Neutron Form Factors

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Abstract

1 Introduction

Nucleon form factors are integral to inelastic scattering experiments–they are measurable quantities that carry important information about the internal structure of nucleons. In electromagnetic interactions, to first order in the electromagnetic coupling constant, the transition amplitude for a point-like Dirac particle from a state ψ_i to a state ψ_f in the presence of an electromagnetic field A^μ is [2]:

$$T_{fi} = -i \int d^4x \; \bar{\psi}_f \gamma^\mu A_\mu \psi_i$$

So we can define the *transition current* for these particles to be:

$$j^\mu_{p \to p'} = \bar{u}(p')\gamma^\mu u(p) e^{i(p'-p)x}$$

Assuming that ψ_i and ψ_f are momentum eigenstates with momenta p and p' (here, γ^μ is a Dirac γ matrix, and u and $\bar{u} = u^\dagger \gamma^0$ are a Dirac bispinor and its adjoint). In general, though, there are two current-conserving independent four-vectors one can construct out of gamma matrices, so a more general electromagnetic transition current appropriate for a composite particle with mass M would be:

$$J^\mu = \bar{U}(P') \left(\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2) \right) U(P) \tag{1}$$

Where $q = P' - P$ is the four momentum transferred by the virtual photon exchanged between a scattering and target particle, and $Q^2 = -q^2 > 0$ (here, $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$). F_1 and F_2 are the Dirac and Pauli form factors respectively. According to Bjorken [get citation], form factors are related to Fourier transforms of density functions describing the internal structure of a composite particle. In the case of a Dirac particle, the Sachs electric and magnetic form factors (G_E and G_M, resp.) are:

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned} \tag{2}$$

Are related to Fourier transforms of the charge densities and magnetic dipole moment densities of the composite particle [CITE]. In the nonrelativistic limit, this is exact—G_E and G_M are Fourier transforms of the electromagnetic densities—but relativistic effects are non-negligible even at low energies and seriously challenge this view. It is more accurate to say that they can be related to transverse charge and magnetic moment densities [MILLER PAPER].

Form factors for protons are well studied [CITATIONS], but the neutron is more elusive, as its net-zero charge makes it difficult to 'hit' in electromagnetic scattering experiments. More recently, better experimental measurements of the electromagnetic form factors have been made [all 3 papers], and several models have been proposed that explain the observations well. Two such models (both proposed in the '80s) are the Princeton bag model, in which nucleons are collections of three quarks confined to a small region [] and the cloudy bag model, which adds effects from a cloud of virtual pions surrounding the quark bag. The corrections from the pion cloud turn out to fit experiment better for the proton, and a more recent calculation by Gerald Miller also calculated accurate form factors for the neutron. In his paper, Miller used light-front perturbation theory to calculate the neutron form factors by including effects from virtual π^0 particles surrounding the neutron, where the interactions between the neutron and pion have undetermined form factors built in to match experimental data.

The goal of this project is to explore a similar model, though instead of an $n^0 \to n^0\pi^0$ process, we have the other allowed isospin-conserving process, $n^0 \to p^+\pi^-$. Additionally, in this perturbative calculation, we are assuming that the proton and pion are point particles to second order in the coupling constant g, and are therefore ignoring the contribution of their form factors to the neutron form factor.

2 The model

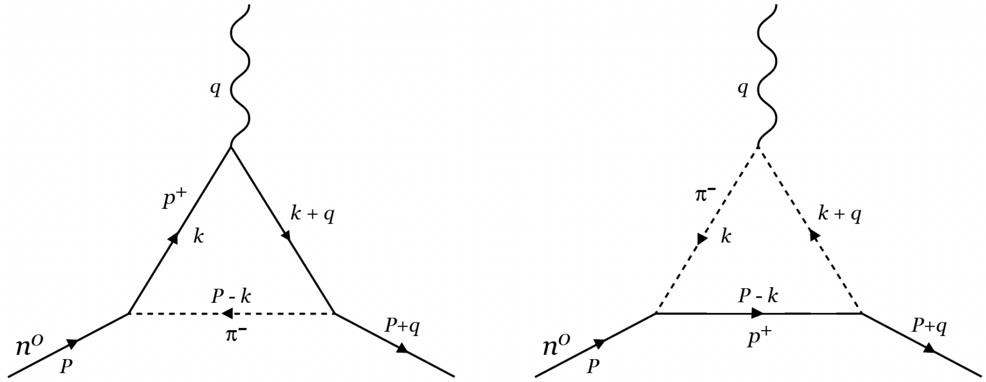
The interaction Lagrangian (adapted from wave equations given in Bjorken and Drell) for the proton, neutron, and pion fields is:

