

The background of the slide is a dark, deep blue space filled with a nebula. The nebula consists of wispy, glowing clouds of gas and dust. In the center, there is a brighter, more concentrated area of light blue and white. To the right of this central area, there is a distinct, glowing red and orange region, suggesting a star or a hot core. The overall effect is a cosmic, ethereal atmosphere.

Electromagnetic Form Factors of Nucleons

Nathan Apfel | Prof. Jerry Miller

2. What is a form factor?

For simple point particles



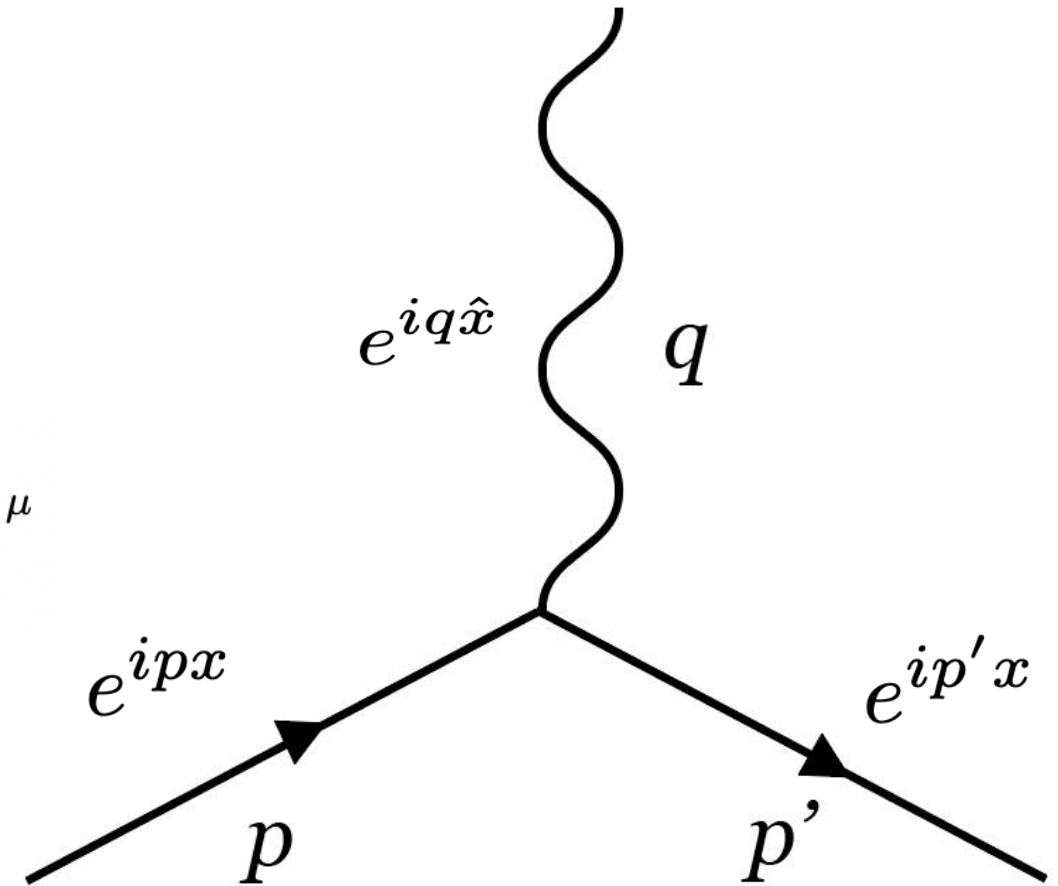
Basically

$$T_{fi} = \langle f | \hat{V} | i \rangle \quad \hat{V} = \hat{j}^\mu A_\mu(\hat{x}) \quad *$$

$$T_{fi} = \int dx \underbrace{\psi'^\dagger(x) \hat{j}^\mu \psi(x)}_{j_{fi}^\mu(x)} A_\mu(x) \quad \hat{j}^\mu = e \left(\overleftarrow{\hat{p}} + \overrightarrow{\hat{p}} \right)^\mu$$

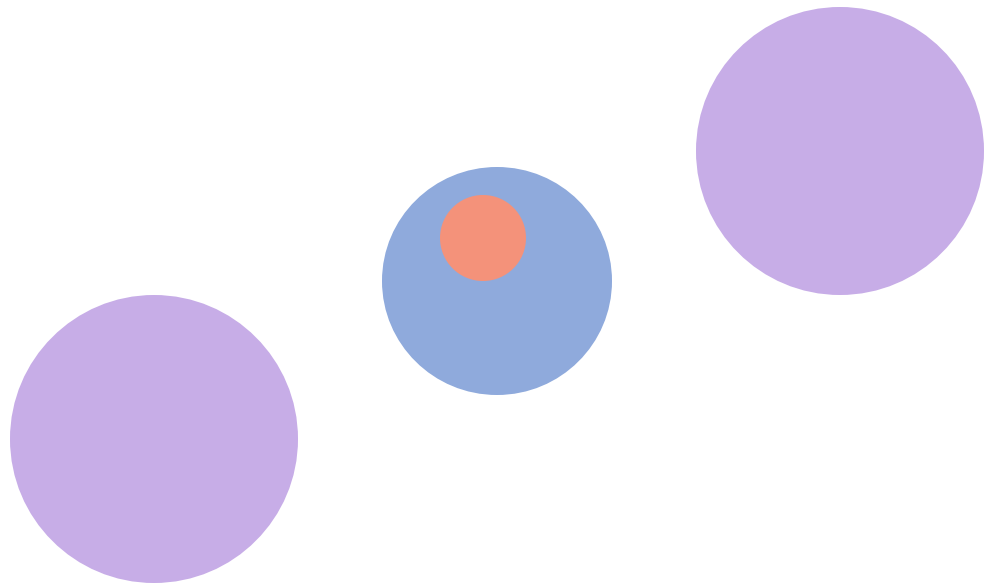
- Plane waves:

$$T_{p \rightarrow p'} = \epsilon_\mu j_{p \rightarrow p'}^\mu(0) \frac{1}{(2\pi)^4} \delta((p + q) - p')$$

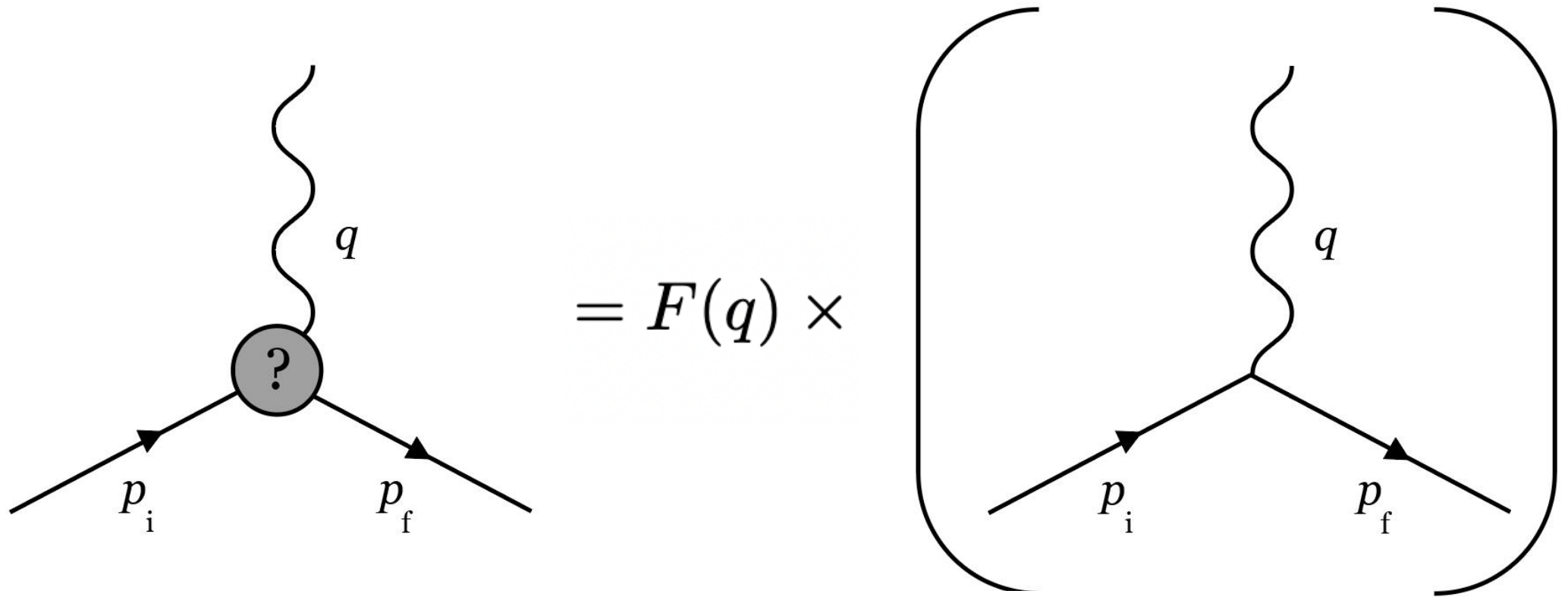


*This is a bit of an abuse of notation, but for conceptual purposes it's close enough

What is a form factor?



What is a form factor?



