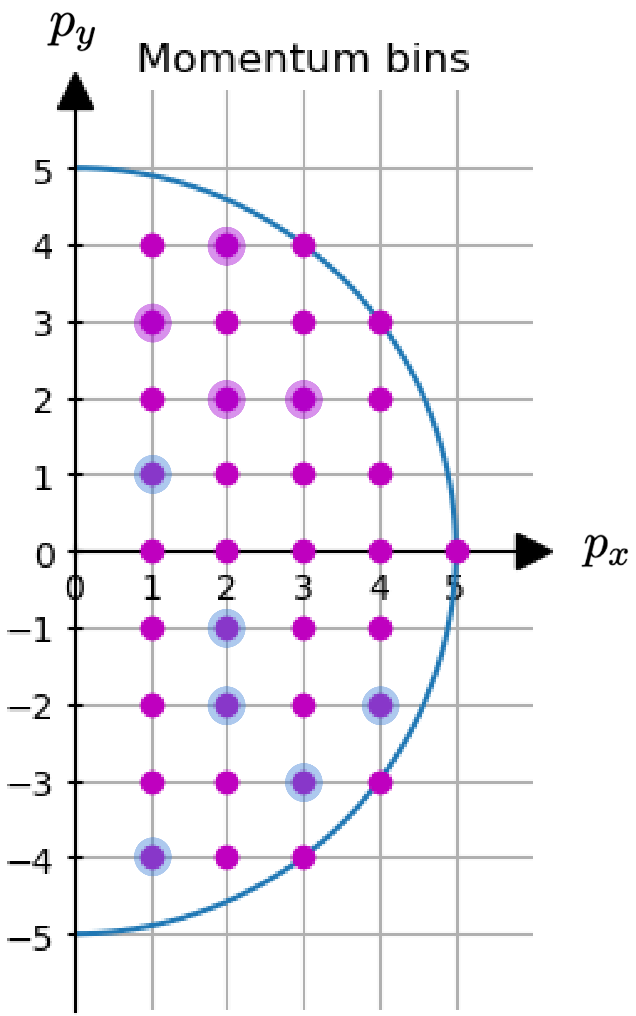
$$p_x = \frac{2\pi}{L} n_x$$

$$p_y = \frac{2\pi}{L} n_y$$

$$p_z = 0$$

$$n_{max} = 5$$

$$x > 0$$



Quantum Kinetic Equations (QKEs)

$$F = \begin{pmatrix} f_{LL} & f_{LR} \\ f_{RL} & f_{RR} \end{pmatrix}$$

$$iDF = [H, F] + iC$$

drift and
force term

vacuum
mass and
forward
scattering
terms

inelastic
collision
term

QKEs

$$\dot{f}_{n_p}(t) = -i[\frac{\Delta m}{4|\bar{p}_n|}R, f_{n_p}(t)] - i\sqrt{2}\frac{G_f}{V}\sum_i \kappa_{ij}[f_{n_q}(t), f_{n_p}(t)] + \text{collision term}$$

time evolution

vacuum mass

forward scattering

inelastic collisions

$$f(t) = \begin{pmatrix} f_{ee}(t) & f_{e\mu}(t) \\ f_{\mu e}(t) & f_{\mu\mu}(t) \end{pmatrix} = \begin{pmatrix} f_1(t) & f_3(t) + if_4(t) \\ f_3(t) - if_4(t) & f_2(t) \end{pmatrix}$$

