Neutrino Many-Body Systems

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INSTITUTE for NUCLEAR THEORY

Outline

1. Overview of neutrino astrophysics
2. Background on second quantization and neutrino many-body systems
3. My work this summer

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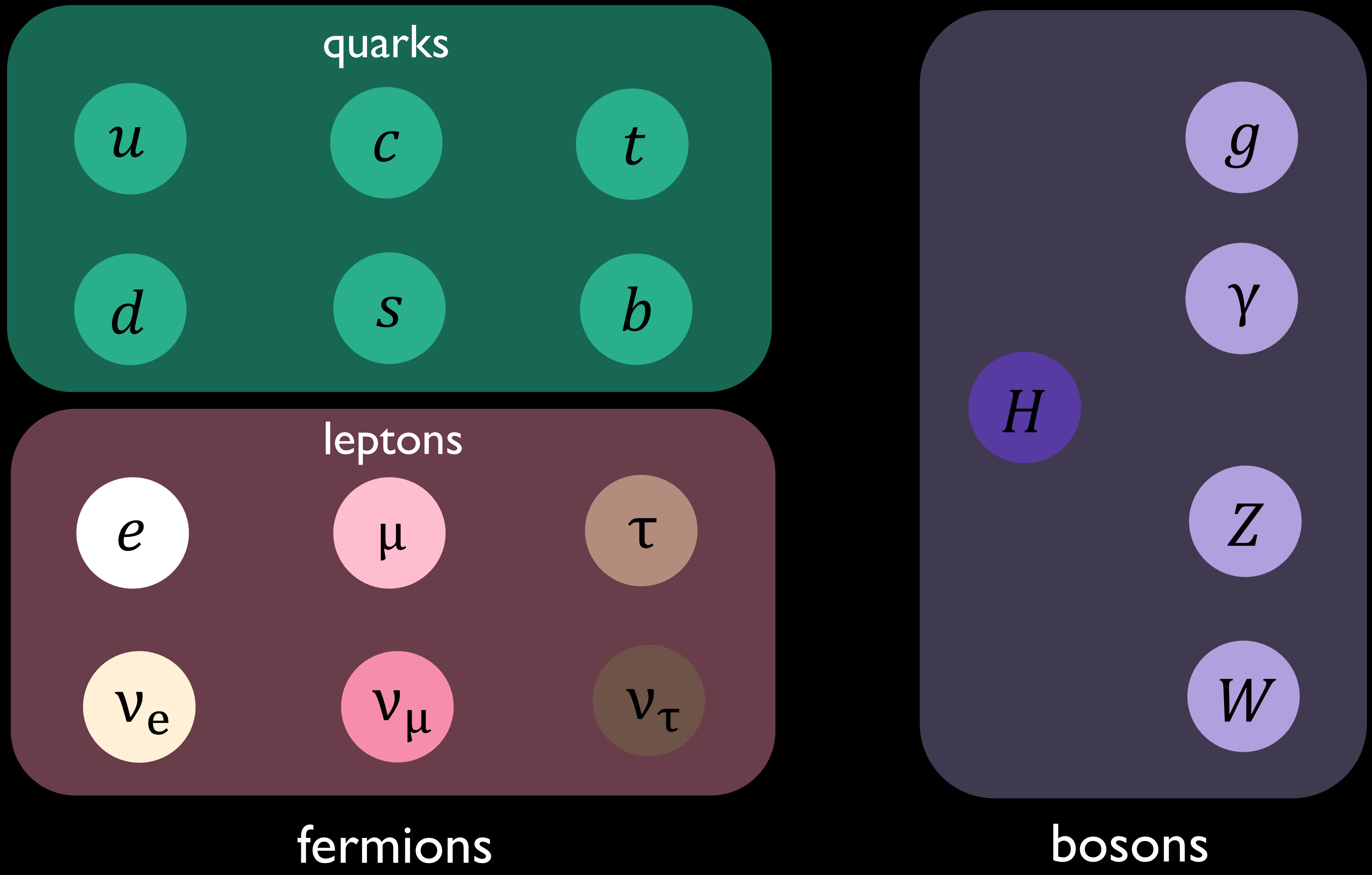
Physics is governed by the Standard Model



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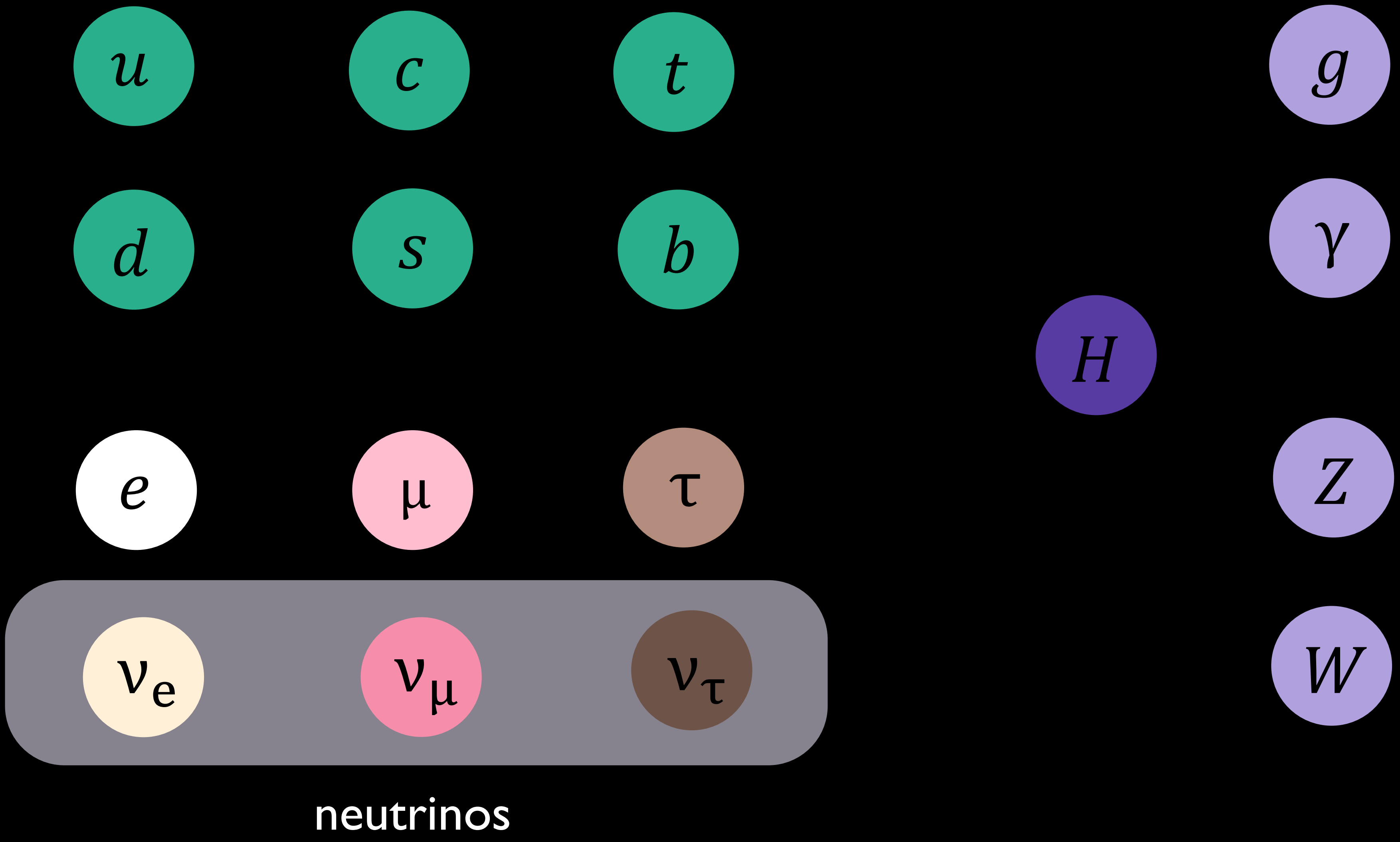


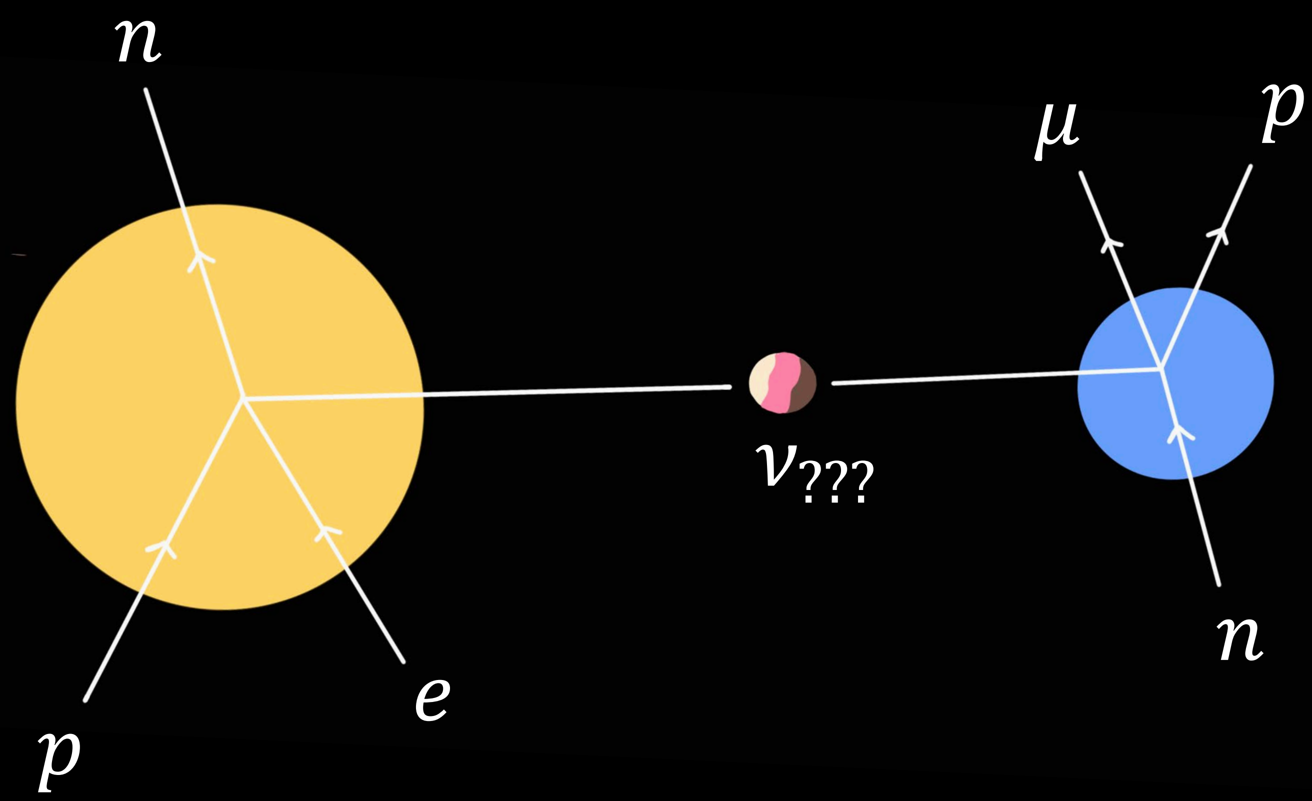
What are neutrinos?