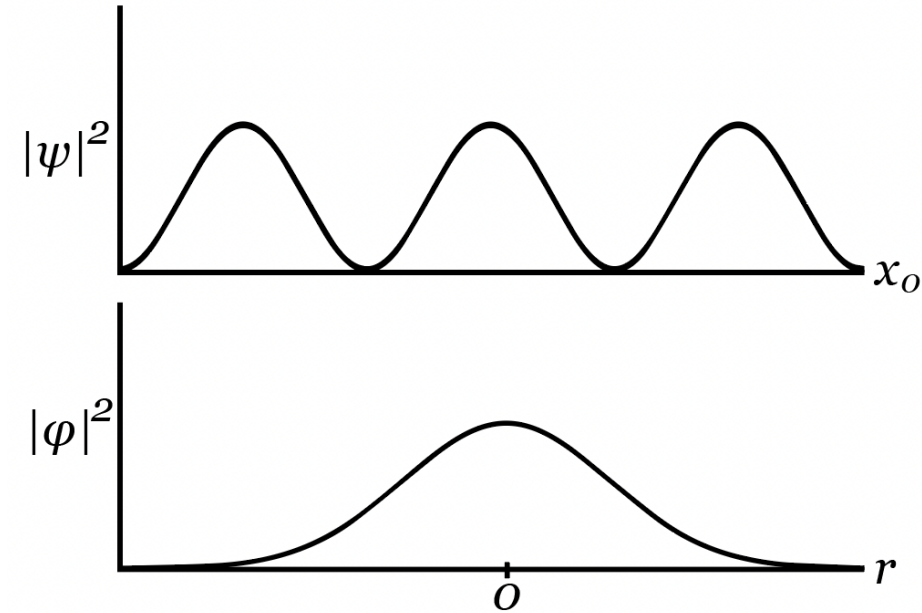
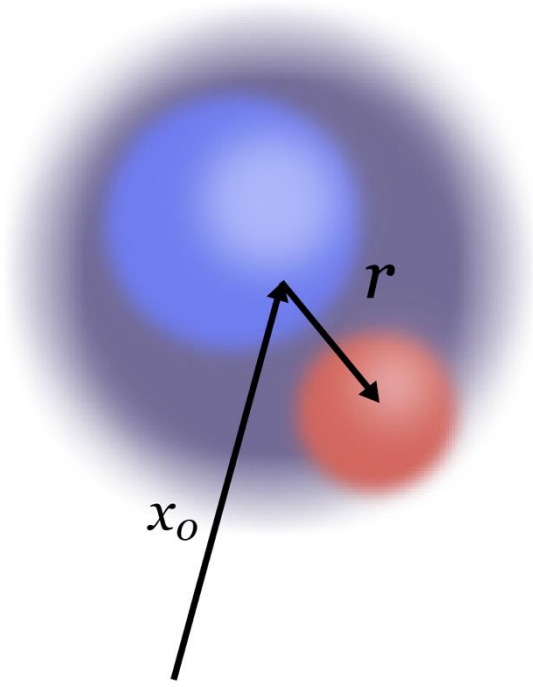
$$j_{fi}^{\mu \text{ composite}} = F(q) \times j_{fi}^{\mu \text{ point}}$$

What's a form factor, physically?

$$x = x_0 + r$$

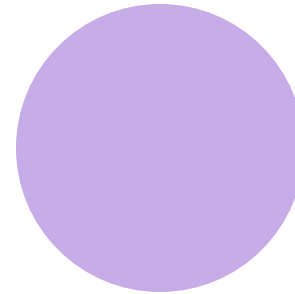
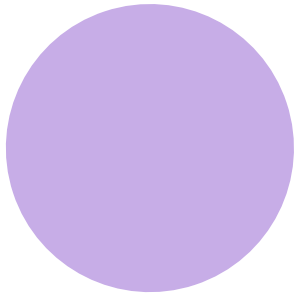
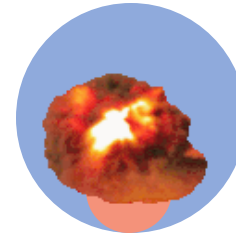
$$\Psi(x_0, r) = \psi(x_0)\phi(r)$$

$$\Psi'(x_0, r) = \psi'(x_0)\phi'(r)$$



$$\underbrace{\langle f | e^{iq\hat{x}} | i \rangle}_{T_{fi}^{\text{composite}}} = \underbrace{\int dx_0 \psi'^{\dagger}(x_0) e^{iqx_0} \psi(x_0)}_{T_{fi}^{\text{point}}} \underbrace{\int dr \phi'^{\dagger}(r) e^{iqr} \phi(r)}_{F(q)}$$

What is a form factor?



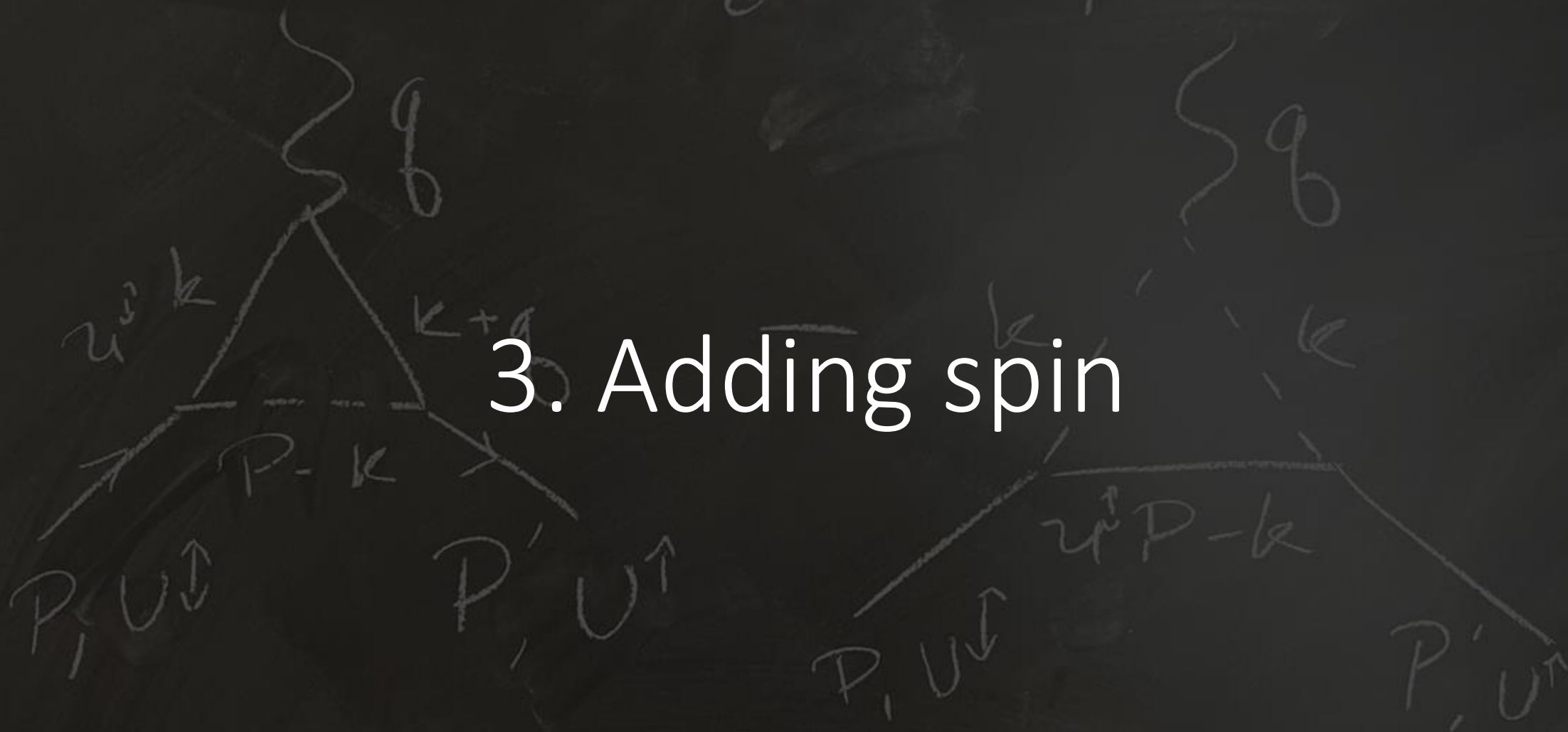
$$F(q) = \int dr \phi'^{\dagger}(r) e^{iqr} \phi(r) \sim \int dr \phi^{\dagger} \phi e^{iqr}$$

A Fourier transform of the density! *

$\rho(r)$	$ F(\vec{q} ^2) $
point like, $f(r) = \frac{1}{4\pi r^2} \delta(r)$	constant, e.g. electron $F(\vec{q} ^2) = 1$
exponential, $f(r) = \frac{a^3}{8\pi} \exp(-ar)$	dipole, e.g. proton $F(\vec{q} ^2) = \left(1 + \frac{ \vec{q} ^2}{a^2}\right)^{-2}$

$$L_{\text{int}} = g \phi \gamma^5 \Sigma_1 \pi + \text{h.c.}$$

3. Adding spin



Form factors with spin

- Klein-Gordon field (without spin):

$$\psi = \psi(x)$$

$$\hat{j}^\mu = e \left(\overleftarrow{\hat{p}} + \overrightarrow{\hat{p}} \right)^\mu$$

- Dirac field (with spin):

$$\psi = u_s \psi(x)$$

$$\hat{j}^\mu = e \gamma^\mu$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}$$

