Theory and Calculation of X-ray spectra

J. J. Kas



Theoretical Spectroscopy Calculations

GOAL: Next Generation Theory
 for Next Generation X-ray Sources

• TALK

I IntroductionII State-of-the-artIII Next generation

Current approximations Ab initio calculations Multi-electron excitations & strong correlations

What is x-ray absorption?



What is x-ray absorption?

Cu Absorption (log-log)



What is x-ray absorption?

Cu Absorption (log-log)



What is EXAFS?



What is EXAFS?

Quantum interference





Qualitative Interpretation



MSE-333 5/8/2013

Introduction

Promise of Next generation Light sources: NSLSII, LCLS ...

Powerful probes of multiple length and time scales Partial solution to DOE 5 Grand Challenges, etc.



BUT Promise cannot be fully exploited without

Next generation theory & software

I Current methods

JULY 2000

Reviews of Modern Physics

VOLUME 72 + NUMBER 3 Published by THE AMERICAN PHYSICAL SOCIETY through the AMERICAN INSTITUTE OF PHysics

THEORETICAL APPROACHES TO X-RAY ABSORPTION FINE STRUCTURE

EXAFS & XANES

Automated & Integrated

XAS codes

FEFF

> 20 yrs development

J. J. Rehr & R.C. Albers Rev. Mod. Phys. 72, 621 (2000)

MEMBER SUBSCRIPTION COPY.

Library or Other Institutions

Use Prohibited Until 2005

Real-space Green's Function (RSGF) Theory VIS – X-ray





Paradigm shift:

Golden rule via Green's Functions $G = 1/(E - h - \Sigma)$

$$\mu(E) \sim -\frac{1}{\pi} \operatorname{Im} \langle \mathbf{i} | \hat{\epsilon} \cdot \mathbf{r}' \operatorname{G}(\mathbf{r}', \mathbf{r}, \mathbf{E}) \hat{\epsilon} \cdot \mathbf{r} | \mathbf{i} \rangle$$

EFFICIENT: No sums over final states !



- Good agreement: *Relativistic* FEFF8 code reproduces all spectral features, *including absence of white line at L₂-edge*.
- Self-consistency essential: position of Fermi level strongly affects white line intensity.

Spectra: RIXS, XES, Compton, ...

PHYSICAL REVIEW B 83, 235114 (2011)

Real-space Green's function approach to resonant inelastic x-ray scattering

J. J. Kas,¹ J. J. Rehr,^{1,*} J. A. Soininen,² and P. Glatzel³

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II. State of the art theory

FEFF9

JJR et al., Comptes Rendus Physique **10**, 548 (2009)

State of the art theory GOAL: ab initio many-body effects *no adjustable parameters* GW Self-energy

Debye Waller factors

RPA core-hole



Need: **multiple codes**: DFT, self-energy, Phonons No one group or code can do it all



* Also used in BSE Improves on final state rule, Z+1, half hole

Many-pole GW Self-energy

Energy dependent $\Sigma(E) = iGW$ – replaces V_{xc} MFP, energy shifts cf. Hedin-Lundqvist plasmon-pole



*J.J. Kas et. al, Phys Rev B 76, 195116 (2007)

Ab initio XAS Debye Waller Factors * $e^{-2\sigma^2 k^2}$

An Initio Determination of Extended X-Ray Absorption Fine Structure Debye-Waller Factors

> Fernando D. Vila, G. Shu, and John J. Rehr Department of Physics, University of Washington, Seattle, WA 98195



H. H. Rossner and H. J. Krappe Hahn-Meitner-Institut Berlin, Glienicker Strasse 100, D-14109 Berlin, Germany (Dated: August 23, 2005)

Many Pole model

$$\sigma^2 = \frac{\hbar}{\mu_i} \int_0^\infty \rho(\omega^2) \coth \frac{\beta \hbar \omega}{2} d\omega$$

for phonons

$$\rho(\omega^2) = \langle Q_i | \delta(\omega^2 - D) Q_i \rangle$$

= {6-step Lanczos recursion]