$$\mathcal{L}_{\text{int}} = g\bar{\Psi}i\gamma^5(\boldsymbol{\tau}.\boldsymbol{\phi})\Psi \tag{3}$$

Where $\Psi = (\psi_p, \psi_n)^T$ is the proton-neutron field, $\boldsymbol{\tau}$ is the isospin operator, and $\boldsymbol{\phi} = (\phi_\pm, \phi_0)$ in isometric-space 'spherical' coordinates represents the $\pi_\pm$ and π_0 fields. For the Neutron, we have the following terms:

$$\mathcal{L}_n = g\bar{\psi}_n i\gamma^5\phi_0\psi_n + \sqrt{2}g\left(\bar{\psi}_p i\gamma^5\phi_+\psi_n + \bar{\psi}_n i\gamma^5\phi_-\psi_p\right) \tag{4}$$

Since $\phi_+ = \phi_-^*$, this is Hermitian. If we take the neutron field to have no electromagnetic interaction to second order in g, then the first term contributes nothing to the electromagnetic form factors and we just have the following two diagrams:



So we have the transition current:

$$
\begin{aligned}
J^\mu &= \bar{U}'\gamma_5 2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{i(\not{k}+\not{q}+M)}{(k+q)^2-M^2+i\epsilon}\gamma^\mu \frac{i(\not{k}+M)}{k^2-M^2+i\epsilon}\frac{i}{(P-k)^2-m^2+i\epsilon}\gamma_5 U \\
&\quad -\bar{U}'\gamma_5 2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{(k+q)^2-m^2+i\epsilon}(2k^\mu+q^\mu)\frac{i}{k^2-m^2+i\epsilon}\frac{i(\not{P}-\not{k}+M)}{(P-k)^2-M^2+i\epsilon}\gamma_5 U
\end{aligned} \tag{5}
$$

Where U is the Dirac spinor for the neutron, M is the neutron and proton mass (which we're taking to be the same), and m is the pion mass.

3 Calculations

The most basic form of this calculation requires no renormalization, as the divergent terms cancel. We begin by using the anticommutation of γ_5 and γ^μ ($\gamma_5\gamma^\mu = -\gamma^\mu\gamma_5$) to cancel the γ_5 terms. Then we can let $\not{P}$ act on the bispinor to the right in the second integral:

$$
\begin{aligned}
J^\mu &= \bar{U}' 2ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{(\not{k}+\not{q}-M)\gamma^\mu(\not{k}-M)}{((k+q)^2-M^2+i\epsilon)(k^2-M^2+i\epsilon)((P-k)^2-m^2+i\epsilon)} U \\
&\quad +\bar{U}' 2ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{(2k^\mu+q^\mu)\not{k}}{((k+q)^2-m^2+i\epsilon)(k^2-m^2+i\epsilon)((P-k)^2-M^2+i\epsilon)} U
\end{aligned} \tag{6}
$$

Then we can combine the propagators using Feynman parameters and shift the origin of the momentum integral to $\kappa^\mu = k^\mu - P^\mu(1-x) + q^\mu(x-y)$:

$$
\begin{aligned}
J^\mu &= \bar{U}' 4ig^2 \int_0^1 dx \int_0^x dy \int \frac{d^4\kappa}{(2\pi)^4} \frac{(\not{\kappa}+\not{P}(1-x)+\not{q}(1-x+y)-M)\gamma^\mu(\not{\kappa}+\not{P}(1-x)-\not{q}(x-y)-M)}{(\kappa^2-Q^2y(x-y)-(1-x)m^2-x^2M^2+i\epsilon)^3} U \\
&\quad +\bar{U}' 4ig^2 \int_0^1 dx \int_0^x dy \int \frac{d^4k}{(2\pi)^4} \frac{(2\kappa^\mu+2P^\mu(1-x)+q^\mu(1-2x+2y))(\not{\kappa}+\not{P}(1-x)-\not{q}(x-y))}{(\kappa^2-Q^2y(x-y)-xm^2-(1-x)^2M^2+i\epsilon)^3} U
\end{aligned} \tag{7}
$$

