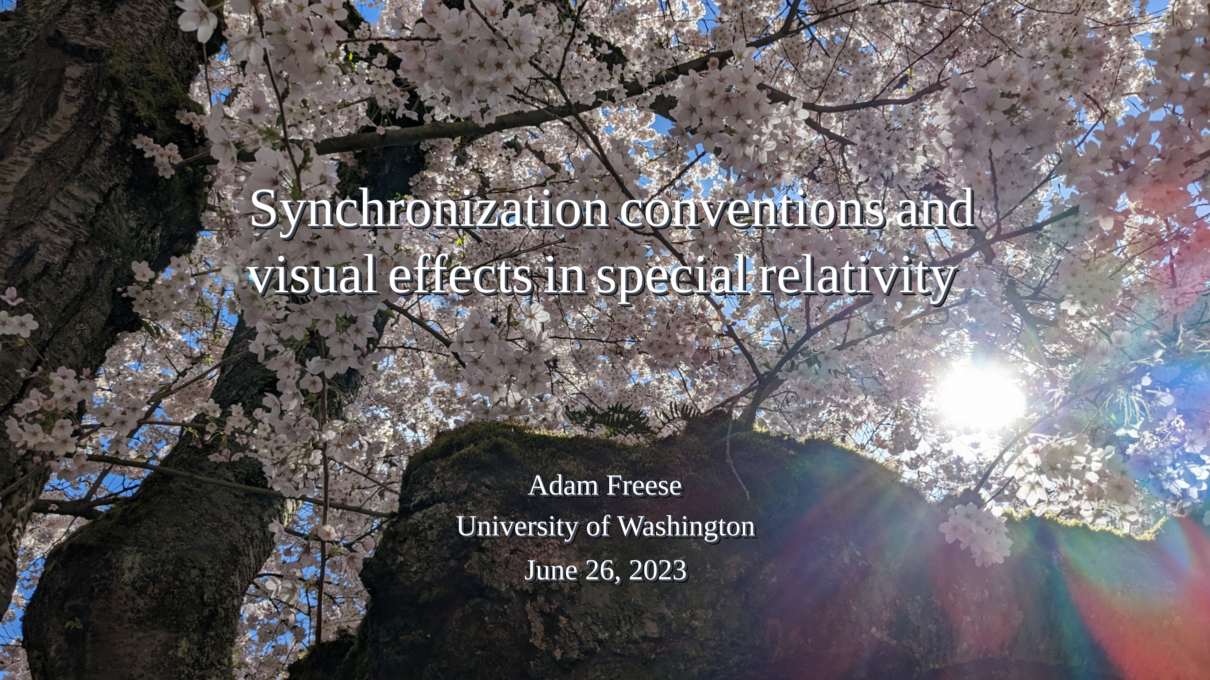

Synchronization conventions and visual effects in special relativity

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June 26, 2023

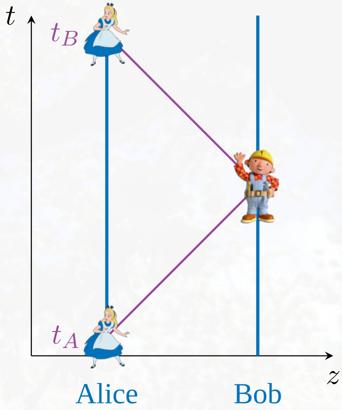
The speed of light

- How would you measure the speed of light?

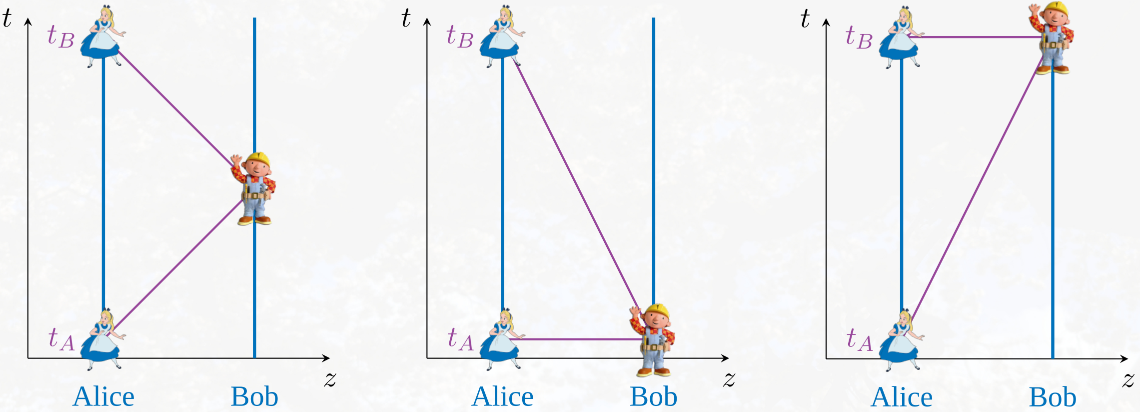
- Station Alice and Bob a fixed distance L apart.
- Alice sends a light signal to Bob, he sends it back.
- Just calculate:

$$c = \frac{2L}{\Delta t}$$

  - Average speed of light during the round trip.
  - Did it move at c in each direction?
  - How can we tell?