Vacuum Mass Term

$$R = \begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$$

$$\dot{f}_{n_p}(t) = -i[\frac{\Delta m}{4|\bar{p_n}|}R, f_{n_p}(t)]$$

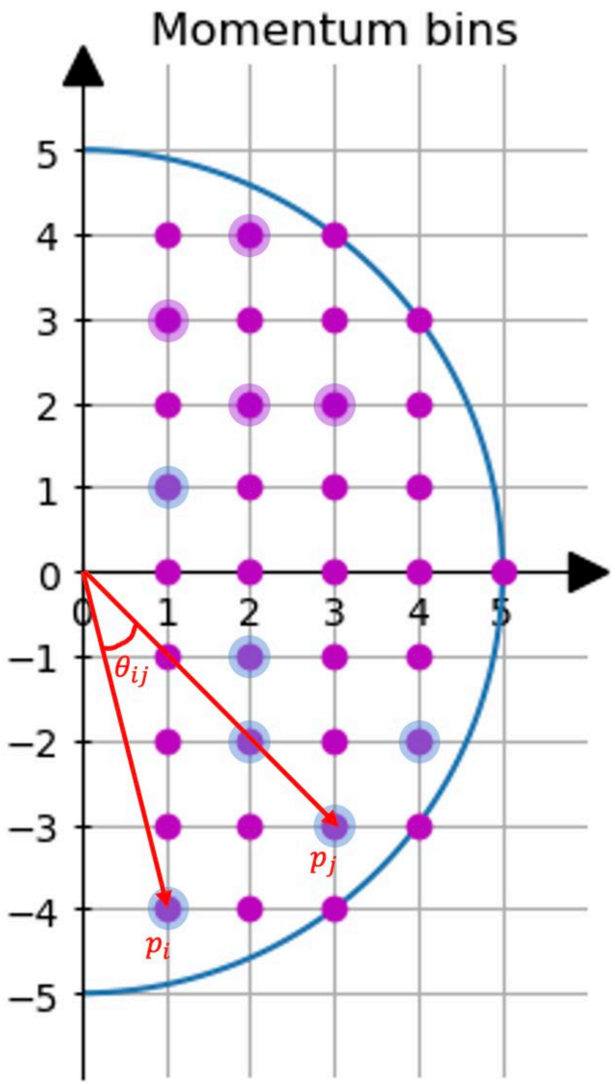
$$f_{n_p}(t) = \begin{pmatrix} 1 - \sin^2(2\theta) sin^2(\frac{\Delta m^2}{4|\bar{p}|}t) & - \\ - & \sin^2(2\theta) sin^2(\frac{\Delta m^2}{4|\bar{p}|}t) \end{pmatrix}$$

Forward Scattering Term

$$\dot{f}_{n_p}(t) = -i[\frac{\Delta m}{4|\bar{p_n}|}R, f_{n_p}(t)] - i\sqrt{2}\frac{G_f}{V}\sum_i \kappa_{ij}[f_{n_q}(t), f_{n_p}(t)]$$

$$\kappa_{ij} = 1 - \cos\theta_{ij}$$

$$\cos\theta_{ij} = \frac{\bar{p_i}\cdot\bar{p_j}}{|\bar{p_i}||\bar{p_j}|}$$



Collision Term

$$\mathcal{C}(\vec{n}_p) = \left(\frac{G_F}{V}\right)^2 V^{1/3} \sum_{\vec{n}_2}\sum_{\vec{n}_3}\sum_{\vec{n}_4} \delta_{\vec{n}_p+\vec{n}_2,\vec{n}_3+\vec{n}_4} (1-\hat{n}_p\cdot\hat{n}_2)(1-\hat{n}_3\cdot\hat{n}_4)$$
$$\times \frac{|\vec{n}_p|+|\vec{n}_2|-|\vec{n}_3|}{\sqrt{(|\vec{n}_p|+|\vec{n}_2|-|\vec{n}_3|)^2-n_{4x}^2-n_{4y}^2}}\,\theta((|\vec{n}_p|+|\vec{n}_2|-|\vec{n}_3|)^2-n_{4x}^2-n_{4y}^2)$$
$$\times \left(\bar{\delta}_{n_{4z},\sqrt{(|\vec{n}_p|+|\vec{n}_2|-|\vec{n}_3|)^2-n_{4x}^2-n_{4y}^2}} - \bar{\delta}_{n_{4z},-\sqrt{(|\vec{n}_p|+|\vec{n}_2|-|\vec{n}_3|)^2-n_{4x}^2-n_{4y}^2}}\right)$$
$$\times \left\{ \left[f_4(I-f_2)+\mathrm{Tr}\Big(f_4(I-f_2)\Big)\right] f_3(I-f_p) + (I-f_p)f_3 \left[(I-f_2)f_4+\mathrm{Tr}\Big((I-f_2)f_4\Big)\right]\right.$$
$$\left. - \left[(I-f_4)f_2+\mathrm{Tr}\Big((I-f_4)f_2\Big)\right](I-f_3)f_p - f_p(I-f_3)\left[f_2(I-f_4)+\mathrm{Tr}\Big(f_2(I-f_4)\Big)\right]\right\}$$