These integrals added are not divergent, but it contains two divergent terms that cancel out, so we have to be careful in the rest of this calculation. To deal with these divergent terms, we can use dimensional regularization: we integrate over $4-\epsilon$ dimensions, then take ϵ to 0.

To procede, we drop all terms odd in κ, and use the following identities:

$$\begin{aligned}
\int d^{4-\epsilon}\kappa\ \kappa^\alpha\kappa^\beta\ f(\kappa^2) &= \int d^{4-\epsilon}\kappa\ \frac{\kappa^2 g^{\alpha\beta}}{4-\epsilon} f(\kappa^2) \qquad (8)\\
\gamma^\alpha\gamma^\mu\gamma_\alpha &= -(2-\epsilon)\gamma^\mu \\
\bar{U}'(2P^\mu + q^\mu)U &= \bar{U}'(2M\gamma^\mu - i\sigma^{\mu\nu}q_\nu)U \\
\gamma^\mu\gamma^\alpha + \gamma^\alpha\gamma^\mu &= 2g^{\mu\alpha} \\
\gamma^\mu\gamma^\alpha &= \frac{1}{2}(\gamma^\mu\gamma^\alpha + \gamma^\alpha\gamma^\mu) + \frac{1}{2}(\gamma^\mu\gamma^\alpha - \gamma^\alpha\gamma^\mu) \\
&= g^{\mu\alpha} - i\sigma^{\mu\alpha} \\
\gamma^\alpha\gamma^\mu\gamma^\beta q_\alpha q_\beta &= 2q^\mu \not{q} - q^2\gamma^\mu \\
&\to Q^2\gamma^\mu \qquad (9)
\end{aligned}$$

Some of the other identities are modified by taking the dimension to be $4-\epsilon$, but they multiply convergent terms in the integral and simplify to the above identities after we take ϵ to 0. We will also introduce a quantity with dimensions of mass μ. The actual value is arbitrary but allows the integral to keep the right units.

This gives:

After dropping terms odd in κ and some Dirac algebra, we get:

$$\begin{aligned}
J^\mu &= \bar{U}'4ig^2\int_0^1 dx\int_{-\frac{x}{2}}^{\frac{x}{2}} dz\int\frac{d^{4-\epsilon}\kappa}{(2\pi)^4}\mu^\epsilon\frac{\gamma^\mu\left(-\frac{2-\epsilon}{4}\kappa^2 + M^2x^2 - Q^2\left(\frac{x^2}{4}-z^2\right)\right) - i\sigma^{\mu\nu}q_\nu Mx^2}{\left(\kappa^2 - Q^2\left(\frac{x^2}{4}-z^2\right) - (1-x)m^2 - x^2M^2 + i\epsilon\right)^3}U \\
&+\bar{U}'4ig^2\int_0^1 dx\int_0^x dy\int\frac{d^{4-\epsilon}k}{(2\pi)^4}\mu^\epsilon\frac{\gamma^\mu\left(\frac{2}{4-\epsilon}\kappa^2 + 2M^2(1-x)^2\right) - i\sigma^{\mu\nu}q_\nu M(1-x)^2}{\left(\kappa^2 - Q^2\left(\frac{x^2}{4}-z^2\right) - xm^2 - (1-x)^2M^2 + i\epsilon\right)^3}U \qquad (10)
\end{aligned}$$