- Tiny particles in the Standard Model
- Only interact via the weak force
- Incredibly abundant
- Come in 3 flavor varieties

ν_e ν_μ ν_τ

Solar neutrinos

- Discrepancy between expected neutrinos based on solar flux and actual neutrino detections
- Instead, neutrinos can change flavor (physicists were only detecting ν_e)



Neutrinos beyond the solar system

- Created in supernovae explosions
- Occur in stellar nucleosynthesis, neutron stars, and black holes
- Around right after the Big Bang

Neutrinos are everywhere! Studying them can help us learn more about different astrophysical phenomena and particle physics

We are interested in simulating systems of many neutrinos and observing how they time-evolve

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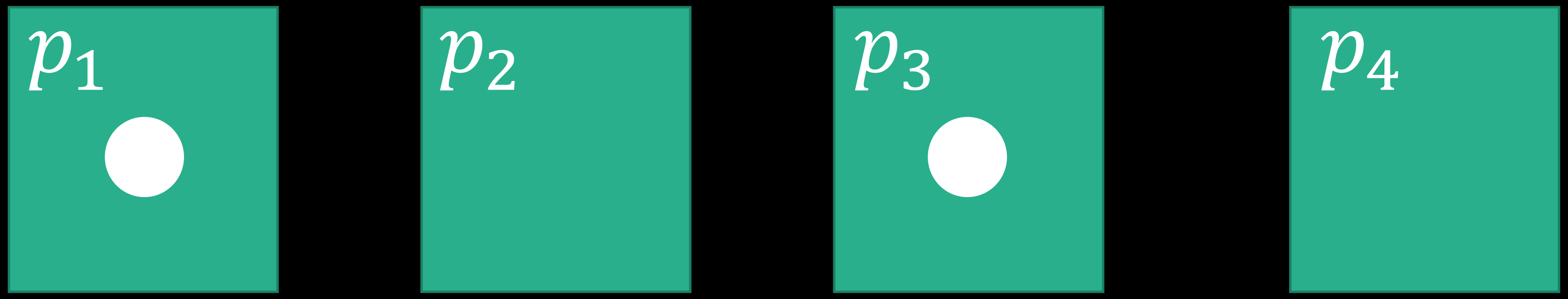
Occupation Number Basis

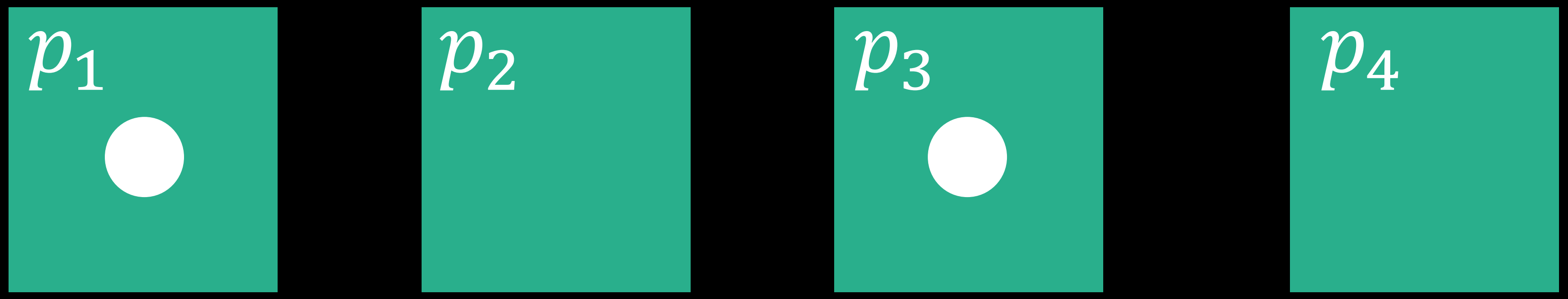
- Many-body systems can be written based on whether a **single-particle state** is occupied
- States correspond to a given **momentum** and **flavor**
- Because fermions cannot occupy the same state, each "bin" will either be 0 (unoccupied) or 1 (occupied)



Occupation Number Basis

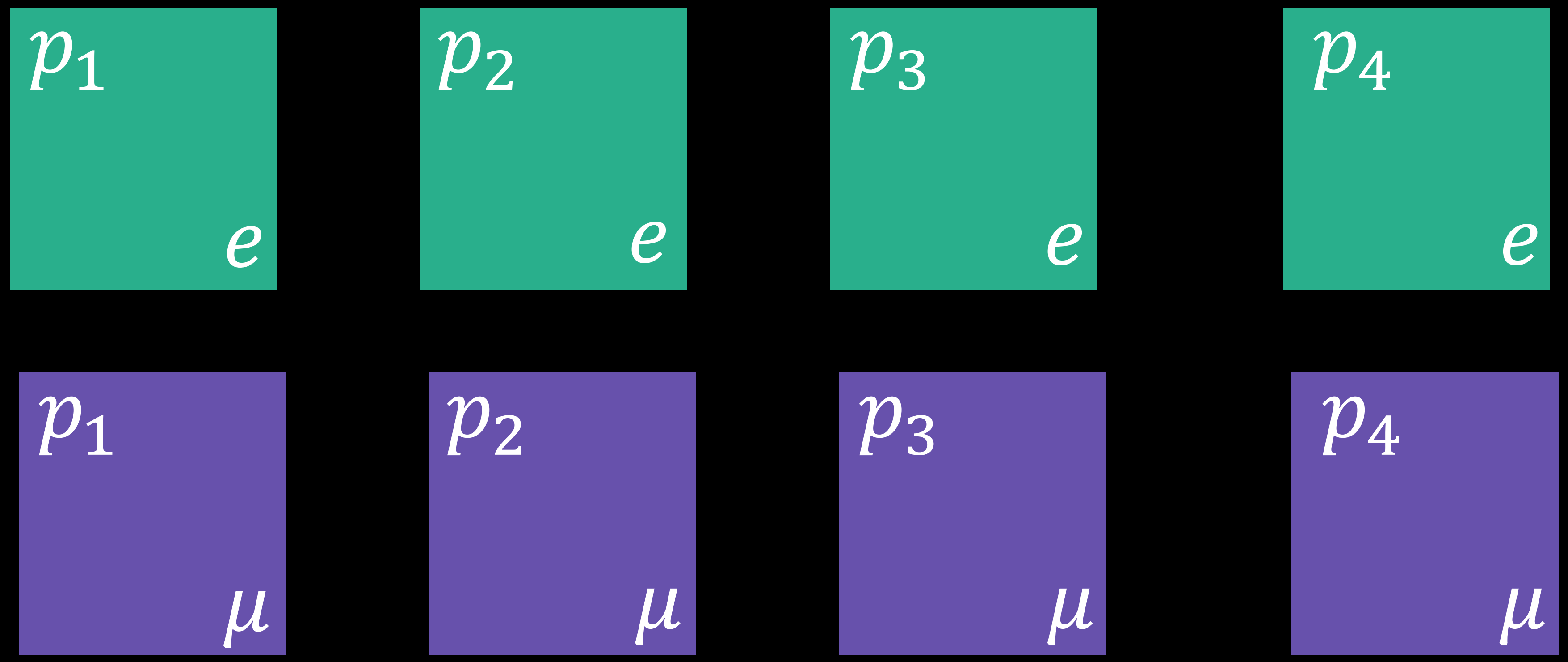
We would write this system like this:

$$\psi = |\,1\;0\;1\;0\,\rangle$$



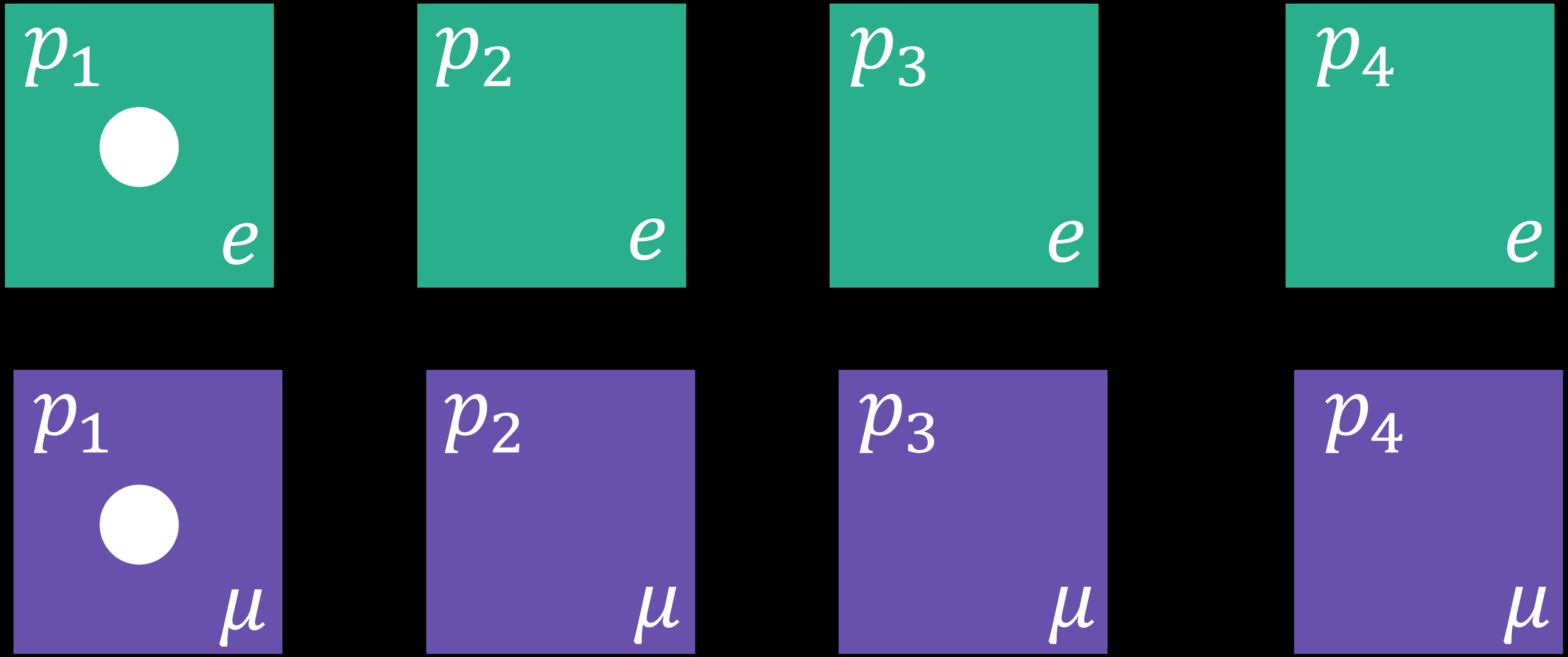
Adding Flavor

Allowing for a 2-flavor system (e and μ), we now have 8 possible states that a neutrino could be in:



Adding Flavor

$$\psi = |\overset{p_1}{\underset{e\ \mu}{1\ 1}}\ \overset{p_2}{0\ 0}\ \overset{p_3}{0\ 0}\ \overset{p_4}{0\ 0}\rangle$$



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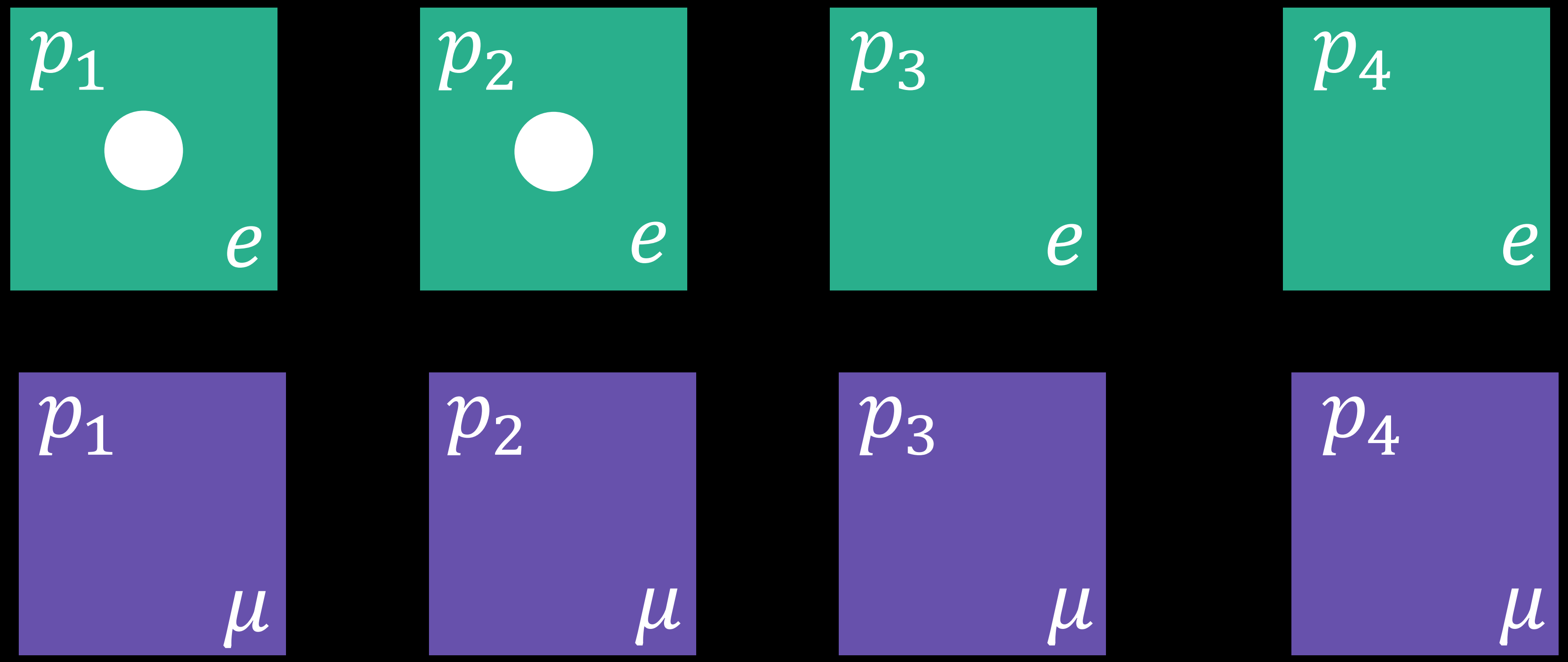
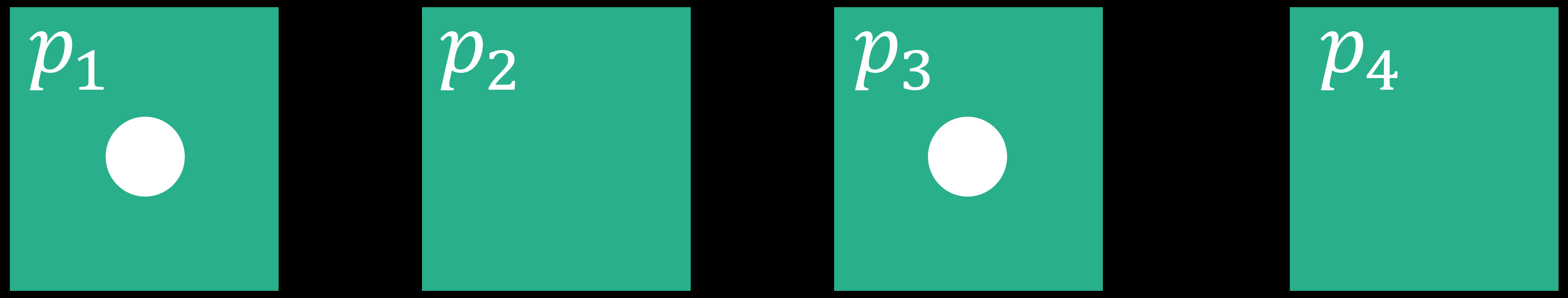


Creation and Annihilation Operators

- $a_i^\dagger$ creates a neutrino in state i
- a_i annihilates a neutrino in state i

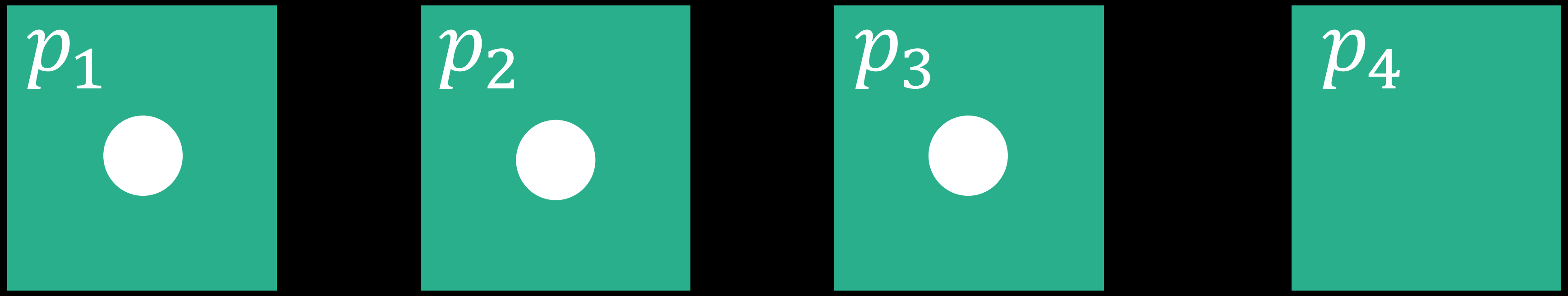
$$\psi = |1\ 0\ 1\ 0\ \rangle$$



Creation and Annihilation Operators

- $a_i^\dagger$ creates a neutrino in state i
- a_i annihilates a neutrino in state i