Form factors with spin

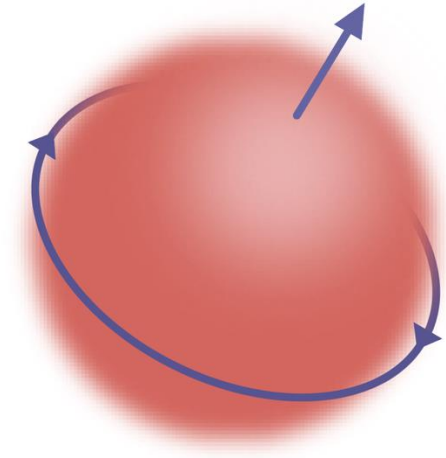
- Klein-Gordon field (without spin):

$$T_{P \rightarrow P'} = \langle P' | \hat{V} | P \rangle$$

- Dirac field (with spin):

$$T_{\uparrow \rightarrow \uparrow} = \langle P', \uparrow | \hat{V} | P, \uparrow \rangle \quad ; S$$

$$T_{\downarrow \rightarrow \uparrow} = \langle P', \uparrow | \hat{V} | P, \downarrow \rangle$$



Form factors with spin

- Klein-Gordon field (without spin):

$$j_{P \rightarrow P'}^\mu = e(P + P') e^{-i(P' - P)x} F(q)$$

- Dirac field (with spin):

$$j_{s \rightarrow s'}^\mu = \bar{u}_{s'}(P') \left(\gamma^\mu F_1(q) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q) \right) u_s(P) e^{-i(P' - P)x}$$

$$\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \quad \bar{u}'_s(P') = u'_s(P')^\dagger \gamma^0$$

Nonrelativistic approximation

$$T_{fi} \sim 2M \langle P', S' | \left(eV(\hat{x}) F_1 - \mathbf{B}(\hat{x}) \cdot \overbrace{\frac{e\hat{\boldsymbol{\sigma}}}{2M} (F_1 + F_2)}^{\hat{\boldsymbol{\mu}}} \right) | P, S \rangle$$

$g/2$

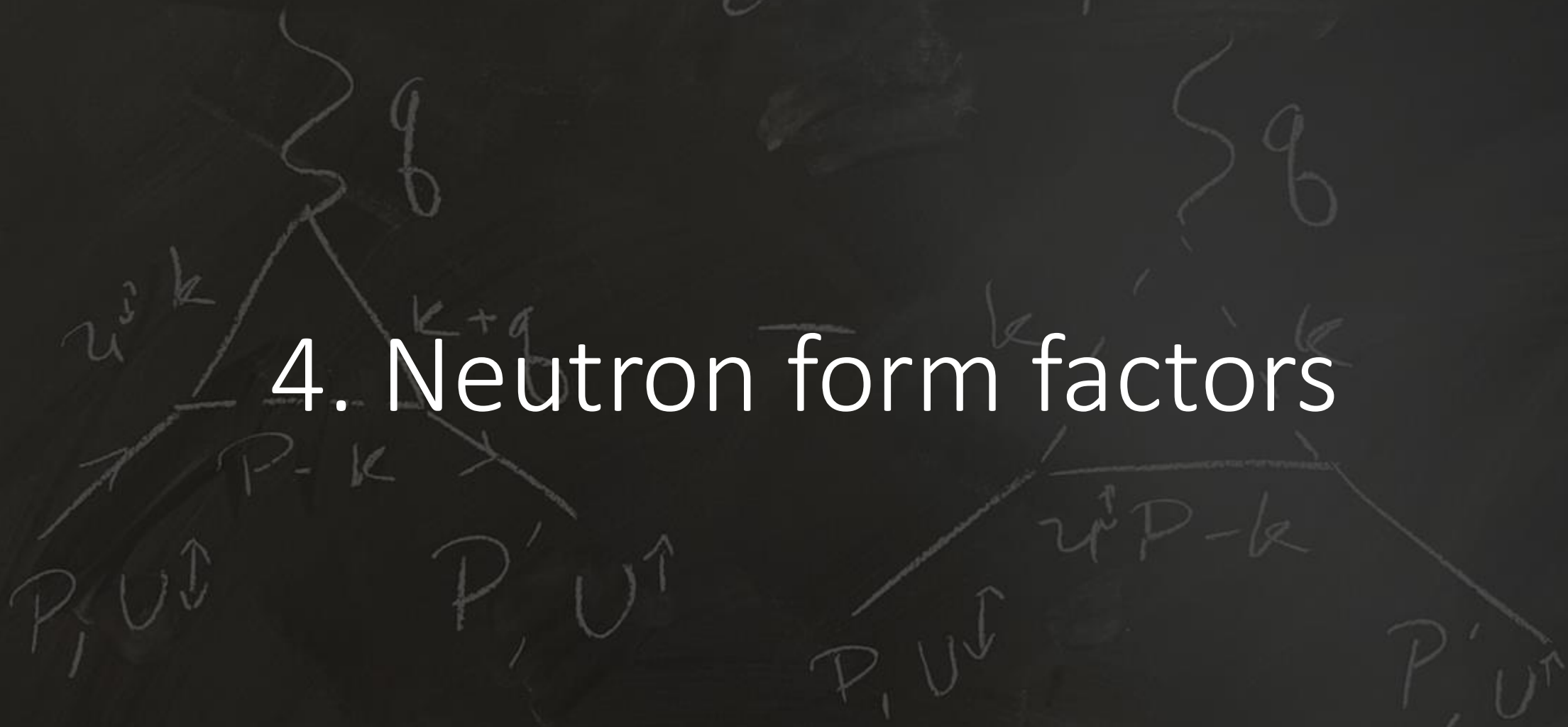
$$F_1(0) \equiv 1, \quad F_2(0) = \frac{g-2}{2}$$

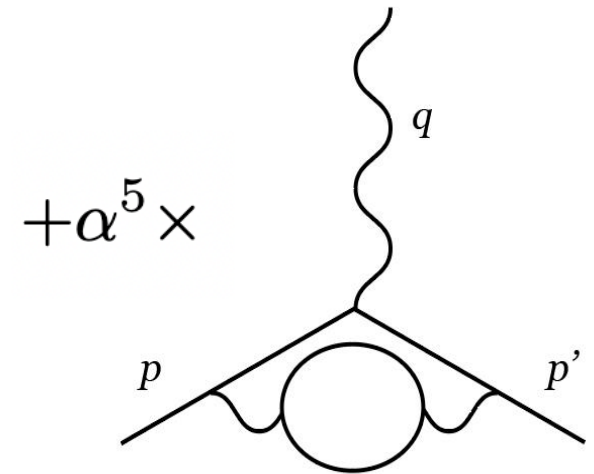
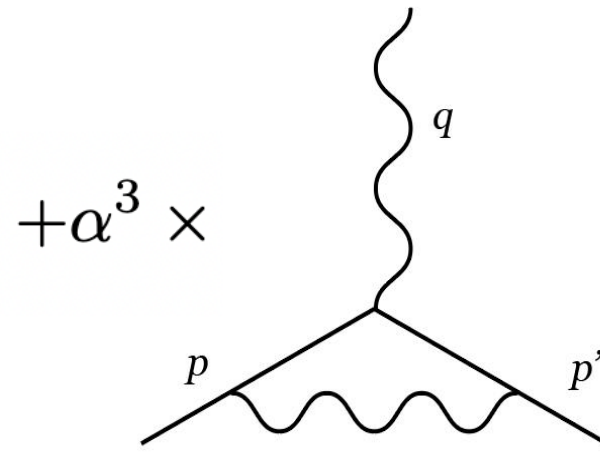
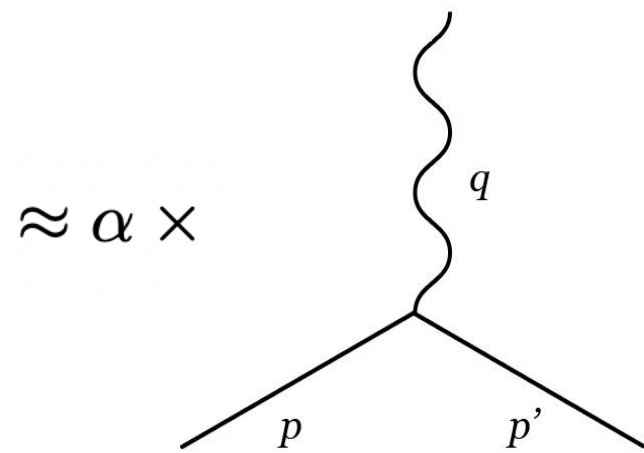
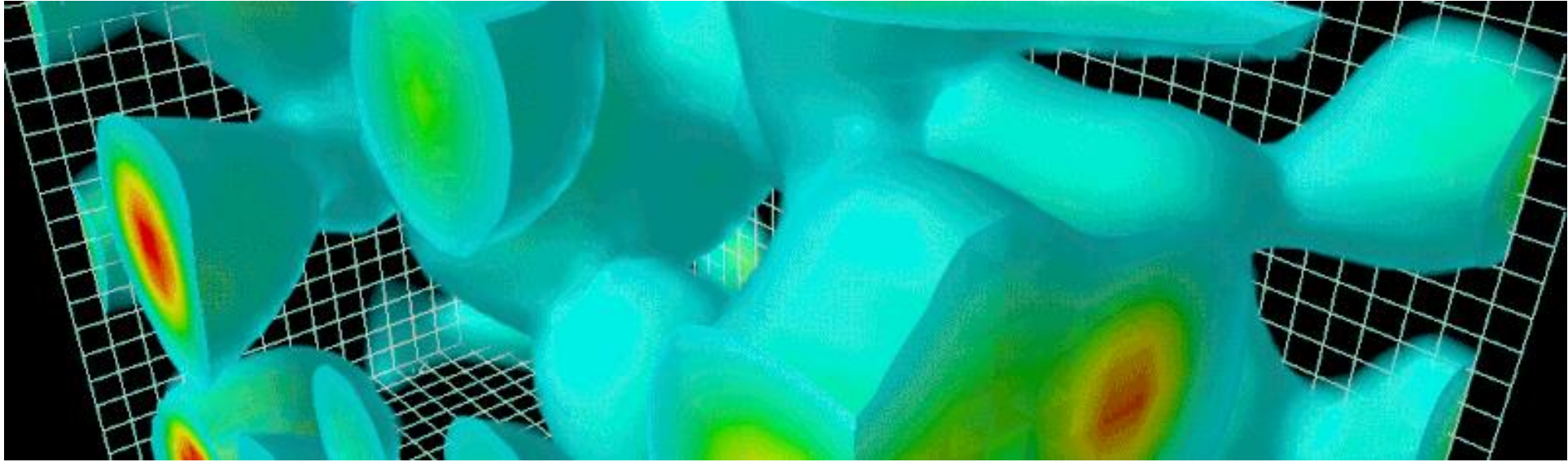
$$a_e = 0.001\,159\,652\,181\,643(764)$$