The one-way speed of light



- What if light is infinitely fast in the Alice→Bob direction?
- Or maybe the Bob→Alice direction?
- How could we tell?

Synchronizing clocks

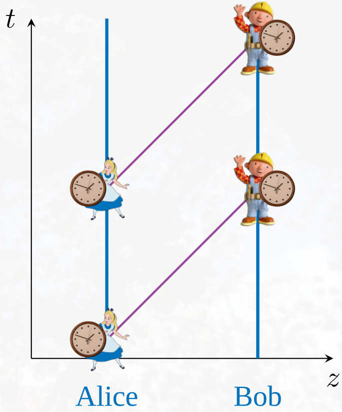
- Alice and Bob synchronize their clocks.
- Alice sends light signal to Bob at time t.
- Bob receives at time $t + \Delta t$.
- Find the one-way speed to be:

$$c_{\mathrm{A}\to\mathrm{B}} = \frac{L}{\Delta t}$$

- But how do they synchronize their clocks?
  - They send light signals, account for time delay:

$$\Delta t = \frac{L}{c_{\mathrm{A}\to\mathrm{B}}}$$

  - Basically, you need to put $c_{\mathrm{A}\to\mathrm{B}}$ in to get $c_{\mathrm{A}\to\mathrm{B}}$.
  - **Circular reasoning**: you get out what you put in.



Simultaneity is just a convention

- Relativity requires *round-trip* speed of light to be invariant.
- Convention that one-way speed of light be c is a *definition*, not an empirical fact.
  - Pointed out in Einstein's 1905 paper.
  - Poincaré said it in 1898 too.
- Redefining "time" coordinate means changing this definition.
  - And we actually do this in some cases!

894 *A. Einstein.*

B durch einen in B befindlichen Beobachter möglich. Es ist aber ohne weitere Festsetzung nicht möglich, ein Ereignis in A mit einem Ereignis in B zeitlich zu vergleichen; wir haben bisher nur eine „A-Zeit" und eine „B-Zeit", aber keine für A und B gemeinsame „Zeit" definiert. Die letztere Zeit kann nun definiert werden, indem man *durch Definition* festsetzt, daß die „Zeit", welche das Licht braucht, um von A nach B zu gelangen, gleich ist der „Zeit", welche es braucht, um von B nach A zu gelangen. Es gehe nämlich ein Lichtstrahl zur „A-Zeit" t_A von A nach B ab, werde zur „B-Zeit" t_B in B gegen A zu reflektiert und gelange zur „A-Zeit" t'_A nach A zurück. Die beiden Uhren laufen definitionsgemäß synchron, wenn

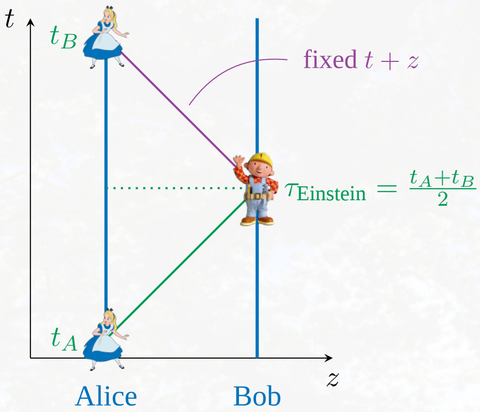
$$t_B - t_A = t'_A - t_B.$$

Einstein, Ann. Phys. 322 (1905) 891

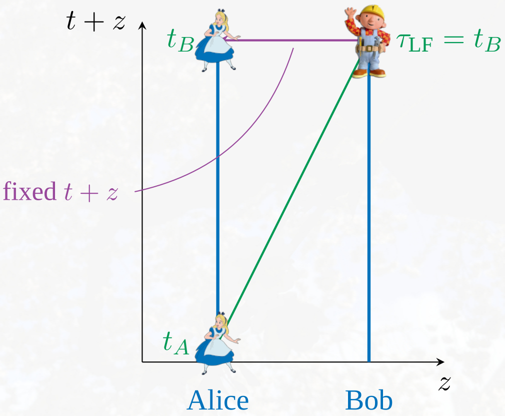
- **Technical review**: Anderson, Stedman & Vetharaniam, Phys. Rept. 295 (1998) 93
- **Didactic overview**: Veritasium, "Why No One Has Measured The Speed of Light" (YouTube)

Synchronization conventions

Einstein synchronization



Light front synchronization

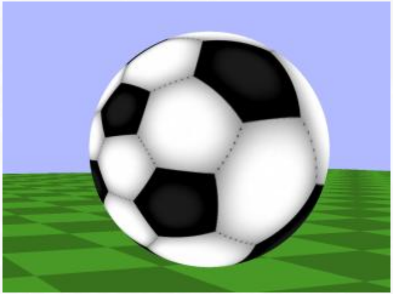


- **Einstein synchronization** defined to be isotropic.
  - Gives you the usual Minkowski coordinates.
- **Light front synchronization** defines hyperplanes with fixed $t + z$ to be "simultaneous."
  - Light travels instantaneously in $-z$ direction *by definition*.

