Neutron Star Equations of State

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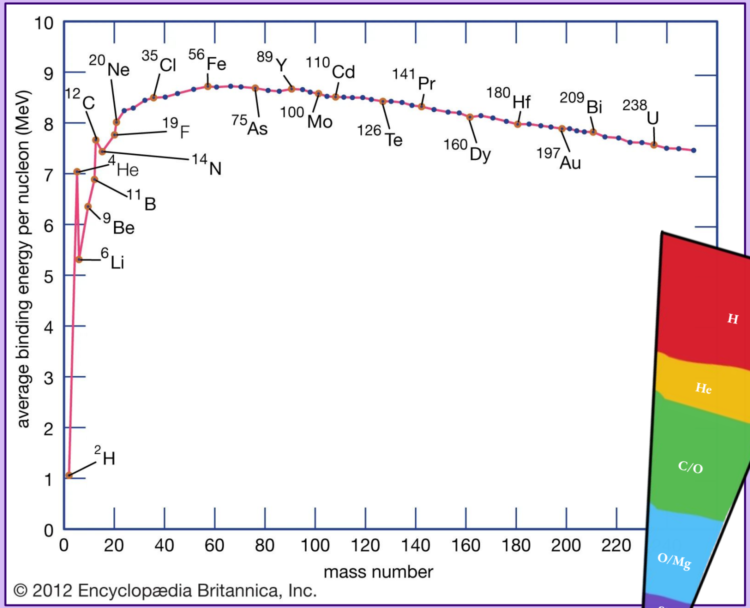
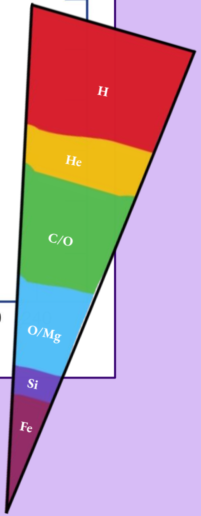
What are neutron stars?

https://www.nasa.gov/feature/goddard/2016/astronomers-find-the-first-wind-nebula-around-a-magnetar

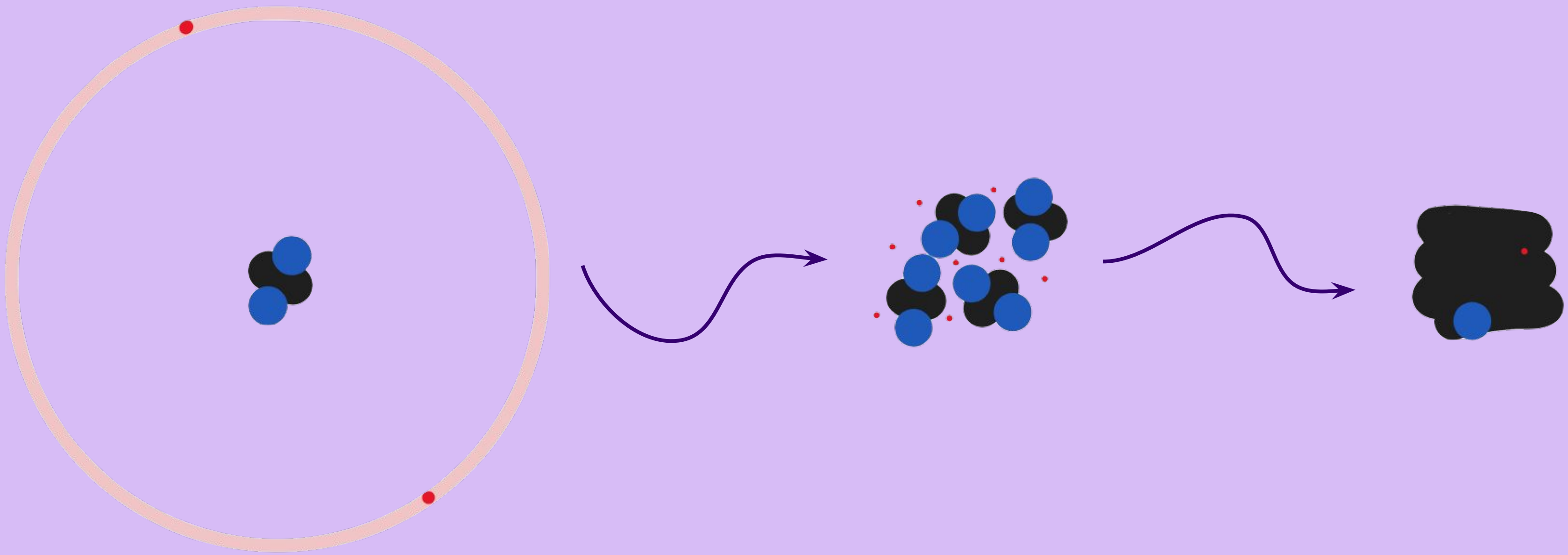
How do they form?