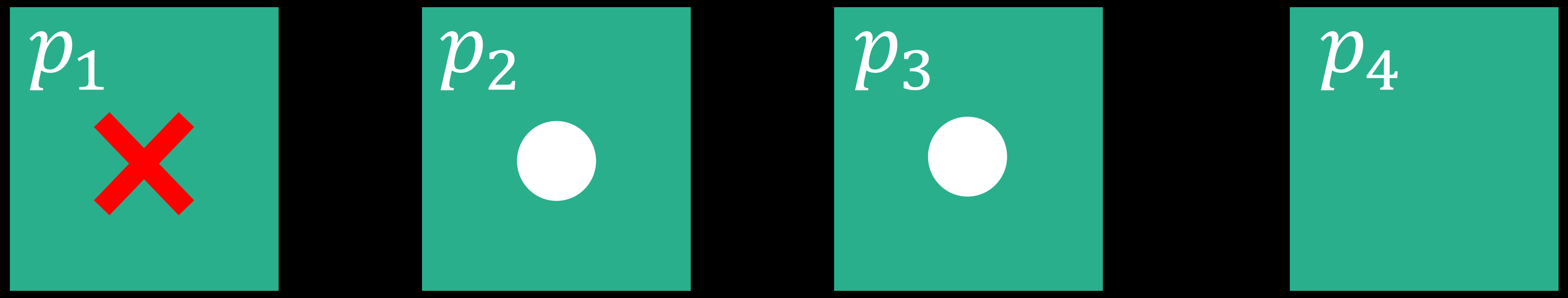
$$a_2^\dagger|1\ 0\ 1\ 0\ \rangle = |\ 1\ 1\ 1\ 0\ \rangle$$



Creation and Annihilation Operators

- $a_i^\dagger$ creates a neutrino in state i
- a_i annihilates a neutrino in state i

$$a_1|1\ 1\ 1\ 0\rangle = |\ 0\ 1\ 1\ 0\ \rangle$$



Creation and Annihilation Operators

- $a_i^\dagger$ creates a neutrino in state i
- a_i annihilates a neutrino in state i

The occupation number basis lets us use creation and annihilation operators more easily!

We also will go on to write the Hamiltonian (expression for the energy of the system) in terms of these operators

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What if we rule out flavor?

Flavor increases the number of possible states by 2^K, where K is the number of momentum modes

For example, with a system of 10 momentum states and 4 neutrinos:

$\binom{10}{4} = 210$ states

$\binom{20}{4} = 4845$ states

Eliminating flavor allows us to probe larger systems and identify how flavor makes neutrino many-body systems behave differently

Constructing the Hamiltonian Matrix

- Hamiltonian is written in terms of operators
- Think of it as a matrix with values telling the system how to get from one state to another

$$H = H_{kin} + H_{vv}$$

Hamiltonian for 1 Flavor

$$H_{kin} = \sum_{p} |p| a_p^\dagger a_p$$

$$H_{\nu\nu} = \frac{G_F}{\sqrt{2}} \sum_{p,\, p',\, q,\, q'} \delta(p + q - p' - q') \times a_{p'}^\dagger a_{q'}^\dagger a_p a_q \times g(p', p, q', q)$$

Constructing the Hamiltonian Matrix

- Example with 4 momentum modes, 2 neutrinos, 1 flavor:
  - There are 6 possible combinations of momenta:
    - p_1, p_2: $\psi = |1\,1\,0\,0\rangle$
    - p_1, p_3: $\psi = |1\,0\,1\,0\rangle$
    - p_1, p_4: $\psi = |1\,0\,0\,1\rangle$
    - p_2, p_3: $\psi = |0\,1\,1\,0\rangle$
    - p_2, p_4: $\psi = |0\,1\,0\,1\rangle$
    - p_3, p_4: $\psi = |0\,0\,1\,1\rangle$

Momentum is conserved; no interaction allowed

final state

initial state

$$
\begin{array}{c c}
 & \begin{array}{cccccc} p_1,p_2 & p_1,p_3 & p_1,p_4 & p_2,p_3 & p_2,p_4 & p_3,p_4 \end{array} \\
\begin{array}{c} p_1,p_2 \\ p_1,p_3 \\ p_1,p_4 \\ p_2,p_3 \\ p_2,p_4 \\ p_3,p_4 \end{array} &
\begin{pmatrix}
\Delta & 0 & 0 & 0 & 0 & 0 \\
0 & \Delta & 0 & 0 & 0 & 0 \\
0 & 0 & \Delta & 0 & 0 & 0 \\
0 & 0 & 0 & \Delta & 0 & 0 \\
0 & 0 & 0 & 0 & \Delta & 0 \\
0 & 0 & 0 & 0 & 0 & \Delta
\end{pmatrix}
\end{array}
$$

Consider momentum pairs where $p_1 + p_2 = p_3 + p_4$

final state

$$
\begin{array}{cc}
 & \begin{array}{cccccc} p_1,p_2 & p_1,p_3 & p_1,p_4 & p_2,p_3 & p_2,p_4 & p_3,p_4 \end{array} \\
\begin{array}{c} p_1,p_2 \\ p_1,p_3 \\ p_1,p_4 \\ p_2,p_3 \\ p_2,p_4 \\ p_3,p_4 \end{array} &
\begin{pmatrix}
\Delta & 0 & 0 & 0 & 0 & \epsilon \\
0 & \Delta & 0 & 0 & 0 & 0 \\
0 & 0 & \Delta & 0 & 0 & 0 \\
0 & 0 & 0 & \Delta & 0 & 0 \\
0 & 0 & 0 & 0 & \Delta & 0 \\
\epsilon & 0 & 0 & 0 & 0 & \Delta
\end{pmatrix}
\end{array}
$$