Where $z = y - x/2$, and terms odd in z have been integrated out. The final manipulation will be to Wick-rotate $\kappa^0 \to i\lambda^0$ so that we can integrate over Euclidean space. This takes $\kappa^2 \to -\lambda^2$ and $d^{4-\epsilon}\kappa \to id^{4-\epsilon}\lambda$.

$$\begin{aligned}
J^\mu &= \bar{U}'4g^2\int_0^1 dx\int_{-\frac{x}{2}}^{\frac{x}{2}} dz\int\frac{d^{4-\epsilon}\lambda}{(2\pi)^4}\mu^\epsilon\frac{\gamma^\mu\left(\frac{2-\epsilon}{4-\epsilon}\lambda^2 + M^2x^2 - Q^2\left(\frac{x^2}{4}-z^2\right)\right) - i\sigma^{\mu\nu}q_\nu Mx^2}{\left(\lambda^2 + Q^2\left(\frac{x^2}{4}-z^2\right) + (1-x)m^2 + x^2M^2\right)^3}U \\
&+\bar{U}'4g^2\int_0^1 dx\int_0^x dy\int\frac{d^{4-\epsilon}\lambda}{(2\pi)^4}\mu^\epsilon\frac{\gamma^\mu\left(-\frac{2}{4-\epsilon}\lambda^2 + 2M^2(1-x)^2\right) - i\sigma^{\mu\nu}q_\nu M(1-x)^2}{\left(\lambda^2 + Q^2\left(\frac{x^2}{4}-z^2\right) + xm^2 + (1-x)^2M^2\right)^3}U \qquad (11)
\end{aligned}$$

Here an overall minus sign has been pulled out from the denominator, and this cancels the minus sign picked up from the i in the numerator times the i from the Wick rotation. We now have expressions for F_1 and F_2, from the definition of the current:

$$J^\mu(0) = \bar{U}' \left(\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2) \right) U \tag{12}$$

We get:

$$F_1(Q^2) = 4g^2 \int_0^1 dx \int_0^x dy \int \frac{d^{4-\epsilon}\lambda}{(2\pi)^4} \mu^\epsilon \left\{ \frac{\frac{2-\epsilon}{4-\epsilon}\lambda^2 + M^2 x^2 - Q^2\left(\frac{x^2}{4} - z^2\right)}{(\lambda^2+\Delta^2)^3} - \frac{\frac{2}{4-\epsilon}\lambda^2 - 2M^2(1-x)^2}{(\lambda^2+\Delta'^2)^3} \right\} \tag{13}$$

$$F_2(Q^2) = -2M^2 \cdot 4g^2 \int_0^1 dx \int_0^x dy \int \frac{d^{4-\epsilon}\lambda}{(2\pi)^4} \mu^\epsilon \left\{ \frac{x^2}{(\lambda^2+\Delta^2)^3} + \frac{(1-x)^2}{(\lambda^2+\Delta'^2)^3} \right\} \tag{14}$$

Where Δ and Δ' are defined as:

$$\begin{aligned} \Delta^2 &= Q^2\left(\frac{x^2}{4} - z^2\right) + (1-x)m^2 + x^2 M^2 \\ &\equiv Q^2\left(\frac{x^2}{4} - z^2\right) + \mathcal{M}^2 \\ \Delta'^2 &= Q^2\left(\frac{x^2}{4} - z^2\right) + xm^2 + (1-x)^2 M^2 \\ &\equiv Q^2\left(\frac{x^2}{4} - z^2\right) + \mathcal{M}'^2 \end{aligned} \tag{15}$$

All the convergent integrals take the following form (taking $\epsilon \to 0$):

$$\int_0^\infty \frac{d^4\lambda}{(2\pi)^4} \frac{1}{(\lambda^2+\Delta_i^2)^3} = \frac{2\pi^2}{(2\pi)^4} \frac{1}{4\Delta_i^2} \tag{16}$$