*Phys. Rev. B 76, 014301 (2007)

Example: XAFS Debye-Waller Factor of Ge



Expt: Dalba et al. (1999)

Real-time Finite *T* **DFT-MD / XAS**

Nanoscale Pt catalysts*

$Pt_{10} / \gamma - Al_2O_3$

Expt. vs DFT/MD structure + FEFF XAS Theory*



*F. Vila, J.J. Rehr, J. Kas, R.G. Nuzzo, A.I. Frenkel, Phys. Rev. B 78, 121404(R) (2008)



CHROMIUM MANGANESE

IRON

COBALT

NICKEL

COPPER

ZINC

²¹

III. Next generation XAS Calculations

- beyond-quasiparticles

Excitonic effects BSE Multi-electron excitations Strong correlations



Plus integrated Codes: DFT, GW/BSE, Real-time etc.

GW Bethe-Salpeter Equation

Bethe-Salpeter equation calculations of core excitation spectra

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E. L. Shirley

National Institute of Standards and Technology (NIST), Gaithersburg, Maryland 20899, USA (Received 29 September 2010; published 4 March 2011)

We present a hybrid approach for Bethe-Salpeter equation (BSE) calculations of core excitation spectra including x-ray absorption (XAS), electron energy loss spectra (EELS), and nonresonant inelastic x-ray scattering (NRIXS). The method is based on *ab initio* wave functions from the plane-wave pseudopotential code ABINIT atomic core-level states and projector augmented wave (PAW) transition matrix elements; the NIST core-leve BSE solver; and a many-pole self-energy model to account for final-state broadening and self-energy shifts Multiplet effects are also approximately accounted for. The approach is implemented using an interface dubbec OCEAN (Obtaining Core Excitations using ABINIT and NBSE). To demonstrate the utility of the code we presen results for the K edges in LiF as probed by XAS and NRIXS, the K edges of KCl as probed by XAS, the Ti L_{2,3} edge in SrTiO₃ as probed by XAS, and the Mg L_{2,3} edge in MgO as probed by XAS. These results are compared with experiment and with other theoretical approaches.

BSE OCEAN*



*Obtaining Core Excitations from ab initio NBSE

Plane-wave, pseudo- potential + PAW + MPSE

cf. EXC, EXC!TING, BerkeleyGW

Many-body Amplitudes in XAS S_{θ}^{2}

Many-body X*S ≈ Convolution

 $\mu(\omega) = \int_0^\infty d\omega' \tilde{A}(\omega, \omega') \mu_{qp}(\omega - \omega')$ $\equiv \langle \mu_{qp}(\omega) \rangle \qquad \text{Na XPS}$

- Source: multi-electron excitations
- Spectral function ~ XPS $A = - \operatorname{Im} G$



Which Green's function ? GW vs Cumulant

J. Phys.: Condens. Matter 11 (1999) R489-R528

On correlation effects in electron spectroscopies and the *GW* approximation

 $\Sigma = i \ GW\Gamma$

 $W = \epsilon^{-1} v$

Lars Hedin Department of Theoretical Physics, Lund University, Sölvegatan 14A, 223 62 Lund, Sweden

GW

Cumulant

 $G(\omega) = G_0 + G_0 \Sigma G \qquad \qquad G(t) = G_0(t) e^{C(t)}$ $\Gamma = 1 \qquad \Sigma^{GW} = iGW \qquad \qquad C \sim \operatorname{Im} \Sigma^{GW}$

Similar ingredients; all many-body effects in Σ^{GW}

Test: Satellites in XPS of simple metals*

Multiple Plasmon Satellites in Na and Al Spectral Functions from *Ab Initio* Cumulant Expansion

F. Aryasetiawan,^{1,2} L. Hedin,¹ and K. Karlsson³ Phys Rev Lett **77**, 2268 (1996)

Good news: GW quasi-particle peak agrees with XPS

Bad news: *GW* has *one* satellite @ wrong energy

XPS has multiple satellites



E (eV)

A(E)

Test 2: Multiple Satellites in XPS of Si

PRL 107, 166401 (2011)