$$a_e = 0.001\,159\,652\,180\,59(13)$$

$$L_{\text{int}} = g \phi \gamma^5 \Sigma_1 \pi + \text{h.c.}$$

4. Neutron form factors

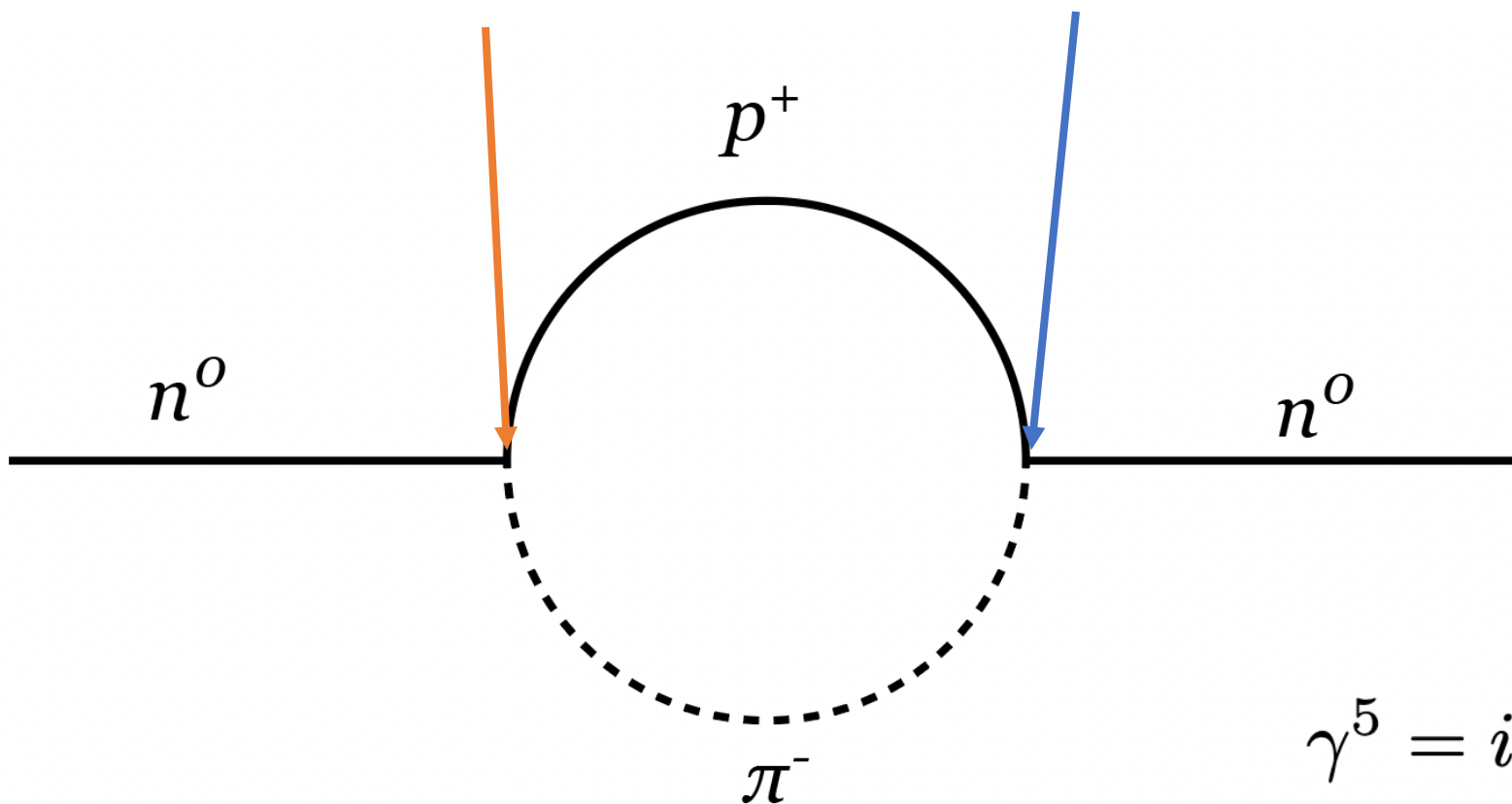




$$\alpha_e \approx 1/137 \quad \alpha_s > 1$$

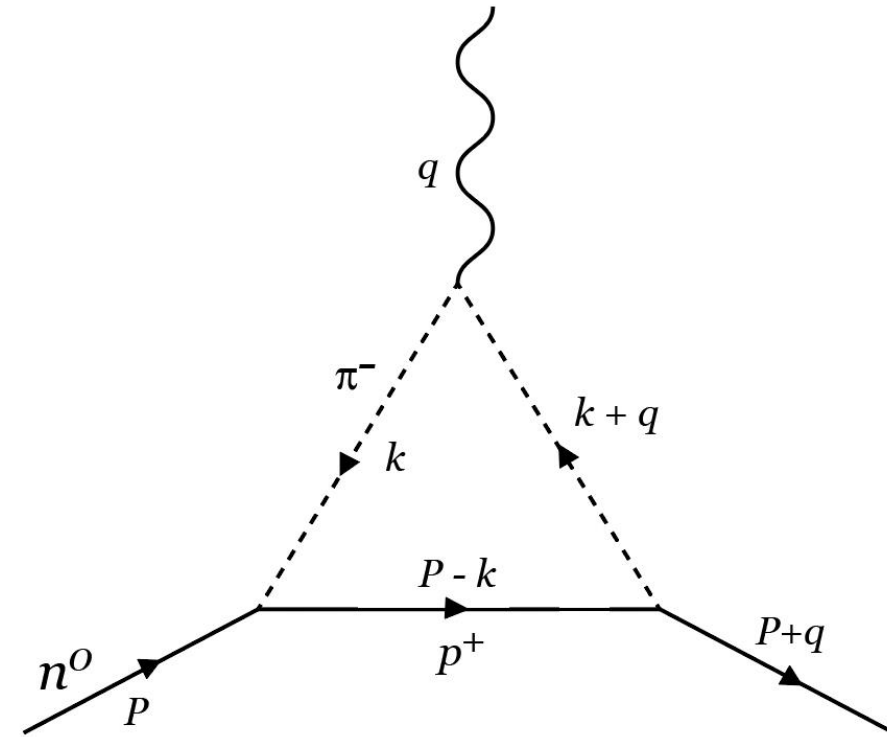
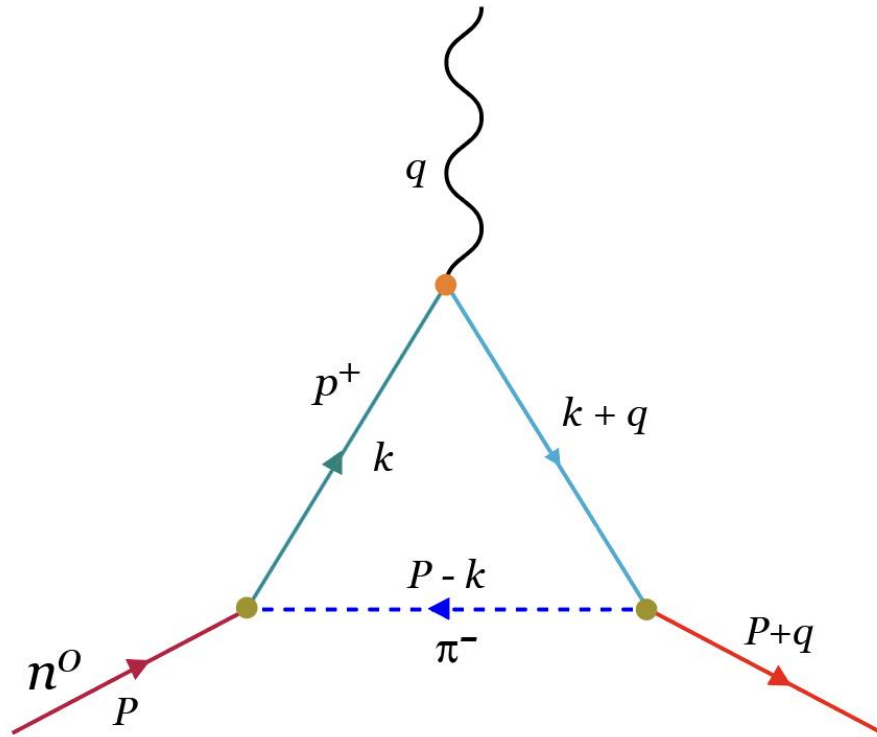
The model: $n^0 \rightarrow p^+ \pi^- \rightarrow n^0$

$$\mathcal{L}_{\text{int}} = ig (\bar{\psi}_p \gamma^5 \psi_n \phi_-^* + \bar{\psi}_n \gamma^5 \psi_p \phi_-) + \dots$$