Outline

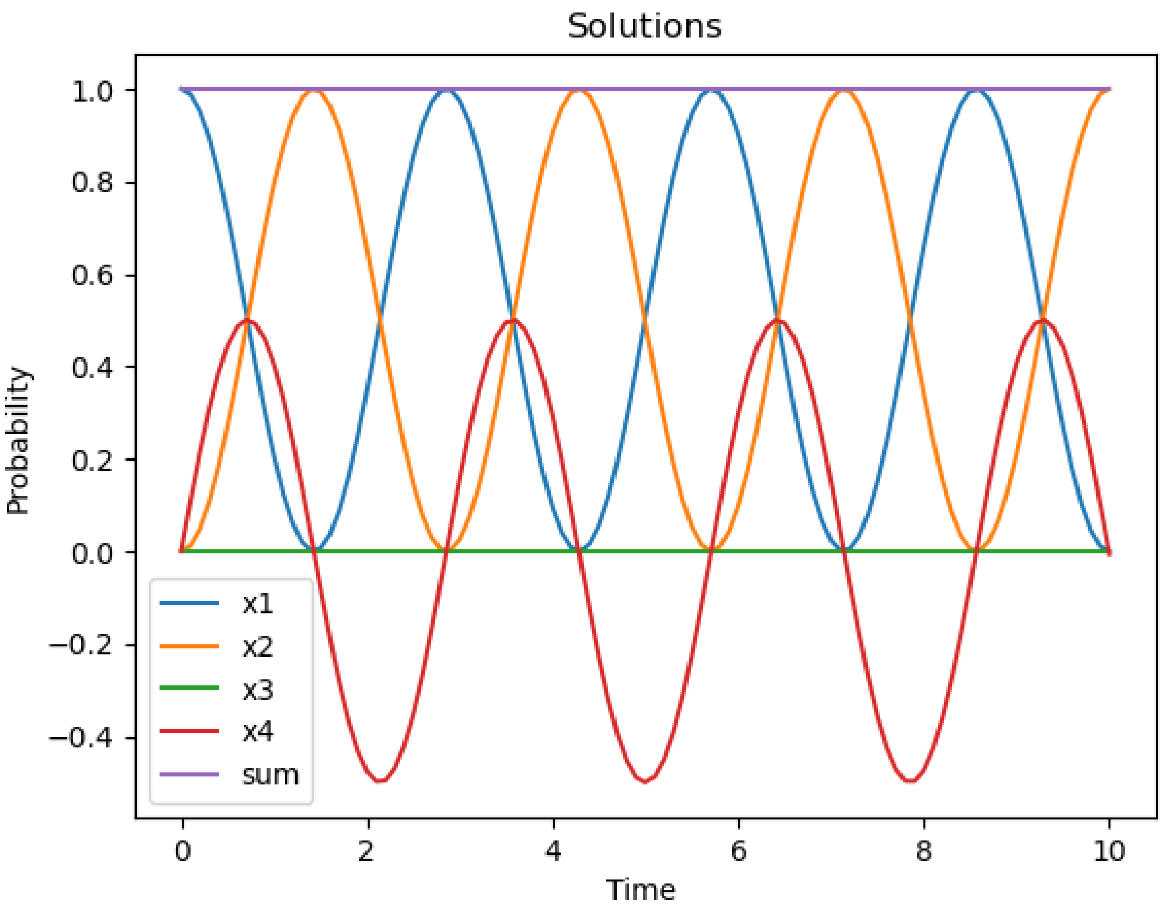
i. Introduction

ii. Quantum Kinetic Equations

iii. Results

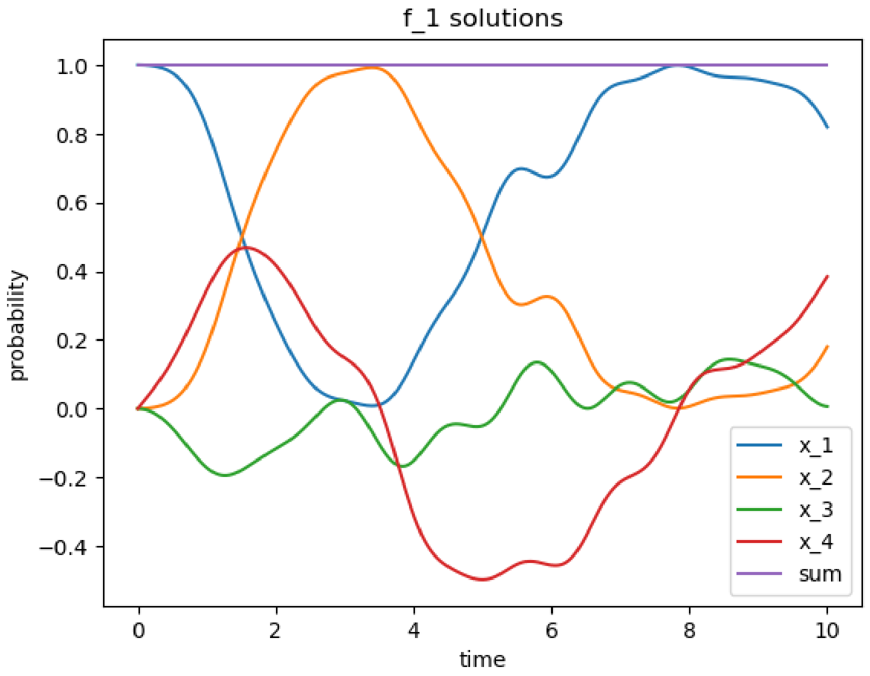
iv. Future Work

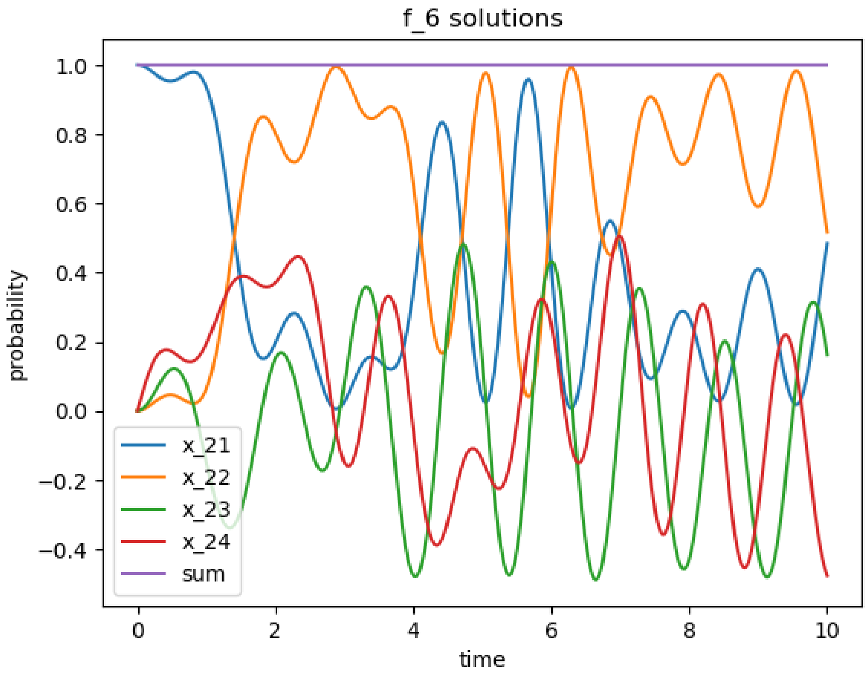
Results: No Coherent Evolution