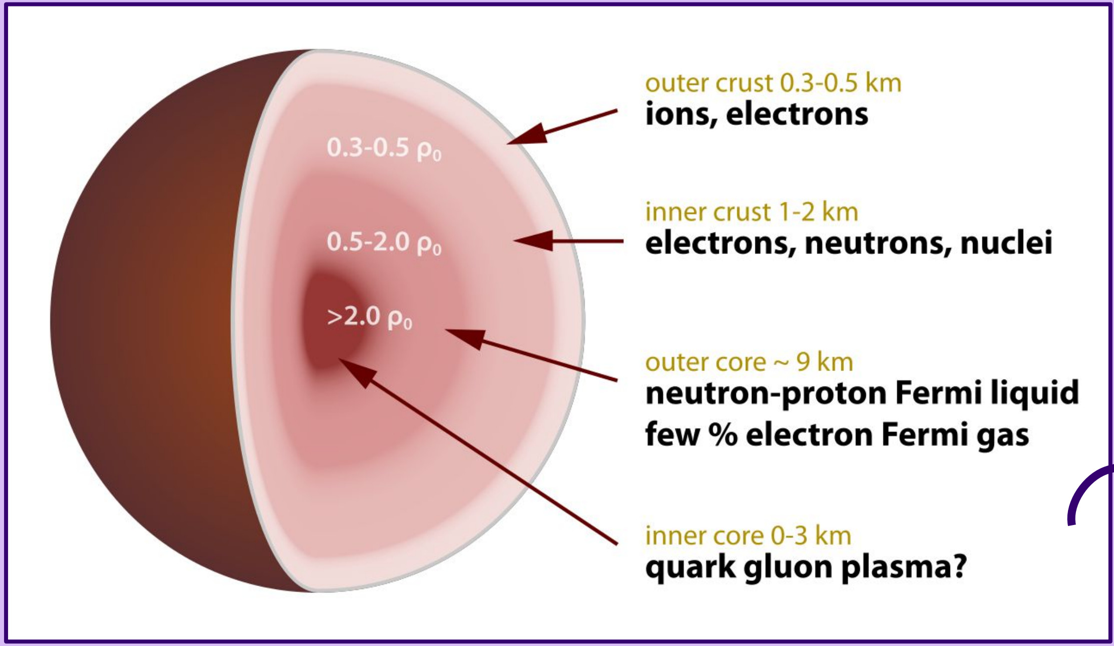


https://sci.esa.int/web/integral/-/48128-multi-wavelength-view-of-the-crab-nebula

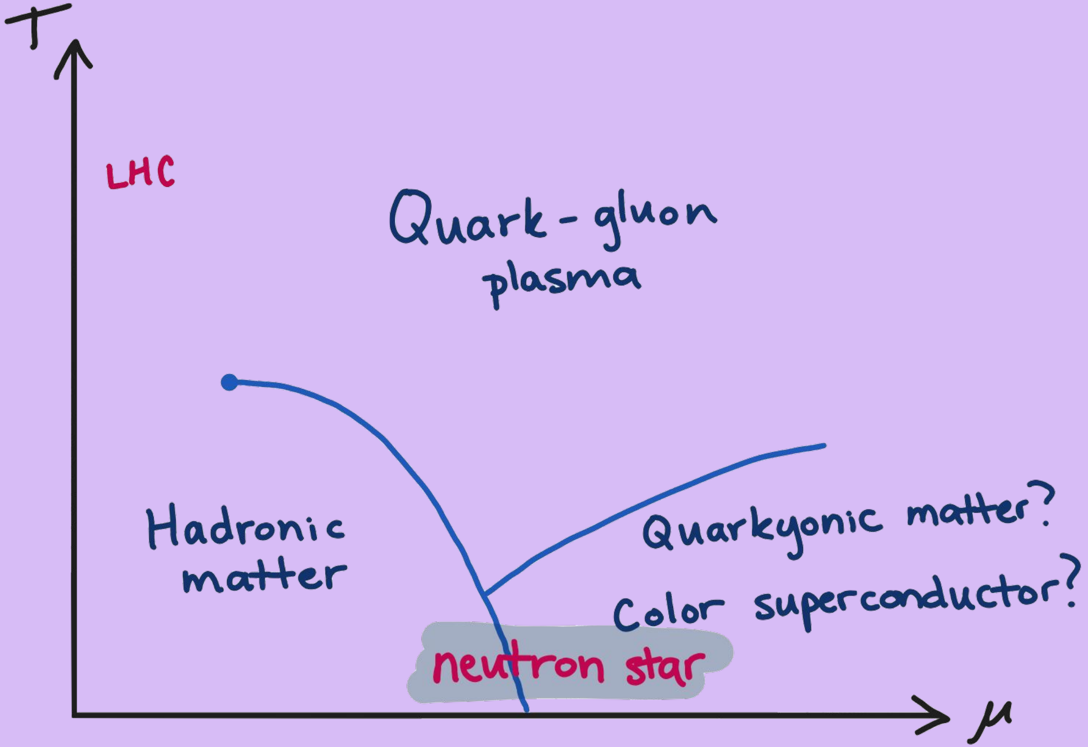
What holds up the collapsed core?



What is a neutron star's structure?



Why do we study neutron stars?



Why now?

Building a neutron star

What tools do we need?

- Structure equations:

- Initial conditions:

- Relationship between pressure and density:

What tools do we need?

- Structure equations:

$$\frac{dp}{dr} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)c^2}\right]\left[1-\frac{2G\mathcal{M}(r)}{c^2 r}\right]^{-1}$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2\rho(r) = \frac{4\pi r^2\epsilon(r)}{c^2}$$

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- Initial conditions:

$$M(r_0) = 0 \qquad p(r_0) = p_0$$

- Relationship between pressure and density:

What tools do we need?

- Structure equations:

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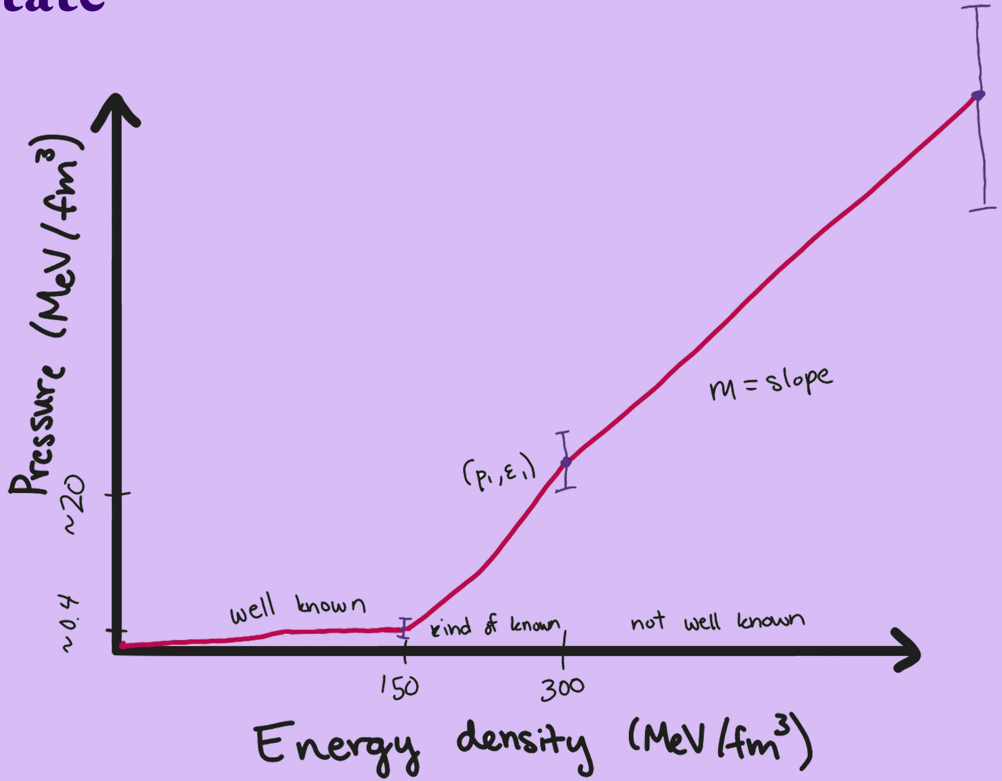
Equation of state (EoS)

What is considered when constructing an EoS?