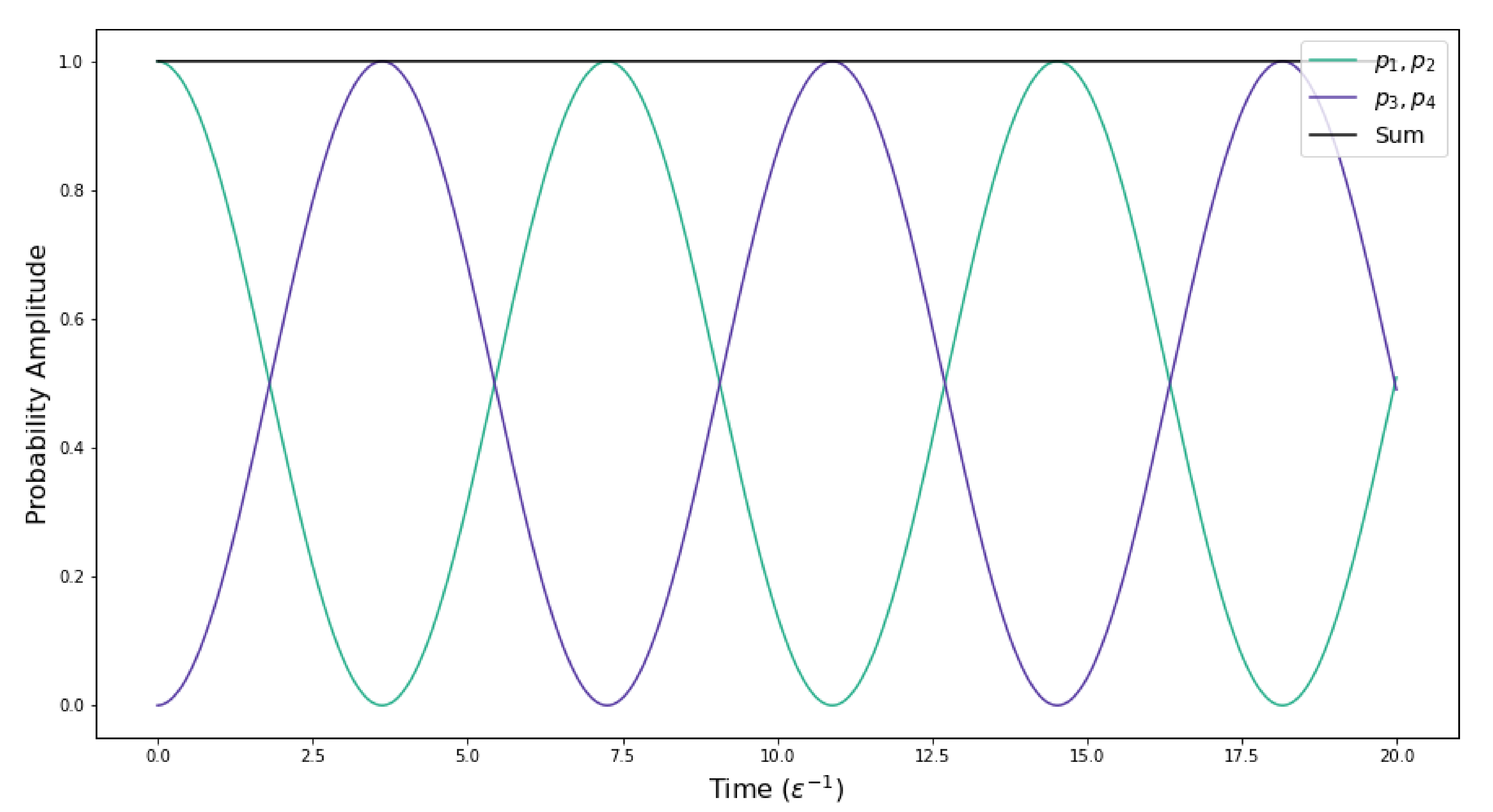
initial state

Recall this term from H_{vv}

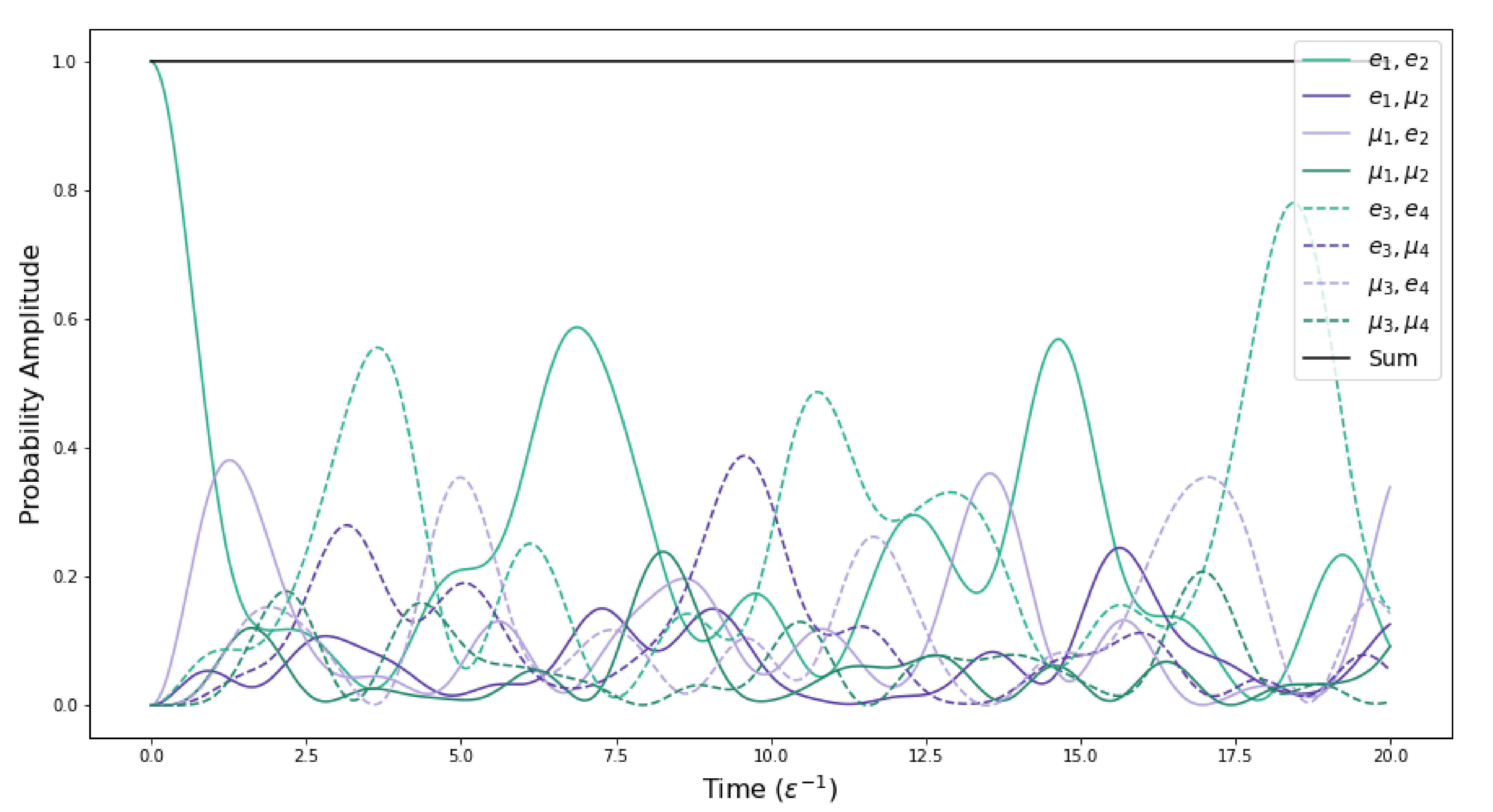
$\delta(p + q - p' - q')$

We can use the information encoded in the Hamiltonian to time-evolve the system!

Flavorless Hamiltonian



Flavorful Hamiltonian



Number Operators

- Number operators represent the probability that a given **momentum bin** is occupied (ie. p_1)
- Different than the Hamiltonian probability amplitudes of a given **many-body state** (ie. $|\ 1\ 1\ 0\ 0\ \rangle$)

Number Operators

- Example with 4 momentum modes, 2 neutrinos, 1 flavor:
  - Each possible state has a probability amplitude that evolves with time:
    - p_1, p_2: $\psi = C_1 |\, 1\ 1\ 0\ 0\, \rangle$
    - p_1, p_3: $\psi = C_2 |\, 1\ 0\ 1\ 0\, \rangle$
    - p_1, p_4: $\psi = C_3 |\, 1\ 0\ 0\ 1\, \rangle$
    - p_2, p_3: $\psi = C_4 |\, 0\ 1\ 1\ 0\, \rangle$
    - p_2, p_4: $\psi = C_5 |\, 0\ 1\ 0\ 1\, \rangle$
    - p_3, p_4: $\psi = C_6 |\, 0\ 0\ 1\ 1\, \rangle$

Number Operators

$$N_1 = |C_1|^2 + |C_2|^2 + |C_3|^2$$
$$N_2 = |C_1|^2 + |C_4|^2 + |C_5|^2$$
$$N_3 = |C_2|^2 + |C_4|^2 + |C_6|^2$$
$$N_4 = |C_3|^2 + |C_5|^2 + |C_6|^2$$

$$p_1, p_2\colon\ \psi = C_1|1\,1\,0\,0\rangle$$
$$p_1, p_3\colon\ \psi = C_2|1\,0\,1\,0\rangle$$
$$p_1, p_4\colon\ \psi = C_3|1\,0\,0\,1\rangle$$
$$p_2, p_3\colon\ \psi = C_4|0\,1\,1\,0\rangle$$
$$p_2, p_4\colon\ \psi = C_5|0\,1\,0\,1\rangle$$
$$p_3, p_4\colon\ \psi = C_6|0\,0\,1\,1\rangle$$