Why consider alternative synchronization schemes?

- It helps show what's really just a matter of convention.
  - Some relativistic effects are artifacts of the synchronization convention
- We can re-examine "paradoxes" from the actual perspectives of the hypothetical observers.
- We can make really cool graphics of what relativistic systems *actually look like*.
  - **Great resource**: `https://www.spacetimetravel.org/`

99% speed of light →



- Actually makes a lot of calculations easier.
  - This is done *all the time* in general relativity.
  - The **light front time** coordinate is used frequently in nuclear and particle physics.

What is time dilation?

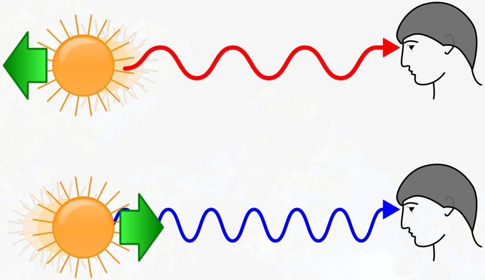
- Time goes slower for moving observers?
  - Would imply there's absolute rest.
  - Bad popsci definition; would never see in physics texts.

- If Alice sees Bob as moving, she also sees his clock going slower?
  - **Empirically false**.
  - If Bob is moving *towards* Alice, she sees his clock going faster! (**Relativistic blueshift**)

- If Bob is moving relative to a **reference frame**, and spatially distant clocks in this frame are synchronized using **Einstein's convention**, then Bob's clock runs slower than the reference frame's time.

Relativistic Doppler effect

- Light from a source is **redshifted** if it's going away from you.
  - Lower frequency.
- Or **blueshifted** if coming towards you.
  - Higher frequency.
- Doppler effect seen in hydrogen spectra of stars, dust clouds, etc.
- Can also use it to selectively hit moving atoms with a laser.
  - Laser cooling
- A **clock** can be tuned to the frequency of light.
  - A clock moving towards you seems to run *faster*.
  - It's getting closer, so less delay between ticks.



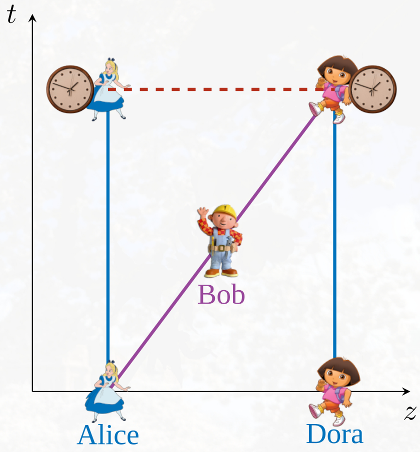
About reference frames

- **Alice** and **Dora** synchronize their clocks by Einstein's convention.
  - The two of them together constitute a **reference frame**.

- **Bob** moves from Alice to Dora.
- A time Δt_B passes on Bob's clock.
- At start: Bob sees Alice's clock reads t_A.
- At end: Bob sees Dora's clock reads t_D.
- **Time dilation** means:

$$\Delta t_B = \sqrt{1 - \frac{v^2}{c^2}}(t_D - t_A) < (t_D - t_A)$$

  - Bob's clock slow compared to *reference frame*.
  - Result depends on Einstein synchronization.



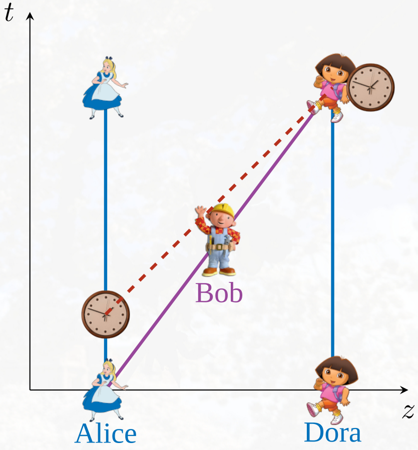
What if we don't use Einstein synchronization?

- What if **Dora** sets her clock to exactly what she sees on **Alice's** clock?
  - Dora *defines* the speed of light to be infinite in the $+z$ direction.