PHYSICAL REVIEW LETTERS

week ending 14 OCTOBER 2011

Valence Electron Photoemission Spectrum of Semiconductors: Ab Initio Description of Multiple Satellites

Matteo Guzzo,^{1,2,*} Giovanna Lani,^{1,2} Francesco Sottile,^{1,2} Pina Romaniello,^{3,2} Matteo Gatti,^{4,2} Joshua J. Kas,⁵ John J. Rehr,^{5,2} Mathieu G. Silly,⁶ Fausto Sirotti,⁶ and Lucia Reining^{1,2,†}



XPS Intensity Ratios in Molecules

PRL 108, 193005 (2012)

PHYSICAL REVIEW LETTERS

week ending 11 MAY 2012

Nonstoichiometric Intensities in Core Photoelectron Spectroscopy

J. Söderström,¹ N. Mårtensson,¹ O. Travnikova,² M. Patanen,² C. Miron,² L. J. Sæthre,³ K. J. Børve,³ J. J. Rehr,⁴ J. J. Kas,⁴ F. D. Vila,⁴ T. D. Thomas,⁵ and S. Svensson¹







<XPS> = XAS

Intensity ratio $\sim Z$

including all losses explains nonstoichiometric ratios

Question: Can the cumulant method work for CT satellites in correlated systems ?

Hedin's answer * MAYBE

"Calculation similar to core case ... but with more complicated fluctuation potentials ...

... not question of principle, but of computational work..."

* L. Hedin, J. Phys.: Condens. Matter **11**, R489 (1999)

Approach: Cumulant expansion in real-time

Real time cumulant approach for charge transfer satellites in x-ray photoemission spectra

J. J. Kas,¹ F. D. Vila,¹ J. J. Rehr,¹ and S. Chambers²

arXiv:1408.2508

¹Dept. of Physics, Univ. of Washington, Seattle, WA 98195-1560 ²Physical Sciences Division, Pacific Northwest National Laboratory, Richland, WA 99352 (Dated: August 7, 2014)

Langreth
$$C(t) = \sum_{q,q'} V_q^* V_{q'} \int d\omega S(q,q',\omega) \frac{e^{i\omega t} - i\omega t - 1}{\omega^2} = \int d\omega \beta(\omega) \frac{e^{i\omega t} - i\omega t - 1}{\omega^2}$$

RT Cumulant: RT-TDDFT

$$S(q,q',\omega) = \int \frac{dt}{2\pi} e^{i\omega t} \langle \rho_q(t) \rho_{q'}(0) \rangle$$



XPS & Real-space Interpretation



Interpretation: satellites due to local ligand-metal charge fluctuations at frequency ~ ω_{ct} in response to transient core-hole potential JJ Kas, FD Vila, JJR & S.Chambers arXiv:1408.2508

Conclusions and Outlook

EXAFS - well understood with current methods RSGF + DFT/MD XANES – semi-quantitative state of the art theory GW, GW/BSE, DW factors, ... XAS, XPS - Multi-electron excitations & correlations via quasi-boson/cumulant theory

Full spectrum calculations possible in arbitrary materials with NG theory and codes.

Rehr group & collaborators



Thanks for Listening!

Many Success Stories

XAS of doped SiC

M Ohkubo et al., Sci. Rep., 2, 831 (2012)





PHYSICS MATERIALS SCIENCE X-ray absorption near edge spectroscopy with a superconducting detector for nitrogen dopants in SiC

M. Ohkubo¹, S. Shiki¹, M. Ukibe¹, N. Matsubayashi¹, Y. Kitajima² & S. Nagamachi³

ANALYTICAL CHEMISTRY

SUBJECT AREAS:

SPECTROSCOPY

"FEFF8.4 played an important role in analysis of XANES spectra of N-doped SiC. I appreciate your effort for developing this fantastic software ..." Masataka Ohkubo

Cumulant Expansion & Spectral Function

$$G_k(t) = e^{i\epsilon_k^0 t} e^{C(t)} \qquad \qquad C(t) = \int d\omega' \beta(\omega') \frac{e^{i\omega' t} - i\omega' t - 1}{\left(\int \omega'^2 \right)^2}$$