Using:

$$\begin{aligned} \int \frac{d^{4-\epsilon}\lambda}{(4\pi)^{4-\epsilon}} \mu^\epsilon \frac{\lambda^2}{(\lambda^2+\Delta_i^2)^3} &= \frac{\mu^\epsilon}{(4\pi)^{2-\epsilon/2}} \left(\frac{4-\epsilon}{2}\right) \frac{\Gamma\left(3 - \frac{4-\epsilon}{2} - 1\right)}{\Gamma(3)} \frac{1}{\Delta_i^{2\left(3-\frac{4-\epsilon}{2}-1\right)}} \\ &\xrightarrow{\epsilon \ll 1} \frac{1}{2}\frac{2\pi^2}{(2\pi)^4} \left(\frac{4-\epsilon}{2}\right) \frac{1}{2!} \left(\frac{2}{\epsilon} - \gamma_E - \ln\left(\frac{\Delta_i^2}{\mu^2}\right)\right) \end{aligned} \tag{17}$$

So the two divergent terms in F_1 are:

$$\int \frac{d^{4-\epsilon}\lambda}{(2\pi)^4} \left(\frac{2-\epsilon}{4-\epsilon} \frac{\lambda^2}{(\lambda^2+\Delta^2)^3} - \frac{2}{4-\epsilon} \frac{\lambda^2}{(\lambda^2+\Delta'^2)^3} \right)$$

$$\begin{aligned} &= \frac{1}{2}\frac{2\pi^2}{(2\pi)^4} \left(\frac{4-\epsilon}{4}\right) \left(\frac{2-\epsilon}{4-\epsilon} \left(\frac{2}{\epsilon} - \gamma_E - \ln\left(\frac{\Delta^2}{\mu^2}\right)\right) - \frac{2}{4-\epsilon}\left(\frac{2}{\epsilon} - \gamma_E - \ln\left(\frac{\Delta'^2}{\mu^2}\right)\right)\right) \\ &= \frac{1}{4}\frac{2\pi^2}{(2\pi)^4} \left(\frac{1-\epsilon/2}{\epsilon} - \frac{1}{\epsilon} - \gamma_E\left(\frac{2-\epsilon}{4} - \frac{1}{2}\right) - \frac{2-\epsilon}{4}\ln\left(\frac{\Delta^2}{\mu^2}\right) + \frac{1}{2}\ln\left(\frac{\Delta'^2}{\mu^2}\right)\right) \\ &\xrightarrow{\epsilon \to 0} \frac{1}{4}\frac{2\pi^2}{(2\pi)^4} \left(-1 - \ln\left(\frac{\Delta^2}{\Delta'^2}\right)\right) \end{aligned} \tag{18}$$

So we have:

$$F_1(Q^2) = \frac{2\pi^2 g^2}{(2\pi)^4}\int_0^1 dx \int_{-\frac{x}{2}}^{\frac{x}{2}} dz \left\{ \frac{M^2x^2 - Q^2\left(\frac{x^2}{4}-z^2\right)}{Q^2\left(\frac{x^2}{4}-z^2\right)+\mathcal{M}^2} + \frac{2M^2(1-x)^2}{Q^2\left(\frac{x^2}{4}-z^2\right)+\mathcal{M}'^2} - 1 + \ln\left(\frac{Q^2\left(\frac{x^2}{4}-z^2\right)+\mathcal{M}'^2}{Q^2\left(\frac{x^2}{4}-z^2\right)+\mathcal{M}^2}\right)\right\} \tag{19}$$

$$F_2(Q^2) = -2M^2\frac{2\pi^2 g^2}{(2\pi)^4}\int_0^1 dx \int_{-\frac{x}{2}}^{\frac{x}{2}} dz \left\{ \frac{x^2}{Q^2\left(\frac{x^2}{4}-z^2\right)+\mathcal{M}^2} + \frac{(1-x)^2}{Q^2\left(\frac{x^2}{4}-z^2\right)+\mathcal{M}'^2}\right\} \tag{20}$$