- **Bob** moves from Alice to Dora.
- A time Δt_{B} passes on Bob's clock.
- At start: Bob sees Alice's clock reads t_{A}.
- At end: Bob sees Dora's clock reads t'_{D}.
- Bob's clock *does not* seem time-dilated!

$$\Delta t_{\mathrm{B}} = \sqrt{\frac{1+v/c}{1-v/c}}(t'_{\mathrm{D}} - t_{\mathrm{A}}) > (t'_{\mathrm{D}} - t_{\mathrm{A}})$$

  - Bob's clock seems to run *faster*!
  - Equivalent to blueshift formula.
  - Result still depends on synchronization rule.



Is time dilation real?

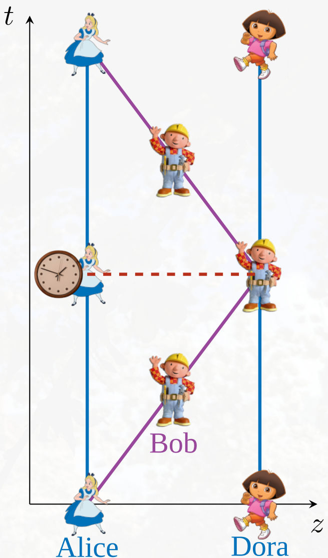
- Under Einstein synchronization, it really happens.
  - Time dilation does describe some aspect of reality.
  - In an alternate universe, it might not happen under Einstein synchronization.
- It does however depend on convention.
  - In another synchronization convention, it might not happen—even in our reality.
- The problem is we're comparing spatially distant events.
  - But what if we compare local events? Enter the **twin paradox**.

The twin paradox

- **Alice** and **Dora** synchronize their clocks by Einstein's convention.
- Start: **Bob** synchronizes his clock to **Alice's**.
- Midpoint: Bob moves to **Dora**, turns around.
- End: Bob meets up with Alice.
- **Time dilation**: Bob's clock should run slower than the reference frame's time.
  - But Alice's clock is synced to the reference frame's time.
  - Bob's clock must be behind Alice's!

$$\Delta t_{\mathrm{B}} = \sqrt{1 - \frac{v^2}{c^2}} \Delta t_{\mathrm{A}} < \Delta t_{\mathrm{A}}$$

- The "paradox": isn't Alice also moving the same way from Bob's perspective?
  - So shouldn't $\Delta t_{\mathrm{A}} < \Delta t_{\mathrm{B}}$? Is this consistent?
  - Actually, this *isn't even Alice's perspective!*



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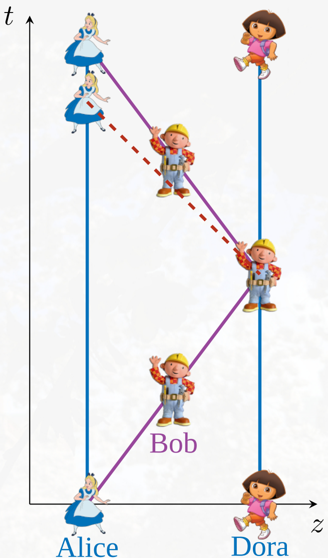
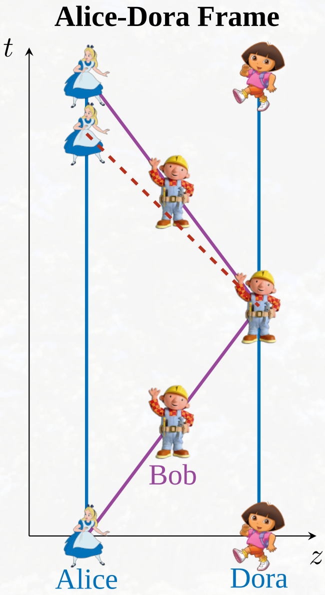
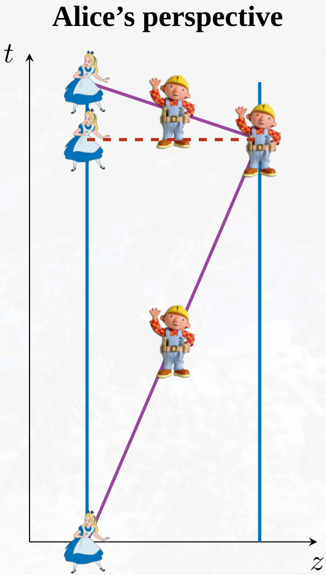
  - So shouldn't $\Delta t_{\mathrm{A}} < \Delta t_{\mathrm{B}}$? Is this consistent?
  - Actually, this *isn't even Alice's perspective!*
  - …because info that Bob turned around is delayed.