Number Operators

$N_1 = |C_1|^2$

$N_2 = |C_1|^2$

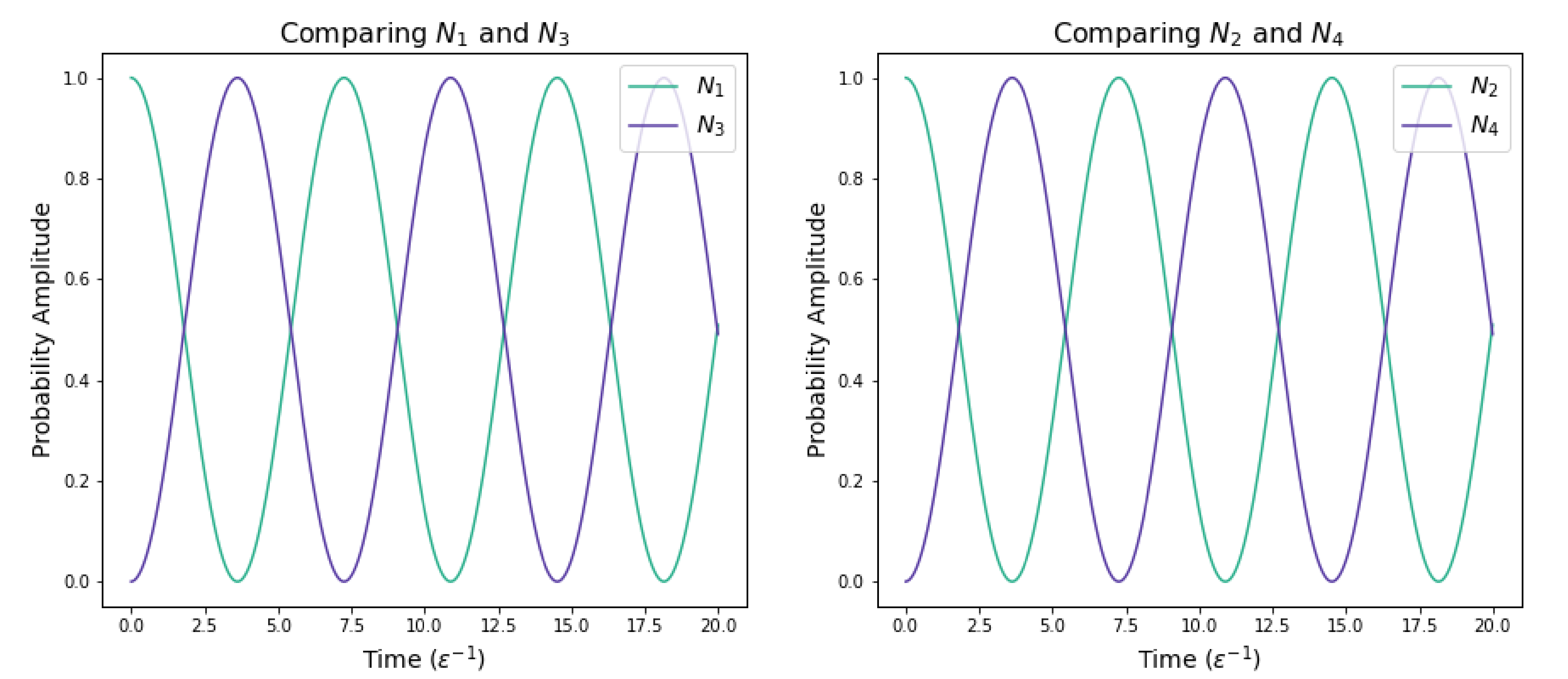
$N_3 = \qquad |C_6|^2$

$N_4 = \qquad |C_6|^2$

p_1, p_2: $\psi = C_1 |\, 1\ 1\ 0\ 0 \,\rangle$

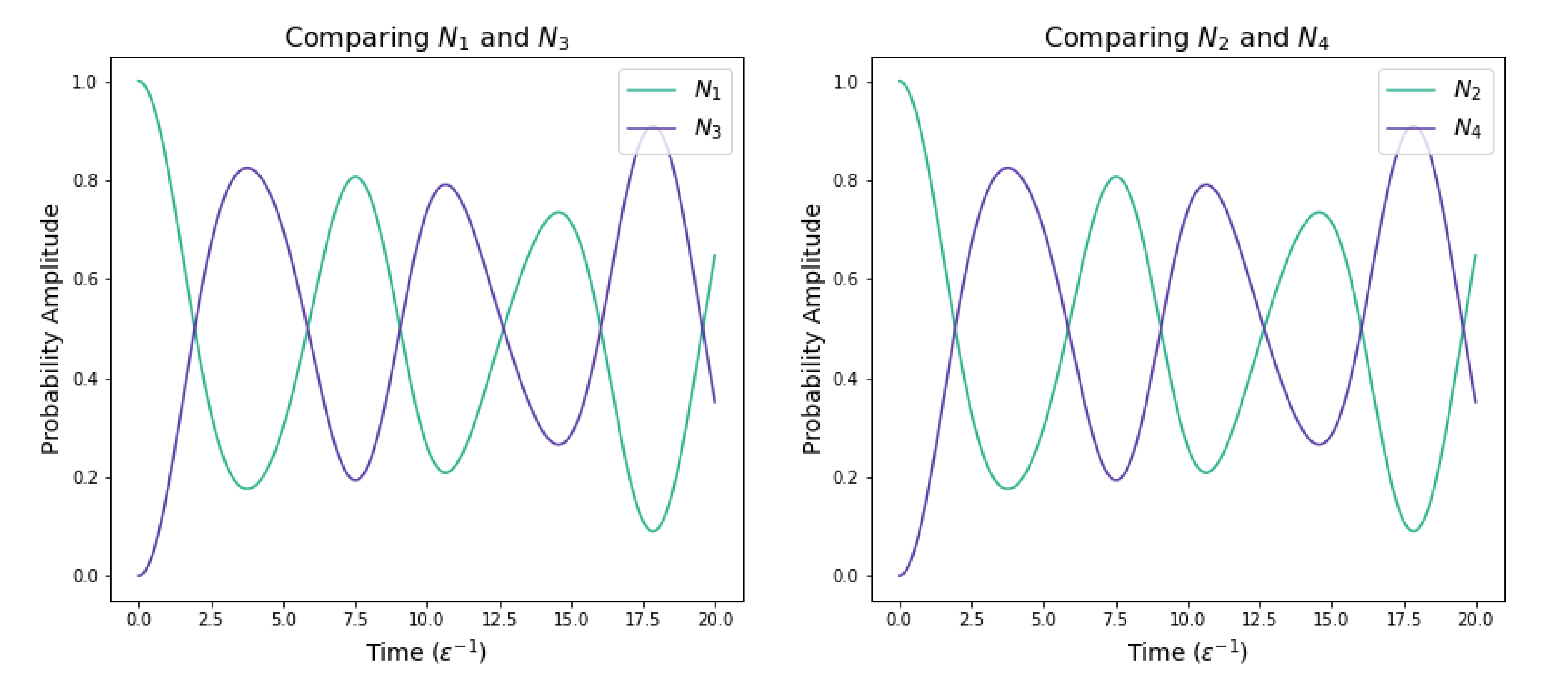
p_3, p_4: $\psi = C_6 |\, 0\ 0\ 1\ 1 \,\rangle$

Graphing N_i



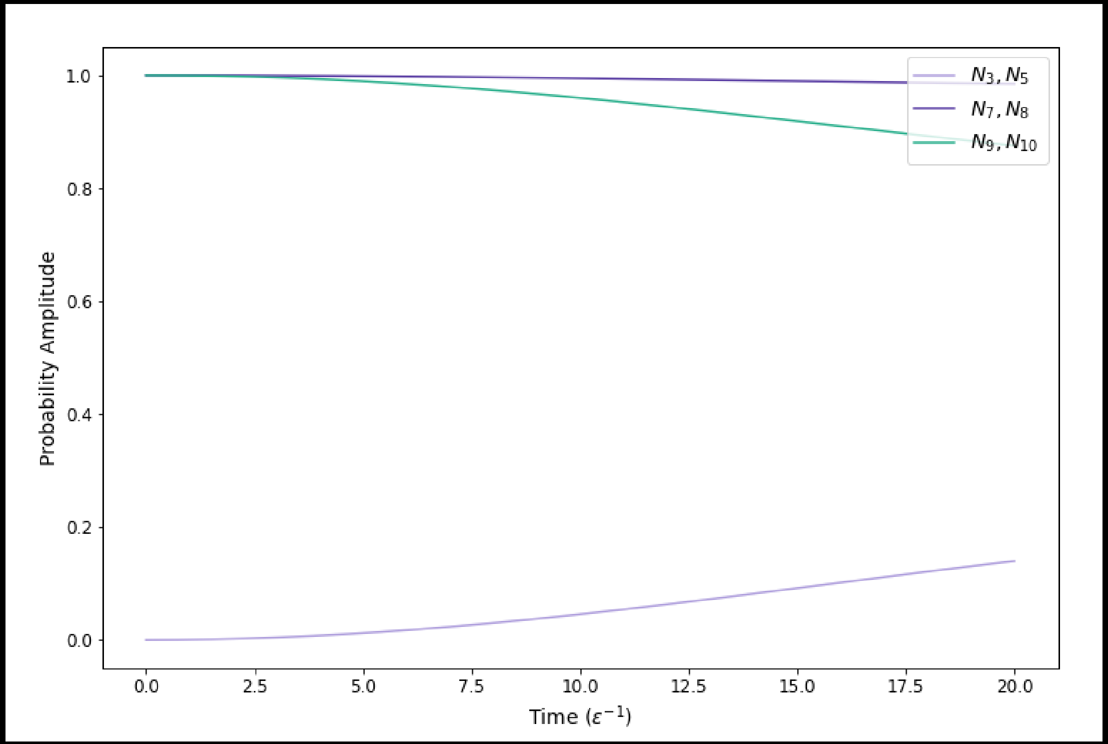
Comparing N_1 and N_3
N_1
N_3
Probability Amplitude
Time (ε^{-1})
0.0
0.2
0.4
0.6
0.8
1.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Comparing N_2 and N_4
N_2
N_4
Probability Amplitude
Time (ε^{-1})

Graphing N_i with Flavor



What happens when states do not talk to each other?

Larger system – neutrinos stay in certain bins



kinetic

$$H = \begin{pmatrix} \Delta & \epsilon \\ \epsilon & -\Delta \end{pmatrix}$$

potential

When $\Delta \gg \epsilon$, neutrinos prefer to stay in their initial state

10 momentum modes, 4 neutrinos, 1 flavor – neutrinos start in p_7, p_8, p_9, p_{10}

What happens when states do not talk to each other?

Implement kinetic energy conservation ($\Delta \rightarrow 0$) to correct for this!

If kinetic energy is conserved, we expect all states to have the same kinetic energy…

…which means that they should all become equally likely.

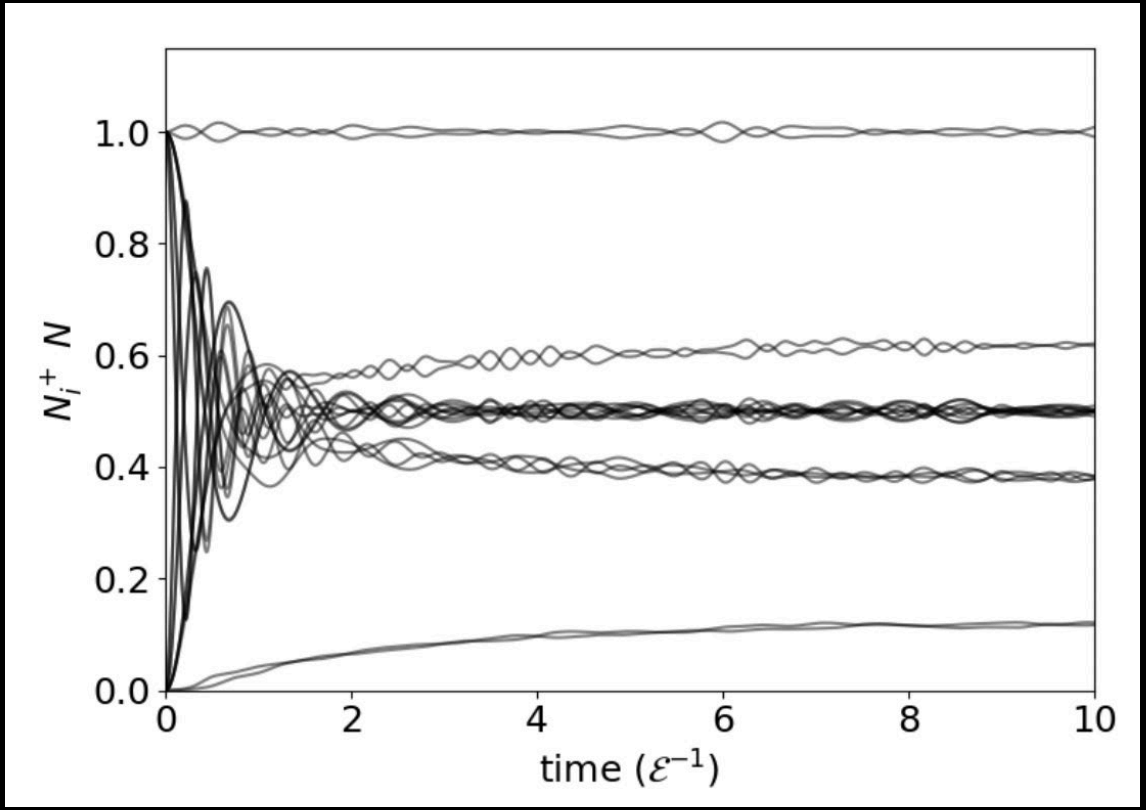
Equilibration

$$\langle N \rangle = \sum_i w_i \langle \psi_i \mid \mathrm{N} \mid \psi_i \rangle$$