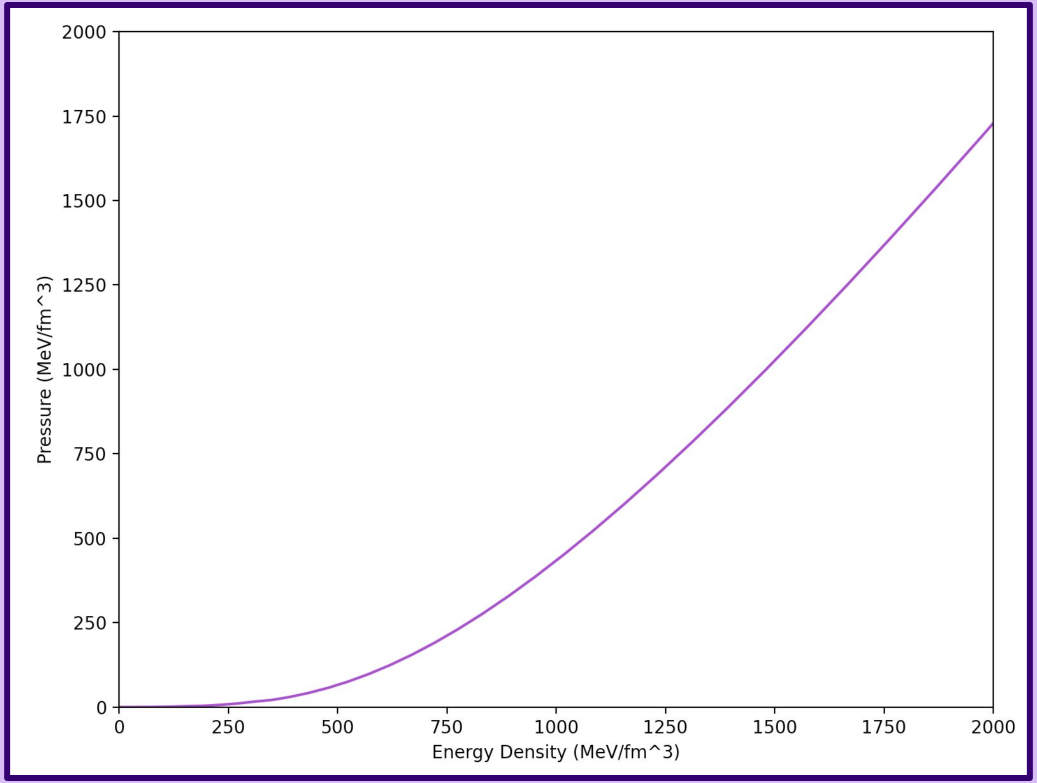
- Proton and electron contributions
- Nucleon-nucleon interactions
  - Effective field theories
  - Nuclei surface tensions
- Coulomb interactions
- Speed of sound
- Different behavior in different sections of the NS

Equation of State

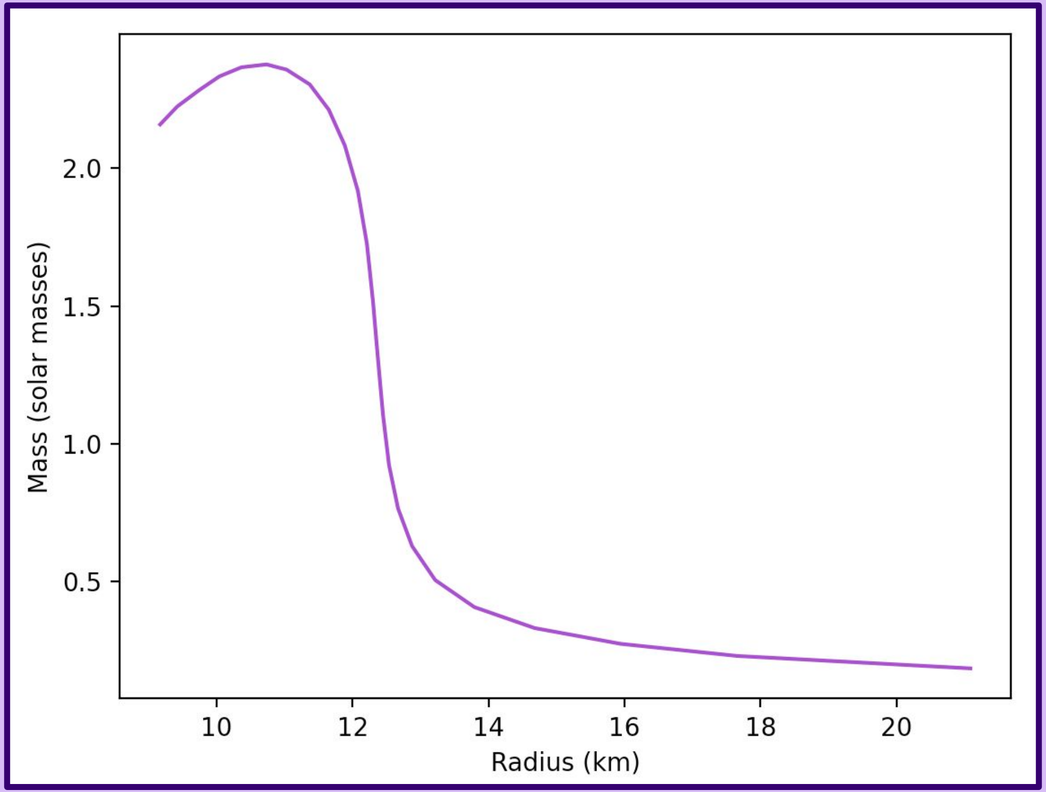


How important is the outer core EOS for the maximum mass of neutron stars?