There are now three distinct integrals over z:

$$\int_{-x/2}^{x/2} dz \frac{1}{\mathcal{M}_i^2 + Q^2\left(\frac{x^2}{4}-z^2\right)} = \frac{2}{Q\sqrt{\mathcal{M}_i^2+\frac{Q^2x^2}{4}}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}_i^2+\frac{Q^2x^2}{4}}}\right) \tag{21}$$

$$\int_{-x/2}^{x/2} dz \frac{Q^2z^2}{\mathcal{M}^2 + Q^2\left(\frac{x^2}{4}-z^2\right)} = \frac{2}{Q}\sqrt{\mathcal{M}^2+\frac{Q^2x^2}{4}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}^2+\frac{Q^2x^2}{4}}}\right) - x$$

$$\int_{-x}^{x} dz \,\ln\left(\frac{\mathcal{M}'^2 + Q^2\left(\frac{x^2}{4}-z^2\right)}{\mathcal{M}^2 + Q^2\left(\frac{x^2}{4}-z^2\right)}\right) =$$

$$\frac{4}{Q}\left(\sqrt{\mathcal{M}'^2+\frac{Q^2x^2}{4}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}'^2+\frac{Q^2x^2}{4}}}\right) - \sqrt{\mathcal{M}^2+\frac{Q^2x^2}{4}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}'^2+\frac{Q^2x^2}{4}}}\right)\right) + x\ln\left(\frac{\mathcal{M}'^2}{\mathcal{M}^2}\right)$$

Note that the second term of the third integral partially cancels the first term of the second integral. This leaves us with:

$$\begin{aligned} F_1(Q^2) &= \frac{\pi^2 g^2}{(2\pi)^4}\int_0^1 dx \left\{ \frac{2}{Q}\frac{4M^2(1-x)^2 + 2m^2x + \frac{Q^2x^2}{2}}{\sqrt{\mathcal{M}'^2+\frac{Q^2x^2}{4}}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}'^2+\frac{Q^2x^2}{4}}}\right)\right. \\ &\left. -\frac{2}{Q}\frac{m^2(1-x) - \frac{Q^2x^2}{2}}{\sqrt{\mathcal{M}^2+\frac{Q^2x^2}{4}}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}^2+\frac{Q^2x^2}{4}}}\right) + x\ln\left(\frac{\mathcal{M}'^2}{\mathcal{M}^2}\right) - 2x\right\} \end{aligned} \tag{22}$$

$$F_2(Q^2) = -2M^2\frac{\pi^2 g^2}{(2\pi)^4}\int_0^1 dx \frac{2}{Q}\left\{ \frac{x^2}{\sqrt{\mathcal{M}^2+\frac{Q^2x^2}{4}}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}^2+\frac{Q^2x^2}{4}}}\right) + \frac{(1-x)^2}{\sqrt{\mathcal{M}'^2+\frac{Q^2x^2}{4}}}\tanh^{-1}\left(\frac{Qx}{2\sqrt{\mathcal{M}'^2+\frac{Q^2x^2}{4}}}\right)\right\} \tag{23}$$

3.1 Form factors at $Q^2 = 0$

The values of F_1 and F_2 at $Q^2 = 0$ are of interest. We can get these directly from equations (13) and (14). Setting $Q = 0$ in both of these equations leaves the integrand independent of z, so integrating over z just gives a multiplicative factor of x:

$$F_1(0) = 4g^2\int_0^1 dx \int \frac{d^{4-\epsilon}\lambda\mu^\epsilon}{(2\pi)^4}\left\{ \frac{x(\frac{(2-\epsilon)\lambda^2}{4-\epsilon} + M^2x^2)}{(\lambda^2+\mathcal{M}^2)^3} - \frac{x(\frac{2\lambda^2}{4-\epsilon} - 2M^2(1-x)^2)}{(\lambda^2+\mathcal{M}'^2)^3}\right\} \tag{24}$$

$$F_2(0) = -2M^2 \frac{8\pi^2 g^2}{(2\pi)^4} \int_0^1 dx \int d\lambda\, \lambda^3 \left\{ \frac{x^3}{(\lambda^2 + \mathcal{M}^2)^3} + \frac{x(1-x)^2}{(\lambda^2 + \mathcal{M}'^2)^3} \right\} \tag{25}$$