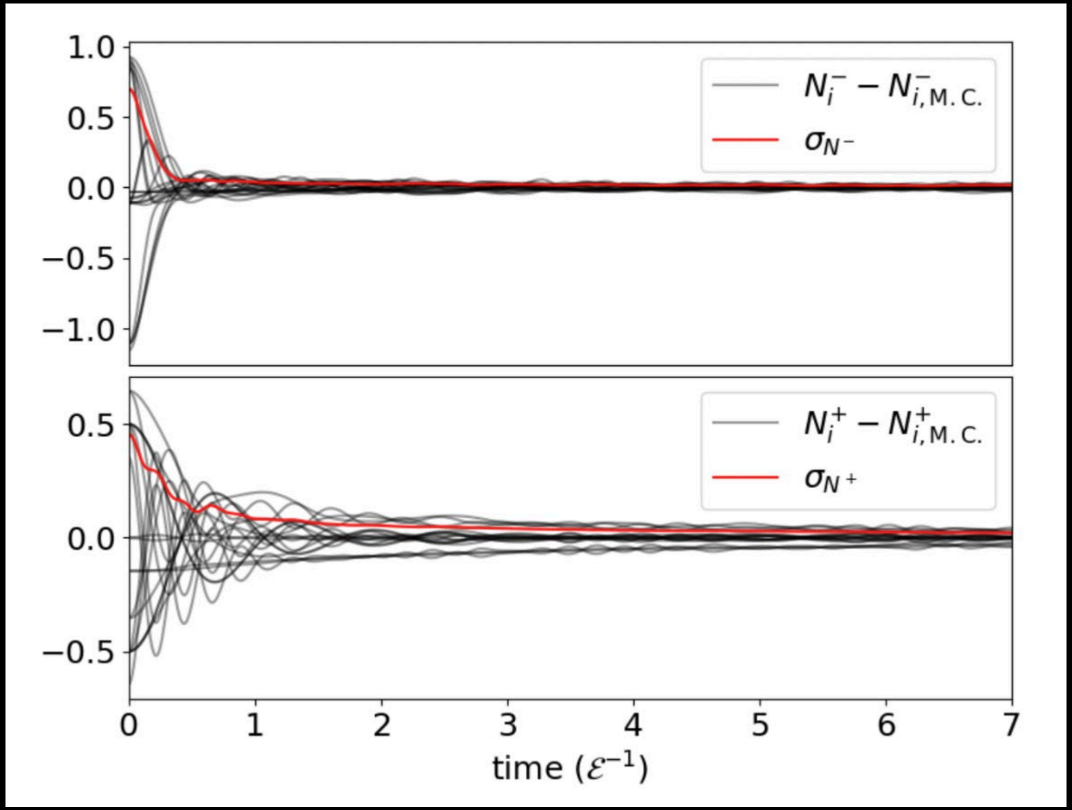
We want to see whether the states equilibrate, or thermalize, without flavor as well as with flavor.