Fancy neutron star model

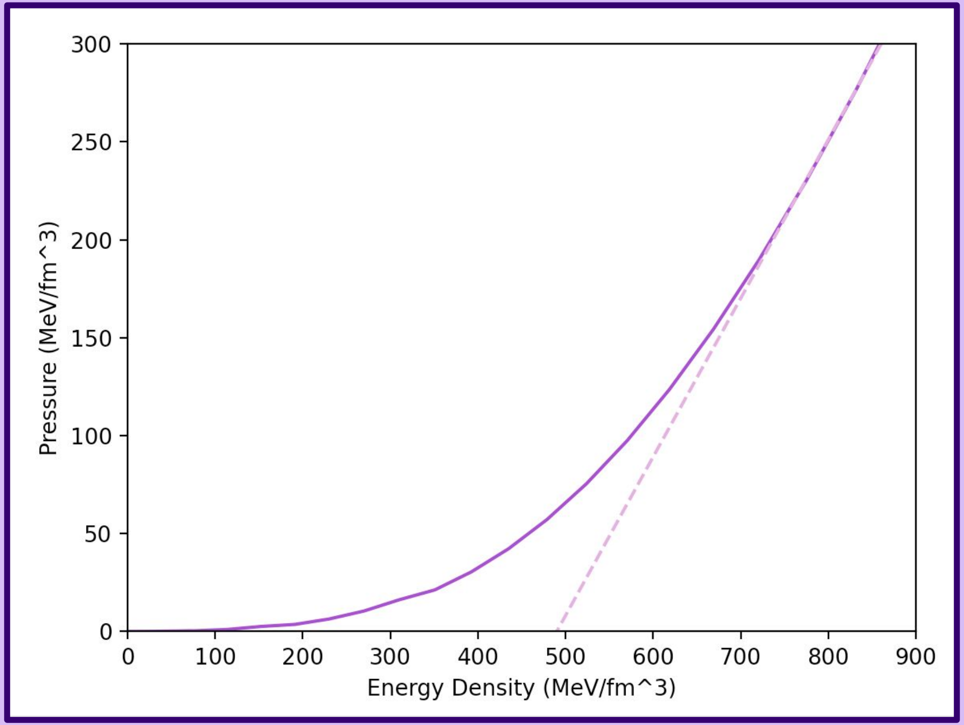


Fancy neutron star model

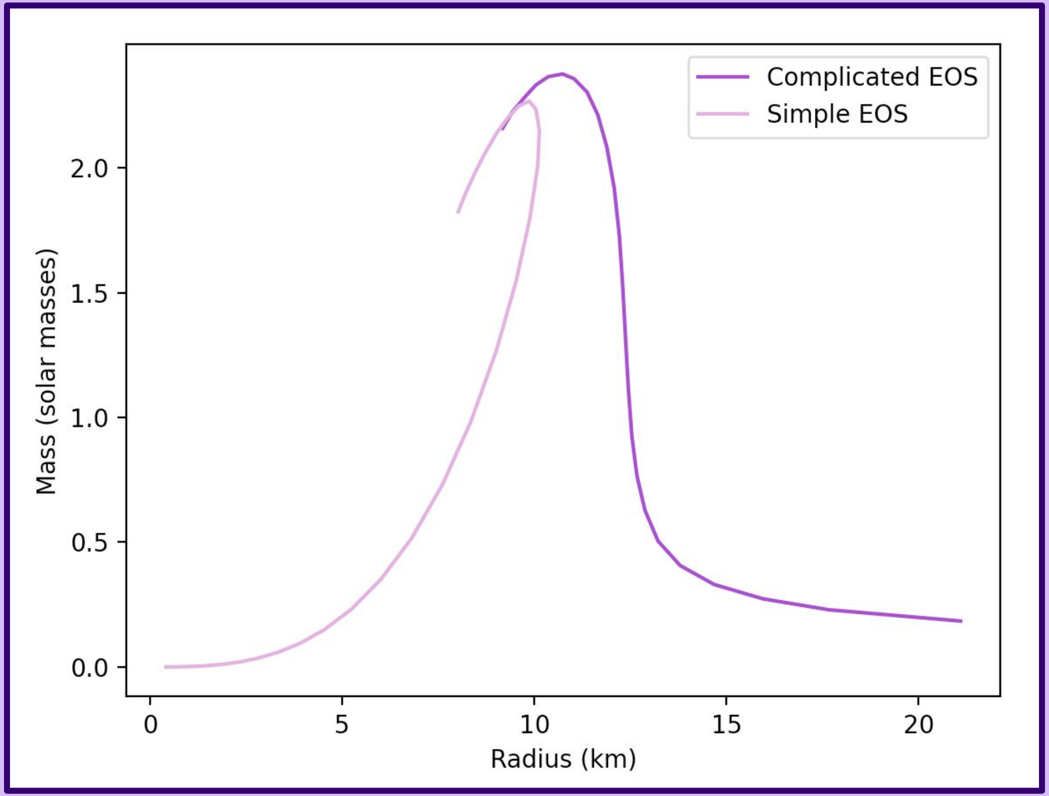


Can we use a simple fit of any high density EOS to extract the mass?