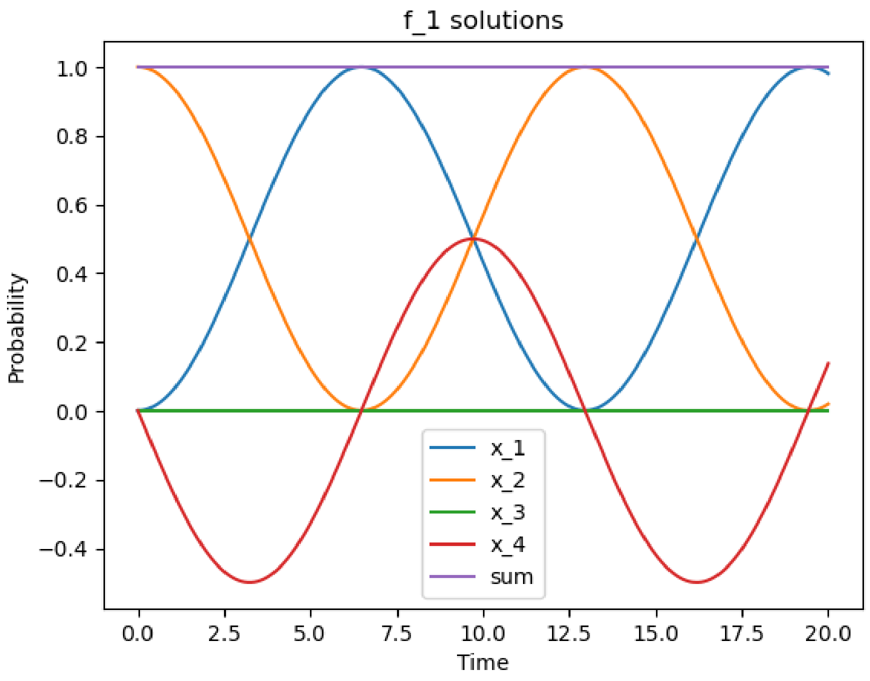
$$f_{n_p}(t) = \begin{pmatrix} 1 - \sin^2(2\theta) sin^2(\frac{\Delta m^2}{4|\bar{p}|} t) & - \\ - & \sin^2(2\theta) sin^2(\frac{\Delta m^2}{4|\bar{p}|} t) \end{pmatrix}$$