Reference frame vs. Alice's perspective





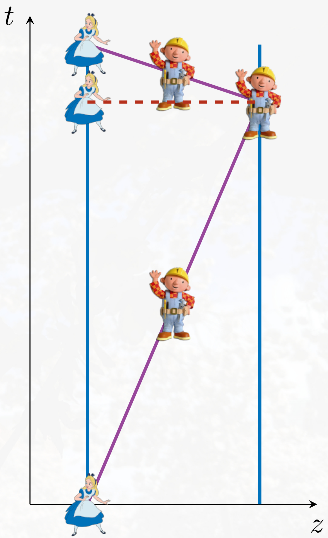
What Alice actually sees

- Start: **Bob** syncs his clock to **Alice's**
- Part 1: Bob slowly moves away for a long time.
  - Bob's clock appears redshifted (slow)
- Part 2: Bob rapidly moves back to Alice.
  - Bob's clock appears blueshifted (fast)
  - But not for very long.
- If you calculate it, reproduce **time dilation**:

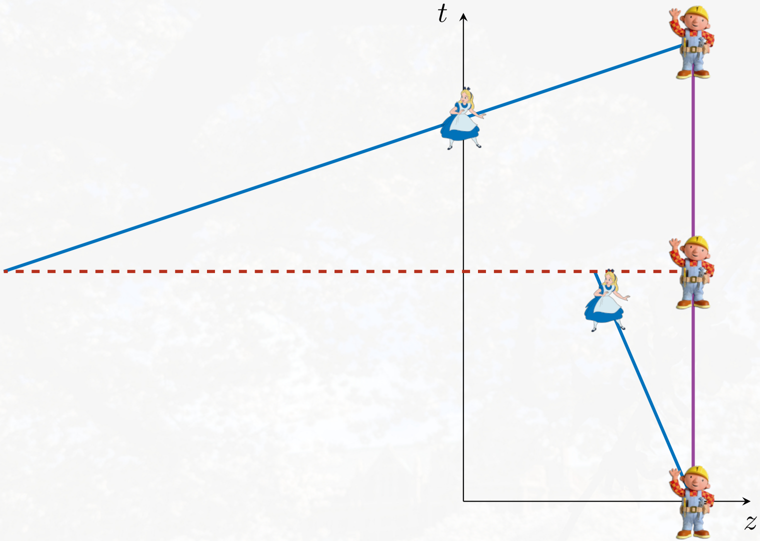
$$\Delta t_{\mathrm{B}} = \sqrt{1 - \frac{v^2}{c^2}}\Delta t_{\mathrm{A}} < \Delta t_{\mathrm{A}}$$

  - We should, since it's locally observable.
- Now, what about Bob's perspective?

Alice's perspective



What Bob sees



- Discontinuity in location from length distortions; Alice looks smaller to Bob after turnaround.

What Bob sees, explained

- Part 1: Alice moves slowly to left.
  - Alice's clock appears redshifted (slow)

$$\Delta t_{\mathrm{A}}^{(1)} = \sqrt{\frac{1 - v/c}{1 + v/c}} \frac{1}{2} \Delta t_{\mathrm{B}} < \frac{1}{2} \Delta t_{\mathrm{B}}$$

- Midpoint: Alice turns around at exactly half time
  - Bob knows when relative motion has changed
- Part 2: Alice moves rapidly to right
  - Alice's clock appears blueshifted (fast)

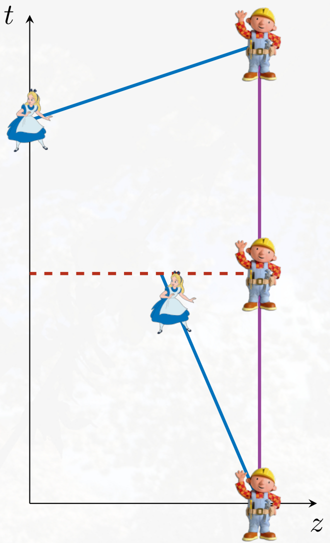
$$\Delta t_{\mathrm{A}}^{(2)} = \sqrt{\frac{1 + v/c}{1 - v/c}} \frac{1}{2} \Delta t_{\mathrm{B}} > \frac{1}{2} \Delta t_{\mathrm{B}}$$

- Add the durations:

$$\Delta t_{\mathrm{A}} = \Delta t_{\mathrm{A}}^{(1)} + \Delta t_{\mathrm{A}}^{(2)} = \frac{1}{\sqrt{1 - v^2/c^2}} \Delta t_{\mathrm{B}} > \Delta t_{\mathrm{B}}$$

  - Consistent with time dilation formula!

Bob's perspective



Resolving the twin paradox

- Alice and Bob can log different times because their perspectives actually differ.
  - Alice sees Bob as blueshifted for a shorter duration.
  - Bob sees Alice as blueshifted for a longer duration.
  - This happens because Bob knows when he's turned around; Alice's knowledge of this is delayed.
- Since Alice's clock is blueshifted longer, it has to log the longer time.
- The standard resolution involves building three reference frames.
  - Alice is always in one frame.
  - Bob starts out in a second frame.
  - Bob boosts to a third frame.
- But frames don't describe observers' actual perspectives.
- Side note: the resolution has **nothing** to do with gravity or acceleration.