$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

Feynman diagrams



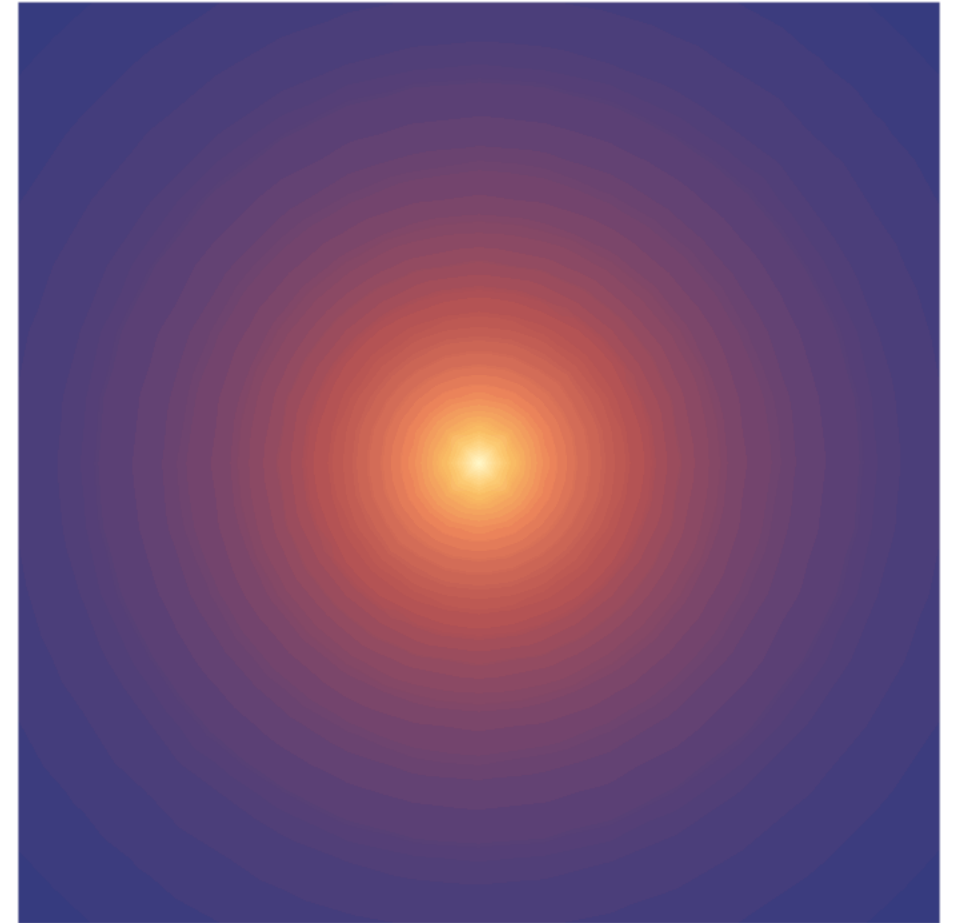
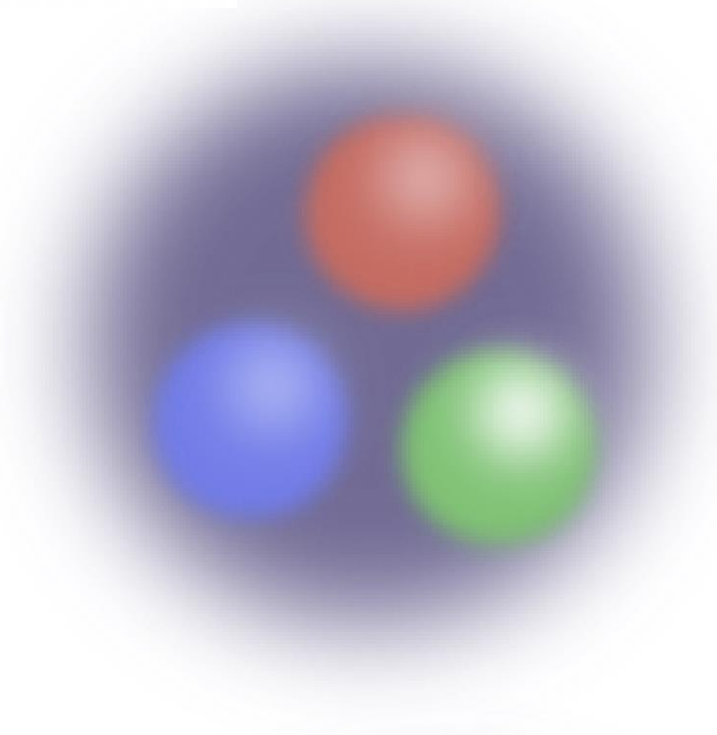
$$\begin{aligned}
 j_{P \rightarrow P+q}^\mu &= -\bar{U}' \gamma_5 g^2 \int \frac{d^4 k}{(2\pi)^4} \frac{i}{(P-k)^2 - m_1^2 + i\epsilon} \frac{i(\not{k} + \not{q} + m_2)}{(k+q)^2 - m_2^2 + i\epsilon} \gamma^\mu \frac{i(\not{k} + m_2)}{k^2 - m_2^2 + i\epsilon} \gamma_5 U \\
 &+ \bar{U}' \gamma_5 g^2 \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m_1^2 + i\epsilon} (2k^\mu + q^\mu) \frac{i}{(k+q)^2 - m_1^2 + i\epsilon} \frac{i(\not{P} - \not{k} + m_2)}{(P-k)^2 - m_2^2 + i\epsilon} \gamma_5 U
 \end{aligned}$$

Pauli-Villars Regularization

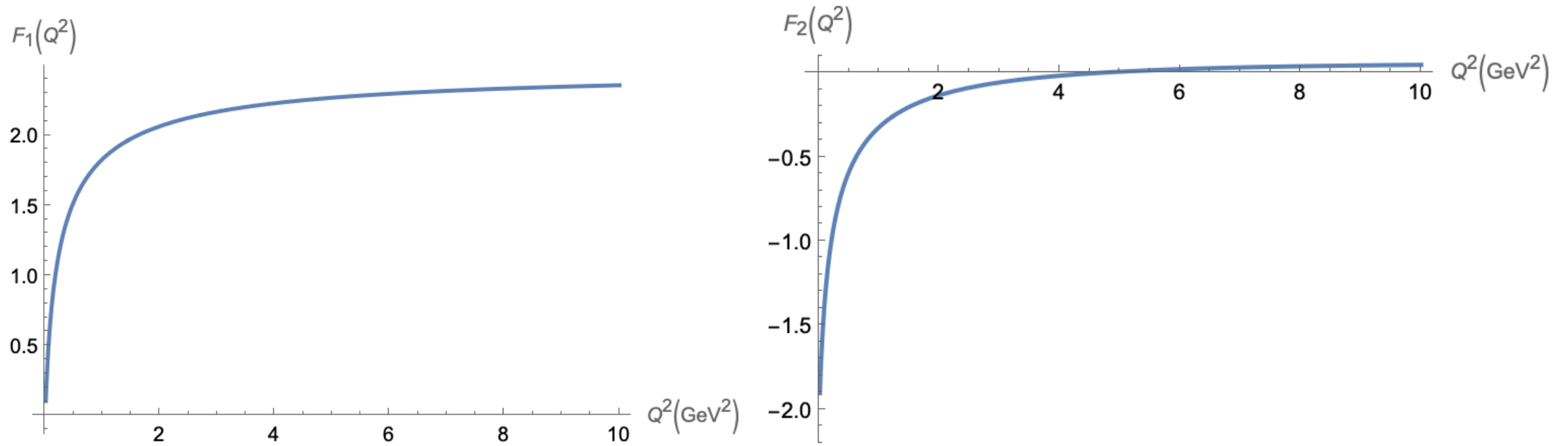
$$j_{P \rightarrow P+q}^\mu = \infty?? \quad \text{Renormalization!}$$

$$\frac{1}{k^2 - m^2} \longrightarrow \frac{1}{k^2 - m^2} - \frac{1}{k^2 - \Lambda^2} \quad \frac{1}{-m_1^2 + i\epsilon} (2k^\mu + q^\mu) \frac{1}{(k -$$