State Equilibration in 2 Flavors

N_i



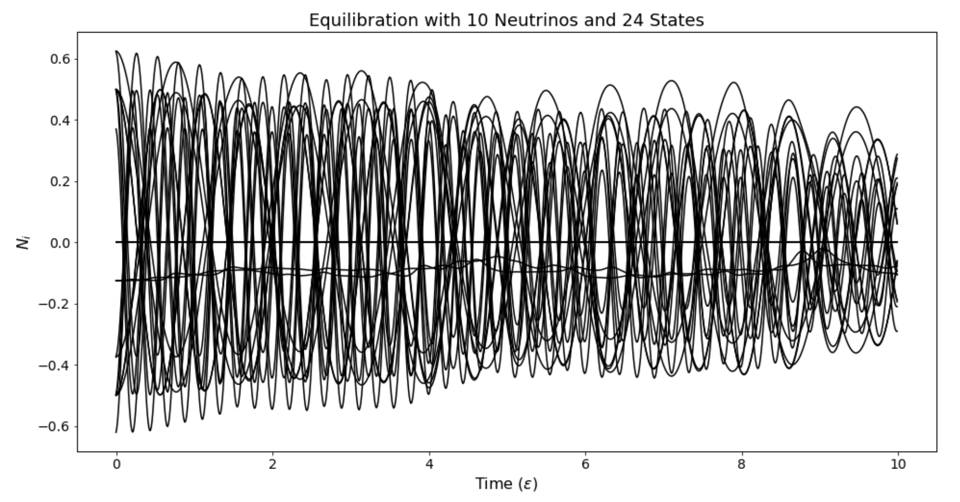
$N_i - N_{i,M.C.}$

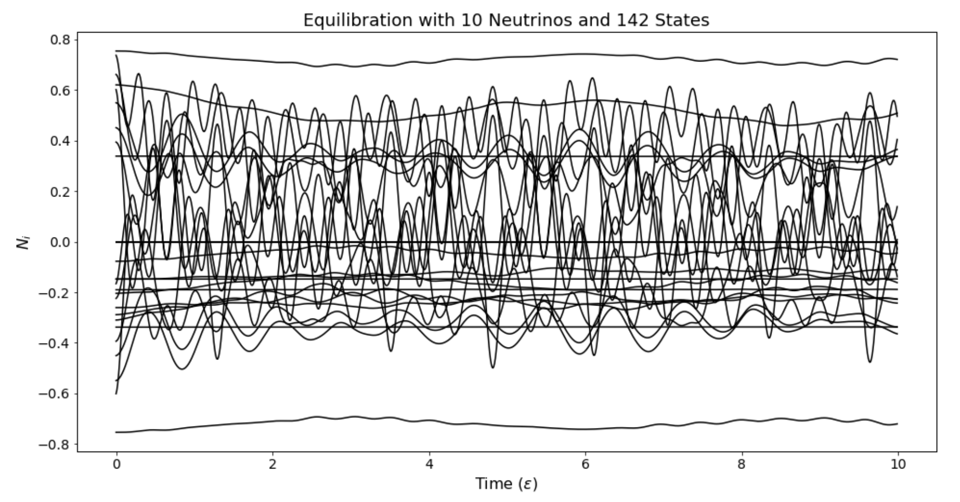


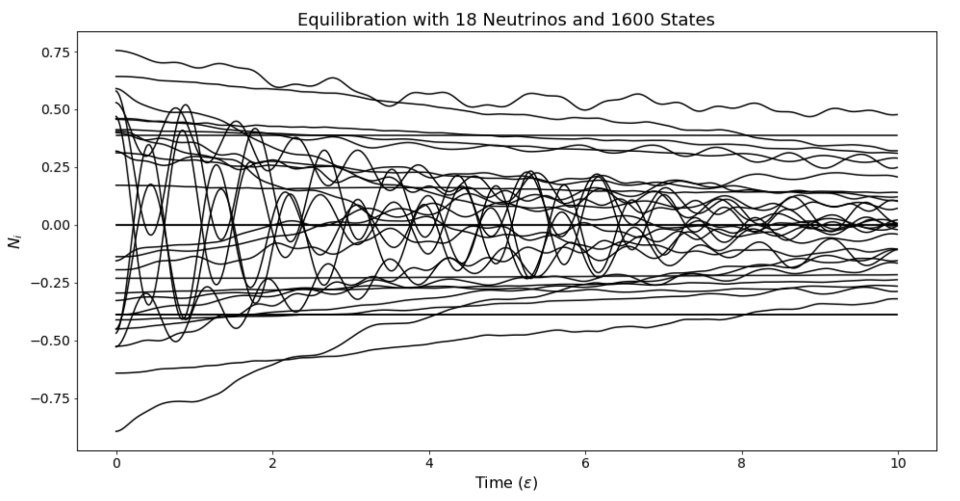
Cirigliano & Sen & Yamauchi 2024

35 momentum modes, 10 neutrinos, 2 flavors

State Equilibration in 1 Flavor







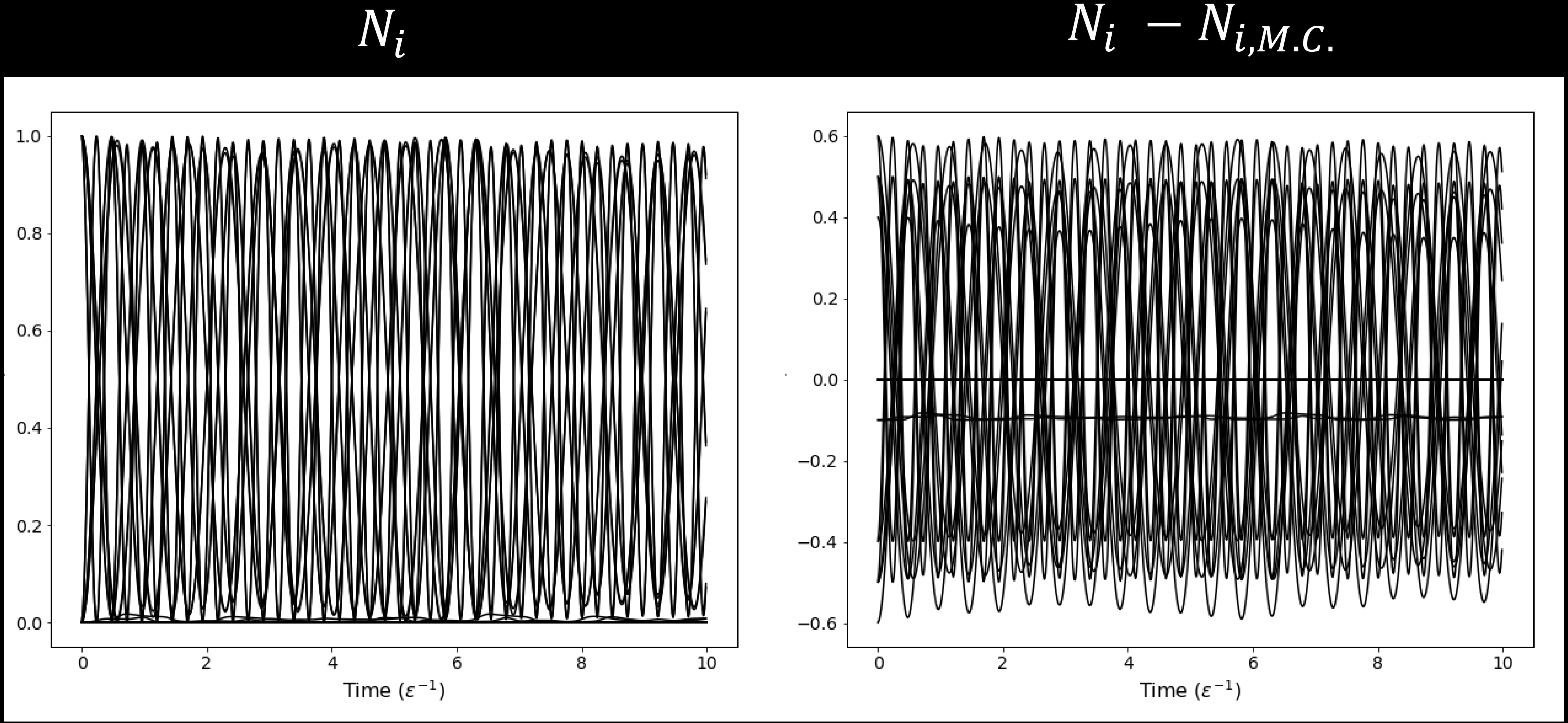
No thermalization is happening!

Why is thermalization only happening in the flavorful case?

- Could it be that the number of possible states needs to be higher?
- Does flavor somehow create more stability or equilibrium in the system?

Look at the same system configuration in 1, 2, and 3 flavors to test

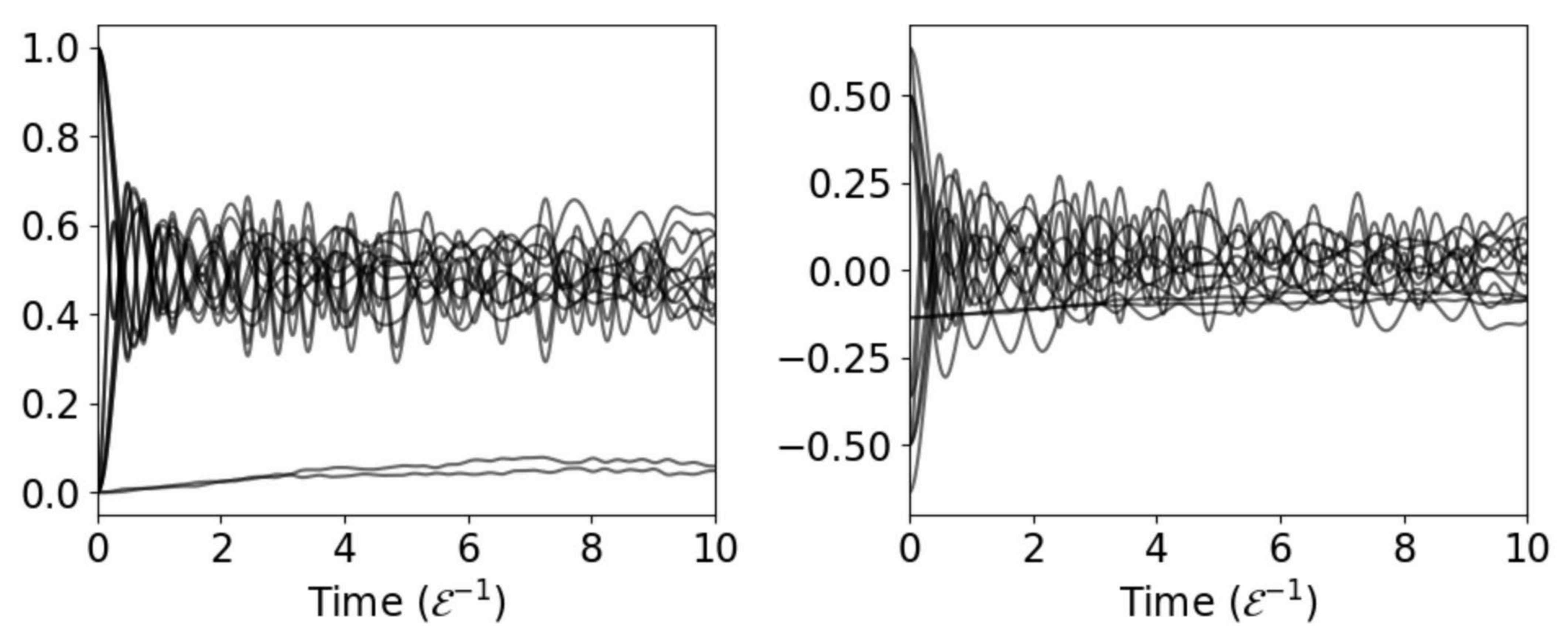
1 flavor: e, e, e, e, e, e



20 momentum modes, 6 neutrinos

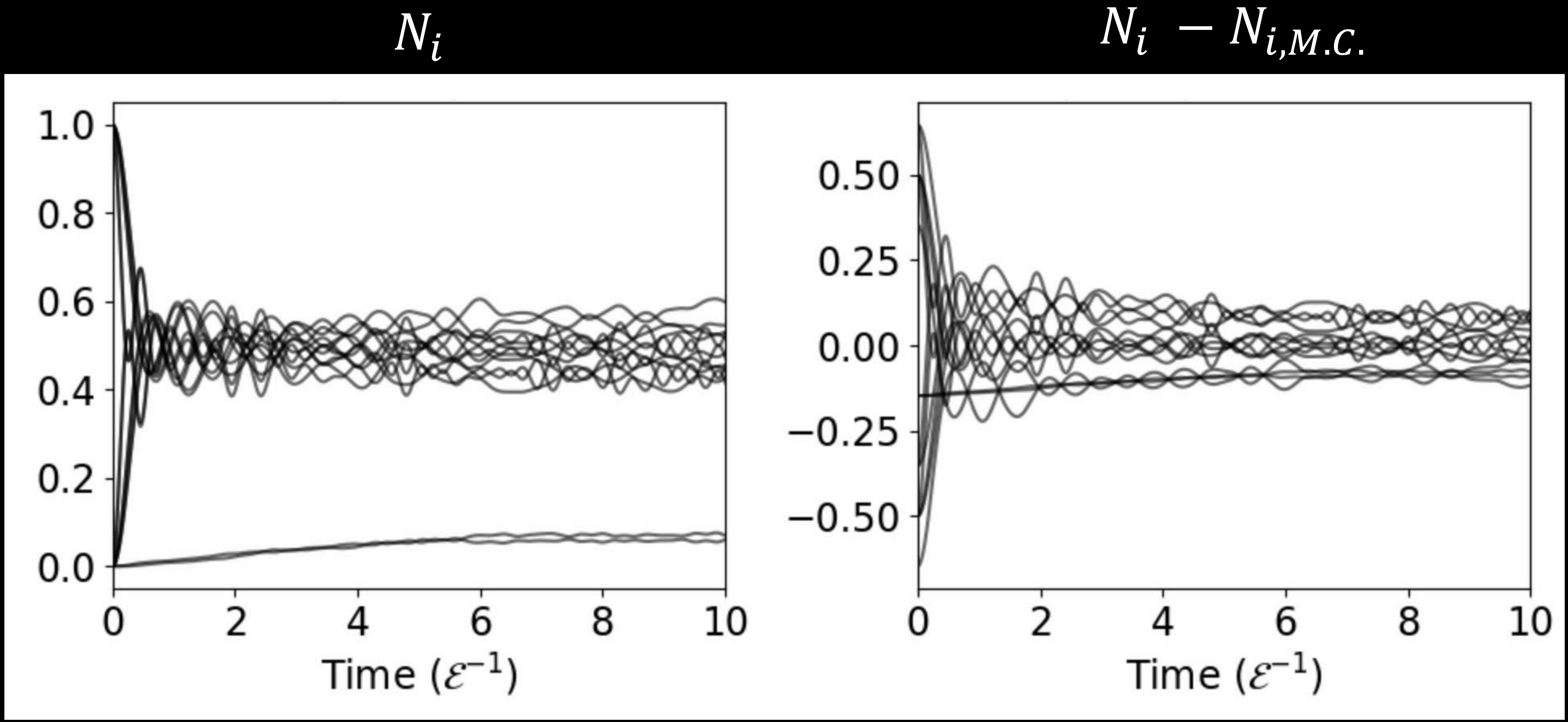
2 flavor: e, e, e, μ, μ, μ

N_i $\quad$ $N_i - N_{i,M.C.}$



20 momentum modes, 6 neutrinos

3 flavor: $e, e, \mu, \mu, \tau, \tau$



20 momentum modes, 6 neutrinos

Next Steps

- Investigate why flavor affects equilibration
  - May have to do with the energy level spacings – can set whether a system equilibrates
- Continue probing larger systems!

Questions?

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