Eddington synchronization

- We've effectively been using an **alternative synchronization convention**.
- Consider anything we're *seeing right now* to be *happening right now*.

$$t_{\text{observer}} = t_{\text{Einstein}} + \frac{r}{c}$$

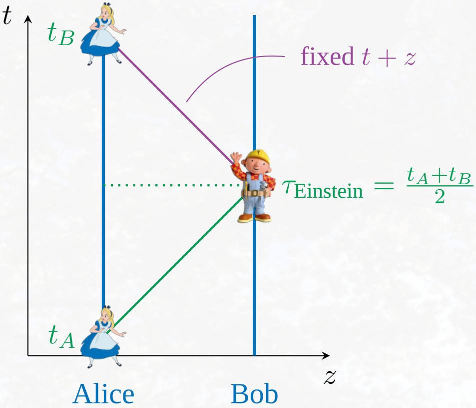
- Actually similar to a time coordinate used by Eddington in 1924

$$t_{\text{Eddington}} = t_{\text{Einstein}} + \frac{r}{c} + \frac{2GM}{c^3} \log\left|\frac{2GM}{c^2 r} - 1\right|$$

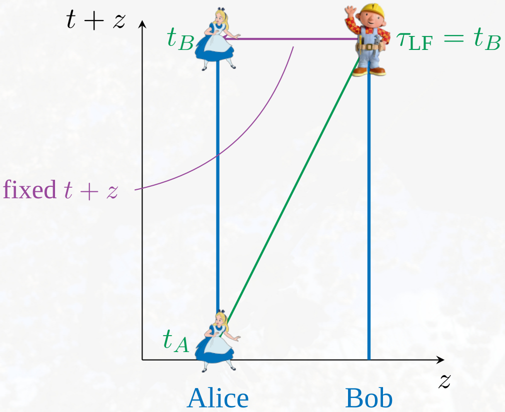
  (We've been considering the case $G \to 0$)
  - Easier to describe trajectories of particles falling into a black hole.
  - Can be used to show the event horizon isn't physically noticeable.
- Alternative ways of defining time pop up all the time in general relativity.

Light front time

Einstein synchronization



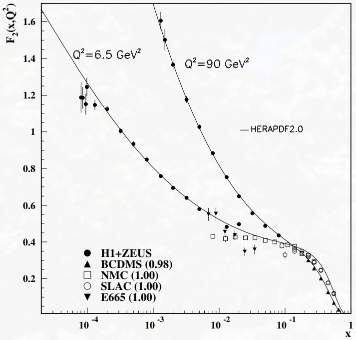
Light front synchronization

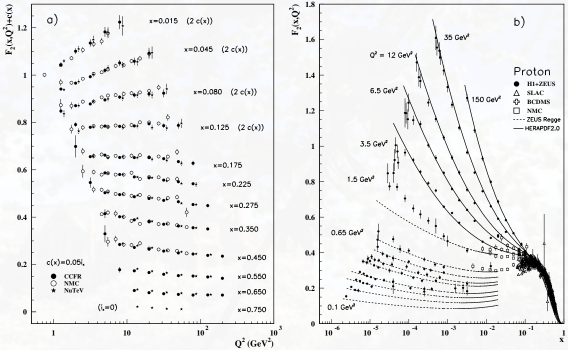


- **Light front synchronization** defines hyperplanes with fixed $t+z$ to be “simultaneous.”
  - Light travels instantaneously in $-z$ direction *by definition.*
  - Basically $G=0$ Eddington time in one direction.
  - First defined by Dirac, Rev.Mod.Phys. 21 (1949)

Why consider this alternative synchronization?

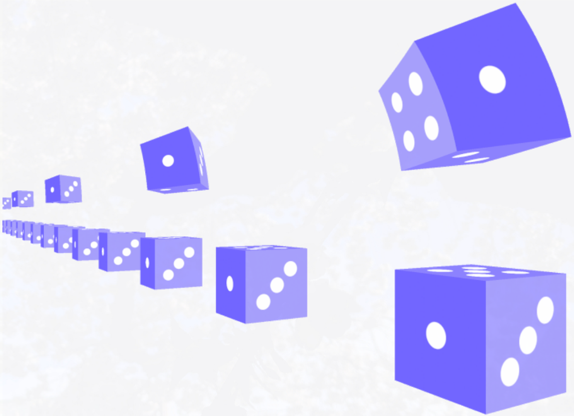
- It helps show what's really just a matter of convention.
- We can re-examine "paradoxes" from the actual perspective of the hypothetical observers.
- We can make really cool graphics of what relativistic systems *actually look like*.
  - **Great resource**: `https://www.spacetimetravel.org/`
- Actually makes a lot of calculations in nuclear & particle physics easier.
  - Particle data group (PDG) plots below: proton structure at fixed **light front time**