Spectral Function

$$A_k(\omega) = \int \frac{dt}{2\pi} e^{i(\omega - \epsilon_k)t} \exp\left\{\int d\omega' \beta(\omega') \frac{e^{i\omega't} - i\omega't - 1}{{\omega'}^2}\right\}$$

*see L. Hedin, J. Phys.: Condens. Matter **11**, R489 (1999) cf. D.C. Langreth *Phys. Rev.* B **1**, 471 (1970) (Linked cluster theorem)

Other properties

Kernel: $\beta(\omega) = |\text{Im } \Sigma_k(\omega)| \sim \text{Im } \varepsilon^{-1} \sim \text{loss function}$ Peaks in $A(\omega) \sim \text{Peaks in loss function}$

$$Z = e^{-\overline{n}}$$
 $\overline{n} = \sum_{q} g_{q}^{2} / \omega_{q}^{2} = \int \beta(\omega) / \omega^{2} d\omega.$

 $\overline{n} = 0.201 r_s^{3/4}$ mean number of shake-up bosons Dimensionless measure of correlation strength $n \sim 0.3$ to 0.4 even in s-p systems !

IV. Problems with Cumulant Expansion*

BAD: Pathologies at Fermi energy*

Time-ordered GF (TO) lacks satellites on both sides of QP peak near Fermi energy

GW lacks multiple satellites

Source: recoil approximation: neglect of "exchange diagrams" in time-ordered cumulant

Electron spectral function





*arXiv:1402.0022

Debye-Waller factors σ^2 in Zr tungstate





FIX: Retarded Cumulant expansion*

PHYSICAL REVIEW B 00, 005100 (2014)

Cumulant expansion of the retarded one-electron Green function

J. J. Kas,^{1,*} J. J. Rehr,^{1,2,†} and L. Reining^{3,2,‡}

 ¹Department of Physics, University of Washington, Seattle, Washington 98195, USA
 ²European Theoretical Spectroscopy Facility (ETSF)
 ³Laboratoire des Solides Irradiés, École Polytechnique, CNRS, CEA-DSM, F-91128 Palaiseau, France (Received 31 January 2014; revised manuscript received 28 July 2014; published xxxxx)

Retarded formalism

$$G_k^R(t) = -i\theta(t)e^{-i\epsilon_k^{HF}t}e^{\tilde{C}_k^R(t)},$$

$$\tilde{C}_k^R(t) = \int d\omega \frac{\beta_k(\omega)}{\omega^2}(e^{-i\omega t} + i\omega t - 1),$$

$$\beta_k(\omega) = \frac{1}{\pi} \left| \operatorname{Im} \Sigma_k^R(\omega + \epsilon_k) \right|,$$



*Phys. Rev. B. (in press, Aug. 2014); arXiv:1402.0022

Retarded Cumulant (GC) gives **GOOD** quasi-particle properties



Many-body Amplitudes in X-ray Spectra

• Many-body X*S ≈ Convolution

 $\mu(\omega) = \int_0^\infty d\omega' \tilde{A}(\omega, \omega') \mu_{qp}(\omega - \omega')$ $\equiv \langle \mu_{qp}(\omega) \rangle$

• Explains crossover: adiabatic

to sudden transition

 $|g_q|^2 = |g_q^{ext}|^2 + |g_q^{intrin}|^2 \cdot 2 g_q^{ext} g_q^{intrin}$

Interference reduces loss!