- Admitting the theory doesn't account for what happens above a certain energy

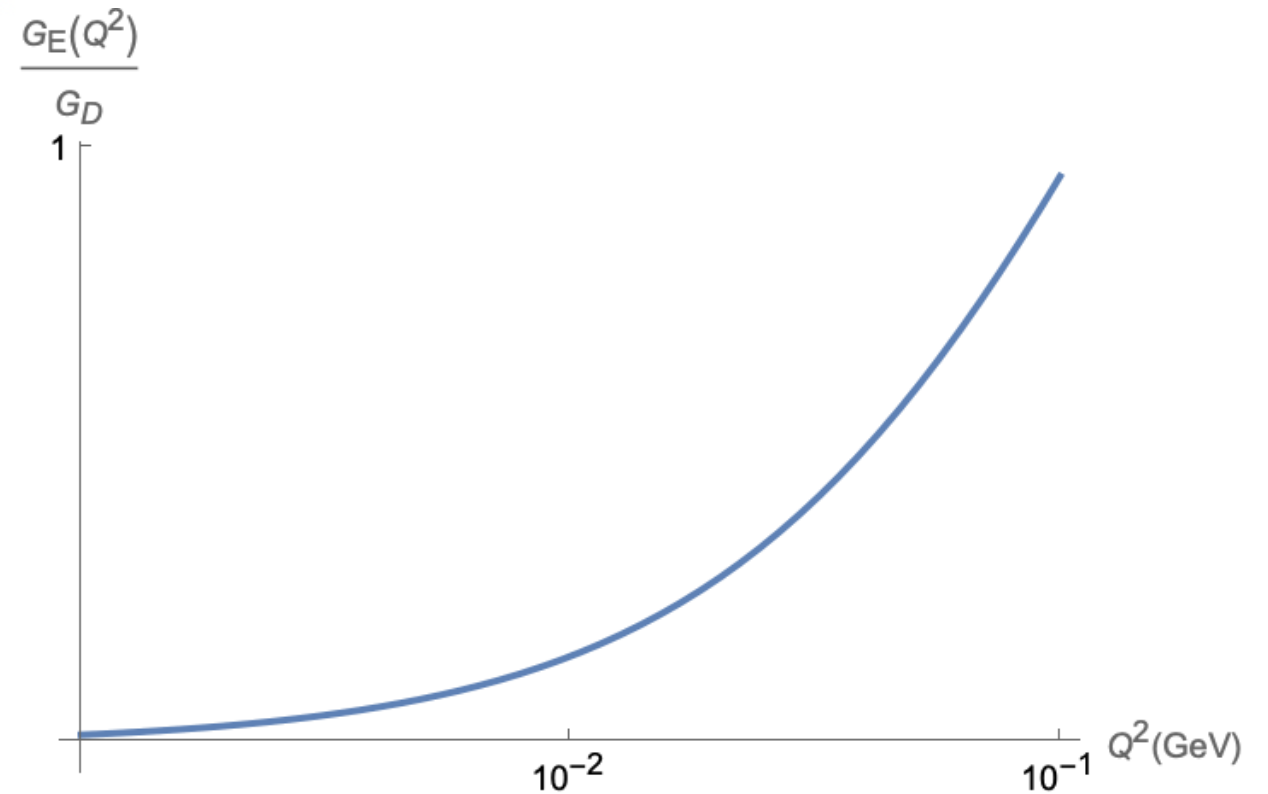
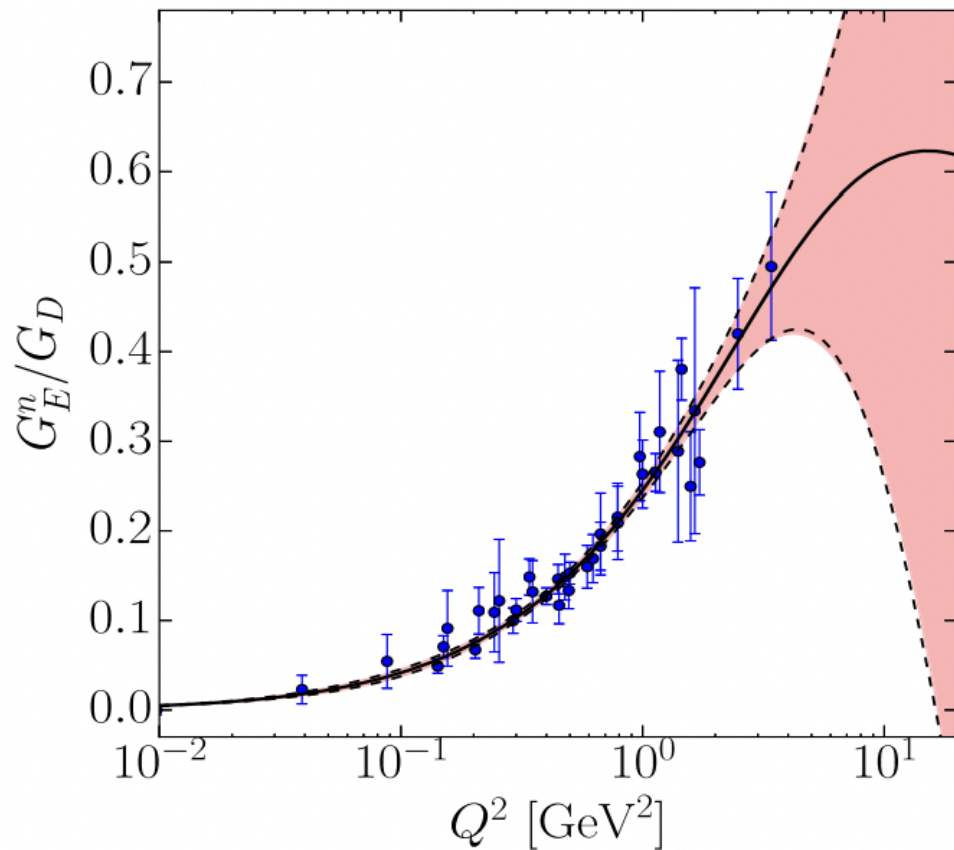


Results—Neutron form factors



Comparing with experiment—electric form factor

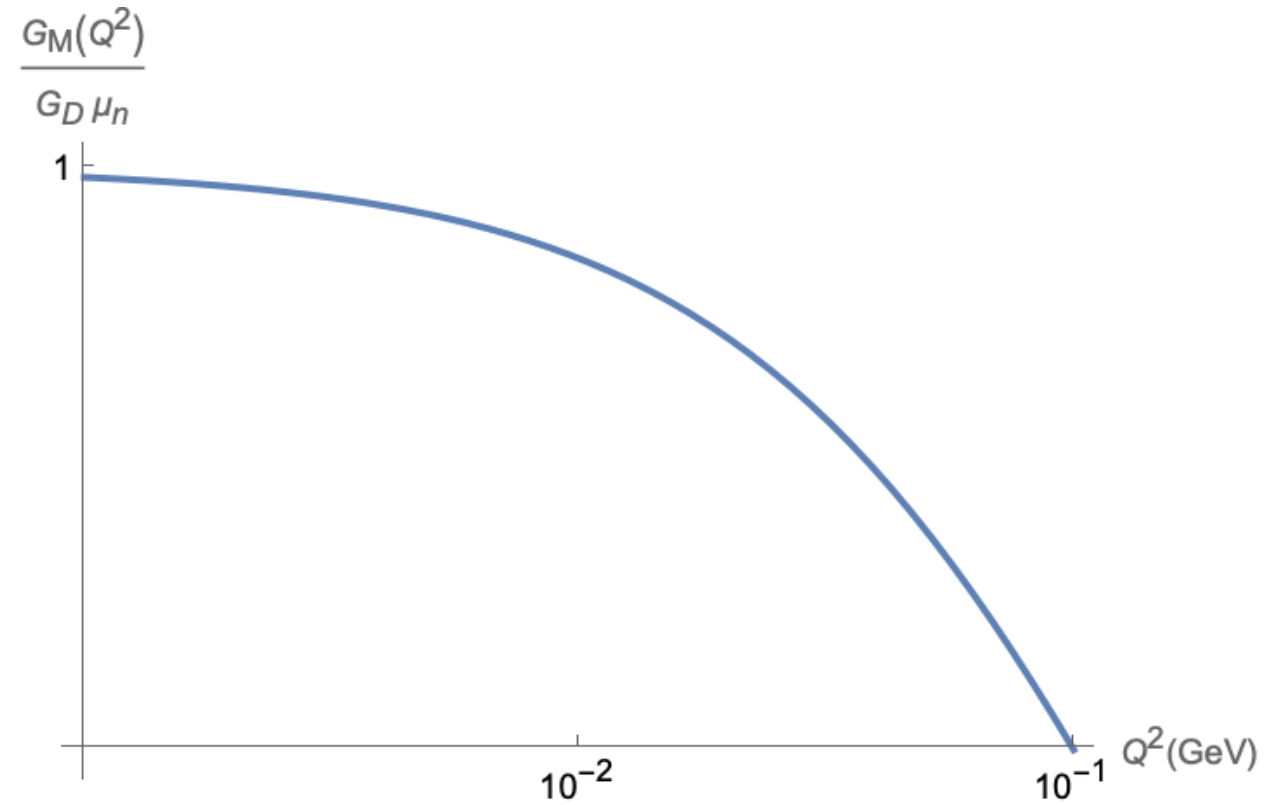
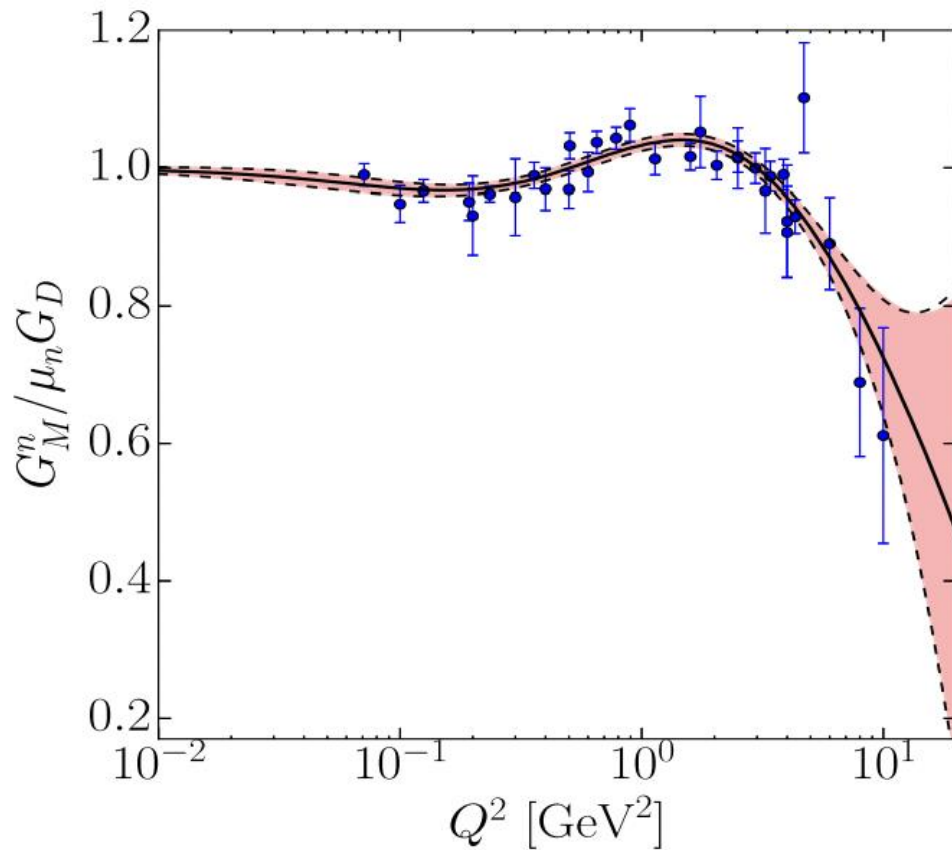
Z. Ye et al. / Physics Letters B 777 (2018) 8–15