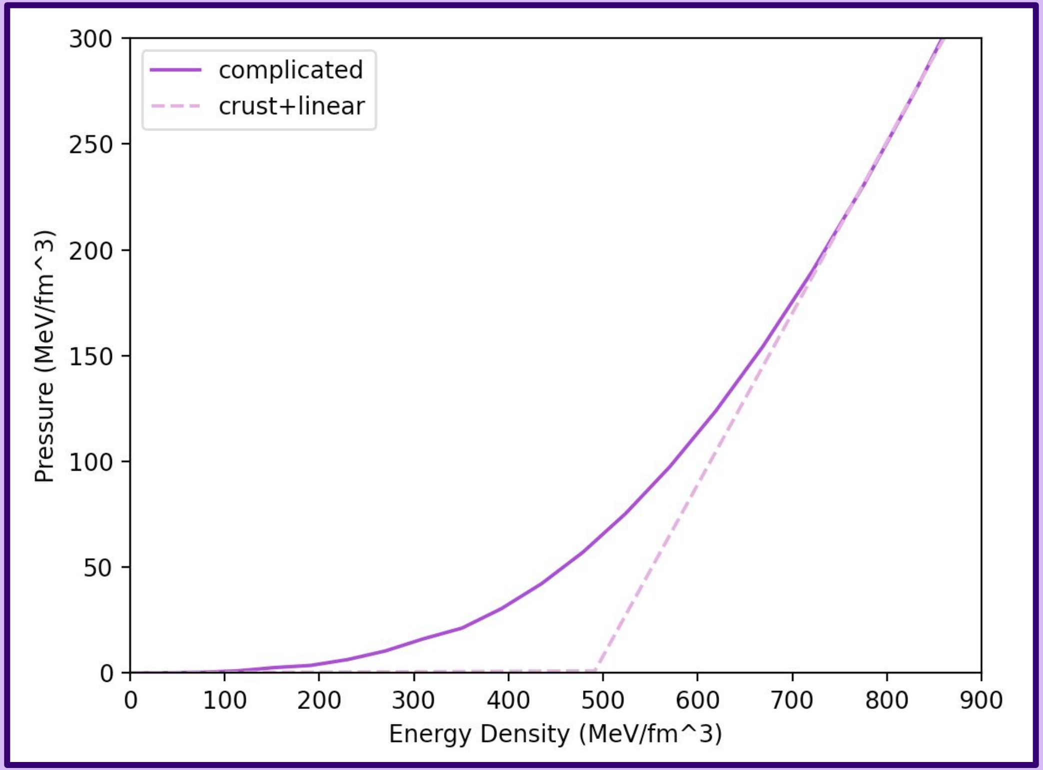
Using a simple, linear fit



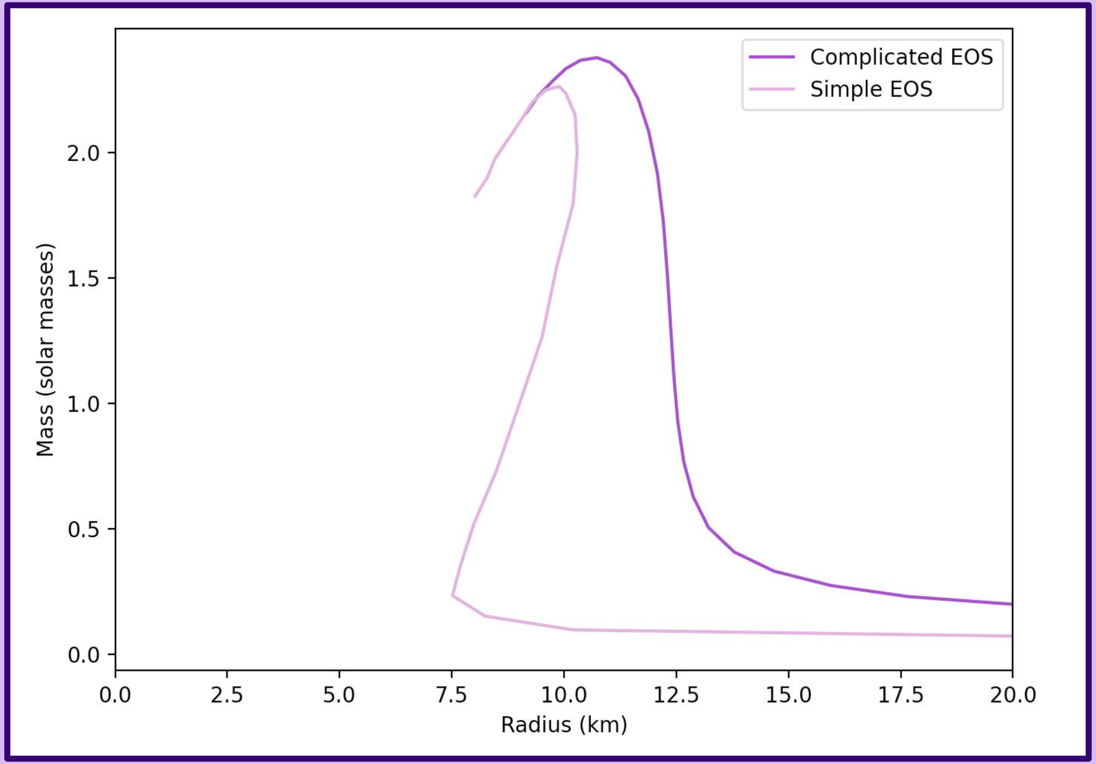
Resulting mass-radius plot



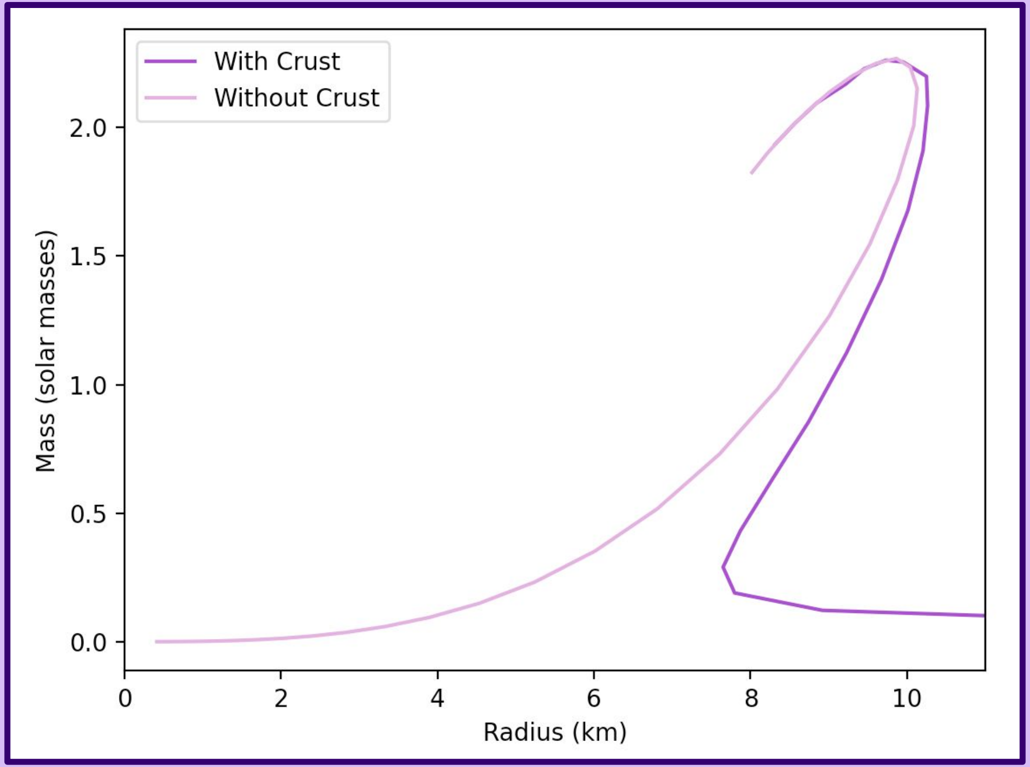