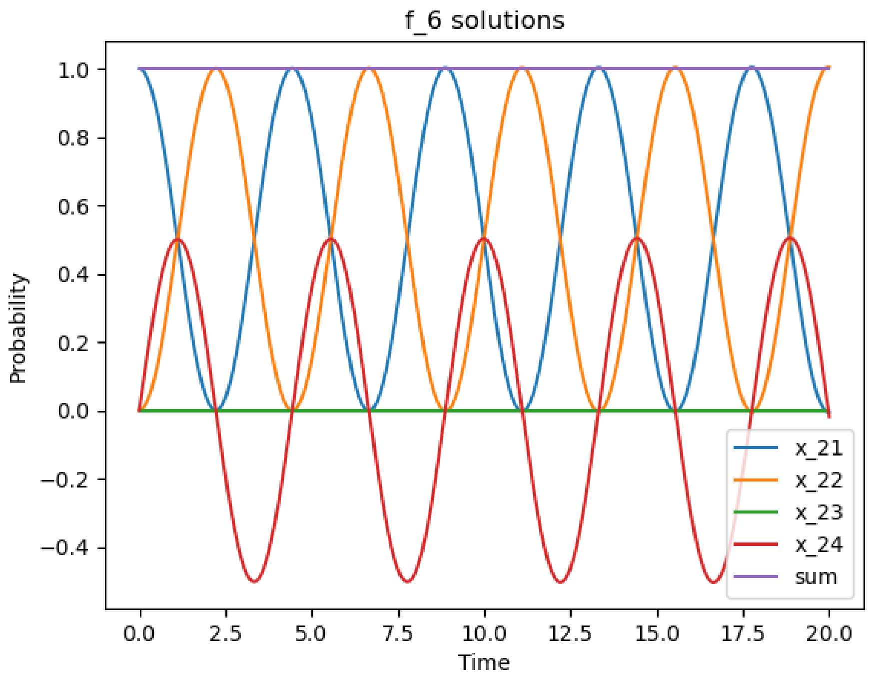
Results: Coherent Evolution



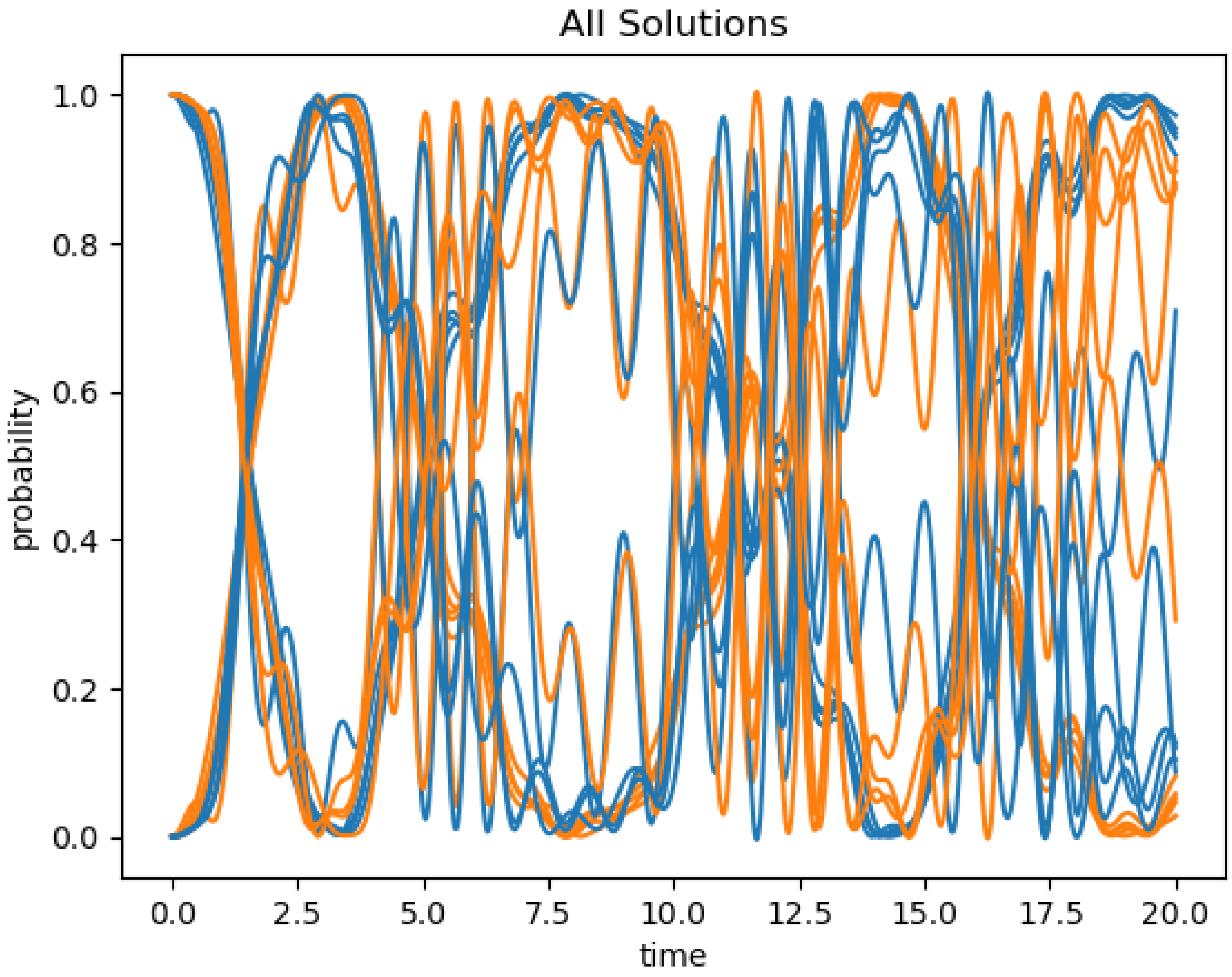


Comparison Between Solutions

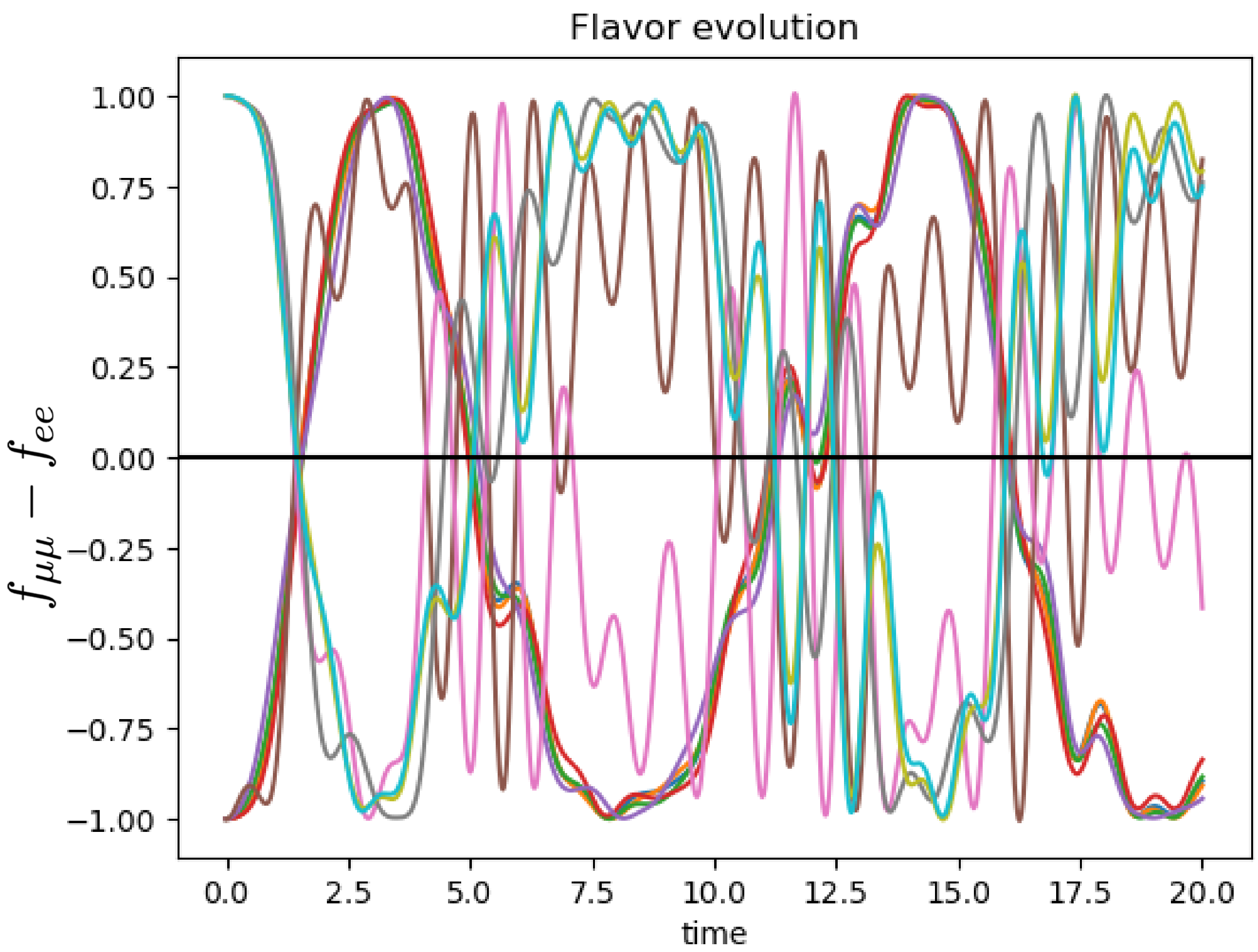




Results: Coherent Evolution