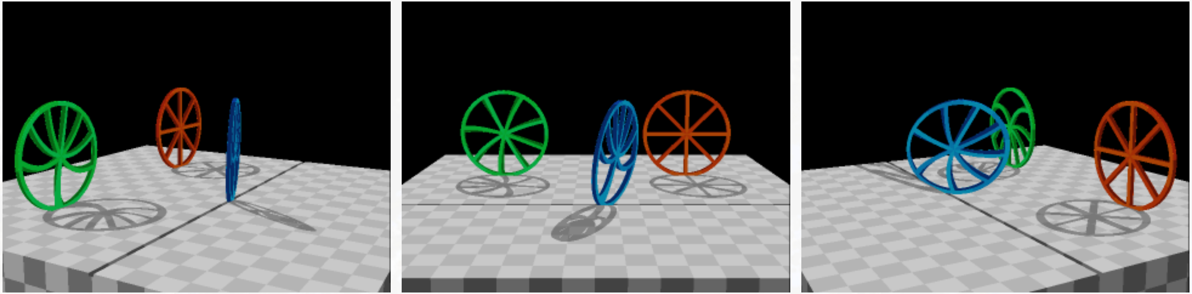
Terrell rotations

- Fast-moving objects *appear rotated*.
  - **Terrell rotation** (PR116, 1959)
- In Minkowski coordinates, contraction+delay:
  - Lorentz contracted length: $\sqrt{1 - v^2/c^2}L$
  - Time delay of $\frac{v}{c}L$ for far side of back.
  - These have a sin-cosine relationship.
- This has implications for particle structure calculations.



Dice images by Ute Kraus, `https://www.spacetimetravel.org/`

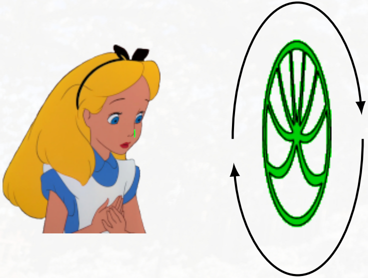
Relativistic wheel



- **Red wheel** is static.
- **Green wheel** is spinning.
- **Blue wheel** is rolling.
- Images from `https://www.spacetimetravel.org/tompkins/8` (has movies!)
- Bunching like in **green wheel** appears in proton & neutron!

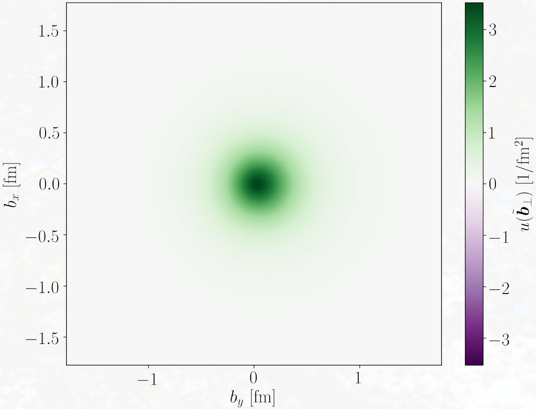
Static wheel

Spinning wheel

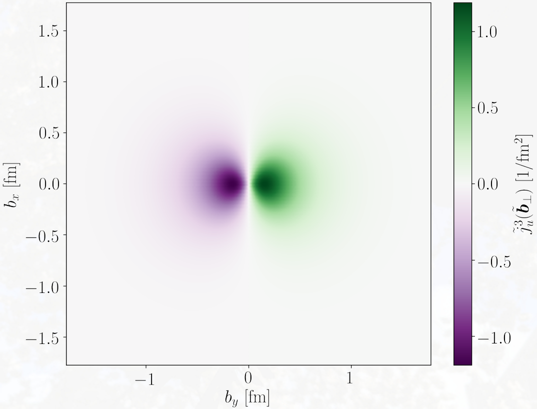


- **Static wheel** has regularly-placed spokes.
- Consider **spinning wheel**, axis oblique to observer.
  - *The wheel is considered at rest.*
- Spokes moving away are **redshifted**.
  - *Appear to* move slower.
  - Pile up; *appear to* become denser.
- Spokes moving towards are **blueshifted**.
  - *Appear to* move faster.
  - *Appear to* become rarer.
- These same distortions are present in nucleons!
  - **Light front densities bake in optical effects.**
- Also see videos at:
  `https://www.spacetimetravel.org/rad`
  (green wheel is relevant case)

Up quark density

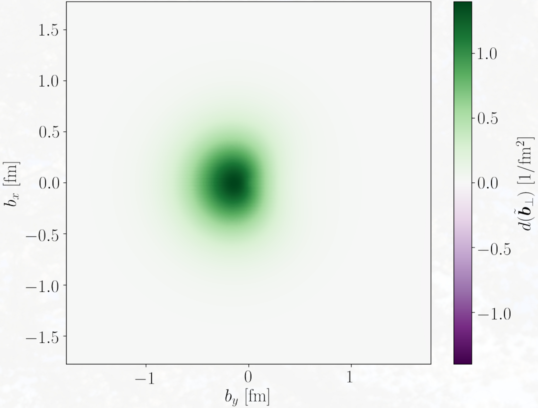


Up quark current ($\tilde{z}$ component)