Using the crust and then a linear fit



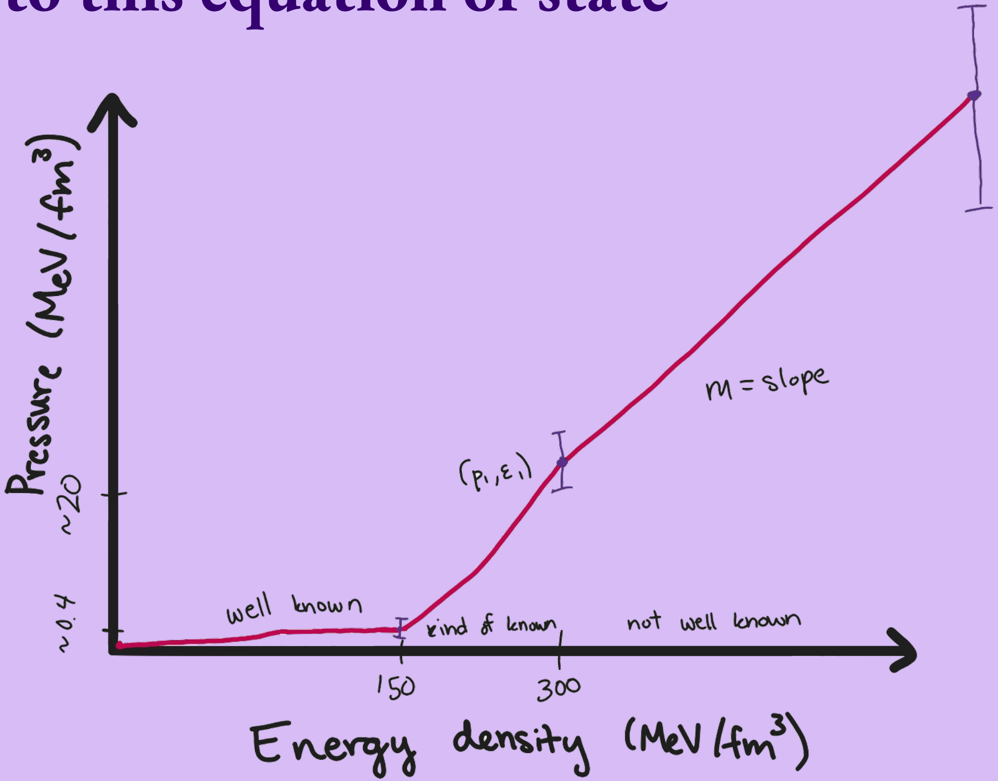
Resulting mass-radius plot



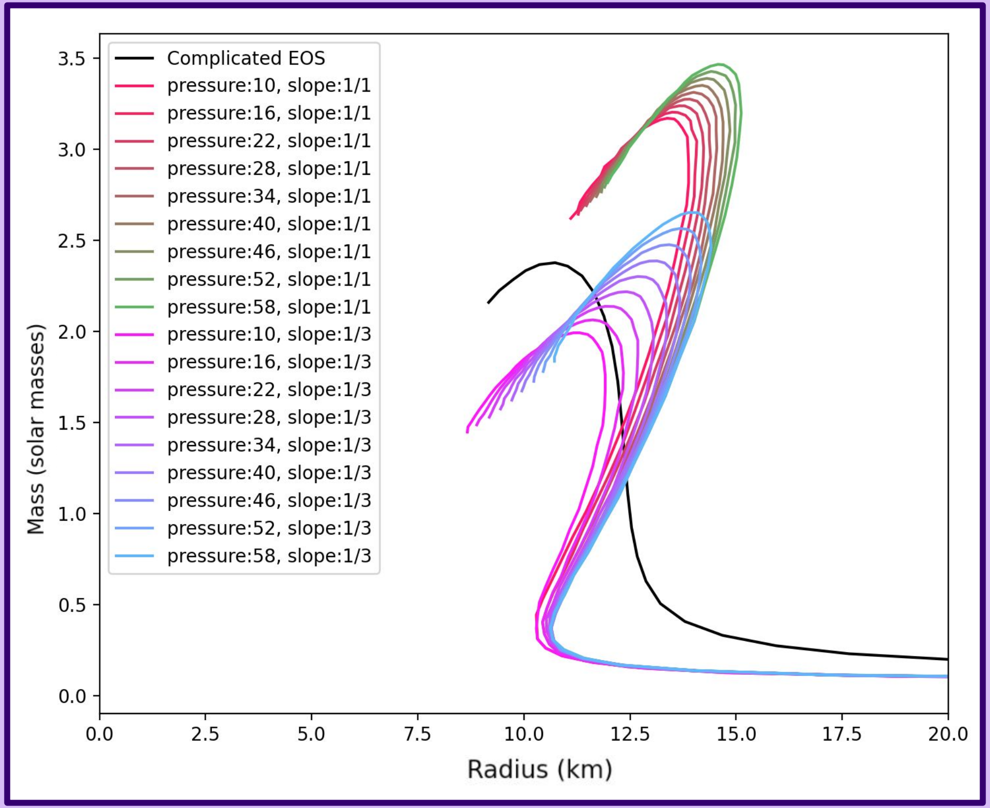
The crust doesn't make a huge difference