(Preliminary)

Comparing with experiment—magnetic form factor

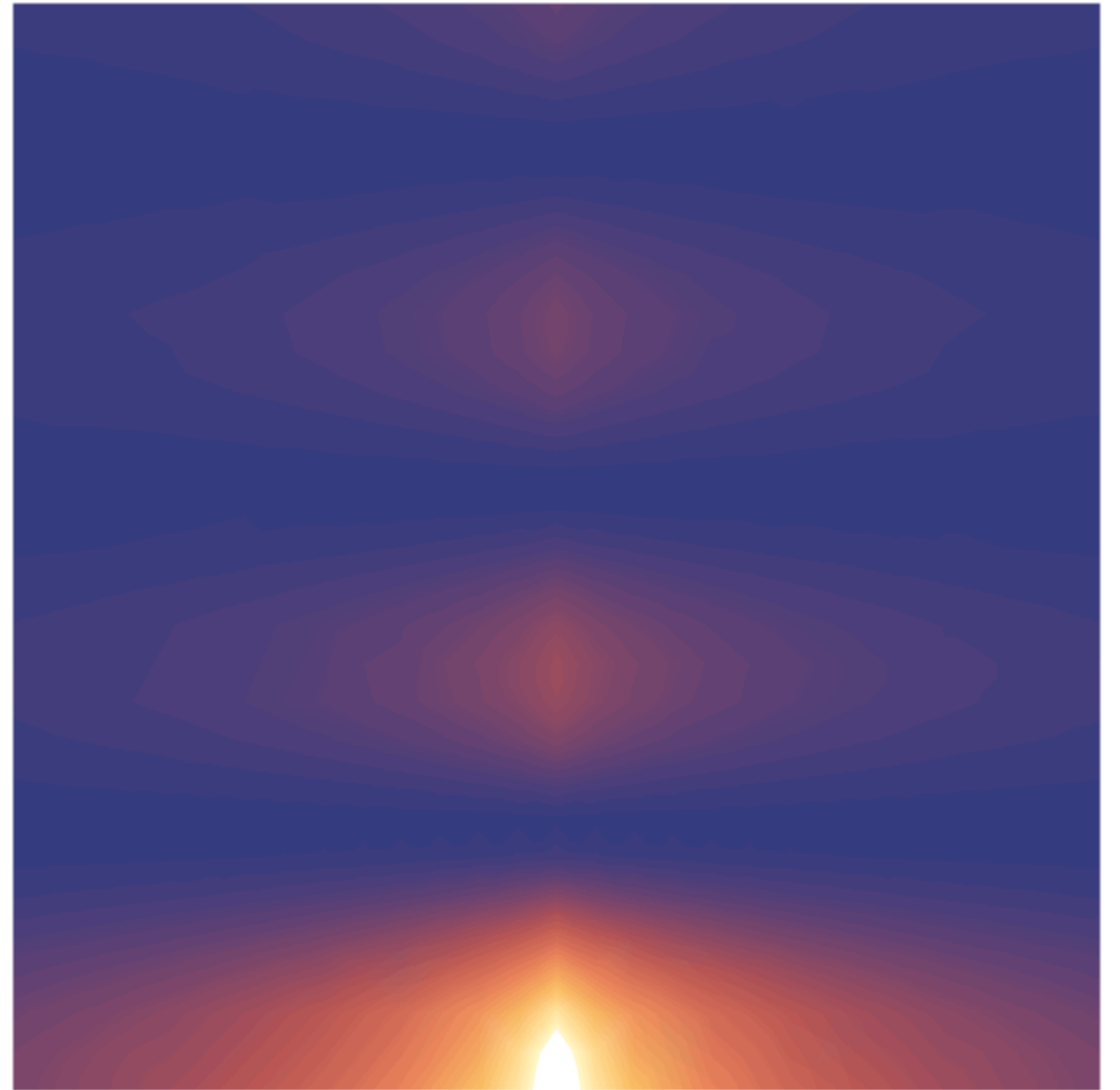
Z. Ye et al. / Physics Letters B 777 (2018) 8–15



(Preliminary)

Next Steps

- More physically-motivated renormalization
- Extracting spatial information—
 - Light-Front wave functions
- Gravitational form factors





Thank you...

- Professor Miller
- All of you!



INSTITUTE for
NUCLEAR THEORY



Questions?

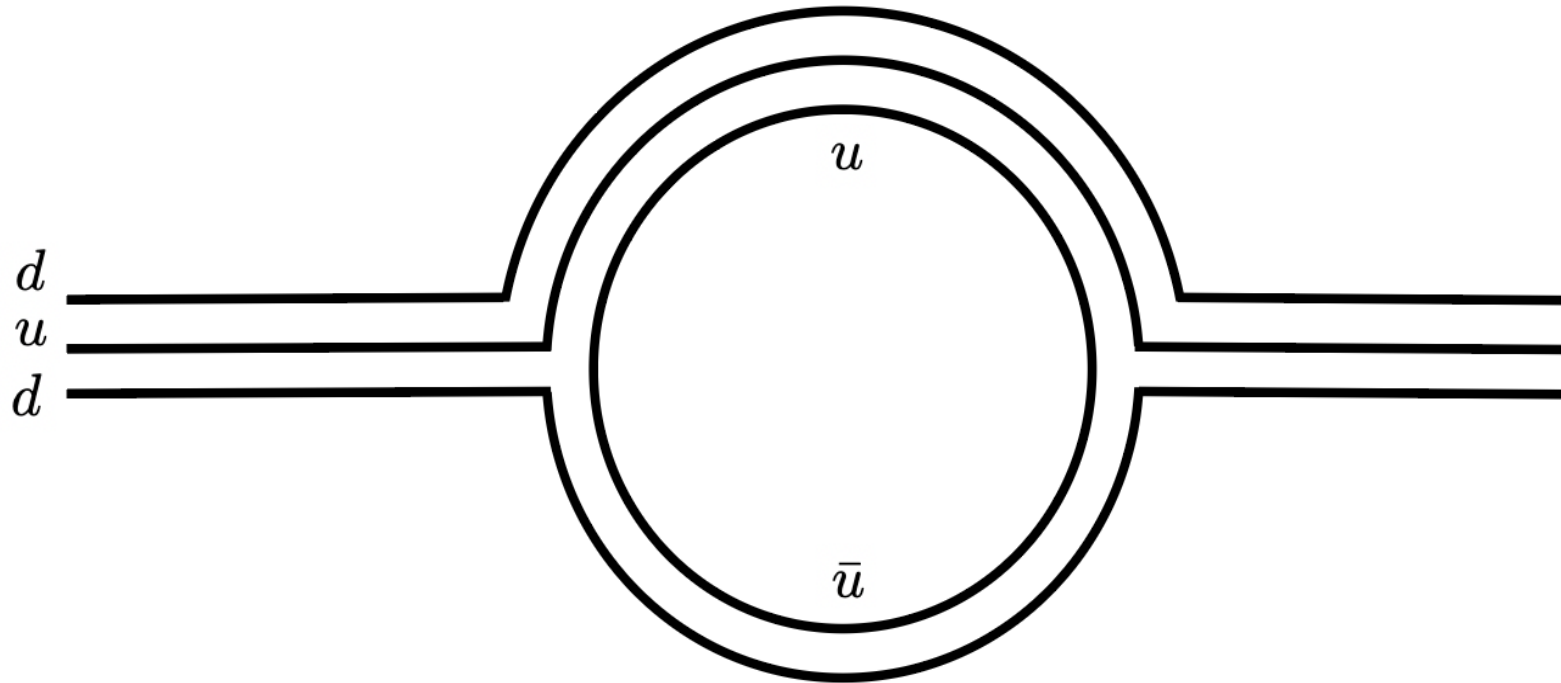
$$L_{\text{int}} = g \phi \gamma^5 \Sigma_1 \pi + \text{h.c.}$$

Backup Slides



The model: $n^0 \rightarrow p^+ \pi^- \rightarrow n^0$

$$\mathcal{L}_{\text{int}} = ig \left(\bar{\psi}_p \gamma^5 \psi_n \phi_-^* + \bar{\psi}_n \gamma^5 \psi_p \phi_- \right) + \dots$$