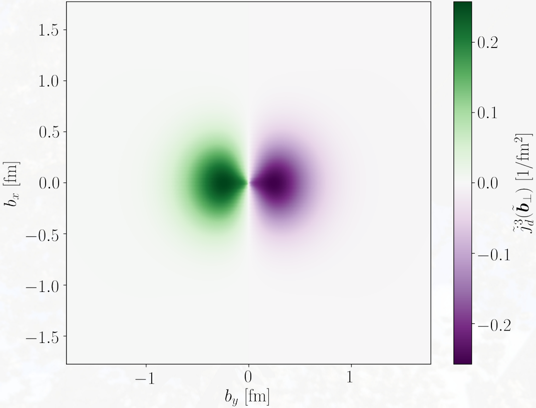


- Proton spin-up along vertical axis; recall right hand rule
- Up quarks orbiting with proton spin
- Purple means towards, green means away.

Down quark density



Down quark current ($\tilde{z}$ component)



- Bigger distortion in down quarks!
- Orbit & bunching in opposite direction from proton spin
- Purple means towards, green means away.

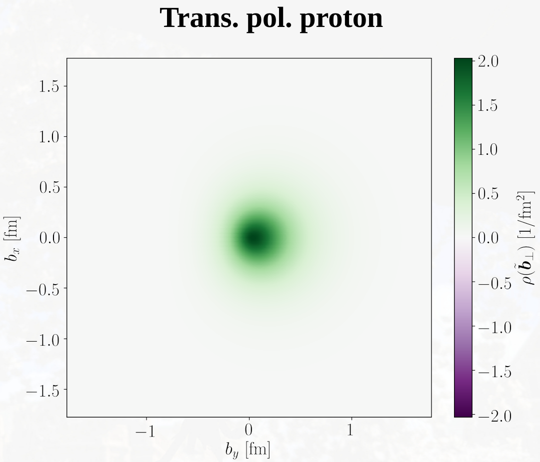
- Up quarks orbit along with proton spin.

$$\langle \omega_u \rangle \approx \quad 0.417\ c/\text{fm} = \quad 125\ \text{ZHz}$$

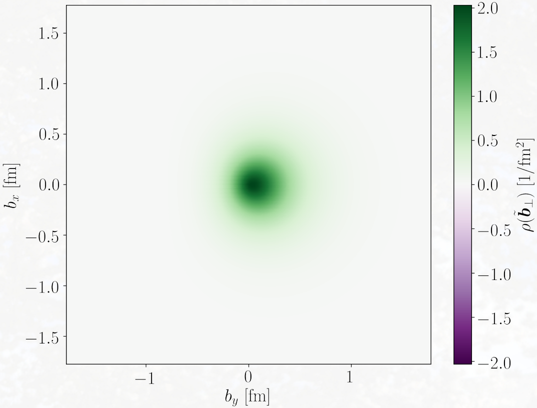
- Down quarks orbit (much faster) against proton spin.

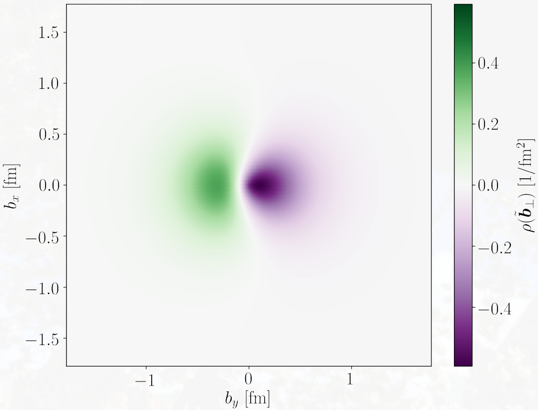
$$\langle \omega_d \rangle \approx -0.922\ c/\text{fm} = -276\ \text{ZHz}$$

- Constructively contribute to *apparent* electric dipole moment.
  - In transversely polarized states.
- Would be what a viewer really sees!
  - Known effect: the relativistic wheel.



Protons and neutrons

Trans. pol. proton 

Trans. pol. neutron 

- Effect more extreme in neutron, since its net charge is zero.

Resources related to this talk

- Video about synchronization conventions:
  Veritasium, “Why No One Has Measured The Speed of Light” (YouTube)
- Visualizations of special and general relativity:
  https://www.spacetimetravel.org/
- Velocity Raptor, a video game demonstrating “seen” relativity in later levels:
  https://testtubegames.com/velocityraptor.html