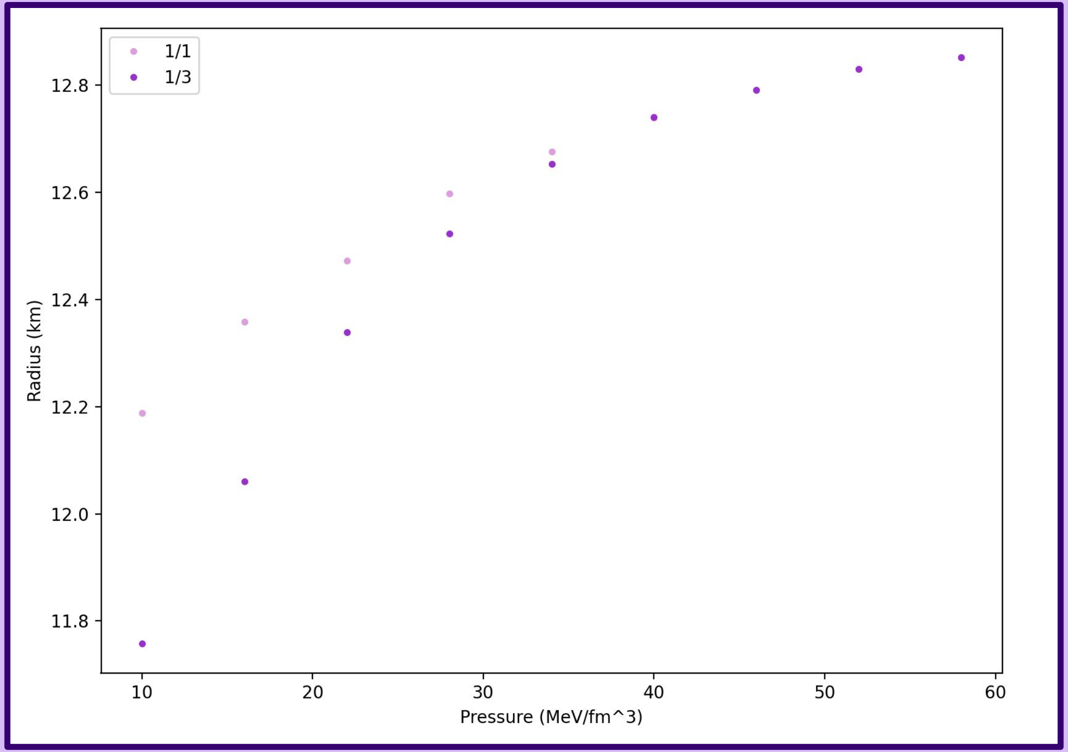
Coming back to this equation of state



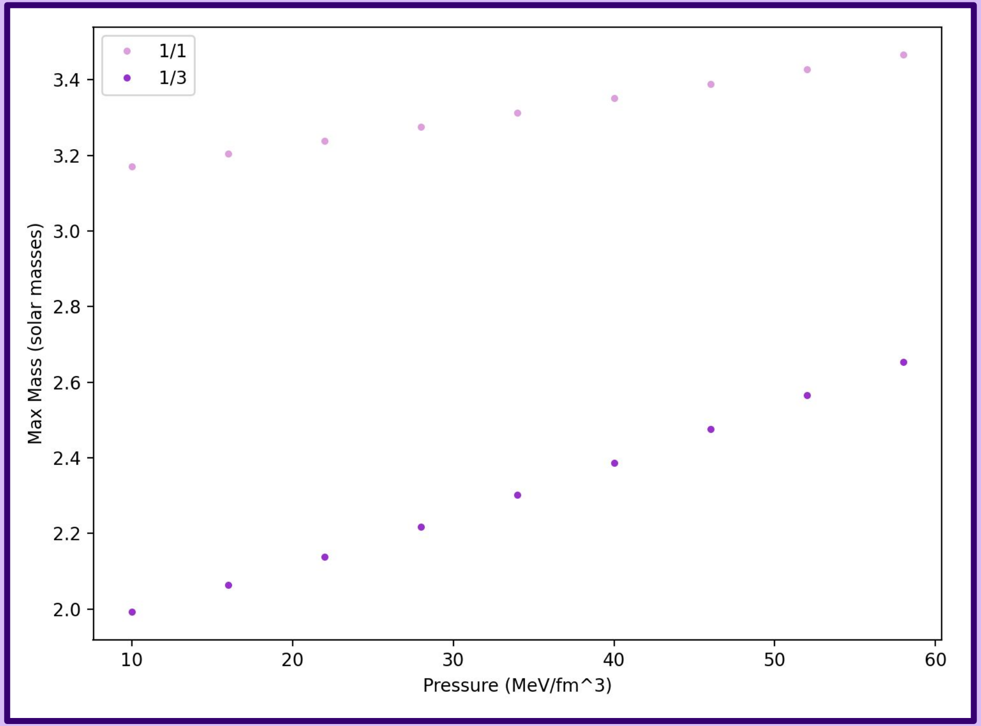
Mass-radius plots varying slope and pressure