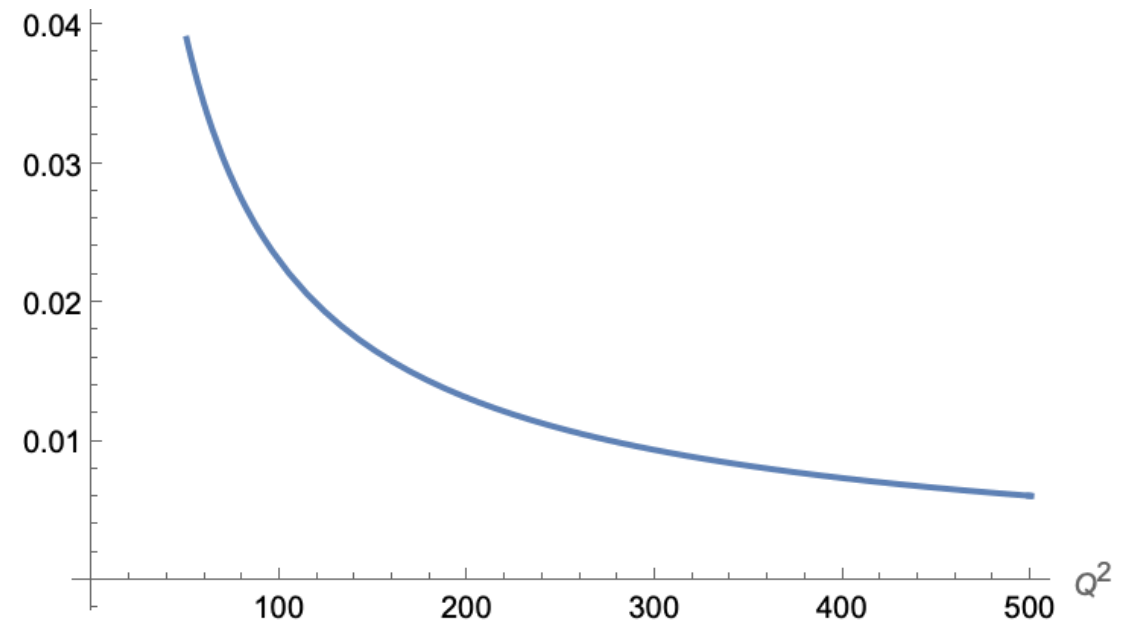
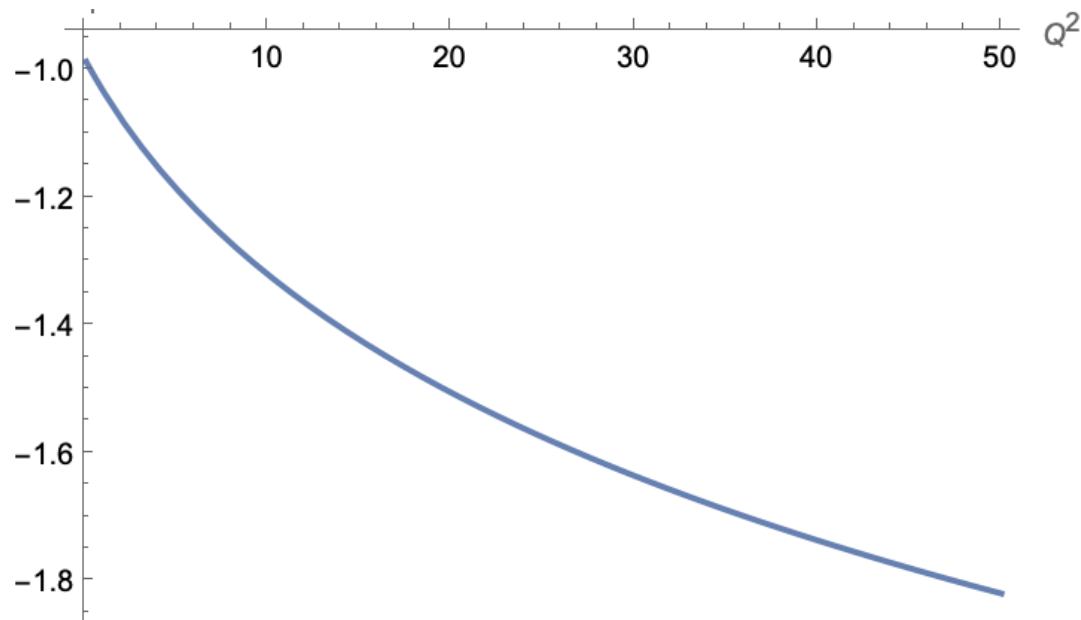
Pauli-Villars Regularization

$$Q^2 \ll \Lambda^2$$

$$\dots - \int_0^1 dx \frac{2}{Q} \sqrt{\mathcal{M}^2 + \frac{Q^2 x^2}{4}} \tanh^{-1} \left(\frac{Qx}{2\sqrt{\mathcal{M}^2 + \frac{Q^2 x^2}{4}}} \right) + \dots$$

$$Q^2 \gg \Lambda^2$$

$$\dots - \frac{\ln(Q)}{Q^2}$$



Light front corrections

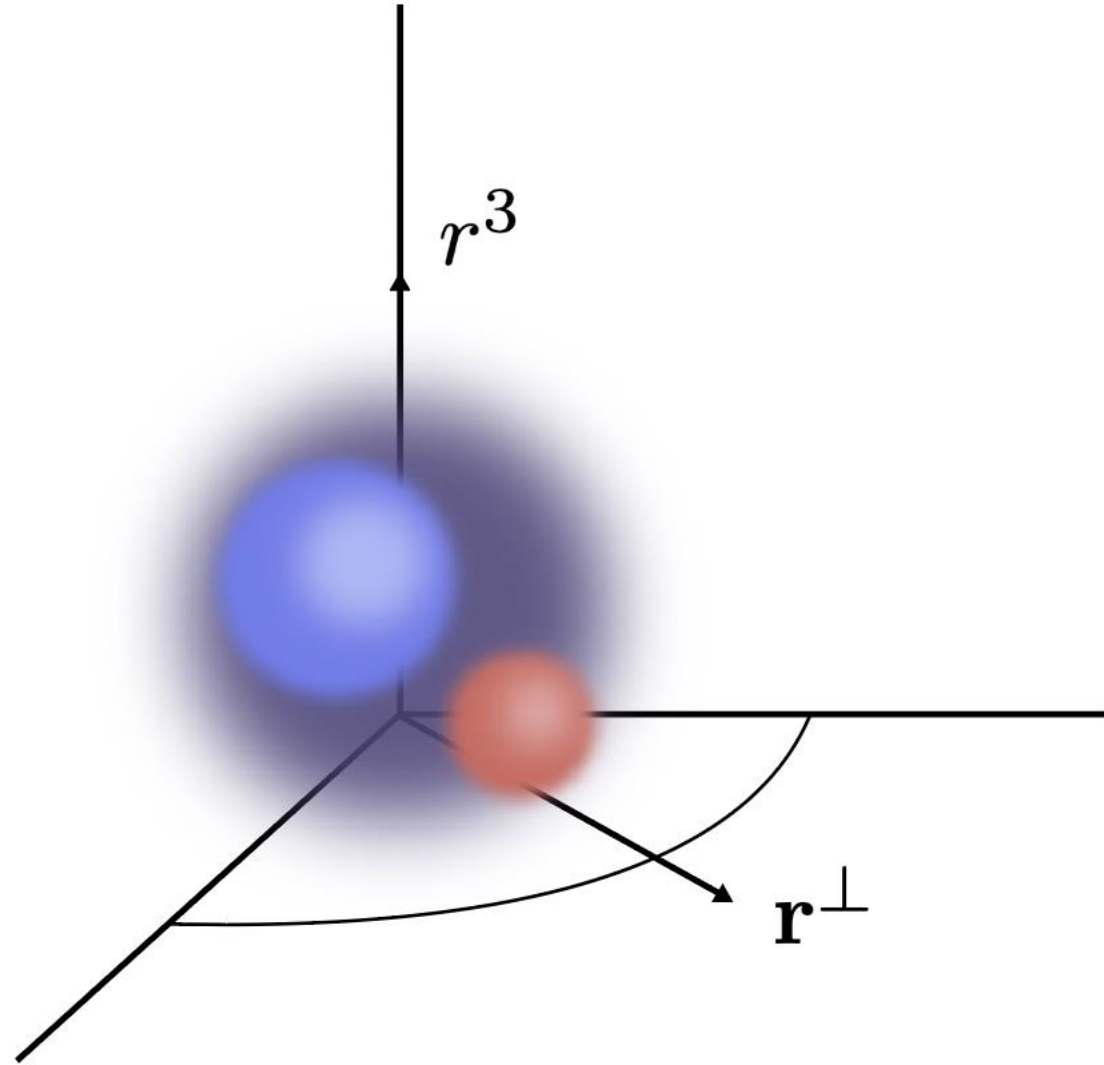
$$\Psi = \Psi(x_0^0, x_0^3, \mathbf{x}_0^\perp, r^0, r^3, \mathbf{r}^\perp)$$

$$x^\pm = x^0 \pm x^3$$

$$x^\pm \xrightarrow{\text{boost}} \gamma(1 + \beta)x^\pm$$

$$x^+ = \text{'time'}$$

$$x^-, \mathbf{x}_\perp = \text{'space'}$$



Light front corrections

$$\Psi = e^{iP^- x^+ / 2} \psi(x^-, x_0^\perp) \phi(x^-, r^\perp)$$

$$q^+ \equiv 0$$

$$T_{fi} \propto \delta(P'^- - P^- - q^-) \int dx^- \int d^2 x_0^\perp \psi'^\dagger e^{iq^\perp x_0^\perp} \psi \int d^2 r^\perp \phi'^\dagger e^{iq^\perp r^\perp} \phi$$

$$F(q) = \int dx^- \int d^2 r^\perp \phi'^\dagger(x^-, r^\perp) e^{iq^\perp r^\perp} \phi(x^-, r^\perp)$$

$$\phi_\perp \equiv \int dx^- \phi(x^-, r^\perp)$$

$$F(q) = \int d^2 r \phi'_\perp{}^\dagger(r^\perp) e^{iq^\perp r^\perp} \phi_\perp(r^\perp)$$