Strong correlation effects: 1. Hubbard corrections

PHYSICAL REVIEW B 85, 165123 (2012)

Hubbard model corrections in real-space x-ray spectroscopy theory

Towfiq Ahmed, J. J. Kas, and J. J. Rehr







cRPA calculated U

FIX: Next generation approach Quasi-Boson Approximation

IDEA: Neutral Excitations - plasmons, phonons, electron-hole pairs, magnons are **bosons** γ

Many-body Model: $|N > = |e^{-}, h, \gamma >$

- Excitations: $H_v = \Sigma_n \omega_n a_n^{\dagger} a_n$
- Electrons: $h' = \Sigma_k \epsilon_k c_k^{\dagger} c_k$
- e-boson coupling $V_{pv} = \Sigma_{nkk'} \left[V_{kk'}^n a_n^\dagger + (V_{kk'}^n)^* a_n \right] c_k^\dagger c_{k'}$
- Core-hole-boson coupling: $V_{vc} = -\Sigma_n V_{bb}^n \left(a_n^{\dagger} + a_n \right)$

 $V^n \rightarrow -\text{Im } \varepsilon^{-1}(\omega_n, q_n)$ "fluctuation potentials"

* L. Hedin, J. Phys.: Condens. Matter **11**, R489 (1999)

GW++: Effective Green's Function - perturbation theory*

 $g_{eff}(\omega) = e^{-a} \left[g'(\omega) + \sum_n \left(\frac{V_{bb}^n}{\omega_n} \right)^2 g'(\omega - \omega_n) - 2\sum_n \frac{V_{bb}^n}{\omega_n} g'(\omega - \omega_n) V^n g'(\omega) \right]$



Leading term: $g'(\omega)$ damped Green's function in presence of core-hole (~ final state rule!)

$$g'(\omega) \equiv [\omega - h' - \Sigma(\omega) + i\gamma]^{-1}$$

*L. Campbell, L. Hedin, J. J. Rehr, and W. Bardyszewski, Phys. Rev. B 65, 064107 (2002)

2. Multiplets a la GW/BSE

Phys. Rev. B 86, 195135 (2012)

BSE calculations of transition metal L-shell spectra

J. Vinson and J. J. Rehr Dept. of Physics, Univ. of Washington Seattle, WA 98195 (Dated: July 3, 2012)

We present *ab initio* Bethe-Salpeter equation (BSE) calculations of the $L_{2,3}$ edges of several insulating and metallic compounds containing Ca, V, Fe, Co, Ni, and Cu, spanning a range of *3d*-electron occupations. Our approach includes the key ingredients of a unified treatment of both extended states and atomic multiplet effects, i.e., self-consistent crystal potentials, ground-state magnetism, *GW* self-energy corrections, spin-orbit terms, and Coulomb interactions between the L_2 and L_3 states. The method is implemented in the OCEAN package, which uses plane-wave pseudopotential wave functions from ABINIT as a basis, a PAW construction for transition matrix elements, and a resolvent formalism for the BSE calculation. The results are in near quantitative agreement with experiment, including both fine-structure at the edges and the non-statistical L_3/L_2 ratios observed in these systems.

GW/BSE + Spin & SO + MPSE + Slater *F*, *G*

Transition metal oxides





Multiplets in SrTiO₃



3. Charge Transfer Satellites

PHYSICAL REVIEW B

VOLUME 60, NUMBER 11

15 SEPTEMBER 1999-I

Transition from the adiabatic to the sudden limit in core-level photoemission: A model study of a localized system

J. D. Lee and O. Gunnarsson Max-Planck Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany

L. Hedin Max-Planck Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany and Department of Theoretical Physics, University of Lund, Sölvegatan 14 A, S-223 62 Lund, Sweden



Approaches: Anderson impurity, DMFT, multiplets,
 + Lots of Sound & Fury + millions of cpu hrs
 BUT still no satisfactory 1st principles theory ...

Simple picture: 2-state model of LGH*

PHYSICAL REVIEW B 89, 085123 (2014)

Charge transfer satellites in x-ray spectra of transition metal oxides

E. Klevak, J. J. Kas, and J. J. Rehr Department of Physics, University of Washington, Seattle, Washington 98195, USA



*Lee, Gunnarsson & Hedin, Phys Rev B 60, 8034 (1999)

Goal: X-ray Spectroscopy Beamline



EU Model



A new facility for research!

European Theoretical Spectroscopy **Facility**

www.etsf.eu

Thanoquanta builds the ETSF European Theoretical Spectroscopy Facility

Network of Excellence

An innovative project...