$\mathcal{M}'$ is identical to $\mathcal{M}$ with x flipped with $1-x$, and since x is integrated over, we can simply substitute x for $1-x$ in the second term of each of these integrals and the fractions combine:

$$F_1(0) = 4g^2 \int_0^1 dx \int \frac{d^{4-\epsilon}\lambda \mu^\epsilon}{(2\pi)^4} \frac{\left(x - \frac{2}{4-\epsilon}\right)\lambda^2 + M^2 x^2 (2-x)}{(\lambda^2 + \mathcal{M}^2)^3} \tag{26}$$

$$F_2(0) = -2M^2 \frac{8\pi^2 g^2}{(2\pi)^4} \int_0^1 dx \int d\lambda\, \lambda^3 \frac{x^2}{(\lambda^2 + \mathcal{M}^2)^3} \tag{27}$$

Now we can integrate over λ and use eq. (17) (ϵ has been taken to 0 after integration).

$$F_1(0) = \frac{2\pi^2 g^2}{(2\pi)^4} \int_0^1 dx \left\{ (2x-1)\left(\ln\left(\frac{\mu^2}{\mathcal{M}^2}\right) - \gamma_E\right) - x + \frac{M^2 x^2 (2-x)}{\mathcal{M}^2} \right\} \tag{28}$$

$$F_2(0) = -2M^2 \frac{2\pi^2 g^2}{(2\pi)^4} \int_0^1 dx \frac{x^2}{\mathcal{M}^2} \tag{29}$$

We have the following term in F_1:

$$\int_0^1 dx (2x-1) \times \text{constants}$$

This integrates to 0. A direct consequence of this is that the mass scale μ doesn't affect the answer, as changing μ only shifts the log by an x-independent amount. For simplicity, let $\mu = M$. and we're left with the following form factors at 0:

$$\begin{aligned}
F_1(0) &= \frac{2\pi^2 g^2}{(2\pi)^4} \int_0^1 dx \left\{ (2x-1)\ln\left(\frac{M^2}{M^2 x^2 + m^2(1-x)}\right) + \frac{M^2 x^2 (2-x)}{M^2 x^2 + m^2(1-x)} - x \right\} \\
&= \frac{2\pi^2 g^2}{(2\pi)^4} \int_0^1 dx \left\{ \frac{2M^2 x - m^2}{M^2 x^2 + m^2(1-x)}(x^2 - x) + \frac{M^2 x^2 (2-x)}{M^2 x^2 + m^2(1-x)} - x \right\} \\
&= -\frac{2\pi^2 g^2}{(2\pi)^4} \int_0^1 dx\, (x - x) \\
&= 0 && (30) \\
F_2(0) &= -2M^2 \frac{2\pi^2 g^2}{(2\pi)^4} \int_0^1 dx \frac{x^2}{M^2 x^2 + m^2(1-x)} && (31)
\end{aligned}$$

Where the natural log was integrated by parts. This means neutrons are uncharged (a perhaps unsurprising result). $F_2(0)$ can be evaluated exactly in the chiral limit ($m/M \to 0$):

$$F_2(0) \to -2 \frac{2\pi^2 g^2}{(2\pi)^4} \tag{32}$$

And numerically for more the more realistic value of $m/M = 1/7$:

$$F_2(0) \approx -1.65 \frac{2\pi^2 g^2}{(2\pi)^4} \tag{33}$$

3.2 Conclusions

The $F_2(0)$ value can be used to determine our theory's coupling constant $\alpha = g^2/4\pi$, as it is related to the experimentally measured gyromagnetic ratio of the neutron. Precisely, it is the neutron's g-factor: $F_2(0) = -3.826....$[1] Therefore:

$$\alpha = \frac{g^2}{4\pi} = -3.82 \frac{2\pi}{-1.65} \approx 14.55$$

The strong coupling constant changes dramatically for low energies, but should be somewhere around 13.5, so this is to be expected.

References

[1] International Committee for Data CODATA. *CODATA 2022: Advances in Data Science and Technology*. Tech. rep. CODATA-Report-2022. Paris, France: Committee on Data for Science and Technology, 2022. URL: `https://physics.nist.gov/cgi-bin/cuu/Value?eqgnn`.

[2] Alan Martin. *Quarks and Leptons*. 1984.