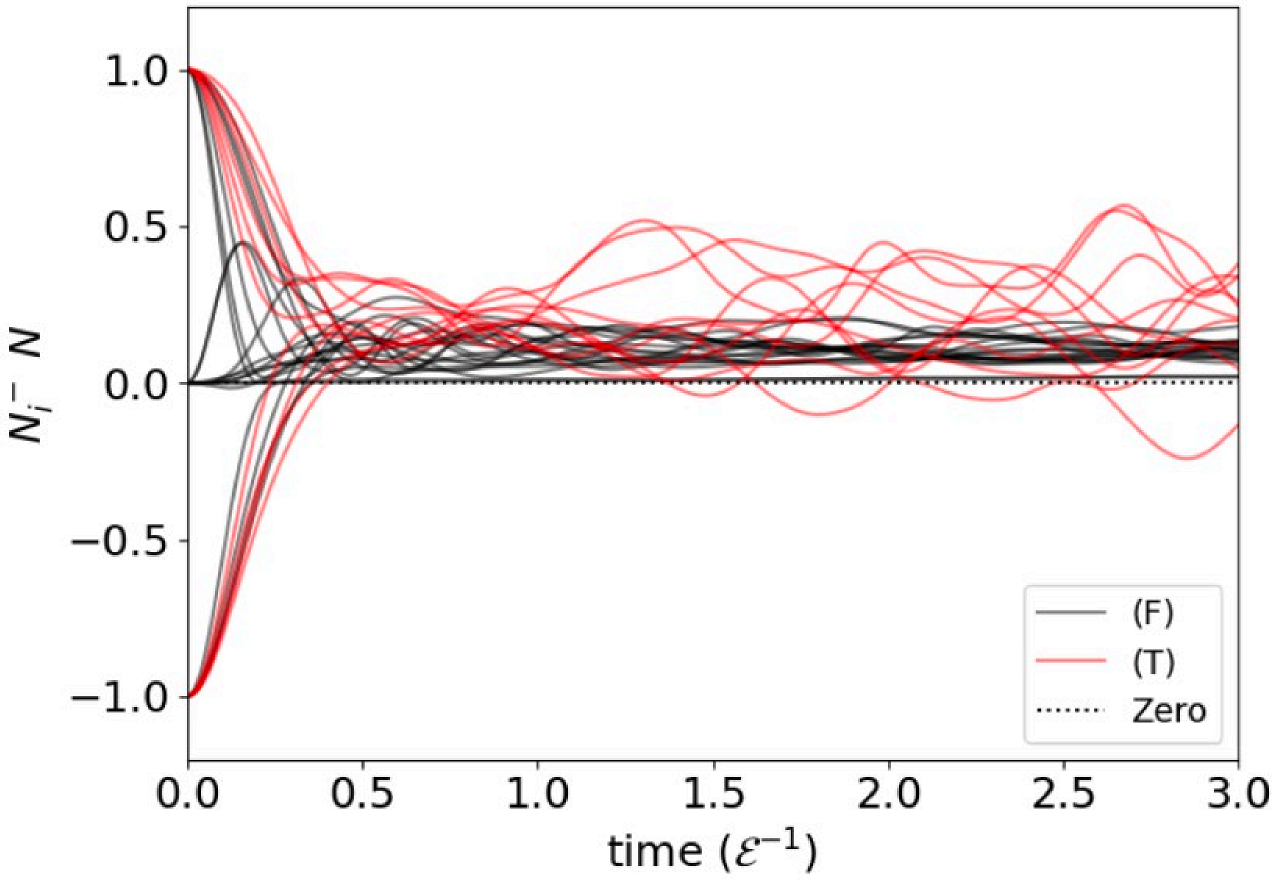


Results: Flavor Evolution



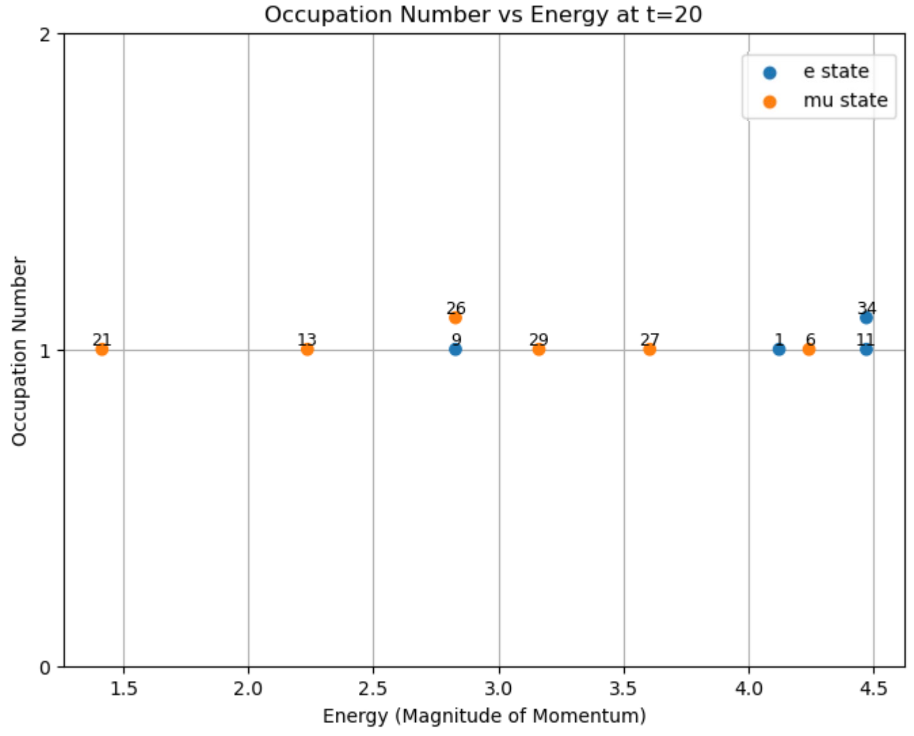
Comparison: Flavor Evolution

- Many body
- With collisions



V. Cirigliano, S. Sen, Y. Yamauchi (2024)

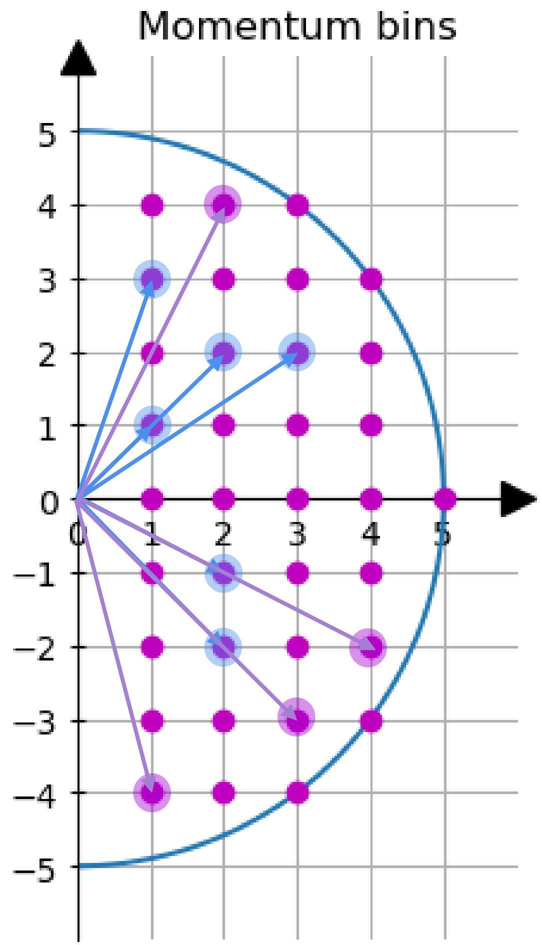
Occupation Number vs. Energy



$$E = |\bar{p}|$$

$$f_{ee} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$f_{\mu\mu} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$



Outline

Future Work

- Collision term
- Compare with quantum many body approach

Thank you!

Special thanks to my advisor, Vincenzo Cirigliano; the REU organizers, Gray Rybka and Arthur Barnard; and my amazing REU cohort!

UNIVERSITY *of* WASHINGTON

INSTITUTE for NUCLEAR THEORY