The Nanoquanta Network of Excellence, partially financed by the European Union under the Sitth Fra mework Programme, is building up a new type of facility called European Theoretical Spectroscopy Facility (ETSF). On the lines of the synchrotron facilities successfully shared by researchers across Europe, the FTSE will offer users the opportunity to henefit men the latest developments in the field of electronic exited states in matter.

To burst the secrets of matter

What happens in matter when it is exposed to ra-What does the response of the marethe radiation – that is, the spectra we spre – leach us? That is what the resear-ers of the FT/F want to know



In matter, the electrons occupy energy levels. When absorbing rediction, they can be premoted to a higher lead, depending on the characteristics of the system. These transitions must then be calculated taking into account the fact that the electron sinternet

relexample of 12g of 5 amond, which contains 86 x 10¹⁸ electrons, shows that the task is huge! Mathematical expressions must then be derived, teking into account, this "many-body" physics in an

Nevertheless much computer power is maxasary to potein a roliable description Freal matter. To do that, researchers develop software which arons atos the expressions into the language of rom **cuters**

1-The core of the ETSF

A network of prominent European Condensed Matter theory groups, which will take responsibility for management of the ETSF and reach out

5 They developed, thanks to a 15 ye us collaboration, many of the theoretical approaches and computational solutions which are commonly used in the framework of calculations of electronic excitations from first principals.

The ETSF will bridge the gap between fundamental research and technological applications

Theoretical spectroscopy and fight against cancer, an exar

- Theory and soft nly to know the properties I have it works and them. of matter, but fort to crisisano tions
- that tochologic important insert tools. For example, the Green Hoote cent Trotein (G-1), which gives the Aeguarda Victoria Lellylish its "Constant, epsew-rance, is large y used in meloo, an biology. But row does the DR assoring light/billy and save and it regions trutter to become fluorescent, one after a while, rotum to its initial state?

A perver understanding of the flucressent (SFR) porphysics, etc.) behaviour problems to improve its use as a biological merver. In the fight ago at conceptional persons to deprove sice "markee" offs ware-autouathing the healthy ones around them.

2-Associate theory groups

A broad and federal community of research groups working on similar topics

- They fully benefit from the wientifk exchange enabled and stimulated by the ETSE and will be able o advertise on the ETSF website and call for proposals just like members of the Core.

3-Users

A large and varied group of researchers from the public or private sector

- Demand for collaboration using the developed offware and support for these calculations is growing rapidly amongst theoreticians and experimentalists, some working at large institutions such as synchrotrons, others in individual laboratories and others in private companies. They benefit from the FTSE in different
- ways, depending on the proximity of the user to the field:
- download software and use it with the help of publications, manuals and tutorials provided by the ETSE:
- participate in training events;
- require engoing consultancy for a project; demand pieces of theory or software that do nul yel exist but are sufficiently relevant to br devised by a member of the ETSF, in collaboration with the proposer if appropriate; propose a relevant project that requires fulltime work of a member of the ETSE.
- Users of the ETSF may already propose pilot projects on different levels, according to their expertise. The first open call for user projects will start in 2005.





re and manuals and to organize training events

European Theoretical Spectroscopy Facility



About the ETSF

→ Beamlines

- · Energy Loss Spectroscopy
- Optics
- Photo-emission Spectroscopy
- Quantum Transport
- Time-resolved Spectroscopy
- Vibrational Spectroscopy
- X-Rays Spectroscopy
- Services
- ▶ Resources
- Funding
- Press
- Impressum

Home

Beamlines

The ETSF is divided into 7 beamlines, each of which is concerned with a specific scientific topic. A beamline coordinator is responsible for the contact with the users of each line. He/She also serves as the contact person for users who want to submit a proposal to the ETSF.

Further details are available on the beamlines' description.

Beamlines and Coordinators

Optics

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Energy Loss Spectroscopy

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Photo-emission Spectroscopy

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Networked "theoretical beamlines"

Vibrational Spectroscopy

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X-Rays Spectroscopy

Prof. John Rehr University of Washington, Seattle, USA jjr@phys.washington.edu