How the radius at 1.4 $M_\odot$ changes

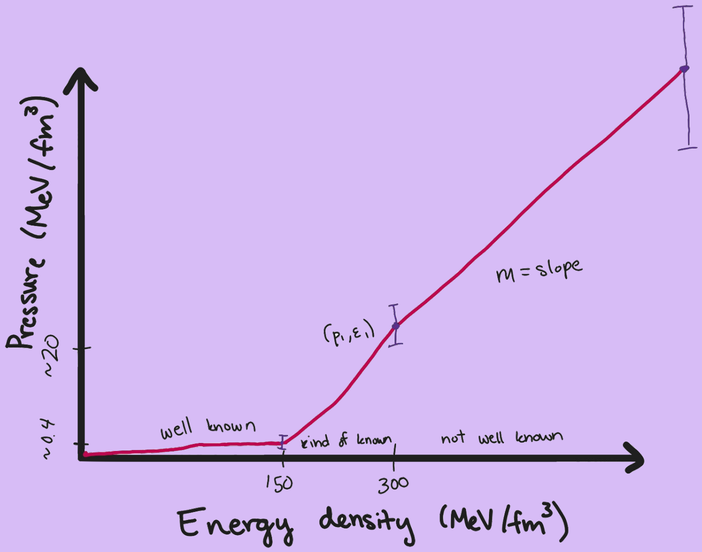


How the maximum mass changes



Conclusion

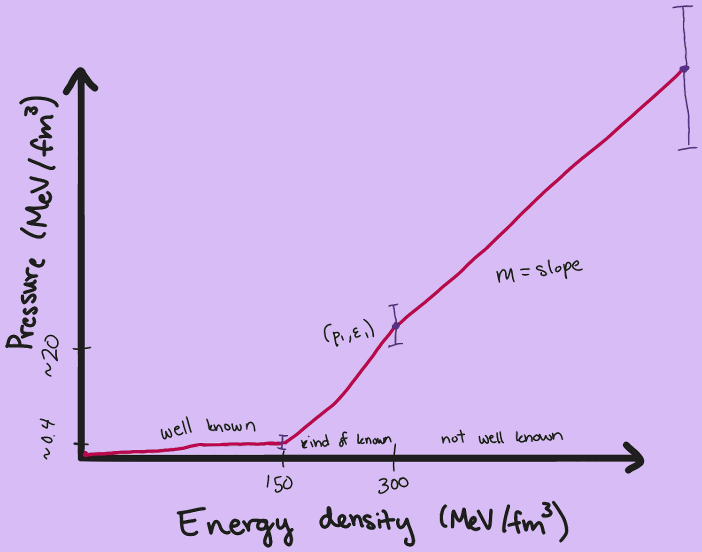
We can extract information from our simplified EoS!



Conclusion

We can extract information from our simplified EoS!

- For the radius at the Chandrasekhar limit, p_1 has a large effect, m has a minor effect
- For the maximum mass, p_1 has a small effect and m has a large effect



Acknowledgements

Thanks to:

- Sanjay Reddy
- The INT
- The NSF
- All of you!

INSTITUTE for NUCLEAR THEORY

Questions?

$$\begin{aligned}\epsilon_{\text{elec}}(k_F) &= \frac{8\pi}{(2\pi\hbar)^3}\int_0^{k_F}(k^2c^2+m_e^2c^4)^{1/2}k^2dk \\ &= \epsilon_0\int_0^{k_F/m_ec}(u^2+1)^{1/2}u^2du \\ &= \frac{\epsilon_0}{8}\left[(2x^3+x)(1+x^2)^{1/2}-\sinh^{-1}(x)\right]\end{aligned}$$

$$\epsilon = nm_NA/Z+\epsilon_{\text{elec}}(k_F)$$

$$p = -\left.\frac{\partial U}{\partial V}\right]_{T=0} = n^2\frac{d(\epsilon/n)}{dn} = n\frac{d\epsilon}{dn}-\epsilon = n\mu-\epsilon$$

$$\begin{aligned}p(k_F) &= \frac{8\pi}{3(2\pi\hbar)^3}\int_0^{k_F}(k^2c^2+m_e^2c^4)^{-1/2}k^4dk \\ &= \frac{\epsilon_0}{3}\int_0^{k_F/m_ec}(u^2+1)^{-1/2}u^4du \\ &= \frac{\epsilon_0}{24}\left[(2x^3-3x)(1+x^2)^{1/2}+3\sinh^{-1}(x)\right]\end{aligned}$$

$$
\begin{aligned}
p(k_F) &= \frac{\epsilon_0}{3}\int_0^{k_F/m_e c} u^3 du = \frac{\epsilon_0}{12}(k_F/m_e c)^4 = \frac{\hbar c}{12\pi^2}\left(\frac{3\pi^2 Z\rho}{m_N A}\right)^{4/3} \\
&\approx K_{\mathrm{rel}}\,\epsilon^{4/3},
\end{aligned}
$$

$$
K_{\mathrm{rel}} = \frac{\hbar c}{12\pi^2}\left(\frac{3\pi^2 Z}{A m_N c^2}\right)^{4/3} .
$$

$$
p = K_{\mathrm{non-rel}}\epsilon^{5/3}, \quad \text{where} \;\; K_{\mathrm{non-rel}} = \frac{\hbar^2}{15\pi^2 m_e}\left(\frac{3\pi^2 Z}{A m_N c^2}\right)^{5/3}
$$

$$
\bar{\epsilon}(\bar{p}) = A_{\mathrm{NR}}\bar{p}^{3/5} + A_{\mathrm{R}}\bar{p}^{3/4}
$$

$$p(n) = n^2 \frac{d}{dn}\left(\frac{\epsilon}{n}\right) = n_0 \left[\frac{2}{3}\left\langle E_F^0 \right\rangle u^{5/3} + \frac{A}{2} u^2 + \frac{B\sigma}{\sigma + 1} u^{\sigma+1}\right]$$

$$\begin{aligned} p(n, x) &= u \frac{d}{du} \epsilon(n, \alpha) - \epsilon(n, \alpha) \\ &= p(n, 0) + n_0 \alpha^2 \left[\frac{2^{2/3} - 1}{5}\left\langle E_F^0 \right\rangle \left(2u^{5/3} - 3u^2\right) + S_0 u^2\right] \end{aligned}$$

$$V_{\mathrm{Nuc}}(u, 0) = \frac{A}{2} u + \frac{B}{\sigma + 1} \frac{u^\sigma}{1 + C u^{\sigma - 1}}$$

$$
\begin{aligned}
\Delta\epsilon_{KE}(n,\alpha) &= \epsilon_{KE}(n,\alpha) - \epsilon_{KE}(n,0) \\
&= n\left\langle E_F\right\rangle \left\{\frac{1}{2}\left[(1+\alpha)^{5/3} + (1-\alpha)^{5/3}\right] - 1\right\} \\
&= n\left\langle E_F\right\rangle \left\{2^{2/3}\left[(1-x)^{5/3} + x^{5/3}\right] - 1\right\} .
\end{aligned}
$$

$$
E(n,\alpha) = E(n,0) + \alpha^2 S(n)
$$

$$
S(u) = (2^{2/3} - 1)\frac{3}{5}\left\langle E_F^0\right\rangle \left(u^{2/3} - F(u)\right) + S_0 F(u)
$$

$$
\langle E_F\rangle = \frac{3}{5}\frac{\hbar^2}{2m_N}\left(\frac{3\pi^2 n}{2}\right)^{2/3}
$$